# Advanced Machine Learning (CS60073)

# Assignment 1: 80 Marks

## Instructions:

- Submit a single jupyter notebook for the entire assignment
- Annotate your solution using markdown before every code cell

Before starting, ensure you have the required tools:

- Install Miniforge (https://github.com/conda-forge/miniforge)
- Install Jupyter Notebook:

```
pip install notebook
```

- Install required packages:

```
pip install numpy scipy matplotlib seaborn pandas
```

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## Part A: Bayesian Parameter Estimation (Conjugate Priors) — *40 marks*

### A.1 Bernoulli with Beta prior *(10 marks)*

- Generate binary data ($y \in \{0,1\}$) with a known success probability.
- Perform Bayesian updating with a **Beta** prior.
- Plot prior, likelihood, and posterior for different priors (informative, uninformative).
- Compute and compare **MLE**, **MAP**, and **posterior mean** estimates.

### A.2 Multinoulli with Dirichlet prior *(10 marks)*

- Simulate categorical data ($K = 3$ classes).
- Update posterior using Dirichlet prior.
- Visualize prior and posterior on a 2D simplex.
- Compute posterior predictive probabilities.

### A.3 Univariate Gaussian *(10 marks)*

- Case 1: Known variance, unknown mean (Gaussian prior).
- Case 2: Unknown mean & variance (Normal-Inverse-Gamma prior).
- Derive analytical posterior and plot prior vs posterior.

### A.4 Multivariate Gaussian *(10 marks)*

- Generate 2D Gaussian data with known mean & covariance.
- Use Normal-Inverse-Wishart prior to update posterior.

- Visualize posterior samples (e.g., covariance ellipses).

## Part B: Probabilistic Supervised Learning — *40 marks*

### B.1 Probabilistic Linear Regression *(15 marks)*

- Generate data from $y = w_0 + w_1 x + \varepsilon,\ \varepsilon \sim \mathcal{N}(0, \sigma^2)$
- Place Gaussian prior over weights and compute posterior analytically.
- Plot posterior over weights as data size increases.
- Compute **posterior predictive distribution** and visualize uncertainty.

### B.2 Binary Logistic Regression (Laplace Approximation) *(15 marks)*

- Generate binary classification data ($y \in \{0,1\}$).
- Assume Gaussian prior over weights.
- Use Laplace approximation to approximate posterior (find MAP, Hessian).
- Sample from approximate posterior and plot decision boundaries.
- Compare with MLE predictions.

### B.3 Multiclass Logistic Regression *(10 marks)*

- Generate synthetic 3-class data.
- Implement Bayesian inference with Gaussian prior (Laplace approximation).
- Compute posterior predictive probabilities.
- Visualize classification boundaries with uncertainty.

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