CSIT504 Module 1 Homework Part 1

1. (Problem 1 on page 12 from Rosen) Which of these statements are propositions? What are the truth values of those that propositions?
   - Boston is the capital of Massachusetts.
   - Miami is the capital of Florida.
   - $2 + 3 = 5$.
   - $5 + 7 = 10$.
   - $x + 2 = 11$.
   - Answer this question.

2. (Problem 17 on page 14 from Rosen) Determine whether each of these conditional statements is true or false.
   - if $1 + 1 = 2$, then $2 + 2 = 5$.
   - if $1 + 1 = 3$, then $2 + 2 = 4$.
   - if $1 + 1 = 3$, then $2 + 2 = 5$.
   - if monkeys can fly, then $1 + 1 = 3$.

3. (Problem 37 on page 15 from Rosen) Construct a truth table for each of the following:
   - $p \to (\bar{q} \vee r)$
   - $\bar{p} \to (q \to r)$
   - $(p \to q) \vee (\bar{p} \to r)$
   - $(p \leftrightarrow q) \vee (\bar{q} \leftrightarrow r)$

4. (Problem 9 on page 35 from Rosen) Show that each of these conditional statements is a tautology by using truth tables.
   - $(p \wedge q) \to p$
   - $\bar{p} \to (p \to q)$
   - $(p \wedge q) \to (p \to q)$
   - $\overline{(p \to q)} \to \bar{q}$

5. (Problem 27 on page 35 from Rosen) Show that $(p \leftrightarrow q)$ and $(p \to q) \wedge (q \to p)$ are logically equivalent.

6. (Problem 31 on page 35 from Rosen) Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

CSIT504 Module 1 Homework Part 2

1. (Problem 29 on page 35 from Rosen) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is tautology.

2. (Problem 5 on page 53 from Rosen) Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

   - $\exists x P(x)$.
   - $\forall x P(x)$.
   - $\exists x \neg P(x)$.
   - $\forall x \neg P(x)$.

3. (Problem 13 on page 53 from Rosen) Determine the truth value of each of the following if the domain consists of all integers.

   - $\forall n(n + 1 > n)$.
   - $\exists n(2n = 3n)$.
   - $\exists n(n = -n)$.
   - $\forall n(3n \leq 4n)$.

4. (Problem 19 on page 54 from Rosen) Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 2, 3, 4, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

   - $\exists x P(x)$.
   - $\forall x P(x)$.
   - $\neg \exists x P(x)$.
   - $\neg \forall x P(x)$.
   - $\forall x((x \neq 3) \to P(x)) \vee \exists x \neg P(x)$.

5. (Problem 25 on page 67 from Rosen) Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

   - $\exists x \forall y(xy = y)$.
   - $\forall x \forall y(((x < 0) \wedge (y < 0) \to (xy > 0))$.
   - $\exists x \exists y((x^2 > y) \wedge (x < y))$.
   - $\forall x \forall y \exists z(x + y = z)$.

6. (Problem 29 on page 67 from Rosen) Suppose the domain of the propositional function $P(x, y)$ consists of all pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write our these propositions using disjunctions and conjunctions.

   - $\forall x \forall y P(x, y)$.
   - $\exists x \exists y P(x, y)$.

- $\exists x \forall y P(x, y)$.
- $\forall x \exists y P(x, y)$.

7. (Problem 31 on page 67 from Rosen) Express the negations of each of these statements so that all negation symbols immediately precede predicates.

    - $\forall x \exists y \forall z T(x, y, z)$.
    - $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$.
    - $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$.
    - $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$.