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Implementation of random finite element method in Plaxis Using API / Python scripting

With examples on slope stability problems

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PROJECT THESIS: TBA4510

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Preface

As a prestudy for my masters thesis this project work is presented as final deliverable in the subject "TBA4510 - Geotechnical Engineering, Specialization Project" at the Department of Civil and Transport Engineering, NTNU during the spring semester of 2021. The course is 7.5 SP credits.

The project was suggested and superviced by Yutao Pan, NTNU.

Here, you give a brief introduction to your work. What it is (e.g., a Project thesis in geotechnics at NTNU as part of the MSc in Civil and Environmental Engineering or Geotechnics and Geohazards), when it was carried out (e.g., during the autumn semester of 2013). If the project has been carried out for a company, you should mention this and also describe the cooperation with the company. You may also describe how the idea to the project was brought up.

Trondheim, 2021-12-20

(Your signature)

Ole Eiesland

Acknowledgment

I would like to thank Yutao Pan for supervising during the project work.

O.E.

Summary and Conclusions

Soil is a complex medium. Its inhomogenous nature means that the physical parameters of soil vary spatialy both vertically and lateraly. Traditionally soil properties is modeled with a representative value, usually some kind of mean value or simmilar. In probabilistic methods, this variability, or uncertainty is taken into account by treating the soil as a random variable sampled from a probability distribution. By using random field theory and statistics one can try to describe how the soil parameters are distributed in space and how they vary with distance. In a slope stability problem, the spatial distribution of the soil strength governing parameters has a direct impact on the development of the failure surface, the falure mechanism and therfore the over all stability of the slope. To simulate stability a finite element program can be used with the soil model parameters input to the finite element mesh based on statistical spatial correlated random fields. To simulate variability and uncertainty, the modeling is repeated many times with different random fields. This is the random finite element method.

Current modern soil modeling software do not support random finite element method, and published reasearch random finite element software code do not have the advanced functionality as modern comercial geotechnical software. How ever, since the random finite element method does not change the way the problem is simulated, only the input parameters change, modern software packages can be used if a way to specify the input and the simulation run parameters can be controlled in an efficient manner. Plaxis, a modern software package devolped by Bently, has capabilities like this through its application program interface and python scripting.

In this project work I present a method to run the random finite element method in Plaxis geothechnical software package using python API scripting interface, and demonstrate the random finite element method on a slope stability problem.

Contents

Chapter 1

Introduction

The first chapter of a well-structured thesis is always an introduction, setting the scene with background, problem description, objectives, limitations, and then looking ahead to summarize what is in the rest of the report. This is the part that readers look at first—*so make sure it hooks them!*

1.1 Background

Soil is a complex medium. Its inhomogenous nature means that the physical parameters of soil vary spatialy both vertically and lateraly. Traditionally soil properties is modeled with a representative value, usually some kind of mean value or simmilar. In probabilistic methods, this variability, or uncertainty is taken into account by treating the soil as a random variable sampled from a probability distribution. By using random field theory and statistics one can try to describe how the soil parameters are distributed in space and how they vary with distance. In a slope stability problem, the spatial distribution of the soil strength governing parameters has a direct impact on the development of the failure surface, the falure mechanism and therfore the over all stability of the slope. To simulate stability a finite element program can be used with the soil model parameters input to the finite element mesh based on statistical spatial correlated random fields. To simulate variability and uncertainty, the modeling is repeated many times with different random fields. This is the random finite element method.

In this section, you should present the problem that you are going to investigate or analyze; why this problem is of interest; what has, so far, been done to solve the problem, and which parts of the problem that remain.

Problem Formulation

Current modern soil modeling software do not support random finite element method, and published reasearch random finite element software code do not have the advanced functionality as modern comercial geotechnical software. How ever, since the random finite element method does not change the way the problem is simulated, only the input parameters change, modern software packages can be used if a way to specify the input and the simulation run parameters can be controlled in an efficient manner. Plaxis, a modern software package devolped by Bently, has capabilities like this through its application program interface and python scripting. It is of great interest to research on spatialy varying soil modeling to utelize the anvanced functions and ease of access of existing software. Gaining this ability will expand the compexity of the problems alowed to be simulated by the RFEM method.

Common causes of discrepancy between the estimated and actual performance of any geotechnical system may be summarized as Cambou 1975; Lee et al. 1983; Mostyn and Li 1993; Phoon and Kulhawy 1999! 1. Variability of the soil properties at a specific site;

You should define your problem in a clear an unambiguous way and explain why this is a problem, why it is of interest—and to whom. It is also important to delimit the problem area.

