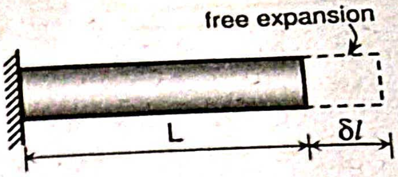
Temperature Stresses and Strains :

When a body is heated, it expands and on cooling, it contracts. If the body is allowed to expand o contract freely with the rise or fall of the temperature, then no stresses will develop in the body. But, if thi deformation of the body due to change in temperature is prevented, then some stresses will develop in the body, such stresses are called **temperature stresses** or **thermal stresses** and the corresponding strains are called temperature strains or thermal strains.

Free Deformation (Free change in length δl) :

Free expansion or contraction of a body due to rise or fall in temperature is called free deformation This deformation is proportional to length of body, change in temperature and coefficient of linear expansion of body material.

Consider a bar of length 'L', heated by 't' °C as shown in fig., then due to temperature rise the increase in length of bar (free expansion of bar) is



$$\boxed{\delta l = \alpha t L}$$

where α = Coefficient of linear expansion, /°C

t = Change in temperature, °C

L = Original length of bar, mm

In this case, no stress will develop in the body, as its free expansion is not prevented.

Prevented Deformation (Prevented change in length δl_p) :

When the free deformation due to change in temperature is prevented by fixing the ends by rig supports, then temperature stresses and strains will develop in the body. If free expansion is prevented, th compressive stresses will develop in the body. If free contraction is prevented, then tensile stresses w develop in the body.

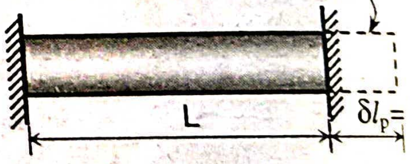
Temperature Stresses and Strains when :

1) Change in length is totally prevented and 2) Change in length is partially prevented

Case I : If Change in length is totally prevented : Consider a bar of length 'L', heated by 't' °C, and its f expansion is totally prevented by another rigid support. Then,

Prevented change in length = Free change in length

i.e. $\delta l_p = \delta l = \alpha t L$



$\therefore$ Temperature Strain, $e = \dfrac{\delta l_p}{L} = \dfrac{\alpha t L}{L}$

$$\therefore \quad \boxed{e = \alpha t}$$

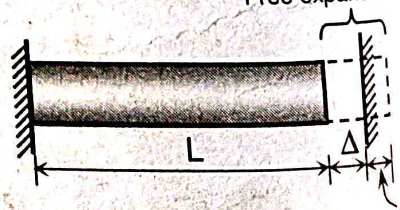
and Temperature Stress, $\sigma = E e$ $\quad \left[\because E = \dfrac{\sigma}{e}\right]$

$$\therefore \quad \boxed{\sigma = \alpha t E}$$

Case II : If change in length is partially prevented : Consider the same bar of length 'L', heated by 't' and if the fixed support yields back (slips back) by some distance 'Δ'. Then,

Prevented change in length = Free change in length – yielding distance

i.e. $\delta l_p = \delta l - \Delta = \alpha t L - \Delta$



$\therefore$ Temperature Strain, $e = \dfrac{\delta l_p}{L}$

$$\therefore \quad \boxed{e = \frac{\alpha t L - \Delta}{L}}$$

and Temperature Stress, $\sigma = E e$ $\quad [\because E = \sigma/e]$

$$\therefore \quad \boxed{\sigma = \left(\frac{\alpha t L - \Delta}{L}\right) E}$$

Temperature Stresses and Strains :

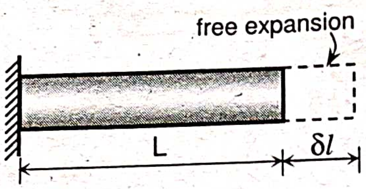
When a body is heated, it expands and on cooling, it contracts. If the body is allowed to expand or contract freely with the rise or fall of the temperature, then no stresses will develop in the body. But, if this deformation of the body due to change in temperature is prevented, then some stresses will develop in the body, such stresses are called **temperature stresses** or **thermal stresses** and the corresponding strains are called temperature strains or thermal strains.

Free Deformation (Free change in length δl) :

Free expansion or contraction of a body due to rise or fall in temperature is called free deformation. This deformation is proportional to length of body, change in temperature and coefficient of linear expansion of body material.

