LESSON 6

- BOOK CHAPTER 7
- (Kinetic energy and Work)

Kinetic Energy:

Kinetic energy *K is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.*

For an object of mass *m* whose speed *v* is well below the speed of light,

$$K = \frac{1}{2}mv^2$$

The SI unit of kinetic energy (and all types of energy) is the ***joule*** *(J), named for James Prescott Joule, an English scientist of the 1800s and defined as*

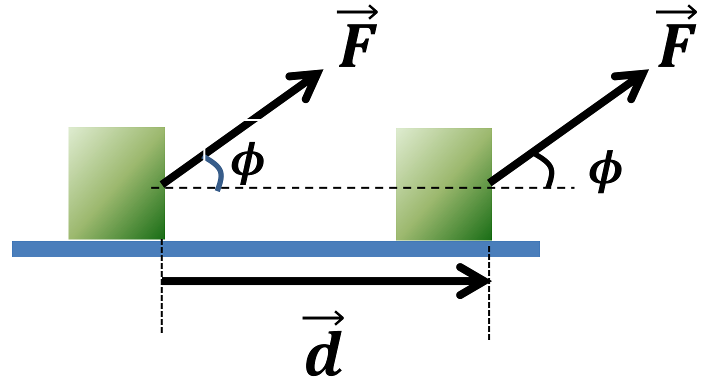
$$1\text{joule} = 1\text{ J} = 1\text{kg}.m^2/s^2$$

Work:

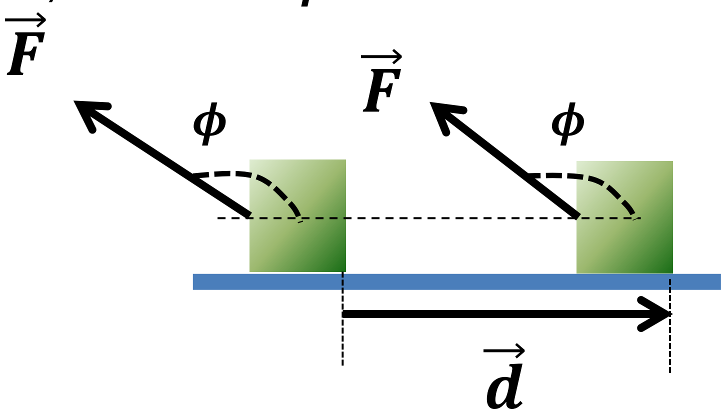
Work *W* is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

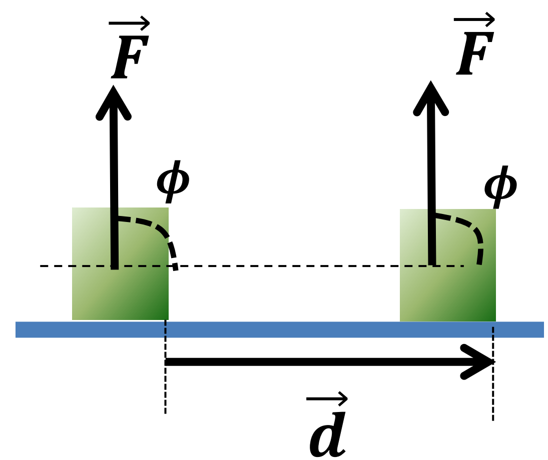
$$\boldsymbol{W = \vec{F}.\vec{d} = Fd\cos\phi}$$



Where ϕ is constant angle between the directions of $\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{d}}$**.** This is positive work, because $\boldsymbol{\phi < 90^0}$



Work is Negative, because $\boldsymbol{90^0 < \phi}$



The force does *no* work on the object, because $\boldsymbol{\phi = 90^0}$

The principle of work-kinetic energy theorem:

For a particle, a change ΔK in the kinetic energy equals the net work *W* done on the particle:

