

<u>EY</u>	(2)	X : 100 110 120 130 140 150 160 170	180 190	(1)
		Y : 45 51 54 61 66 70	<u>75 78</u> 85, 89	

Find ~~predicted~~ regression of product yield  
on the basis of temperature

S.R.	X	$d_x = X - 150$		$\sum d_n$	y	$d_y = Y - 70$	$\sum d_y$	$\sum d_n d_y$	$\sum d_y^2$	$\sum d_n^2$
		$d_x$	$d_x^2$							
1	100	-50	2500	2000	45	-25	29	1150	1290	100
2	110	-40	1600	1600	51	-19	31	760	760	90
3	120	-30	900	900	54	-16	36	480	480	80
4	130	-20	900	1000	61	-9	81	180	180	40
5	140	-10	100	100	66	0	0	0	0	40
6	150	0	0	100	70	4	16	160	160	40
7	158	.	4	10	74	4	64	450	450	40
8	160	.	0	20	78	8	64	225	225	40
9	170	.	30	30	85	15	225	360	360	40
10	180	.	60	100	89	19	361	1760	1760	40
11	190	.	50	2500	89	19	361	261	261	40

$$\begin{aligned} \sum d_x &= -50 \\ \sum d_n &= 8500 \end{aligned}$$

$$\begin{aligned} \sum d_n d_y &= -27 \\ \sum d_y^2 &= 2005 \end{aligned}$$

$$\bar{x} = \frac{\sum x}{n} =$$

$$\bar{x} = \frac{\sum x}{n} = 150 + 5 \times A - 150, \quad B = 70$$

$$b_{xy} = \frac{\sum d_n d_y - \overline{d_n} \overline{d_y}}{\sum d_n^2 - \overline{d_n^2}} = \frac{4120 - \frac{(-50)(-27)}{10}}{8500 - \frac{(-50)^2}{10}}$$

$$= 0.483$$

$$b_{xx} = \frac{\sum d_n^2 d_y - \sum d_n \sum d_y}{\sum d_y^2 - \overline{d_y^2}} = \frac{4120 - \frac{(-50)(-27)}{10}}{2005 - \frac{(-27)^2}{10}} = 2.06$$

$$\bar{x} = 145, \bar{y} = 67.3 \quad \bar{x} = \frac{\sum x}{n} = \frac{1550}{10} = 155$$

Linear regression or  $y$  on  $x$   $\bar{y} = \frac{\sum y}{n} = \frac{673}{10} = 67.3$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$A = 150 \quad \textcircled{1}$$

$$B = 70$$

$$y - 67.3 = 0.683(x - 145)$$

$$y = 0.683x - 2.735$$

$x$  on  $y$

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 145 = 2.06(y - 67.3)$$

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Ex: Given the eqn of the two  
regression lines are  $3x + 2y = 26$

and  $b_{xy} = 31$ . Find value of  $x$  if

(i) correlation coefficient between  $x$  &  $y$

(ii)  $b_y$ , if  $b_n = 3$

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Sol: mean  $\bar{x}, \bar{y}$

$$x = 5, y = 5.7$$

## line of regression ②

①

Ex ① the regression lines of a sample are

$$x+6y=6 \text{ and } 3x+2y=10 \text{ Find}$$

① Sample means  $\bar{x}$  and  $\bar{y}$  ②  $y$  ③  $y$  when  $x=12$

13, 11, 18

Soln  $x+6y=6$

$$3x+2y=10$$

$$x=3, y=\frac{1}{2} \quad \bar{x}=3, \bar{y}=\frac{1}{2}$$

If the line  $x+6y=6$  is the line of regression of  $y$  on  $x$  is  $6y = -x + 6$

$$y = -\frac{1}{6}x + 1 \therefore b_{yx} = -\frac{1}{6}$$

If the line  $3x+2y=10$  is the line of regression

$$\text{of } x \text{ on } y \text{ is } 3x = -2y + 10$$

$$x = -\frac{2}{3}y + \frac{10}{3} \therefore b_{xy} = -\frac{2}{3}$$

$$\gamma = \sqrt{b_{yx}b_{xy}} = \sqrt{\left(-\frac{1}{6}\right)\left(-\frac{2}{3}\right)} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$$b_{yx} \text{ & } b_{xy} \text{ are negative } \gamma = -\frac{1}{3}$$

$$\text{if } x=12, y = -\frac{x}{6} + 1 = -\frac{12}{6} + 1 = -2 + 1 = -1$$

Ex ② If the tangent of the angle made by the line of regression of  $y$  on  $x$  is 0.6 and  $b_{xy} = 2$  find

Find the correlation coefficient bet'  $x$  &  $y$

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(2)

Soln If the eqn of line of regression  
of  $y$  on  $x$  is  $y - \bar{y} = b_{yx}(x - \bar{x})$

