3. Recursion Tree Problems

Q. For each of the following recurrence relations, give a short formula for the cost of the root node, the total cost of the leaves, and the total cost of all the internal nodes.

(a) $\boldsymbol{T(n) = 4T(n/3) + O(n\sqrt{n})}$

Cost of the root node: $O(n^{1.5})$

Total cost of the leaves: $4^{\log_3 n} O(1)$

Total cost of all the internal nodes: $\sum_{i=1}^{log_3 n - 1} 4^i O(n^{1.5} / 3^i)$

(b) $\boldsymbol{T(n) = 3T(n/3) + O(n)}$

Cost of the root node: $O(n)$

Total cost of the leaves: $3^{\log_3 n} O(1) = n^{\log_3 3} O(1) = n\, O(1)$

Total cost of all the internal nodes: $\sum_{i=1}^{log_3 n - 1} 3^i O(n / 3^i)$