Probability Distributions

Random variables

set of all possible out comes of an experiment is called sample space. denoted by S

$$P(A) = \frac{\text{no. of favourable out comes}}{\text{no. of possible out comes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

eg. coin is tossed probability of head obtained

$S = \{H\ T\}$

$n(S) = 2$

A = Head obtained $= H$

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

$$0 \leq P(A) \leq 1$$

the variable may take discrete values. it is called random variables.

variables used to denote the numerical values of the outcome of an experiment is called random variable r.v.

$X = 0, 1, 2, 3 \ldots$

(i) Let X be a r.v. if X takes finite or countably infinite values $x_0, x_1, x_2 \ldots$ then X is called discrete r.v.

(ii) Let X be a r.v. If X takes uncountably infinite values in a given interval then X is called continuous r.v.

<u>probability distribution of a d.r.v.</u>

Let X be a d.r.v. Let $x_1, x_2 \ldots x_n$ be the possible values of X with each possible outcome x_i we associate a number $p(x_i) = P(X = x_i)$ called probability of x_i

The numbers $p(x_i)$ $i = 1, 2 \ldots n$ must satisfy the conditions

(i) $p(x_i) \geq 0$ for all i

(ii) $\sum_{i=1}^{n} p(x_i) = 1$

The function p is called the probability function or p.m.f. or p.d.f. of a r.v. X. set of pairs (x_i, p_i) is called the probability distribution of X.

X :	x_1	x_2	x_3	- -	x_n
$p(x_i)$:	$p(x_1)$	$p(x_2)$	...		$p(x_n)$

$\therefore$

X :	x_1	x_2	x_3	- -	x_n
$p(x_i)$:	p_1	p_2	p_3		p_n

Ex (1) Find the probability distribution of number of heads (X) obtained when a fair coin is tossed 4 times

Soln: Coin is tossed 4 times

$= 2^4 = 16$

$n(S) = 16$

$S = \{$ HHHH, HHHT, HHTH, HHTT,
HTHH, HTHT, HTTH, HTTT,
THHH, THHT, THTH, THTT,
TTHH, TTHT, TTTH, TTTT $\}$

$n = 4$

~~P(X)~~ $\quad P(x) = \frac{nCr}{n(S)}$

Probability distribution of X

$X:$	0	1	2	3	4
$P(X=x):$	$\frac{4C0}{16}$	$\frac{4C1}{16}$	$\frac{4C2}{16}$	$\frac{4C3}{16}$	$\frac{4C4}{16}$
$P(X=x):$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

$\therefore$

$X:$	0	1	2	3	4
$P(X=x):$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Ex 2 Write down the probability distribution of the sum of numbers appearing on the basis of two unbiased dice

Soln: Two dice are thrown

$n(S) = 6^2 = 36$

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

Sum 2	appears	once
Sum 3	appears	two times
4	appears	3 times
5		4
6		5
7		6
8		5
9		4
10		3
11		2
12		1

P. D. B

x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Space for Marks	Question No.	
	(1)	The p.d.f. of r.v. X is

X : 0 1 2 3 4 5 6

$P(X=x)$: k $3k$ $5k$ $7k$ $9k$ $11k$ $13k$

Find k, $P(X<4)$, $P(3\le X\le 6)$

Sol: Given X is d.r.v.

$\sum P(x_i)=1$

$k+3k+5k+7k+9k+11k+13k=1$

$49k=1$

$\therefore k=\frac{1}{49}$

x	0	1	2	3	4	5	6
$P(x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

$P(X<4) = P(0)+P(1)+P(2)+P(3)$

$= \frac{1}{49}+\frac{3}{49}+\frac{5}{49}+\frac{7}{49} = \frac{16}{49}$

$P(3< X\le 6) = P(4)+P(5)+P(6)$

$= \frac{9}{49}+\frac{11}{49}+\frac{13}{49}$

$= \frac{33}{49}$

(1) The probability function of d.r.v. X

X : 0 1 2 3 4 5 6 7

$P(X=x)$: 0 c $2c$ $2c$ $3c$ c^2 $2c^2$ $7c^2+c$

Find c, $P(X\ge 6)$, $P(X<6)$