Literature Survey

The random finite element method (RFEM) has been in use since the mid-1990s [e.g, see Griffiths and Fenton, 1993]. RFEM combines random field theory to represent the spatialy varying soil with finite element method (FEM) for deformation analysis. Stochastic analysis in FEM methods can be built into the finite element equations themselvs [e.g., see Vanmarcke and Grigoriu, 1983], or a Monte Carlo approach can be used were multiple realizations of different spatial soil models can be analyzed together. The Monte Carlo approach can be computationaly demanding, but is very flexible by utilizing arbitrary FEM code changing just the input. The Monte Carlo RFEM and its application to many geotechnical problems including seepage, bearing capacity, earth pressure and settlement is described in more detail in book by Fenton, Griffiths, et al. [2008]. Example of RFEM analysis for slope stability is presented by [see Fenton, Griffiths, et al., 2008, Chapter 13] based on FEM code developed by Smith, Griffiths, and Margetts [2013]. The RFEM code is publicly available and extensive research has been conducted using it. A list of all but the most recent publications using the code is available here: (`http://random.engmath.dal.ca/rfem/rfem_pubs.html`). The RFEM code is for two-dimensional plane strain analysis of elastic perfectly plastic soils with a Mohr Coulomb failure criterion. For a detailed discussion of the method [e.g, see Griffiths and Lane, 1999]. The limitation of the failure criteria and the soil model can restrict the application of the method to more complex soils who displays different characteristics. Software with a range of failure criteria and soil models exist, such as Plaxis by Bently [Brinkgreve et al., 2010]. Plaxis do not have random field functionality.

Optum G2, (Krabbenhøft and Lyamin, 2014)

What Remains to be Done?

To be abel to describe more complex geotechnical problems, a method to unify material models and numerical methods is needed. Implimentation of new soil models into FEM code is not straight forward. Plaxis alows for python scripting through an application programing interface (API). The RFEM Monte Carlo method could be implemented in Plaxis by scripting random field input and automating simulation.

After you have defined and delimited your problem – and presented the relevant results found in the literature within this field, you should sum up which parts of the problem that remain to be solved.

1.2 Objectives

The main objectives of this project are

1. Implement The RFEM Monte Carlo method in Plaxis using python API interface

2. Demonstrate and verify the implementation on a simple slope stability problem

3. Reproduce literature results?

4. Comparison to analytical results

1.3 Limitations

The field of random behavior of soils in geotechnical problems is extensive and this project work scope only scratches in the surface. The methodology of finite element numerical computations and random fields is only described briefly and beyond the scope of this project course to go into in any detail.

The python code presented is not attempted optimized in any way, the focus is on proof of concept. The result presented is a tool, further research and application will prove its usability.

1.4 Approach

A literature search was conducted to get an overview of the current implementations of the RFEM code. Source code and acompanying documentation was also studied where available. The main part of the project was to code the implementation of the RFEM method into the Plaxis 2D software using the python scripting API. Study of the Plaxis 2D software manual and online documentation was key to get familiar with Plaxis and the API functions to controll and automate the program execution from the python code. Example problems from literature is used to verify the validity of the implementation. Results from other software is reproduced to compare results. Analytical results is compared to the implementation.

1.5 Structure of the Report

The rest of the report is structured as follows. Chapter 2 gives an brief introduction to random field theory, finite element analysis and application to slope stability problems. In chapter 3 the implementation of the RFEM method in Plaxis using python API script is discussed. Chapter 4 compares results of simulations using the python API implementation of the RFEM method in Plaxis. Chapter 5 gives a summary and discussion of the results and main findings of the project work.

Chapter 2

Theory

The content of this chapter will vary with the topic of your thesis.

2.1 Random Fields

To model the spatial variability of soils random fields are used. To describe the random field of a soil parameter, e. g. the undrained shear strength of the soil, three parameters are commonly used. The Mean, μ, Standart deviation, σ of the soil parameters underlying probability distribution and the Spatial correlation length, θ also known as the scale of fluctuation. The mean is a measure of around which value the soil strength parameter is distributed. The standard deviation tells how much the values in the soil random fields varies in values. The spatial correlation length, or scale of fluctuation, is a measure of how similar in strength points in the spatial random field are depending on how far away the points are from each other. Large spatial correlation length vary smoothly and varying slowly, while scale of fluctuation is jagged and rappidly varying. Soil samples taken close together is more likely to be similar than soil samples taken far apart. Du to the process of soil deposition the soil tend to have different properties in differing direction. We say that soil is anisotropic. Typically for layered soils, the soil properties are more similar in the horizontal direction than in the vertical direction. This anisotropy can be represented by having a different scale of fluctuation in the horozontal directial than in the vertical direction.