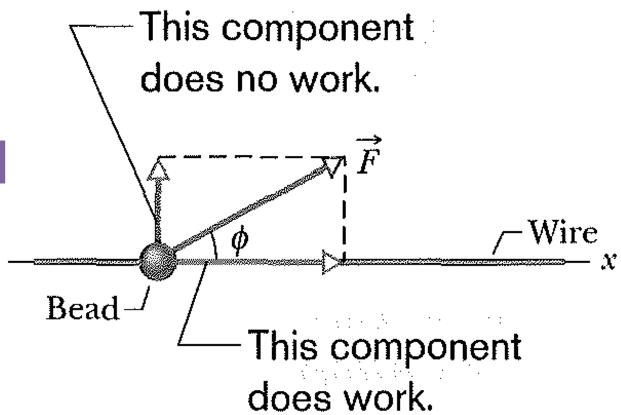
$$\boldsymbol{\Delta K = K_f - K_i = W}$$

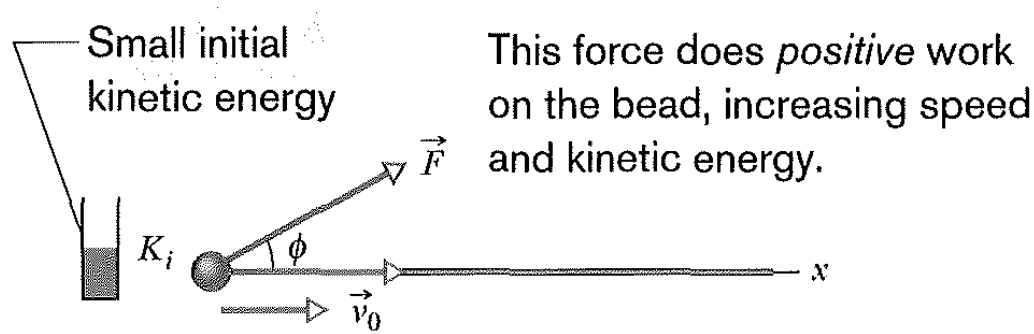
This is known as work-kinetic energy theorem, in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done.

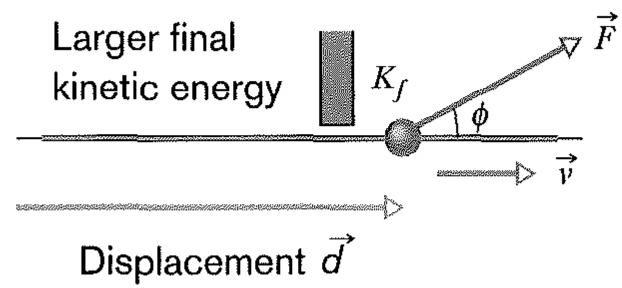
$$1\ joule = (\text{newton})(1\ \text{meter}) = 1\text{N.m} = 0.738\ \text{ft.lb}$$

The SI unit of work is joule, the same as kinetic energy. The corresponding unit in the British system is the foot-pound (ft.lb). 1joule is equivalent to

Finding an Expression for Work-Kinetic energy:







Considering a bead that can slide along a frictionless wire that is stretched along a horizontal *x* axis.

A constant force $\vec{F}$, directed at an angle ϕ to the wire, accelerates the bead along the wire.

Using Newton's law we can write,

$$F_x = ma_x \qquad \text{..............} (1)$$

$where\ m\ is\ mass\ of\ the\ bead$ and $a_x\ is\ the\ acceleration\ along\ x - axis.$

Using the equation for motion with constant acceleration, we can write

$$v^2 = v_0^2 + 2a_x d$$

$$v^2 - v_0^2 = 2a_x d$$

$$a_x = \frac{(v^2 - v_0^2)}{2d} \qquad \text{.............. (2)}$$

Substituting the value of a_x in equation (1) we can write,

$$F_x = m\left(\frac{v^2 - v_0^2}{2d}\right)$$

$$F_x d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\boxed{W = K_f - K_i}$$

in which $K_i = \frac{1}{2}mv_0^2$ is the initial kinetic energy of the particle and $K_f = \frac{1}{2}mv^2$ is the kinetic energy after the work, $W = F_x d$ is done.