Consider a bar of length 'L', heated by 't' ^{o}C as shown in fig., then due to temperature rise the increase in length of bar (free expansion of bar) is

$$\boxed{\delta l = \alpha t L}$$



where α = Coefficient of linear expansion, /^{o}C
t = Change in temperature, ^{o}C
L = Original length of bar, mm

In this case, no stress will develop in the body, as its free expansion is not prevented.

σ/e = E
L×E/e

Prevented Deformation (Prevented change in length δl_p) :

When the free deformation due to change in temperature is prevented by fixing the ends by rigid supports, then temperature stresses and strains will develop in the body. If free expansion is prevented, then compressive stresses will develop in the body. If free contraction is prevented, then tensile stresses will develop in the body.

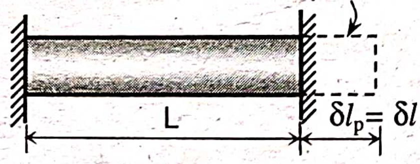
Temperature Stresses and Strains when :

1) Change in length is totally prevented and 2) Change in length is partially prevented

Case I : If Change in length is totally prevented : Consider a bar of length 'L', heated by 't' ^{o}C, and its free expansion is totally prevented by another rigid support. Then,

Prevented change in length = Free change in length

i.e. $$\delta l_p = \delta l = \alpha t L$$



$$\therefore \quad \text{Temperature Strain, } e = \frac{\delta l_p}{L} = \frac{\alpha t L}{L}$$

$$\therefore \quad \boxed{e = \alpha t}$$

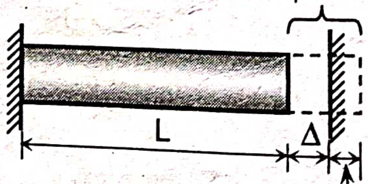
$$\text{and Temperature Stress, } \sigma = E e \qquad \left[\because E = \frac{\sigma}{e}\right]$$

$$\therefore \quad \boxed{\sigma = \alpha t E}$$

Case II : If change in length is partially prevented : Consider the same bar of length 'L', heated by 't' ^{o}C, and if the fixed support yields back (slips back) by some distance 'Δ'. Then,

Prevented change in length = Free change in length – yielding distance

i.e. $$\delta l_p = \delta l - \Delta = \alpha t L - \Delta$$



$$\therefore \quad \text{Temperature Strain, } e = \frac{\delta l_p}{L}$$

$$\therefore \quad \boxed{e = \frac{\alpha t L - \Delta}{L}}$$

$$\text{and Temperature Stress, } \sigma = E e \qquad [\because E = \sigma/e]$$

$$\therefore \quad \boxed{\sigma = \left(\frac{\alpha t L - \Delta}{L}\right) E}$$

2) Temperature Stresses in Composite Bars - In Parallel :

Consider two bars of different materials, one of copper and another of steel, connected in parallel as shown in fig. 1.

Let α_c and α_s = Coefficient of linear expansion of copper and steel respectively.

We know that $\alpha_c > \alpha_s$. If the temperature of the bars is increased by t ^{O}C, then both bars will expand. Since $\alpha_c > \alpha_s$, therefore expansion of copper will be more than expansion of steel (i.e. $\alpha_c\, t\, L_c > \alpha_s\, t\, L_s$) as shown in fig. 2.

But, if the bars are connected to each other firmly, then copper will apply tensile force **P** on steel and steel will apply compressive force **P** on copper and the total elongation of both bars will be same as shown in fig. 3.

