## Composite Bar :

A bar made up of two or more different materials, joined together is called a composite bar. The bars are joined in such a way that the system extends or contracts as one unit, equally. when subjected to tension or compression.

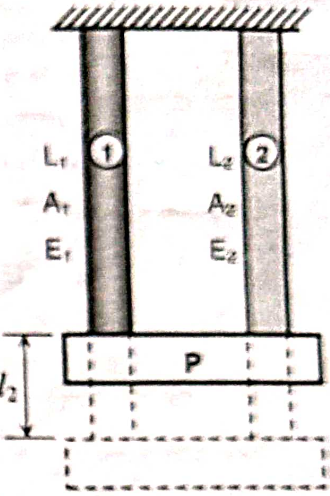
Consider two rods of different materials subjected to load P as shown in fig.

Let $P$ = Total load on the bars,

$L_1, L_2$ = Length of bar 1 and bar 2,

$A_1, A_2$ = Area of bar 1 and bar 2,

$E_1, E_2$ = Young's modulus of bar 1 and bar 2.




Following two conditions should be used for solving examples on composite bars :

**1) Change in length of bar 1 = Change in length of bar 2**

i.e. $\boxed{\delta l_1 = \delta l_2}$

$$\therefore \quad \frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\therefore \quad \frac{\sigma_1 L_1}{E_1} = \frac{\sigma_2 L_2}{E_2} \qquad \left[\because \ \sigma = \frac{P}{A}\right]$$

when $L_1 = L_2$

$$\therefore \quad \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \qquad (\text{i.e. } e_1 = e_2)$$

$$\therefore \quad \sigma_1 = \frac{E_1}{E_2}\sigma_2$$

$$\therefore \quad \sigma_1 = m\sigma_2 \qquad \left(\text{where } m = \text{modular ratio} = \frac{E_1}{E_2}\right)$$

**2) Total load = Load carried by bar 1 + Load carried by bar 2**

i.e. $\boxed{P = P_1 + P_2}$

$$\therefore \quad P = \sigma_1 A_1 + \sigma_2 A_2$$

From condition (1), relation between $\sigma_1$ and $\sigma_2$ can be obtained and from condition (2) values of $\sigma_1$ and $\sigma_2$ can be obtained.

## Modular Ratio (m) :

The ratio of Young's modulus of two different materials is called ***modular ratio***. It is denoted by 'm'.

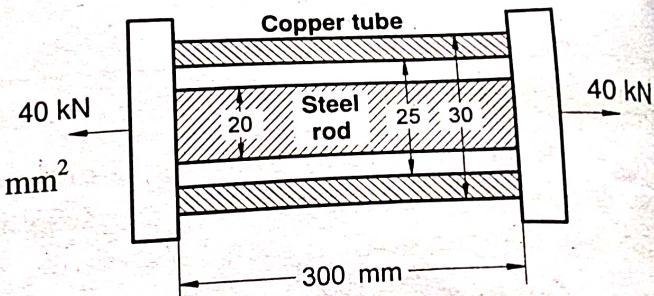
$$\therefore \quad m = \frac{E_1}{E_2}$$

Where, $E_1$ = Young's modulus of material (1) and

$E_2$ = Young's modulus of material (2)

**1. A steel rod of 20 mm diameter and 300 mm long is enclosed centrally inside a hollow copper tube of external diameter 30 mm and internal diameter 25 mm. The ends of the tube and rods are brazed together, and composite bar is subjected to an axial pull of 40 kN. Find the stresses developed in rod and tube. Also find the extension of the rod.**
**Take $E_s = 200\ GN/m^2$ and $E_c = 100\ GN/m^2$.** **B[N 05]**

**Solution :** Given : $d_s = 20$ mm, $D_c = 30$ mm, $d_c = 25$ mm, $L_s = L_c = 300$ mm, $P = 40\ kN = 40 \times 10^3\ N\ (T)$,
$E_s = 200 \times 10^3\ N/mm^2$, $E_c = 100 \times 10^3\ N/mm^2$




$$\therefore \quad A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} 20^2 = 100\ \pi\ mm^2$$

$$\therefore \quad A_c = \frac{\pi}{4}(D_c^2 - d_c^2) = \frac{\pi}{4}(30^2 - 25^2) = 68.75\ \pi\ mm^2$$

