Revelation of SAO 78380 Duplicity from Processing of Photoelectric Lunar Occultation Curve by Tikhonov's Regularization Method

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Abstract: The results of the processing of the photoelectric lunar occultation curve of the star SAO 78380 are presented. Observation was made with 100 cm reflector of Mt. Maidanak observatory (Uzbekistan, longitude is $4^h 27^m 36^s$, latitude - 39° 41'18") in the spectral band "R" with a time resolution of 1 ms. Processing was carried out by applying Tikhonov's regularization method in order to solve a Fredholm integral equation with respect to the brightness strip distribution over the source observed along the lunar limb normal. The results obtained give evidence of the duplicity of the star with 20 arcmilliseconds angular separation between its components in projection along perpendicular to the lunar limb and magnitude difference 1.7^m.

Since 1988 photoelectric observations of the lunar occultations of stars with a time resolution of 1 ms were begun at the Mt. Maidanak observatory of the Sternberg Astronomical Institute in Uzbekistan (altitude is about 2600 m). Description of the photometer and accompanying equipment used during observations, and of a program for precomputation of the circumstances of occultations of stars by the Moon is given in the paper [2]. It is known that one of the main kinds of information which could be obtained from lunar occultation observations is revelation of duplicity or multiplicity of the stars under investigation and determination of their parameters. We present here the results obtained when one of the photoelectric occultation curves recorded was processed.

Occultation of the star SAO 78380 = BD +28 1120 which has visual magnitude about 9.0^m and spectral class K2 [5] was observed with 100 cm reflector of Mt. Maidanak observatory (Uzbekistan) on April 21, 1988 at $22^h01^m52^s$ UT. Unfortunately recording of the time of occultation with more high accuracy was not carried out because of technical reasons. The occultation point's position angle on the lunar limb was P=53°. Computational velocity of the lunar limb motion along its radius-vector in the occultation point was about 525 m/s. Observation was obtained in the spectral band "R" which is close to standard one. Photoelectrical occultation curve of about 300 ms period which is a part of record obtained is presented in Fig.1.

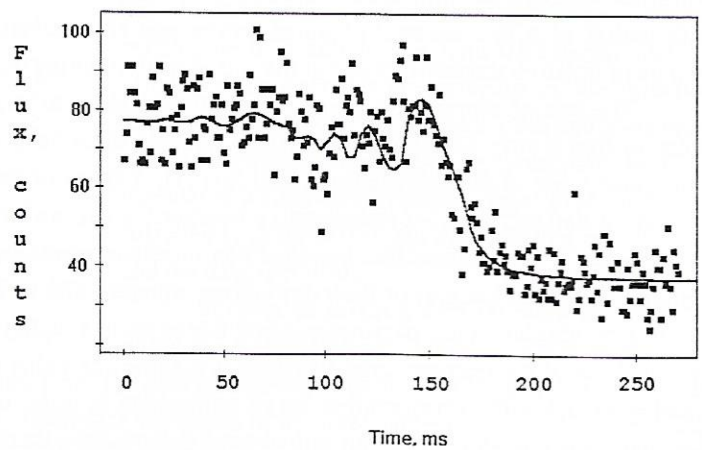

Figure 1.
Photoelectric occultation curve of SAO 78380 recorded with a time resolution of 1 ms, and model diffraction curve for occultation of a double star, corresponding to the strip brightness distribution found from processing of the data.

There are a few various algorithms for processing a data of photoelectric lunar occultation observations of stars. One can represent an occultation curve recorded, when it is free from a stochastic noise as a convolution of the one-dimensional strip brightness distribution across the source under study with the Fresnel diffraction curve for a heterochromatic point-like source. Thus mathematically the problem is reduced to search of the solution of Fredholm integral equation of the 1st kind [1]. The problems of solution stability and of convergence arise when equations of this type are examined. Various methods exist for finding the solution desired which are based on the various assumptions.

A method of fitting the best finite-number-dimensional parametric model is the most widely employed [4]. When the method is applied the strip brightness distribution across the object observed (as a rule, across a stellar disk) is assumed to be known. This method yieldes reliable results when the model used is described by a small number of parameters, for instance in determining of angular diameter of a single star from photoelectric occultation diffraction curve which was recorded. But when we deal with a possible close duplicity of the star investigated, a number of model parameters increases, hence convergence of fitting algorithm in the presence of noise in the data recorded becomes worse, and the problem of statistical assurance of the results obtained becomes considerably more complex.

