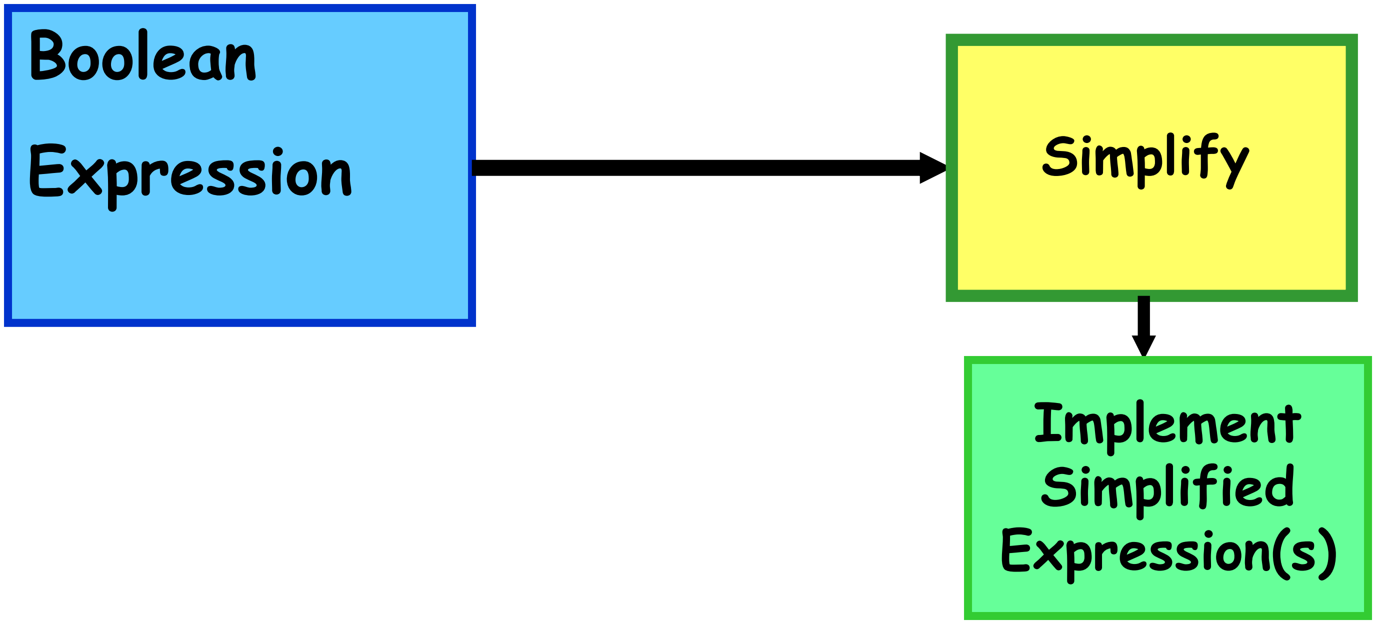
Boolean Simplification

Two ways …

1. A Boolean expression is given
2. A Logic Circuit is given.

1. A Boolean Expression is given

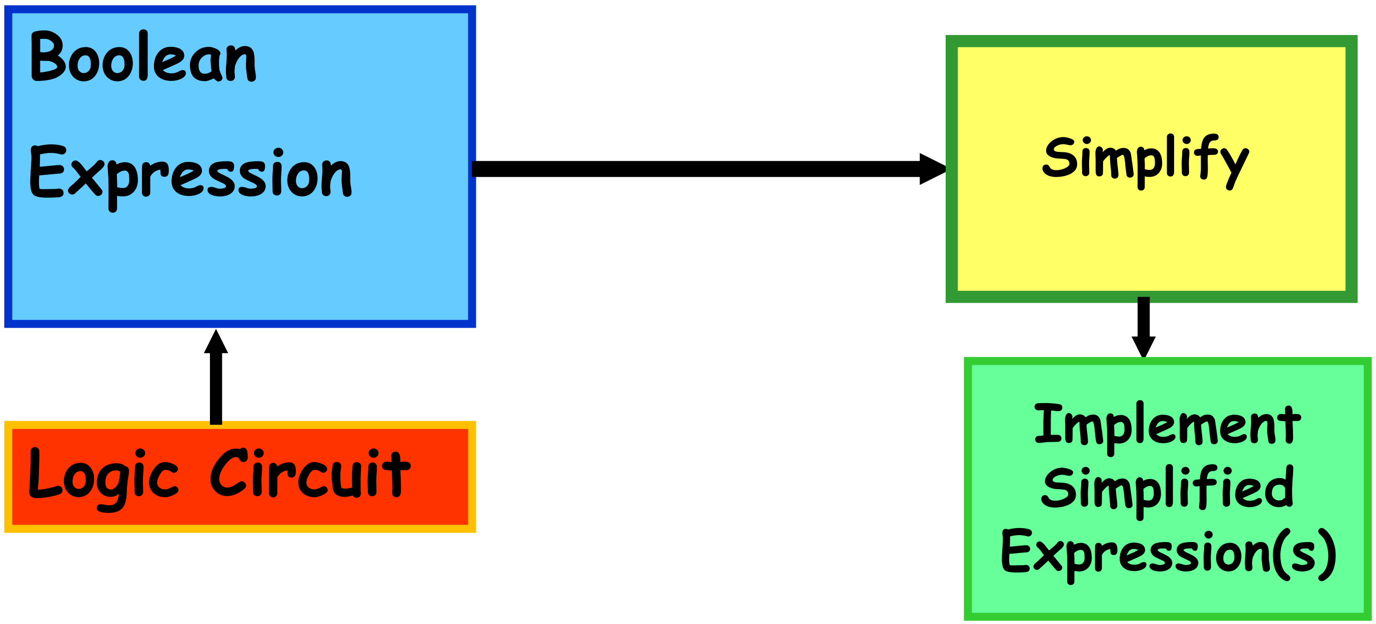


How can we simplify?

