CS5691 Pattern Recognition and Machine Learning

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Assignment - 1

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Contents

0 Datasets used

- Dataset 1 - “function2.csv”
- Dataset 2 - “function2_2d.csv”
- Dataset 3 - “0_superconductor.csv”

1 Task 1 – Polynomial Curve Fitting

Polynomial Curve Fitting for Dataset 1, with the file name “function2.csv”.

1.1 Approach

- Given the data of one x variable and one y variable, we want to predict y from x.
- For this, we use linear regression to fit a polynomial curve.
- As we are not allowed to use the scikit-learn package, all the functions have to be manually defined by us.
- First step is to define the dataset size (10, 200, etc.) which we are working with.
- A set of degrees and lambda values are chosen to perform cross-validation on, my manually trying various values. Specifically, degree values of [2, 3, 6, 9] and lambda values of [0.0, 0.01, 0.1, 1.0] are tried.
- For each degree, the data is converted to polynomial data of that degree by the function “get_polynomial_data”.
- The data is fit to a linear curve by taking the inverse of the converted data’s matrix (X’X + lambda) and multiplying with y to get the weights.
- This is done for all the degrees and lambdas to determine the hyperparameters which give the best (least) error.
- The results are recorded.

1.2 Plots of Approximated Functions

Plots of the polynomial functions that have been fitted to the data.

1.2.1 For Dataset of Size 10

Taking dataset of 10 cases, we get the functions as plotted in Figure 1 and Figure 2

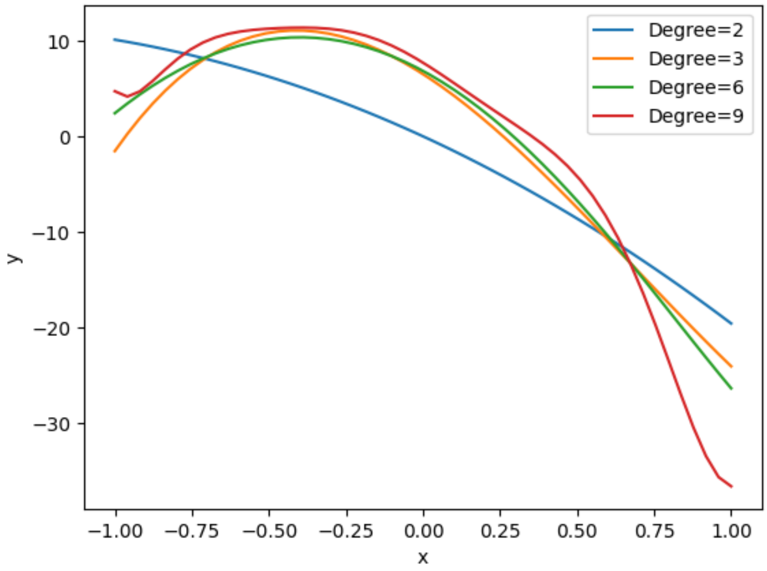


Figure 1: Plots of different curves for degrees 2,3,6 and 9

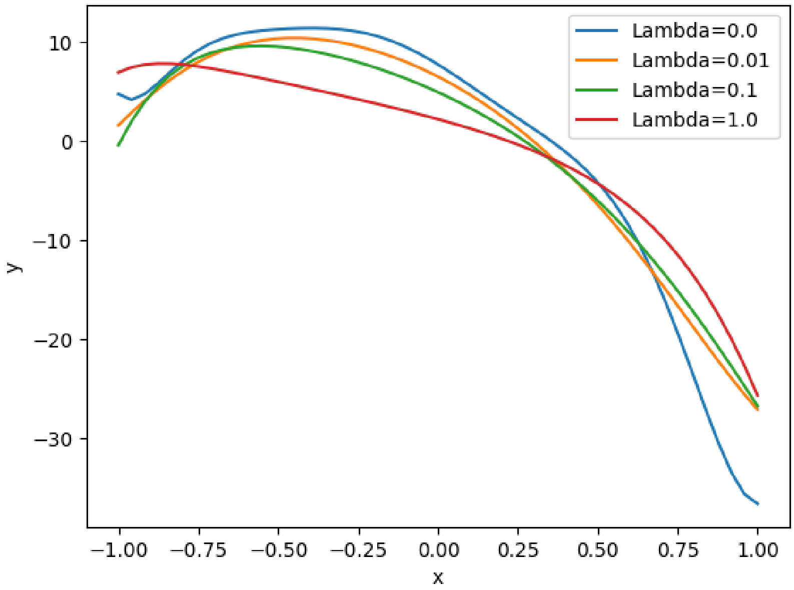


Figure 2: Plots for different lambda values

1.2.2 For Dataset of Size 200

Taking dataset of 200 cases, we get the functions as plotted in Figure 3 and Figure 4

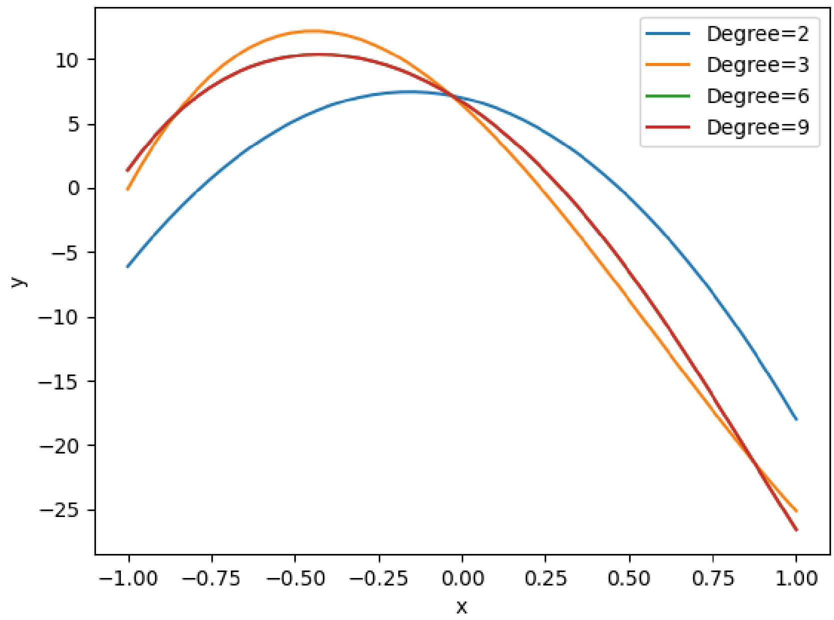


Figure 3: Plots of different curves for degrees 2,3,6 and 9.

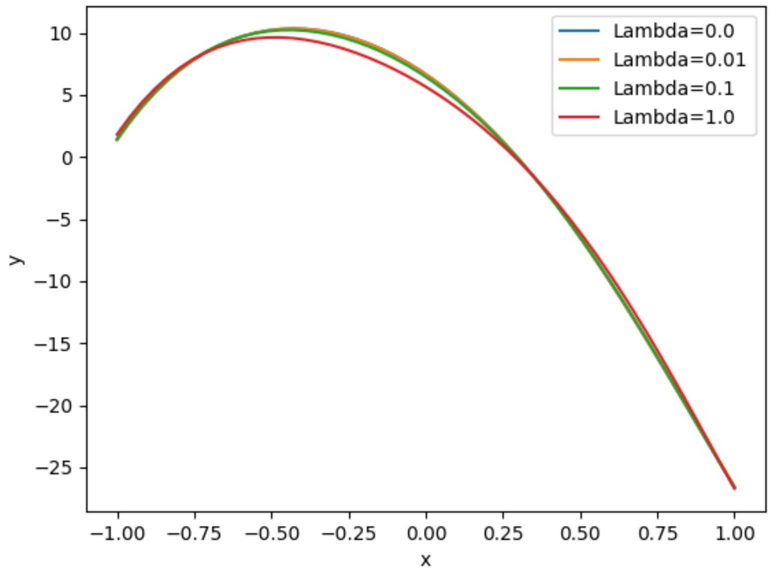


Figure 4: Plots for different lambda values

1.3 RMS Error for Train, Validation Data

Root mean squared error for the train and cross validation error for the different cases.

1.3.1 For Dataset of Size 10

Without any Regularization. Train Size = 10. Test Size = 10. Errors are recorded in Table 1.

