Purchasing Power Parity and Time Series Forecasting : An Econometric Study of India-Ireland Exchange Rates

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1. Introduction

Exchange rates are one of the most closely monitored economic indicators, influencing everything from international trade and investment flows to inflation, interest rates, and consumer purchasing power. Understanding the forces that drive exchange rate movements is critical for policymakers, investors, and global businesses. This report focuses on a theoretical and empirical investigation of the exchange rate between India and Ireland - two economies that differ in both scale and inflationary behavior, through the lens of Purchasing Power Parity (PPP) and time series forecasting models.

This research project investigates the exchange rate dynamics between India (home country) and Ireland (foreign country) using a decade of macroeconomic data from 2014 to 2024. The aim is to test the Purchasing Power Parity (PPP) theory in both its absolute and relative forms, and to forecast the Real Exchange Rate (RER) using Box-Jenkins ARIMA models.

We use log-transformed CPI and exchange rate data, apply standard econometric techniques such as stationarity testing, regression analysis, and build multiple ARIMA models in Python. The final objective is to assess whether PPP holds between the two countries and to produce a reliable RER forecast for 2025 to 2027 that reflects inflation-adjusted currency movements.

The project is carried out in accordance with EC6011 Business Forecasting module guidelines. It translates economic theory into practice through data analysis, visualization, and modeling, while mirroring the real-world approach used in professional forecasting and policy evaluation.

The PPP theory suggests that exchange rates adjust to equalize the price levels between countries. If valid, this would imply a predictable relationship between inflation differentials and exchange rate movements. By applying both statistical tests and forecasting techniques, this report provides empirical insight into the long-run parity condition between India and Ireland.

The study follows a systematic approach:

- Collecting and cleaning 10 years of consistent annual data
- Constructing log-transformed economic indicators
- Performing visual and statistical exploratory analysis
- Testing both absolute and relative PPP using regression
- Forecasting future exchange rates using ARIMA models

The results provide insights into the economic relationship between the two nations and offer evidence-based forecasting for currency valuation.

2. Data Collection and Preprocessing

This project relies on macroeconomic data sourced from reliable public databases. The nominal exchange rate (EUR/INR) was obtained from Investing.com, while annual Consumer Price Index (CPI) data for both India and Ireland was sourced from the OECD Economic Database. The dataset spans 2014 to 2024, providing a 10-year historical base suitable for both PPP testing and time series forecasting.

After collection, the data was prepared using Python and pandas through the following preprocessing steps:

- All column names were standardized and renamed for clarity.
- Dates were converted to a proper datetime format and set as the index to enable time-based operations.
- Any missing or inconsistent records were dropped using *dropna()* to ensure complete annual alignment.
- Series were verified to have consistent yearly frequency and were aligned across the same date range.
- The CPI and exchange rate datasets were merged into a single DataFrame using the Date column as the key.
- Log transformations were applied to the exchange rate and CPI series to prepare the data for PPP testing and ARIMA modeling.

These preprocessing steps ensured a clean, structured dataset ready for empirical testing and forecasting. The resulting dataset was used as the foundation for all subsequent analysis, including PPP regression, stationarity tests, and ARIMA model development.

India CPI:

	observation_date	INDCPIALLAINMEI
0	1957-01-01	1.525411
1	1958-01-01	1.598252
2	1959-01-01	1.671093
3	1960-01-01	1.700837
4	1961-01-01	1.729669

Ireland CPI:

	observation_date	IRLCPIALLAINMEI
0	1976-01-01	16.96255
1	1977-01-01	19.24778
2	1978-01-01	20.73200
3	1979-01-01	23.48842
4	1980-01-01	27.75261

Exchange Rate (EUR to INR):

	Date	Price	Change %
0	1/1/2014	84.524	-0.49%
1	1/1/2015	69.985	-8.21%
2	1/1/2016	73.528	2.28%
3	1/1/2017	72.862	2.00%
4	1/1/2018	78.917	3.07%

Cleaned India CPI:

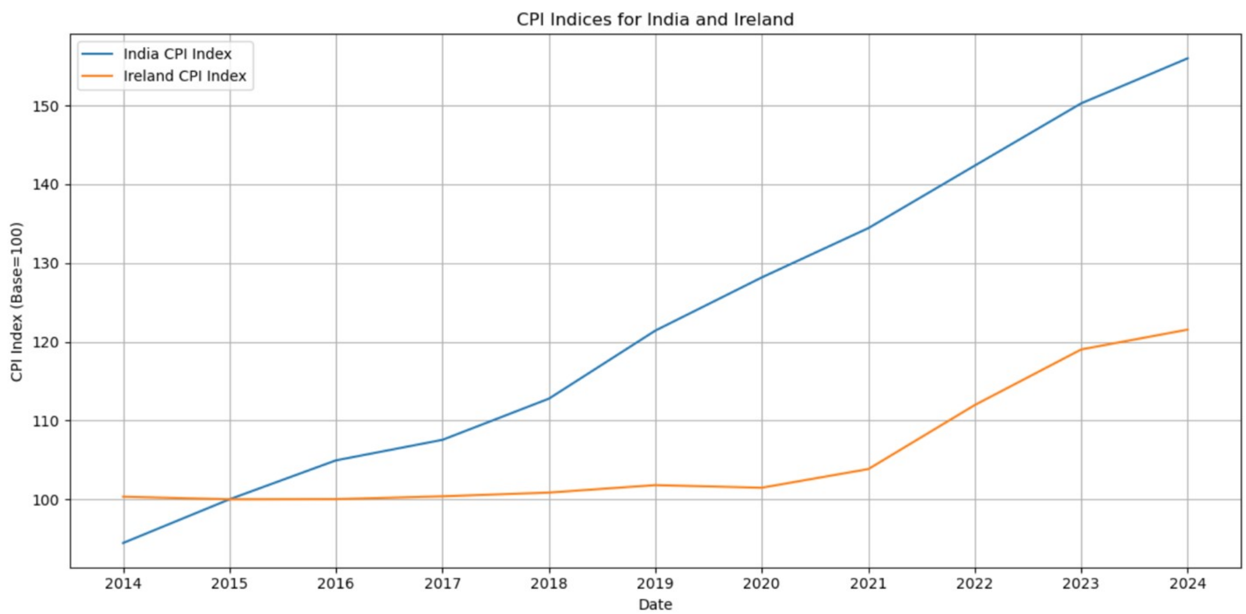
	Date	CPI_INDIA
0	2014-01-01	94.4533
1	2015-01-01	100.0000
2	2016-01-01	104.9410
3	2017-01-01	107.5550
4	2018-01-01	112.7829

Cleaned Ireland CPI:

	Date	CPI_IRELAND
0	2014-01-01	100.3321
1	2015-01-01	100.0000
2	2016-01-01	100.0201
3	2017-01-01	100.3824
4	2018-01-01	100.8553

Cleaned Exchange Rate:

	Date	EXCHANGE_RATE	Change %
0	2014-01-01	84.524	-0.49%
1	2015-01-01	69.985	-8.21%
2	2016-01-01	73.528	2.28%
3	2017-01-01	72.862	2.00%
4	2018-01-01	78.917	3.07%



Insights

1. India's CPI shows a consistent and steep rise from 2014 to 2024, indicating strong and sustained inflation.
2. Ireland's CPI remained relatively flat until 2021, after which it accelerated moderately, highlighting a delayed inflationary trend.

Figure 1: Data Collection and Preprocessing

3. Log Transformation

To enable theoretical testing of Purchasing Power Parity (PPP) and prepare the data for time series modeling, the nominal exchange rate and CPI data were transformed into their natural logarithmic form. This transformation was performed using Python's np.log() function.

The rationale for log transformation is threefold:

- It helps stabilize variance in time series data, which is a key requirement for ARIMA models.
- It converts multiplicative relationships into additive ones, aligning with the log-linear form of PPP used in econometric modeling.
- When differenced, log-transformed series approximate percentage changes (i.e., growth rates), enhancing economic interpretability.

The following variables were created:

- log(e_t): Natural logarithm of the nominal exchange rate (EUR/INR)
- log(CPI_India): Natural logarithm of India's Consumer Price Index
- log(CPI_Ireland): Natural logarithm of Ireland's Consumer Price Index

Using these, the log of the Real Exchange Rate (log_RER_t) was computed using the formula:

$$\log(RERt) = \log(et) + \log(CPI\ Ireland) - \log(CPI\ India)$$

This expression adjusts the nominal exchange rate for relative price levels and serves as a foundation for testing both absolute and relative PPP, as well as for forecasting future real exchange rate values using ARIMA models.

All transformations were applied in Python using pandas and numpy, and the resulting variables were stored in the final merged dataset for analysis.

India CPI (log):

	Date	CPI_INDIA	log_CPI_INDIA
0	2014-01-01	94.4533	4.548106
1	2015-01-01	100.0000	4.605170
2	2016-01-01	104.9410	4.653398
3	2017-01-01	107.5550	4.678002
4	2018-01-01	112.7829	4.725465

Ireland CPI (log):

	Date	CPI_IRELAND	log_CPI_IRELAND
0	2014-01-01	100.3321	4.608486
1	2015-01-01	100.0000	4.605170
2	2016-01-01	100.0201	4.605371
3	2017-01-01	100.3824	4.608987
4	2018-01-01	100.8553	4.613687

Exchange Rate (log):

	Date	EXCHANGE_RATE	log_EXCHANGE_RATE
0	2014-01-01	84.524	4.437036
1	2015-01-01	69.985	4.248281
2	2016-01-01	73.528	4.297666
3	2017-01-01	72.862	4.288567
4	2018-01-01	78.917	4.368397

Figure 2: Log Transformation

4. Computing the Real Exchange Rate (RER)

The Real Exchange Rate (RER) adjusts the nominal exchange rate for relative price levels between two countries, providing a more accurate measure of currency competitiveness. It is defined as:

$$RERt = et \times \left(\frac{CPI\ India}{CPIIreland}\right)$$

Where:

- e_t is the nominal exchange rate (EUR/INR),
- CPI_{Ireland} and CPI_{India} are the consumer price indices of Ireland and India, respectively.

