

# Assessment of Assignment Problem using Hungarian Method

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## Abstract:

**Assignment Problem corresponds with the product distribution between demand points and supply points. Many algorithms were suggested to find the optimal result. The purpose of this study is to propose an appropriate model to explore the solution to the assignment problem. This paper focuses on the Hungarian Method. This study has conducted a detailed case study to find a feasible solution to the assignment problem. The computational outcomes suggested that the Hungarian Method provides an optimal solution and can handle any complex scenario. The findings of this study can be used as a reference for decision-makers to mitigate production-related risk and adapt to sustainable market changes.**

## Keywords:

**Assignment Problem, Hungarian Method, Genetic algorithm, Balanced and Unbalanced Problem**

## 1. INTRODUCTION

In today's world, where critical upgrades in the assignment of items from various sources to various destinations are required, the assignment of an appropriate vehicle has become a focus for organizations. (Xiang et al. 2021). Drivers waste an average of approximately 107 hours in a year due to huge traffic on the road and pay around \$2243 per year for not assigning vehicles to the appropriate parking place (Woodyard 2018). Drivers' failure to estimate the right assignment of vehicles which costs more than \$20 billion or \$97 per driver in a year only in the USA (McCoy 2017). A review which is funded by the World Bank, assessed a yearly financial expense of a queue of cars of around 47 billion LE (8 billion USD) in the "Greater Cairo Region" in 2010, bringing about a per capita cost of about 2400 LE (400 USD). Such expense is assessed to be approximately 15% of the total per capita GDP (Nakat, Herrera, and Cherkaoui 2013).

Furthermore, a bottleneck is a congested area in a manufacturing system (such as an assembly line or a computer network) that causes the system to stop or move very slowly. Inefficiencies in the bottleneck frequently result in delays and

higher production costs. Bottlenecks can disrupt the flow of the manufacturing process and significantly increase the time and cost of production. Bottlenecks are more likely to occur when a company begins the production process for a new product. This is due to the possibility of process issues that the company must identify and resolve; this condition necessitates additional examination and fine-tuning. Controlling the production process, anticipating potential bottlenecks, and devising effective solutions should be the primary concern of any company. One of the most common causes of bottlenecks is the failure to assign the appropriate machine to the appropriate location. This consequence signifies assignment problems that create an obstacle to provide many commodities in the right place on time. To put

light on this unbearable situation this study is analyzing a comparison of assignment problems while considering the minimization of the total cost of supplying and fulfilling the demand.

The main focus of this study is to create an optimal solution to an assignment problem and provide an illustration of an appropriate approach. This paper is organized as: section 2 summarizes the literature review. Section 3 explains the proposed methodology whereas section 4 describes the conclusion along with recommendations for further research.

## 2. LITERATURE SURVEY

Many practitioners and researchers used the Hungarian method in the past to solve assignment problems (Kuhn 1955); (Chopra et al. 2017). The existing Hungarian method for solving unbalanced assignment problems is based on the assumption that some jobs should be assigned to dummy or pseudo machines, but those jobs are left unexecuted by the dummy machines in the Hungarian method. However, it is sometimes impractical in real-world situations. (Rabbani, Khan, and Quddoos 2019) proposed a modified 'Hungarian method' for solving unbalanced assignment problems without leaving any job unexecuted. The method works by finding rows with just one zero and crossing out the other zero for each respective column. The stepwise algorithm for the