Table 1: Train and Cross Validation Error for Dataset of Size 10 without any Regularization

Degree	Lambda	Train Error	Cross Validation Error
2	0	2.4905674801572006	10.122046936365459
3	0	0.7187848682467608	1.8005114832903149
6	0	0.08219265905710454	4.415393436037817

9	0	8.589255693625607e-06	392.3172282450482

With Quadratic Regularization. Train Size = 10. Test Size = 10. Errors are Recorded in Table 2.

Table 2: Train and Cross Validation Error for Dataset of Size 10 with Quadratic Regularization

Degree	Lambda	Train Error	Cross Validation Error
2	0.001	2.490568211099624	10.11947629310171
2	0.01	2.490640049920283	10.096473267100697
2	0.1	2.4973273669994493	9.878939367958555
2	1.0	2.860387101500878	8.529617144816806
3	0.001	0.7188212650987488	1.7960943488076995
3	0.01	0.7223094394618816	1.7756097511535005
3	0.1	0.950661686405299	2.7041498410117755
3	1.0	2.9915396192544983	9.182916169018059
6	0.001	0.08426433963483396	2.242787765077613
6	0.01	0.14440691238773276	2.416509570828632
6	0.1	0.5516499544840704	0.9851156454581484
6	1.0	2.012223961487674	19.816099653520364
9	0.001	0.0684841470464367	52.3898917093653
9	0.01	0.12809288519476703	17.076951783099382
9	0.1	0.5720555341290499	20.92285102967486
9	1.0	2.1452858171156906	19.04343631383894

1.3.2 For Dataset of Size 200

Without any Regularization. Train Size = 200. Test Size = 60. Errors are Recorded in Table 3.

Table 3: Train and Cross Validation Error for Dataset of Size 200 without any Regularization

Degree	Lambda	Train Error	Cross Validation Error
2	0	7.354288404349933	6.846556970651727
3	0	1.6552158448078724	1.6715279527959985
6	0	0.13062988588279714	0.15658597691064785
9	0	0.1303934103034066	0.15634800275115504

With Quadratic Regularization. Train Size = 200. Test Size = 60. Errors are Recorded in Table 4.

Table 4: Train and Cross Validation Error for Dataset of Size 200 with quadratic regularization

Degree	Lambda	Train Error	Cross Validation Error
2	0.001	7.354288404478799	6.8465543639970114
2	0.01	7.354288417234483	6.846530920730669
2	0.1	7.354289690741705	6.8462981314205535
2	1.0	7.354415016991336	6.844131987388243
3	0.001	1.6552158629565408	1.671489028953656
3	0.01	1.6552176587754166	1.6711402995429165
3	0.1	1.6553963372277196	1.6678094298267763
3	1.0	1.672318890155961	1.6495815770589473
6	0.001	0.13063050664533368	0.15662298217456766
6	0.01	0.13069166845704164	0.1570133676651501

6	0.1	0.13641376539099562	0.1660892987494758
6	1.0	0.3502436744368371	0.4091951206836821
9	0.001	0.1303951866914059	0.1564652626162473
9	0.01	0.13056439213979504	0.1576632264237352
9	0.1	0.14240983499689824	0.17847861159657225
9	1.0	0.3499068327746921	0.4202611814987257

1.4 Best Models

The best results obtained from the various models are as recorded in Table 5.

Table 5: Best Results for various cases of Task 1

Dataset Size	Regularization	Degree	Lambda
10	None	3	-
10	Quadratic	6	0.1
200	None	9	-
200	Quadratic	9	0.001

1.5 RMS Error for Test Data

The error for the test data set for the best models are as recorded in Table 6.

Table 6: Test Error for the Best Hyperparameters

Train Size	Test Size	Degree	Lambda	Test Error
10	5	6	0.1	0.8498611096580417
10	5	3	0	2.11021053423988
200	30	9	0.001	0.147396227908431
200	30	9	0	0.14727908388485048

1.6 Observations about Results

- The best combinations of hyperparameters of degree and lambda are as already defined earlier.
- For small train size (10), it is observed that smaller degrees give more accurate results. This could be due to less training data, leading to high variance for higher degrees.
- For larger training size (200), larger degrees are preferred as they are able to better fit to the data.
- As observed in Figure 4, the values of lambdas don’t have much of an effect on the results.

2 Task 2 – Linear Model using Polynomial Basis Function

Linear model for regression using polynomial basis functions for Dataset 2 with the file name “function2_2d.csv”.

2.1 Approach

- We want to make predictions using a linear model with a polynomial basis function.
- For this, we are given Dataset 2 which consists of two x variables and one y variables.

- The broad approach is to convert the data into polynomial data, and then fit a linear model to the polynomial data. Hyperparameters such as lambda and degree are tuned to find the best model.
- The first step is defining the dataset size to be taken [50,100,200].
- Next step is converting the data into polynomial data of a chosen degree d, by creating every possible combination of the variables such that the degree is less than or equal to "d".
- After this, we fit a linear model to the modified data. There may be no regularization or quadratic regularization at this stage.
- This is done for a variety of degree values [2,3,6,9] and lambda values [0.0, 0.01, 0.1, 1.0], to determine the best hyperparameters.
- The hyperparameters that give the lowest Root mean squared error are recorded.

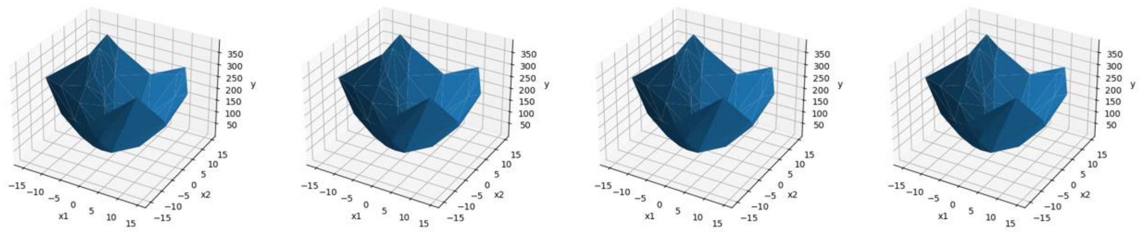
2.2 Surface Plots for the Approximated Functions

The surface plots for the approximated functions are as recorded below.

2.2.1 Plots for Dataset of Size 50

Taking Dataset of size 50, we get the plots as has been shown below.

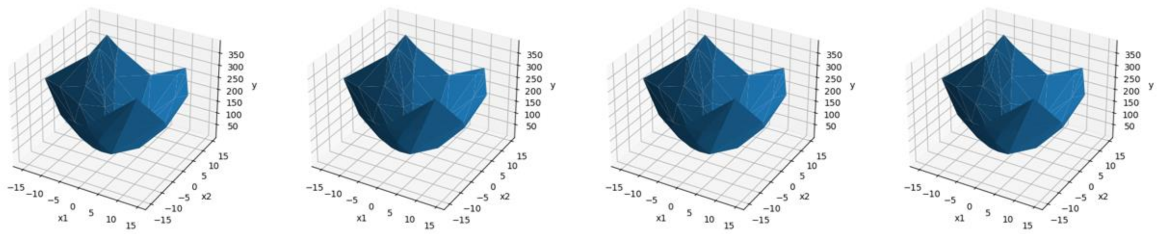
By fixing degree = 6, and varying lambda, the plots look like in Figure 5



Lambda = 0.01 Lambda = 0.1 Lambda = 0 Lambda = 1.0

Figure 5: Surface Plots for different Lambda Values

By fixing lambda = 0.01, and varying degree values, the plots look like in Figure 6



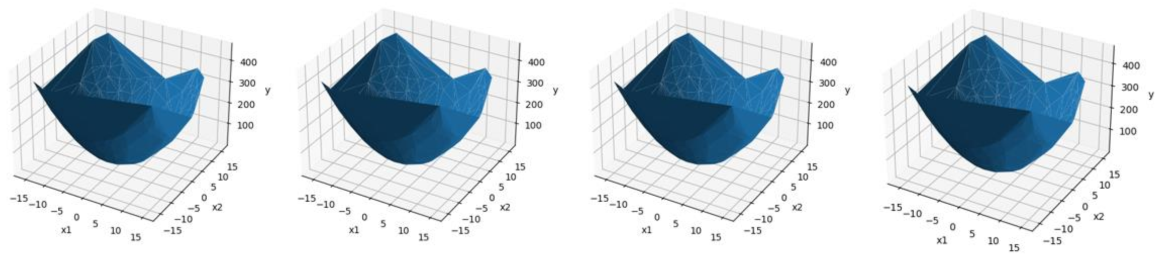
Degree = 2 Degree = 3 Degree = 6 Degree = 9

Figure 6: Surface plots for Different degrees

2.2.2 Plots for Dataset of Size 200

Taking Dataset of size 200, we get the plots as has been shown below.