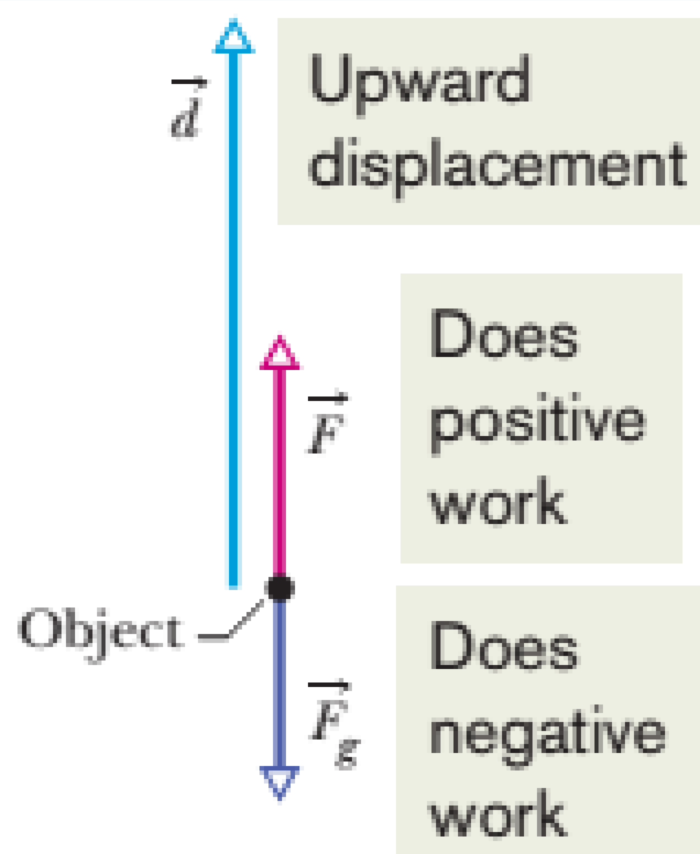
Work Done by the Gravitational Force:

We know that the work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$\boldsymbol{W = \vec{F}.\vec{d} = Fd\cos\phi}$$

Accordingly, the work W_g done by the gravitational force $\vec{F}_g$ on a object (particle/body) of mass *m* as the object moves through a displacement $\vec{d}$ is given by