proposed methodology is developed and programmed in Java SE 11. Afterward, the proposed method is compared with three other methods, and it is shown that it yields the best results. (Kumar 2006) proposed a method to solve the unbalanced assignment problem that is capable of solving the unbalanced problem by assigning all jobs to the machine in an optimal way. This method divides the unbalanced matrix into several balanced submatrices, which are subsequently solved using the Hungarian method. This method has a drawback in that it often fails to provide a minimal total cost. Instead of using polynomial complexity, (Iampang, Boonjing, and Chanvarasuth 2010) introduced a new cost and space-efficient solution for unbalanced assignment problems. This approach uses linear space complexity. The proposed method offers a lower optimal cost than (Kumar 2006) according to an experiment with 100,000 cost matrices. For the unbalanced assignment problem, (Wang et al. 2021) developed a graph-based twin cost matrices method with an improved ant colony optimization algorithm. It can solve assignment problems uniformly, whether they are balanced or unbalanced, constrained or unconstrained. The twin cost matrices with independent pheromones correlate AP (Assignment problem) and TSP (Travelling Salesman Problem). The mutation method makes it easier to get to the optimal solution by reducing the likelihood that the ant colony may fall in the local optimum. According to experiments the method produces superior outcomes when compared to other existing methods.

A study on hospital layout design remodeling was undertaken as a Quadratic Assignment Problem (QAP) with geodesic distances, which is a configurational problem that results in inefficient transportation operations for patients, medical personnel, and material logistics (Cubukcuoglu et al. 2021). The internal transportation processes between interrelated facilities are minimized by renovating existing hospitals using Computer-Aided Design (CAD). Homayouni and Fontes (2021) addressed an extension of the flexible job shop scheduling problem by considering that jobs need to be moved around the shop floor by a set of vehicles which involves assigning each production operation to one of the alternative machines. In this study, mixed integer linear programming model was utilized for the problem and showed efficiency at solving small-sized instances to optimality. A furthermore study has been conducted on the neural graph-matching network. A QAP network directly learns with the affinity matrix (equivalently the association graph) whereby the matching problem is translated into a constrained vertex classification task is presented in another study (Wang, Yan, and Yang 2022). The embedding network approach is used to achieve and even exceed state-of-the-art graph matching and QAP solvers with a significantly reduced time cost. For the well-known QAP, (Dokeroglu, Sevinc, and Cosar (2019) introduced hybrid Artificial Bee Colony (ABC) optimization techniques. The robust tabu search approach is used to model bee exploration and

exploitation activities. And, from the QAPLIB library, 125 of 134 benchmark issue instances are solved optimally, with a 0.27% variation indicated for 9 big problem cases that could not be handled optimally.

Xiang and Liu (2021) integrated berth allocation and quay crane assignment problem considered uncertainties in the late arrival of ships and inflation of container quantity which is based on historical data. A robust model has been formulated with a weighted max penalty function. With the help of the decomposition method, the problem is solved by containing a deterministic master problem and a stochastic sub-problem. The applied method showed that it can handle the uncertainties more than the robust, deterministic which has the terms of total expected cost, total vessel delays, and utilization rates of the berth and quay crane and making it the most attractive one. Another study has been started considering an airport gate assignment problem that assigns a set of aircraft to a set of gates (Karsu, Azizoğlu, and Alanlı 2021). This study determined to make aircraft gate assignments that would work to minimize the total

walking distance traveled by the passengers. With the mixed-integer nonlinear programming model, it had been linearized and developed with bound algorithm, beam search, and filtered beam search algorithms. In this recent study, the facility layout problem (FLP) dealt with optimal assignments which helped to minimize the transportation cost (Hameed et al. 2021). It can be added to the hospital facility layout problem that targeted comprehensive clinics, laboratories, and radiology units. The proposed method of hybrid algorithm gathered the DDE and tabu search (TS) which was performed with implemented benchmark instances from the QAPLIB website. 42 optimal and 52 instances were found through the proposed method.

