correlation (2)

Ex (1) calculate the coefficient of correlation between x & y

x : 3 5 4 6 2

y : 3 4 5 6

x	x^2	y	y^2	xy
3	9	3	9	9
5	25	4	16	20
4	16	5	25	20
6	36	2	4	12
2	4	6	36	12

$\Sigma x = 20$ $\Sigma x^2 = 90$ $\Sigma y = 20$ $\Sigma y^2 = 90$ $\Sigma xy = 73$

$\bar{x} = \frac{\Sigma x}{n} = \frac{20}{5} = 4$

$\bar{y} = \frac{\Sigma y}{N} = \frac{20}{5} = 4$

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{N}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{N}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{N}\right)}}$$

$$= \frac{73 - \frac{(20)(20)}{5}}{\sqrt{\left(90 - \frac{(20)^2}{5}\right)\left(90 - \frac{(20)^2}{5}\right)}}$$

$= 73 - 80 = -7 = -7$

$\sqrt{90-80}\ \sqrt{90-80}\ \sqrt{10}\ \sqrt{10}\ 10$

$= -0.7$

HW - Find r

X:	2	3	4	7	4	
Y:	8	7	3	1	1	19

AM = 0.81

Spearman's Rank Correlation

$$R = 1 - \frac{6 \Sigma d_i^2}{n^3 - n}$$

d_i : difference in ranks

EX(1): Compute Spearman's rank correlation coefficient

X:	18	20	34	52	12	
Y:	39	23	35	18	46	16

Soln. calculation of R both are RT

Sr. no	X	rank R_1	Y	Rank R_2	$d_i = R_1 - R_2$	d_i^2
	18	4	39	2	2	4
	20	3	23	4	-1	1
	34	2	35	3	-1	1
	52	1	18	5	-4	16
	12	5	46	1	4	16

$\Sigma d_i^2 = 38$

$n = 5$

$$R = 1 - 6\frac{\Sigma d_i^2}{N^3 - N} = 1 - \frac{6(38)}{5^3 - 5} = 1 - \frac{6(38)}{120}$$

$$= \frac{20 - 38}{20} = \frac{-18}{20} = -\frac{9}{10} = -0.9$$

(3) (3) Calculate Spearman's rank correlation

Height in inches : 60 62 64 66 68 70 72 74

Wt in lbs : 92 83 101 110 128 119 137 146

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Soln:

Sr no	X	R_1	Y	R_2	$d_i = R_1 - R_2$	d_i^2
1	60	8	92	7	1	1
2	62	7	83	8	-1	1
3	64	6	101	6	0	0
4	66	5	110	5	0	0
5	68	4	128	3	1	1
6	70	3	119	4	-1	1
7	72	2	137	2	0	0
8	74	1	146	1	0	0

$\Sigma d_i^2 = 4$

$N = 8$

$$R = 1 - \frac{6 \Sigma d_i^2}{N^3 - N}$$

$$= 1 - \frac{6(4)}{8^3 - 8} = 1 - \frac{6(4)}{8(63)}$$

$$= \frac{60}{63} = 0.952$$

Find r

X : 12 17 22 27 32

Y : 113 119 117 115 121

r = 0.6

Equal ranks

Some time they have same value
. 4 here equal rank

4 & 5th item ~~are~~ have same value 45

$\frac{4+5}{2} = 4.5$ $\quad m_1 = 2$

$\frac{4+5+6}{3} = 5$ $\quad m_2 = 3$

$$R = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) \cdots\right]}{n^3 - n}$$

EX (3) Obtain rank correlation coefficient

x :	10	12	18	18	15	40
y :	12	18	25	25	50	25

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Soln:

x	R_1	y	R_2	$d_i = R_1 - R_2$	d_i^2
10	6	12	6	0	0
12	5	18	5	0	0
18	2.5	25	3	-0.5	0.25
18	2.5	25	3	-0.5	0.25
15	4	50	1	3	9
40	1	25	2	-2	4