Remark:Difference between homogenous media and isotropic

2.1.1 SRM - Spectral Representation Method

Various methods exist for generating random fields that can be used to represent spatialy variable soil. [see e. g. Fenton, Griffiths, et al., 2008, Chapter 6]. The methods differ in their efficiency and accuracy and complexity e. g. ability to describe anisotropy. In this project work the spectral representation method is used. It is showed by Shinozuka and Deodatis [1996] how to simulate multi-dimensional homogeneous stochastic fields using the spectral representation method. Sample functions of the stochastic field can be generated using a cosine series formula. These sample functions accurately reflect the prescribed probabilistic characteristics of the stochastic field when the number of terms in the cosine series is large.

$$f(x,y) = \sqrt{2}\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} A_{n_1 n_2}[cos(\kappa_{1_{n_1}} x_1 + \kappa_{2_{n_2}} x_2 + \Phi^{(1)}_{n_1 n_2}) + cos(\kappa_{1_{n_1}} x_1 - \kappa_{2_{n_2}} x_2 + \Phi^{(2)}_{n_1 n_2})] \quad (2.1)$$

where:

$$A_{n_1 n_2} = \sqrt{2 S_{f_0 f_0}(\kappa_{1_{n_1}}, \kappa_{2_{n_2}}) \Delta\kappa_1 \Delta\kappa_2} \quad (2.2)$$

$$\kappa_{1_{n_1}} = n_1 \Delta\kappa_1 \quad \text{and} \quad \kappa_{2_{n_2}} = n_2 \Delta\kappa_2 \quad (2.3)$$

$$\Delta\kappa_1 = \frac{\kappa_{1u}}{N_1} \quad \text{and} \quad \Delta\kappa_2 = \frac{\kappa_{2u}}{N_2} \quad (2.4)$$

and:

$$A_{0 n_2} = A_{n_1 0} = 0 \quad \text{for} \quad n_1 = n_2 = 0, 1, \ldots, N_1 - 1 \quad (2.5)$$

See chapter 3 for details on implementation.

2.2 Finite Element Method

The Finite Element method is a numerical method that can be used to solve a multitude of geotechnical and other engineering problems. Being a numerical method, FEM gives approximate solution, care sould be used when constructing the model to be calculated. Things to keep in mind when running FEM analysis is the volume extent and the boundry interface of the model, element type, density of mesh and criteria of convergence. In comparison to limit equilibrium methods (LEM), FEM methods can give deformations in a state before ultimate limit state i. e. before failure. FEM have no prior assumtions of the falure mechanism i.e. no assumption of a circular falure surface in a LEM slope stability analysis. It is a very attractive property of FEM analysis combined with spatial variable soil, to allow slope falure to develop naturally by finding the path of weakest soil.

The main principles of the FEM is to devide the soil into smaller elements, then develop description of what happening in each element, then add the behaviour of each element together to find the behaviour of the whole volume.

Seven steps of a finite element program is described by Nordal [2020]:

1. Element modeling, equations are built for each element, integration in points inside the elemnt is used to form stiffness matix

2. Global modelling, all element siffness matrixes are assembled into a system of equations to form a global stiffness matrix. An incremental load vector is found.

3. Equation solving of the global equation system for the load increment. This gives a displacement increment

4. Stress evaluation of the calculated displacement increment. The displacments are used to find the strains wich is used to find the stress increments

5. Testing for numerical accuracy. If the calculations show too high unbalanced forces, it will be necessary to adjust the load increment and/or add more iterations. if so step 1-5 must be recalculated. When the results converges the program proceeds to step 6.

6. Updating of results by adding the deformations and stress to form total deformations and total stresses.

7. Calculation of new load increment. The response of the new load increment is found by repeating 1 to 6. New load increment is repeatedly added until the specified external load is reached, or failure occur.

Remark:Plaxis measures the fraction of the applied loadstep according to step 7 above in a variabel wich in this project is accessed to automatically tell if we have faliure or not.

