

Regression ①

①

line of regression of Y on x is

$$Y = a + bx$$

line of regression of X on Y is

$$X = a + bY$$

eqn of line of regression of Y on x is

$$Y - \bar{Y} = b_{YX} (x - \bar{x})$$

$$Y - \bar{Y} = r \frac{\sigma_Y}{\sigma_X} (x - \bar{x})$$

$$b_{YX} = r \frac{\sigma_Y}{\sigma_X} = \text{regression coefficient}$$

of Y on x

eqn of line of regression of X on Y is

$$X - \bar{x} = b_{XY} (Y - \bar{Y})$$
$$= r \frac{\sigma_X}{\sigma_Y} (Y - \bar{Y})$$

$$b_{XY} = r \frac{\sigma_X}{\sigma_Y} = \text{regression coefficient}$$

of X on Y

① line of regression of Y on x

$$Y = a + bx$$

$$\Sigma Y = a \Sigma 1 + b \Sigma x$$

$$\Sigma xY = a \Sigma x + b \Sigma x^2$$

⑥ line of regression of x on y

①

$$x = a + by$$

$$\sum x = aN + b\sum y$$

$$\sum xy = a\sum y + b\sum y^2$$

are normal eqs

Ex: 1 Find the equations of the lines of regression

$$x: 5 \ 6 \ 7 \ 8 \ 9$$

$$y: 2 \ 4 \ 5 \ 6 \ 8 \quad \text{also find } r$$

Soln.	Sr No.	x	x^2	y	y^2	xy
	1	5	25	2	4	10
	2	6	36	4	16	24
	3	7	49	5	25	35
	4	8	64	6	36	48
	5	9	81	8	64	72
		$\sum x$	$\sum x^2$	$\sum y$	$\sum y^2$	$\sum xy$
		$N=5$	$= 255$	$= 25$	$= 145$	$= 189$

line of regression of y on x is

$$y = a + bx$$

$$\sum y = aN + b\sum x \quad \therefore 25 = 5a + 35b \quad \text{--- (1)}$$

$$\sum xy = a\sum x + b\sum x^2 \quad 189 = 35a + 255b \quad \text{--- (2)}$$

$$a = -4.8, \quad b = 1.4$$

$$y = -4.8 + 1.4x$$

line of regression on x and y

(3)

$$x = a + by$$

$$\sum x = an + b \sum y \quad \therefore 35 = 5a + 25b \quad (3)$$

$$\sum xy = a \sum y + b \sum y^2 \quad 189 = 25a + 125b \quad (4)$$

$$a = 2.2, b = 0.56$$

$$x = 2.2 + 0.56y$$

$$b_{yx} = 1.4, b_{xy} = 0.56$$

$$r = \sqrt{b_{yx} b_{xy}} = \sqrt{(1.4)(0.56)} = 0.88$$

$$(1) \quad b_{yx} = \frac{\sum xy}{\sum x^2} \quad b_{xy} = \frac{\sum xy}{\sum y^2}$$

$$x = x - \bar{x}, y = y - \bar{y}$$

$$(2) \quad dx = x - \bar{x}, dy = y - \bar{y}$$

$$b_{yx} = \frac{\sum dx dy - \frac{\sum dx \sum dy}{N}}{\sum dx^2 - \frac{(\sum dx)^2}{N}}$$

$$b_{xy} = \frac{\sum dx dy - \frac{\sum dx \sum dy}{N}}{\sum dy^2 - \frac{(\sum dy)^2}{N}}$$

(iii) x, y actual values

$$b_{yx} = \frac{\sum xy - \frac{\sum x \sum y}{N}}{\sum x^2 - \frac{(\sum x)^2}{N}}$$

$$b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{N}}{\sum y^2 - \frac{(\sum y)^2}{N}}$$

$$r = \pm \sqrt{b_{yx} b_{xy}}$$

properties

(3)

$$(1) \quad b^2 x^2 + b^2 y^2 = r^2$$

$$r = \sqrt{b^2 x^2 + b^2 y^2}$$

both $b^2 x^2$, $b^2 y^2$ are positive or both are negative

$$(2) \quad b^2 x^2 \leq \frac{1}{b^2 y^2}$$

$$(3) \quad \frac{b^2 x^2 + b^2 y^2}{2} \geq r$$