$b_{yx} = 0.6$  is the slope of regression.

$$b_{yx} = \gamma \frac{\sigma_y}{\sigma_x} \quad \sigma_y = 2 \sigma_x$$

$$0.6 = \gamma \frac{2 \sigma_x}{\sigma_x} = 2\gamma$$

$$\gamma = 0.3$$

Ex(3) It is given that the means of  $x$  &  $y$   
are 5 and 10. If the line of regression  
of  $y$  on  $x$  is parallel to the line  $20y = 9x + 40$   
estimate the value of  $y$  for  $x = 30$  15

Soln  $\bar{x} = 5, \bar{y} = 10 \therefore$  line of regression

of  $y$  on  $x$  is  $20y = 9x + 40$

$$y = \frac{9}{20}x + 2$$

$$b_{yx} = \frac{9}{20}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 10 = \frac{9}{20}(x - 5)$$

$$y = 10 + \frac{9}{20}x - \frac{9}{4}$$

$$y = \frac{31}{4} + \frac{9}{20}x$$

$$x = 30, y = \frac{31}{4} + \frac{9}{20}(30) = \frac{31}{4} + \frac{54}{4} = \frac{85}{4}$$

$$= 21.25$$

## line of regression ②

①

Ex ① the regression lines of a sample are

$$x+6y=6 \text{ and } 3x+2y=10 \text{ Find}$$

① Sample means  $\bar{x}$  and  $\bar{y}$  ②  $y$  ③  $y$  when  $x=12$

13, 15, 18

$$\text{Soh } x+6y=6$$

$$3x+2y=10$$

$$x=3, y=\frac{1}{2} \quad \bar{x}=3, \bar{y}=\frac{1}{2}$$

If the line  $x+6y=6$  is the line of regression of  $y$  on  $x$  is  $6y = -x + 6$

$$y = -\frac{1}{6}x + 1 \therefore b_{yx} = -\frac{1}{6}$$

If the line  $3x+2y=10$  is the line of regression

$$\text{of } x \text{ on } y \text{ is } 3x = -2y + 10$$

$$x = -\frac{2}{3}y + \frac{10}{3} \therefore b_{xy} = -\frac{2}{3}$$

$$\gamma = \sqrt{b_{yx} b_{xy}} = \sqrt{\left(-\frac{1}{6}\right) \left(-\frac{2}{3}\right)} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$b_{yx}$  &  $b_{xy}$  are negative  $\gamma = -\frac{1}{3}$

$$\text{if } x=12, y = -\frac{x}{6} + 1 = -\frac{12}{6} + 1 = -2 + 1 = -1$$

Ex ② If the tangent of the angle made by the line of regression of  $y$  on  $x$  is 0.6 and  $b_{xy} = 2$  find

Find the correlation coefficient betw  $x$  &  $y$

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# Curve Fitting

(3)

Ex: Fit a st. line  $y = a + bx$

$x: 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$y: 49 \quad 54 \quad 60 \quad 73 \quad 80 \quad 86$

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Soln	$x$	$y$	$x^2$	$xy$
	1	49	1	49
	2	54	4	108
	3	60	9	180
	4	73	16	292
	5	80	25	400
	6	81	36	516
	$\sum x = 21$	$\sum y = 402$	$\sum x^2 = 91$	$\sum xy = 1545$
$N=6$				

$$y = a + bx$$

$$\sum y = aN + b\sum x \quad \therefore 402 = 6a + 21b \quad \text{---(1)}$$

$$\sum xy = a\sum x + b\sum x^2 \quad 1545 = 21a + 91b \quad \text{---(2)}$$

$$\sum x = 21$$

$$6a + 21b = 402 \quad \text{---(1)}$$

$$21a + 91b = 1545 \quad \text{---(2)}$$

$$a = 39.38, b = 7.89$$

$$y = 39.38 + 7.89x$$

Ex ② Fit a straight line (4)

X:	1965	1966	1967	1968	1969
Y:	125	140	165	195	200

$$\text{Soln. } \bar{x} = 1967, \bar{y} = 165$$

X	Y	$x = X - 1967$	$y = Y - 165$	$x^2$	$XY$
1965	125	-2	-40	4	80
1966	140	-1	-25	1	25
1967	165	0	0	0	0
1968	195	1	30	1	30
1969	200	2	35	4	70

