Exam: Bayesian Statistics

Belgian Falcons

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Frequentist Approach

Linear regression, t-test and ANOVA boil down to the same regression equation:

Regression:

$$Y = \beta_0 + \beta_1 X_{1,(\text{continuous})} + \epsilon \tag{1}$$

t-test:

$$Y = \beta_0 + \beta_1 X_{1,(\text{dummy variable})} + \epsilon \tag{2}$$

ANOVA:

$$Y = \beta_0 + \beta_1 X_{1,(\text{dummy variable})} + \beta_2 X_{2,(\text{dummy variable})} + \epsilon \tag{3}$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2)$

Note: The t-test is a two-sample case of ANOVA

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Parameter Estimation

- Estimate the magnitude of the parameter
- Determine whether the credible interval is significantly different from the null value
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Model Comparison

- Decide which of two hypothetical prior distributions is least incredible
- One prior expresses the parameter value as the null value
- Alternative prior expresses that hypothesis could be any value

Parameter Estimation

- Determine HDI: via JAGS or conjugate analysis
- Reject or accept parameter value based on location to ROPE
- Investigate with prior sensitivity

Model Comparison

Compare $Pr(H_0|data)$ with $Pr(H_a|data)$

$$\frac{Pr(H_0|data)}{Pr(H_a|data)} = \frac{Pr(data|H_0) \times Pr(H_0)}{Pr(data|H_a) \times Pr(H_a)} \tag{4}$$

$$\frac{Pr(H_0|data)}{Pr(H_a|data)} = \text{BayesFactor} \times \frac{Pr(H_0)}{Pr(H_a)} \tag{5}$$

Model Comparison

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Use conjugate Normal Inverse Gamma prior, i.e. $NIG(a, d, \mathbf{m}, \mathbf{V})$

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$$B = \frac{|\mathbf{V}|^{\frac{1}{2}} |\mathbf{V}_{\mathbf{A}}^{\star}|^{\frac{1}{2}}}{|\mathbf{V}_A|^{\frac{1}{2}} |\mathbf{V}^{\star}|^{\frac{1}{2}}} \left(\frac{a^{\star}}{a_A^{\star}} \right)^{\frac{d^{\star}}{2}} \tag{6}$$

where

$$\mathbf{V}^{\star} = (\mathbf{V}^{-1} + \mathbf{X}'\mathbf{X})^{-1} \tag{7}$$

$$\mathbf{m}^{\star} = (\mathbf{V}^{-1} + \mathbf{X}'\mathbf{X})^{-1}(\mathbf{V}^{-1}\mathbf{m} + \mathbf{X}'\mathbf{y}) \tag{8}$$

$$a^{\star} = a + \mathbf{m}'\mathbf{V}^{-1}\mathbf{m} + \mathbf{y}'\mathbf{y} - (\mathbf{m}^{\star})'(\mathbf{V}^{\star})^{-1}\mathbf{m}^{\star} \tag{9}$$

Tip: read the accompanying chapter

Model Comparison

- Investigate the prior sensitivity
- Choose prior parameters such that the parameter with all density on the null value (spike-shaped density)
- Choose weakly informed alternative parameters such that parameter could be any value (broad density)

Hint: Which parameter governs this shape?

Model Comparison

Not clear? Try exercise with fair coin

Beta-Bernouilli

The Bayes Factor for Beta Bernouilli is the quotient of marginal likelihoods:

$$\frac{p(z, N|H_a)}{p(z, N|H_0)} = \frac{B(z + a_a, N - z + b_a)/B(a_a, b_a)}{B(z + a_0, N - z + b_0)/B(a_0, b_0)}$$

where N is the number of trials and z the number of heads.

Model Comparison

Beta-Bernouilli

Try out different hypotheses (i.e. different priors)

```
## Specify trials
# N = number of samples
# z = number of heads
# simulate data
N <- 150
z <- rbinom(size = N, prob = 0.5, 1)
# null hypothesis: coin is fair. i.e. theta=0.5, i.e spiked at 0.5
a_null <- 500
b_null <- 500
# alt. hypothesis: coin is not fair. i.e. theta=/=0.5 or uniform density
a_alt <- 1
b_alt <- 1
# determine bayes factor
pD_null <- beta(a=z+a_null,b=n-z+b_null)/beta(a=a_null,b=b_null)
pD_alt <- beta(a=z+a_alt,b=n-z+b_alt)/beta(a=a_alt,b=b_alt)
pD_alt/pD_null
```

Good luck!