OMKAR SAVANT - 43

GROUP NO - 6

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Group 6

* INTRODUCTION :

In mathematics, a differential equation is an equation that relates one or more function and their derivatives. In the applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the D.E defines a relationship between two. Because such are extremely common, differential equation plays a prominent role in many disciplines including engineering, physics, economics and biology.

The study of differential equation consist mainly of the study of their solutions (the set of functions that satisfy the equation), and of the properties of their solutions. Only the simpliest differential equation may be determined without computing them exactly.

If a closed-form expressions for the solutions may be numerically approximated using computers.

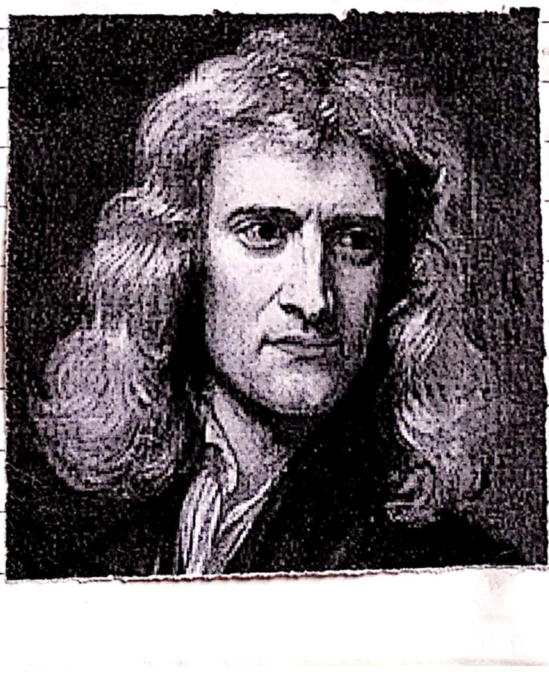
* HISTORY:

Differential equation First came into existance with the inventation of calculus by Newton and Leibiniz. In chapter two of his 1671 work "Methodus Floxonium et Serierum Infinitariuw" Issac Newton listed three Kinds of Differential Equation.

$$\frac{dy}{dx} = f(x) \qquad \frac{dy}{dx} = f(x, y)$$

$$x_1 \frac{\partial y}{\partial x_1} + x_2 \frac{\partial y}{\partial x_2} = y$$

In all these cases, y is an unknown function of x, and f is a given function.
He solves these examples and others using infinite series and discusses the non-uniqueness of solution.



Issac Newton

★ Order of Differential Equation :

Differential Equation are classified on the basis of order. Order of a differential equation is the order of the highest order derivative (also known as differential coefficient) present in equation.

For example : $\frac{d^3y}{dx^3} + 3x\frac{dy}{dx} = e^y$

In this equation the order of the highest derivative is 3 hence this is a third order differential equation.

1] First order differential Equation :
The order of highest derivative in case of first order differential Equation is :
i] A linear equation has order.
ii] In case of linear Differential Equation, the first derivative is the highest order derivative.

Example : $\frac{dy}{dx} + (x^2+5)y = \frac{x}{5}$

This represent a first order Differential Equation.

2] Second order Differential Equation:
When the order of the highest derivatives is 2, then it is Second order Differential Eqn.

Example:

$$\frac{d^2y}{dx^2} + (x^2+3x)y = a$$

In this example, the order of the highest derivative is 2. Therefore it is a Differential equation.

3] Third order D.E:
When the order of the highest derivative is 3, then it is third order D.E.

Example:

$$\frac{dy}{dx} + (x^3+6x)y = a$$

In this example, the order of the highest derivative is 3. Therefore it is a differential equation.

Degree of differential Equation:

The degree of D.E is represented by the power of the highest order derivative in the given differential equation.
The differential equation must be a polynomial equation in derivative for degree to be defined.

Example 1:-

$$\frac{d^4y}{dx^4} + \left(\frac{d^2y}{dx^2}\right)^2 - 3\frac{dy}{dx} + y = 9$$

Here, the exponent of the highest order derivative is one and the given differential equation is polynomial equation in derivative. Hence, the degree of this equation is 1.

Example 2:-

$$\left(\frac{d^3y}{dx^3}\right)^2 + y = 0$$

The order of this equation is 3 and the degree is 2.

* Formation of Differential Equation:

Steps :

1] Differentiate the given function with respect to the independant variable present in equation.

2] Keep differentiating times in such a way that (n+1) equation are obtained.

3] Using (n+1) equations obtained, eliminate constants (C_1, C_2, C_3).

The general solution of the differential equation is the relation between variable x and y which is obtained after removing the derivative where the relation contain arbitary constant to denote the order of an equation. If particular values are given to arbitary constant, the general solution of differential equations are obtained. In order to solve the first order Differential Equations of first degree some standard formula's are available to get general solutions.

★ Methods of solving Differential Equation :

1] Variable Separable form.
2] Method of variable Sepration form by suitable substitution.
3] Homogenous Differential Equation.
4] Exact Differential Equation.
5] Linear Differential Equation
6] Non-Homogeneous Differential Equation.
7] Bernoulli's Differential Equation.

1] Variable Separable form :

A Differential Equation is said to be separable if the variables can be separated. That is a separable equation is one that can be written in form.

$F(y)\,dy = G(x)\,dx$

Once this is done, all that is needed to solve the equation is to integrate both sides. The method for solving separable equation can therefore be summarized as follows :

Seprate the variable and Integrate.

2] Linear Differential Equation.

