

# Regression

①

line of regression of  $y$  on  $x$

$$y = a + bx$$

line of regression of  $x$  on  $y$

$$x = a + by$$

either line of regression of  $y$  on  $x$ ,  
or

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$y - \bar{y} = r \frac{b_y}{b_x} (x - \bar{x})$$

$b_{yx} = r \frac{\partial y}{\partial x}$  = regression coefficient  
of  $y$  on  $x$

either line of regression of  $x$  on  $y$ ,

$$x - \bar{x} = b_{xy} (y - \bar{y})$$
  
$$= r \frac{b_x}{b_y} (y - \bar{y})$$

$b_{xy} = r \frac{\partial x}{\partial y}$  = regression coefficient  
of  $x$  on  $y$

① line of regression of  $y$  on  $x$

$$y = a + bx$$

$$\Sigma y = a \Sigma x + b \Sigma x^2$$

$$\Sigma xy = a \Sigma x + b \Sigma x^2$$

Q. line of regression of x on y

(1)

$$x = a + b y$$

$$\sum x = aN + b \sum y$$

$$\sum xy = a \sum y + b \sum y^2$$

are normal equations

Ex: Q Find the equations of the lines  
of regression

x : 5 6 7 8 9

y : 2 4 5 6 8 also find r

Soln.	sr no.	x	$x^2$	y	$y^2$	$xy$
	1	5	25	2	4	10
	2	6	36	4	16	24
	3	7	49	5	25	35
	4	8	64	6	36	48
	5	9	81	8	64	72
		$\Sigma x$	$\Sigma x^2$	$\Sigma y$	$\Sigma y^2$	$\Sigma xy$
	$N=5$	$= 35$	$= 255$	$= 25$	$= 145$	$= 189$

line of regression of y on x is

$$y = a + b x$$

$$\sum y = aN + b \sum x \therefore 25 = 5a + 35b \quad \text{--- (1)}$$

$$\sum xy = a \sum x + b \sum x^2 \quad 189 = 35a + 255b \quad \text{--- (2)}$$

$$a = -4.8, \quad b = 1.4$$

$$\underline{y = -4.8 + 1.4x}$$

line of regression of X on Y

(3)

$$Y = a + bT$$

$$\sum Y = aN + b\sum T \quad \therefore 35 = 5a + 25b - (3)$$

$$\sum XY = a\sum T + b\sum T^2 \quad 189 = 25a + 125b - (4)$$

$$a = 2.2 \quad b = 0.16$$

$$Y = 2.2 + 0.16T$$

$$b_{yx} = 1.4 \quad b_{xy} = 0.56$$

$$\gamma = \sqrt{b_{yx} b_{xy}} = \sqrt{(1.4)(0.56)} = 0.88$$

(i)  $b_{yx} = \frac{\sum XY}{\sum T^2} \quad b_{xy} = \frac{\sum XT}{\sum T^2}$

$$x = T - \bar{T}, \quad y = Y - \bar{Y}$$

(ii)  $dX = \bar{x} - x, \quad dY = \bar{y} - y$

$$b_{yx} = \frac{\sum dX dY - \frac{1}{N} \sum dX \sum dY}{\sum d^2 X - \frac{(\sum dX)^2}{N}}$$

$$b_{xy} = \frac{\sum dY dX - \frac{1}{N} \sum dX \cdot \sum dY}{\sum d^2 Y - \frac{(\sum dY)^2}{N}}$$

(iii)  $x, y$  actual values

$$b_{yx} = \frac{\sum XY - \frac{1}{N} \sum X \sum Y}{\sum X^2 - \frac{(\sum X)^2}{N}}$$

$$b_{xy} = \frac{\sum XY - \frac{1}{N} \sum X \sum Y}{\sum Y^2 - \frac{(\sum Y)^2}{N}}$$

$$\left. \begin{array}{l} \gamma = \pm \sqrt{b_{yx} b_{xy}} \end{array} \right\}$$

(3)

properly

$$\textcircled{1} \quad b_{yx} b_{xy} = r^2$$

$$r = \sqrt{b_{yx} b_{xy}}$$

both  $b_{yx}, b_{xy}$  are positive or both are negative

$$\textcircled{2} \quad b_{yx} \leq \frac{1}{b_{xy}}$$

$$\textcircled{1.1} \quad \frac{b_{yx} + b_{xy}}{2} \geq r$$