# Lecture 38 - Adams integration

- Multi-step methods

Heun's method → self-starting

→ non-self-starting → requires previous step value

↓

predictor → open integration

→ same order of truncation error

corrector → closed integration

error estimate using the predictor & corrector

Predictor

Predictor modifier → use estimates of local truncation error

Corrector → iterative

Corrector modifier

Adams integration

Use as much information about the function from the previous steps

Predictor → Open formula → $y_i$

Corrector → Closed formula → $y^0_{i+1}$

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(3) ⇒ $y_{i+1} = y_i + h\left[f_i + \frac{f_i}{2} - \frac{f_{i-1}}{2} + \frac{f''_i h^2}{4} + \frac{f''_i h^2}{6} + O(h^3) + \dots\right]$

$$= y_i + h\left[\underbrace{\frac{3f_i}{2} - \frac{f_{i-1}}{2}}_{\text{derivative}} + \underbrace{\frac{5f''_i}{12}h^2}_{\text{truncation term}} + \dots\right] + O(h^4)$$

second order Adams-Bashforth

Predictor - Adams-Bashforth

$$y_{i+1} = y_i + f_i h + f'_i \frac{h^2}{2!} + f''_i \frac{h^3}{3!} + \dots$$

$$y_{i+1} = y_i + h\left[\underline{f_i} + \underline{f'_i} \frac{h}{2} + f''_i \frac{h^2}{6} + \dots\right] \to (1)$$

$$f'_i = \underbrace{\frac{f_i - f_{i-1}}{h}}_{\text{derivative}} + \underbrace{\frac{f''_i h}{2}}_{\text{Truncation term}} + O(h^2) \to (2)$$

Sub (2) in (1)

$$y_{i+1} = y_i + h\left[f_i + \frac{h}{2}\left(\frac{f_i - f_{i-1}}{h} + \frac{f''_i h}{2} + O(h^2)\right) + f''_i \frac{h^2}{6} + \dots\right] \to (3)$$

Corrector - Adams-Moulton method                Adams-Bashforth Adams-Moulton - (ABAM)

Closed formula

$$y_i = y_{i+1} - f_{i+1} h + \frac{f'_{i+1} h^2}{2!} - \frac{f''_{i+1} h^3}{3!} + \dots$$

$$y_{i+1} = y_i + h \Big[ \underbrace{f_{i+1} - \frac{h}{2} f'_{i+1} + \frac{h}{6} f''_{i+1}}_{i+1} + \dots \Big] \quad \text{— } (4)$$

$$f'_{i+1} = \underbrace{\frac{f_{i+1} - f_i}{h}}_{\text{derivative}} + \underbrace{f''_{i+1}}_{\text{truncation term}} \frac{h}{2} + O(h^2) \quad \text{— } (5)$$

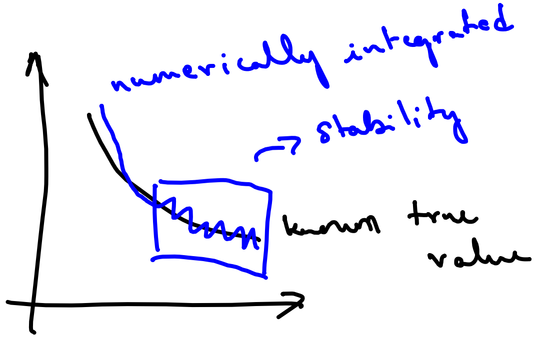
Sub (5) in (4)

$$y_{i+1} = y_i + h \Big[ \underbrace{f_{i+1} - \frac{h}{2} \Big[ \frac{f_{i+1} - f_i}{h}}_{\text{derivative}} + f''_{i+1} \frac{h}{2} + O(h^2) \Big] + \frac{h}{6} f''_{i+1} + \dots \Big]$$

$$y_{i+1} = y_i + h \Big[ \frac{1}{2} f_{i+1} + \frac{1}{2} f_i \Big] - \underbrace{\frac{1}{12} h^3 f''_{i+1}}_{\text{truncation}} - O(h^4)$$

# Numerical differencing

$$
\begin{array}{lllll}
f_{i-4} & & & & \\
f_{i-3} & \nabla f_{i-3} = \frac{f_{i-3} - f_{i-4}}{h} & & & \\
f_{i-2} & \nabla f_{i-2} & \nabla^2 f_{i-2} & & \\
f_{i-1} & \nabla f_{i-1} & \nabla^2 f_{i-1} & \nabla^3 f_{i-1} & \\
f_{i} & \nabla f_{i} & \nabla^2 f_{i} & \nabla^3 f_{i-1} & \nabla^4 f_{i}
\end{array}
$$




## Adams - Bashforth

$$y_{i+1} = y_i + h \sum_{k=0}^{n-1} \alpha_k f_{i-k} + O(h^{n+1})$$

Newton's polynomial

## Adams - Moulton

$$y_{i+1} = y_i + h \sum_{k=0}^{n-1} \beta_k f_{i+1-k} + O(h^{n+1})$$

$$\left.\begin{array}{l}\alpha_k \\ \beta_k\end{array}\right\} \text{ can be derived from recursive relations}$$

Stability → the integrated values have to be bounded.