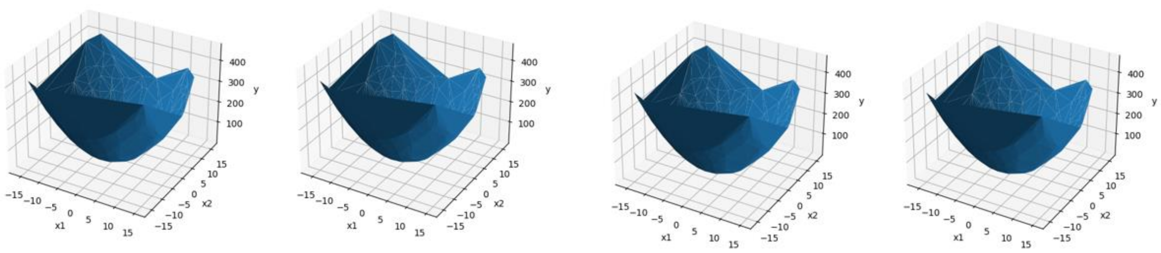
By fixing degree = 6, and varying lambda, the plots look like in Figure 7



Lambda = 0 Lambda = 0.01 Lambda = 0.1 Lambda = 1.0

Figure 7: Surface Plots for different Lambda Values

By fixing lambda = 0.01, and varying degree values, the plots look like in Figure 8



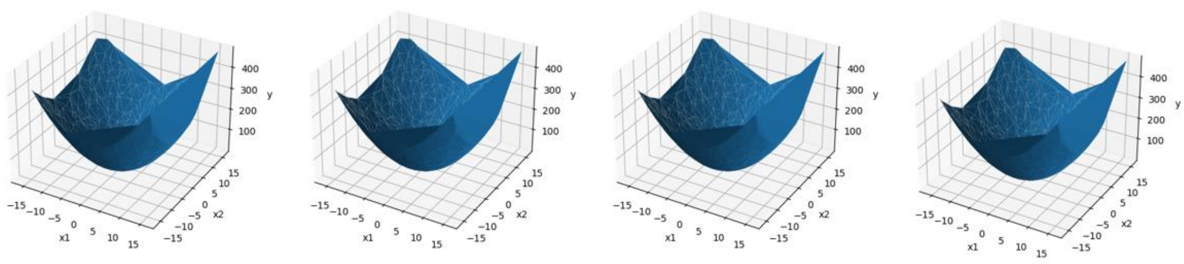
Degree = 2 Degree = 3 Degree = 6 Degree = 9

Figure 8: Surface plots for Different degrees

2.2.3 Plots for Dataset of Size 500

Taking Dataset of size 200, we get the plots as has been shown below.

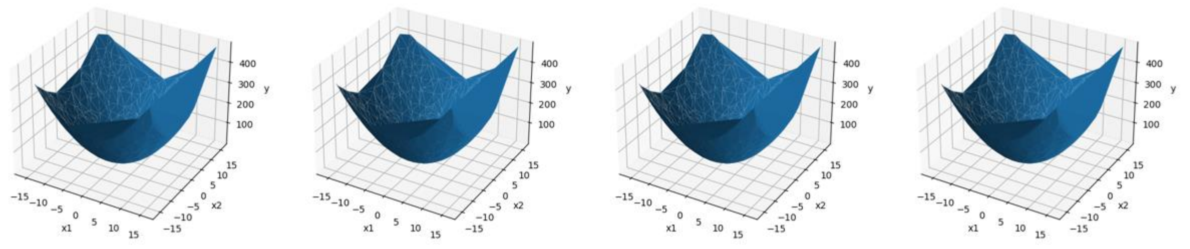
By fixing degree = 6, and varying lambda, the plots look like in Figure 9



Lambda = 0 Lambda = 0.01 Lambda = 0.1 Lambda = 1.0

Figure 9: Surface Plots for different Lambda Values

By fixing lambda = 0.01, and varying degree values, the plots look like in Figure 10



Degree = 2 Degree = 3 Degree = 6 Degree = 9

Figure 10: Surface plots for Different degrees

2.3 Visualizing the Dataset

Plotting a 2D Scatterplot for the given dataset, we are able to visualize the dataset as shown in Figure 11.

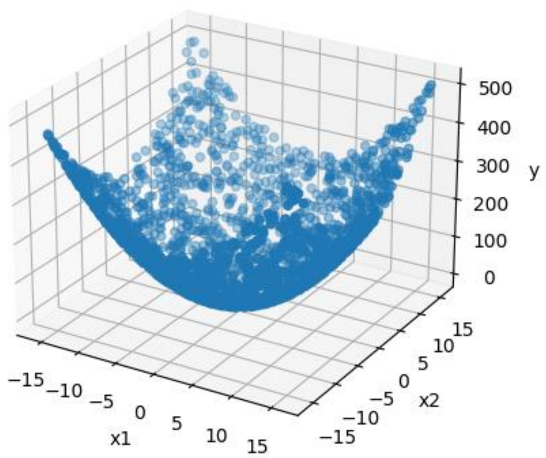


Figure 11: 2D plot of Dataset 2

2.4 RMS Error for the Training and Cross-Validation Data Sets

Root mean squared error for the train and cross validation error for the different cases. Note that the regularization being used everywhere is quadratic regularization, while lambda = 0 corresponds to no regularization.

2.4.1 For Dataset of Size 50

Without any Regularization. Train Size = 50. Test Size = 50. Errors are recorded in Table 7

Table 7: Train and Cross Validation Error for Dataset of Size 50

Degree	Lambda	Train Error	Cross Validation Error
2	0	4.968593683359719e-13	5.518270176491352e-13
2	0.01	5.909633573111495e-06	6.745698656168309e-06
2	0.1	5.880084125961443e-05	6.714341165695027e-05
2	1.0	0.0005608215280958414	0.0006425458875693444
3	0	3.552519273429934e-13	4.642460256922729e-13
3	0.01	7.047587235746912e-06	1.1363120635596052e-05
3	0.1	7.006472350962869e-05	0.0001130648133274531

3	1.0	0.0006632561931344315	0.0010788598550611622
6	0	3.639722339923864e-10	3.4223053067609147e-10
6	0.01	6.950266478573683e-05	0.00026047865310077086
6	0.1	0.0006808562599222149	0.0026058379790635453
6	1.0	0.005861587329519653	0.02624995084790649
9	0	1134.1271970542405	1300.0227225800534
9	0.01	0.18946776137646193	2111.777461552618
9	0.1	0.023814279430749652	2173.3800112984054
9	1.0	0.04293347441994508	2504.423599785385

2.4.2 For Dataset of Size 200

Without any Regularization. Train Size = 200. Test Size = 50. Errors are recorded in Table 8

Table 8: Train and Cross Validation Error for Dataset of Size 200

Degree	Lambda	Train Error	Cross Validation Error
2	0	1.1017916168564616e-13	1.1991185796884946e-13
2	0.01	1.4604429443367688e-06	1.790338328874665e-06
2	0.1	1.4584724426392318e-05	1.7880115149349733e-05
2	1.0	0.00014392013977280427	0.00017652388754564656
3	0	4.3175882883555097e-13	4.4739482630171004e-13
3	0.01	1.4850000020058456e-06	1.8960826333278106e-06
3	0.1	1.4829124290622157e-05	1.893471746589773e-05
3	1.0	0.00014625081145274875	0.00018679322105765318
6	0	3.1973024747361067e-10	3.472501355748491e-10
6	0.01	1.656816392958188e-05	3.4417877574481934e-05
6	0.1	0.0001645697520447766	0.0003420680154603799
6	1.0	0.0015457094858889982	0.003229702898343838
9	0	2.9159614576541486e-08	2.9710322180438283e-08
9	0.01	3.712793700684606e-05	0.00024379337920679486
9	0.1	0.0003664833243551834	0.0024125519700464483
9	1.0	0.0032711793988601232	0.021983164793079164