Boolean expression simplification algorithm

- Put the Boolean expression into sum of-products (SOP) form
- Apply the known Boolean simplification rules
- Implement.

2. A Logic Circuit is given



How can we simplify a Logic Circuit?

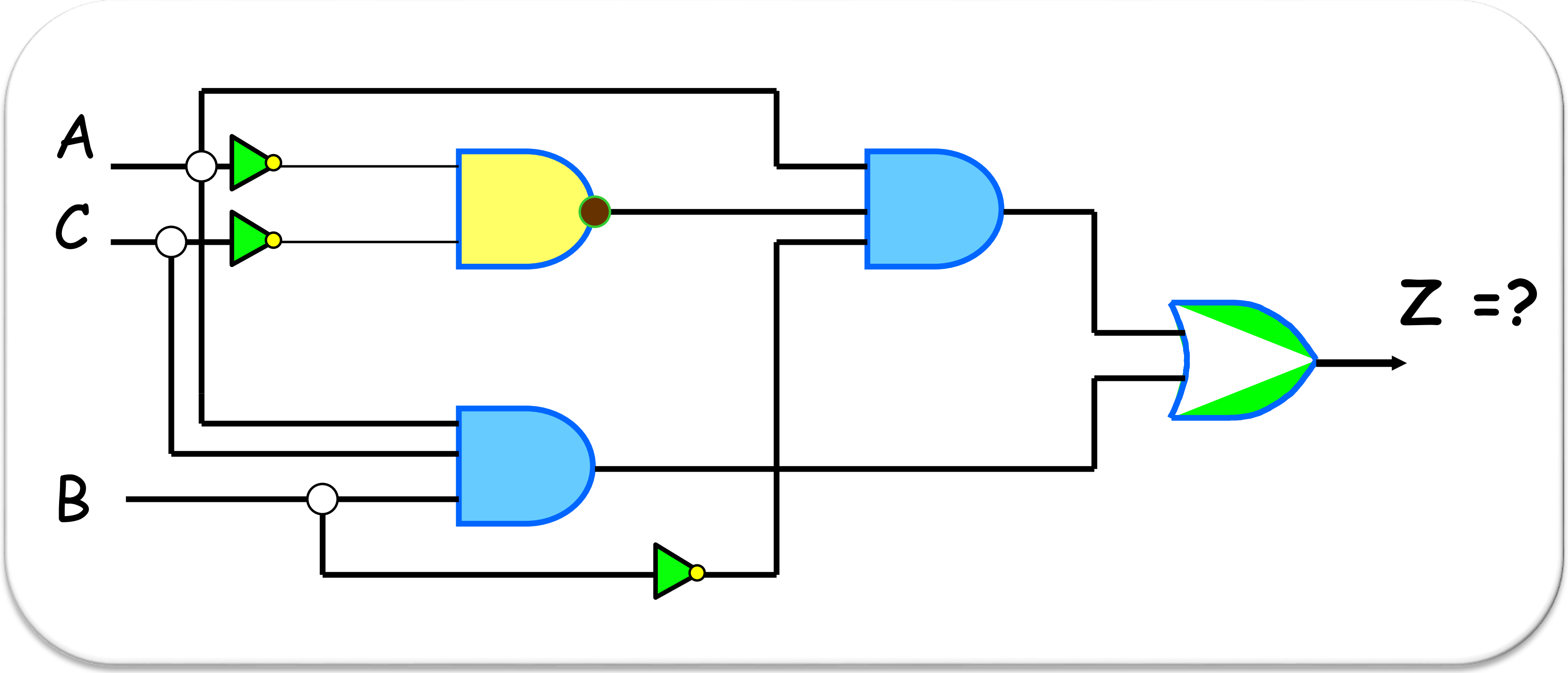
Logic Circuit simplification algorithm

- Derive the output Boolean expression
- Apply the known Boolean simplification rules
- Implement.

