

(1)

Ex (12)	X :	100	110	120	130	140	150	160	170	
	Y :	45	51	54	61	66	70	74	78	85, 89
								180	190	

Find the linear regression of product yield on the basis of temperature

Soln:

Sr	X	$dX = X - 150$	dY	Y	$dY = Y - 70$	dX^2	dY^2	$dX dY$
1	100	-50	2500	45	-25	2500	625	-1250
2	110	-40	1600	51	-19	1600	361	-760
3	120	-30	900	54	-16	900	256	-480
4	130	-20	400	61	-9	400	81	-180
5	140	-10	100	66	-4	100	16	-40
6	150	0	0	70	0	0	0	0
7	160	10	100	74	4	100	16	160
8	170	20	400	78	8	400	64	480
9	180	30	900	85	15	900	225	1350
10	190	40	1600	89	19	1600	361	1760
		ΣdX = -50	ΣdX² = 8500	ΣY = 705	ΣdY = -27	ΣdY² = 2005	ΣdX dY = 4120	

$$\bar{X} = \frac{\Sigma X}{N}$$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{1500}{10} = 150 \quad A = 150, B = 70$$

$$b_{YX} = \frac{\Sigma dX dY}{\Sigma dX^2} = \frac{4120 - \frac{(-50)(-27)}{10}}{8500 - \frac{(-50)^2}{10}}$$

$$= 0.483$$

$$b_{XY} = \frac{\Sigma dX dY}{\Sigma dY^2} = \frac{4120 - \frac{(-50)(-27)}{10}}{2005 - \frac{(-27)^2}{10}} = 0.206$$

$$\bar{x} = 145, \bar{y} = 67.3 \quad \bar{x} = \frac{\sum x}{n} = \frac{1550}{10} = 155$$

$$\text{Linear regression of } y \text{ on } x \quad \bar{y} = \frac{\sum y}{n} = \frac{673}{10} = 67.3$$

$$A = 150 \quad \textcircled{D}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$B = 70$$

$$y - 67.3 = 0.483(x - 145)$$

$$y = 0.483x - 2.735$$

$x \text{ or } y$

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 145 = 2.06(y - 67.3)$$

~~or~~

Ex: Given The eqn of the two regression lines are $3x + 2y = 26$

and $6x + y = 31$. Find Correlation of x & y

(i) correlation coefficient between x & y

(ii) b_{yx} if $n = 3$

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Sol: mean \bar{x}, \bar{y}

$$x = 4, y = 7$$

line of regression ②

①

EX① the regression lines of a sample are

$$x + 6y = 6 \text{ and } 3x + 2y = 10 \text{ Find}$$

① Sample means \bar{x} and \bar{y} ② r ③ Find y when $x = 12$

15, 15, 18

Soln $x + 6y = 6$

$$3x + 2y = 10$$

$$x = 3, y = \frac{1}{2} \quad \bar{x} = 3, \bar{y} = \frac{1}{2}$$

If the line $x + 6y = 6$ is the line of regression of y on x is $6y = -x + 6$

$$y = -\frac{1}{6}x + 1 \therefore b_{yx} = -\frac{1}{6}$$

If the line $3x + 2y = 10$ is the line of regression

of x on y is $3x = -2y + 10$

$$x = -\frac{2}{3}y + \frac{10}{3} \therefore b_{xy} = -\frac{2}{3}$$

$$r = \sqrt{b_{yx} b_{xy}} = \sqrt{\left(-\frac{1}{6}\right)\left(-\frac{2}{3}\right)} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

b_{yx} and b_{xy} are negative $r = -\frac{1}{3}$

$$\text{if } x = 12, y = -\frac{x}{6} + 1 = -\frac{12}{6} + 1 = -2 + 1 = -1$$

EX② If the tangent of the angle made by the line of regression of y on x is 0.6 and $3y = 2x$

Find the correlation coefficient betⁿ x and y

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(2)

Soln If the eqn of line of regression of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$b_{yx} = 0.6$ is the slope of regression.