$\Sigma d_i^2 = 13.50$

$n = 6$

in x: 18 occurs two time $\frac{2+3}{2} = 2.5$ $\quad m_1 = 2$

in y: 25 occurs 3 time $\frac{2+3+4}{3} = 3$ $\quad m_2 = 3$

$$R = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right.}{n^3 - n}$$

$$R = 1 - \frac{6\left[13.5 + \frac{1}{12}(8-2) + \frac{1}{12}(27-3)\right]}{6^3 - 6}$$

$$= 1 - \frac{6\left[13.5 + \frac{1}{2} + 2\right]}{6(36-1)}$$

$$= 1 - \frac{16}{35} = 1 - 0.4571 = 0.5429$$

Ex(6) Find Rank correlation coefficient

x :	32	55	49	60	43	37	43	49	10	20
y :	40	30	70	20	30	50	72	60	45	25

Soln

x	R_1	y	R_2	$R_1 - R_2$	d_1^2
32	8	40	6	2	4
55	2	30	7.5	-5.5	
49	3.5	70	2	1.5	
60	1	20	10	-9	
43	5.5	30	7.5	-2	
37	7	50	4	3	
43	5.5	~~60~~ 72	1	4.5	
49	3.5	60	3	0.5	
10	10	45	5	5	
20	9	25	9	0	

$\Sigma d_1^2 = 176$

x 49 occurs 2 times $\frac{3+4}{2} = 3.5$

43 occurs 2 times $\frac{5+6}{2} = 5.5$

y : 30 occurs three times $\frac{7+8}{2} = 7.5$

$$R = 1 - 6\left(\Sigma d_1^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \frac{1}{12}(m_3^3 - m_3)\right) \Big/ (N^3 - N)$$

$N = 10$

$$R = 1 - \frac{6\left[176 + \frac{1}{12}(8-2) + \frac{1}{12}(8-4) + \frac{1}{12}(82-31)\right]}{1000-10} \quad (6)$$

$$= 1 - \frac{6\left[176 + 0.5 + 0.5 + 0.5\right]}{990}$$

$$= 1 - \frac{6\left[177.5\right]}{165}$$

$$= 1 - 1.076 = -0.076$$

Ex (5) If $r_{xy} = 0.4$, $cov(x,y) = 1.6$, $\sigma^2_y = 25$

Find σ_x

(ii) If $R_{xy} = 0.143$ and sum of the difference between the ranks is 48 find N

Soln $r_{xy} = 0.4$, $cov(x,y) = 1.6$, $\sigma^2_y = 25$

$$r_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$0.4 = \frac{1.6}{\sigma_x \cdot 5}$$

$$\sigma_x = \frac{1.6}{5(0.4)} = \frac{4}{5} = 0.8$$

(ii) $R = 0.143$, $\sum d_i^2 = 48$

$$R = 1 - \frac{6\sum d_i^2}{n^3 - n} \quad \therefore 0.143 = 1 - \frac{6(48)}{n^3 - n}$$

$$\frac{288}{n^3 - n} = 1 - 0.143 = 0.857$$

$$N^3 - N = \frac{288}{0.857} = 336$$

$$N^3 - N - 336 = 0$$

$$N = 7$$

Ex (6) Calculate the correlation coefficient between x & y

$N = 10$, $\Sigma x = 140$, $\Sigma y = 150$, $\Sigma (x-10)^2 = 180$

$\Sigma (y-15)^2 = 215$, $\Sigma (x-10)(y-15) = 60$

Soln $\Sigma d_x^2 = 180$, $\Sigma d_y^2 = 215$,

$\Sigma d_x d_y = 60$

$$\bar{x} = A + \frac{\Sigma d_x}{N} \qquad \bar{x} = \frac{\Sigma x}{N} = \frac{140}{10} = 14$$

$$14 = 10 + \frac{\Sigma d_x}{10} \qquad \Sigma d_x = 40$$

$$\bar{y} = \frac{\Sigma y}{N} =$$

$$\bar{y} = B + \frac{\Sigma d_y}{N} \qquad 15 = 15 + \frac{\Sigma d_y}{10} \qquad \Sigma d_y = 10$$

$$r = \frac{\Sigma d_x d_y - \frac{\Sigma d_x \Sigma d_y}{N}}{\sqrt{\left(\Sigma d_x^2 - \frac{(\Sigma d_x)^2}{N}\right)\left(\Sigma d_y^2 - \frac{(\Sigma d_y)^2}{N}\right)}}$$

$$= \frac{60 - \frac{(40)(10)}{10}}{\sqrt{\left(180 - \frac{(40)^2}{10}\right)\left(215 - \frac{(10)^2}{10}\right)}}$$

$$= \frac{60 - 40}{\sqrt{(180 - 160)(215 - 10)}} = \frac{20}{\sqrt{(20)(205)}}$$

$$= 0.715$$