LESSON 3

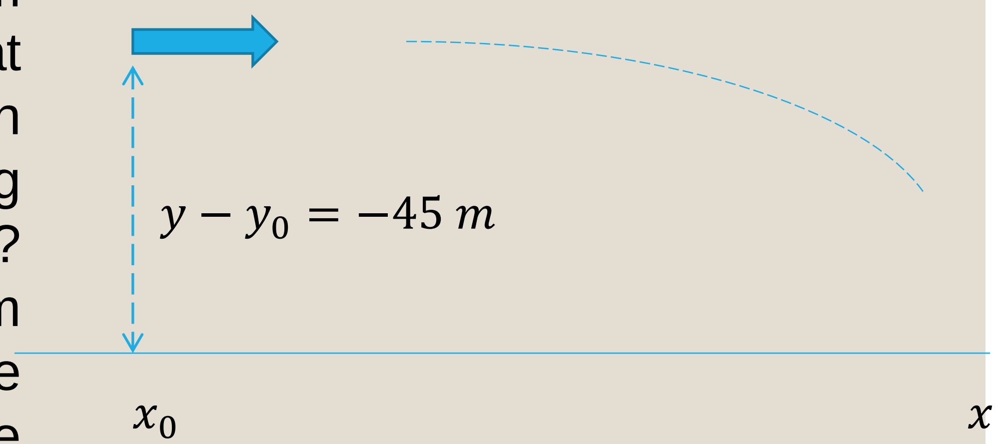
- BOOK CHAPTER 4

  (Projectile Motion)

Problem 23 (Book chapter 4):

A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

$v_0 = 250\ m/s$ and $\theta_0 = 0^0$



Answer: (a) We know

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2} g t^2$$

$$-45 = (v_0\ sin\ 0^0)\ t - 4.9t^2$$

$$-45 = 0 - 4.9t^2$$

$$\boxed{t = \sqrt{\frac{45}{4.9}} = 3.03\ s}$$

(b) We know $x - x_0 = (v_0 \cos\theta_0)\ t$

$$x - x_0 = (250)(\cos 0^0)\ (3.03)$$

$$x - x_0 = (250)(1)(3.03)$$

$$\boxed{x - x_0 = 757.50\ m}$$

(c) We know $v_y = v_0 \sin\theta_0 - gt$

$$v_y = 250(\sin 0^0) - (9.8)\,(3.03)$$

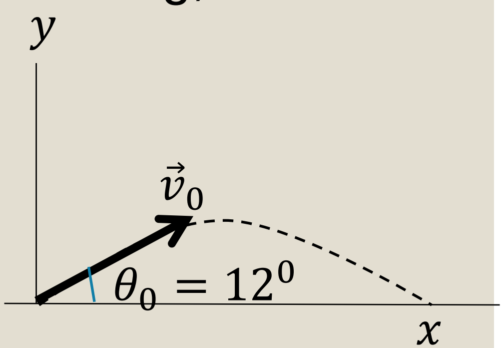
$$v_y = 0 - (9.8)\,(3.03)$$

$$v_y = -29.69 \text{ m/s}$$

The magnitude of v_y is 29.69 m/s

Problem 25 (Book chapter 4):

The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 12.0º to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, determine his take-off speed.



$R = x - x_0 = 77\ m$

Answer:

Since the take-off and landing heights are the same, that is $y - y_0 = 0$, we can use the formula

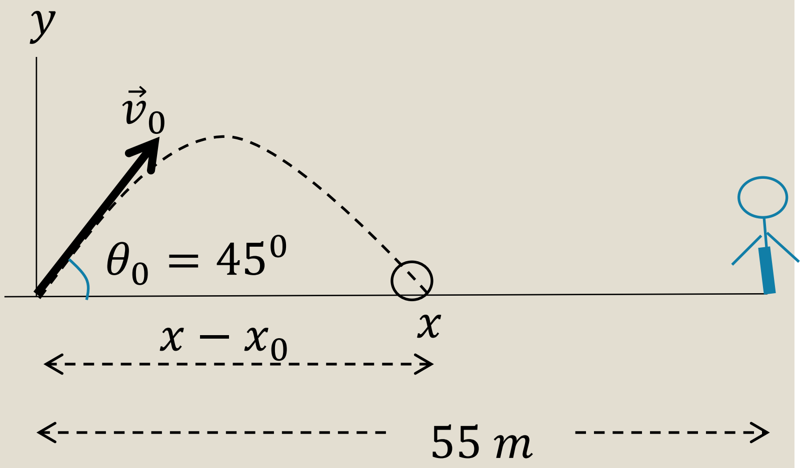
$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad or \quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}}$$

$$or \ \ v_0 = \sqrt{\frac{(77)(9.8)}{\sin 24^0}}$$

$$v_0 = \sqrt{\frac{754.6}{0.4067}} = 43.07\ m/s$$

Problem 30 (Book chapter 4):

A soccer ball is kicked from the ground with an initial speed of 19.5 m/s at an upward angle of 45°. A player 55 m away in the direction of the kick starts running to meet the ball at that instant. What must be his average speed if he is to meet the ball just before it hits the ground?