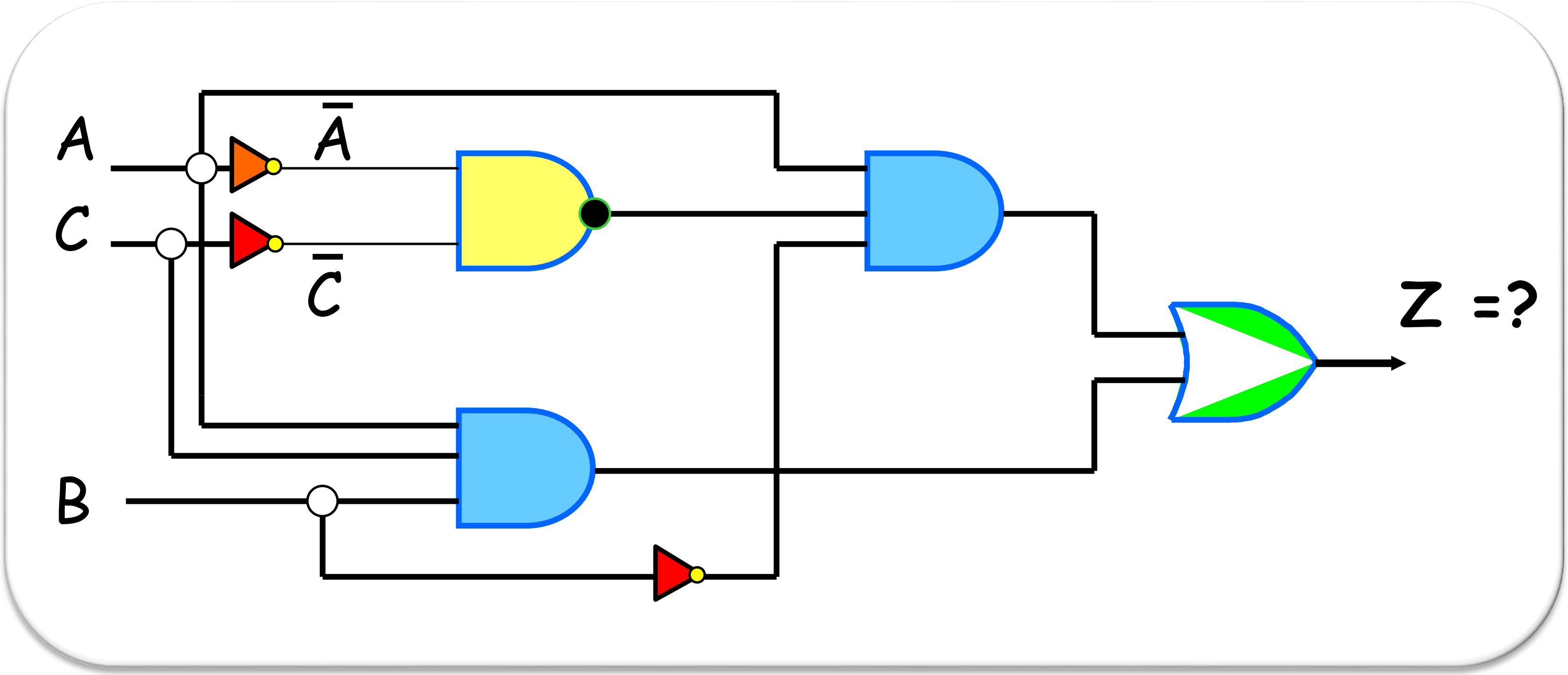
Example

Logic Circuit; Derive Z

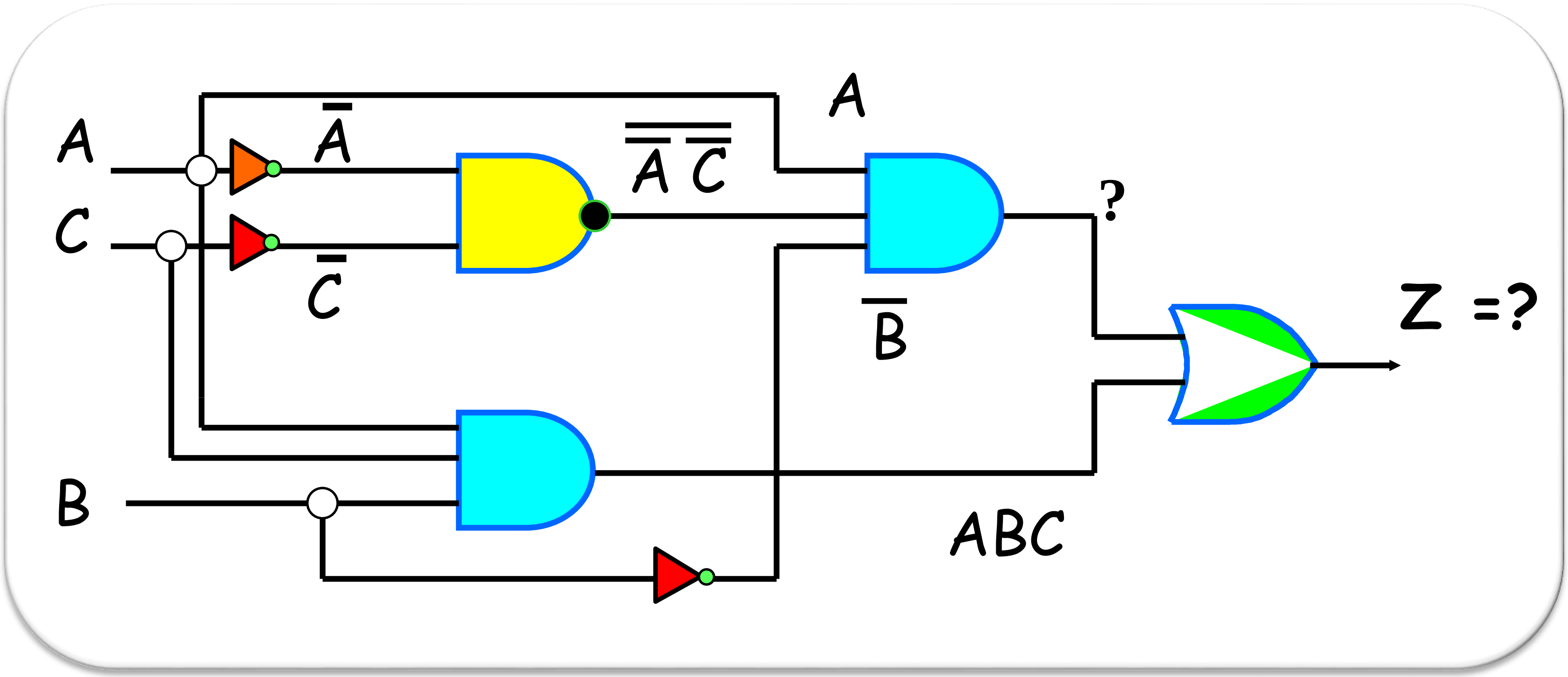
Example



Derive Z

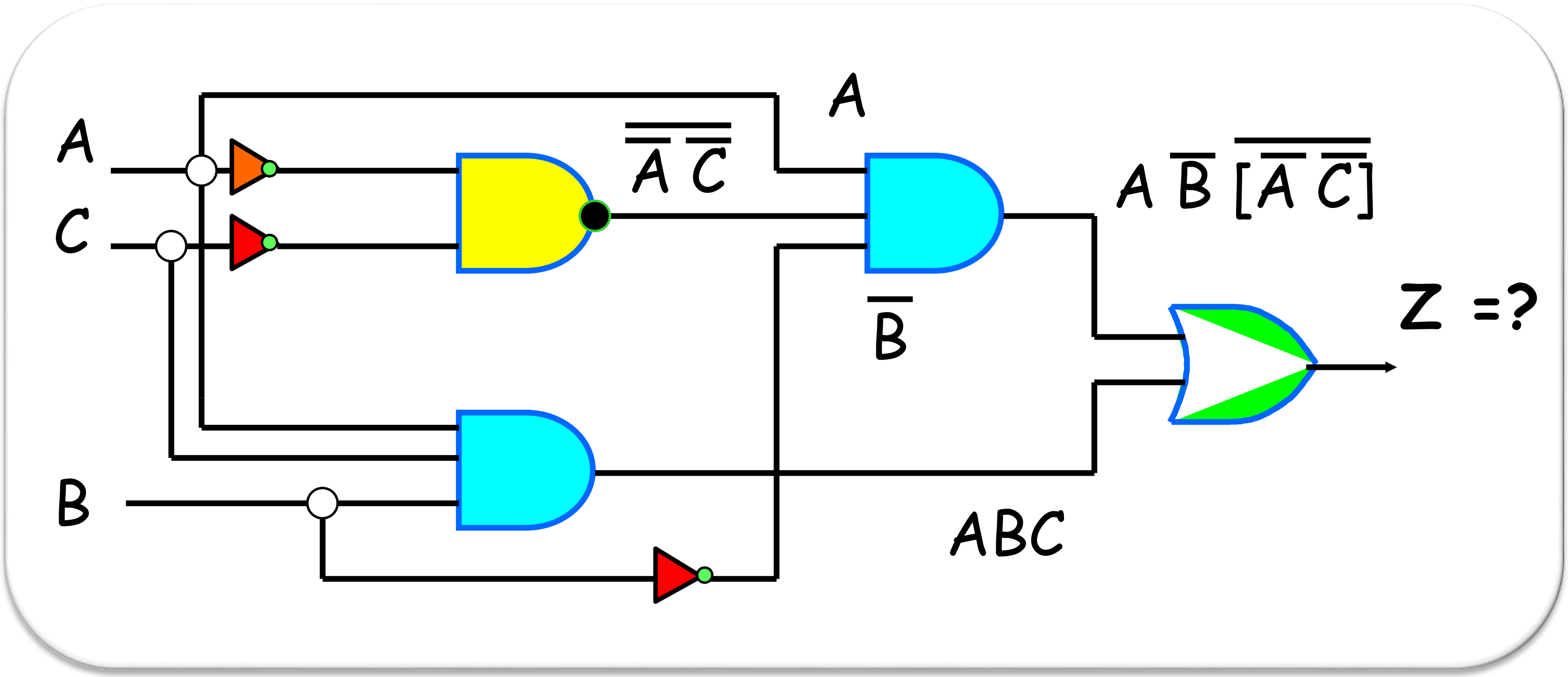


Derive Z

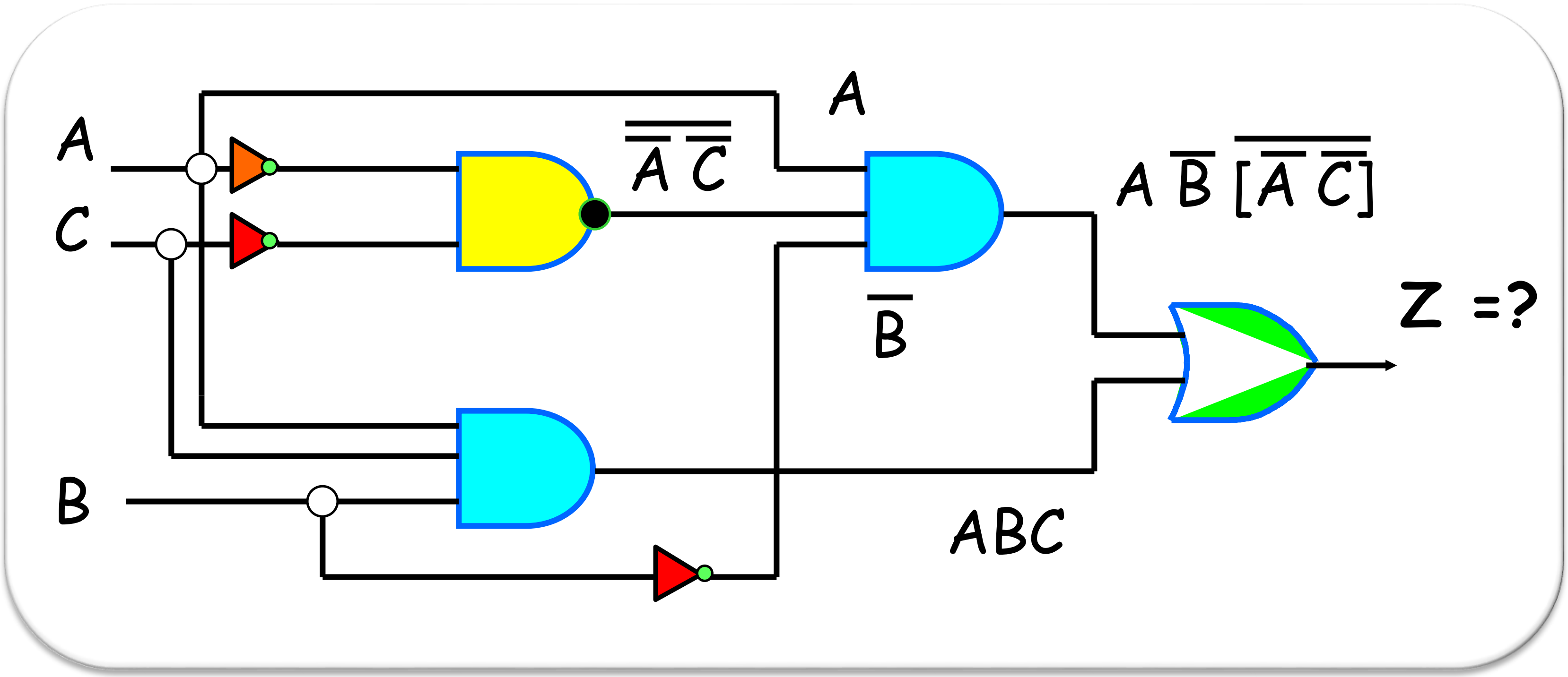


A
$\overline{A}$
$\overline{\overline{A}\,\overline{C}}$
A
C
?
$\overline{C}$
Z =?
$\overline{B}$
B
ABC

Finally the output expression is …



Output expression



$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

Simplify the Boolean expression

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\overline{C}}\,]$$

- $\overline{x+y} = \overline{x}\bullet\overline{y}$
- $\overline{x\bullet y} = \overline{x}+\overline{y}$

DeMorgan theorem

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

- $\overline{x+y} = \overline{x}\bullet\overline{y}$
- $\overline{x\bullet y} = \overline{x}+\overline{y}$

