interactive ik gradient descent

September 22, 2024

1	Agility	-	Solution
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1.1.1 Assumptions:

None of the chains, arms or legs, touching the gorund by default. Because that would create closed loops. The base is attaching to the world/ground by a 6 dof joint. Its a virtual joint to represent the floating base.

1.1.2 Preliminaries 1 and 2:

r + 3 dof for base 7 dof for left arm 7 dof for right arm 6 dof for left leg 6 dof for right leg $q_robot = 29 + r$

Euler Angles:

The minimal representation of the oriention is Euler Angles but they suffer from Gimbal Lock. Its 3 element for orientation. This is the min set of coordinate we can describe an orientation with. Thus:

r = 3q_robot = 29 + 3 = 32-D (32 dimension space)

Quaternion:

Quaternion is not the minimal representation of orientation. We can think of it as a sphere in 4D, a 4-element vector whose norm is one. So we have 4 elements with one constraint which means 3 dof but represented in 4D space.

r = 4q_robot = 29 + 4 = 33-D with one constraint

There is another representation like axis-angle as well.

Rotation Matrix:

9 elements but because of orthonormality there are 6 constraints resulting in 3 dof but representd in 9D. This is the singularity-free representation where each orientation can be fully defined with one unique rotation matrix.

r = 9

 $q_robot = 29 + 9 = 38-D$ with 6 constraints.

To fully determine the configuration of the robot, we need minimum of 32 values, or maximum of 38 values with respecting the constraints.

1.1.3 Preliminaries 3:

Minimum set of coordinates that we can use to define the C-Space is the dof of the robot. Hence, the answer is 32

1.1.4 Preliminaries 4:

Assumptions: - 6 joints to represent the floating base - Task space is 6D. For each serial chain attached to the base, we have one Jacobian. The first 6 columns are the same for all of them as they represent the base, we call it $J_{6\times 6}^b$.

superscripts:

b: base

la: left arm

ra: right arm

ll: left leg

rl: right leg

Left Arm:

Left Arm:
$$V_{6\times 1}^{la} = [J_{6\times 6}^b|J_{6\times 7}^{la}] \begin{bmatrix} \dot{q}_{6\times 1}^b \\ \\ \\ \dot{q}_{7\times 1}^{la} \end{bmatrix}$$

Right Arm:

Right Arm:
$$V_{6\times 1}^{ra} = [J_{6\times 6}^b|J_{6\times 7}^{ra}] \begin{bmatrix} \dot{q}_{6\times 1}^b \\ \\ \\ \dot{q}_{7\times 1}^{ra} \end{bmatrix}$$

Left Leg:
$$V_{6\times 1}^{ll} = [J_{6\times 6}^b|J_{6\times 6}^{ll}] \begin{bmatrix} \dot{q}_{6\times 1}^b \\ \\ \\ \dot{d}_{7\times 1}^{ll} \end{bmatrix}$$

Right Leg:

Right Leg:
$$V_{6\times 1}^{rl} = [J_{6\times 6}^b|J_{6\times 6}^{rl}] \begin{bmatrix} \dot{q}_{6\times 1}^b \\ \\ \\ \dot{q}_{7\times 1}^{rl} \end{bmatrix}$$

Or we could put all in one big Jacobian:

$$V_{24\times 1} = \begin{bmatrix} J_{6\times 6}^b & J_{6\times 7}^{la} & 0 & 0 & 0 \\ & & & & \\ J_{6\times 6}^b & 0 & J_{6\times 7}^{ra} & 0 & 0 \\ & & & \\ J_{6\times 6}^b & 0 & 0 & J_{6\times 6}^{ll} & 0 \\ & & & \\ J_{6\times 6}^b & 0 & 0 & 0 & J_{6\times 6}^{rl} \end{bmatrix}_{24\times 32} \begin{bmatrix} \dot{q}_{6\times 1}^b \\ & & \\ \dot{q}_{7\times 1}^{ra} \\ & & \\ \dot{q}_{7\times 1}^{rl} \\ & & \\ \dot{q}_{7\times 1}^{rl} \end{bmatrix}_{32\times 1}$$

