CSIT504 Module 5 Homework

1. (Problem 17 on page 91 from Rosen) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
   - a proof by contraposition.
   - a proof by contradiction.

2. (Problem 7 on page 91 from Rosen) Use a direct proof to show that every odd integer is the difference between two squares.

3. (Problem 13 on page 91 from Rosen) Prove that if x is irrational, then $1/x$ is irrational.

4. (Problem 3 on page 329 from Rosen) Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$ for the positive integer n.
   - What is the statement $P(1)$?
   - Show that $P(1)$ is true, completing the basis step of the proof.
   - What is the inductive hypothesis?
   - What do you need to prove in the inductive step?
   - Complete the inductive step, identifying where you use the inductive hypothesis.
   - Explain why these steps show that this formula is true whenever n is a positive integer.

5. (Problem 5 on page 329 from Rosen) Prove that $1^2 + 3^2 + 5^2 \cdots + (2n+1)^2 = (n+1)(2n+1)(2n+3/3$ whenever n is a positive integer

6. (Problem 19 on page 330 from Rosen) Let $P(n)$ be the statement that

   $$1 + 1/4 + 1/9 + \cdots + 1/n^2 < 2 - 1/n,$$

   where n is an integer greater than 1.
   - What is the statement $P(2)$?
   - Show that $P(2)$ is true, completing the basis step of the proof.
   - What is the inductive hypothesis?
   - What do you need to prove in the inductive step?
   - Complete the inductive step, identifying where you use the inductive hypothesis.
   - Explain why these steps show that this formula is true whenever n is an integer greater than 1.

7. (Problem 3 on page 341 from Rosen) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 8$.

- Show that the statements $P(8)$, $P(9)$, and $P(10)$ are true, completing the basis step of the proof.
- What is the inductive hypothesis of the proof?
- What do you need to prove in the inductive step?
- Complete the inductive step for $k \geq 10$.
- Explain why these steps show that this formula is true whenever $n \geq 8$.