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Implementation of random finite element method in Plaxis Using API / Python scripting

With examples on slope stability problems

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PROJECT THESIS: TBA4510

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Preface

As a prestudy for my masters thesis this project work is presented as final deliverable in the subject "TBA4510 - Geotechnical Engineering, Specialization Project" at the Department of Civil and Transport Engineering, NTNU during the spring semester of 2021. The course is 7.5 SP credits.

The project was suggested and supervised by Yutao Pan, NTNU.

Here, you give a brief introduction to your work. What it is (e.g., a Project thesis in geotechnics at NTNU as part of the MSc in Civil and Environmental Engineering or Geotechnics and Geohazards), when it was carried out (e.g., during the autumn semester of 2013). If the project has been carried out for a company, you should mention this and also describe the cooperation with the company. You may also describe how the idea to the project was brought up.

Trondheim, 2021-12-20

(Your signature)

Ole Eiesland

Acknowledgment

I would like to thank Yutao Pan for supervising during the project work.

O.E.

Summary and Conclusions

Soil is a complex medium. Its inhomogenous nature means that the physical parameters of soil vary spatially both vertically and laterally. Traditionally soil properties are modeled with a representative value, usually some kind of mean value or similar. In probabilistic methods, this variability, or uncertainty is taken into account by treating the soil as a random variable sampled from a probability distribution. By using random field theory and statistics one can try to describe how the soil parameters are distributed in space and how they vary with distance. In a slope stability problem, the spatial distribution of the soil strength governing parameters has a direct impact on the development of the failure surface, the failure mechanism and therefore the overall stability of the slope. To simulate stability a finite element program can be used with the soil model parameters input to the finite element mesh based on statistical spatial correlated random fields. To simulate variability and uncertainty, the modeling is repeated many times with different random fields. This is the random finite element method.

Current modern soil modeling software do not support random finite element method, and published research random finite element software code do not have the advanced functionality as modern commercial geotechnical software. However, since the random finite element method does not change the way the problem is simulated, only the input parameters change, modern software packages can be used if a way to specify the input and the simulation run parameters can be controlled in an efficient manner. Plaxis, a modern software package developed by Bentley, has capabilities like this through its application program interface and python scripting.

In this project work I present a method to run the random finite element method in Plaxis geotechnical software package using python API scripting interface, and demonstrate the random finite element method on a slope stability problem.

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Chapter 1

Introduction

The first chapter of a well-structured thesis is always an introduction, setting the scene with background, problem description, objectives, limitations, and then looking ahead to summarize what is in the rest of the report. This is the part that readers look at first—*so make sure it hooks them!*

1.1 Background

Soil is a complex medium. Its inhomogenous nature means that the physical parameters of soil vary spatially both vertically and laterally. Traditionally soil properties is modeled with a representative value, usually some kind of mean value or similar. In probabilistic methods, this variability, or uncertainty is taken into account by treating the soil as a random variable sampled from a probability distribution. By using random field theory and statistics one can try to describe how the soil parameters are distributed in space and how they vary with distance. In a slope stability problem, the spatial distribution of the soil strength governing parameters has a direct impact on the development of the failure surface, the failure mechanism and therefore the overall stability of the slope. To simulate stability a finite element program can be used with the soil model parameters input to the finite element mesh based on statistical spatial correlated random fields. To simulate variability and uncertainty, the modeling is repeated many times with different random fields. This is the random finite element method.

In this section, you should present the problem that you are going to investigate or analyze; why this problem is of interest; what has, so far, been done to solve the problem, and which parts of the problem that remain.

Problem Formulation

Current modern soil modeling software do not support random finite element method, and published reasearch random finite element software code do not have the advanced functionality as modern comercial geotechnical software. How ever, since the random finite element method does not change the way the problem is simulated, only the input parameters change, modern software packages can be used if a way to specify the input and the simulation run parameters can be controlled in an efficient manner. Plaxis, a modern software package devolped by Bently, has capabilities like this through its application program interface and python scripting. It is of great interest to research on spatially varying soil modeling to utelize the anvanced functions and ease of access of existing software. Gaining this ability will expand the compexity of the problems alowed to be simulated by the RFEM method.

