Lecture 34 - Solving ODE

Integration → system defined by ODE

$$y_{i+1} = y_i + \boxed{\text{Slope} \times \text{step-size}}$$

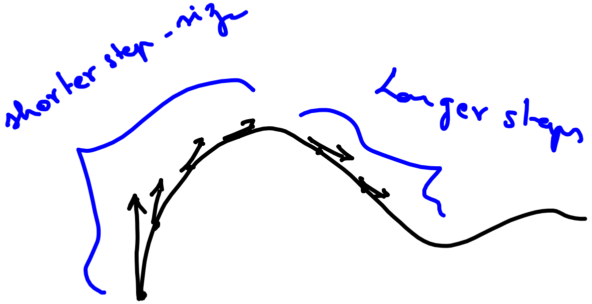
First-order Taylor

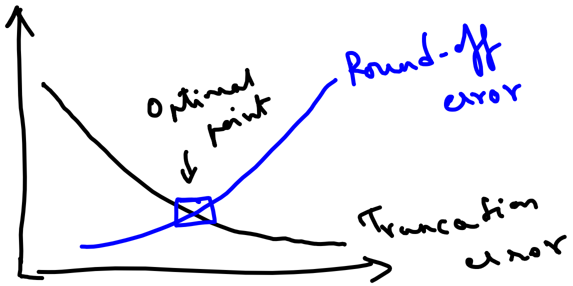
Modifying this term to improve accuracy

↓ Single-step

↘ Multi-step

} Adaptive technique





Euler's method

ODE $\rightarrow$ $f(x, y)$ $\rightarrow$ unknown?

$\downarrow$

independent variable
free to choose

$$y_{i+1} = y_i + \underbrace{f(x_i, y_i)}_{\text{first order ODE}} h$$

$\rightarrow$ First order Taylor series approximation

Initial value $\rightarrow$ preciso, observed, measured
$\rightarrow$ imprecise, inaccurate, modelled

Boundary value

Orbit determination $\rightarrow$ determine the orbit precisely
$\rightarrow$ Orbit is dependent on the gravitational & non-gravitational forces

$$\frac{d^3y}{dx^3}$$

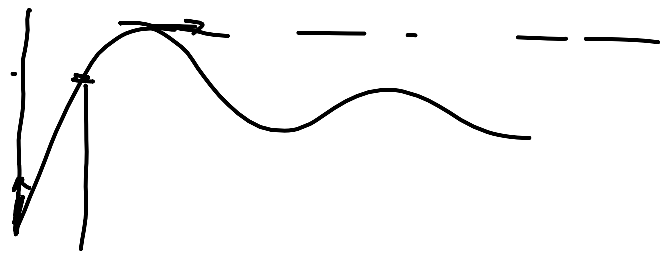
$\downarrow$

$f(x, y''')$

$f(x, y'')$

$f(x, y')$

$$y_{i+1} = y_i + f(x_i, y_i)\,h$$ single step



Heun's method

$$y^0_{i+1} = y_i + f(x_i, y_i)\,h$$ step 1 Predicting $y_{i+1} \to y^0_{i+1}$

$$f(x_{i+1}, y^0_{i+1})$$

$$\downarrow$$

$$\bar{f}(x, y) = \frac{f(x_i, y_i) + f(x_{i+1}, y^0_{i+1})}{2}$$

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y^0_{i+1})}{2}\,h$$ Step 2 Correcting $y_{i+1} \to$ correct y^0_{i+1}

The mid-point method

$$y_{i+1/2} = y_i + f(x_i, y_i)\, h/2$$

$$y_{i+1} = y_i + f(x_{i+1/2}, y_{i+1/2})\, h$$

Multi-step method

Runge-Kutta (RK)

$$y_{i+1} = y_i + \phi(x_i, y_i, h)\, h$$

Single-step

Accurate

Slow

n-th order Runge-Kutta

$$\phi = a_1 k_1 + a_2 k_2 + \dots + a_n k_n$$

2nd order

$$\phi = a_1 k_1 + a_2 k_2 \rightarrow k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + p_i h,\ y_i + q_{11} k_1 h) \rightarrow \text{intermediate, recursive}$$

Algorithm

Driver function

↳ h, x_i, y_i

↳ ODE for evaluating $f(x_i, y_i)$

↳ Integration method