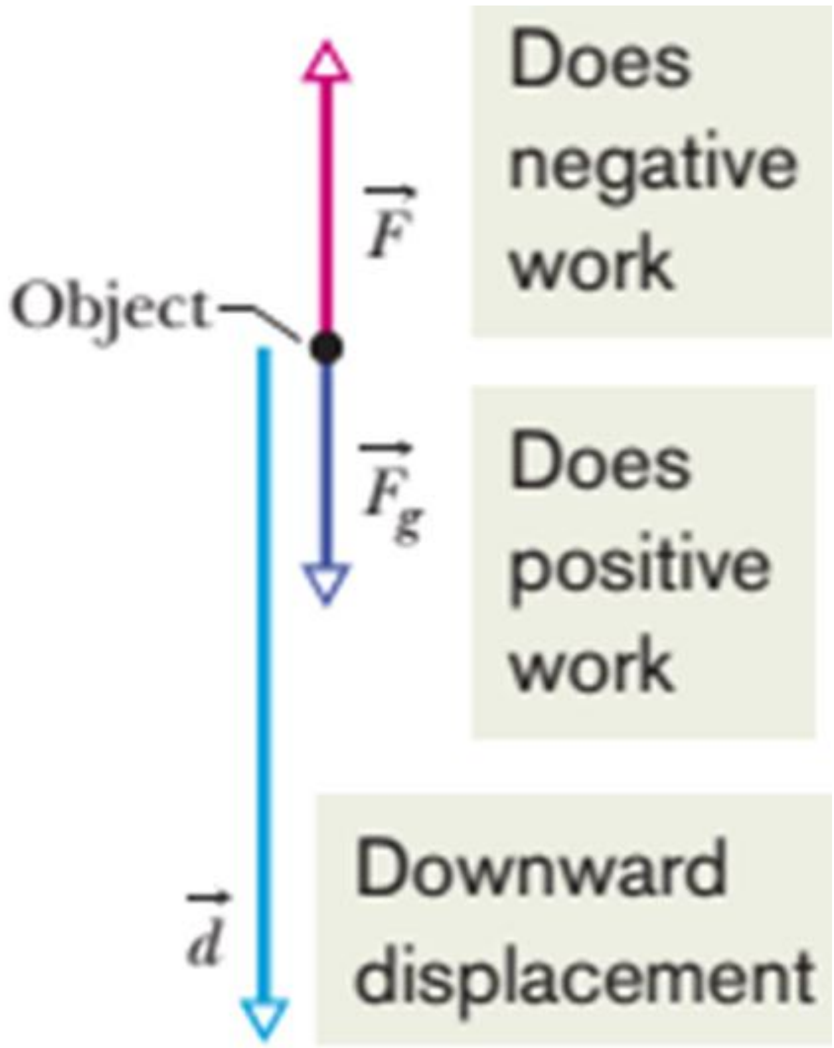
$$W_g = F_g d\cos\phi$$



Where ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

- **For rising an object:** Force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$ (as shown in the adjacent figure). Hence, W_g becomes

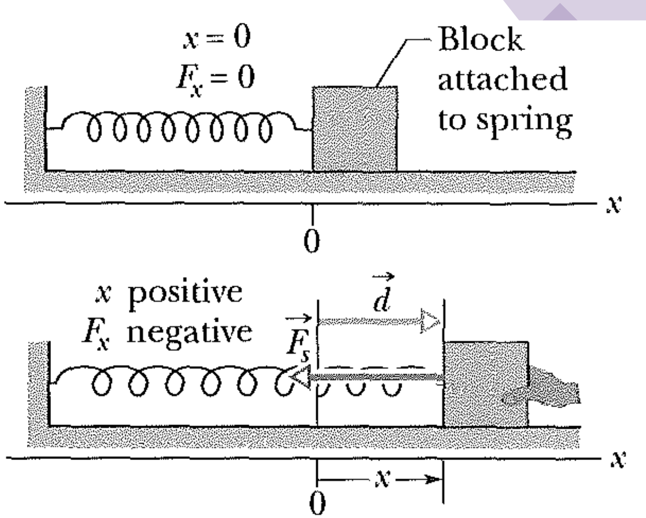
$$W_g = F_g d\cos 180^0 = mgd(-1) = -mgd$$



- **For lowering an object:** Force $\vec{F}_g$ is directed along the displacement $\vec{d}$ (as shown in the adjacent figure). Hence, W_g becomes

$$W_g = F_g d \cos 0^0$$
$$= mgd(+1) = mgd$$

The Spring Force:

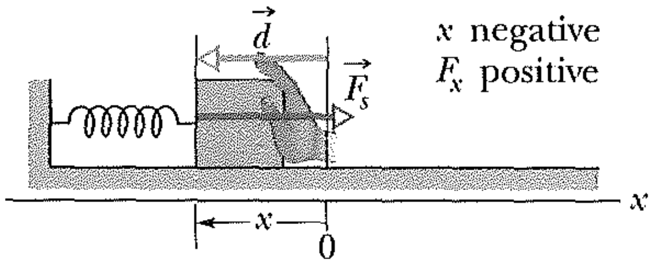


The force $\vec{F}_s$ from a spring is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in its relaxed state (neither compressed nor extended). The *spring force* $\vec{F}_s$ is given by

$$\vec{F}_s = -k\vec{d}$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant k is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring.