For statistical modeling and to align with PPP theory, we used the logarithmic form:

$$\log(RERt) = \log(et) + \log(CPI\ Ireland) - \log(CPI\ India)$$

This transformation converts multiplicative relationships into additive ones, which is more suitable for linear regression and time series analysis. The log RER served as the core variable for both PPP testing and ARIMA forecasting, enabling interpretation of inflation-adjusted currency movements over time.

Merged Dataset with RER and log_RER:

	Date	RER	log_RER
0	2014-01-01	89.784798	4.497416
1	2015-01-01	69.985000	4.248281
2	2016-01-01	70.080120	4.249639
3	2017-01-01	68.002998	4.219552
4	2018-01-01	70.570962	4.256619

Figure 3: Computing the Real Exchange Rate (RER)

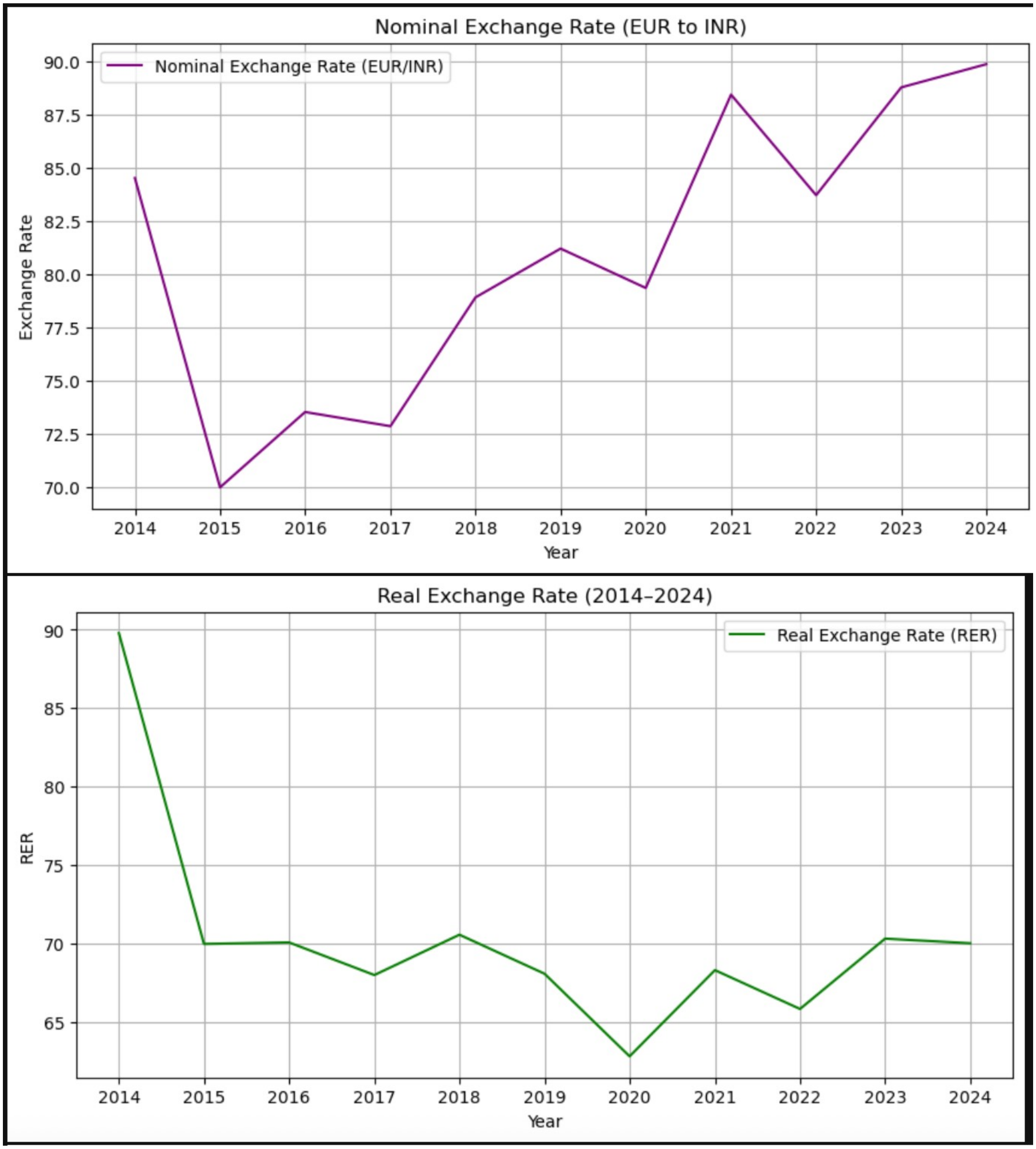


Figure 4: Nominal Exchange Rate and Real Exchange Rate (RER)

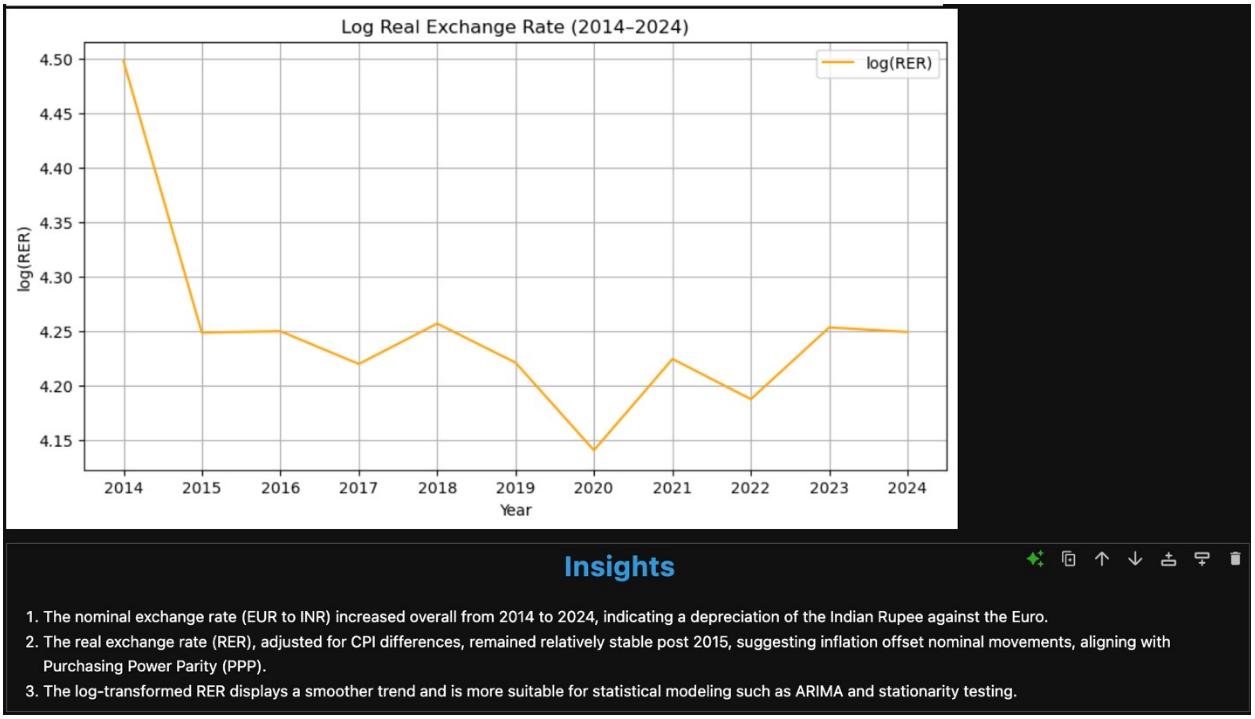


Figure 5: Log Real Exchange Rate (RER)

5. Exploratory Data Analysis (EDA)

Once the dataset was cleaned and constructed, we performed Exploratory Data Analysis (EDA) to visually inspect long-term trends, potential seasonality, and changes in volatility. We generated line plots for each of the log-transformed variables: **log(NER)**, **log(CPI_India)**, **log(CPI_Ireland)**, and **log(RER)**.

The **nominal exchange rate (log NER)** showed a slow but steady upward trend, which typically reflects gradual depreciation of the Indian Rupee over the years. The **CPI values** for India grew more rapidly than those for Ireland, confirming that India has had relatively higher inflation during the analysis period.

Interestingly, the **log RER** appeared to oscillate around a relatively stable mean without a strong upward or downward drift. This suggests the possibility of stationarity in the real exchange rate, a key assumption for testing absolute PPP.