<u>Find $\sigma_s = ?, \sigma_c = ?, \delta l_s = \delta l_c = ?$</u>

**Condition (1) :** Change in length of steel rod = Change in length of copper tube

i.e. $\boxed{\delta l_s = \delta l_c}$

or $e_s = e_c$ $\quad (\because L_s = L_c)$

$$\therefore \quad \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \quad \sigma_s = \frac{E_s}{E_c}\sigma_c = \frac{200 \times 10^3}{100 \times 10^3}\sigma_c$$

$$\therefore \quad \sigma_s = 2\sigma_c \qquad \text{.......... (1)}$$

**Condition (2) :** Total load = Load carried by steel rod + Load carried by copper tube

i.e. $\boxed{P = P_s + P_c}$

$$\therefore \quad P = \sigma_s A_s + \sigma_c A_c \qquad \text{.......... (2)}$$

$$\therefore \quad 40 \times 10^3 = 2\sigma_c \times 100\ \pi + \sigma_c \times 68.75\ \pi \quad = 844.30\ \sigma_c$$

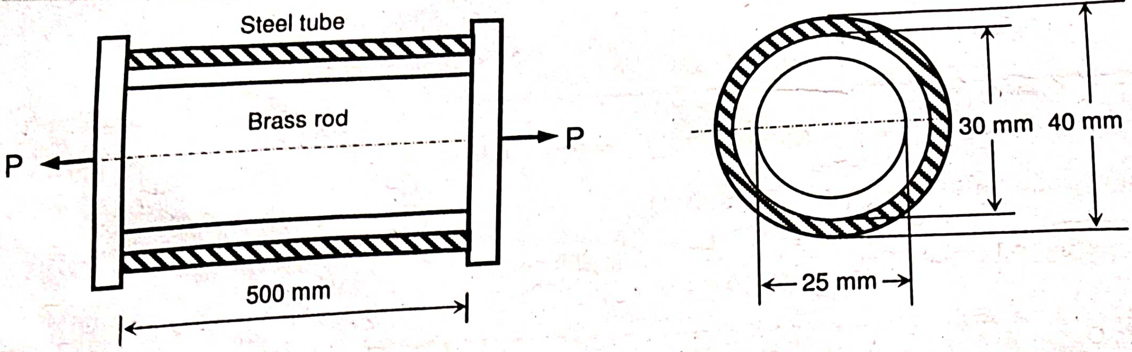
$$\therefore \quad \sigma_c = \frac{40 \times 10^3}{844.30} = \mathbf{47.37\ N/mm^2}\ \text{(Tensile)}$$

and $\sigma_s = 2\sigma_c = 2 \times 47.37 = \mathbf{94.75\ N/mm^2}$ (Tensile)

Extension of rod,

$$\delta l_s = \frac{\sigma_s L_s}{E_s} = \frac{94.75 \times 300}{200 \times 10^3} = \mathbf{0.142\ mm}$$

**4. A composite bar is made up of a brass rod of 25 mm dia. enclosed in a steel tube of 40 mm external dia. and 30 mm dia. as shown in fig. The rod and tube being co-axial and equal in length are securely fixed at each end. If the stresses in brass and steel are not toexceed 70 MPa and 120 MPa respectively. Find the load (P) the composite bar can safely carry. Also find the change in length, if the composite bar is 500 mm long. Take $E_s$ = 200 GPa and $E_b$ = 80 GPa.** **Sol.C.[N 13**




**Solution :** Given : $d_b = 25$ mm, $D_s = 40$ mm, $d_s = 30$ mm,

$$\therefore A_b = \frac{\pi}{4} d_b^2 = \frac{\pi}{4} 25^2 = 156.25\pi \text{ mm}^2 \qquad \therefore A_s = \frac{\pi}{4}(D_s^2 - d_s^2) = \frac{\pi}{4}(40^2 - 30^2) = 175\pi \text{ mm}^2$$

$L_b = L_s = 500$ mm,

$\sigma_{b(allowable)} = 70$ N/mm², $\sigma_{s(allowable)} = 120$ N/mm²

$E_b = 80 \times 10^3$ N/mm², $E_s = 200 \times 10^3$ N/mm²

Find P = ?

