2. Asymptotic Bound Proofs

a) $2n^2 + 5n\,log_2\,n \;\in O(n^2)$

We need to show that:

There are positive constants c and n_0 such that
$0 \leq 2n^2 + 5n\,log_2\,n \;\leq cn^2$ for all $n \geq n_0$

Now, if we get a value for c such that the above equation holds true for all the n

Let's choose c = 3.

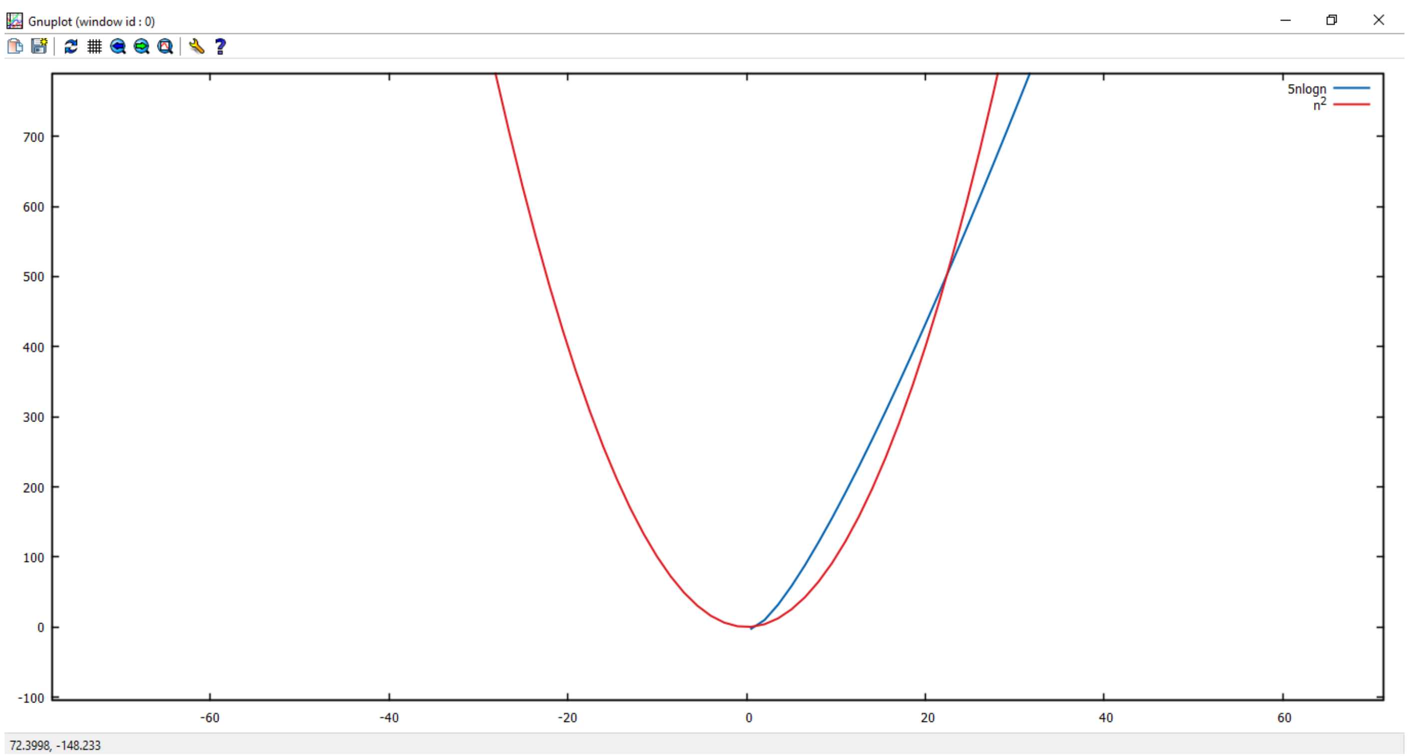
$$2n^2 + 5n\,log_2\,n \;\leq 3n^2$$

$$2n^2 + 5n\,log_2\,n \;\leq 2n^2 + \;n^2$$

This inequality holds as long as:

$2n^2 \leq 2n^2$ .... true for any n

$5n\,log_2\,n \;\leq n^2$ .... true as long as $n \geq 22.45$



Thus, for c = 3 and n_0 = 22.45; the above inequality holds true. That's we wanted to prove.

b) If $f(n) \in \omega(g(n))$ then $f(n) \in \Omega(g(n))$

We need to show that:

There are positive constants c and n_0 such that

$$0 \le cg(n) \le f(n) \text{ for all } n \ge n_0$$

But, we're given that:

For any positive c, there exists an n_0 such that

$$0 \le cg(n) < f(n) \text{ for all } n \ge n_0$$

ω notation is like a > b and Ω notation is like a ≥ b

e.g. a = 10 and b = 5, then according to ω, a > b. This also fulfils the condition of Ω (a ≥ b).

So, for every a and b, this condition holds true.

So, we can say that

$$\text{If } f(n) \in \omega(g(n)) \text{ then } f(n) \in \Omega(g(n))$$