$$b_{yx} = r \frac{s_y}{s_x} \quad b_y = 2s_x$$

$$0.6 = r \frac{2s_x}{s_x} = 2r$$

$$r = 0.3$$

Ex(3) It is given that the means of x & y are 5 and 10. If the line of regression of y on x is parallel to the line $20y = 9x + 40$ estimate the value of y for $x = 30$ 15

Soln $\bar{x} = 5, \bar{y} = 10 \therefore$ line of regression

$$\text{of } y \text{ on } x \text{ is } 20y = 9x + 40$$

$$y = \frac{9}{20}x + 2$$

$$b_{yx} = \frac{9}{20}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 10 = \frac{9}{20}(x - 5)$$

$$y = 10 + \frac{9}{20}x - \frac{9}{4}$$

$$y = \frac{31}{4} + \frac{9}{20}x$$

$$x = 30, y = \frac{31}{4} + \frac{9}{20}(30) = \frac{31}{4} + \frac{55}{4} = \frac{85}{4}$$

$$= 21.25$$

Line of regression ②

①

EX① the regression lines of a sample are

$$x + 6y = 6 \text{ and } 3x + 2y = 10 \text{ Find}$$

① Sample means \bar{x} and \bar{y} ② r ③ Find y when $x = 12$

1, 1, 18

Soln $x + 6y = 6$

$$3x + 2y = 10$$

$$x = 3, y = \frac{1}{2} \quad \bar{x} = 3, \bar{y} = \frac{1}{2}$$

If the line $x + 6y = 6$ is the line of regression of y on x is $6y = -x + 6$

$$y = -\frac{1}{6}x + 1 \therefore b_{yx} = -\frac{1}{6}$$

If the line $3x + 2y = 10$ is the line of regression

of x on y is $3x = -2y + 10$

$$x = -\frac{2}{3}y + \frac{10}{3} \therefore b_{xy} = -\frac{2}{3}$$

$$r = \sqrt{b_{yx} b_{xy}} = \sqrt{\left(-\frac{1}{6}\right)\left(-\frac{2}{3}\right)} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

b_{yx} & b_{xy} are negative $r = -\frac{1}{3}$

$$\text{if } x = 12, y = -\frac{x}{6} + 1 = -\frac{12}{6} + 1 = -2 + 1 = -1$$

EX② If the tangent of the angle made by the line of regression of y on x is 0.6 and $b_y = 2$ or

Find the correlation coefficient betⁿ x & y

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Curve Fitting

(3)

Ex: Fit a st. line $y = a + bx$

$x:$ 1 2 3 4 5 6
 $y:$ 49 54 60 73 80 86 15

Soln

x	y	x^2	xy
1	49	1	49
2	54	4	108
3	60	9	180
4	73	16	292
5	80	25	400
6	86	36	516
$\Sigma x = 21$	$\Sigma y = 402$	$\Sigma x^2 = 91$	$\Sigma xy = 1545$
$N = 6$			

$$y = a + bx$$

$$\Sigma y = aN + b\Sigma x \quad \therefore 402 = 6a + 21b \quad \text{--- (1)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad 1545 = 21a + 91b \quad \text{--- (2)}$$

$$\therefore 6a + 21b = 402 \quad \text{--- (1)}$$

$$21a + 91b = 1545 \quad \text{--- (2)}$$

$$a = 39.38, b = 7.89$$

$$y = 39.38 + 7.89x$$

Ex ② Fit a straight line

④

X:	1965	1966	1967	1968	1969
Y:	125	140	165	195	200

Soln. $\bar{x} = 1967, \bar{y} = 165$

X	Y	$x = X - 1967$	$y = Y - 165$	x^2	xy
1965	125	-2	-40	4	-80
1966	140	-1	-25	1	-25
1967	165	0	0	0	0
1968	195	1	30	1	30
1969	200	2	35	4	70
				$\sum x^2 = 10$	$\sum xy = 205$
				$\sum x = 0$	$\sum y = 0$

$$Y = a + bx$$

$$\sum Y = aN + b\sum X$$

$$\sum XY = a\sum X + b\sum X^2$$

$$0 = 5a + 0 \quad a = 0$$

$$205 = 0 + b \cdot 10$$

$$b = \frac{205}{10} = 20.5$$

$$Y = 20.5X$$

$$Y - 165 = 20.5(X - 1967)$$

$$Y = 165 + 20.5X - (20.5)(1967)$$

$$Y = 20.5X + 165 - 40323.5$$

$$Y = 20.5X - 40158.5$$

Fitting a parabola

(5)