Chapter 3

RFEM implementation in Plaxis 2D using python API

The content of this chapter will vary with the topic of your thesis.

3.1 SRM implementation

In generating the random soil strength field an implementation of the SRM method written in MATLAB code by Yutao Pan out of Deodatis, Shinozuka, and Papageorgiou [1990] is converted to python code.

```
import numpy as np

def incmatrix(genl1,genl2):
    m = len(genl1)
    n = len(genl2)
    M = None #to become the incidence matrix
    VT = np.zeros((n*m,1), int)  #dummy variable

    #compute the bitwise xor matrix
    M1 = bitxormatrix(genl1)
    M2 = np.triu(bitxormatrix(genl2),1)

    for i in range(m-1):
        for j in range(i+1, m):
            [r,c] = np.where(M2 == M1[i,j])
            for k in range(len(r)):
                VT[(i)*n + r[k]] = 1;
                VT[(i)*n + c[k]] = 1;
```

```
                VT[(j)*n + r[k]] = 1;
                VT[(j)*n + c[k]] = 1;

                if M is None:
                    M = np.copy(VT)
                else:
                    M = np.concatenate((M, VT), 1)

                VT = np.zeros((n*m,1), int)

    return M
```

Listing 3.1: Python example

3.2 Local averaging

When i. e. a clay sample is sheard in the laboratory to detrmine strength parameters, failure develops over the whole sample when the bonds of the sample yield. The measured strength is a function of the average bond strength of the sample. The greater the sample size the stronger is the averaging effect. Input parameters in modeling, in the case of RFEM, the mean, standard deviation and spatial correlation length are assumed to be point measures. Therefore when populating a RFEM model spatial averaging needs to be taken into account since the element sizes is in general much grater than the size of the sample from wich the parameter was derived. It can be shown [Vanmarcke, 2010] that the the reduction in variance due to local averaging is given by:

$$\sigma_A = \sigma\sqrt{\gamma} \tag{3.1}$$

where σ_A is the new spatially averaged variance wich is to be used when drawing samples from the distribution to put into the finite element mesh and γ is the variance reduction function, defined for a rectangular element as:

$$\gamma = \frac{4}{T_x^2 T_y^2}\int_0^{T_x}\int_0^{T_y}(T_x-\tau_x)(T_y-\tau_y)\rho(\tau_x,\tau_y)d\tau_x d\tau_y \tag{3.2}$$

where T_x and T_y is the size of the element in the x and y direction respectivly, ρ is the correlation function and τ_x and τ_y is the difference between the respectivly the x and y coordinates of any two points in the random field.

γ has the value of 1.0 when $T = 0$ [see Fenton et al., 2008, chapter 3]. Setting $T = \alpha\theta$ (i.e some scalar α times the scale of fluctuation θ) leads to the conclution that elements much smaller than the scale of fluctuation is affected to small degree by variance reduction.

In this project work this realization is used, but to what degree is left to be discussed.

3.3 Monte Carlo

Monte Carlo is a method that can estimate the means, varances and probabilities of the responses of complex systems to random input [see Fenton et al., 2008, chapter 6.6]. Consider the random response of a system $g(X_1, X_2)$ where X_1 and X_2 are random variables. The system fails if the the value of $g(X_1, X_2) > g_{critical}$. Monte Carlo simulates a sequence of realizations of X_1 and X_2, evaluates $g(X_1, X_2)$ and checks if $g(X_1, X_2) > g_{critical}$.

The method is very versitile and can be applied to most kinds of systems. Drawbacks is that there are no analytical solution, if the system, e.g. the input is changed, the simulation must be rerun, we can not predict the response to a change in input. Also, to simulate rear events, alot of simulations is needed wich can be computational demanding.

3.4 Plaxis 2D

The following procedure to run the RFEM Monte Carlo analysis is implemented in Plaxis using the python API. The user specify the input soil parameters, the problem geometry and the desired number of realizations. One realization is one stability simulation on one random field. The procedure is run fully autonomus without user interaction.

1. The script start by creating a new empty Plaxis project
2. Next, the problem geometry is generated i.e. the slope height, inclination and extent.
3. The random field representing the soil property is generated based on the user specified mean, standart deviation (or CoV) and scale of fluctuation
4. The soil elements are populated with the soil parameter values from the random field.
5. Then Plaxis grids the geometry generating the FEM mesh

6. Next a new phase is added after the initaial phase, activating the geometry represented by the soil

7. Finaly the deformations are calculated and a result is stored, i.e. failure or no filure in a slope stability problem

8. Now step 3 to 7 is repeated generating a new realization of the random field, a user specified number of times

9. In the end statistics are gathered and written to a file, and plots of the results can be displayed

Chapter 4

Results

To test the validity of the implementation a series of slope stability simulations is run. The results from these simulations is presented in the following sections.

