HW - 2

Q Given :- Matrix 'M' $\in R^{d \times d}$
det ~~ate~~ vector 'x' $\in R^d$

Prove :- $E[\cancel{X}^T M x] = x^T E[M] x$

ans :- Given :-
M is a ~~random~~ random variable (r.v.)
and, $M \in R^{d \times d}$
also,
'x' is a deterministic vector of format :-

$$x \in R^d$$

hence,

$$M = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ A_{21} & - & & \vdots \\ \vdots & & & \\ A_{d1} & \cdots & & A_{dd} \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

To prove :- $E[x^T M x] = x^T E[M] x$

We know that,

$$E[x^T M x] = E\left[\sum_{i,j=1}^{d} x_i M_{ij} x_j\right]$$

$\therefore$ Since 'x' is a deterministic vector, and M is the only random~~able~~ variable (r.v.) in the equation. ~~the~~ ~~can~~ ~~as~~ $E[x]$ will be constant and the above equation can take following form :-

$$\Rightarrow \quad E\left[\sum_{i,j=1}^{d} x_i m_{ij} x_j\right] = \sum_{i,j=1}^{d} x_i \, E[m_{ij}] \, x_j$$

$$= \sum_{i,j=1}^{d} x_i x_j \, . \, E[m_{ij}]$$

$$= x^2 \, E[m_{ij}]$$

$$= x^T x \, E[m_{ij}] \quad \ldots\ldots \left(\text{for det. vector } x^2 = x^T x\right)$$

$$= x^T \, E[m_{ij}] \, x$$

Thus,

$$\Rightarrow \quad E[x^T M x] = x^T E[M] x$$