$$y = a + bx + cx^2$$

$$x \neq a$$

$$\sum y = aN + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

are normal eqns

Ex: Fit a second degree parabola

curve

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

LL

Soln Since the values of x are odd

we change $x = x - \text{middle term}$

$$x = x - 5$$

$$y = y$$

The eqn of the parabola $y = a + bx + cx^2$

$$\sum y = aN + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

are normal eqns

Calculation of Σx , Σx^2 , Σx^3

Calculation of Σx , Σx^2 , Σx^3

(6)

Sr. No.	x	$x-5$	x^2	x^3	x^4	$x=y$	xy	x^2y
1	1	-4	16	-64	256	2	-8	32
2	2	-3	9	-27	81	6	-18	54
3	3	-2	4	-8	16	7	-14	28
4	4	-1	1	-1	1	8	-8	8
5	5	0	0	0	0	10	0	0
6	5	0	0	0	0	10	0	0
7	6	1	1	1	1	11	11	11
8	7	2	4	8	16	11	22	24
9	8	3	9	27	81	10	30	90
	9	4	16	64	256	9	36	144

$$n=9 \quad \Sigma x=0 \quad \Sigma x^2=60 \quad \Sigma x^3=0 \quad \Sigma x^4=708 \quad \Sigma x=74 \quad \Sigma xy=51 \quad \Sigma x^2y=411$$

$$74 = 9a + 60c \quad \text{--- (1)}$$

$$51 = 60b \quad \text{--- (2)} \quad b = 0.85$$

$$411 = 60a + 708c \quad \text{--- (3)}$$

$$a = 10.004, \quad c = -0.263 = -0.27$$

$$y = 10 + 0.85x - 0.27x^2 \quad \begin{matrix} x = x-5 \\ y = y \end{matrix}$$

$$y = 10 + 0.85(x-5) - 0.27(x-5)^2$$

$$y = -1 + 3.55x - 0.27x^2$$

Ex ② Fit a second degree parabola

⑦

Curve and find production in 1982

Year x : 1974 75 76 77 78 79 80 81
 production y : 12 14 26 42 40 50 52 53
 in t/m

Soln Since values of x are even then

$$x = x - (\text{mean of middle term}) = x - \frac{(1972+1978)}{2} = x - \frac{(1972+1978)}{2}$$

Sr no	x	x^2	x^3	x^4	y	xy	x^2y
1	1974	-7	-343	2401	12	-84	588
2	1975	-5	-125	625	14	-70	350
3	1976	-3	-27	81	26	-78	234
4	1977	-1	-1	1	42	-42	42
5	1978	1	1	1	40	40	40
6	1979	3	27	81	50	150	450
7	1980	5	125	625	52	260	1300
8	1981	7	343	2401	53	371	2597

$$\begin{aligned} \sum x^2 &= 168 \\ \sum x^3 &= 0 \\ \sum x^4 &= 6216 \\ \sum y &= 489 \\ \sum xy &= 547 \\ \sum x^2y &= 561 \end{aligned}$$

eqn of parabola

$$y = a + bx + cx^2$$

$$\sum y = a \sum 1 + b \sum x + c \sum x^2$$

$$\sum xy = a \sum x + b \sum x^2 + c \sum x^3$$

$$\sum x^2y = a \sum x^2 + b \sum x^3 + c \sum x^4$$

$$289 = 89 + 168C \quad - (1)$$

(8)

$$347 = 168b \quad - (2) \quad b = 3.2559$$

$$5601 = 168a + 621CC \quad - (3)$$

$$a = 39.7811 \quad C = -0.1741$$

$$Y = 39.78 + 3.2559x - 0.1741x^2$$

$$x = (x - 1977.5)^2 \quad Y = y$$

$$y = 39.78 + 3.2559(x - 1977.5)^2 - 0.17(x - 1977.5)^2$$

$$y = -2671997.77 + 2695.92x - 0.68x^2$$

$$x = 1982$$

$$y = 55.35$$