4.1 Homogenous isotropic soil - Zero variance

To check the base case of spatialy invariant soil with known analytical solutions. This is to validate the input to the random field generation, the FEM mesh and calculation. The simulation is a elasto-plastic FEM simulation with Mohr-Columb material model using Plaxis undraind(C) behaviour. 15-Node triangular FEM elements are used. The slope is 5 meter high with a 2:1 gradient. The soil parameters is presented in Table 4.1. The resulting random field, or in this particular case a constant field, is shown in Figure 4.1 and the 2:1 slope geometry and Plaxis 2D mesh is shown in Figure 4.2.

Table 4.1: Soil parameters for Homognus isotropic soil

Soil model	Mohr Columb - Undrained(C)		
Statistical Soil Parameters	Mean μ	Coefficient of Variation $CoV = \frac{\sigma}{\mu}$	Scale of fluctuation θ
Unit weight, $\gamma_{sat} = \gamma unsat$	20 kN/m^3	0	-
Modulus of elasticity, E	10 MPa	0	-
Poissons ratio, ν	0.49	0	-
Undrained Shear Strength,S_u	20 kPa	0	-

Running a Plaxis c-reduction calculation phase, plot in 4.3, on the uniform soil slope gives a Factor of Safety, $F_s = 1.14$. The corresponding F_s optaind by slope stability charts after Janbu [1968] is $F_s = 1.16$.

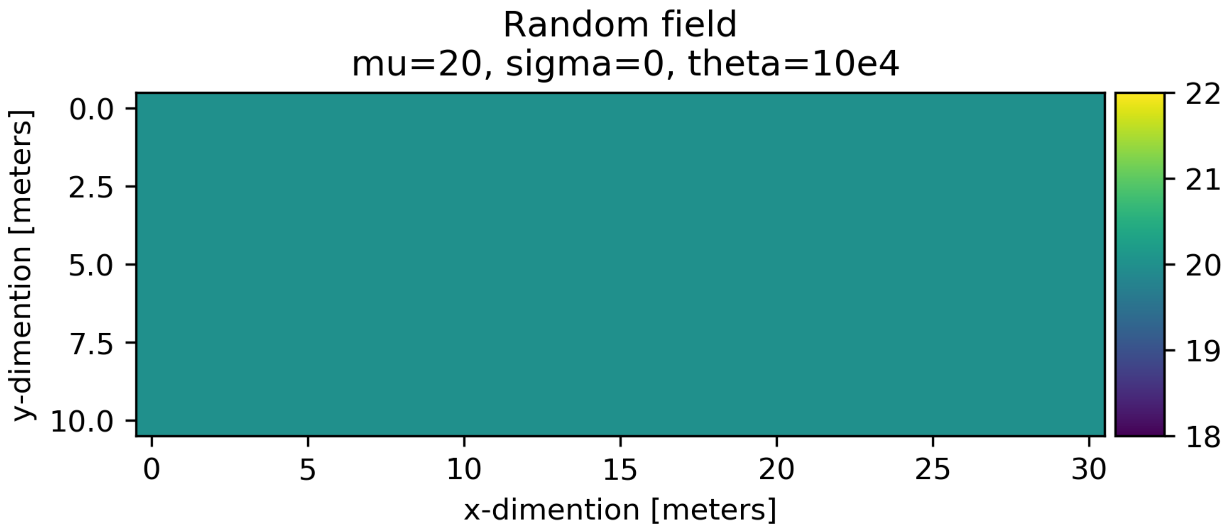

Figure 4.1: S_u Random field, in this particular case the soil strength is uniform