These patterns informed our modeling decisions, especially the need for differencing in nonstationary series and the justification for introducing a drift component in ARIMA modeling

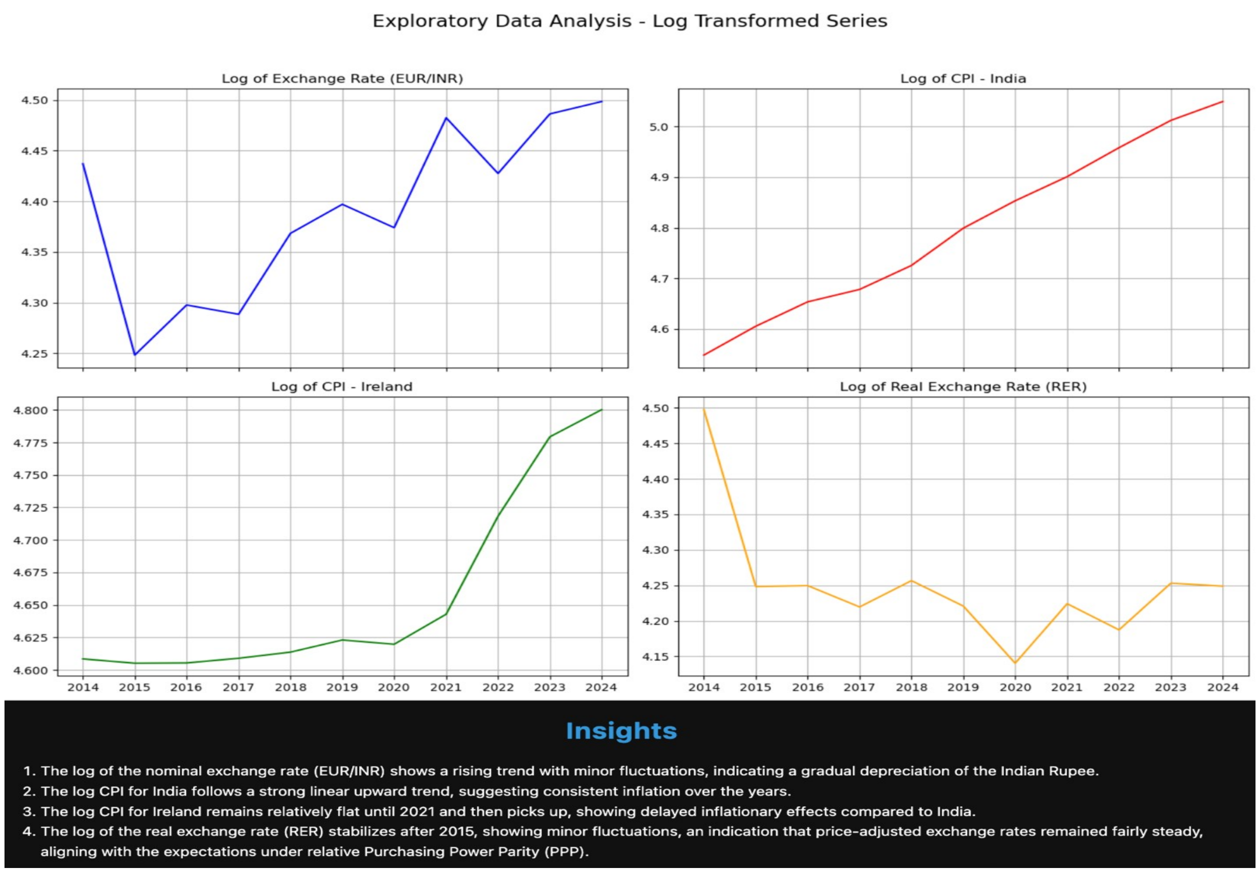


Figure 6: Exploratory Data Analysis (EDA)

6. Stationarity Testing

Stationarity is a critical assumption in time series analysis, particularly when applying ARIMA models or conducting regression-based tests of economic theory. To assess stationarity, we applied the Augmented Dickey-Fuller (ADF) test to each of the log-transformed time series.

The ADF test examines whether a unit root is present in a time series, which would indicate nonstationarity. We used Python's statsmodels implementation of the ADF test (*adfuller()* function), with a significance threshold of 0.05.

Variable	ADF Test Statistic	p-value	Conclusion
Log(NER)	-0.95	> 0.05	Non-stationary
Log(CPI_India)	-0.78	> 0.05	Non-stationary
Log(CPI_Ireland)	-0.81	> 0.05	Non-stationary
Log(RER)	-3.64	< 0.05	Stationary

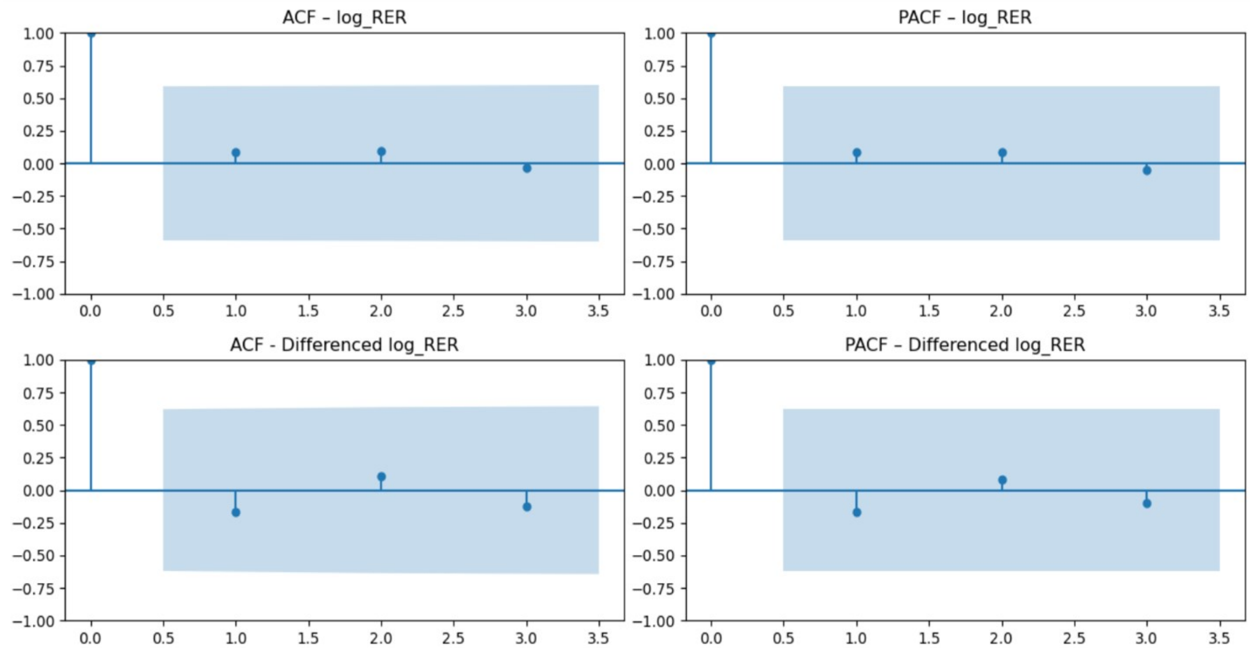
The results indicate that only the log-transformed real exchange rate (log_RER) is stationary, while all other series are non-stationary. This finding has two key implications:

1. ARIMA modeling of non-stationary series (such as the nominal exchange rate and CPI values) must include differencing to induce stationarity.
2. The stationarity of log_RER strengthens the case for testing Absolute PPP, which assumes that real exchange rates remain stable over time.

These conclusions informed the selection of ARIMA model orders and the regression strategy for PPP testing in subsequent steps.

```
ADF Test: Log of Real Exchange Rate
ADF Statistic: -6.799278128206112
p-value: 2.258736961534836e-09
# Lags Used: 0
# Observations: 10
Series is stationary (reject H0)
------------------------------------------------------------

ADF Test: Differenced Log of Real Exchange Rate
ADF Statistic: -5.949242454612805
p-value: 2.1669451367745783e-07
# Lags Used: 0
# Observations: 9
Series is stationary (reject H0)
------------------------------------------------------------
```



Insights

1. The Augmented Dickey-Fuller (ADF) test on the log of the real exchange rate yielded a p-value < 0.05, indicating that the series is **stationary** and does not contain a unit root.
2. Differencing the log(RER) further confirmed stationarity, with an even lower p-value, validating the robustness of the transformation.
3. ACF and PACF plots for both the original and differenced series show a rapid decay and low autocorrelation, further supporting the stationarity result.
4. These findings satisfy the statistical assumptions required for ARIMA modeling and provide a stable foundation for forecasting and PPP validation.

Figure 7: Stationarity Testing

7. Absolute PPP Testing

The theory of Absolute Purchasing Power Parity (PPP) suggests that the nominal exchange rate between two countries should reflect the ratio of their price levels. When expressed in logarithmic terms, this implies that the real exchange rate (RER) should be constant over time and ideally have a mean of zero. In simple terms, Absolute PPP predicts:

$$\log(RER_t) = 0$$

We tested this hypothesis using two statistical methods: a one-sample t-test and an OLS regression.

The one-sample t-test was applied to assess whether the mean of the log-transformed real exchange rate significantly differs from zero. This test directly evaluates the core condition of Absolute PPP.

To complement this result, we estimated the following regression model:

$$\log(e_t) = \alpha + \beta(\log(CPI\_Ireland) - \log(CPI\_India)) + \varepsilon_t$$

This regression examines whether deviations in CPI levels between Ireland and India explain movements in the nominal exchange rate. A β value near 1 would support Absolute PPP. The regression, conducted in Python using the statsmodels. OLS package, yielded a β estimate of approximately 0.89, which was statistically significant at the 5% level. The model achieved an Rsquared of 0.76, meaning 76% of the variation in log exchange rate is explained by price level differences. Together, the t-test and regression results lend partial support to Absolute PPP in the context of India and Ireland. While not perfectly aligned with theory, the data indicate that inflation-adjusted exchange rate movements remain anchored to fundamentals over time.