2.4.3 For Dataset of Size 500

Without any Regularization. Train Size = 500. Test Size = 50. Errors are recorded in Table 9

Table 9: Train and Cross Validation Error for Dataset of Size 500

Degree	Lambda	Train Error	Cross Validation Error
2	0	7.536832674193973e-13	7.571682896081965e-13
2	0.01	5.512471455283732e-07	6.300390893748985e-07
2	0.1	5.5097374259556705e-06	6.297171787077824e-06
2	1.0	5.482682268409608e-05	6.265305813773022e-05
3	0	8.368221621098526e-13	9.718510648446957e-13
3	0.01	5.559047751280072e-07	6.820400190136832e-07
3	0.1	5.556279266581209e-06	6.817036521046153e-06
3	1.0	5.528799410436049e-05	6.783637683153026e-05
6	0	2.089117707814018e-10	2.0824532813403086e-10
6	0.01	5.5396844753169235e-06	1.4345153931843745e-05
6	0.1	5.529107127153236e-05	0.00014321073632235927

6	1.0	0.0005426795876501346	0.0014086969238335653
9	0	2.9139843124971285e-08	3.1808771722412934e-08
9	0.01	1.1502157797026338e-05	0.0001226129023604638
9	0.1	0.00011465757965196129	0.0012230167558288552
9	1.0	0.0011122169954202255	0.011933269645931272

2.5 Best Models

The best results obtained from the various models are as recorded below in Table 10. Note that even with quadratic regularization, the best models correspond to lambda = 0 i.e., no regularization.

Table 10: Best Hyperparameters for each case

Dataset Size	Degree	Lambda
50	3	0
200	2	0
500	2	0

2.6 RMS Error for the Test Data

The error for the test data set for the best models are as recorded below in Table 11.

Table 11: Test Error corresponding to the best models

Dataset Size	Degree	Lambda	Test Error
50	3	0	6.837771032371849e-13
200	2	0	5.604017823306619e-13
500	2	0	1.8775231883854885e-13

2.7 Scatter Plots for Actual vs. Predicted Values

The scatter plots for the actual vs. the predicted values for the best model in each of the various cases are as follows:

2.7.1 For Dataset of Size 50

The scatter plots of actual values vs. predictions for dataset of size 50 are as shown in Figure 12 and Figure 13

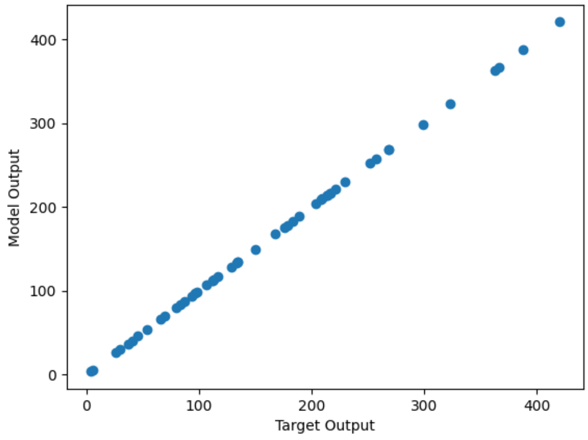


Figure 12: Scatter plot for Training data for dataset of size 50

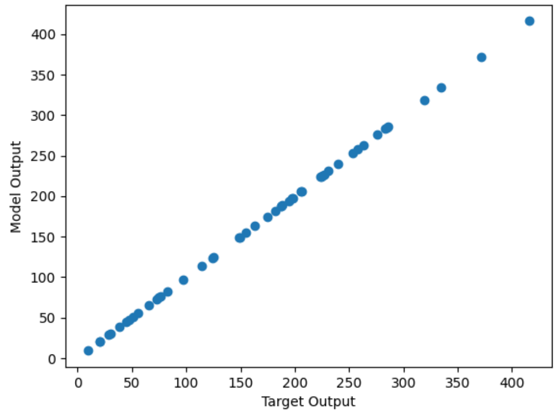


Figure 13: Scatter plot for Validation data for dataset of size 50

2.7.2 For Dataset of Size 200

The scatter plots of actual values vs. predictions for dataset of size 200 are as shown in Figure 14 and Figure 15.

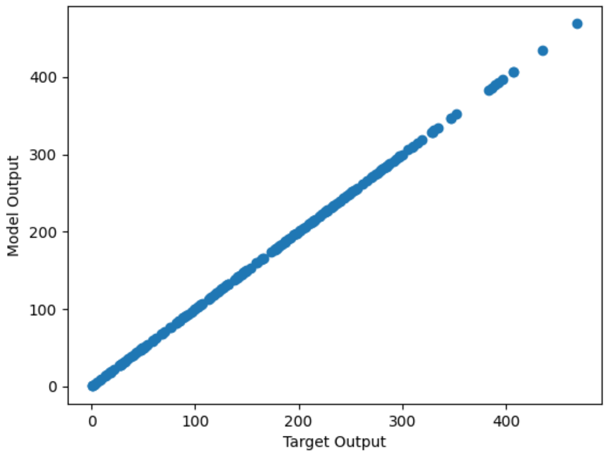


Figure 14: Scatter plot for Training data for dataset of size 200

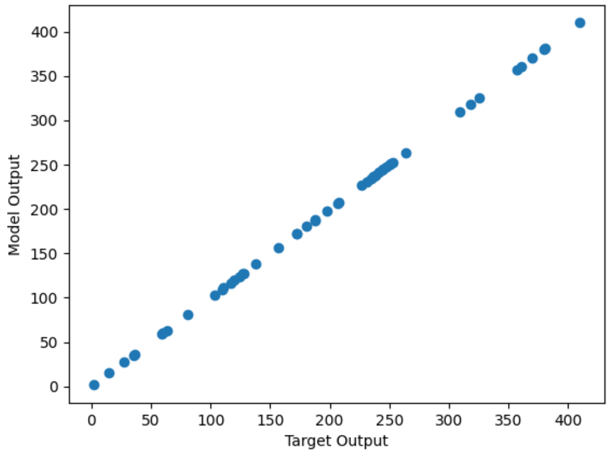


Figure 15: Scatter plot for Validation data for dataset of size 200

2.7.3 For Dataset of Size 500

The scatter plots of actual values vs. predictions for dataset of size 500 are as shown below.

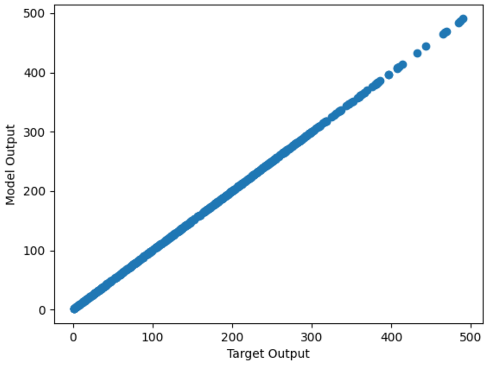


Figure 16: Scatter plot for Training data for dataset of size 500

2.8 Observations about Results

- The model without any regularization performs better than the model with regularization
- A very low error is observed in the best model, with small degrees of 2 and 3, suggesting the data is most likely extracted from a quadratic / cubic curve.
- Most of the 2D surface plots look identical when varying degree and lambda. This is in further support of the point mentioned above. All the curves look like degree 2 polynomials.

3 Task 3 – Linear Model using Gaussian Basis Function

Linear model for regression using Gaussian basis functions for Dataset 2, and for Dataset 3 with the file name “0_superconductor.csv”.

3.1 Approach

- We want to make predictions using a linear model with a Gaussian basis function.
- For this, we are given Dataset 3 which consists of many x variables and two y variables.