Common causes of discrepancy between the estimated and actual performance of any geotechnical system may be summarized as Cambou 1975; Lee et al. 1983; Mostyn and Li 1993; Phoon and Kulhawy 1999! 1. Variability of the soil properties at a specific site;

You should define your problem in a clear an unambiguous way and explain why this is a problem, why it is of interest—and to whom. It is also important to delimit the problem area.

Literature Survey

The random finite element method (RFEM) has been in use since the mid-1990s [e.g, see [Griffiths and Fenton, 1993](#)]. RFEM combines random field theory to represent the spatially varying soil with finite element method (FEM) for deformation analysis. Stochastic analysis in FEM methods can be built into the finite element equations themselvs [e.g., see [Vanmarcke and Grigoriu, 1983](#)], or a Monte Carlo approach can be used were multiple realizations of different spatial soil models can be analyzed together. The Monte Carlo approach can be computationally demanding, but is very flexible by utilizing arbitrary FEM code changing just the input. The Monte Carlo RFEM and its application to many geotechnical problems including seepage, bearing capacity, earth pressure and settlement is described in more detail in book by [Fen-](#)

ton, Griffiths, et al. [2008]. Example of RFEM analysis for slope stability is presented by [see Fenton, Griffiths, et al., 2008, Chapter 13] based on FEM code developed by Smith, Griffiths, and Margetts [2013]. The RFEM code is publicly available and extensive research has been conducted using it. A list of all but the most recent publications using the code is available here: (http://random.engmath.dal.ca/rfem/rfem_pubs.html). The RFEM code is for two-dimensional plane strain analysis of elastic perfectly plastic soils with a Mohr Coulomb failure criterion. For a detailed discussion of the method [e.g. see Griffiths and Lane, 1999]. The limitation of the failure criteria and the soil model can restrict the application of the method to more complex soils who displays different characteristics. Software with a range of failure criteria and soil models exist, such as Plaxis by Bentley [Brinkgreve et al., 2010]. Plaxis do not have random field functionality.

Optum G2, (Krabbenhøft and Lyamin, 2014)

What Remains to be Done?

To be able to describe more complex geotechnical problems, a method to unify material models and numerical methods is needed. Implementation of new soil models into FEM code is not straight forward. Plaxis allows for python scripting through an application programming interface (API). The RFEM Monte Carlo method could be implemented in Plaxis by scripting random field input and automating simulation.

After you have defined and delimited your problem – and presented the relevant results found in the literature within this field, you should sum up which parts of the problem that remain to be solved.

1.2 Objectives

The main objectives of this project are

1. Implement The RFEM Monte Carlo method in Plaxis using python API interface
2. Demonstrate and verify the implementation on a simple slope stability problem
3. Reproduce literature results?
4. Comparison to analytical results

1.3 Limitations

The field of random behavior of soils in geotechnical problems is extensive and this project work scope only scratches in the surface. The methodology of finite element numerical computations and random fields is only described briefly and beyond the scope of this project course to go into in any detail.

The python code presented is not attempted optimized in any way, the focus is on proof of concept. The result presented is a tool, further research and application will prove its usability.

1.4 Approach

A literature search was conducted to get an overview of the current implementations of the RFEM code. Source code and accompanying documentation was also studied where available. The main part of the project was to code the implementation of the RFEM method into the Plaxis 2D software using the python scripting API. Study of the Plaxis 2D software manual and online documentation was key to get familiar with Plaxis and the API functions to control and automate the program execution from the python code. Example problems from literature is used to verify the validity of the implementation. Results from other software is reproduced to compare results. Analytical results is compared to the implementation.

1.5 Structure of the Report

The rest of the report is structured as follows. Chapter 2 gives an brief introduction to random field theory, finite element analysis and application to slope stability problems. In chapter 3 the implementation of the RFEM method in Plaxis using python API script is discussed. Chapter 4 compares results of simulations using the python API implementation of the RFEM method in Plaxis. Chapter 5 gives a summary and discussion of the results and main findings of the project work.

Chapter 2

Theory

The content of this chapter will vary with the topic of your thesis.