 $V_{24\times1}$ contains the spatial velcoties of the 4 end-effectors, namely: left_hand, right_hand, left_leg, right_leg

In general, the columns of jacobian represent the joint axes of the robot (direction and moment wrt to end-effector). Also the robot works in a 6DOF space (3 translation + 3 rotation). Now we may have different scenarios:

- Case1: robot DOF = 6. the jacobian is 6x6. Number of rows and columns is the same - Case2: robot DOF < 6. underactuated robot. In this case J is 6xn and n<6. Its a thin matrix, more rows than columns - Case3: robot DOF > 6. redundant robot. In this case J is 6xn and n>6. Its a fat matrix with more columns than rows

For underactuated robots, we may not be able to find solutions, for redundant robots we have infinite solutions and for case 1, we have inverse of the jacobian as a square matrix but this inverse is not guarantee to exist; for example at singularities.

1.1.5 Preliminaries 5:

We need to express the Jacobian with respect to base_link, then we can $V_{6\times 1}^{la}=[J_{6\times 7}^{la}]\dot{q}_{7\times 1}^{la}$ Calculating jacobian geometrically, we know that the columns are joint directions and moment. So what we need to do it to express those base_link not world.

If the those joint and moments are expressed in world frame we can do a converstion like following to express them in base_link:

 T_{WB} : homogeneous transformation between wrold to base_link

 z_W : direction of a joint in the left arm manipulator expressed in wrold

 $z_B = T_{WB}^{-1} z_W$

1.2 Gradient-Based Optimiztion for IK problem:

This question is asking to solve inverse kinematics with a gradient-based optimization method.

For solutions to 1, 2, 3, and 5, please look at the following sections that include the code. I put comment there where needed to clarify, its a working code in Drake. If you have Drake up and running you can run each section to see the result. If not I am sending videos to show how these codes work in simulation.

1.2.1 GBO 4

The gradient descent leads the direction for the next iteration given and initial configuration state like q_{init} . We use the gradient to know which direction to go. It updates the q (joint values) based on the error in the task space. Also, there is learning rate called α which determines how big of a step we want to take twoard the min. For manipulators, the gradient is the Jacobian to project/ convert the error in task-space to joint space. In general we update the joint values with:

$$\Delta \theta = -\alpha (\frac{\partial F}{\partial \theta})^T$$

F is the objective function.

1.2.2 GBO 6

The main effect of a mobile base is increasing the dof. This makes the problem harder because the search space for optimization is higher. Although optimization based motion planning are pretty scalable

```
[1]: import numpy as np
     import time
     from IPython.display import clear_output, SVG, display
     from matplotlib import pyplot as plt
     from pydrake.all import (
         AddMultibodyPlantSceneGraph,
         Box,
         Cylinder,
         DiagramBuilder,
         InverseKinematics,
         MeshcatVisualizer,
         MeshcatVisualizerParams,
         RigidTransform,
         Role,
         RollPitchYaw,
         RotationMatrix,
         Solve,
         StartMeshcat,
         JacobianWrtVariable,
         Parser,
         RevoluteJoint,
         JointSliders
     )
     from manipulation.meshcat_utils import ( # TODO(russt): switch to pydrake_
      →version
         _MeshcatPoseSliders,
     from manipulation.scenarios import AddIiwa, AddShape, AddWsg
     from manipulation.utils import ConfigureParser, running_as_notebook
     from scipy.spatial.transform import Rotation as R
```

```
[2]: # Start the visualizer.
meshcat = StartMeshcat()
```

INFO:drake:Meshcat listening for connections at http://localhost:7001

1.2.3 GBO 1, 2, 3. First Approach.

Min distance in C-space. We can define the cost function as the quadratic of the error in joint space. Subject to satisfying Jacobian relationship.

$$min\frac{1}{2}\Delta\theta^{T}\Delta\theta$$
$$s.t.\Delta x = J(\theta)\Delta\theta$$

With Largrange multiplier we can solve the problem which is:

$$\Delta \theta = \alpha J^{\#} \Delta x$$

We need the pseudo inverse of the jacobian to use in the iteration for updating the joint values. So the gradient descent basically looks like:

$$\theta_{k+1} = \theta_k + \alpha J^{\#} \Delta x$$

using the pseduo inverse $J^{\#}$ makes the optmization process to look for the shortest path in C-space. As a result, this method converges faster relative to the other one but matrix inversion is necessary here.