$\sum x = 0$     $\sum y = 0$

$\sum x^2 = 10$     $\sum XY = 205$

$$Y = a + bx$$

$$\sum Y = a n + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

$$0 = 5a + 0 \quad a = 0$$

$$205 = 0 + b 10 \quad b = 20.5$$

$$b = \frac{205}{10} = 20.5$$

$$Y = 20.5x$$

$$\therefore Y - 165 = 20.5(x - 1967)$$

$$Y = 165 + 20.5x - 20.5(1967)$$

$$Y = 165 + 20.5x - 40323.5$$

$$Y = 20.5x + 165 - 40323.5$$

$$Y = 20.5x - 40158.5$$

## Fitting a parabola

(5)

$$y = afx^2 + bx + c$$

~~xf = a~~

$$\sum Y = an + b\sum x + c\sum x^2$$

$$\sum xY = ax\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2Y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

are normal eqns

Ex: Fit a second degree parabol.

curve

x :	1	2	3	4	5	6	7	8	9	
y :	2	6	7	8	10	11	11	10	9	LL

Soln since the values of x are odd

we change  $x = x - \text{middle term}$

$$x = x - 5$$

$$Y = y$$

The eqn of the parabola  $y = afx^2 + bx + c$

$$\sum Y = an + b\sum x + c\sum x^2$$

$$\sum xY = ax\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2Y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

are normal eqns

Calculation of  $\Sigma x$ ,  $\Sigma x^2$ ,  $\Sigma x^3$

(6)

$Sx$	$x$	$x = x - 5$	$x^2$	$x^3$	$x^4$	$y = y$	$xy$	$x^2y$
1	1	-4	16	-64	256	2	-8	32
2	2	-3	9	-27	81	6	-18	54
3	3	-2	4	-8	16	7	-14	28
4	4	-1	1	-1	1	8	-8	8
5	5	0	0	0	0	10	0	0
6	6	1	1	1	1	11	11	11
7	7	2	4	8	16	11	22	44
8	8	3	9	27	81	10	30	90
9	9	4	16	64	256	9	36	144

$$n=9 \quad \Sigma x=0 \quad \Sigma x^2=60 \quad \Sigma x^3=0 \quad \Sigma x^4=708 \quad \Sigma x^5=74 \quad \Sigma x^6=51 \quad \Sigma x^7=411$$

$$74 = 3a + 60c \quad \text{--- (1)}$$

$$51 = 6ab \quad \text{--- (2)} \quad b = 0.85$$

$$411 = 60ac + 708c \quad \text{--- (3)}$$

$$a = 10.00h^2 = 10, \quad c = -0.263 = -0.27$$

$$y = 10 + 0.85x - 0.27x^2 \quad \begin{matrix} x = x - 5 \\ y = y \end{matrix}$$

$$y = 10 + 0.85(x-5) - 0.27(x-5)^2$$

$$y = -1 + 3.55x - 0.27x^2$$

Ex(2) Fit a second degree parabol.

(7)

Carry out and find production in 1982

Year  $x$  : 1974 75 76 77 78 79 80 81  
 Production  $y$  : 12 14 20 42 40 50 52 53  
 in tone

Soln Since values of  $x$  are even take

$$x = x - (\text{mean of middle term}) = x - \frac{(1977+1978)}{2} - \frac{(1977-1974)}{2}$$

SR No	$x$	$x = 2(x - 1977.5)$	$x^2$	$x^3$	$x^4$	$y = 1$	$xy$	$x^2y$
1	1974	-7	49	-343	2401	12	-84	588
2	1975	-5	25	-125	625	14	-70	350
3	1976	-3	9	-27	81	20	-60	134
4	1977	-1	1	-1	1	42	-42	42
5	1978	1	1	1	1	40	-40	40
6	1979	3	9	27	81	50	-150	450
7	1980	5	25	125	625	52	-260	1300
8	1981	7	49	343	2401	53	-371	2597

$$\begin{aligned} \sum x^2 &= 168 \\ \sum x^3 &= 0 \\ \sum x^4 &= 168 \end{aligned}$$

$$\begin{aligned} \sum y &= 289 \\ \sum xy &= 561 \\ \sum x^2y &= 561 \end{aligned}$$

Method of Parabola

$$y = a + bx + cx^2$$

$$\sum y = a \sum 1 + b \sum x + c \sum x^2$$

$$\sum xy = a \sum 1 + b \sum x + c \sum x^2$$

$$\sum x^2y = a \sum 1 + b \sum x + c \sum x^3$$

(8)

$$289 = 89 + 168c \quad - (1)$$

$$547 = 168b \quad - (2) \quad b = 3.2559$$

$$5601 = 168a + 6211c \quad - (3)$$

$$a = 39.7811 \quad c = -0.1761$$

$$Y = 39.78 + 3.2559x - 0.1761x^2$$

$$x = (x - 1977.5)^2 \quad Y = y$$

$$y = 39.78 + 3.2559(x - 1977.5)^2 - 0.17(x - 1977.5)^4$$

$$y = -2671997.77 + 2695.92x - 0.682x^2$$

$$x = 1982$$

$$y = 55.35$$

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