Sum-Of-Products (SOP) form

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

$$= ABC + A\overline{B}A + A\overline{B}C$$

Factor-out: AC

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

$$= ABC + A\overline{B}A + A\overline{B}C$$

$$= AC[B+\overline{B}] + A\overline{B}$$

Simplified logic expression

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

$$= ABC + A\overline{B}A + A\overline{B}C$$

$$= AC[B+\overline{B}] + A\overline{B}$$

$$= AC + A\overline{B}$$

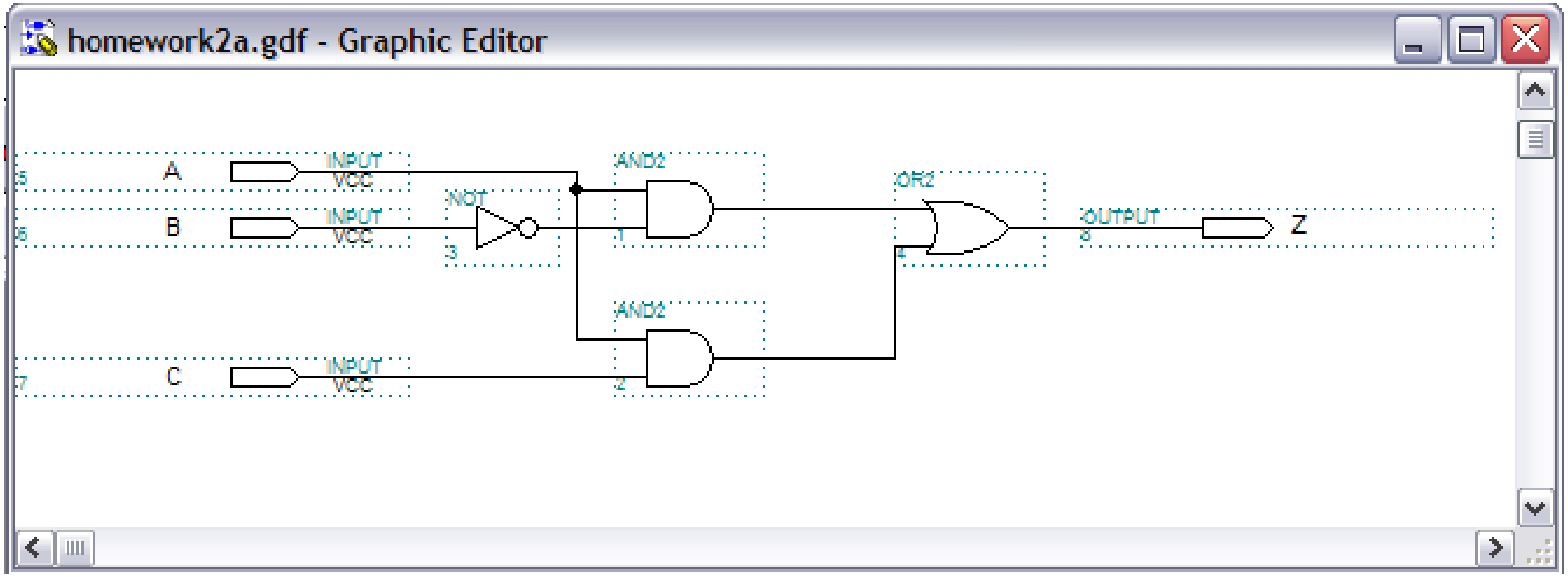
Implement (gates) the expression

$$Z = AC + A\overline{B}$$

2 Minutes …

Simplified logic circuit (gates)

$$Z = AC + A\overline{B}$$



Two-level implementation

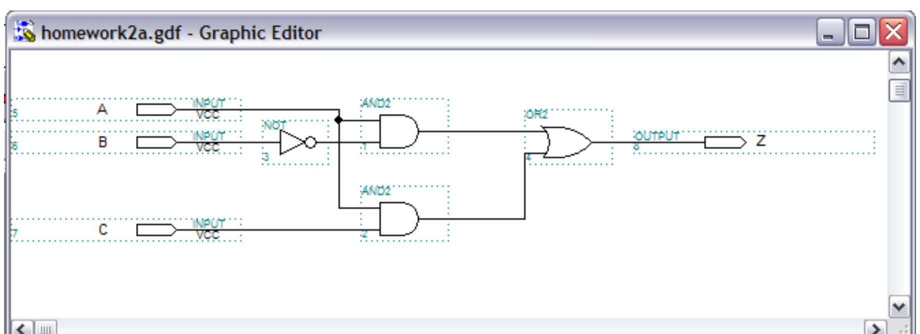
VHDL code for: $Z = AC + A\bar{B}$

homework2a_vhdl.vhd - Text Editor

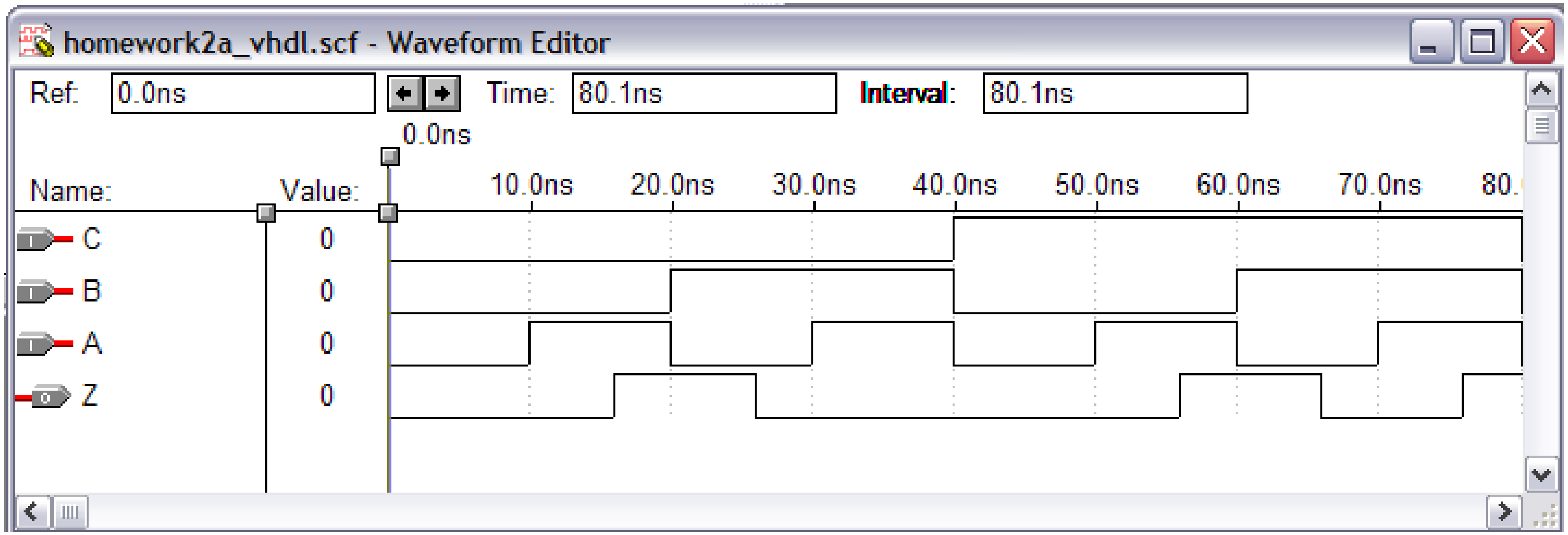
```
entity homework2a_vhdl is
    port (a,b,c: in bit;
          z:  out bit);
end homework2a_vhdl;

architecture dataflow of homework2a_vhdl is
begin
    z<=(a and not b) or (a and c);
end dataflow;
```

Line 8 Col 33 INS



Simulation



homework2a_vhdl.scf - Waveform Editor
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0.0ns
Name:
Value:
10.0ns
20.0ns
30.0ns
40.0ns
50.0ns
60.0ns
70.0ns
80.
C
B
A
Z
0
0
0
0

