Q.1 1.81, 1.24, 1.06, 1.80, 2.15, 1.93, 1.97, 1.69, 1.91,
1.86, 1.08, 2.53, 1.12, 2.74, 1.69, 1.23, 1.24,
1.80, 2.30, 1.54.

data in Ascending order
1.06, 1.08, 1.12, 1.23, 1.24, 1.24, 1.54, 1.59, 1.69,
1.80, 1.80, 1.81, 1.86, 1.91, 1.93, 1.97, 2.15, 2.3,
2.53, 2.74.

Intervals.	observed freq	expected freq
1-1.5	6	5
1.5-2	10	5
2-2.5	2	5
2.5-3	2	5

uniform distribution.
expected value = $\frac{20}{4} = 5$.

H_0 = samples are uniformly distributed.

$$\sum_{i=1}^{4} \frac{(o_i - e_i)^2}{e_i} = \frac{(6-5)^2}{5} + \frac{(10-5)^2}{5} + \frac{(2-5)^2}{5} + \frac{(2-5)^2}{5}$$

$$= \frac{1}{5} + \frac{25}{5} + \frac{9}{5} + \frac{9}{5} = 8.8.$$

$\chi^2_{3,0.05} = 7.815$.

& $8.8 > 7.815$ ∴ Null Hypothesis is rejected & samples has not been generated from $U(1,3)$

Q.2.

years	22	25	14	6	3
rainstorms annually	0	1	2	3	4

soln - poisson distribution is

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} ; x = 0,1,2,3,\ldots$$

we need the value of λ

$$\text{mean} = \frac{\Sigma f x}{\Sigma f} = \frac{83}{70} = 1.185.$$

x	f	fx
0	22	0
1	25	25
2	14	28
3	6	18
4	3	12
	$\Sigma f = 70$	$\Sigma fx = 83$

Null Hypothesis H_0 = The poisson distribution is best fit to the given info

calculate E & χ^2

$$E = 70 \times \frac{e^{-1.185} \times 1.185^{x}}{x!}$$

rainstorms	years(O)	(E)	$(O-E)^2/E$
0	22	21.40	0.0166
1	25	25.36	0.00515
2	14	15.02	0.0701
3	6	5.93	0.00069
4	3	1.75	0.8766.
			$\Sigma = 0.9693$

From chi-square table $\chi^2_{3,0.05} = 7.815$.

Since computed value of W is 0.9693 which is less than the 7.815, we Accept the null hypothesis

Q.3. pdf of two parameter exponential distribution

$$f(x) = \frac{1}{\sigma} e^{-\frac{(x-\mu)}{\sigma}}$$

cdf of the distribution

$$F(x) = \frac{1}{\sigma} \int_{\mu}^{x} e^{-\left(\frac{x-\mu}{\sigma}\right)} dx$$

$$= \frac{1}{\sigma} \int_{\mu}^{x} e^{-x/\sigma} \times e^{\mu/\sigma} dx.$$

$$= \frac{e^{\mu/\sigma}}{\sigma} \int_{\mu}^{x} e^{-x/\sigma} dx$$

$$= -e^{\mu/\sigma} \left[e^{-x/\sigma}\right]_{\mu}^{x}$$

$$= -e^{\mu/\sigma} \left[e^{-x/\sigma} - e^{-\mu/\sigma}\right]$$

$$= e^{\mu/\sigma} \left[e^{-\mu/\sigma} - e^{-x/\sigma}\right]$$

$$= 1 - e^{-\left(\frac{x-\mu}{\sigma}\right)}$$

$$1 - e^{-\frac{(x-\mu)}{\sigma}} = U.$$

$$e^{-\left(\frac{x-\mu}{\sigma}\right)} = 1-U$$

$$-\left(\frac{x-\mu}{\sigma}\right) = \log(1-U)$$

$$x - \mu = -\sigma \log(1-U)$$

$$x = \mu - \sigma \log(1-U)$$

$$f(x/\theta) = \frac{1}{\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)}. \quad \mu \leq x < \infty, \ \sigma > 0.$$

$$\theta(\mu, \sigma) \qquad \cdot \Theta = \{(\mu, \sigma), \mu \in R, \sigma > 0\}.$$

A sample of size $n=30$ is generated ~~from~~ in R program

	observed freq
$x \leq 2$	0
$2 \leq x \leq 3$	22
$3 \leq x < 4$	4
$4 \leq x \leq 5$	2
$x \geq 5$	2

expected values . $E = np$.

O	observed	expected	$(O-E)^2/E$
$x \leq 2$	0	0	0
$2 \leq x \leq 3$	22	18.964	9.217/18.964
$3 \leq x < 4$	4	6.976	8856/6.976
$4 \leq x \leq 5$	2	2.566	0.320/2.566
$x \geq 5$	2	1.494	0.256/1.494
			$\Sigma = 2.054$

$\chi^2_{4,0.05} = 9.488$.

$2.054 < 9.488$ $\therefore$ we Accept the null Hypothesis

Thus we can conclude that the generated sample follows $f(x/\theta_n)$