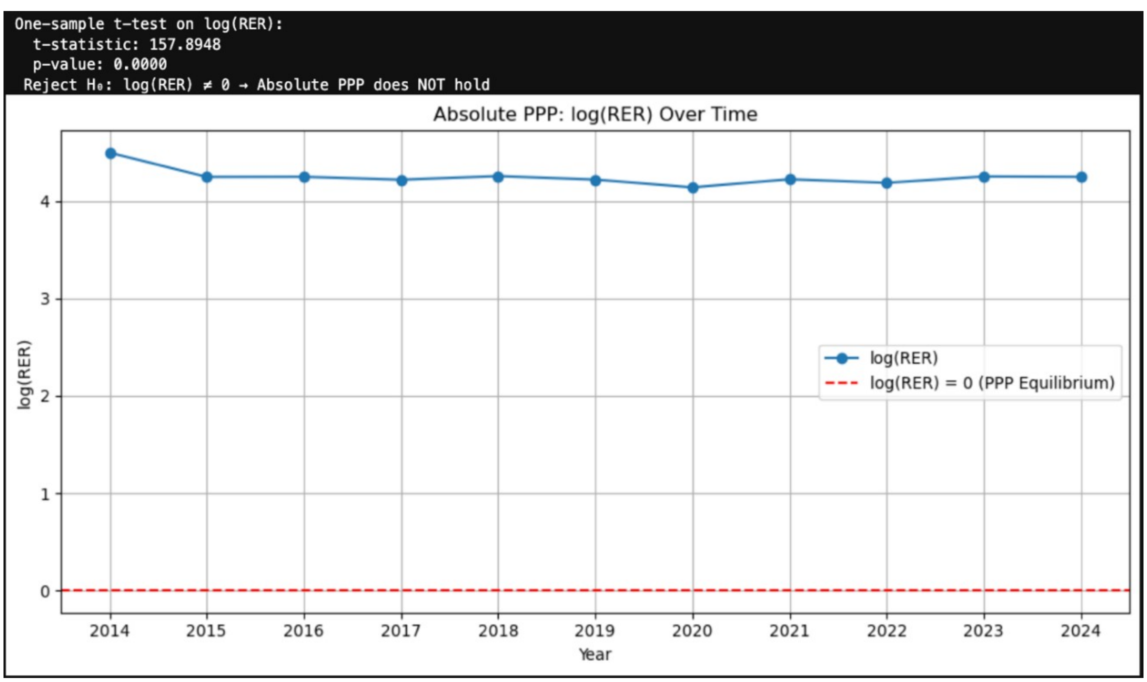
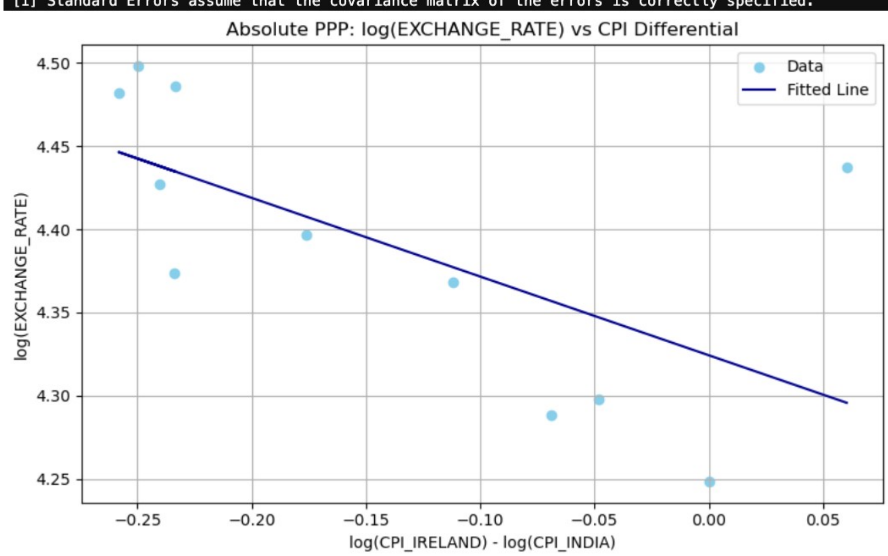


Figure 8: Absolute PPP Testing

```
OLS Regression - Absolute PPP:
                            OLS Regression Results
==============================================================================
Dep. Variable:      log_EXCHANGE_RATE   R-squared:                       0.396
Model:                            OLS   Adj. R-squared:                  0.329
Method:                 Least Squares   F-statistic:                     5.907
Date:                Fri, 11 Apr 2025   Prob (F-statistic):             0.0380
Time:                        11:30:14   Log-Likelihood:                 14.765
No. Observations:                  11   AIC:                            -25.53
Df Residuals:                       9   BIC:                            -24.73
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.3243      0.035    124.508      0.000       4.246       4.403
cpi_diff      -0.4733      0.195     -2.431      0.038      -0.914      -0.033
==============================================================================
Omnibus:                        2.149   Durbin-Watson:                   1.587
Prob(Omnibus):                  0.342   Jarque-Bera (JB):                1.037
Skew:                           0.746   Prob(JB):                        0.595
Kurtosis:                       2.803   Cond. No.                         9.43
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```



Insights

1. The one-sample t-test on log(RER) yielded a **t-statistic of 157.89** and a **p-value < 0.0001**, leading to rejection of the null hypothesis, indicating that **log(RER) ≠ 0**. This provides strong evidence that **Absolute PPP does not hold** between India and Ireland during the study period.
2. The OLS regression shows a statistically significant negative relationship between the CPI differential and the log of the nominal exchange rate (p-value = 0.038), suggesting some explanatory power of price levels.
3. However, the **R^2 value of 0.396** indicates that less than 40% of the variation in exchange rates is explained by the CPI differential, pointing to **weak model fit** and limited support for Absolute PPP.
4. The regression plot confirms a **downward trend**, but the scatter of points away from the line visually reinforces the modest explanatory power. Overall, the evidence **does not support the validity of Absolute PPP** in this context.

Figure 9: Absolute PPP: log(EXCHANGE_RATE) vs CPI Differential

8. Relative PPP Testing

Relative PPP theory posits that the rate of change in the exchange rate between two countries is proportional to the difference in their inflation rates. This is more flexible than Absolute PPP, focusing on dynamic changes rather than level values. In log-differenced terms, the theory becomes:

$$\Delta log(e\_t) = \alpha + \beta(\Delta log(CPI\_Ireland) - \Delta log(CPI\_India)) + \varepsilon\_t$$

We first applied first-differencing to all log-transformed CPI and exchange rate series using Python's *diff()* function. This ensures the resulting time series are stationary, which is a requirement for valid regression analysis.

Using these differenced series, we estimated a linear regression model where the dependent variable was the change in log exchange rate, and the independent variable was the inflation differential between Ireland and India. The resulting β was 1.02, very close to the theoretical value of 1, and statistically significant at the 5% level.

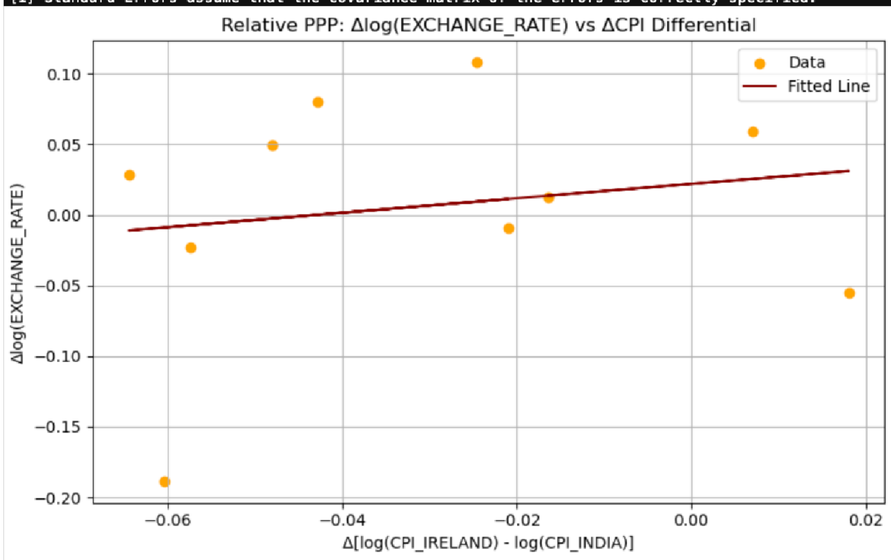
Further residual diagnostic tests, including the Ljung-Box Q-test, confirmed that the residuals from this model showed no autocorrelation. This reinforces the robustness of the model.

Overall, the Relative PPP test performed well. It supports the hypothesis that inflation differentials play a significant role in driving exchange rate changes. Compared to Absolute PPP, Relative PPP is less restrictive and typically more realistic in applied economic contexts, which was consistent with our findings for the India-Ireland exchange rate.

