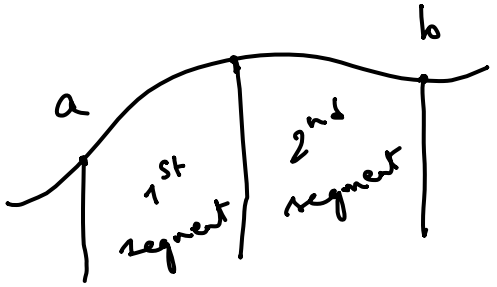
Lecture 30 - Numerical Integration



→ Trapezoidal rule → slope between a and b

→ Simpson's rule → polynomial fit b/w a and b

$f(x_k)$ → Lagrange polynomial → do the integration of the polynomial.

Evenly spaced data

even-segments x odd number of points then the integral formula is more accurate

Multiple application

$$\int_a^b f(x)\,dx = \int_a^{a_1} f(x)\,dx + \int_{a_1}^{a_2} f(x)\,dx + \int_{a_2}^{a_3} f(x)\,dx + \int_{a_3}^{b} f(x)\,dx$$

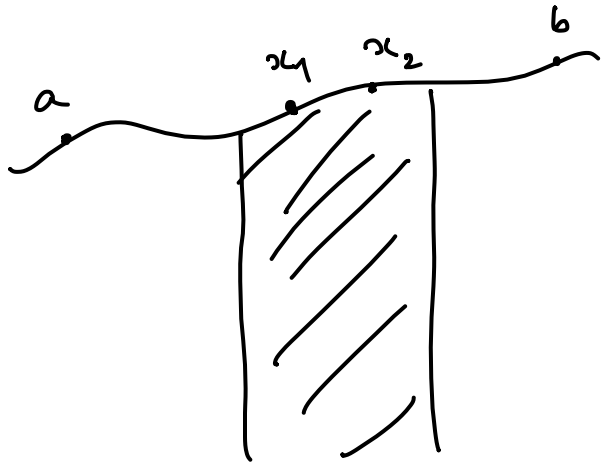
Unevenly spaced data

- sampling is uneven
- data gaps
- outliers in data
- pseudorandom

Trapezoidal rule

Simpson's rule

Open Integrals



Rectangular rule

$$I = (b-a) f(x_1)$$

$$= (b-a) \frac{f(x_1) + f(x_2)}{2}$$

Integral behaviors

Numerical integrations of Equations

Analytical Equation known

Analytical integration → not feasible

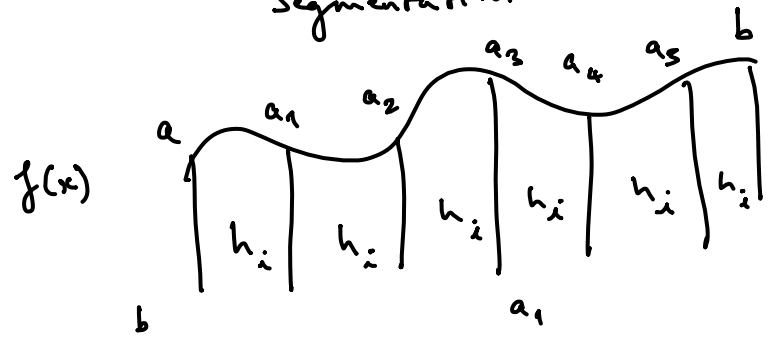
Numerically integrate

We can play around with the data spacing

$$I = I(h_1) + E(h_1) = I(h_2) + E(h_2)$$

$$\frac{E(h_1)}{E(h_2)} = \frac{h_1^2}{h_2^2}$$

Segmentation



$$I = \int_a^b f(x)\,dx = \int_a^{a_1} f(x)\,dx + \ldots + \int_{a_5}^{b} f(x)\,dx$$

Trapezoid

Assumption: Trapezoidal rule

$$E(\xi) = -\frac{(b-a)^3}{12n^2} f''(\xi)$$

Simpson's rule 2, 3, 4, 5, 6

$$= -\frac{(b-a)}{12} h^2 f''(\xi) \qquad \frac{(b-a)}{n} = h \Rightarrow n = \frac{(b-a)}{h}$$

$$\bar{f}''(\xi) = \frac{1}{n}\sum f''(\xi)$$

$$\frac{E(h_1)}{E(h_2)} = \frac{h_1^2}{h_2^2}$$

$$E(h_2) = h_2^2 \frac{E(h_1)}{h_1^2}$$

$$I = I(h_2) + h_2^2 \frac{E(h_1)}{h_1^2} = I(h_1) + E(h_1)$$

$$\Rightarrow \quad E(h_1) = \frac{I(h_2) - I(h_1)}{\left(1 - h_2^2/h_1^2\right)}$$

$$I = I(h_2) + \hat{E}(h_2)$$

Richard Extrapolation

Romberg Integration $= h_i = \frac{h_0}{2^i}$

$$E(h_1) = \frac{h_1^2}{h_2^2} E(h_2)$$

$$I(h_1) + \frac{h_1^2}{h_2^2} E(h_2) = I(h_2) + E(h_2)$$

$$\hat{E}(h_2) = \frac{I(h_2) - I(h_1)}{\left(\frac{h_1^2}{h_2^2} - 1\right)}$$

Adaptive techniques

- Understand the variation of the slope / change
- Use smaller segments in regions of higher change

  vice-verse

- Integration techniques
  - Romberg → function expression known
  - Newton-Cotes of varying orders → function tabulated

SIAM Reviews

Numerische Mathematik

JSTOR