4.2 Homogenus anisotropic soil - Spatial correlation length 50 and 10, COV=0.3

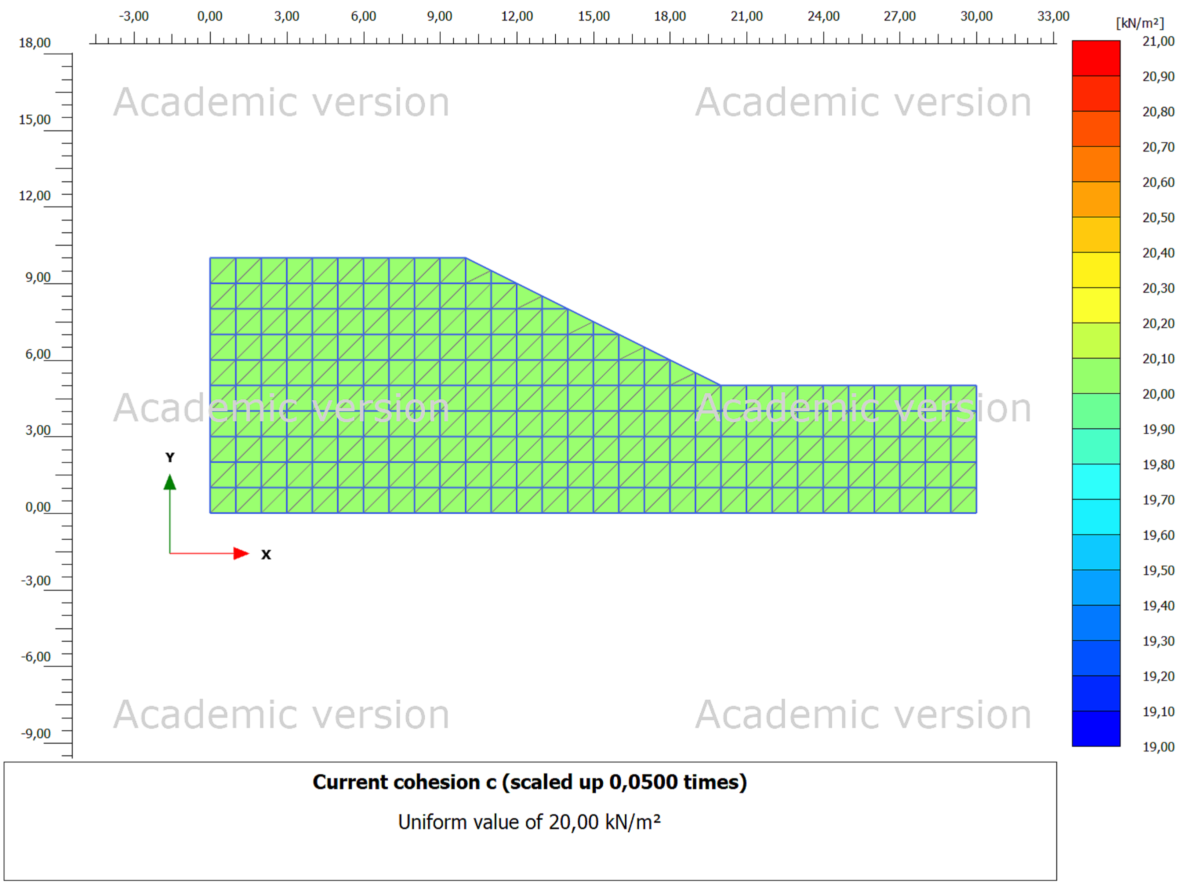

Figure 4.2: Slope geometry, with soil strength property from the random field maped to the soil Plaxis soil elements and triangular FEM elements displayed