The Kuka arm here is redundant manipulator and psedue inverse of the Jacobian would be a better approach to find the closest q values in the next step. There are infinite solution basically but psedudo inverse will give us the one that is the min in least square sense.

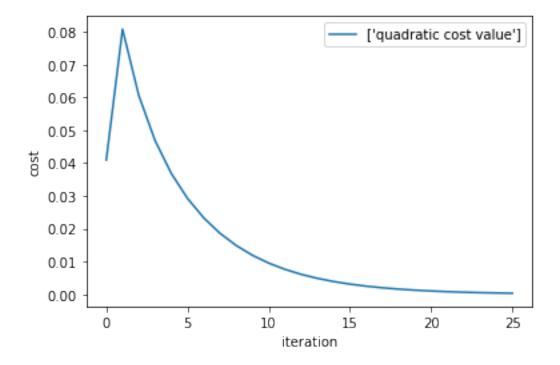
```
r_error = rct.as_rotvec() # angle * axis

out = np.zeros(6)
out[:3] = r_error
out[3:] = position_error
return out
```

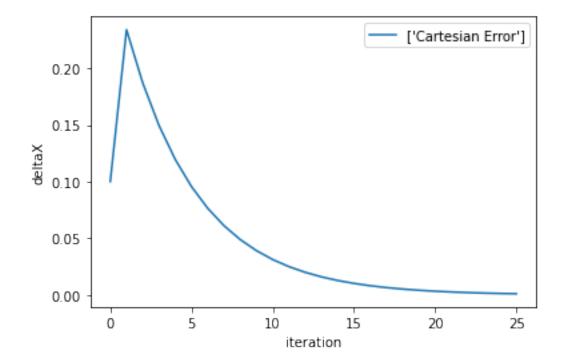
```
[]: def teleop_inverse_kinematics():
         # params
         alpha = 0.2 # learning rate
         max_iteration = 100
        min_deltaX = 1e-3 # error threshold in Cartesian space between current pose_
      →of the end-effector and the target
         np.set_printoptions(precision=3, suppress=True)
         builder = DiagramBuilder()
         plant, scene graph = AddMultibodyPlantSceneGraph(builder, time step=0.001)
         iiwa = AddIiwa(plant)
         wsg = AddWsg(plant, iiwa, welded=True)
         plant.Finalize()
         visualizer = MeshcatVisualizer.AddToBuilder(
             builder,
             scene_graph,
             meshcat,
             MeshcatVisualizerParams(delete_prefix_initialization_event=False),
         )
         diagram = builder.Build()
         context = diagram.CreateDefaultContext()
         plant_context = plant.GetMyContextFromRoot(context)
         q0 = plant.GetPositions(plant_context)
         gripper_frame = plant.GetFrameByName("body", wsg)
         base_frame = plant.GetBodyByName("iiwa_link_0").body_frame()
         def my_callback(context, pose):
             # target pose
             p_target = pose.translation()
             quat_target = R.from_matrix(pose.rotation().matrix()).as_quat()
             # initial state
             q_current = q0
             it = 0
```

```
costs = []
       errors = [] # Cartesian error
       while it < max_iteration:</pre>
           # forward kineamtics. get gripper frame
           transform = gripper_frame.CalcPoseInWorld(plant_context)
           quat_current = R.from_matrix(transform.rotation().matrix()).
→as_quat()
          p_current = transform.translation()
           # check error between current pose of the end-effector and the
→target pose
           deltaX = calcDeltaX(p_current, quat_current, p_target, quat_target)
           error = np.linalg.norm(deltaX)
           if error < min_deltaX:</pre>
               break
           # calc jacobain
           J = plant.CalcJacobianSpatialVelocity(
               context=plant_context,
               with_respect_to=JacobianWrtVariable.kQDot,
               frame_B=gripper_frame,
               p_BoBp_B=np.zeros(3),
               frame_A=plant.world_frame(),
               frame_E=plant.world_frame()
           )
           # calculate the next joint values by gradient descent equation
           q_current = q_current.reshape(7,1)
           q_next = q_current + alpha * np.linalg.pinv(J) @ deltaX.reshape(6,1)
           # update the robot in the scene
           plant.SetPositions(plant_context, q_next)
           cost = calcCost(q_current, q_next)
           costs.append(cost)
           errors.append(error)
           q_current = q_next
           it += 1
      plt.figure()
      plt.plot(range(it), costs, label=["quadratic cost value"])
      plt.xlabel("iteration")
      plt.ylabel("cost")
      plt.legend()
```

```
display(plt.show())
       plt.figure()
       plt.plot(range(it), errors, label=["Cartesian Error"])
       plt.xlabel("iteration")
       plt.ylabel("deltaX")
       plt.legend()
       display(plt.show())
       if it == max_iteration:
            print('did not converge in {} iterations. Cost is at: {}'.
 →format(max_iteration, cost))
        else:
            print('converged at iteration {}. Catesian error: {} cost: {}'.
 →format(it, error, cost))
            print('solution:\n{}'.format(q_current.T))
        clear_output(wait=True)
   meshcat.DeleteAddedControls()
   sliders = _MeshcatPoseSliders(meshcat)
   sliders.SetPose(
       plant.EvalBodyPoseInWorld(plant_context, plant.GetBodyByName("body",_
 ⇔wsg))
   sliders.Run(visualizer, context, my_callback)
teleop_inverse_kinematics()
```



