

1. For this problem, you'll be comparing functions that describe the running times of two different algorithms (well, at least we'll be pretending they do). For each question, you'll need to do two things. Report which of the two algorithms is asymptotically faster (i.e., which function has a slower rate of growth). Also, report the trade-off point between the two functions, the smallest value n_0 such that, for all $n \geq n_0$, the faster-growing function has a larger value at n than the slower-growing function. Give this answer rounded to one fractional digit of precision.

a) $f_1(n) = 0.5n^2 - 10n$
 $g_1(n) = 3n$

Here, at the tradeoff point, the values of these two functions will be equal as they intersect over there. So, making $f_1(n)$ and $g_1(n)$ equal, we get

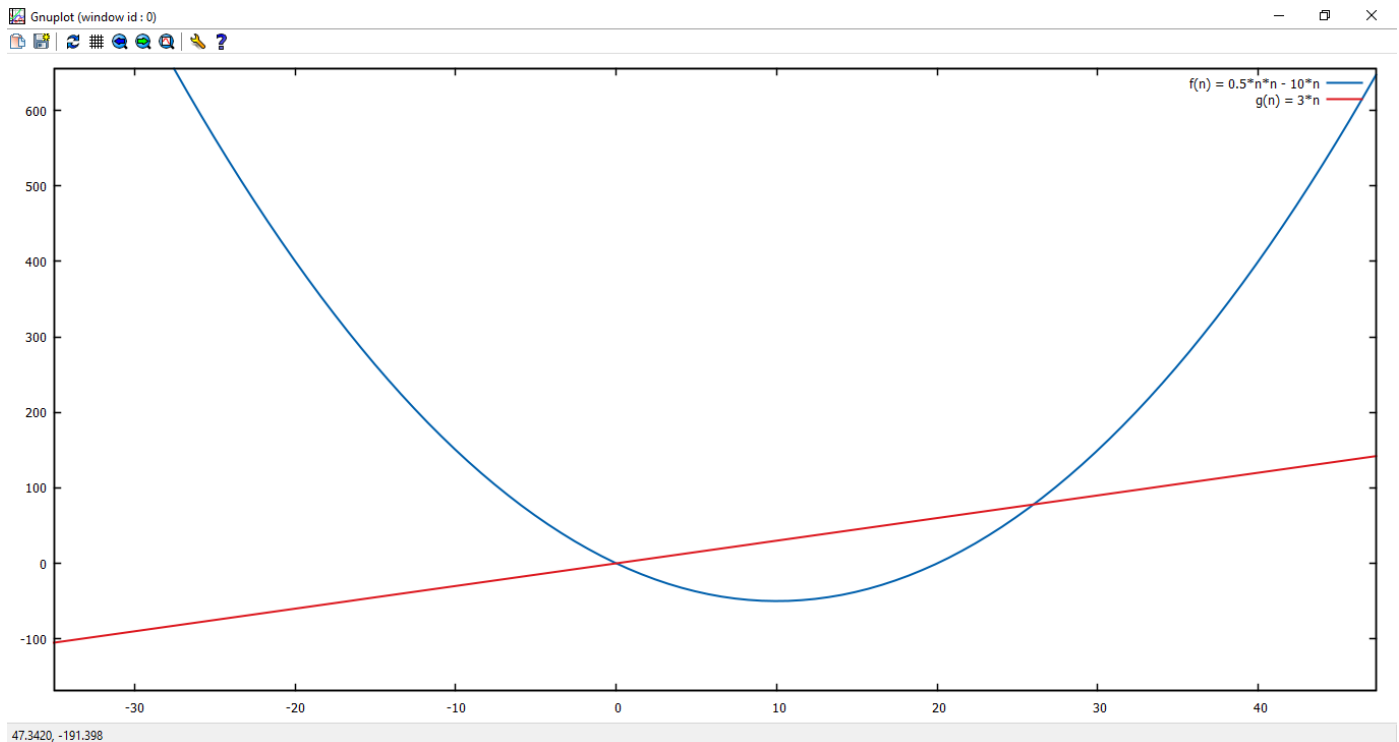
$$\begin{array}{rcl} 0.5n^2 - 10n & = & 3n \\ 0.5n^2 & = & 13n \\ 0.5n & = & 13 \\ n & = & 26 \end{array}$$

Thus, $n_0 = 26$ (trade-off point)

For every $n \geq n_0$, $f_1(n)$ grows faster than $g_1(n)$.

Therefore, $g_1(n)$ is asymptotically faster than $f_1(n)$ (i.e. it has a slower rate of growth)

Also, if we plot the functions, then we can see that after $n=26$, there is no 'n' where $g_1(n)$ is faster growing than $f_1(n)$.



b) $f_2(n) = 3n \log_2 n$
 $g_2(n) = 2n^{1.5}$

Here, at the trade-off point n_0 , we can say that these functions have equal values.

$$3n \log_2 n = 2n^{1.5}$$

$$3 \log_2 n = 2n^{0.5}$$

$$n = 98.8 \text{ (rounded to one fractional digit)}$$

Thus, $n_0 = 98.8$ (trade-off point)

For every $n \geq n_0$, $g_1(n)$ grows faster than $f_1(n)$.

Therefore, $f_1(n)$ is **asymptotically faster** than $g_1(n)$ (i.e. it has a **slower rate of growth**)

Also, if we plot the functions, then we can see that after $n=98.8$, there is no 'n' where $f_2(n)$ is faster growing than $g_2(n)$.