Answer: Here, $y - y_0 = 0$

We use the following formula to find the time of flight of the ball.

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2} g t^2$$

$$0 = (19.5)\,(\sin\ 45^0)\ t - 4.9\ t^2$$

$$(19.5)(0.707)\ t = 4.9\ t^2$$

$$t = \frac{13.787}{4.9} = 2.81\ s$$

The player must take the time 2.81 s to meet the ball.

We need to find $x - x_0$ *to obtain the distance travled by the player.*

$$x - x_0 = (v_0 \cos\theta_0)\ t = (19.5)(\cos 45^0)(2.81)$$

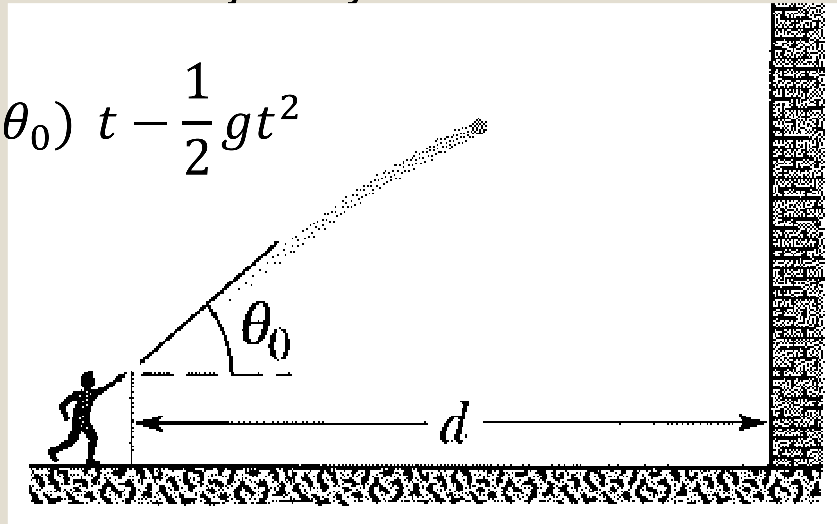
$$x - x_0 = 38.74\ m$$

$$Average\ speed\ of\ the\ player = \frac{Distance\ traveled\ by\ the\ player}{Time\ taken\ by\ the\ player} = \frac{55 - 38.74}{2.81} = 5.786\ \mathrm{m/s}$$

Problem 32 (Book chapter 4):

You throw a ball toward a wall at speed 25.0 m/s and at angle 40.0° above the horizontal (as shown in the figure). The wall is distance *d*= 22.0 m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

Answer: (a) We know $y - y_0 = (v_0 \sin\theta_0)\ t - \dfrac{1}{2} g t^2$



$$y - y_0 = (25)(\sin\ 40^0)\ t - 4.9\ t^2$$

$$y - y_0 = (25)(0.6428)t - 4.9\ t^2$$

$$y - y_0 = 16.07t - 4.9\ t^2$$

To find t we use the following formula,

$$x - x_0 = (v_0 \cos\theta_0)\ t$$

$$t = \frac{x - x_0}{v_0 \cos\theta_0} = \frac{22}{25\ cos 40^0} = \frac{22}{(25)(0.7660)}$$

$$t = 1.149\ s$$

Given $v_0 = 25\ m/s$; $\theta_0 = 40^0$

$$x - x_0 = d = 22\ m$$

$(a)\ y - y_0 = ?\ \ (b)\ \ v_x = ?\quad and\ (c)\ \ v_y = ?$

$(d)\ Did\ the\ ball\ pass\ the\ highest\ point?$

Therefore,

$$y - y_0 = (16.07)(1.149) - (4.9)(1.149)^2 = 18.46 - 6.469 = 11.99\ m$$

(b) We know $v_x = v_{0x} = v_0 \cos\theta_0 = 25\cos 40^0 = (25)(0.766) = 19.15\, m/s$

