Module 3

Bayes’Theorem

1. Introduction to Bayes’ Theorem

Bayes’ Theorem is a fundamental concept in probability theory that helps us update our beliefs when new evidence is available. It is widely used in:

- Medical diagnosis
- Spam email filtering
- Fraud detection
- Machine learning
- Risk analysis

Bayes’ Theorem tells us how to calculate the probability of an event based on prior knowledge and new data.

The mathematical formula of Bayes’ Theorem is:

$P(A|B)$ = Posterior probability (Probability of A given B)

$P(B|A)$ = Likelihood

$P(A)$ = Prior probability

$P(B)$ = Total probability of evidence

2. Real-World Scenario: Disease Testing

Scenario Description

Suppose a disease affects 1% of a population.

A medical test has:

99% accuracy for detecting the disease (True Positive Rate)

95% accuracy for identifying healthy people (True Negative Rate)

We want to calculate:

If a person tests positive, what is the probability that they actually have the disease?

Step 1: Define Probabilities

Let:

D = Person has disease

T = Test result is positive

Given:

P(D) = 0.01 (1% prevalence)

P(T|D) = 0.99 (True positive rate)

P(T|not D) = 0.05 (False positive rate)

We need to find:

P(D|T)

3. Applying Bayes' Theorem

Using the formula:

First, calculate P(T), the total probability of testing positive.

This happens in two cases:

Person has disease and test is positive

Person does not have disease but test is false positive

So,

Now substitute values:

1. $P(\neg D) = 1 - 0.01 = 0.99$
2. $P(T) = (0.99 \times 0.01) + (0.05 \times 0.99)$
3. $P(T) = 0.0099 + 0.0495$
4. $P(T) = 0.0594$

Now apply Bayes formula:

5.$(D|T) = (0.99 \times 0.01) / 0.0594$

6.$P(D|T) = 0.0099 / 0.0594$

7.$P(D|T) \approx 0.1667$

So the probability is:

16.67%

4. Interpretation and Real-Life Meaning

Even though the test is 99% accurate, the probability that a person actually has the disease after testing positive is only 16.67%

Key Insights

- High test accuracy does not guarantee high certainty.
- Disease prevalence (prior probability) plays a major role.
- Bayes' Theorem helps doctors interpret test results correctly.
- Without Bayesian reasoning, we may overestimate disease risk.
- Real-Life Applications Beyond Medical Testing

Bayes' Theorem is also used in:

1. Spam Filtering

Email services like Gmail use Bayesian filtering to detect spam messages based on word probability.

2. Fraud Detection

Banks use Bayesian models to detect suspicious transactions.

3. Machine Learning

Naive Bayes classifier is widely used for classification problems.

Conclusion

This medical testing example shows that Bayes' Theorem is extremely important in real-world decision making.

Even with a highly accurate test:

The final probability depends heavily on prior conditions.

Rare events produce lower posterior probabilities.

Bayesian reasoning prevents misinterpretation of data.

Thus, Bayes' Theorem provides a mathematical framework to update beliefs using evidence and is essential in healthcare, AI, and data science.

Name: Omkar Ramdas Naik

USN:01SU24CS099

Section:B