2.1 Random Fields

To model the spatial variability of soils random fields are used. To describe the random field of a soil parameter, e. g. the undrained shear strength of the soil, three parameters are commonly used. The Mean, μ , Standard deviation, σ of the soil parameters underlying probability distribution and the Spatial correlation length, θ also known as the scale of fluctuation. The mean is a measure of around which value the soil strength parameter is distributed. The standard deviation tells how much the values in the soil random fields varies in values. The spatial correlation length, or scale of fluctuation, is a measure of how similar in strength points in the spatial random field are depending on how far away the points are from each other. Large spatial correlation length vary smoothly and varying slowly, while scale of fluctuation is jagged and rapidly varying. Soil samples taken close together is more likely to be similar than soil samples taken far apart. Due to the process of soil deposition the soil tend to have different properties in differing direction. We say that soil is anisotropic. Typically for layered soils, the soil properties are more similar in the horizontal direction than in the vertical direction. This anisotropy can be represented by having a different scale of fluctuation in the horizontal direction than in the vertical direction.

Remark: Difference between homogenous media and isotropic

2.1.1 SRM - Spectral Representation Method

Various methods exist for generating random fields that can be used to represent spatially variable soil. [see e. g. [Fenton, Griffiths, et al., 2008](#), Chapter 6]. The methods differ in their efficiency and accuracy and complexity e. g. ability to describe anisotropy. In this project work the spectral representation method is used. It is showed by [Shinozuka and Deodatis \[1996\]](#) how to simulate multi-dimensional homogeneous stochastic fields using the spectral representation method. Sample functions of the stochastic field can be generated using a cosine series formula. These sample functions accurately reflect the prescribed probabilistic characteristics of the stochastic field when the number of terms in the cosine series is large.

$$f(x, y) = \sqrt{2} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} A_{n_1 n_2} [\cos(\kappa_{1n_1} x_1 + \kappa_{2n_2} x_2 + \Phi_{n_1 n_2}^{(1)}) + \cos(\kappa_{1n_1} x_1 - \kappa_{2n_2} x_2 + \Phi_{n_1 n_2}^{(2)})] \quad (2.1)$$

where:

$$A_{n_1 n_2} = \sqrt{2S_{f_0 f_0}(\kappa_{1n_1}, \kappa_{2n_2}) \Delta \kappa_1 \Delta \kappa_2} \quad (2.2)$$

$$\kappa_{1n_1} = n_1 \Delta \kappa_1 \quad \text{and} \quad \kappa_{2n_2} = n_2 \Delta \kappa_2 \quad (2.3)$$

$$\Delta \kappa_1 = \frac{\kappa_{1u}}{N_1} \quad \text{and} \quad \Delta \kappa_2 = \frac{\kappa_{2u}}{N_2} \quad (2.4)$$

and:

$$A_{0n_2} = A_{n_1 0} = 0 \quad \text{for} \quad n_1 = n_2 = 0, 1, \dots, N_1 - 1 \quad (2.5)$$

See chapter 3 for details on implementation.

2.2 Finite Element Method

The Finite Element method is a numerical method that can be used to solve a multitude of geotechnical and other engineering problems. Being a numerical method, FEM gives approximate solution, care should be used when constructing the model to be calculated. Things to

keep in mind when running FEM analysis is the volume extent and the boundary interface of the model, element type, density of mesh and criteria of convergence. In comparison to limit equilibrium methods (LEM), FEM methods can give deformations in a state before ultimate limit state i. e. before failure. FEM have no prior assumptions of the failure mechanism i.e. no assumption of a circular failure surface in a LEM slope stability analysis. It is a very attractive property of FEM analysis combined with spatial variable soil, to allow slope failure to develop naturally by finding the path of weakest soil.

The main principles of the FEM is to divide the soil into smaller elements, then develop description of what happening in each element, then add the behaviour of each element together to find the behaviour of the whole volume.

Seven steps of a finite element program is described by [Nordal \[2020\]](#):

1. Element modeling, equations are built for each element, integration in points inside the element is used to form stiffness matrix
2. Global modelling, all element stiffness matrixes are assembled into a system of equations to form a global stiffness matrix. An incremental load vector is found.
3. Equation solving of the global equation system for the load increment. This gives a displacement increment
4. Stress evaluation of the calculated displacement increment. The displacements are used to find the strains which is used to find the stress increments
5. Testing for numerical accuracy. If the calculations show too high unbalanced forces, it will be necessary to adjust the load increment and/or add more iterations. If so step 1-5 must be recalculated. When the results converge the program proceeds to step 6.
6. Updating of results by adding the deformations and stress to form total deformations and total stresses.
7. Calculation of new load increment. The response of the new load increment is found by repeating 1 to 6. New load increment is repeatedly added until the specified external load is reached, or failure occurs.

