1. Substitution Method Proof

To show that the solution to the $T(n) = 5T(n\ /\ 3) + \Theta\ (n^2)$ is $O(n^2)$

To show that $T(n)\ \in\ O(n^2)$ means to show that there is a constant c such that $T(n)\ \leq\ cn^2$ for $n\ \geq n_0$

Base case: $T(n_0)\ \leq\ c$ for some n_0
Inductive case: $T(n/3)\ \leq\ c(n/3)^2$

If 'a' is the upper-bound constant associated with the $\Theta\ (n^2)$, then

$$
\begin{aligned}
T(n) &\leq 5T(n/3)\ + an^2 \\
&\leq 5c(n/3)^2 + an^2 \\
&\leq 5cn^2/9 + an^2 \\
&\leq (5\tfrac{c}{9} + a)n^2
\end{aligned}
$$

As long as $a \leq \frac{4c}{9}$, the above inequality becomes

$T(n)\quad \leq cn^2$ ... (provided $a \leq \frac{4c}{9}$ for sufficiently large n)

(For base case, $T(1) \leq c$)