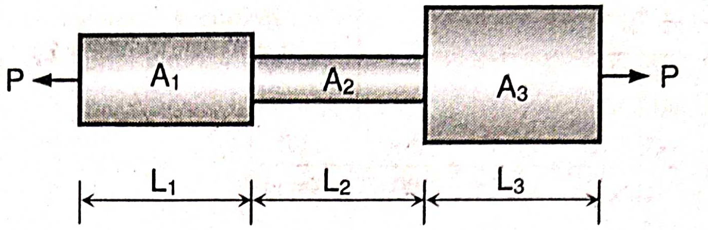
Bars of Varying Cross-section :

Sometimes a bar is made up of different materials, different lengths and different cross-sectional areas as shown in fig.



In such cases, though each section is subjected to the same axial load P, the stresses, strains and changes in lengths for each section will be different. Total change in length is equal to the sum of changes in lengths of individual section.

Let P = Axial load acting on the bar

L_1, L_2 & L_3 = Length of section 1, 2 & 3

A_1, A_2 & A_3 = Cross-sectional area of section 1, 2 & 3

E_1, E_2 & E_3 = Young's modulus of section 1, 2 & 3

Then stresses in section 1, 2 & 3 are

$$\sigma_1 = \frac{P}{A_1}, \qquad \sigma_2 = \frac{P}{A_2} \quad \text{and} \quad \sigma_3 = \frac{P}{A_3}$$

Total change in length of the bar

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\therefore \quad \boxed{\delta l = \frac{PL_1}{A_1E_1} + \frac{PL_2}{A_2E_2} + \frac{PL_3}{A_3E_3}}$$

$$\therefore \quad \delta l = P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3}\right]$$

If material of each section is same, then value of E remains same for all sections.

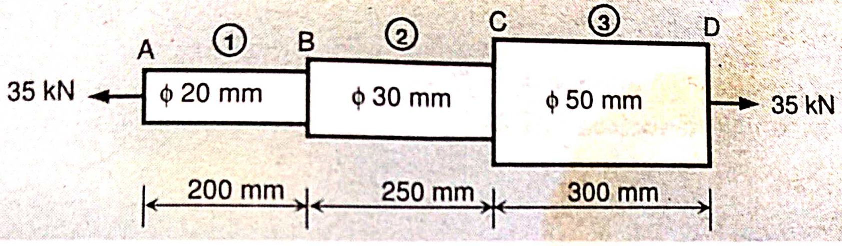
$$\therefore \quad \delta l = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right] \qquad (\because E_1 = E_2 = E_3 = E)$$

2. An axial pull of 35 kN is acting on a bar consisting of three lengths as shown in fig. If the Young's modulus $E = 2.1 \times 10^5$ N/mm^2, determine

i) Total elongation of the bar

ii) Stresses in each section.

B.C.[M 13], AM[S 07]



Solution : Given : $P = 35$ kN $= 35000$ N, $E = 2.1 \times 10^5$ N/mm^2

$L_1 = 200$ mm, $L_2 = 250$ mm, $L_3 = 300$ mm

$d_1 = 20$ mm, $d_2 = 30$ mm, $d_3 = 50$ mm

$$\therefore \quad A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} 20^2 = 100\pi \text{ mm}^2 = 314.15 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} 30^2 = 225\pi \text{ mm}^2 = 706.85 \text{ mm}^2$$

$$A_3 = \frac{\pi}{4} d_3^2 = \frac{\pi}{4} 50^2 = 625\pi \text{ mm}^2 = 1963.49 \text{ mm}^2$$

Find δl, σ_1, σ_2 and $\sigma_3 = ?$

i) Total elongation of the bar

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\therefore \quad \delta l = \frac{PL_1}{A_1E_1} + \frac{PL_2}{A_2E_2} + \frac{PL_3}{A_3E_3}$$

$$= \frac{P}{E}\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}+\frac{L_3}{A_3}\right] \qquad (\because E_1 = E_2 = E_3 = E)$$

$$= \frac{35000}{2.1\times10^5}\left[\frac{200}{100\,\pi}+\frac{250}{225\,\pi}+\frac{300}{625\,\pi}\right]$$

$$\therefore \quad \delta l = \mathbf{0.1905\ mm\ (Elongation)}$$

ii) Stresses in each section

Stress in portion 1

$$\sigma_1 = \frac{P}{A_1} = \frac{35000}{100\pi} = \mathbf{111.40\ N/mm^2}\ \text{(Tensile)}$$

