Dual Manipulator:

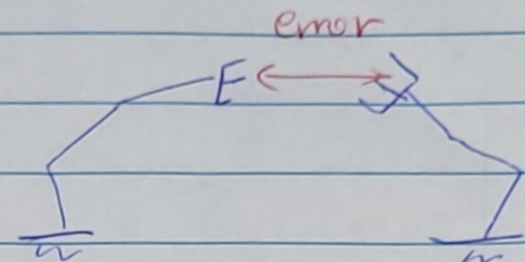
$x_l = F_l(\theta_l) \qquad x_r = F_r(\theta_r)$

FK for left arm FK for right arm

Optimization:

$$\min F = \frac{1}{2}\left(F_l(\theta_l) - F_r(\theta_r)\right)^T \left(F_l(\theta_l) - F_r(\theta_r)\right)$$

$$\theta = \begin{bmatrix} \theta_l \\ \theta_r \end{bmatrix}_{14\times1}$$



$$\frac{\partial F}{\partial \theta} = \left(\frac{\partial F_l}{\partial \theta} - \frac{\partial F_r}{\partial \theta}\right)^T (F_l - F_r)$$

F_l is NOT dependent on θ_r. F_r is NOT dependent on θ_l

$$\frac{\partial F_l}{\partial \theta} = \begin{bmatrix} J^l_{6\times7} & 0_{6\times7} \end{bmatrix}_{6\times14} \qquad \frac{\partial F_r}{\partial \theta} = \begin{bmatrix} 0_{6\times7} & J^r_{6\times7} \end{bmatrix}_{6\times14}$$

Jacobian of the left arm

Zero matrix

Jacobian of the right arm.

$$\frac{\partial F}{\partial \theta} = \begin{bmatrix} J^l & -J^r \end{bmatrix}^T_{6\times14} \Delta X_{6\times1}$$

$F_l - F_r$

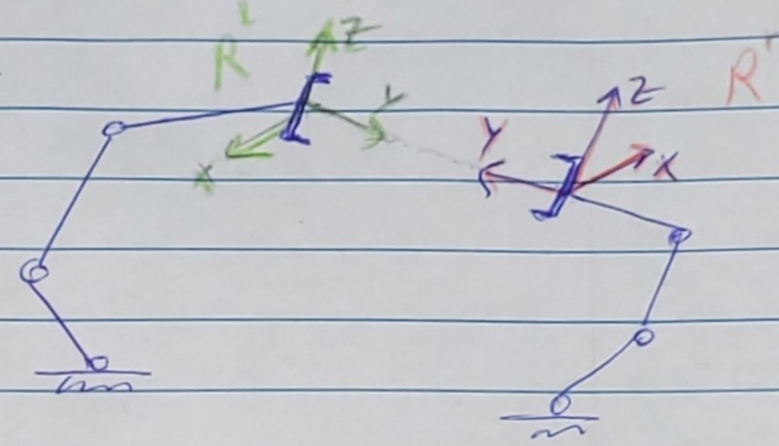
Gradient Descent:

$$\Delta\theta = -\alpha \frac{\partial F}{\partial \theta}^{T}$$

$$\Delta\theta = -\alpha \underbrace{[J^{l} \; -J^{r}]^{T}}_{\text{our new Jacobian, } J} \Delta X$$

$$\boxed{\underset{14\times1}{\theta_{k+1}} = \underset{14\times1}{\theta_{k}} + \alpha \underset{6\times14}{J^{T}} \underset{6\times1}{\Delta X}}$$

Now, if we want the orientations of the grippers to be facing each other the orientation error part of the ΔX vector should include an extra transformation



$$R^{r} R_{z,\pi} = \hat{R}^{r}$$

So ΔX:

position error: $P_{error} = P_e - P_r$

rotation error

$$R_{error} = R^{r} (\hat{R}^{e})^{-1}$$

Convert rotation matrix to axis-angle

$$R_{error} \rightarrow angle_{error},\ axis_{error}$$

$$\Delta X = \begin{bmatrix} (angle)_{error}\ axis_{error} \\ P_{error} \end{bmatrix}$$