- The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement.
- The SI unit for k is the newton per meter



If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, we can write

$$F_x = -kx$$

The Work Done by a Spring Force:

The net work W_s done by the spring (from x_i to x_f) is

$$W_s = \int_{x_i}^{x_f} -F_x\, dx = \int_{x_i}^{x_f} -kx\, dx$$

[Where $|F_x| = kx$]

$$W_s = -k \int_{x_i}^{x_f} x\, dx$$

$$W_s = -k \left| \frac{x^2}{2} \right|_{x=x_i}^{x=x_f} = -\frac{1}{2} k\left(x_f^2 - x_i^2\right)$$

$$W_s = -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_i^2$$

If $x_i = 0$ and if we assume that $x_f = x$, the above equation becomes

$$\boxed{\boldsymbol{W_s = -\frac{1}{2} k x^2}}$$

Problem 1 (Book chapter 7)

A proton (mass $m = 1.67 \times 10^{-27} kg$) is being accelerated along a straight line at $3.6 \times 10^{15} m/s^2$ in a machine. If the proton has an initial speed of $2.4 \times 10^7\ m/s$ and travels 3.5 cm, what then is (a) its speed and (b) the increase in its kinetic energy?

Answer: Here, initial speed, $v_i = 2.4 \times 10^7\ m/s$ and the distance traveled by the proton, $s = 3.5\ cm = 0.035\ m$ and We assume final speed is v_f.

(a) We use the formula, $v_f^2 = v_i^2 + 2as = (2.4 \times 10^7)^2 + 2(3.6 \times 10^{15})(0.035)$

$$v_f^2 = 5.76 \times 10^{14} + 2.52 \times 10^{14} = 8.28 \times 10^{14}\ \frac{m^2}{s^2}$$

$$\boxed{\boldsymbol{v_f = 2.88 \times 10^7 m/s}}$$

(b) The change (increase) in kinetic energy,

$$\Delta K = K_f - K_i = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

$$\Delta K = \frac{1.67 \times 10^{-27}}{2}\left((2.88 \times 10^7)^2 - (2.4 \times 10^7)^2\ \right)$$

$$\Delta K = \frac{1.67 \times 10^{-27}}{2} (8.29 \times 10^{14} - 5.76 \times 10^{14})$$

$$\Delta K = \frac{1.67 \times 10^{-27} \times 2.53 \times 10^{14}}{2} = \frac{4.23 \times 10^{-13}}{2}$$

$$\boxed{\boldsymbol{\Delta K = 2.115 \times 10^{-13}\ J}}$$

Problem 9 (Book chapter 7)

The only force acting on a 2.0 kg canister that is moving in an x-*y* plane has a magnitude of 5.0 N. The canister initially has a velocity of 4.0 m/s in the positive *x* direction and some time later has a velocity of 6.0 m/s in the positive *y* direction. How much work is done on the canister by the 5.0 N force during this time?

Answer: We use the formula for work-kinetic energy theorem, which is

$$W = \Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

In the above formula, speed is required; whatever the directions.

$$W = \frac{2}{2}(6^2 - 4^2) = 36 - 16 = 20 \text{ J}$$

$$\boxed{W = 20 \text{ J}}$$

Given

$m = 2\ kg$

$v_i = 4\ m/s$

$v_f = 6\ m/s$

$W = ?$

Problem 11 (Book chapter 7)

A 12.0 N force with a fixed orientation does work on a particle as the particle moves through the three-dimensional displacement $\vec{d} = \left(2\hat{\imath} - 4\hat{\jmath} + 3\hat{k}\right) m$. What is the angle between the force and the displacement if the change in the particle's kinetic energy is (a) +30.0 J and (b) - 30.0 J?

Answer: Here, we use the work-kinetic energy relation, which is

$$\boldsymbol{\Delta K = W = \vec{F}.\vec{d} = Fd\; cos\; \phi}$$

Where ϕ is the angle between force $\vec{F}$ and displacement $\vec{d}$.

