SRM Institute of Science and Technology

College of Engineering and Technology Kattankulathur-603 203

Department of Mathematics

21MAB102T-Advanced Calculus and Complex Analysis

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|-------|------|--------|----|
| Tiito | rial | Sheet. | -7 |

| Sl.No. | Part-B (8 Marks) | Answers |
|--------|---|---|
| Q1. | If \vec{r} is the position vector of the point (x,y,z) find (i) $\nabla \left(\frac{1}{r}\right)$. (ii) $\nabla (\log r)$. | $-\frac{r}{r^3}, \frac{\vec{r}}{r^3}$ |
| Q2. | Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = \vec{r} $ and hence deduce $\nabla^2\left(\frac{1}{r}\right)$ | |
| Q3. | Find the unit normal vector to the surface (i) $x^2 + xy + z^2 = 4$ at (1,-1,2). (ii) $xy^2z^3 = 1$ at (1,1,1). | $\frac{\vec{i}+\vec{j}+4\vec{k}}{3\sqrt{2}}, \frac{\vec{i}+2\vec{j}+3\vec{k}}{\sqrt{14}}$ |
| Q4. | Evaluate $\int_C \vec{F} \cdot dr$, for (i) $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, C is curve in the xy plane $y = 2x^2$, from $(0,0)$ to $(1,2)$. (ii) $\vec{F} = 5xy\vec{i} + 2y\vec{j}$, C is the part of the curve in the xy plane $y = x^2$, from $x = 1$ to $x = 2$. | $-\frac{7}{6}$, $\frac{135}{4}$ |
| Q5. | Determine whether the following vectors are conservative field. (i) $\vec{F} = (2xy+z^2)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ (ii) $\vec{F} = (e^xz-2xy)\vec{i}+(x^2-1)\vec{j}+(e^x+z)\vec{k}$ | |
| Sl.No. | Part-C (15 Marks) | Answers |
| Q6. | Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in the xy plane bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = b$. | |