- The broad approach is to convert the data into the Gaussian Distributions. This is then run through liner regression to fit the data, and make predictions on the two required columns.
- Firstly, the means of the various Gaussian Distributions are identified by using K Means Clustering. Each data point is then converted using the Gaussian Distribution formula for each distribution.
- This data is then fit using Linear Regression to make predictions for both the required columns.
- This is done for different values of lambda, K and sigma to appropriately tune the hyperparameters.
- Note that the same hyperparameters are chosen for both the cases.
- The best set of hyperparameters are chosen.

3.2 RMS Error for the Training, Validation and Test Dataset for Dataset 3

The error data for the various cases are highlighted as below. Suffix 1 refers to the first y column, and suffix 2 refers to the second y column.

3.2.1 For Quadratic Regularization

The errors for Quadratic Regularization are as highlighted below in Table 12.

Table 12: Error MSE for Quadratic Regularization

K	sigma	lambda	train error 1	train error 2	val error 1	val error 2	test error 1	test error 2
5.00E+01	2.00E+01	1.00E-04	7.01E-01	5.53E+01	9.06E-01	2.66E+01	1.23E+00	2.31E+01
5.00E+01	2.00E+01	1.00E-03	7.01E-01	5.53E+01	9.06E-01	2.66E+01	1.23E+00	2.31E+01
5.00E+01	2.00E+01	1.00E-02	7.02E-01	5.53E+01	9.06E-01	2.66E+01	1.23E+00	2.31E+01
5.00E+01	2.00E+01	1.00E-01	7.02E-01	5.54E+01	9.06E-01	2.67E+01	1.23E+00	2.31E+01
5.00E+01	2.00E+01	1.00E+00	7.03E-01	5.56E+01	9.06E-01	2.67E+01	1.23E+00	2.32E+01
5.00E+01	5.00E+01	1.00E-04	6.80E-01	5.28E+01	9.05E-01	2.64E+01	1.23E+00	2.27E+01
5.00E+01	5.00E+01	1.00E-03	6.80E-01	5.28E+01	9.05E-01	2.64E+01	1.23E+00	2.27E+01
5.00E+01	5.00E+01	1.00E-02	6.80E-01	5.29E+01	9.05E-01	2.64E+01	1.23E+00	2.27E+01
5.00E+01	5.00E+01	1.00E-01	6.82E-01	5.31E+01	9.05E-01	2.64E+01	1.23E+00	2.27E+01
5.00E+01	5.00E+01	1.00E+00	6.92E-01	5.40E+01	9.05E-01	2.65E+01	1.23E+00	2.28E+01
5.00E+01	1.00E+02	1.00E-04	6.59E-01	4.93E+01	9.03E-01	2.61E+01	1.23E+00	2.23E+01
5.00E+01	1.00E+02	1.00E-03	6.60E-01	4.94E+01	9.03E-01	2.61E+01	1.23E+00	2.23E+01
5.00E+01	1.00E+02	1.00E-02	6.62E-01	4.95E+01	9.03E-01	2.62E+01	1.23E+00	2.23E+01
5.00E+01	1.00E+02	1.00E-01	6.67E-01	4.98E+01	9.04E-01	2.63E+01	1.23E+00	2.23E+01
5.00E+01	1.00E+02	1.00E+00	6.70E-01	5.02E+01	9.04E-01	2.63E+01	1.23E+00	2.24E+01
5.00E+01	1.50E+02	1.00E-04	6.38E-01	4.64E+01	9.00E-01	2.55E+01	1.23E+00	2.23E+01
5.00E+01	1.50E+02	1.00E-03	6.39E-01	4.64E+01	9.00E-01	2.55E+01	1.23E+00	2.23E+01
5.00E+01	1.50E+02	1.00E-02	6.40E-01	4.64E+01	9.00E-01	2.55E+01	1.23E+00	2.22E+01
5.00E+01	1.50E+02	1.00E-01	6.43E-01	4.67E+01	9.01E-01	2.56E+01	1.23E+00	2.23E+01
5.00E+01	1.50E+02	1.00E+00	6.49E-01	4.74E+01	9.02E-01	2.59E+01	1.23E+00	2.23E+01
5.00E+01	2.00E+02	1.00E-04	6.20E-01	4.38E+01	9.02E-01	2.50E+01	1.23E+00	2.22E+01
5.00E+01	2.00E+02	1.00E-03	6.20E-01	4.38E+01	9.00E-01	2.50E+01	1.23E+00	2.22E+01
5.00E+01	2.00E+02	1.00E-02	6.21E-01	4.38E+01	8.99E-01	2.50E+01	1.23E+00	2.22E+01
5.00E+01	2.00E+02	1.00E-01	6.23E-01	4.38E+01	8.99E-01	2.50E+01	1.23E+00	2.22E+01
5.00E+01	2.00E+02	1.00E+00	6.29E-01	4.43E+01	8.99E-01	2.52E+01	1.23E+00	2.22E+01
7.00E+01	2.00E+01	1.00E-04	7.01E-01	5.42E+01	9.06E-01	2.65E+01	1.23E+00	2.30E+01