There are two main sources of a noise in photoelectric occultation observations: quantum noise of the light flux and atmospheric scintillation. The latter phenomenon manifests itself in the light intensity fluctuations on the telescope aperture. When we try to solve a Fredholm integral equation with respect to the brightness strip distribution it is necessary to take into account that differing noise factors affect in different ways the required function of interest. As it is known from numerical experiments, Poisson noise leads to smoothing of this brightness strip distribution curve, while stellar scintillation could perturb its form. Thus an obtaining of a reasonably trustworthy solution of the equation mentioned turns out to be very complicated problem, and we should use all available additional information in order to solve a matter of duplicity of the star in question.

With this purpose we can use the following a priori information:

1) light source occulted by the Moon has a small finite angular size and positive brightness. Hence, one-dimensional strip brightness distribution along the lunar limb normal can be described as non-negative bounded function $b(x)$, which is defined on the finite interval: $b(x)=0$ if $x<a$ or $x>b$, and $0\leq b(x)\leq M$ (M is some positive value) if $a\leq x\leq b$;

2) data recorded $S(t)$ may be represented as a sum of a diffraction light curve $S0(t)$ corresponding to the flux behaviour during occultation process without any noise influence, and function $N(t)$ describing the noise that is being present:

$$S(t) = S0(t) + N(t) \tag{1}$$

3) one may consider a quantity of the sum of values $N(t)$ squared to be known. It is quite fair assumption since we can determine some statistical properties of the available noise from the photoelectric record with period of a few seconds obtained before and after occultation section of data, and account for a change of noise characteristics due to the flux downfall during occultation.

The use of aforesaid a priori information enables to apply Tikhonov's regularization method in order to find a strip distribution $b(x)$ from integral equation mentioned, i.e. to restore the function $b(x)$ from processing of the observational data $S(t)$ [1]. A range of possible solutions $b(x)$ belongs to the space of interval-defined bounded non-negative functions. Next, we have to confine a space of possible solutions to the space W of differentiable bounded non-negative functions, which have both finite integral of values squared and finite integral of their derivatives squared. The level of "smoothness" of the solution $b(x)$ is defined by regularization parameter α and has to be in keeping with the accuracy of the initial data $S(t)$. By the form of the restored strip distribution $b(x)$ we can judge about the validity of application of one or another model with corresponding set of parameters in order to data processing by means of fitting the best parametric model, and about influence of noise factors on the results derived. In particular, in the case of a double star the resulting function $b(x)$ must show two-maxima contour which cannot be explained by influence of noise factors.

To describe the method used we can write a basic equation, which relates functions b(x) and S(t) one with another. It can be written in operator form as

$$Ab = S \tag{2}$$

where A – integral operator of a convolution of Fresnel diffraction curve for a heterochromatic point-like source with the function b(x) sought-for. This operator involves the apparature effects which consist in the factors of finite width of spectral band, of finite telescope aperture and of finite integration time in the flux measurements. Next, let F(x) belongs to the space W, described above, and $F'(x)=f(x)$. Let us define the norm L[F] of function F(x) as integral of F(x) squared over all possible values of x. Then we introduce the norm W[F] in the space W as

$$W[F] = L[F] + L[f] \tag{3}$$

Tikhonov's functional can be written now as

$$T[b] = L[\ Ab\text{-}S\ ] + \alpha \cdot W[b] \tag{4}$$

This functional should be minimized in order to find its extremal function b(x) in W space, which could be designated as the solution desired. In searching for this solution we have used the conjugate gradient's projection algorithm [6]. The value of parameter α should be fitted such that the sum of deviations (i.e. values N(t) in eq.(1)) squared of data recorded from model curve S0(t) for solution founded (this sum is equal to L[Ab-S]) would be in accord with a priori value. In that case the "smoothness" of the solution b(x) will be in keeping with the accuracy of initial data S(t) and, as it was proved, an extremal function b(x) of functional T will converge to the true solution of eq.(2), when the data accuracy will become higher [6].

In processing of SAO 78380 occultation curve a blackbody spectrum of the star's radiation with the temperature 4590 K was assumed, in accord with characteristic temperature of K2 stars. Such approximation is quite permissible because the solution b(x) is a weak function of the spectral energy distribution in stellar radiation.

