

Laplace transform - I

Let $f(t)$ be a function of t $t \geq 0$ is defined by definite integral $\int_0^{\infty} e^{-st} f(t) dt$ if it exists is called Laplace transform of $f(t)$ is denoted by $L\{f(t)\}$

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s) \text{ or } f(s) = \phi(s)$$

$$t \rightarrow s$$

L.T. of standard functions

$$\textcircled{1} \quad L\{e^{at}\} = \frac{1}{s-a}$$

$$L\{\bar{e}^{at}\} = \frac{1}{s+a}$$

$$\textcircled{2} \quad L[1] = \frac{1}{s}$$

$$\textcircled{3} \quad L\{t^n\} = \frac{n!}{s^{n+1}} \quad \text{if } n \text{ is a non-negative integer}$$

$$= \frac{T_{n+1}}{s^{n+1}} \quad \text{if } n \text{ is a fraction.}$$

$$\textcircled{4} \quad \Gamma_{n+1} = n! \quad \text{if } n \text{ is a non-negative integer}$$

$$= n \Gamma_n \quad \text{if } n \text{ is a fraction}$$

$$\text{eg. } \Gamma_4 = \Gamma_3 + 1 = 3! = 1 \cdot 2 \cdot 3 = 6$$

$$\Gamma_{\frac{3}{2}} = \Gamma_{\frac{1}{2} + 1} = \frac{1}{2} \Gamma_{\frac{1}{2}} = \frac{\pi}{2} \therefore \Gamma_{\frac{1}{2}} = \sqrt{\pi}$$

$$\text{eg. } \Gamma_{\frac{9}{2}} = \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \Gamma_{\frac{1}{2}} = \frac{105 \sqrt{\pi}}{16}$$

$$(4) L[\sin kt] = \frac{k}{s^2 + k^2}$$

$$(5) L[f(\cosh kt)] = \frac{s}{s^2 + k^2}$$

$$(6) L\{\sinh kt\} = \frac{k}{\frac{s}{2} - k^2}$$

$$(7) L[\cosh kt] = \frac{s}{\frac{s}{2} - k^2}$$

$$(8) L[af_1(t) + b f_2(t)] = a L[f_1(t)] + b L[f_2(t)]$$

$$(9) \text{Find } L^{-1} \text{ of } (\sin 2t - \cos 2t)^2$$

$$(1) 2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$(2) 2 \cot A \sin B = \sin(A+B) - \sin(A-B)$$

$$(3) 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$(4) \frac{2 \sin A \cos I}{2 \sin A \sin B} = \cos(A-B) - \cos(A+I)$$

~~Sin + Cos Sin~~

$$\sin 2x = 2 \sin x \cos x, \quad \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$\sin^2 x + \cos^2 x = 1$$

$$(1) \sin^2 x = \frac{1 - \cos 2x}{2}, \quad \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$(2) \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

$$\sin^3 x = \underline{3 \sin x - \sin 3x}$$

$$\cos^3 x = \underline{\frac{3 \cos x + \cos 3x}{4}}$$

$$\cosh^2 x = \frac{1 + \cosh 2x}{2}, \quad \sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\sqrt{1 + \sin x} = \sqrt{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}$$

$$= \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2} = \sin \frac{x}{2} + \cos \frac{x}{2}$$

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Ex(1) Find L.T. of $(\sin 2t - \cos 2t)^2$

$$\text{Soln: } L[(\sin 2t - \cos 2t)^2]$$

$$= L[\underline{\sin^2 2t} - 2\sin 2t \cos 2t + \underline{\cos^2 2t}]$$

$$= L[1 - \sin 2(2t)]$$

$$= L[1] - L[\sin 4t]$$

$$= \frac{1}{s} - \frac{s}{s^2 + 16}$$

Ex(2) Find L.T. of $\sqrt{1 + \sin t}$

$$\text{Soln: } L[\sqrt{1 + \sin t}]$$

$$= L[\sqrt{\underline{\sin^2 \frac{t}{2}} + \underline{\cos^2 \frac{t}{2}} + 2\sin \frac{t}{2} \cos \frac{t}{2}}]$$