(a) $\Delta K = Fd\; cos\; \phi$

$$\cos\phi = \frac{\Delta K}{Fd}$$

$$\phi = \cos^{-1}\frac{\Delta K}{Fd} = \cos^{-1}\frac{30}{(12)(5.385)} = \cos^{-1} 0.464$$

$$\boxed{\boldsymbol{\phi = 62.35^0}}$$

Given

$\Delta K = +30\, J$ for (a)

$\Delta K = -30\, J$ for (b)

$|F| = 12\, N$

$\vec{d} = \left(2\hat{\imath} - 4\hat{\jmath} + 3\hat{k}\right)$ m

$d = \sqrt{(2)^2 + (-4)^2 + (3)^2}$

$d = 5.385\, m$

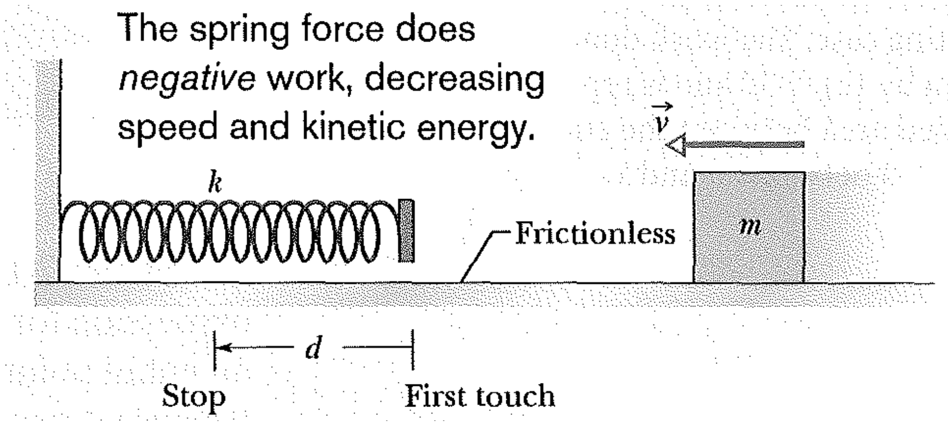
(b) For $\Delta K = -30\,J$

$$\phi = \cos^{-1}\frac{\Delta K}{Fd} = \cos^{-1}\frac{-30}{(12)(5.385)} = \cos^{-1} -0.464$$

$$\boxed{\phi = 117.65^0}$$

Sample Problem 7.06 (page 161) Home work

In the Figure below, a cumin canister of mass $m = 0.40\ kg$ slides across a horizontal frictionless counter with speed v = 0.50 *m/s.* It then runs into and compresses a spring of spring constant k = 750 *N/m.* When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?



Hints: use the formula for work done by Spring.

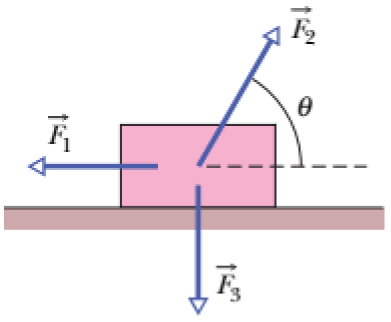
$$W = K_f - K_i = -\frac{1}{2}kd^2$$

Let's try

1. [Chap 7 - problem 2]: If a Saturn V rocket with an Apollo spacecraft attached had a combined mass of 2.9×10^5 kg and reached a speed of 11.2 km/s, how much kinetic energy would it then have?

2. [Chap 7 - problem 8]: A ice block floating in a river is pushed through a displacement $\vec{d} = (15\, m)\hat{\imath} - (12\, m)\hat{\jmath}$ along a straight embankment by rushing water, which exerts a force $\vec{F} = (210\, N)\hat{\imath} - (150\, N)\hat{\jmath}$ on the block. How much work does the force do on the block during the displacement?

3. Figure below shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F1$ = 5.00 N, $F2$ = 9.00 N, and $F3$ = 3.00 N, and the indicated angle is θ = 60.0°. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?



4. [Chap 7 - problem 41]: A single force acts on a 3.0 kg particle-like object whose position is given by $x = 3.0\, t - 4.0\, t^2 + 1.0\, t^3$, with x in meters and t in seconds. Find the work done by the force from t = 0 to t = 4.0 s.

Thank You