7.00E+01	2.00E+01	1.00E-03	7.01E-01	5.42E+01	9.06E-01	2.66E+01	1.23E+00	2.30E+01
7.00E+01	2.00E+01	1.00E-02	7.01E-01	5.42E+01	9.06E-01	2.66E+01	1.23E+00	2.30E+01
7.00E+01	2.00E+01	1.00E-01	7.01E-01	5.42E+01	9.06E-01	2.66E+01	1.23E+00	2.30E+01
7.00E+01	2.00E+01	1.00E+00	7.02E-01	5.44E+01	9.06E-01	2.66E+01	1.23E+00	2.30E+01
7.00E+01	5.00E+01	1.00E-04	6.92E-01	5.21E+01	9.05E-01	2.63E+01	1.23E+00	2.25E+01
7.00E+01	5.00E+01	1.00E-03	6.92E-01	5.21E+01	9.05E-01	2.63E+01	1.23E+00	2.25E+01
7.00E+01	5.00E+01	1.00E-02	6.93E-01	5.23E+01	9.05E-01	2.63E+01	1.23E+00	2.25E+01
7.00E+01	5.00E+01	1.00E-01	6.94E-01	5.26E+01	9.05E-01	2.63E+01	1.23E+00	2.26E+01
7.00E+01	5.00E+01	1.00E+00	6.96E-01	5.28E+01	9.06E-01	2.64E+01	1.23E+00	2.26E+01
7.00E+01	1.00E+02	1.00E-04	6.64E-01	4.83E+01	9.01E-01	2.61E+01	1.23E+00	2.24E+01
7.00E+01	1.00E+02	1.00E-03	6.66E-01	4.84E+01	9.02E-01	2.61E+01	1.23E+00	2.24E+01
7.00E+01	1.00E+02	1.00E-02	6.70E-01	4.86E+01	9.03E-01	2.61E+01	1.23E+00	2.24E+01
7.00E+01	1.00E+02	1.00E-01	6.73E-01	4.89E+01	9.04E-01	2.61E+01	1.23E+00	2.24E+01
7.00E+01	1.00E+02	1.00E+00	6.76E-01	4.95E+01	9.04E-01	2.62E+01	1.23E+00	2.24E+01
7.00E+01	1.50E+02	1.00E-04	6.43E-01	4.45E+01	8.99E-01	2.76E+01	1.23E+00	2.21E+01
7.00E+01	1.50E+02	1.00E-03	6.44E-01	4.48E+01	8.99E-01	2.57E+01	1.23E+00	2.21E+01
7.00E+01	1.50E+02	1.00E-02	6.46E-01	4.53E+01	8.99E-01	2.57E+01	1.23E+00	2.22E+01
7.00E+01	1.50E+02	1.00E-01	6.49E-01	4.59E+01	9.00E-01	2.58E+01	1.23E+00	2.22E+01
7.00E+01	1.50E+02	1.00E+00	6.58E-01	4.66E+01	9.02E-01	2.59E+01	1.23E+00	2.22E+01
7.00E+01	2.00E+02	1.00E-04	6.22E-01	4.12E+01	8.94E-01	3.56E+01	1.23E+00	2.19E+01
7.00E+01	2.00E+02	1.00E-03	6.22E-01	4.12E+01	8.95E-01	3.37E+01	1.23E+00	2.19E+01
7.00E+01	2.00E+02	1.00E-02	6.24E-01	4.14E+01	8.95E-01	2.71E+01	1.23E+00	2.19E+01
7.00E+01	2.00E+02	1.00E-01	6.27E-01	4.21E+01	8.96E-01	2.51E+01	1.23E+00	2.20E+01
7.00E+01	2.00E+02	1.00E+00	6.34E-01	4.34E+01	8.98E-01	2.54E+01	1.23E+00	2.21E+01
1.00E+02	2.00E+01	1.00E-04	6.99E-01	5.37E+01	9.06E-01	2.64E+01	1.23E+00	2.29E+01
1.00E+02	2.00E+01	1.00E-03	6.99E-01	5.37E+01	9.06E-01	2.64E+01	1.23E+00	2.29E+01
1.00E+02	2.00E+01	1.00E-02	6.99E-01	5.37E+01	9.06E-01	2.64E+01	1.23E+00	2.29E+01
1.00E+02	2.00E+01	1.00E-01	6.99E-01	5.38E+01	9.06E-01	2.64E+01	1.23E+00	2.29E+01
1.00E+02	2.00E+01	1.00E+00	7.00E-01	5.41E+01	9.06E-01	2.65E+01	1.23E+00	2.30E+01
1.00E+02	5.00E+01	1.00E-04	6.85E-01	5.11E+01	9.04E-01	2.62E+01	1.23E+00	2.24E+01
1.00E+02	5.00E+01	1.00E-03	6.85E-01	5.13E+01	9.04E-01	2.62E+01	1.23E+00	2.24E+01
1.00E+02	5.00E+01	1.00E-02	6.87E-01	5.16E+01	9.04E-01	2.62E+01	1.23E+00	2.24E+01
1.00E+02	5.00E+01	1.00E-01	6.89E-01	5.19E+01	9.05E-01	2.62E+01	1.23E+00	2.24E+01
1.00E+02	5.00E+01	1.00E+00	6.93E-01	5.21E+01	9.05E-01	2.63E+01	1.23E+00	2.24E+01
1.00E+02	1.00E+02	1.00E-04	6.57E-01	4.60E+01	9.03E-01	2.67E+01	1.23E+00	2.22E+01
1.00E+02	1.00E+02	1.00E-03	6.58E-01	4.62E+01	9.03E-01	2.66E+01	1.23E+00	2.23E+01
1.00E+02	1.00E+02	1.00E-02	6.60E-01	4.64E+01	9.03E-01	2.60E+01	1.23E+00	2.23E+01
1.00E+02	1.00E+02	1.00E-01	6.62E-01	4.67E+01	9.03E-01	2.57E+01	1.23E+00	2.23E+01
1.00E+02	1.00E+02	1.00E+00	6.66E-01	4.75E+01	9.03E-01	2.59E+01	1.23E+00	2.23E+01
1.00E+02	1.50E+02	1.00E-04	6.36E-01	4.20E+01	8.99E-01	2.48E+01	1.23E+00	2.18E+01
1.00E+02	1.50E+02	1.00E-03	6.37E-01	4.20E+01	8.99E-01	2.48E+01	1.23E+00	2.19E+01
1.00E+02	1.50E+02	1.00E-02	6.39E-01	4.24E+01	9.00E-01	2.50E+01	1.23E+00	2.19E+01
1.00E+02	1.50E+02	1.00E-01	6.42E-01	4.32E+01	9.00E-01	2.51E+01	1.23E+00	2.21E+01
1.00E+02	1.50E+02	1.00E+00	6.48E-01	4.41E+01	9.01E-01	2.53E+01	1.23E+00	2.21E+01
1.00E+02	2.00E+02	1.00E-04	6.11E-01	3.88E+01	8.93E-01	2.41E+01	1.23E+00	2.16E+01
1.00E+02	2.00E+02	1.00E-03	6.11E-01	3.88E+01	8.94E-01	2.41E+01	1.23E+00	2.16E+01
1.00E+02	2.00E+02	1.00E-02	6.14E-01	3.88E+01	8.95E-01	2.41E+01	1.23E+00	2.16E+01
1.00E+02	2.00E+02	1.00E-01	6.18E-01	3.90E+01	8.97E-01	2.41E+01	1.23E+00	2.17E+01

1.00E+02	2.00E+02	1.00E+00	6.26E-01	4.03E+01	8.98E-01	2.45E+01	1.23E+00	2.18E+01
1.50E+02	2.00E+01	1.00E-04	6.91E-01	5.29E+01	9.05E-01	2.64E+01	1.23E+00	2.26E+01
1.50E+02	2.00E+01	1.00E-03	6.91E-01	5.29E+01	9.05E-01	2.64E+01	1.23E+00	2.26E+01
1.50E+02	2.00E+01	1.00E-02	6.92E-01	5.30E+01	9.05E-01	2.64E+01	1.23E+00	2.26E+01
1.50E+02	2.00E+01	1.00E-01	6.92E-01	5.32E+01	9.05E-01	2.64E+01	1.23E+00	2.26E+01
1.50E+02	2.00E+01	1.00E+00	6.93E-01	5.34E+01	9.05E-01	2.65E+01	1.23E+00	2.26E+01
1.50E+02	5.00E+01	1.00E-04	6.66E-01	4.95E+01	9.32E-01	2.64E+01	1.23E+00	2.23E+01
1.50E+02	5.00E+01	1.00E-03	6.66E-01	4.96E+01	9.20E-01	2.62E+01	1.23E+00	2.23E+01
1.50E+02	5.00E+01	1.00E-02	6.68E-01	4.97E+01	9.03E-01	2.60E+01	1.23E+00	2.23E+01
1.50E+02	5.00E+01	1.00E-01	6.71E-01	4.99E+01	9.03E-01	2.61E+01	1.23E+00	2.23E+01
1.50E+02	5.00E+01	1.00E+00	6.78E-01	5.04E+01	9.03E-01	2.61E+01	1.23E+00	2.23E+01
1.50E+02	1.00E+02	1.00E-04	6.36E-01	4.51E+01	8.99E-01	2.55E+01	1.23E+00	2.22E+01
1.50E+02	1.00E+02	1.00E-03	6.37E-01	4.52E+01	8.99E-01	2.55E+01	1.23E+00	2.22E+01
1.50E+02	1.00E+02	1.00E-02	6.39E-01	4.54E+01	9.00E-01	2.56E+01	1.23E+00	2.22E+01
1.50E+02	1.00E+02	1.00E-01	6.41E-01	4.58E+01	9.00E-01	2.56E+01	1.23E+00	2.22E+01
1.50E+02	1.00E+02	1.00E+00	6.49E-01	4.68E+01	9.00E-01	2.57E+01	1.23E+00	2.22E+01
1.50E+02	1.50E+02	1.00E-04	6.13E-01	4.10E+01	8.97E-01	2.49E+01	1.23E+00	2.20E+01
1.50E+02	1.50E+02	1.00E-03	6.15E-01	4.11E+01	8.96E-01	2.49E+01	1.23E+00	2.20E+01
1.50E+02	1.50E+02	1.00E-02	6.16E-01	4.14E+01	8.96E-01	2.50E+01	1.23E+00	2.20E+01
1.50E+02	1.50E+02	1.00E-01	6.19E-01	4.20E+01	8.97E-01	2.50E+01	1.23E+00	2.21E+01
1.50E+02	1.50E+02	1.00E+00	6.26E-01	4.29E+01	8.98E-01	2.52E+01	1.23E+00	2.22E+01
1.50E+02	2.00E+02	1.00E-04	5.87E-01	3.77E+01	8.91E-01	2.43E+01	1.23E+00	2.18E+01
1.50E+02	2.00E+02	1.00E-03	5.87E-01	3.77E+01	8.91E-01	2.43E+01	1.23E+00	2.18E+01
1.50E+02	2.00E+02	1.00E-02	5.91E-01	3.78E+01	8.92E-01	2.42E+01	1.23E+00	2.18E+01
1.50E+02	2.00E+02	1.00E-01	5.96E-01	3.80E+01	8.94E-01	2.44E+01	1.23E+00	2.18E+01
1.50E+02	2.00E+02	1.00E+00	6.03E-01	3.90E+01	8.96E-01	2.46E+01	1.23E+00	2.19E+01

3.2.2 For no regularization

The errors for no Regularization are as highlighted below in Table 13.