Simplification Examples

Simplify the expression …

$$ABC+AB\overline{C}+A\overline{B}C$$

New Example

$ABC+AB\overline{C}+A\overline{B}C$

$$= AB(C + \overline{C}) + A\overline{B}C$$

$ABC+AB\overline{C}+A\overline{B}C$

$= AB(C+\overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$ABC + AB\overline{C} + A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$ABC + AB\overline{C} + A\overline{B}C$

$$= AB(C + \overline{C}) + A\overline{B}C$$

$$= AB + A\overline{B}C$$

$$= A(B + \overline{B}C)$$

$$= A(B + C) = AB + AC$$

$ABC+AB\overline{C}+A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C) = AB + AC$

Another proof:

$ABC+AB\overline{C}+A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C) = AB + AC$

Another proof:

Because: $A + A = A$

$ABC + AB\overline{C} + A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C) = AB + AC$

Another proof:

$= ABC + ABC + AB\overline{C} + A\overline{B}C$

Because: $A + A = A$

(Add another ABC)

$ABC + AB\overline{C} + A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C) = AB + AC$

Another proof:

Because: $A + A = A$

$= ABC + ABC + AB\overline{C} + A\overline{B}C$

$= AB(C + \overline{C}) + AC(B + \overline{B})$

$ABC+AB\overline{C}+A\overline{B}C$

$$= AB(C+\overline{C}) + A\overline{B}C$$

$$= AB + A\overline{B}C$$

$$= A(B + \overline{B}C)$$

$$= A(B + C) = AB + AC$$

Another proof:

Because: $A + A = A$

$$= ABC + ABC + AB\overline{C} + A\overline{B}C$$

$$= AB(C + \overline{C}) + AC(B + \overline{B})$$

$$= AB + AC$$

Simplify the expression

$$\bar{A}\, C\, [\, \overline{\bar{A}\, B\, D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

- $\overline{x+y} = \bar{x} \bullet \bar{y}$
- $\overline{x \bullet y} = \bar{x} + \bar{y}$

New Example

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,\bar{\bar{A}} + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,A + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,A + \bar{A}\,C\,\bar{B} + \bar{A}\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

Simplify the expression

$$\bar{A}\, C\, [\,\overline{\bar{A}\, B\, D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, [\,A + \bar{B} + \bar{D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, A + \bar{A}\, C\, \bar{B} + \bar{A}\, C\, \bar{D} + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$(A\,\bar{A} = 0)$,

Simplify the expression

$$\bar{A}\,C\,[\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,A + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,A + \bar{A}\,C\,\bar{B} + \bar{A}\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$(A\,\bar{A} = 0)$, take 2nd with 5th and 3rd with 4th terms.

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,A + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,A + \bar{A}\,C\,\bar{B} + \bar{A}\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

($A\,\bar{A} = 0$), take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B}\,C\,[\,A + \bar{A}\,] + \bar{A}\,\bar{D}\,[\,C + B\,\bar{C}\,]$$

Simplify the expression

$$\bar{A} C [\overline{\bar{A} B D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$= \bar{A} C [A + \bar{B} + \bar{D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$= \bar{A} C A + \bar{A} C \bar{B} + \bar{A} C \bar{D} + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$(A \bar{A} = 0)$, take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B} C [A + \bar{A}] + \bar{A} \bar{D} [C + B \bar{C}]$$

$$= \bar{B} C + \bar{A} \bar{D} [C + B]$$

Final result

$$\bar{A}\, C\, [\, \overline{\bar{A}\, B\, D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, [\, A + \bar{B} + \bar{D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, A + \bar{A}\, C\, \bar{B} + \bar{A}\, C\, \bar{D} + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

($A\,\bar{A} = 0$), take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B}\, C\, [\, A + \bar{A}\,] + \bar{A}\, \bar{D}\, [\, C + B\, \bar{C}\,]$$

$$= \bar{B}\, C + \bar{A}\, \bar{D}\, [\, C + B\,] = \bar{B}C + \bar{A}\bar{D}C + \bar{A}\bar{D}B$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

New Example

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

$$= B\overline{D}[\overline{A} + A + 1]$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + B\ B\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}\ B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

$$= B\overline{D}[\overline{A} + A + 1]$$

$$= B\overline{D}[1 + 1]$$

Final result

$$(\overline{A} + B)(A + B + D)\,\overline{D}$$

$$= [\,\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD\,]\,\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

$$= B\overline{D}\,[\,\overline{A} + A + 1\,]$$

$$= B\overline{D}\,[\,1 + 1\,]$$

$$= B\overline{D}\,(1) = B\overline{D}$$

Another example

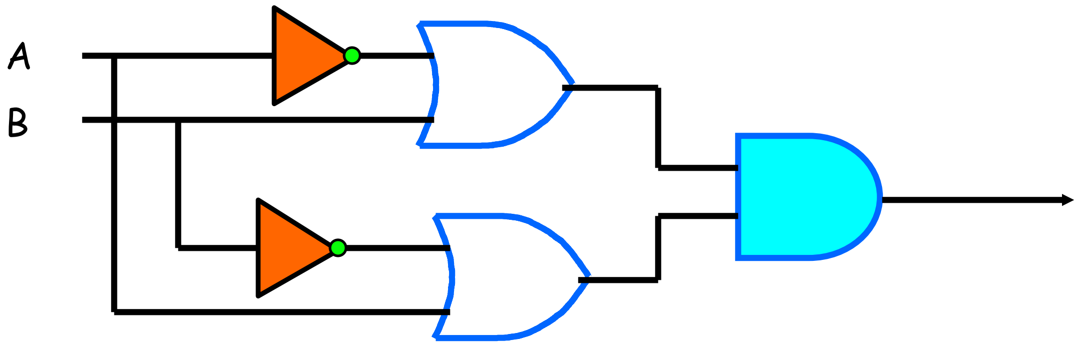
Given: $(\overline{A}+B)(\overline{B}+A)$