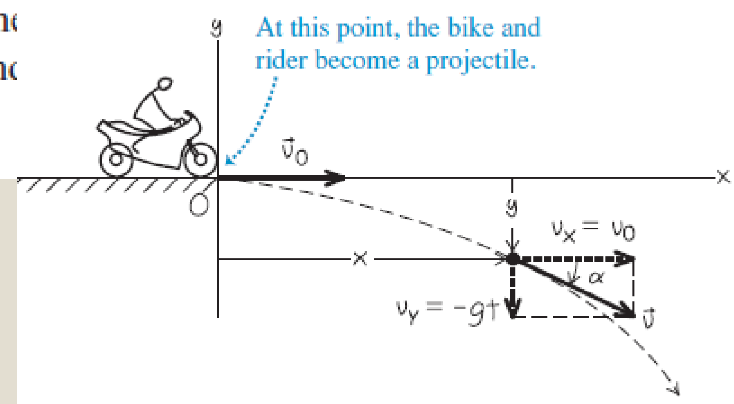
(c) We know $v_y = v_0 \sin\theta_0 - gt = 25 \sin 40^0 - (9.8)(1.149)$

$$v_y = (25)(0.6428) - 11.26 = 4.81 \text{ m/s}$$

(d) Since v_y *is positive, that is,* $v_y > 0$, *the ball did not reach to the highest point on hitting the wall.*

Example 3.6 A body projected horizontally

A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff, and velocity 0.50 s after it leaves the edge of the cliff.



SOLUTION

IDENTIFY and SET UP: Figure 3.22 shows our sketch of the motorcycle's trajectory. He is in projectile motion as soon as he leaves the edge of the cliff, which we choose to be the origin of coordinates so $x_0 = 0$ and $y_0 = 0$. His initial velocity $\vec{v}_0$ at the edge of the cliff is horizontal (that is, $\alpha_0 = 0$), so its components are $v_{0x} = v_0 \cos \alpha_0 = 9.0\ \mathrm{m/s}$ and $v_{0y} = v_0 \sin \alpha_0 = 0$. To find the motorcycle's position at $t = 0.50$ s, we use Eqs. (3.20) and (3.21); we then find the distance from the origin using Eq. (3.24). Finally, we use Eqs. (3.22) and (3.23) to find the velocity components at $t = 0.50$ s.

EXECUTE: From Eqs. (3.20) and (3.21), the motorcycle's x- and y-coordinates at $t = 0.50$ s are

$$x = v_{0x}t = (9.0\ \mathrm{m/s})(0.50\ \mathrm{s}) = 4.5\ \mathrm{m}$$

$$y = -\tfrac{1}{2}gt^2 = -\tfrac{1}{2}(9.80\ \mathrm{m/s^2})(0.50\ \mathrm{s})^2 = -1.2\ \mathrm{m}$$

The negative value of y shows that the motorcycle is below its starting point.

From Eq. (3.24), the motorcycle's distance from the origin at $t = 0.50$ s is

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5\ \mathrm{m})^2 + (-1.2\ \mathrm{m})^2} = 4.7\ \mathrm{m}$$

From Eqs. (3.22) and (3.23), the velocity components at

The motorcycle has the same horizontal velocity v_x as when it left the cliff at $t = 0$, but in addition there is a downward (negative) vertical velocity v_y. The velocity vector at $t = 0.50$ s is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (9.0\ \mathrm{m/s})\hat{\imath} + (-4.9\ \mathrm{m/s})\hat{\jmath}$$

From Eq. (3.25), the speed (magnitude of the velocity) at $t = 0.50$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0\ \mathrm{m/s})^2 + (-4.9\ \mathrm{m/s})^2} = 10.2\ \mathrm{m/s}$$

From Eq. (3.26), the angle α of the velocity vector is

$$\alpha = \arctan\frac{v_y}{v_x} = \arctan\left(\frac{-4.9\ \mathrm{m/s}}{9.0\ \mathrm{m/s}}\right) = -29^\circ$$

The velocity is 29° below the horizontal.

EVALUATE: Just as in Fig. 3.17, the motorcycle's horizontal motion

Let's Practice !!

1. [Chap 4 - problem 21]: A dart is thrown horizontally with an initial speed of 10 m/s toward point P, the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P, 0.19 s later. (a)What is the distance PQ? (b) How far away from the dart board is the dart?
2. [Chap 4 - problem 24]: In the 1991 World Track and Field Championships in Tokyo, Mike Powell jumped 8.95 m, breaking by a full 5 cm the 23-year long-jump record set by Bob Beamon. Assume that Powell's speed on takeoff was 9.5 m/s (about equal to that of a sprinter) and that g 9.80 m/s2 in Tokyo. How much less was Powell's range than the maximum possible range for a particle launched at the same speed?
3. [Chap 4 - problem 29]: A projectile's launch speed is five times its speed at maximum height. Find launch angle θ_0 .
4. [Chap 4 - problem 36]: During a tennis match, a player serves the ball at 23.6 m/s, with the center of the ball leaving the racquet horizontally 2.37 m above the court surface. The net is 12 m away and 0.90 m high. When the ball reaches the net, (a) does the ball clear it and (b) what is the distance between the center of the ball and the top of the net? Suppose that, instead, the ball is served as before but now it leaves the racquet at 5.00° below the horizontal. When the ball reaches the net, (c) does the ball clear it and (d) what now is the distance between the center of the ball and the top of the net?

Thank You