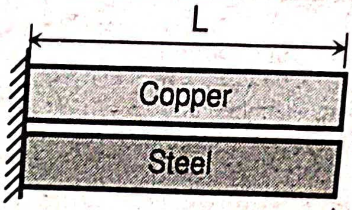


Fig. 1

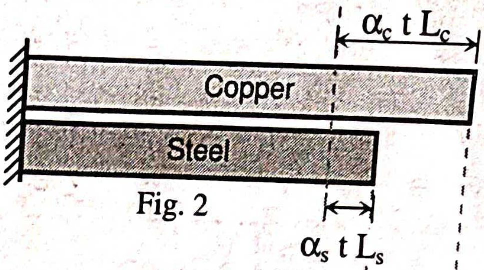


Fig. 2

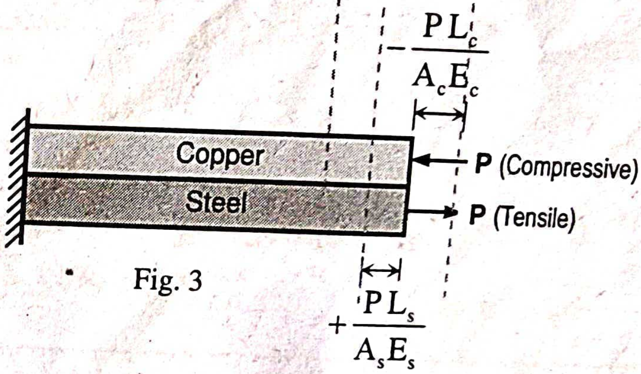


Fig. 3

$\therefore$ **Total elongation of copper = Total elongation of steel**

But, Total elongation of copper = Elongation of copper due to temp. – Contraction due to compressive force P

$$= \alpha_c\, t L_c - \frac{P L_c}{A_c E_c} \qquad \text{..........(1)}$$

And Total elongation of steel = Elongation of steel due to temp. + Elongation due to tensile force P

$$= \alpha_s\, t L_s + \frac{P L_s}{A_s E_s} \qquad \text{..........(2)}$$

Therefore,

$$\alpha_c\, t L_c - \frac{P L_c}{A_c E_c} = \alpha_s\, t L_s + \frac{P L_s}{A_s E_s}$$

or

$$\boxed{\alpha_c\, t L_c - \alpha_s\, t L_s = \frac{P L_c}{A_c E_c} + \frac{P L_s}{A_s E_s}}$$

or

$$t(\alpha_c - \alpha_s) = \frac{P}{A_c E_c} + \frac{P}{A_s E_s} \qquad [\because L_c = L_s]$$

or

$$t(\alpha_c - \alpha_s) = \frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} \qquad \left[\because \sigma = \frac{P}{A}\right]$$

Type 6.3

Problems on Temperature Stresses in Composite Bars – In Parallel

1. A steel tube of 30 mm external diameter and 20 mm internal diameter encloses a copper rod of 14 mm diameter to which it is rigidly joined at each end. Find the stresses in steel tube and copper rod, if the temperature of the composite bar is increased by 180°C.
Take : $E_s = 2.1 \times 10^5$ N/mm², $E_c = 1.1 \times 10^5$ N/mm².
$\alpha_s = 1.1 \times 10^{-5}$/°C, $\alpha_c = 1.8 \times 10^{-5}$/°C. **B[M 09] , A[M 10]**

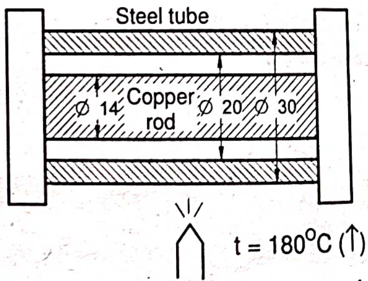
Solution : Given : $D_s = 30$ mm, $d_s = 20$ mm, $d_c = 14$ mm, $t = 180°C$

$$\therefore A_s = \frac{\pi}{4}(D_s^2 - d_s^2) = \frac{\pi}{4}(30^2 - 20^2) = 125\pi \text{ mm}^2$$

$$\therefore A_c = \frac{\pi}{4} d_c^2 = \frac{\pi}{4} 14^2 = 49\pi \text{ mm}^2$$

$E_s = 2.1 \times 10^5$ N/mm², $E_c = 1.1 \times 10^5$ N/mm²

$\alpha_s = 1.1 \times 10^{-5}$/°C, $\alpha_c = 1.8 \times 10^{-5}$/°C



As the temperature increases and $\alpha_c > \alpha_s$, copper rod will be in compression and steel tube will be in tension. Under equilibrium condition,

Tensile force on steel = Compressive force on copper

$$\therefore \quad P_s = P_c = P$$

Using relation

$$\boxed{\alpha_c\, t L_c - \alpha_s\, t L_s = \frac{P L_c}{A_c E_c} + \frac{P L_s}{A_s E_s}}$$

$$\therefore \quad t(\alpha_c - \alpha_s) = \frac{P}{A_c E_c} + \frac{P}{A_s E_s} \qquad [\because L_c = L_s]$$

$$\therefore \quad 180(1.8\times10^{-5} - 1.1\times10^{-5}) = P\left[\frac{1}{49\pi \times 1.1 \times 10^5} + \frac{1}{125\pi \times 2.1 \times 10^5}\right]$$

$$\therefore \quad P = \mathbf{17701.17\ N} = 17.70 \text{ kN}$$

$\therefore$ Stress in copper $\quad \sigma_c = \frac{P}{A_c} = \frac{17701.17}{49\pi} =$ **114.98 N/mm²** (Compressive)