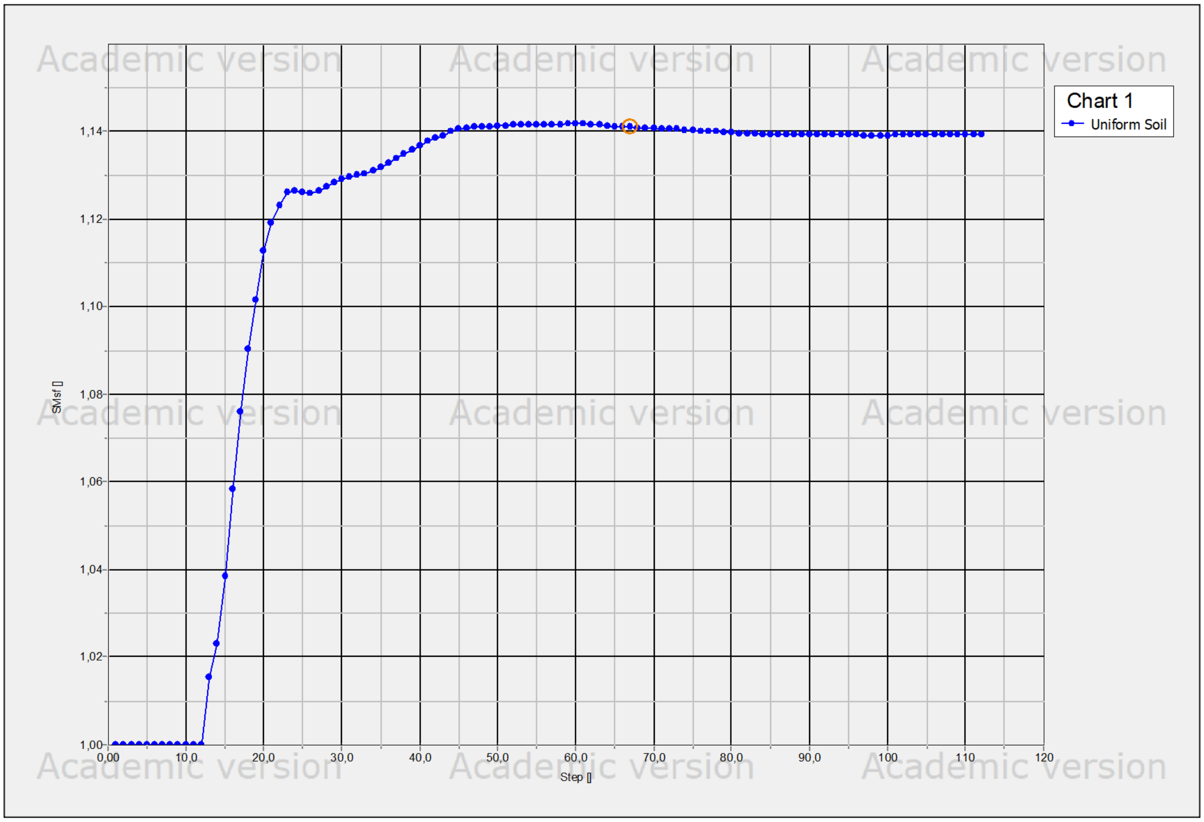

Figure 4.3: Plaxis Safety factor

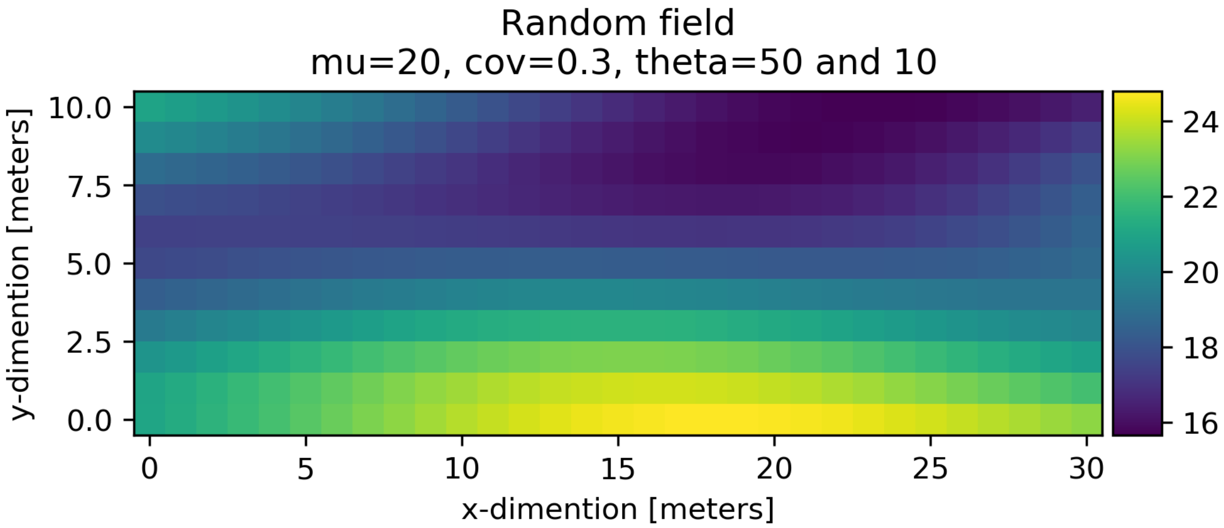

Figure 4.4: Random field

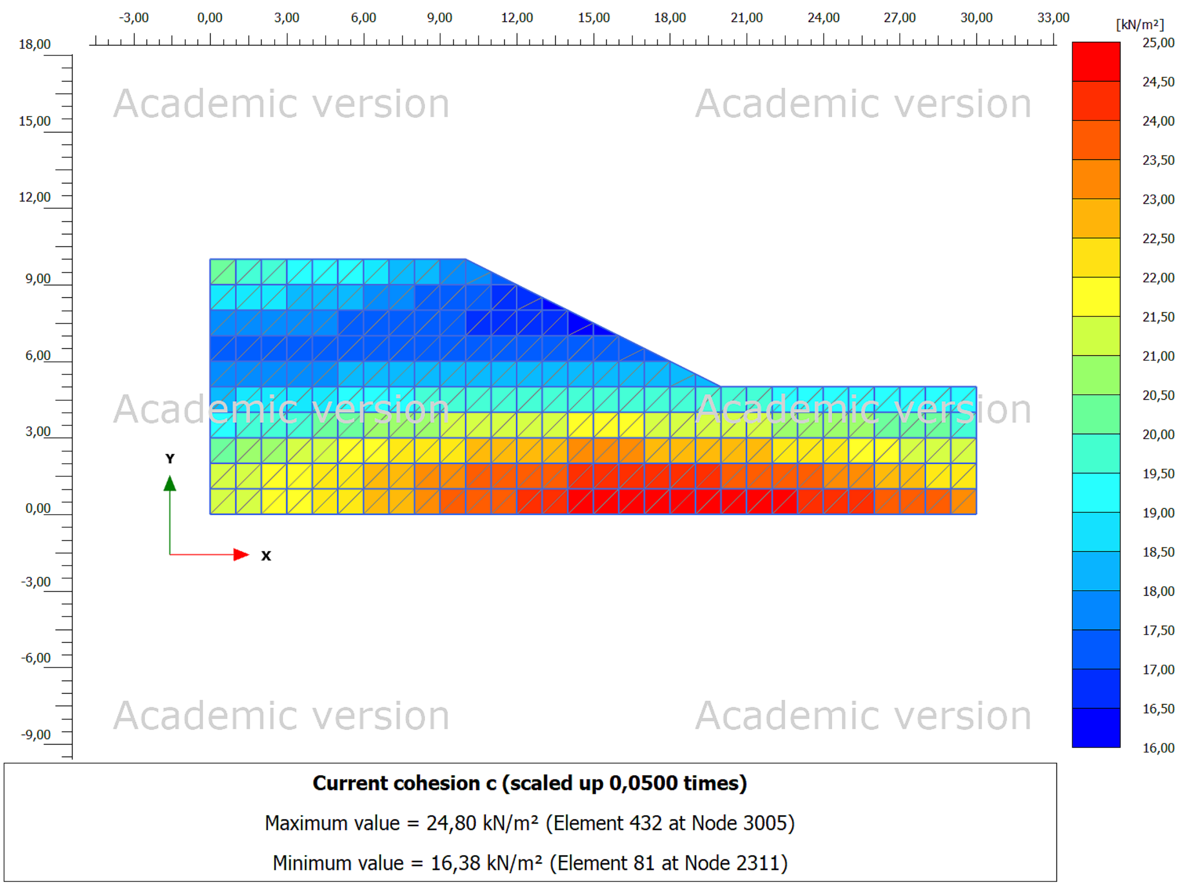

Figure 4.5: Slope geometry

Chapter 5

Summary and Recommendations for Further Work

In this final chapter you should sum up what you have done and which results you have got. You should also discuss your findings, and give recommendations for further work.

5.1 Summary and Conclusions

Here, you present a brief summary of your work and list the main results you have got. You should give comments to each of the objectives in Chapter 1 and state whether or not you have met the objective. If you have not met the objective, you should explain why (e.g., data not available, too difficult).

This section is similar to the Summary and Conclusions in the beginning of your report, but more detailed—referring to the the various sections in the report.

5.2 Discussion

Here, you may discuss your findings, their strengths and limitations.

5.3 Recommendations for Further Work

You should give recommendations to possible extensions to your work. The recommendations should be as specific as possible, preferably with an objective and an indication of a possible approach.

The recommendations may be classified as:

- Short-term
- Medium-term
- Long-term

Appendix A

Acronyms

FEM Finite Element Method

RFEM Random Finite Element Method

SRM Spectral Representation Method

CoV Coefficient of Variation

Appendix B

Additional Information

This is an example of an Appendix. You can write an Appendix in the same way as a chapter, with sections, subsections, and so on.

B.1 Introduction

B.1.1 More Details

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