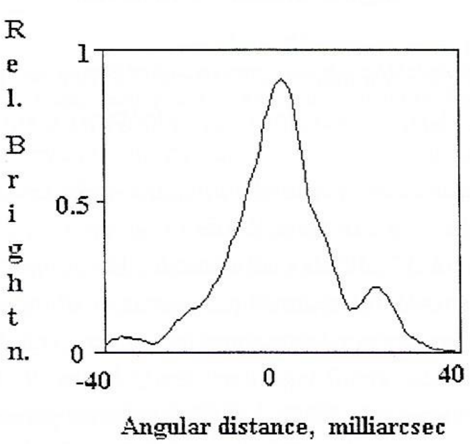

Figure 2.
The one-dimensional strip brightness distribution b(x) restored by processing of the initial observational data.

Fig.2 shows the one-dimensional strip brightness distribution across the source observed which results from processing of the data presented in Fig.1, by procedure described above. This function b(x) has two maxima, and its form remains steady when regularization parameter α varies within reasonable limits. Thus we have a serious basis to presume a close duplicity of the star. Corresponding model diffraction curve for occultation of a double star with 0.02 arcsec angular separation between its components in the direction along a perpendicular to the lunar limb and their magnitude difference about 1.7^m is presented in Fig.1 together with the observational data. One can see a sufficiently good agreement between this model curve and the data recorded.

In order to investigate a problem of reliability of the result obtained we studied an effect of the available stochastic noise on the solution found from the data processing. The main difficulty here is to estimate effects of atmospheric scintillation. Stochastic variations of the light intensity due to this phenomenon result in the noise function N(t) appears to be partially correlated. Hence a statistics of the light flux recorded will be essentially different from the Poisson one. There are algorithms, which allow to take into account explicitly an information on the statistics of the light flux fluctuations attributed to atmospheric scintillation (for instance, some features of correlation functions), but they are very complicated and require a great computations (see, for example, Ref. [3]). Therefore we have chosen the line of using the available record of the really registered noise.

We took the model occultation curve for a point-like single star, and superimposed the real noise recorded on this curve, producing by such a way a number of man-made realizations of occultation process, which could be observed in principle with given characteristics of noise. With this purpose we have used the available photoelectric record of the noise fluctuations of the light flux which was obtained long before occultation section of data. Here it is necessary to take into consideration that the noise recorded after occultation is the Poisson noise, but the noise recorded before occultation has much greater variance and includes a powerful scintillation component with other statistics. For this reason the Poisson noise from the part of record which was obtained later than occultation occured was superimposed on the background section of theoretical diffraction curve.

We have produced 4 various man-made noisy occultation curves and have restored the strip brightness distributions b1(x) by their processing, which are shown in Fig.3. We can see that these brightness distributions b1(x) are much more close to exponential distribution than the function b(x) found from original occultation curve. And if we would superimpose a pure Poisson noise on the model diffraction curve for a point-like source we would obtain a restored function b2(x) which is very similar to exponential distribution (as it follows from Tikhonov's regularization theory).

As it was mentioned above the distribution b(x) recovered from actually observed occultation curve shows two-maxima profile which is evidently different from a single exponential one. It is important to emphasize that the area under secondary maximum of the strip distribution curve b(x) obtained from initial data, reaches about 12-13% fraction of the area under main maximum of this curve, just as the areas under secondary maxima of the curves b1(x) recovered from the man-made realizations (if we consider these secondary maxima to be actual) account for a fraction of the area under main one, which is smaller by a factor of 2-2.5.

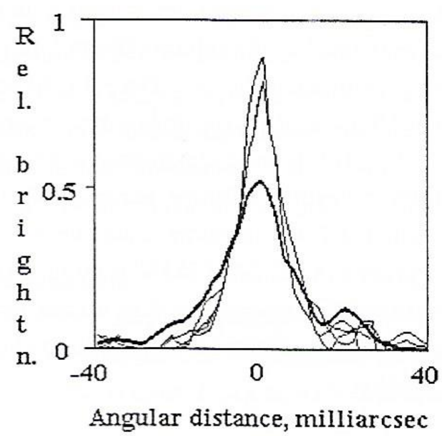

Figure 3.
The strip brightness distributions recovered from actually observed occultation curve (b(x), heavy line), and from the man-made noisy occultation curves (b1(x), thin lines).

Thus we have rather convincing evidence for the star SAO 78380 is really double. The projected angular separation between its components was about 20 milliarcsec, the assumed position angle of the direction along which the Moon's limb appeared to scan the object occulted was about 53 degrees, and the magnitude difference between the components is about 1.7^{m} .

An analysis of the available observational data concerning SAO 78380, and further investigations of this star are required.

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