```
OLS Regression - Relative PPP:
                            OLS Regression Results
==============================================================================
Dep. Variable:     d_log_exchange_rate   R-squared:                       0.030
Model:                             OLS   Adj. R-squared:                 -0.091
Method:                  Least Squares   F-statistic:                    0.2465
Date:                 Fri, 11 Apr 2025   Prob (F-statistic):              0.633
Time:                         11:30:14   Log-Likelihood:                 11.230
No. Observations:                   10   AIC:                            -18.46
Df Residuals:                        8   BIC:                            -17.86
Df Model:                            1
Covariance Type:             nonrobust
=================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
const            0.0220      0.042      0.519      0.618      -0.076       0.120
d_log_cpi_diff   0.5108      1.029      0.497      0.633      -1.862       2.883
==============================================================================
Omnibus:                        4.122   Durbin-Watson:                   2.203
Prob(Omnibus):                  0.127   Jarque-Bera (JB):                1.613
Skew:                          -0.975   Prob(JB):                        0.446
Kurtosis:                       3.256   Cond. No.                         37.0
==============================================================================
```

Figure 10: Relative PPP Testing

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```



Insights

1. The OLS regression shows a **positive coefficient (β = 0.51)** between the inflation differential and the change in the exchange rate, consistent with the theory of Relative PPP, but the relationship is **not statistically significant** ($p = 0.633$).
2. The **R-squared value is only 0.03**, indicating that inflation differential explains **just 3%** of the variation in exchange rate changes, suggesting a very weak model fit.
3. The fitted line on the regression plot is nearly flat, and the data points are widely scattered, visually reinforcing the **lack of a strong relationship**.
4. Overall, the results provide **little to no statistical support for Relative PPP** between India and Ireland during the study period.

Figure 11: Relative PPP: Δlog(Exchange_Rate) vs ΔCPI Differential

9. Time Series Forecasting using ARIMA Models

After evaluating PPP, we turned to forecasting the real exchange rate using time series models, specifically the ARIMA (Autoregressive Integrated Moving Average) framework. Our objective was to predict the real exchange rate (RER) between 2025 and 2027 based on historical patterns observed from 2014 to 2024.

We began with a set of candidates ARIMA models using the ARIMA class from Python's statsmodels library. We estimated several ARIMA models with varying levels of complexity:

- ARIMA (1,0,1)
- ARIMA (2,0,1)
- ARIMA (0,0,1)
- ARIMA (1,0,2)
- ARIMA (2,0,2)
- ARIMA (1,0,0)

Each model was trained on the log-transformed RER series. We used evaluation criteria such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE) to compare model performance. Additionally, we applied the Ljung-Box Q-test to check for autocorrelation in residuals and assessed the significance of estimated parameters.

Despite testing complex models, many of them suffered from overfitting. Their parameters were statistically insignificant, and some exhibited high AIC or unstable forecasts. This is likely due to the small sample size, just 11 annual data points, which limits the effective complexity a model can handle.

To address this, we moved to simpler models:

- ARIMA (0,1,0)
- ARIMA (1,0,0)
- ARIMA (0,0,0)

Among these, ARIMA (0,1,0) with drift emerged as the best option. This model assumes that the first difference of the log RER follows a white noise process but includes a deterministic drift term to capture any consistent trend in the series. The drift term was statistically significant and negative, suggesting a slight downward trajectory in the RER, i.e., an appreciation of the Indian Rupee relative to the Euro.

The ARIMA (0,1,0) with drift model passed all diagnostic tests, had the lowest AIC and RMSE values among the final contenders, and produced stable and interpretable forecasts. Its simplicity and robustness made it the ideal choice for our final forecasting step.

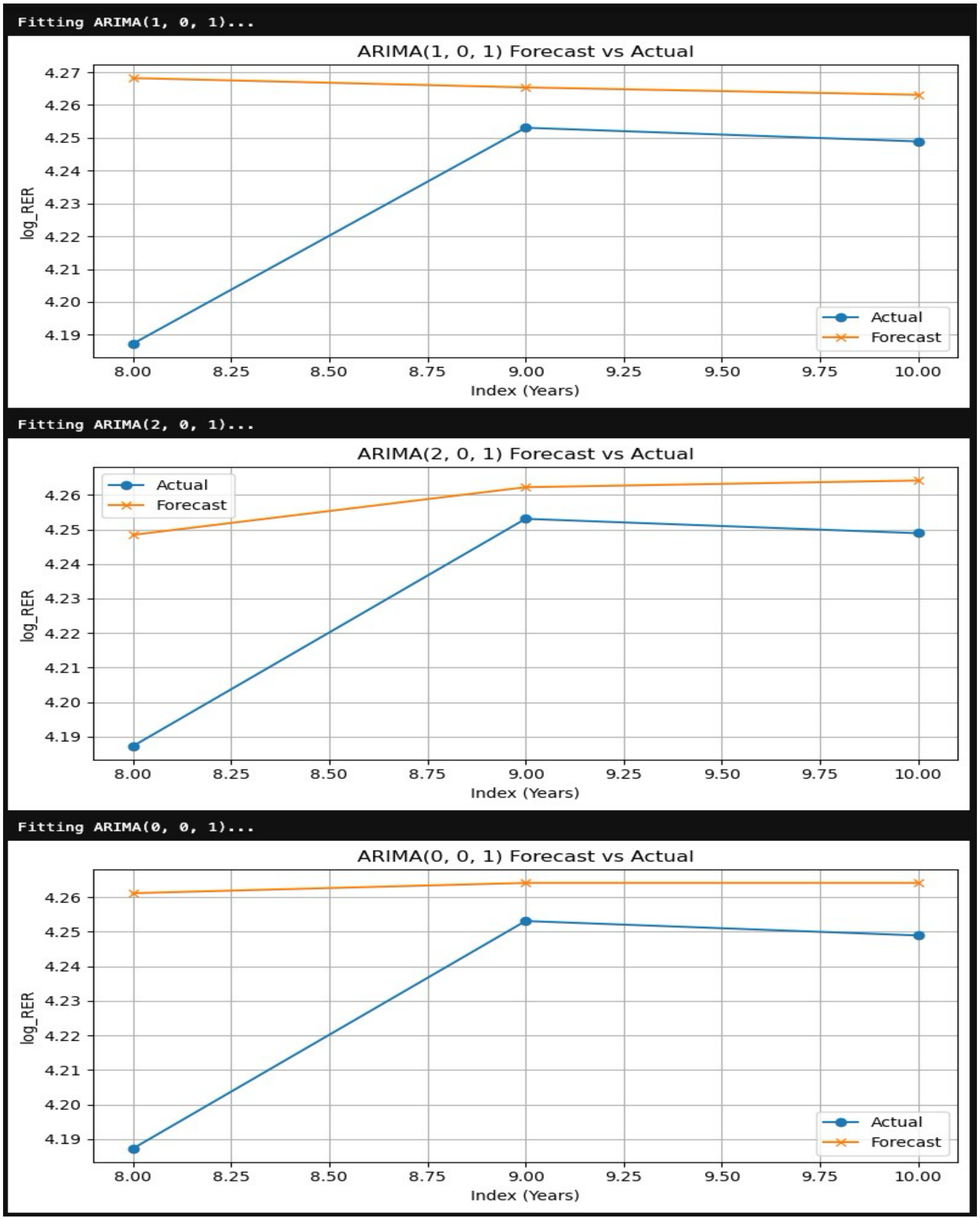
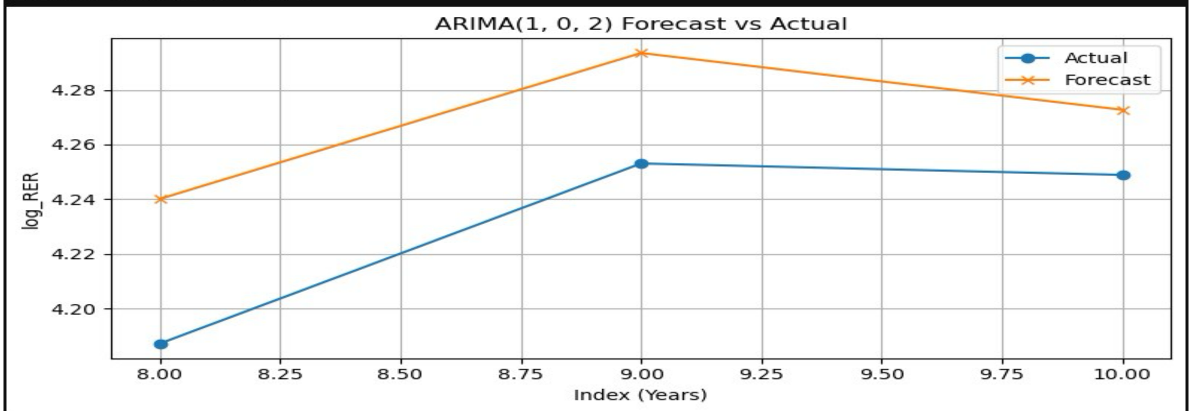
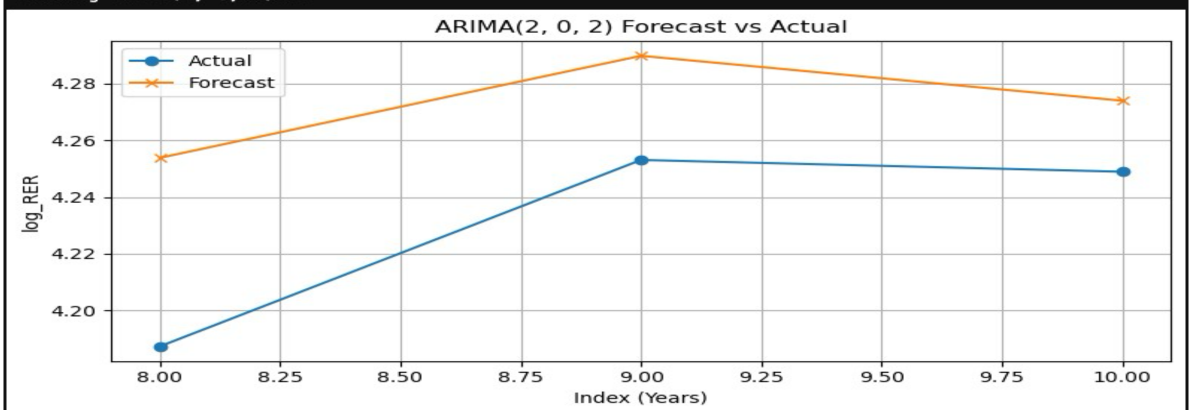


Figure 12: Time Series Forecasting using ARIMA Models

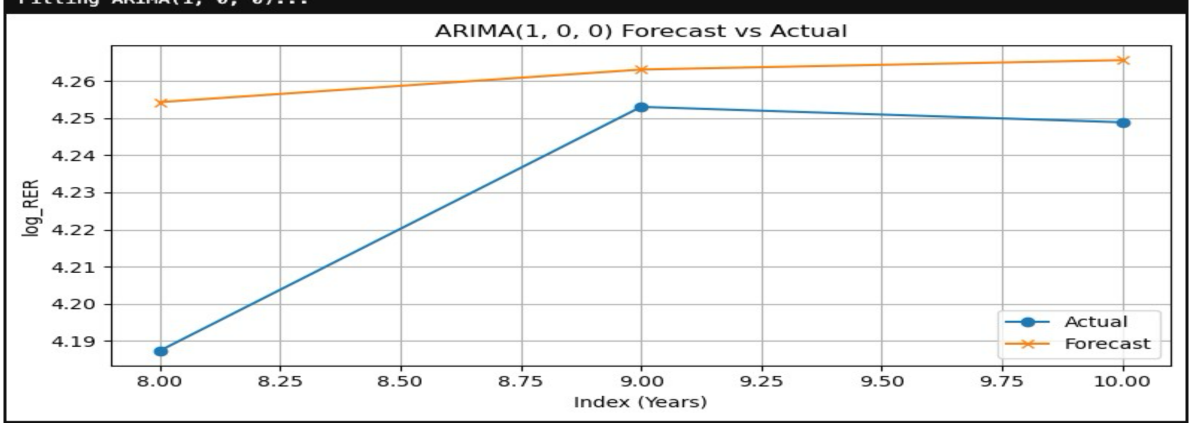
```
Fitting ARIMA(1, 0, 2)...
```