$\therefore$ Stress in steel $\quad \sigma_s = \frac{P}{A_s} = \frac{17701.17}{125\pi} =$ **45.07 N/mm²** (Tension)

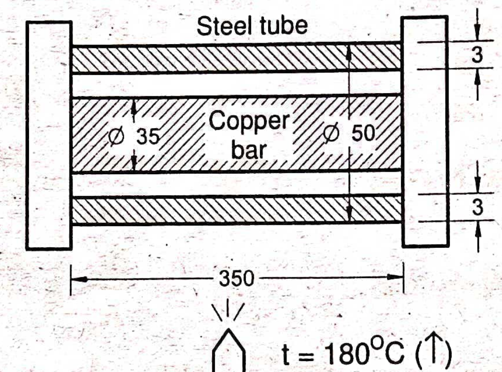
2. **A steel tube of 50 mm external diameter and 3 mm thickness encloses centrally a solid copper bar of 35 mm diameter. The bar and the tube are rigidly connected at their ends at a temperature of 20°C. Find the stress in each metal, when heated to 170°C. Also find the increase in length, if the original length of the assembly is 350 mm.**

Take : $\alpha_c = 1.7 \times 10^{-5}/°C$; $\alpha_s = 1.08 \times 10^{-5}/°C$;

$E_c = 100$ GPa ; $E_s = 200$ GPa. **S[N 08]**

Solution : Given : $D_s = 50$ mm, $t_k = 3$ mm, $\therefore d_s = D_s - t_k = 50 - 2(3) = 44$ mm, $d_c = 35$ mm,

$t_i = 20^oC$, $t_f = 170^oC$, $\therefore t = t_f - t_i = 170 - 20 = 150^oC\ (\uparrow)$



$$\therefore A_s = \frac{\pi}{4}(D_s^2 - d_s^2) = \frac{\pi}{4}(50^2 - 44^2) = 141\pi \text{ mm}^2$$

$$\therefore A_c = \frac{\pi}{4}d_c^2 = \frac{\pi}{4}35^2 = 306.25\pi \text{ mm}^2$$

$\alpha_c = 1.7 \times 10^{-5}/°C$; $\alpha_s = 1.08 \times 10^{-5}/°C$;

$E_c = 100$ GPa ; $E_s = 200$ GPa.

As the temperature increases and $\alpha_c > \alpha_s$, copper bar will be in compression and steel tube will be in tension.

Under equilibrium condition,

Tensile force on steel = Compressive force on copper

$$\therefore \quad P_s = P_c = P$$

Using relation $$\boxed{\alpha_c\, t\, L_c - \alpha_s\, t\, L_s = \frac{P L_c}{A_c E_c} + \frac{P L_s}{A_s E_s}}$$

$$\therefore \quad t(\alpha_c - \alpha_s) = \frac{P}{A_c E_c} + \frac{P}{A_s E_s} \qquad [\because L_c = L_s]$$

$$\therefore \quad 150(1.7\times10^{-5} - 1.08\times10^{-5}) = P\left[\frac{1}{306.25\pi \times 100 \times 10^3} + \frac{1}{141\pi \times 200 \times 10^3}\right]$$

$$\therefore \quad P = 42893.95 \text{ N} = 42.89 \text{ kN}$$

$\therefore$ Stress in copper $\sigma_c = \frac{P}{A_c} = \frac{42893.95}{306.25\,\pi} =$ **44.58 N/mm²** (Compressive)

$\therefore$ Stress in steel $\sigma_s = \frac{P}{A_s} = \frac{42893.95}{141\pi} =$ **96.83 N/mm²** (Tension)

Increase in length of assembly = Total elongation of copper or steel

$$\therefore \quad \delta l = \alpha_c\, t\, L_c - \frac{P L_c}{A_c E_c} \quad \text{or} \quad \alpha_s\, t\, L_s + \frac{P L_s}{A_s E_s}$$

$$\therefore \quad \delta l = 1.7 \times 10^{-5} \times 150 \times 350 - \frac{42893.95 \times 350}{306.25\pi \times 100 \times 10^3}$$

$$\therefore \quad \delta l = \mathbf{0.736\ mm}$$