1. Implement using basic (AND, OR, NOT) gates
2. Simplify
3. Implement the simplified expression

New Example

1. First implementation

$$(\overline{A}+B)(\overline{B}+A)$$



2. Simplify

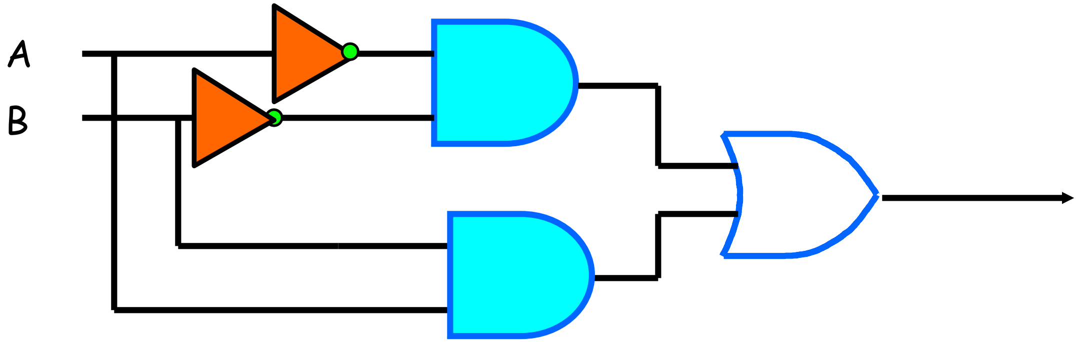
$$(\overline{A}+B)(\overline{B}+A) = \overline{A}\,\overline{B} + \overline{A}A + \overline{B}B + BA$$

$$= \overline{A}\,\overline{B} + AB$$

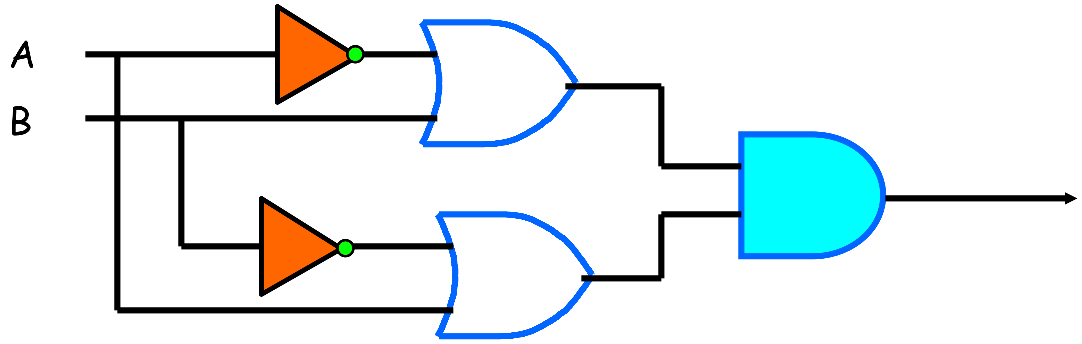
3. Second implementation

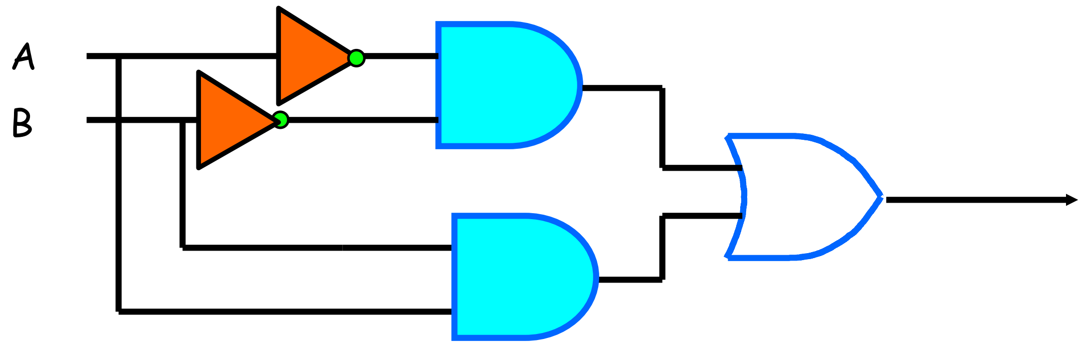
$$(\overline{A}+B)(\overline{B}+A) = \overline{A}\,\overline{B} + \overline{A}A + \overline{B}B + BA$$

$$= \overline{A}\,\overline{B} + AB$$



Both implementations are equivalent





Simplify the expression

$$Z = A\overline{C}\,\overline{D} + \overline{A}BD + A\overline{B}CD$$

New Example

Z = Z

$$Z = A\overline{C}\,\overline{D} + \overline{A}BD + A\overline{B}CD$$

Can not be simplified

Therefore … 3 cases …

- The result is a simplified expression
- The result is an equivalent expression
- The expression can not be simplified.