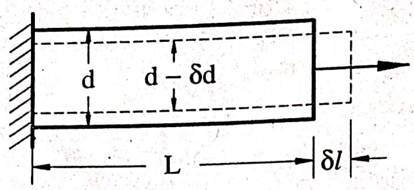
Linear Strain and Lateral Strain :

When a rod is subjected to an axial tensile load, its length increases (i.e. linear deformation), but at the same time there is decrease in diameter of the rod (i.e. lateral deformation). Therefore, there are strains in linear and lateral directions. Refer fig.



Linear Strain :

The strain in the direction of applied force is known as ***linear strain*** or *longitudinal strain* or *primary strain*. It is the ratio of change in linear dimension to the original linear dimension. It is denoted by **e**. It has no unit. It may be positive or negative.

$$\text{Linear strain} = \frac{\text{Change in linear dimension}}{\text{Original linear dimension}}$$

For a bar subjected to axial load,

$$\boxed{\text{Linear strain} \quad e = \frac{\delta l}{L}}$$ where δl = Change in length, L = Original length

Lateral Strain :

The strain in the direction at right angles to the direction of applied force is known as ***lateral strain*** or secondary strain. It is the ratio of change in lateral dimension to the original lateral dimension.

Lateral strain is always opposite to linear strain. i.e. if linear strain is tensile (+ve), then lateral strain is compressive (−ve). It is denoted by e_L. It has no unit. It may be positive or negative.

$$\text{Lateral strain} = \frac{\text{Change in lateral dimension}}{\text{Original lateral dimension}}$$

For a circular bar,

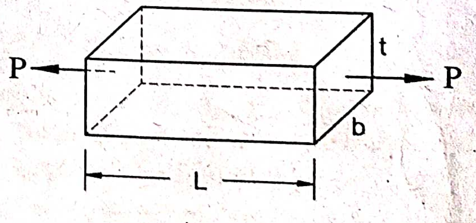
$$\boxed{\text{Lateral strain} \quad e_L = \frac{\delta d}{d}}$$ where δd = Change in diameter, d = Original diameter

For a rectangular bar,

$$\boxed{\text{Lateral strain} \quad e_L = \frac{\delta b}{b} = \frac{\delta t}{t}}$$ where δb = Change in width, b = Original width
δt = Change in thickness, t = Original thickness

Consider a bar of length **L**, width **b** and thickness **t** subjected to an axial tensile load P as shown in fig. Due to tensile load P, the length of the bar increases by amount δl. But, width and thickness decreases by amount δb and δt.

Therefore,



$$\text{Linear strain} \quad e = \frac{\delta l}{L} \text{ (Tensile) and}$$

$$\text{Lateral strain} \quad e_L = \frac{\delta b}{b} = \frac{\delta t}{t} \text{ (Compressive)}$$

Poisson's Ratio :

When a homogeneous material is loaded within its elastic limit, the ratio of the lateral strain to the linear strain is constant and is known as ***Poisson's ratio***. The value of Poisson's ratio for different materials varies from 0.25 to 0.35. It is denoted by μ or $1/m$.

$$\therefore \quad \text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}}$$

$$\therefore \quad \mu \text{ or } \frac{1}{m} = \frac{e_L}{e}$$

$$\therefore \quad \boxed{\text{Lateral strain} \quad e_L = \mu \times e \text{ or } \frac{1}{m} \times e}$$

As lateral strain is always opposite to linear strain, hence algebraically, lateral strain is written as

$$\text{Lateral strain} \quad e_L = -\mu \times e$$

Volumetric Strain :

When a body is subjected to external forces on its faces, there will be change in its volume. The ratio of change in volume to the original volume is known as ***volumetric strain***. It is denoted by e_V.

Volumetric strain $\boxed{e_V = \dfrac{\delta V}{V}}$ where δV = Change in volume, V = Original volume

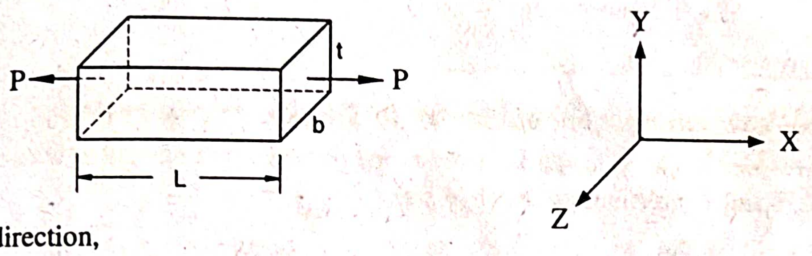
Volumetric strain is the algebraic sum of all axial or linear strains.

i.e. $$\boxed{e_V = e_x + e_y + e_z}$$

where
e_x = strain in X–direction
e_y = strain in Y–direction
e_z = strain in Z–direction

Volumetric Strain in a rectangular bar subjected to Uni-axial stress system :

Consider a rectangular bar of length L, width b and thickness t subjected to a tensile force P along X-direction as shown in fig.