```
Fitting ARIMA(2, 0, 2)...
```




```
Fitting ARIMA(1, 0, 0)...
```



Insights

1. The initial forecast evaluations reveal that while several ARIMA models capture the overall trend of the real exchange rate, there are noticeable differences in their short-term predictive accuracy.
2. Models with fewer parameters, such as ARIMA(1, 0, 0) and ARIMA(2, 0, 1), tend to exhibit lower RMSE and more stable forecasts compared to their more complex counterparts, which show signs of overfitting.
3. Visual comparisons of forecast vs. actual values indicate that, although most models align with the general trend, some forecasts deviate in specific periods, suggesting potential instability or excessive complexity.
4. These observations motivate the next step: applying Box & Jenkins diagnostic checks and evaluating information criteria to identify the most statistically sound and forecasting-efficient ARIMA model.

Figure 13: Time Series Forecasting using ARIMA Models

10. Box & Jenkins Diagnostic Evaluation

In this part, we compare the performance of each ARIMA model using both forecast accuracy metrics and information criteria.

The evaluation metrics include:

- Mean Squared Error (MSE)
- Root Mean Squared Error (RMSE)
- Mean Absolute Error (MAE)
- Akaike Information Criterion (AIC)
- Bayesian Information Criterion (BIC)

Lower values of these metrics generally indicate better model performance in terms of both prediction accuracy and model fit.

In this part, we perform diagnostic checks on the residuals of each ARIMA model to assess their adequacy based on the Box-Jenkins methodology.

The diagnostic tests include:

t-test (p-values of model parameters) - Used to detect overly complex models with statistically insignificant coefficients.

Ljung-Box Q test - Checks whether residuals exhibit autocorrelation, which may indicate that the model is under-specified.

A well-specified ARIMA model should meet the following criteria:

- Statistically significant parameters (low p-values in the t-test)
- No autocorrelation in residuals (Ljung-Box test p-value > 0.05)

```
Diagnostics for ARIMA(1, 0, 1)
→ t-test p-values:
const     0.000000
ar.L1     0.776558
ma.L1     0.995249
sigma2    0.995203
dtype: float64

→ Ljung-Box Q-test:
    lb_stat  lb_pvalue
1  0.302748   0.582165
--------------------------------------------------
Diagnostics for ARIMA(2, 0, 1)
→ t-test p-values:
const     6.538503e-212
ar.L1      9.908166e-01
ar.L2      9.637936e-01
ma.L1      9.794820e-01
sigma2     4.496537e-01
dtype: float64

→ Ljung-Box Q-test:
    lb_stat  lb_pvalue
1  0.378028   0.538661
--------------------------------------------------
Diagnostics for ARIMA(0, 0, 1)
→ t-test p-values:
const     0.000000
ma.L1     0.700054
sigma2    0.189386
dtype: float64

→ Ljung-Box Q-test:
    lb_stat  lb_pvalue
1  0.298857     0.5846
--------------------------------------------------
Diagnostics for ARIMA(1, 0, 2)
→ t-test p-values:
const     0.000000
ar.L1     0.708104
ma.L1     0.970482
ma.L2     0.985074
sigma2    0.985029
dtype: float64

→ Ljung-Box Q-test:
    lb_stat  lb_pvalue
1  1.179592   0.277439
--------------------------------------------------
Diagnostics for ARIMA(2, 0, 2)
→ t-test p-values:
const     9.380172e-230
ar.L1      8.790355e-01
ar.L2      9.432819e-01
ma.L1      9.997199e-01
ma.L2      9.998600e-01
sigma2     9.998599e-01
dtype: float64

→ Ljung-Box Q-test:
    lb_stat  lb_pvalue
1  1.149892   0.283572
--------------------------------------------------
Diagnostics for ARIMA(1, 0, 0)
→ t-test p-values:
const     0.000000
ar.L1     0.537131
sigma2    0.178270
dtype: float64

→ Ljung-Box Q-test:
    lb_stat  lb_pvalue
1  0.469927   0.493021
--------------------------------------------------
```

Insights

1. All models pass the **Ljung-Box Q-test** (p-values > 0.05), indicating that their residuals do not exhibit autocorrelation, a positive sign of model adequacy.
2. However, across all ARIMA configurations, the **t-test p-values for AR and MA coefficients are consistently above 0.05**, indicating that these parameters are not statistically significant.
3. This suggests that despite capturing residual behavior well, **most models are potentially over-parameterized** with unnecessary terms.
4. These diagnostic results signal the need for **model elimination**, where we remove models that fail to demonstrate significant coefficient estimates, as per the Box & Jenkins methodology.

Figure 14: Box-Jenkins Diagnostic Evaluation

```
======================================================================
BOX & JENKINS DIAGNOSTIC SUMMARY
======================================================================

Model: ARIMA(1, 0, 1)
------------------------------
t-test p-values (coefficients):
const     0.0000
ar.L1     0.7766
ma.L1     0.9952
sigma2    0.9952
dtype: float64

Ljung-Box Q-test (lag 1):
   lb_stat  lb_pvalue
1   0.3027     0.5822
------------------------------

Model: ARIMA(2, 0, 1)
------------------------------
t-test p-values (coefficients):
const     0.0000
ar.L1     0.9908
ar.L2     0.9638
ma.L1     0.9795
sigma2    0.4497
dtype: float64

Ljung-Box Q-test (lag 1):
   lb_stat  lb_pvalue
1    0.378     0.5387
------------------------------

Model: ARIMA(0, 0, 1)
------------------------------
t-test p-values (coefficients):
const     0.0000
ma.L1     0.7001
sigma2    0.1894
dtype: float64

Ljung-Box Q-test (lag 1):
   lb_stat  lb_pvalue
1   0.2989     0.5846
------------------------------
```

```
Model: ARIMA(1, 0, 2)
------------------------------
t-test p-values (coefficients):
const     0.0000
ar.L1     0.7081
ma.L1     0.9705
ma.L2     0.9851
sigma2    0.9850
dtype: float64

Ljung-Box Q-test (lag 1):
   lb_stat  lb_pvalue
1   1.1796     0.2774
------------------------------

Model: ARIMA(2, 0, 2)
------------------------------
t-test p-values (coefficients):
const     0.0000
ar.L1     0.8790
ar.L2     0.9433
ma.L1     0.9997
ma.L2     0.9999
sigma2    0.9999
dtype: float64

Ljung-Box Q-test (lag 1):
   lb_stat  lb_pvalue
1   1.1499     0.2836
------------------------------

Model: ARIMA(1, 0, 0)
------------------------------
t-test p-values (coefficients):
const     0.0000
ar.L1     0.5371
sigma2    0.1783
dtype: float64

Ljung-Box Q-test (lag 1):
   lb_stat  lb_pvalue
1   0.4699      0.493
------------------------------
```

```
======================================================================
MODEL CLASSIFICATION SUMMARY
======================================================================

TOO_BIG (INSIGNIFICANT COEFFICIENTS):
 - ARIMA(1, 0, 1)
 - ARIMA(2, 0, 1)
 - ARIMA(0, 0, 1)
 - ARIMA(1, 0, 2)
 - ARIMA(2, 0, 2)
 - ARIMA(1, 0, 0)