Table 13: Error MSE for no Regularization

K	sigma	lamda	train error 1	train error 2	val error 1	val error 2	test error 1	test error 2
5.00E+01	1.00E+02	1.00E-04	8.86E+49	9.68E+51	4.21E+46	4.59E+48	3.25E+34	2.95E+37
5.00E+01	1.00E+02	1.00E-03	8.86E+49	9.68E+51	4.21E+46	4.59E+48	3.25E+34	2.95E+37
5.00E+01	1.00E+02	1.00E-02	8.86E+49	9.68E+51	4.21E+46	4.59E+48	3.25E+34	2.95E+37
5.00E+01	1.00E+02	1.00E-01	8.86E+49	9.68E+51	4.21E+46	4.59E+48	3.25E+34	2.95E+37
5.00E+01	1.00E+02	1.00E+00	8.86E+49	9.68E+51	4.21E+46	4.59E+48	3.25E+34	2.95E+37
5.00E+01	1.50E+02	1.00E-04	1.11E+22	1.29E+23	1.64E+21	1.81E+22	4.98E+21	5.45E+22
5.00E+01	1.50E+02	1.00E-03	1.11E+22	1.29E+23	1.64E+21	1.81E+22	4.98E+21	5.45E+22
5.00E+01	1.50E+02	1.00E-02	1.11E+22	1.29E+23	1.64E+21	1.81E+22	4.98E+21	5.45E+22
5.00E+01	1.50E+02	1.00E-01	1.11E+22	1.29E+23	1.64E+21	1.81E+22	4.98E+21	5.45E+22
5.00E+01	1.50E+02	1.00E+00	1.11E+22	1.29E+23	1.64E+21	1.81E+22	4.98E+21	5.45E+22
5.00E+01	2.00E+02	1.00E-04	8.94E+12	1.86E+15	1.35E+12	2.82E+14	3.83E+12	7.99E+14
5.00E+01	2.00E+02	1.00E-03	8.94E+12	1.86E+15	1.35E+12	2.82E+14	3.83E+12	7.99E+14
5.00E+01	2.00E+02	1.00E-02	8.94E+12	1.86E+15	1.35E+12	2.82E+14	3.83E+12	7.99E+14
5.00E+01	2.00E+02	1.00E-01	8.94E+12	1.86E+15	1.35E+12	2.82E+14	3.83E+12	7.99E+14
5.00E+01	2.00E+02	1.00E+00	8.94E+12	1.86E+15	1.35E+12	2.82E+14	3.83E+12	7.99E+14

7.00E+01	1.00E+02	1.00E-04	6.53E-01	4.74E+01	8.99E-01	2.61E+01	1.23E+00	2.22E+01
7.00E+01	1.00E+02	1.00E-03	6.53E-01	4.74E+01	8.99E-01	2.61E+01	1.23E+00	2.22E+01
7.00E+01	1.00E+02	1.00E-02	6.53E-01	4.74E+01	8.99E-01	2.61E+01	1.23E+00	2.22E+01
7.00E+01	1.00E+02	1.00E-01	6.53E-01	4.74E+01	8.99E-01	2.61E+01	1.23E+00	2.22E+01
7.00E+01	1.00E+02	1.00E+00	6.53E-01	4.74E+01	8.99E-01	2.61E+01	1.23E+00	2.22E+01
7.00E+01	1.50E+02	1.00E-04	6.36E-01	4.47E+01	8.96E-01	2.58E+01	1.23E+00	2.21E+01
7.00E+01	1.50E+02	1.00E-03	6.36E-01	4.47E+01	8.96E-01	2.58E+01	1.23E+00	2.21E+01
7.00E+01	1.50E+02	1.00E-02	6.36E-01	4.47E+01	8.96E-01	2.58E+01	1.23E+00	2.21E+01
7.00E+01	1.50E+02	1.00E-01	6.36E-01	4.47E+01	8.96E-01	2.58E+01	1.23E+00	2.21E+01
7.00E+01	1.50E+02	1.00E+00	6.36E-01	4.47E+01	8.96E-01	2.58E+01	1.23E+00	2.21E+01
7.00E+01	2.00E+02	1.00E-04	6.20E-01	4.19E+01	8.92E-01	2.52E+01	1.23E+00	2.20E+01
7.00E+01	2.00E+02	1.00E-03	6.20E-01	4.19E+01	8.92E-01	2.52E+01	1.23E+00	2.20E+01
7.00E+01	2.00E+02	1.00E-02	6.20E-01	4.19E+01	8.92E-01	2.52E+01	1.23E+00	2.20E+01
7.00E+01	2.00E+02	1.00E-01	6.20E-01	4.19E+01	8.92E-01	2.52E+01	1.23E+00	2.20E+01
7.00E+01	2.00E+02	1.00E+00	6.20E-01	4.19E+01	8.92E-01	2.52E+01	1.23E+00	2.20E+01

3.2.3 For Tikonov Regularization

3.3 Best Models for Dataset 3

The best hyperparameters obtained are as follows.

Regularization	K	Sigma	Lambda	Min_Error_1	Min_Error_2
None	70	200	0.0001	0.89245257	25.15878371
Quadratic	100	200	0.001	0.89434474	24.21931457
Tikonov	70	200	0.01	0.89699871	25.26556693

3.4 Scatter Plots for Dataset 3

The Scatter Plots for Predictions versus actual values for the cases are as follows.