Ngo et al. (2021) created a model for the assignment problem of scheduling classes of FPT University lecturers in Vietnam. Using a compromise programming approach, the model is transformed into a single objective model. Afterward, a genetic algorithm is provided for the model which can generate a calendar incorporating lecturer schedules while ensuring associated conditions. Based on differential evolution and self-adaptive multi-task particle swarm optimization (SaMTPSO), an effective Evolutionary Multi-task Optimization (EMTO) solver is designed in a study (Zheng et al. 2021). After that, the algorithm is used to resolve the weapon-target assignment problem on two test suites (MTO and WTA-MTO benchmark), and it is compared to other relevant algorithms to demonstrate the algorithm's viability in resolving WTA issues. In another paper, a study is undertaken on a storage assignment problem caused by a shortage of volume in container terminal yards, and a storage-sharing approach between container terminals and dry ports is proposed as a solution (Hu et al. 2021). A multiple-objective mixed

integer programming model is developed, with the goals of lowering travel distance, balancing, and maximizing shared storage, and the problem is solved using the Non-dominated Sorting Algorithm II (NSGA-II).

## METHODOLOGY

Assignment problem is a special case of transportation problem where each supply point should be assigned to a demand point and each demand point should be made. It is used in determining which employee and machine should be assigned to which job. To solve the assignment problem, the Hungarian method can be utilized.

### Assumptions:

- The number of assignees and the number of tasks are the same.
- Each assignee is to be assigned to exactly one task.
- Each task is to be presented by exactly one assignee.
- There is a cost  $C_{ij}$  associated with assignee  $i$  ( $i=1,2,3,\dots,n$ ), performing task  $j$  ( $j=1,2,3,\dots,n$ ).
- The objective is to determine how all  $n$  assignments should be made to minimize the total cost.

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

## Balanced Assignment Problem

The balanced assignment problem is one in which the number of facilities and the number of jobs is equal. The objective is to assign jobs to machines for the least amount of money possible if there are "n" jobs to complete on "m" machines (i.e., one job to one machine) (or maximum profit). based on the idea that each machine is capable of carrying out every work, albeit with varying degrees of efficiency. Also, the following assumption must be considered:

- $m = n$ , which means jobs and machines are equal in numbers.
- No job can be given to more than one machine.
- There is a cost associated with each machine and job.

The balanced assignment problem is solved using Hungarian method in the following steps given below. Table 1 illustrates

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

the initial matrix table of a balanced assignment problem.

Table 1. Initial matrix table of a balanced assignment problem

Since the matrix of table 1 is a square matrix, the problem is balanced. Table 2 to table 5 present the steps required to determine the appropriate job assignment to the machine.

1. Subtracting the minimum element from all the elements in the respective rows, the new table will be:

Table 2. Matrix table after step 1

Table 2 represents the matrix after completing the 1<sup>st</sup> step.

2. Then subtract the minimum value from all values in the respective column.

Table 3. Matrix table after step 2

Table 3 represents the matrix after completing the 2nd step.

3. Draw a minimum number of horizontal and vertical values to cover all zeros.

- a. If  $N = n$ ,  $n$ =order of matrix

Then an optimal solution can be made

- b. If  $N < n$ , then go to the next step

Table 4. Matrix table after step 3

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

Machine Job	1	2	3	4	5
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	0	0	0	0	0

4. Determine the smallest uncovered element  $x$ . Write uncovered value = uncovered value -  $x$

a. Intersection value = intersection value +  $x$

b. Line values (Other values) as same

The number of lines drawn to cover zero is  $N = 4 < \text{orders of matrix} = 5$ , so this is not the optimal result. Again 5. Go to step 3.

Table 5 shows that no lines are drawn to cover all zeroes = 5 and order of matrix = 5. Therefore, we can form an assignment.

To recapitulate, total cost =  $8+12+4+6+12 = 42$

### Unbalanced Assignment Problem

The current Hungarian method for resolving unbalanced assignment issues is predicated on the idea that some tasks should be delegated to dummy or pseudo-machines. The tasks delegated to the dummy machines are not carried out. One might be interested in running all the jobs on actual machines in real-world scenarios. In the unbalanced assignment issue, a special subclass of the transportation problem, the goal is to find the best distribution of the number of jobs ( $n$ ) to the number of machines ( $m$ ), where  $n \neq m$ . The presumptions listed below are taken into account:

- There are more jobs than machines, or  $n > m$ .
- No job can be given to more than one machine
- There is a cost associated with each machine and job.