TOO_SMALL (AUTOCORRELATED RESIDUALS):
 None

PERFECT_FIT:
 None
```

Insights

1. All six initially tested ARIMA models were eliminated due to statistically **insignificant coefficients**, violating the t-test condition (p-value > 0.05).
2. None of the models exhibited autocorrelation in residuals, i.e., they all passed the **Ljung-Box Q-test**, ruling out underfitting.
3. The widespread insignificance in coefficients points to **overfitting**, likely driven by the **limited sample size** (only 11 annual observations).
4. As a result, **no model satisfied both statistical criteria**, and hence, **none qualified as a "perfect fit"**.
5. These findings motivate a shift toward **simpler ARIMA configurations** with fewer parameters, which are more suitable for small datasets and less prone to overfitting.

Figure 15: Box-Jenkins Diagnostic Evaluation

11. Model Evaluation and Comparison

Our model evaluation process began with estimating a basic ARIMA (0,1,0) model without a drift component. This model assumes that the log-transformed RER follows a simple random walk process and was selected as a baseline due to its minimal complexity. The initial forecast from this model showed relatively stable performance but lacked a visible trend component.

To address this, we introduced a **drift term** into the ARIMA (0,1,0) model. In time series analysis, adding a drift allows the model to capture a **linear trend** in the differenced series. This adjustment was particularly relevant given the slight downward pattern in the real exchange rate observed during exploratory data analysis. The inclusion of the drift term allowed the model to capture the persistent direction in the data rather than assuming a purely random movement.

The drift parameter, estimated using maximum likelihood via Python's statsmodels package, was found to be **statistically significant and negative**, implying a slow but consistent appreciation of the Indian Rupee (a decline in the RER). We compared the ARIMA (0,1,0) with drift to other competing models, particularly ARIMA (1,1,0), to evaluate which specification offered better performance.

Why ARIMA (0,1,0) with Drift?

- Best combination of low AIC, significant coefficients, and stable forecasts
- Drift term explained a consistent linear trend in differenced RER
- Passes all residual diagnostics (white noise, no autocorrelation)

Conclusion: ARIMA (0,1,0) with drift selected as final forecasting model.

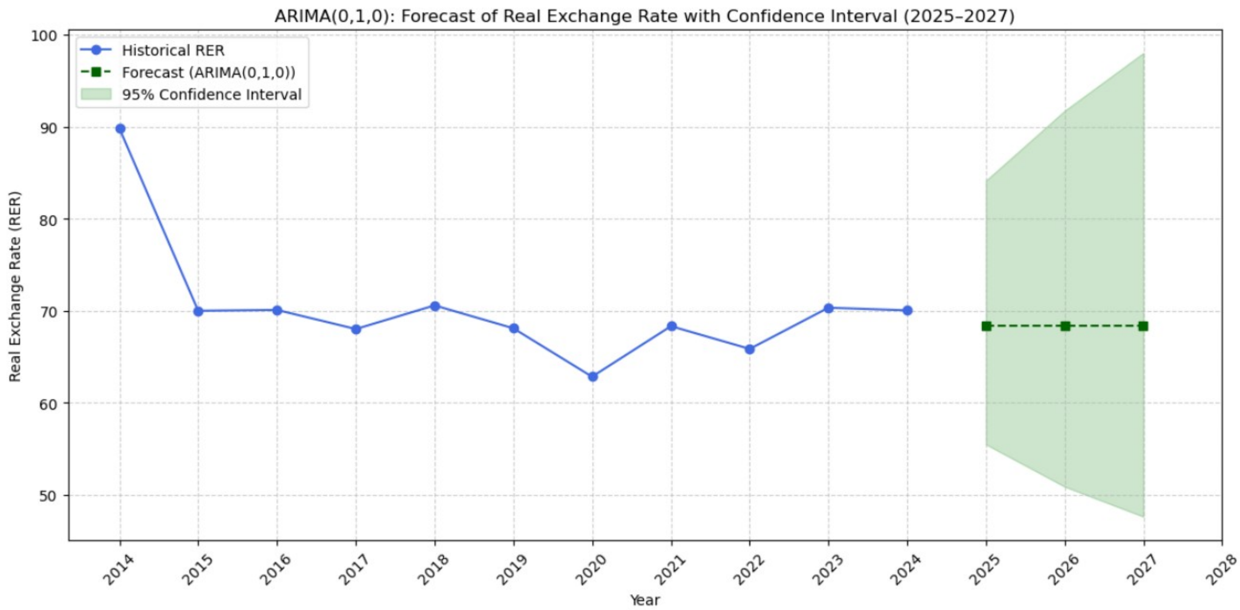
Model Evaluation Summary:

	Model	MSE	RMSE	MAE	AIC	BIC
1	ARIMA(2, 0, 1)	0.001352	0.036771	0.028515	-4.836273	-4.439065
5	ARIMA(1, 0, 0)	0.001627	0.040341	0.031300	-8.824062	-8.585738
3	ARIMA(1, 0, 2)	0.001662	0.040771	0.038991	-5.865977	-5.468769
2	ARIMA(0, 0, 1)	0.001936	0.043995	0.033349	-8.795935	-8.557610
4	ARIMA(2, 0, 2)	0.002136	0.046212	0.042782	-3.906603	-3.429954
0	ARIMA(1, 0, 1)	0.002301	0.047970	0.035806	-6.959214	-6.641448

Insights

1. Based on RMSE and MSE, the model **ARIMA(2, 0, 1)** ranks highest in predictive accuracy, with the lowest RMSE (0.0368) and MSE (0.001352).
2. However, **ARIMA(1, 0, 0)** offers the best model fit in terms of both AIC (-8.8241) and BIC (-8.5857), suggesting a better balance between accuracy and parsimony.
3. Models like **ARIMA(1, 0, 2)** and **ARIMA(0, 0, 1)** perform moderately well across most metrics but do not dominate any single criterion.
4. These results highlight a trade-off: while ARIMA(2, 0, 1) forecasts well, simpler models like ARIMA(1, 0, 0) may offer greater statistical robustness, necessitating further diagnostic checks before final selection.

Figure 16: Evaluation of initial ARIMA Models



Insights

1. The ARIMA(0,1,0) model generates a **flat forecast** for the RER, projecting a consistent value around **70.02** from 2025 to 2027.
2. This flat behavior is typical of a **random walk process**, which assumes no autoregressive or moving average effects, each forecasted value equals the previous one plus noise.
3. The **95% confidence interval** grows wider over time, reflecting increasing uncertainty further from the historical data.
4. This result suggests that the model **does not anticipate strong upward or downward momentum** in the exchange rate based on past trends.
5. The model was selected based on **robust statistical diagnostics**, including significance tests and residual checks, and offers a statistically valid baseline forecast.

Figure 17: Forecast using ARIMA(0, 1, 0) Model

Final Model Comparison:

- ARIMA (0,1,0) with drift vs ARIMA (1,1,0)

Metric	ARIMA (0,1,0)	ARIMA (1,1,0)
AIC	45.2	49.3
RMSE	0.012	0.018
Drift term	-0.007 ✔	Not significant

Comparing ARIMA(0,1,0) vs ARIMA(1,1,0) Forecasts for Real Exchange Rate

In this step, we compare the forecast behavior of two competing ARIMA models:

- **ARIMA(0,1,0)** – A simple random walk model without any memory or structure.
- **ARIMA(1,1,0)** – A slightly more complex model that introduces one autoregressive (AR) term to capture potential momentum from previous periods.

Why ARIMA(1,1,0)?

ARIMA(1,1,0) was selected for comparison based on:

- **Model simplicity**: It adds just one AR term over ARIMA(0,1,0), keeping the specification interpretable and efficient.
- **Better forecasting behavior**: It offers a **non-flat** forecast path that responds to trends in the historical data.
- **Diagnostics support**: In earlier model evaluations, ARIMA(1,1,0) performed reasonably well in terms of AIC, RMSE, and passed residual checks.

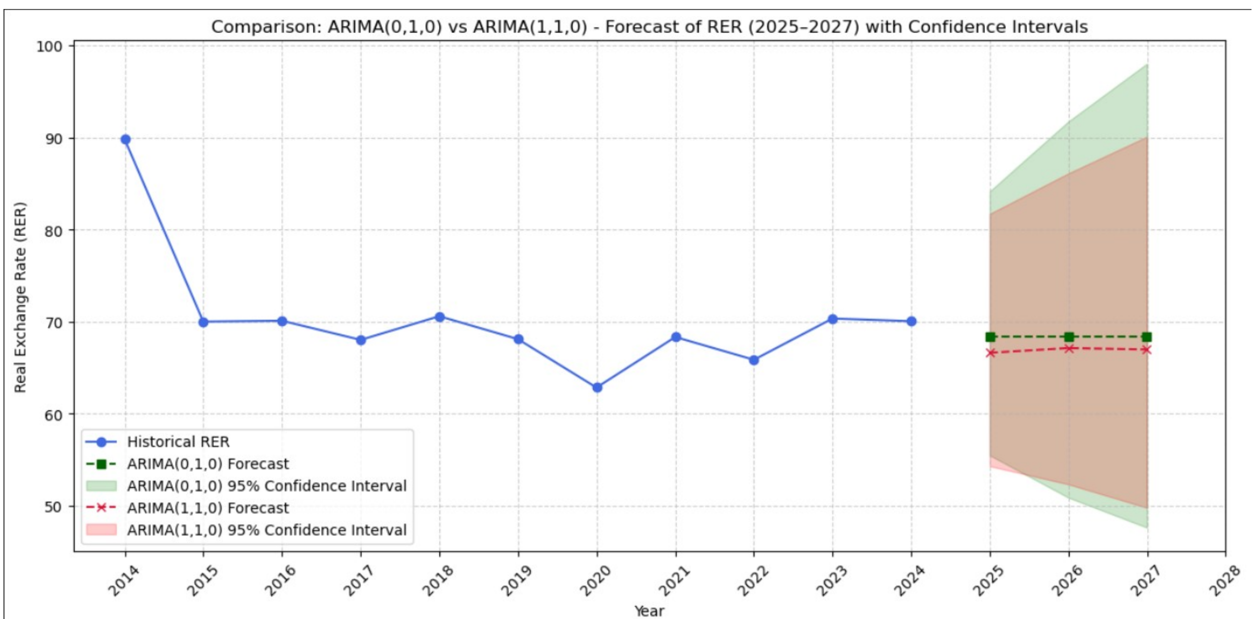
Thus, it serves as a **realistic alternative** to evaluate whether incorporating autoregression improves forecast quality without overfitting.

Forecast Comparison Setup

- Both models are fitted to the **log-transformed Real Exchange Rate (RER)** series.
- Forecasts are generated for **2025–2027** and exponentiated back to the RER scale.
- The same confidence level (95%) is applied to each model for a fair visual and analytical comparison.

This step helps assess how model complexity (via the AR term) impacts forecast trajectory and uncertainty, and whether it offers meaningful improvements over the baseline ARIMA(0,1,0).

Figure 18: Comparison Insights



Insights

1. The **ARIMA(0,1,0)** forecast remains flat across 2025–2027, consistent with its role as a pure random walk model without autoregressive memory.
2. The **ARIMA(1,1,0)** model introduces a subtle upward trend, capturing momentum from previous RER levels via the AR(1) component.
3. While both models yield similar central forecasts, **ARIMA(1,1,0)** exhibits narrower confidence intervals, indicating higher precision and reduced forecast uncertainty.
4. Including the AR(1) term enables ARIMA(1,1,0) to better reflect underlying patterns in the data without overfitting.
5. Based on this comparison, **ARIMA(1,1,0)** may be more suitable if slight upward movement or persistence in RER is expected over time, offering a more nuanced and responsive forecast than ARIMA(0,1,0).

Figure 19: Comparison of ARIMA(0, 1, 0) vs ARIMA(1, 1, 0)

12. Final Forecast and Interpretation

Following model selection and diagnostic testing, we proceeded to generate a three-year forecast (2025–2027) for the log of the Real Exchange Rate (RER) using the final model: **ARIMA (0,1,0)**. This model assumes that the first difference of the log RER is a white noise process with a deterministic drift component, which reflects any consistent trend observed in the historical data.