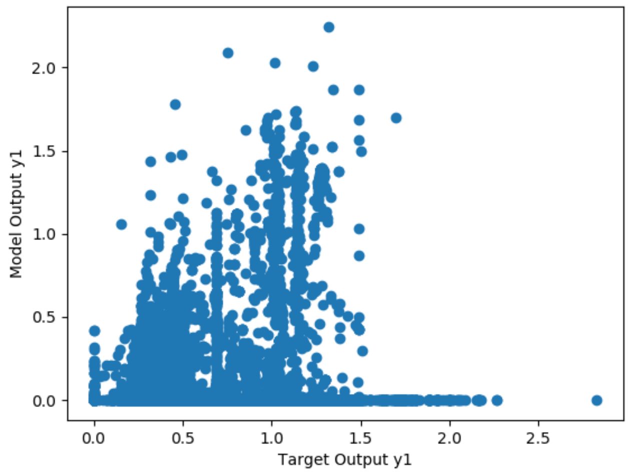


Figure 17: Train - No Regularization - Column 1

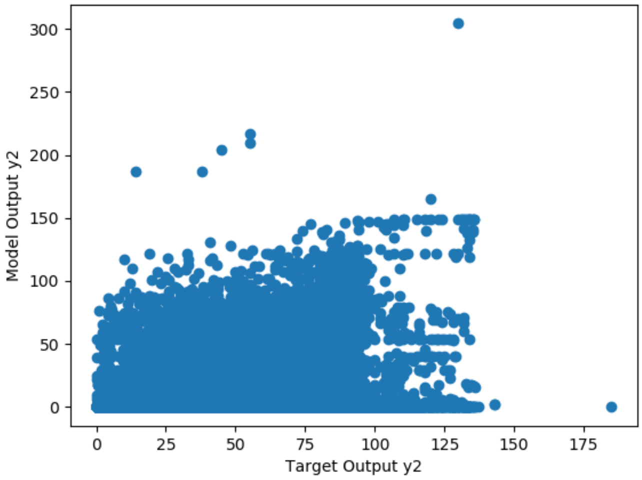


Figure 18: Train - No Regularization - Column 2

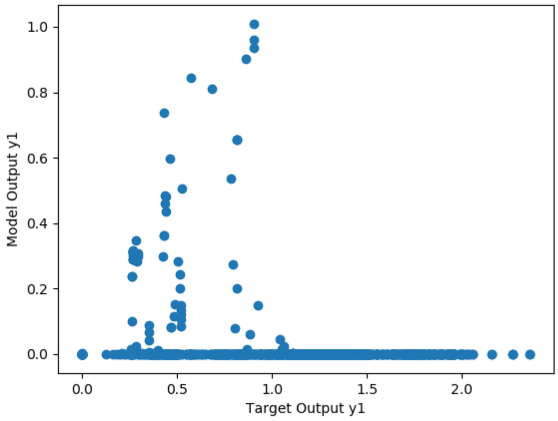


Figure 19: Test - No Regularization - Column 1

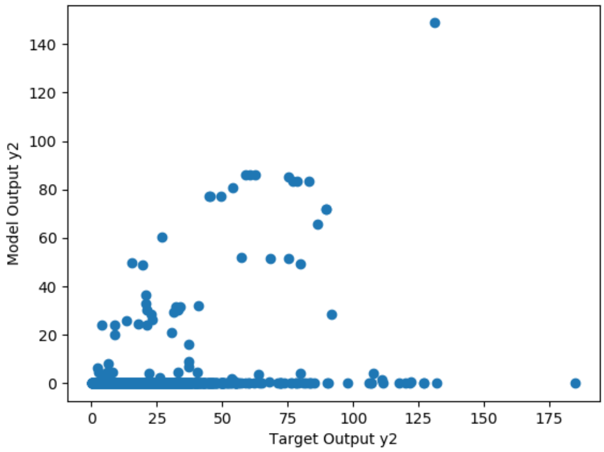


Figure 20: Test - No Regularization - Column 2

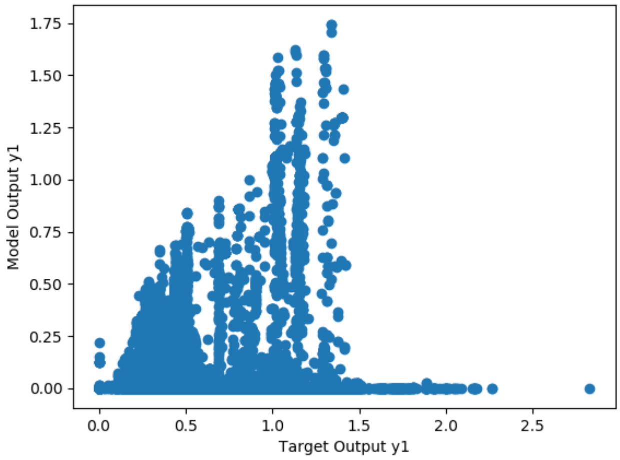


Figure 21: Train - Quadratic Regularization - Column 1

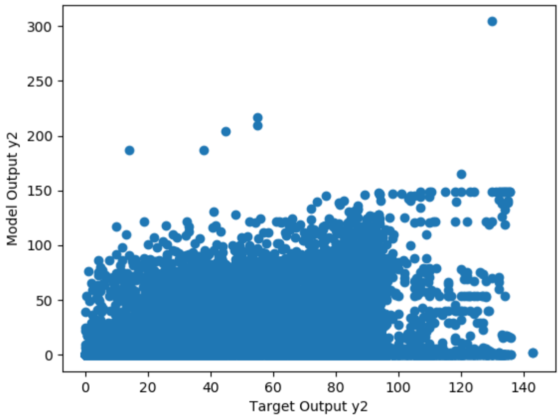


Figure 22: Train - Quadratic Regularization - Column 2

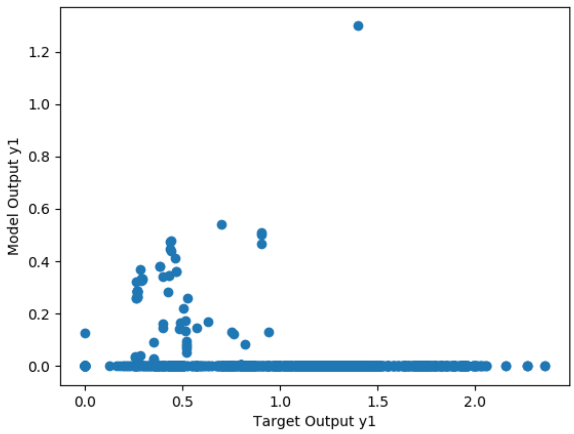


Figure 23: Test - Quadratic Regularization - Column 1

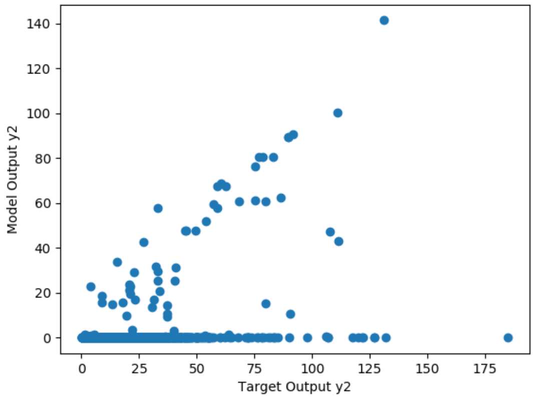


Figure 24: Test - Quadratic Regularization - Column 2

3.5 Scatter Plots for Dataset 2

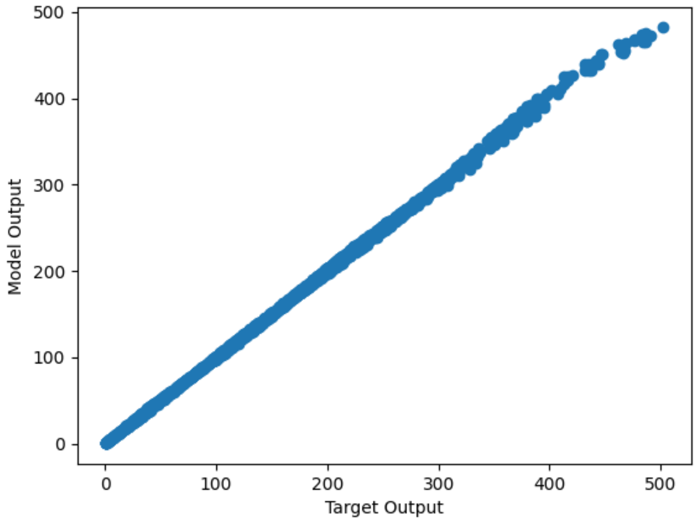


Figure 25: Train Scatter Plot for Quadratic Regularization

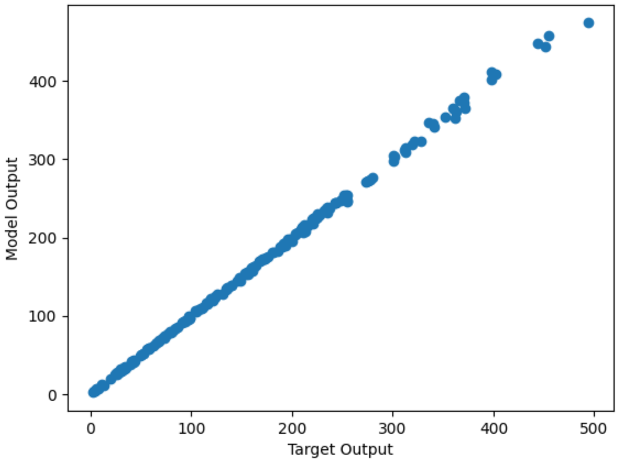


Figure 26: Test Scatter Plot for Quadratic Regularization

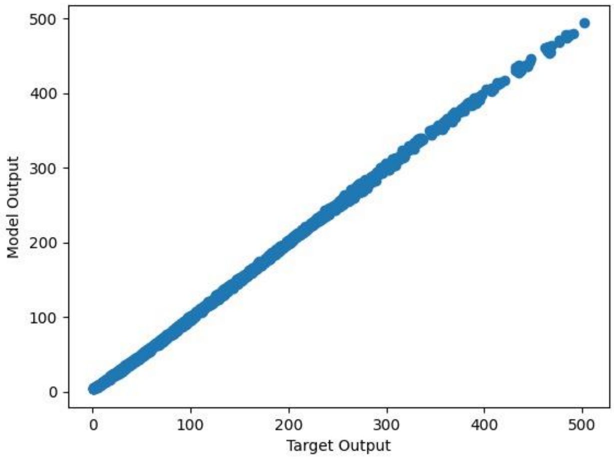


Figure 27: Train Scatter Plot for Tikonov Regularization

3.6 Best Models for Dataset 2

Regularization	K	Sigma	Lambda
Quadratic	100	8	0.0001
Tikonov	25	20	0.0001