None



${\tt None}$

```
converged at iteration 26. Catesian error: 0.0008798673916370972 cost:
0.0003357250553648192
solution:
[[-0.28     0.165 -0.193 -1.481     0.151     1.279 -0.504]]
```

1.2.4 GBO 1, 2, 3. Second Approach.

Min distance in Cartesian space for objective function.

Forward kinematics:

```
\begin{aligned} x &= F(\theta) \\ \Delta x &= x_{target} - F(\theta) \end{aligned}
```

Optimization problem:

$$min\frac{1}{2}\Delta x^T\Delta x$$

No inverse matrix needed, its a simpler computation but needs many iterations to converge. The reason is that it does not aim at find the shortest path in C-space necessarily. Cost function does not inforce that.

The gradient descent basically looks like:

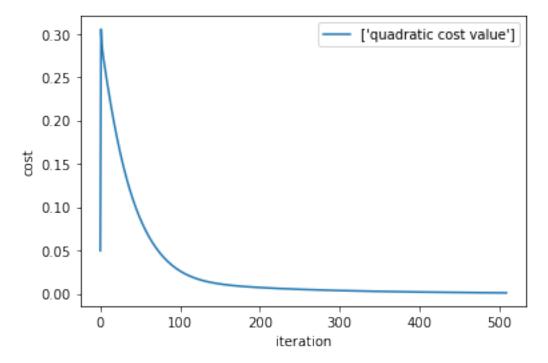
$$\theta_{k+1} = \theta_k + \alpha J^T \Delta x$$

```
[3]: def calcCost(deltaX):
         return 1/2*np.linalg.norm(deltaX)
     def calcDeltaX(p_current, quat_current, p_target, quat_target) -> np.ndarray:
         Calculates error in Cartesian space
         Arguments:
             x: current pose.
             y: target pose
         Returns:
             difference in pose
         # position current to target
         position_error = p_target - p_current
         # Rc, Rtarget,. Rc * Rct = Rt => Rct = Rt * Rc^-1
         # quaternion current to target
         r_current_inv = R.from_quat(quat_current).inv()
         rct = R.from_quat(quat_target) * r_current_inv
         r_error = rct.as_rotvec() # angle * axis
         out = np.zeros(6)
         out[:3] = r_error
```

```
out[3:] = position_error
return out
```

```
[]: def teleop_inverse_kinematics():
         # params
         alpha = 0.2
         max_iteration = 800
         min_deltaX = 1e-3
         builder = DiagramBuilder()
         plant, scene_graph = AddMultibodyPlantSceneGraph(builder, time_step=0.001)
         iiwa = AddIiwa(plant)
         wsg = AddWsg(plant, iiwa, welded=True)
         plant.Finalize()
         visualizer = MeshcatVisualizer.AddToBuilder(
             builder,
             scene_graph,
             meshcat,
             MeshcatVisualizerParams(delete_prefix_initialization_event=False),
         )
         diagram = builder.Build()
         context = diagram.CreateDefaultContext()
         plant_context = plant.GetMyContextFromRoot(context)
         q0 = plant.GetPositions(plant_context)
         gripper_frame = plant.GetFrameByName("body", wsg)
         base_frame = plant.GetBodyByName("iiwa_link_0").body_frame()
         # Frame.CalcPoseInWorld().translation()
         def my_callback(context, pose):
             # target pose
             p_target = pose.translation()
             quat_target = R.from_matrix(pose.rotation().matrix()).as_quat()
             # initial state
             q_current = q0
             it = 0
             costs = [] # Cartesian Error
             while it < max_iteration:</pre>
                 # forward kineamtics. get gripper frame
                 transform = gripper_frame.CalcPoseInWorld(plant_context)
```

```
quat_current = R.from_matrix(transform.rotation().matrix()).
→as_quat()
          p_current = transform.translation()
          # check error between current pose of the end-effector and the
⇒target pose
          deltaX = calcDeltaX(p_current, quat_current, p_target, quat_target)
          cost = calcCost(deltaX)
          if cost < min_deltaX:</pre>
              break
          # calc jacobain
          J = plant.CalcJacobianSpatialVelocity(
              context=plant_context,
              with_respect_to=JacobianWrtVariable.kQDot,
              frame_B=gripper_frame,
              p_BoBp_B=np.zeros(3),
              frame_A=plant.world_frame(),
              frame_E=plant.world_frame()
          )
          # calculate the next joint values by gradient descent equation
          q_current = q_current.reshape(7,1)
          q_next = q_current + alpha * J.T @ deltaX.reshape(6,1)
          # update the robot in the scene
          plant.SetPositions(plant_context, q_next)
          costs.append(cost)
          q_current = q_next
          it += 1
      plt.figure()
      plt.plot(range(it), costs, label=["quadratic cost value"])
      plt.xlabel("iteration")
      plt.ylabel("cost")
      plt.legend()
      display(plt.show())
      if it == max_iteration:
          print('did not converge in {} iterations. Cost is at: {}'.
→format(max_iteration, cost))
      else:
          print('converged at iteration {}. Cost: {}'.format(it, cost))
```



None

1.2.5 GBO 5: Dual manipulators

Min distance in Cartesian space between the two end-effector for objective function. We do not have to have the orientatio no of the end-effector of the two robots match. We may want the direction of the gripper to be parallel for example instead of having the whole orientation matching. There

is a fixed transform between the two end-effector in the error function.

