4. Master Method

Q. Use the master method to give a solution to each of the following recurrences. Briefly explain your application of the master method (e.g., which case applies to each problem). ($\ominus$ represents big theta)

a) $\boldsymbol{T(n) = 2T(n/4) + \ominus(\sqrt{n})}$

Cost of the root node = $\sqrt{n} = n^{0.5}$

Total cost of the leaves = $n^{\log_4^2} = n^{0.5}$

Thus, cost of the root node and the total cost of the leaves are equal. This is the second case of Master Method. $\boldsymbol{T(n) = \ominus(\sqrt{n}\, log\, n)}$

b) $\boldsymbol{T(n) = 9T(n/3) + \ominus(n)}$

Cost of the root node = n

Total cost of the leaves = $n^{\log_3^9} = n^2$

Thus, the total cost of the leaves dominates the cost of the root node. This is the first case of Master Method. $\boldsymbol{T(n) = \ominus(n^2)}$

c) $\boldsymbol{T(n) = 5T(n/2) + \ominus(n^2)}$

Cost of the root node = n^2

Total cost of the leaves = $n^{\log_2^5} = n^{2.32}$

Thus, the total cost of the leaves dominates the cost of the root node. This is the first case of Master Method. $\boldsymbol{T(n) = \ominus(n^{2.32})}$

d) $\boldsymbol{T(n) = 2T(n/2) + \ominus(n\sqrt{n})}$

Cost of the root node = $n\sqrt{n} = n^{1.5}$

Total cost of the leaves = $n^{\log_2^2} = n$

Thus, cost of the root node dominates the total cost of the leaves. This is the third case of Master Method. $\boldsymbol{T(n) = \ominus(n^{1.5})}$