Homework #1 (part 1)

1. The maximal relative error of the volume of a ball is allowed within 1%. What's the maximal relative error in measuring the radius of the ball?

2. Given $a \neq 0$, $b \neq 0$, $b^2 - 4ac > 0$, one use a computer to solve $ax^2 + bx + c = 0$ by using

$$x_1 = (-b - \text{sign}(b)\sqrt{b^2 - 4ac})/(2a),$$
$$x_2 = c/(ax_1).$$

Explain its advantage over the common formula.

3. Given the equation $x^2 - 40x + 1 = 0$, find its roots to five significant digits using $\sqrt{399} \approx 19.975$, correctly rounded to five digits.

4. Give exact ways of avoiding loss-of-significance errors in the following computations.

(a) $\log(x+1) - \log x$ with large x

(b) $\sin x - \sin y$ with $x \approx y$

(c) $\tan x - \tan y$ with $x \approx y$

(d) $\dfrac{1 - \cos x}{x^2}$ with $x \approx 0$

(e) $\sqrt[3]{1+x} - 1$ with $x \approx 0$