For the math, look at dual.png

```
[3]: def calcCost(deltaX):
        return 1/2*np.linalg.norm(deltaX)
     def calcDeltaX(p_current, quat_current, p_target, quat_target) -> np.ndarray:
         Calculates error in Cartesian space
         Arguments:
             x: current pose.
             y: target pose
         Returns:
            difference in pose
         # position current to target
         position_error = p_target - p_current
         # creat transformation to make the angles facing each other. Rotate around
      \rightarrow z then, you have y-axis
         # facing which means the grippers are facing
         angle = np.pi
         axis = np.array([0,0,1])
         quat = np.zeros(4)
         quat[:3] = axis * np.sin(angle/2) # qx,qy,qz,qw
         quat[3] = np.cos(angle/2)
         rot = R.from_quat(quat)
         quat_target = R.from_quat(quat_target)
         quat_target = (quat_target * rot).as_quat()
         # Rc, Rtarget,. Rc * Rct = Rt => Rct = Rt * Rc^-1
         # quaternion current to target
         r_current_inv = R.from_quat(quat_current).inv()
         rct = R.from_quat(quat_target) * r_current_inv
         r_error = rct.as_rotvec() # angle * axis
         out = np.zeros(6)
         out[:3] = r_error # now this rotation error here contains the fixed_
      →transformation to have the grippers facing each other
         out[3:] = position_error
         return out
     def AddIiwaAt(plant, q0, base_location=[0,0,0], model_instance_name="iiwa_7"):
        parser = Parser(plant)
```

```
iiwa = parser.AddModelsFromUrl(
        f"package://drake_models/iiwa_description/sdf/iiwa7_no_collision.sdf"
    [0]
    plant.RenameModelInstance(iiwa, model_instance_name)
    plant.WeldFrames(plant.world_frame(),
                    plant.GetFrameByName("iiwa_link_0", iiwa),
                    RigidTransform(base_location)
                )
    # Set default positions:
    index = 0
    for joint_index in plant.GetJointIndices(iiwa):
        joint = plant.get_mutable_joint(joint_index)
        if isinstance(joint, RevoluteJoint):
            joint.set_default_angle(q0[index])
            index += 1
    return iiwa
def AddWsgWithName(plant, iiwa_model_instance,
                roll=np.pi / 2.0, welded=False, sphere=False, name='gripper'):
    parser = Parser(plant)
    ConfigureParser(parser)
    if welded:
        if sphere:
            file = "package://manipulation/schunk_wsg_50_welded_fingers_sphere.
 ⇔sdf"
            file = "package://manipulation/schunk_wsg_50_welded_fingers.sdf"
    else:
            "package://drake_models/wsg_50_description/sdf/
 ⇔schunk_wsg_50_with_tip.sdf"
    directives = f"""
directives:
- add_model:
    name: gripper
    file: {file}
0.00
    gripper = parser.AddModelsFromString(directives, ".dmd.yaml")[0]
    plant.RenameModelInstance(gripper, name)
    X_7G = RigidTransform(RollPitchYaw(np.pi / 2.0, 0, roll), [0, 0, 0.09])
```

