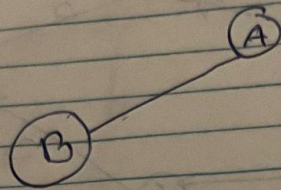


Lab 11

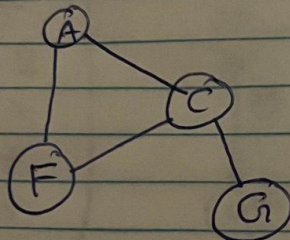
Student ID	Name
614760	Omkar Nath Chaudhary
614732	Sushil Subedi
614604	Rahul Niraula
614600	Shrawan Adhikari

Q1.

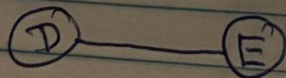
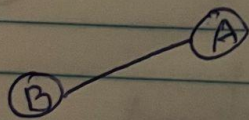
a) Let $U = \{A, B\}$, Draw $G(U)$



b) Let $W = \{A, C, G, F\}$, draw $G(W)$

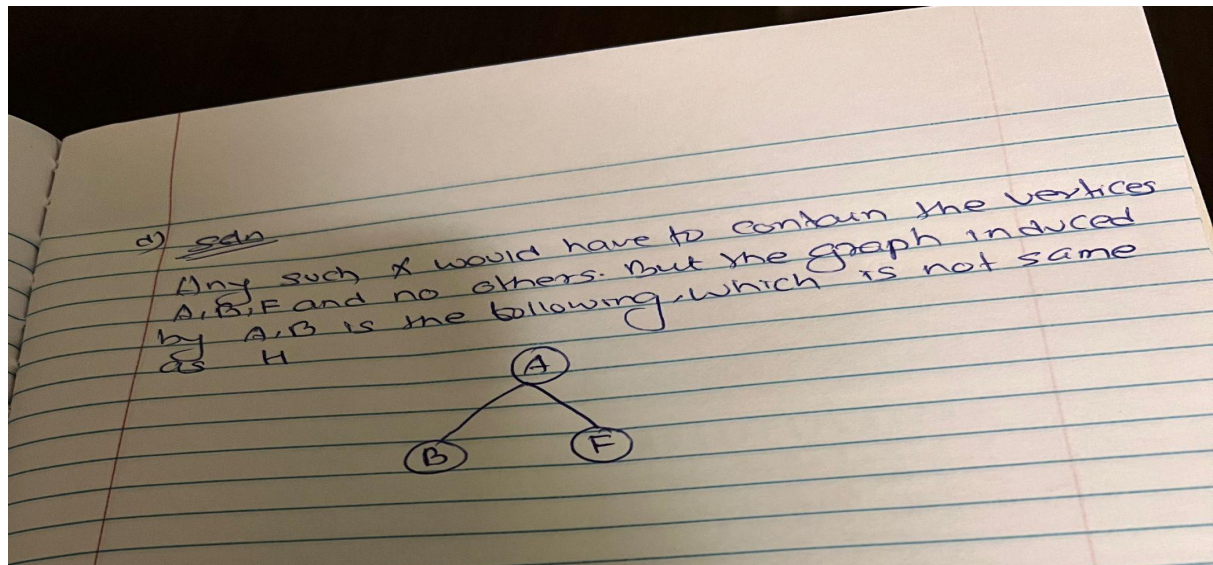


c) Let $Y = \{A, B, D, E\}$, Draw $G(Y)$



d) Consider

1.d solution :



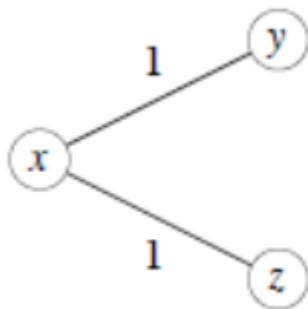
Q2.

Show that a graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique light edge crossing the cut. Show that the converse is not true by giving a counterexample.

Solution:

Suppose that for every cut of G , there is a unique light edge crossing the cut. Let us consider two minimum spanning tree, T and T' of G . We will show that every edge of T is also in T' which means that T and T' are the same tree and hence there is a unique minimum spanning tree.

Consider any edge (u,v) subset of T . If we remove (u,v) from T , then T becomes disconnected, resulting in a cut $(S, V-S)$. The edge (u,v) is a light edge crossing the cut $(S, V-S)$. Now consider the edge (x,y) subset of T' that crosses $(S, V-S)$. It is a light crossing edge on this cut too. Since the light edge crossing $(S, V-S)$ is unique, the edges (u,v) and (x,y) are the same edge. Thus, (u,v) subset of T' . Since we chose (u,v) arbitrarily, every edge in T is also in T' .



The graph is its own minimum spanning tree and so the minimum spanning tree is unique. Consider the cut($\{x\}, \{y,z\}$). Both of the edges (x,y) and (x,z) are light edges crossing the cut and they are both light edges.

Q3. The following graph has a Hamiltonian cycle. Find it.

Solution: This topic is still not taught So skipping for further lab.

Q4.

Consider the problem of computing a *maximum* spanning tree, namely the spanning tree that maximizes the sum of edge costs. Do Prim and Kruskal's algorithm work for this problem (assuming of course that we choose the crossing edge with maximum cost)?

Solution:

Yes, all the proofs are the same. Just flip the inequalities. Another way of seeing that there's no difference in the problems is to just flip the sign on all the edges. Finding the maximum spanning tree is the same problem as finding the minimum spanning tree in a graph which had costs negated (relative to the originals). As we saw from the previous question, having negative costs doesn't change the correctness of the MST algorithms.