CSIT 504 Midterm 2

1. (15 pts) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - 2$, for $n = 1,2,3, \ldots$, and suppose that $a_0 = 2$.
   a) What are a_1, a_2, a_3?
   b) Please solve its recurrence relation and initial condition to generate a closed formula for the sequence.

2. (15 pts) Find a **div** m and a **mod** m when
   a) $a = 18, \ m = 3$
   b) $a = -88, \ m = 13$

3. (15 pts) Determine
   a) whether 75 is congruent to 11 modulo 8? Please explain the reason.
   b) whether 24 is congruent to 14 modulo 7? Please explain the reason.

4. (10 pts) Find the prime factorizations of 200 and 67.

5. (15 pts) What are the Greatest Common Divisor and the Least Common Multiple of these pairs of integers?
   a) 19, 3
   b) $4^3 \times 7^2, \ \ 2^3 \times 4^2 \times 7^5$

6. (15 pts) Encode the message "stop at noon" using the function f(x) = (x + 5) **mod** 26.

7. (15 pts) Use the Principle of Mathematical Induction to prove that for $n \geqslant 0$

$$1 + 3 + 9 + 27 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$$