Here stress in X-direction,

$$\sigma_x = \frac{P}{A} = \frac{P}{b \times t}$$

Since there is no load in Y and Z direction, stresses in Y and Z directions are zero.

i.e. $$\sigma_y = \sigma_z = 0$$

Strain in X-direction = Linear strain due to σ_x

$$e_x = \frac{\sigma_x}{E}$$

Strain in Y-direction = Lateral strain due to σ_x

$$e_y = -\mu \times e_x = -\mu \frac{\sigma_x}{E}$$

Strain in Z-direction = Lateral strain due to σ_x

$$e_z = -\mu \times e_x = -\mu \frac{\sigma_x}{E}$$

∴ Volumetric strain

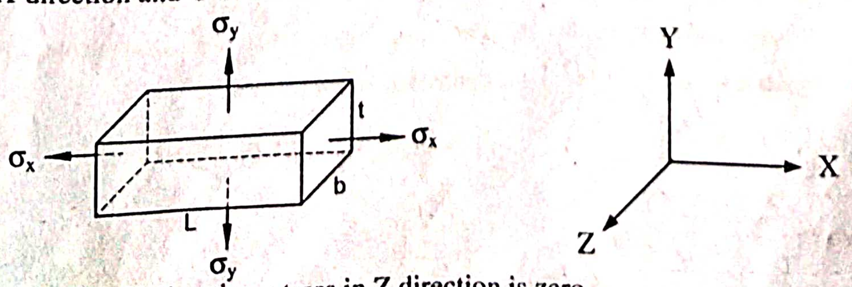
$$e_V = e_x + e_y + e_z$$

∴ $$e_V = \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E}$$

∴ $$\boxed{e_V = \frac{\delta V}{V} = \frac{\sigma_x}{E}(1 - 2\mu)} = e_x(1 - 2\mu)$$

Volumetric Strain in a rectangular bar subjected to Bi-axial stress system:

Consider a rectangular bar of length L, width b and thickness t subjected to a two tensile stresses σ_x and σ_y along X-direction and Y-direction as shown in fig.



Since there is no load in Z direction, stress in Z direction is zero.

i.e. $$\sigma_z = 0$$

Strain in X-direction = Linear strain due to σ_x + Lateral strain due to σ_y

$$e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

Strain in Y-direction = Linear strain due to σ_y + Lateral strain due to σ_x

$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

Strain in Z-direction = Lateral strain due to σ_x + Lateral strain due to σ_y

$$e_z = -\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

∴ Volumetric strain

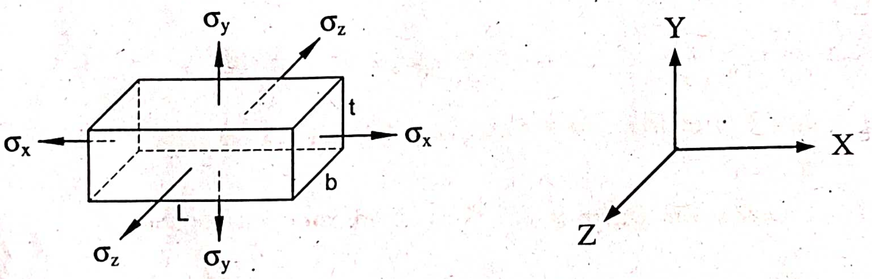
$$e_V = e_x + e_y + e_z$$

∴ $$e_V = \left[\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right] + \left[\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}\right] + \left[-\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right]$$

∴ $$\boxed{e_V = \frac{\delta V}{V} = \frac{\sigma_x + \sigma_y}{E}(1-2\mu)}$$

Volumetric Strain in a rectangular bar subjected to Tri-axial stress system :

Consider a rectangular bar of length L, width b and thickness t subjected to a three tensile stre σ_x, σ_y and σ_z along X, Y and Z directions as shown in fig.