Stress in portion 2

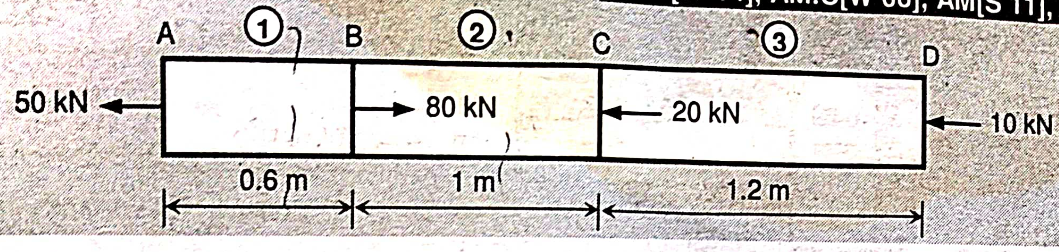
$$\sigma_2 = \frac{P}{A_2} = \frac{35000}{225\pi} = \mathbf{49.51\ N/mm^2}\ \text{(Tensile)}$$

Stress in portion 3

$$\sigma_3 = \frac{P}{A_3} = \frac{35000}{625\pi} = \mathbf{17.82\ N/mm^2}\ \text{(Tensile)}$$

1. A brass bar having cross-sectional area of 1000 mm² is subjected to axial forces shown in fig. Find the total change in length of the bar. E = 100 GN/m².

AM[M 10], S[N 03], B.Tech.[D 12], AM.C[W 14], AM.C[W 06], AM[S 11], B.C.[D 13], Sh.[N 12]

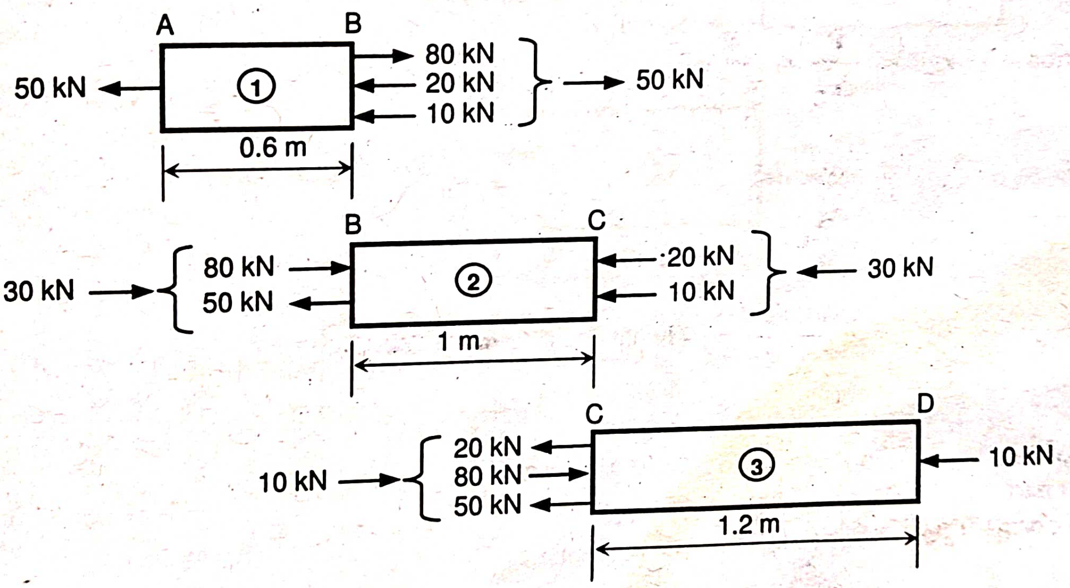


Solution : Given : $A = 1000\ \text{mm}^2$, $E = 100\ \text{GN/m}^2 = 100 \times 10^3\ \text{N/mm}^2$

$L_1 = 600$ mm, $L_2 = 1000$ mm, $L_3 = 1200$ mm   Find $\delta l = ?$

Note that bar is in equilibrium (sum of → forces = sum of ← forces).

Free Body Diagram (F.B.D.) of each part is as shown in fig. (Considering resultant forces acting at left and right side of each part).



Thus, forces on each part

$P_1 = +50$ kN (Tensile), $P_2 = -30$ kN (Compressive), $P_3 = -10$ kN (Compressive)

We know that, total change in length of bar

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\therefore \quad \delta l = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}$$