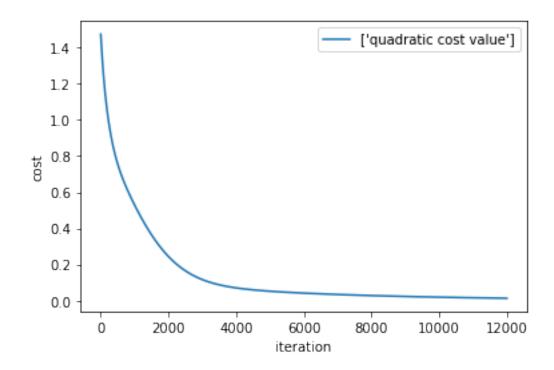
```
plant.WeldFrames(
    plant.GetFrameByName("iiwa_link_7", iiwa_model_instance),
    plant.GetFrameByName("body", gripper),
    X_7G,
)
return gripper
```

```
[8]: def teleop_inverse_kinematics():
         meshcat.Delete()
         meshcat.DeleteAddedControls()
         # params
         alpha = 0.001
         max_iteration = 12000
         min_deltaX = 1e-3
         builder = DiagramBuilder()
         plant, scene_graph = AddMultibodyPlantSceneGraph(builder, time_step=0.001)
         q0_1 = [0.8, 0.1, 0, -0.8, 0, 1.0, 0]
         iiwa_1 = AddIiwaAt(plant, q0_1, [0.0, 0.5, 0.0], "kuka1")
         wsg_1 = AddWsgWithName(plant, iiwa_1, welded=True, name="gripper1")
         q0_2 = [0.1, 0.4, 0, -1.2, 0, 1.0, 0]
         iiwa_2 = AddIiwaAt(plant, q0_2, [0.0, -0.5, 0.0], "kuka2")
         wsg_2 = AddWsgWithName(plant, iiwa_2, welded=True, name="gripper2")
         plant.Finalize()
         visualizer = MeshcatVisualizer.AddToBuilder(
             builder,
             scene_graph,
             meshcat,
             MeshcatVisualizerParams(delete_prefix_initialization_event=False),
         )
         # sliders = builder.AddSystem(JointSliders(meshcat, plant))
         diagram = builder.Build()
         context = diagram.CreateDefaultContext()
         plant_context = plant.GetMyContextFromRoot(context)
         q0_1 = plant.GetPositions(plant_context, iiwa_1)
         gripper_frame_1 = plant.GetFrameByName("body", wsg_1)
         base_frame_1 = plant.GetBodyByName("iiwa_link_0", iiwa_1).body_frame()
         q0_2 = plant.GetPositions(plant_context, iiwa_2)
```

```
gripper_frame_2 = plant.GetFrameByName("body", wsg_2)
  base_frame 2 = plant.GetBodyByName("iiwa_link_0", iiwa_2).body_frame()
  # to show the robots at their inital configurations
  diagram.ForcedPublish(context)
  time.sleep(3)
  # initial state
  q current = np.zeros(14)
  q_current[:7] = q0_1
  q_current[7:] = q0_2
  q_current.reshape(14,1)
  it = 0
  costs = [] # Cartesian Error
  while it < max_iteration:</pre>
      # forward kineamtics. get gripper frame
      transform_1 = gripper_frame_1.CalcPoseInWorld(plant_context)
      quat_current_1 = R.from_matrix(transform_1.rotation().matrix()).
⇔as_quat()
      p_current_1 = transform_1.translation()
