CSIT504_51SP21 DISCRETE MATH IN COMPUTING

FINAL EXAM

1. **(10 pts)** Find the value of each of these four quantities.
     (a) $P(8,2)$ (b) $C(17,3)$

2. **(15 pts)** Please use mathematical induction to prove that

$$1 + 4 + 7 + 10 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}$$

   where $n \geq 1$, In your answer, please clearly indicate Basis Step, Inductive Step, and Conclusion Step.

3. **(15 pts)** A class consists of 20 sophomores and 15 freshmen.
   a) If the class needs to form a committee of size five. How many committees are possible?
   b) How many committees are possible if the committee must have three sophomores and two freshmen?
   c) How many ways are there to choose a president, a vice president, a secretary, and a treasurer.

4. **(15 pts)** Determine whether this proposition is a tautology: $((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$. Please explain the reasons.

5. **(15 pts)** In the following questions (a) and (b), please determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive
   a) The relation R on $\{w, x, y, z\}$ where $R = \{(w,w), (w,x), (x,w), (x,x), (x,z), (y,y), (z,y), (z,z)\}$. And please include the reason for your answer.
   b) The relation R on $\mathbb{Z}$ (integer) where aRb means $|a - b| \leq 1$. And please include the reason for your answer.

6. **(10 pts)** Draw the directed graph for the relation defined by the matrix $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$

7. **(10 pts)** Consider the following theorem: If n is an even integer, then n + 1 is odd. Give a proof by contraposition of this theorem.

8. **(10 pts)** Find three subsets of {1,2,3,4,5,6,7,8,9} such that the intersection of any two has size 3 and the intersection of all three has size 1. Please explain your answer.