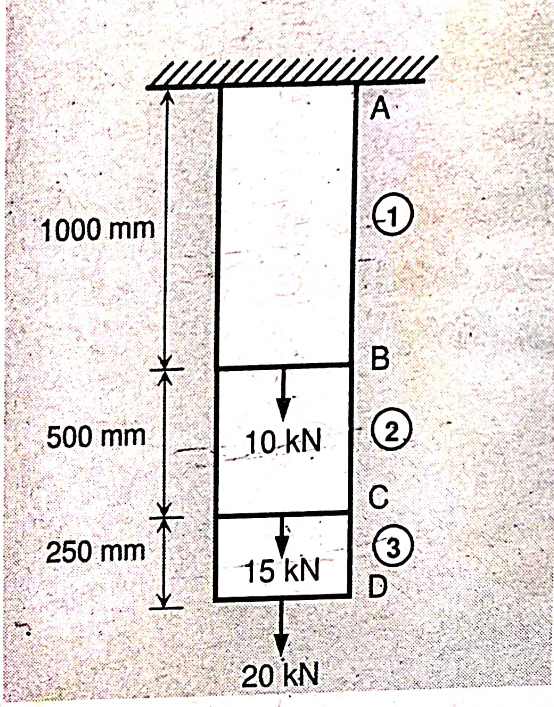
Here $A_1 = A_2 = A_3 = A$ and $E_1 = E_2 = E_3 = E$

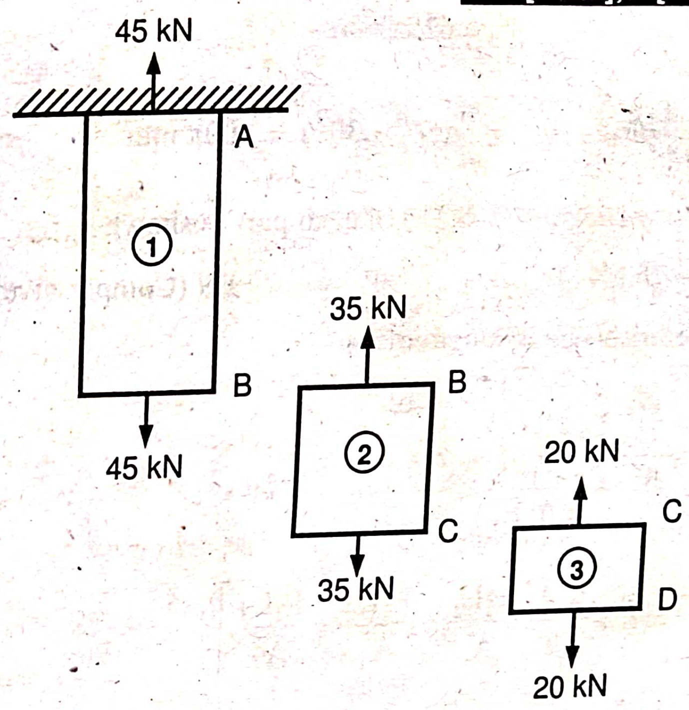
$$\therefore \quad \delta l = \frac{1}{AE}\left[P_1 L_1 + P_2 L_2 + P_3 L_3\right]$$

$$\therefore \quad \delta l = \frac{1}{1000 \times 100 \times 10^3}\left[50 \times 10^3 \times 600 + (-30 \times 10^3) \times 1000 + (-10 \times 10^3) \times 1200\right]$$

$$\therefore \quad \delta l = \mathbf{-0.12\ mm} \quad \text{i.e. } 0.12 \text{ mm (Contraction)}$$

3. A steel bar ABCD of cross sectional area 500 mm^2 is acted upon by forces as shown in fig. Neglecting effect of self-weight of the bars, find the change in length of bar AD. Take E = 200 GPa. **B.C.[M 14], B[N 96], B[M 93]**





Solution : Given : $A = 500\ mm^2$, $E = 200\ GPa = 200 \times 10^3\ N/mm^2$

$L_1 = 1000$ mm, $L_2 = 500$ mm, $L_3 = 250$ mm Find $\delta l = ?$

Consider Free Body Diagram (F.B.D.) of each part as shown in fig.

Thus, forces on each part

$$P_1 = +45\ kN = +45 \times 10^3\ N\ (\text{Tensile})$$
$$P_2 = +35\ kN = +35 \times 10^3\ N\ (\text{Tensile})$$
$$P_3 = +20\ kN = +20 \times 10^3\ N\ (\text{Tensile})$$

We know that, total change in length of bar

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\therefore \quad \delta l = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}$$

Here $A_1 = A_2 = A_3 = A$ and $E_1 = E_2 = E_3 = E$

$$\therefore \quad \delta l = \frac{1}{AE}[P_1 L_1 + P_2 L_2 + P_3 L_3]$$

$$\therefore \quad \delta l = \frac{1}{500 \times 200 \times 10^3}\left[45 \times 10^3 \times 1000 + 35 \times 10^3 \times 500 + 20 \times 10^3 \times 250\right]$$

$$\therefore \quad \delta l = \mathbf{0.675\ mm\ (Elongation)}$$