The unbalanced assignment problem is solved using the Hungarian method in the following steps given below.

Table 7. Initial matrix table of an Unbalanced Assignment Problem

Machine Job	1	2	3	4	5
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13

Since the matrix of table 7 is not a square matrix, the problem is unbalanced. A dummy job (job 5) has been added with corresponding entities zero to make it a square matrix. Table 9 to table 11 present the steps required to determine the appropriate job assignment to the machine.

Machine Job	1	2	3	4	5
1	5	0	5	10	8
2	0	6	12	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Table 8. The new matrix after adding a dummy row

Table 8 is the new modified matrix which is a balanced

Job	Machine	Cost
1	2	8
2	5	12
3	3	4
4	1	6
5	4	12

matrix after adding the dummy row to table 7.

1. Subtracting the minimum element from all the elements in the respective rows, the new table will be:

Table 9. Matrix table after step 1

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	0	0	0	0	0

Table 9 represents the matrix after completing step 1.

2. Then subtract the minimum value from all values in the respective column.

Table 10. Matrix table after step 2

Table 10 is representing the matrix after completing step 2.

Table 11. Matrix table after 1st scanning

Table 11 is the matrix after 1<sup>st</sup> scanning. In this matrix, no of lines drawn to cover all zeroes = 5 and order of matrix = 5. So, we can form an assignment from this table.

Table 12. Final table matrix

Table 12 is the final matrix table after completing all steps.

Table 13. Result of the Unbalanced Assignment Problem

Table 13 represents which machine is assigned for which job.

**Therefore, the total cost = 8+9+4+6+0 =27**

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	0	0	0	0	0

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	0	0	0	0	0

Machine Job	1	2	3	4	5
1	5	0	8	10	11
2	<input type="checkbox"/> 0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	0	0	0	0	0

## 1. RESULT

### Hybrid Algorithm (Hungarian+Genetic)

1. Initialize the parameters:
  - Set the population size, mutation rate, maximum number of generations, and no-improvement limit.
  - Generate a random cost matrix of appropriate size.
2. Perform preprocessing on the cost matrix:
  - Adjust the size of the matrix as needed.
  - Create a random cost matrix using NumPy.
  - Copy the created matrix.
3. Implement the steps for solving the assignment problem using a genetic algorithm:
  - Define helper functions for creating an individual, calculating fitness, performing crossover, and applying mutation.
  - Initialize the population with the provided job list and randomly created individuals.
  - Iterate through the generations while checking the termination criteria.
  - Evaluate fitness values and select the top individuals for crossover and mutation.
  - Perform crossover and mutation operations on the selected individuals.
  - Update the population with the newly generated individuals.
  - Track the best fitness and the number of generations without improvement.
  - Output the optimal solution and related information.
4. Visualize the optimal assignment using a heatmap.
5. Optionally, include additional functionalities such as main function integration and linear programming model optimization.
6. Ensure all necessary libraries and dependencies are installed for successful execution.
7. Test the algorithm with different cost matrices and parameters to evaluate its performance and effectiveness.
8. Optimize the algorithm as needed to enhance its efficiency and solution quality.

## CONCLUSION

This study has explored the intricacies of task allocation in balanced and unbalanced assignment problems. While the genetic algorithm proves effective for small cost matrices, its scalability for larger datasets presents a challenge. Future research should enhance its scalability by modifying mutation and crossover functions.

The genetic algorithm excels in balanced assignments but may encounter issues when jobs outnumber agents. Future work should focus on tailored solutions for unbalanced assignments and strategies for handling dummy elements.

Machine learning integration and hybrid approaches, combining the genetic algorithm with other optimization methods, offer potential enhancements for task allocation efficiency and accuracy.

In summary, future research must address scalability, unbalanced scenarios, dummy element handling, and innovative methodologies to maximize the genetic algorithm's potential for task allocation.

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