$$= L[(\sin \frac{t}{2} + \cos \frac{t}{2})^2]^{\frac{1}{2}}$$

$$= L[\sin \frac{t}{2}] + L[\cos \frac{t}{2}]$$

$$= \frac{\frac{1}{2}}{\frac{s^2 + 1}{4}} + \frac{s}{s^2 + \frac{1}{4}} = \frac{\frac{1}{2} - \frac{1}{4}}{4s^2 + 1} + \frac{4s}{4s^2 + 1}$$

$$= \frac{2 + 4s}{4s^2 + 1}$$

Ex(3) Find L.T. of $e^{2t} + 4t^3 - \sin t \cos 3t$

$$\text{Soln: } L[e^{2t} + 4t^3 - \sin t \cos 3t]$$

$$= L[e^{2t} + 4(t^3) - \frac{1}{2} L[2\sin t \cos 3t]]$$

$$= \frac{1}{s-2} + 4 \cdot \frac{3!}{s^3 + 1} - \frac{1}{2} L[\sin 5t + \sin(4t)]$$

$$= \frac{1}{s-2} + \frac{24}{s^3 + 1} - \frac{1}{2} [\frac{5}{s^2 + 25} - \frac{1}{s^2 + 1}]$$

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H.W

$$\text{Q) } L[\cos t, \cos 2t, \cos 3t]$$

$$\text{Q) Find } L[\sin^5 t], L[\sinh^3 t]$$

$$\text{SOLN: } \sin \text{nat} = \frac{e^{iat} - e^{-iat}}{2i}$$

$$(\sin t)^5 = \left(\frac{e^{it} - e^{-it}}{2i} \right)^5$$

$$\begin{aligned} \sin^5 t &= \frac{1}{32i} \left[5(0(e^{it})^5 (e^{-it})^0 - 5(1(e^{it})^4 (e^{-it})^1 \right. \\ &\quad \left. + 5(2(e^{it})^3 (e^{-it})^2 - 5(3(e^{it})^2 (e^{-it})^3 \right. \\ &\quad \left. + 5(4(e^{it})^1 (e^{-it})^4 - 5(5(e^{it})^0 (e^{-it})^5) \right] \end{aligned}$$

$$\begin{aligned} \sin^5 t &= \frac{1}{32i} \left[\frac{e^{5it} - e^{-5it}}{2i} - 5 \frac{e^{4it} - e^{-4it}}{2i} \right. \\ &\quad \left. + 10 \frac{e^{3it} - e^{-3it}}{2i} - 10 \frac{e^{2it} - e^{-2it}}{2i} \right. \\ &\quad \left. + 10 \frac{e^{it} - e^{-it}}{2i} - 5 \frac{e^{it} - e^{-it}}{2i} \right] \end{aligned}$$

$$= \frac{1}{32i} \left[\frac{e^{5it} - e^{-5it}}{2i} - 5 \frac{e^{3it} - e^{-3it}}{2i} + 10 \frac{e^{it} - e^{-it}}{2i} \right]$$

$$\begin{aligned} &= \frac{1}{16} \left[\left(\frac{e^{5it} - e^{-5it}}{2i} \right) - 5 \left(\frac{e^{3it} - e^{-3it}}{2i} \right) \right. \\ &\quad \left. + 10 \left(\frac{e^{it} - e^{-it}}{2i} \right) \right] \end{aligned}$$

$$\sin^5 t = \frac{1}{16} [\sin 5t - 5 \sin 3t + 10 \sin t]$$

$$L[\sin^5 t] = \frac{1}{16} [L[\sin 5t] - 5 L[\sin 3t]]$$

$$+ 10 L[\sin t]$$

$$= \frac{1}{16} \left[\frac{5}{s^2 + 25} - 5 \cdot \frac{3}{s^2 + 9} + 10 \cdot \frac{1}{s^2 + 1} \right]$$

$$= \frac{5}{16} \left[\frac{1}{s^2 + 25} - \frac{3}{s^2 + 9} + \frac{2}{s^2 + 1} \right]$$