**Condition (1) :** Change in length of steel tube = Change in length of brass rod

i.e. $\boxed{\delta l_s = \delta l_b}$

or $e_s = e_b$ $\quad (\because L_s = L_b)$

$$\therefore \quad \frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\therefore \quad \sigma_s = \frac{E_s}{E_b}\sigma_b = \frac{200 \times 10^3}{80 \times 10^3}\sigma_b$$

$$\therefore \quad \sigma_s = 2.5\,\sigma_b \quad \text{.......... (1)}$$

In equation (1),

If we put $\sigma_b$ = 70 N/mm², then $\sigma_s$ = 175 N/mm² > $\sigma_{s(allowable)}$ = 120 N/mm²; hence not safe.

If we put $\sigma_s$ = 120 N/mm², then $\sigma_b$ = 48 N/mm² < $\sigma_{s(allowable)}$ = 70 N/mm²; hence safe.

$\therefore$ $\sigma_s$ = 120 N/mm² and $\sigma_b$ = 48 N/mm²

**Condition (2) :** Total load = Load carried by steel tube + Load carried by brass rod

i.e. $\boxed{P = P_s + P_b}$

$$\therefore \quad P = \sigma_s A_s + \sigma_b A_b \quad \text{.......... (2)}$$

$$\therefore \quad P = 120 \times 175\pi + 48 \times 156.25\pi = 89535.39 \text{ N} = \mathbf{895.35 \text{ kN}}$$

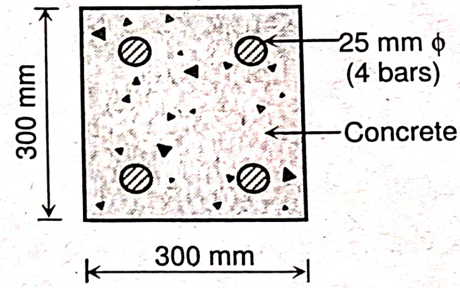
Change in length of composite bar (i.e. $\delta l_s$ or $\delta l_b$)

$$\delta l = \delta l_s = \frac{\sigma_s L_s}{E_s} = \frac{120 \times 500}{200 \times 10^3} = \mathbf{0.3 \text{ mm}} \text{ (elongation)}$$

**5. A load of 400 kN is applied on a short column 300 mm × 300 mm. The column is reinforced by steel bars of 25 mm diameter and 4 nos. If the modulus of elasticity for steel is 15 times that of concrete, find the stresses in concrete and steel.** **B.C.[N 06], B.Tech.[N 06]**

**Solution :** Given : $P = 400\text{ kN} = 400 \times 10^3\text{ N (C)}$, $d_s = 25\text{ mm}$, No. of bars = 4,

$$\therefore \quad A_s = 4\times\frac{\pi}{4}d_s^{\,2} = 4\times\frac{\pi}{4}25^2 = 625\pi\text{ mm}^2 = 1963.49\text{ mm}^2$$




$$\therefore \quad A_c = (300\times300) - A_s = (300\times300) - 625\pi = 88036.50\text{ mm}^2$$

$$E_s = 15\,E_c, \quad \therefore \text{ Modular ratio } m = 15.$$

Find $\sigma_s$ , $\sigma_c$ = ?

**Condition (1) :** Change in length of steel bar = Change in length of concrete

i.e. $\boxed{\delta l_s = \delta l_c}$

or $e_s = e_c \qquad (\because L_s = L_c)$

$$\therefore \quad \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \quad \sigma_s = \frac{E_s}{E_c}\sigma_c$$

$$\therefore \quad \sigma_s = m\,\sigma_c = 15\,\sigma_c \qquad \ldots\ldots\ldots (1)$$

**Condition (2) :** Total load = Load carried by steel rods + Load carried by concrete

i.e. $\boxed{P = P_s + P_c}$

$$\therefore \quad P = \sigma_s A_s + \sigma_c A_c \qquad \ldots\ldots\ldots (2)$$

$$\therefore \quad 400\times10^3 = 15\,\sigma_c\times625\pi + \sigma_c\times88036.50$$

$$\therefore \quad 400\times10^3 = 117488.93\,\sigma_c$$

$$\therefore \quad \sigma_c = \frac{400\times10^3}{117488.93} = \mathbf{3.40\ N/mm^2}\text{ (Compressive)}$$

Stress developed in concrete is 3.40 N/mm² (Compressive)

From equation (1)

and $\sigma_s = 15\,\sigma_c = 15\times3.40 = \mathbf{51.06\ N/mm^2}$ (Compressive)

Stress developed in steel is 51.06 N/mm² (Compressive)