A linear Differential Equation is a differential equation that is defined by a linear polynomial in the unknown function and its derivatives, that is an equation of the form.

$$a_0(x)y + a_1(x)y' + a_2(x)y'' + \dots + a_n(x)y^{(n)} + b(x) = 0$$

where $a_0(x)$, $a_n(x)$ and $b(x)$ are arbitary differential function that do not need to be linear, and $y' \dots y^{(n)}$ are the sucessive derivatives of an unknown function y of the variable x.

A linear Differential Equation or a System of linear equation such that the associated homogeneous equation have constant Coefficients may be solved by quadracture, which means that Solution can be expressed in terms of integral.

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* Examples

1] $\sec^2 y \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$. Find the particular solution if $y = \frac{\pi}{4}$ at $x = \frac{\pi}{4}$

→

$\sec^2 x \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

$$\log|\tan x| = -\log|\tan y| + \log C$$

$$\log|\tan x| + \log|\tan y| = \log C$$

$$\log(\tan x \cdot \tan y) = \log C \longrightarrow ①$$

$$\tan 45 \cdot \tan 45 = C$$

$$\boxed{1 = C}$$

Put in ①

$$\tan x \cdot \tan y = 1$$

2] $\frac{dy}{dx} = e^{x-y} + xe^{-y}$

$\frac{dy}{dx} = e^{x-y} + xe^{-y}$

$\frac{dy}{dx} = e^{x} \cdot e^{-y} + xe^{-y}$

$\frac{dy}{e^{-y}} = e^{x} + x\,dx$

$e^{+y}\,dy = e^{x} + x\,dx$

Integrate

$e^{y} = e^{x} + \frac{x^2}{2} + C$

3] $\frac{dy}{dx} + y\cot x = \cos x$

$\longrightarrow$

$P = \cot x$ $\quad Q = \cos x$

Soln is given by -

$y \cdot If = \int Q \cdot If + dx + C$

$If = e^{\int P dx}$

$= e^{\int \cos x \, dx}$

$= \cancel{e^{\log}} \sin x$

$= \sin x$

$y \sin x = \int \cos x \cdot \sin x \, dx + C$

Put $\sin x = t$

$\cos x \, dx = dt$

$= \int \frac{t}{2} dt + C$

$= \frac{t^2}{2} + C$

$y \sin x = \frac{\sin x}{2} + C$

4] $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$

→

$\frac{dy}{dx} + (\cot x)y = \operatorname{cosec} x$

$P = \cot x$, $Q = \operatorname{cosec} x$

$\therefore$ IF $= e^{\int P dx}$

$= e^{\int \cot x}$

$= e^{\log \sin x}$

$= \sin x$

Soln is given by

$y \cdot \sin x = \int \operatorname{cosec} x \cdot \sin x \, dx$

$= \int \frac{1}{\cancel{\sin x}} \cdot \cancel{\sin x} \, dx$

$= \int 1 \, dx$

$y \cdot \sin x = x + C$

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* Concept :

i] A delay differential Equation (DDE) is an equation for a function of a single variable, usually called time, in which the derivative of the function at a certain time, is given in terms of value of function at earlier time.

ii] A Stochastic-Differential Equation (SDE) is an equation in which the unknown quantity is the stochastic process and the equation involves some known stochastic process.

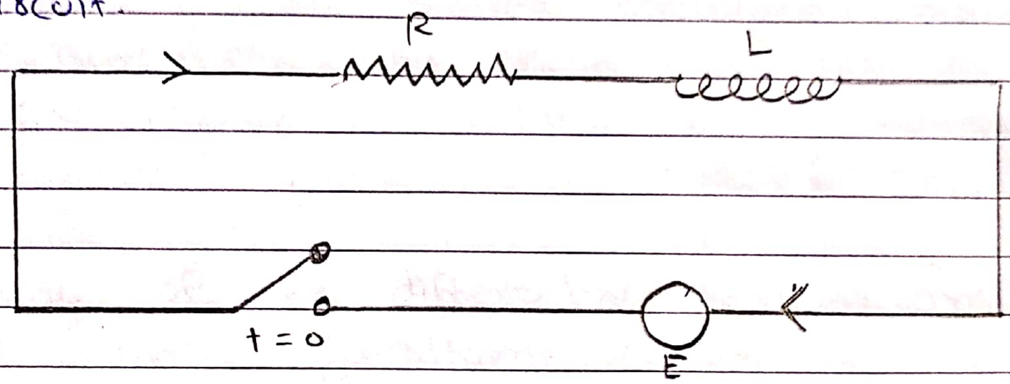
iii] An Intro-Differential Equation (IDE) is an equation that combines aspects of a differential equation and an integral equation.

iv] A Stochastic Partial Differential Equation (SPDE) is an equation that generalize SDE's to include space-time noise processes, with application in quantum theory.

v] A Differential Algebraic equation (DAE) is a differential equation comprising differential and Algebraic terms, given in Implicit form.

* Applications:

i] Exponential growth - Population.
ii] Exponential decay - Radioactive material
iii] Falling object.
iv] Newton's law of cooling.
v] R.L Circuit.



vi] Sepration of variable
vii] To control motion of Aeroplane.
viii] To calculate Heart beat.
ix] To study about space.
x] Used in Biomedical science

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* Conclusion:

Differential Equations plays major role in applications of science and engineering. It arises in wide variety of engineering applications for example: electromagnetic theory, signal processing, computational Fluid dynamics, etc. This equations can be Solved using either analytically or numerical method.

Since many of the differential equations arising in real life applications cannot be Solved analytically or we can say that their analytical Solution does not exist, for such type of problems certain numerical methods exist in the literature and are then presented for solving differential equations.