      transform_2 = gripper_frame_2.CalcPoseInWorld(plant_context)
      quat_current_2 = R.from_matrix(transform_2.rotation().matrix()).
→as_quat()
      p current 2 = transform 2.translation()
      # check error between current pose of the end-effector and the targetu
→pose
      deltaX = calcDeltaX(p_current_1, quat_current_1, p_current_2,_

¬quat_current_2)
      # deltaX = deltaX[3:] # we must care only about the position
      cost = calcCost(deltaX)
      if cost < min deltaX:</pre>
          break
      # calc jacobain
      J1 = plant.CalcJacobianSpatialVelocity(
           context=plant_context,
          with_respect_to=JacobianWrtVariable.kQDot,
          frame_B=gripper_frame_1,
          p_BoBp_B=np.zeros(3),
          frame_A=plant.world_frame(),
          frame_E=plant.world_frame()
```

```
# print(J1.shape)
        np.set_printoptions(precision=3, suppress=True)
        # print(J1)
        J2 = plant.CalcJacobianSpatialVelocity(
            context=plant_context,
            with_respect_to=JacobianWrtVariable.kQDot,
            frame_B=gripper_frame_2,
            p_BoBp_B=np.zeros(3),
            frame_A=plant.world_frame(),
            frame_E=plant.world_frame()
        # print(J2)
        # calculate the next joint values by gradient descent equation
        J = J1 - J2
        \# J = J[3:,:]
        q_next = q_current.reshape(14,1) + alpha * J.T @ deltaX.reshape(6,1) #_
 \hookrightarrow J.T, np.linalq.pinv(J)
        # update the robot in the scene
        plant.SetPositions(plant context, q next)
        diagram.ForcedPublish(context)
        costs.append(cost)
        q_current = q_next
        it += 1
    plt.figure()
    plt.plot(range(it), costs, label=["quadratic cost value"])
    plt.xlabel("iteration")
    plt.ylabel("cost")
    plt.legend()
    display(plt.show())
    if it == max iteration:
        print('did not converge in {} iterations. Cost is at: {}'.
 →format(max_iteration, cost))
    else:
        print('converged at iteration {}. Cost: {}'.format(it, cost))
        print('solution:\n{}'.format(q_current.T))
    clear_output(wait=True)
    diagram.ForcedPublish(context)
teleop_inverse_kinematics()
```



None did not converge in 12000 iterations. Cost is at: 0.013131934177912747