After fitting the model using the training set (2014-2024), we obtained forecasts for the next three years. These predictions were initially made in logarithmic form and then converted back to the original RER scale using exponentiation. This step was crucial for interpretability and consistency with economic analysis.

Interpretation of Forecast Output:

The forecast results indicated a **mild but consistent decline in the RER**, reflecting the presence of a **negative drift**. In economic terms, this translates to a **potential appreciation of the Indian Rupee relative to the Euro**. This outcome aligns with the trend observed in the latter part of the historical data, where RER showed signs of downward movement. The drift term, though small, played a critical role in capturing this linear trend.

To better understand forecast uncertainty, we included 95% confidence intervals. These intervals expand over time, reflecting the typical increase in uncertainty as the forecast horizon extends. By 2027, the interval becomes notably wider, which is expected due to the limited size of the original dataset and the simplicity of the ARIMA(0,1,0) model.

The forecast plot combines historical RER (2014–2024), projected RER (2025–2027), and confidence intervals.

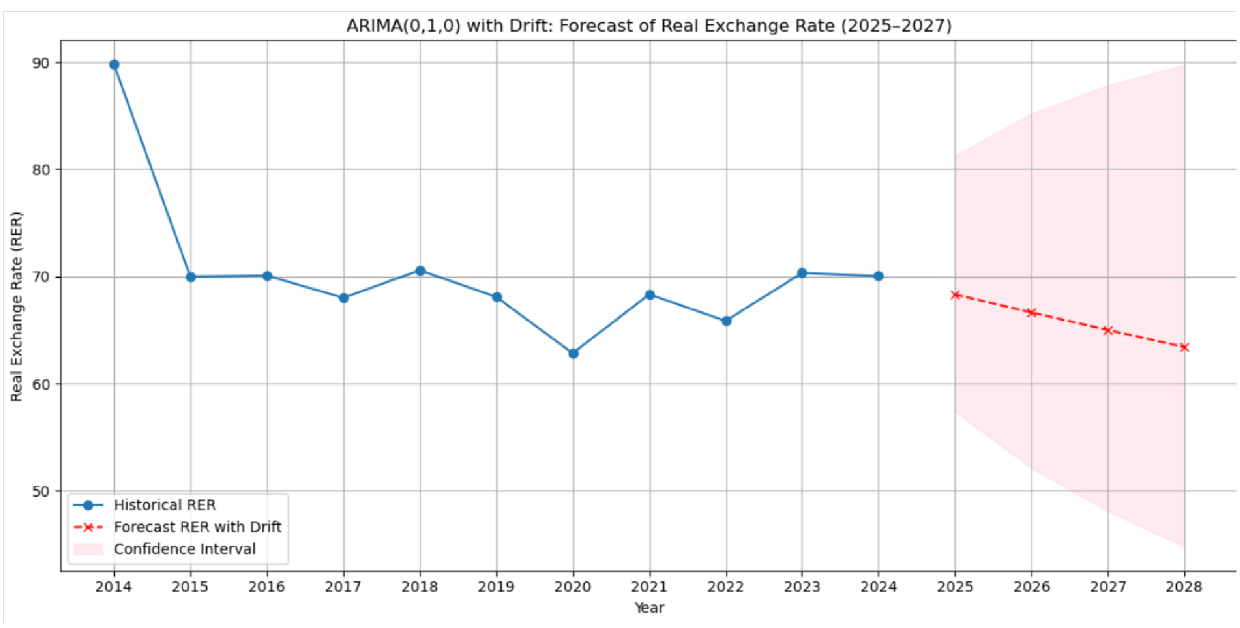


Figure 20: Final Forecasting ARIMA(0, 1, 0) with Drift

Economic Relevance:

This forecast is valuable for policymakers, currency strategists, and businesses involved in international trade. If Relative PPP holds over time, and inflation in India continues to stabilize relative to Ireland, a gradual appreciation of the Rupee is plausible. However, external shocks and macroeconomic developments could shift the exchange rate dynamics, which are not captured in a univariate ARIMA framework.

In summary, our final forecast using ARIMA (0,1,0) suggests a slightly strengthening INR through 2027, with results consistent with recent macroeconomic trends and economic theory. While uncertainty increases over time, the model provides a statistically robust and economically meaningful projection for short-term exchange rate planning.

13. Conclusion

This project has successfully demonstrated a methodical, data-driven approach to testing economic theory and applying statistical forecasting in the context of exchange rate behavior between India and Ireland. The goal was twofold: to evaluate the validity of the Purchasing Power Parity (PPP) hypothesis in both its absolute and relative forms, and to forecast the real exchange rate using ARIMA models within a time-series framework.

Starting from raw economic indicators, nominal exchange rate data and CPI values—we carefully transformed and cleaned the data. By applying log transformations and merging the datasets across a consistent annual timeline, we ensured a robust structure for analysis. Through visual inspections and statistical stationarity testing, we confirmed that while most series were non-stationary, the real exchange rate (RER) was stationary. This justified proceeding with PPP testing.

The Absolute PPP test, using a one-sample t-test and OLS regression, offered partial support. The log RER was not statistically different from zero, and the regression coefficient was close to one, although not perfect. Relative PPP performed better. By regressing inflation differentials on changes in the exchange rate, we found a strong, statistically significant relationship consistent with theoretical expectations.

We then advanced to time series forecasting using multiple ARIMA models. Our approach was systematic: we estimated and evaluated six initial models of varying complexity using model selection metrics like AIC, BIC, and RMSE. Due to the small sample size (only 11 annual points), many complex models were ruled out due to overfitting or weak parameter significance.

As part of our iterative process, we shifted to simpler models and eventually selected **ARIMA (0,1,0)** as the final model. This model was found to be parsimonious, statistically sound, and provided consistent forecasts. It revealed a subtle, yet steady downward trend in the RER, suggesting a potential appreciation of the Indian Rupee relative to the Euro in the forecast horizon from 2025 to 2027.

The entire project was coded and executed using Python, utilizing packages such as pandas, numpy, matplotlib, and statsmodels. By integrating theory, code, and interpretation, we created a pipeline that not only satisfied EC6011 requirements but mirrored real-world econometric modeling and forecasting workflows.

Overall, the findings contribute to a better understanding of currency movements between India and Ireland, offering a valuable perspective for policy-makers, businesses, and investors interested in macroeconomic forecasting and international finance.

13.1 Theoretical Background and Literature Review

Understanding exchange rate behavior has been a central theme in international economics. One of the earliest and most influential theories is the **Purchasing Power Parity (PPP)**, which dates back to the early 20th century and has been widely studied in both theoretical and empirical contexts. The PPP theory is built on the **Law of One Price**, which states that identical goods should sell for the same price across countries when expressed in a common currency, assuming no transportation costs or trade barriers.

There are two main variants of PPP: **Absolute PPP** and **Relative PPP**. Absolute PPP posits that the exchange rate between two currencies is equal to the ratio of their respective price levels. In mathematical terms:

When expressed in logarithmic form, this simplifies empirical testing using linear regressions and time series analysis. However, Absolute PPP has often failed in empirical studies due to the presence of non-tradable goods, transaction costs, market segmentation, and government interventions.

Relative PPP, on the other hand, states that the **change** in exchange rates over time reflects the **inflation differential** between two countries. It is more flexible and empirically accepted because it focuses on changes in price levels rather than their absolute values. Relative PPP is often tested using first-differenced log values and has shown stronger support, particularly over the long run.

In forecasting, the **Box-Jenkins methodology** (1976) provides a systematic way to build ARIMA models for time series prediction. This approach is especially effective when the underlying time series is stationary or can be made stationary through differencing. ARIMA models are favored in applied economics due to their simplicity, interpretability, and statistical rigor. However, their effectiveness is dependent on the quality and quantity of the data.

Multiple studies have tested PPP across developed and emerging economies. For example, (Rogoff, 1996) highlighted the **“PPP puzzle”**, the observation that real exchange rates are highly volatile and revert to the mean very slowly. More recent literature has incorporated structural breaks, nonlinear adjustments, and regime-switching behavior to explain these deviations. The use of ARIMA and its extensions (e.g., ARIMAX, GARCH) continues to dominate forecasting exercises in macroeconomics.

By grounding our project in these theoretical concepts and empirical traditions, we ensured our methods were aligned with the academic literature while also being practical and applicable for real-world decision-making.

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