Remark:Plaxis measures the fraction of the applied loadstep according to step 7 above in a variable which in this project is accessed to automatically tell if we have failure or not.

Chapter 3

RFEM implementation in Plaxis 2D using python API

The content of this chapter will vary with the topic of your thesis.

3.1 SRM implementation

In generating the random soil strength field an implementation of the SRM method written in MATLAB code by Yutao Pan out of [Deodatis, Shinozuka, and Papageorgiou \[1990\]](#) is converted to python code.

```
1 import numpy as np
2
3 def incmatrix(genl1, genl2):
4     m = len(genl1)
5     n = len(genl2)
6     M = None #to become the incidence matrix
7     VT = np.zeros((n*m,1), int) #dummy variable
8
9     #compute the bitwise xor matrix
10    M1 = bitxormatrix(genl1)
11    M2 = np.triu(bitxormatrix(genl2),1)
12
13    for i in range(m-1):
14        for j in range(i+1, m):
15            [r,c] = np.where(M2 == M1[i,j])
16            for k in range(len(r)):
17                VT[(i)*n + r[k]] = 1;
18                VT[(i)*n + c[k]] = 1;
```

```

19         VT[(j)*n + r[k]] = 1;
20         VT[(j)*n + c[k]] = 1;
21
22         if M is None:
23             M = np.copy(VT)
24         else:
25             M = np.concatenate((M, VT), 1)
26
27         VT = np.zeros((n*m,1), int)
28
29     return M

```

Listing 3.1: Python example

3.2 Local averaging

When i. e. a clay sample is sheared in the laboratory to determine strength parameters, failure develops over the whole sample when the bonds of the sample yield. The measured strength is a function of the average bond strength of the sample. The greater the sample size the stronger is the averaging effect. Input parameters in modeling, in the case of RFEM, the mean, standard deviation and spatial correlation length are assumed to be point measures. Therefore when populating a RFEM model spatial averaging needs to be taken into account since the element sizes is in general much greater than the size of the sample from which the parameter was derived. It can be shown [?] that the the reduction in variance due to local averaging is given by:

$$\sigma_A = \sigma \sqrt{\gamma} \quad (3.1)$$

where σ_A is the new spatially averaged variance which is to be put into the finite element mesh and γ is the variance reduction function, defined for a rectangular element as:

$$\gamma = \frac{4}{T_x^2 T_y^2} \int_0^{T_x} \int_0^{T_y} (T_x - \tau_x)(T_y - \tau_y) \rho(\tau_x, \tau_y) d\tau_x d\tau_y \quad (3.2)$$

where T is the size of the element in the x and y direction ρ is the correlation function and τ is the difference between the x and y coordinates of any two points in the random field.

It can further be seen that the value of γ goes to 1 when T goes to 0. So setting $T = a\theta$ leads to the conclusion that elements much smaller than the scale of fluctuation is not affected by

variance reduction. In this project work this realization is used.

3.3 Plaxis 2D

3.4 Monte Carlo

Chapter 4

Results

The content of this chapter will vary with the topic of your thesis.

4.1 Homogenous soil - Infinite spatial correlation length

Chapter 5

Summary and Recommendations for Further Work

In this final chapter you should sum up what you have done and which results you have got. You should also discuss your findings, and give recommendations for further work.

5.1 Summary and Conclusions

Here, you present a brief summary of your work and list the main results you have got. You should give comments to each of the objectives in Chapter 1 and state whether or not you have met the objective. If you have not met the objective, you should explain why (e.g., data not available, too difficult).

This section is similar to the Summary and Conclusions in the beginning of your report, but more detailed—referring to the the various sections in the report.

5.2 Discussion

Here, you may discuss your findings, their strengths and limitations.

5.3 Recommendations for Further Work

You should give recommendations to possible extensions to your work. The recommendations should be as specific as possible, preferably with an objective and an indication of a possible

approach.

The recommendations may be classified as:

- Short-term
- Medium-term
- Long-term

Appendix A

Acronyms

FEM Finite Element Method

RFEM Random Finite Element Method

SRM Spectral Representation Method

Appendix B

Additional Information

This is an example of an Appendix. You can write an Appendix in the same way as a chapter, with sections, subsections, and so on.

B.1 Introduction

B.1.1 More Details

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