Strain in X-direction = Linear strain due to σ_x + Lateral strains due to σ_y and σ_z

$$e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

Strain in Y-direction = Linear strain due to σ_y + Lateral strains due to σ_x and σ_z

$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

Strain in Z-direction = Linear strain due to σ_z + Lateral strains due to σ_x and σ_y

$$e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

∴ Volumetric strain

$$e_V = e_x + e_y + e_z$$

∴ $$e_V = \left[\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right] + \left[\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}\right] + \left[\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right]$$

∴ $$\boxed{e_V = \frac{\delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)}$$

Generalized Hooke's law

The generalized Hooke's law shows the linear relationship beween stress and strain.

For 2-Dimensional stress system

$$e_x = \frac{1}{E}[\sigma_x - \mu\sigma_y]$$

$$e_y = \frac{1}{E}[\sigma_y - \mu\sigma_x]$$

$$e_z = \frac{1}{E}[-\mu(\sigma_x + \sigma_y)]$$

For 3-Dimensional stress system

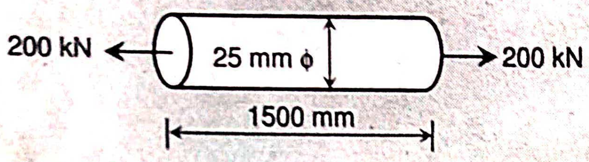
$$e_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)]$$

$$e_y = \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_z)]$$

$$e_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)]$$

1. A bar of steel 25 mm diameter, 1500 mm long is subjected to an axial tensile force of 200 kN. If $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$, calculate

1) Normal stress,
2) Linear strain,
3) Lateral strain,
4) Change in length of bar,
5) Change in diameter of bar, and
6) Change in volume of bar.



B.C.[M 15]

Solution : Given : $d = 25$ mm, $L = 1500$ mm, $P = 200$ kN $= 200 \times 10^3$ N (T), $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$

$\therefore A = \frac{\pi}{4} d^2 = \frac{\pi}{4} 25^2 = 156.25\pi \text{ mm}^2$ and $V = A \times L = 156.25\pi \times 1500 = 234375\pi \text{ mm}^3$

1) Normal stress $\sigma = \frac{P}{A} = \frac{200 \times 10^3}{156.25\pi} =$ **407.43 N/mm² (T)**

2) Linear strain $e = \frac{\sigma}{E} = \frac{636.61}{2 \times 10^5} =$ **2.03718×10^{-3} (T)**

3) Lateral strain $e_L = \mu \times e = 0.3 \times 2.03718 \times 10^{-3} =$ **6.1115×10^{-4} (C)**

4) As Linear strain $e = \frac{\delta l}{L}$

$\therefore$ Change in length $\delta l = e \times L = 2.03718 \times 10^{-3} \times 1500 =$ **3.055 mm** (elongation)

5) As Lateral strain $e_L = \frac{\delta d}{d}$

$\therefore$ Change in dia. $\delta d = e_L \times d = 6.1115 \times 10^{-4} \times 20 =$ **0.0152 mm** (contraction)

6) As Volumetric Strain $e_V = \frac{\delta V}{V} = e(1 - 2\mu) = 2.03718 \times 10^{-3}(1 - 2 \times 0.3) = 8.1487 \times 10^{-4}$

$\therefore$ Change in volume $\delta V = e_V \times V = 8.1487 \times 10^{-4} \times 234375\pi =$ **600 mm³** (expansion)

6. **A bar of steel 80 mm × 80 mm × 200 mm long is subjected to a tensile load of 350 kN along the longitudinal axis and tensile loads of 750 kN and 500 kN on lateral faces. Find the change in the dimensions of the bar and change in volume.**
Take : $E = 2.1 \times 10^5$ N/mm^2, $\mu = 0.3$. **AM[W 14], B[M 09], B[N 07]**

Solution : Given :

$L = 200$ mm, $b = 80$ mm, $t = 80$ mm,

$P_x = 350$ kN $= 350 \times 10^3$ N (T),

$P_y = 750$ kN $= 750 \times 10^3$ N (T),

$P_z = 500$ kN $= 500 \times 10^3$ N (T),

$E = 2.1 \times 10^5$ N/mm^2, $\mu = 0.3$,

<u>Find δl, δb, δt, and δV = ?</u>