CSIT504 Module 7 Homework

1. (Problem 1 on page 581 from Rosen) List all the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if

   (a) $a = b$.

   (b) $a + b = 4$.

   (c) $a > b$.

   (d) $a|b$.

   (e) $gcd(a, b) = 1$.

   (f) $lcm(a, b) = 2$.

2. (Problem 5 on page 581 from Rosen) Determine whether the relation R on the set of all Web pages is is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

   (a) everyone who has visited Web page a has also visited Web page b.

   (b) there are no common links found on both Web page a and Web page b.

   (c) there is at least one common link on Web page a and Web page b.

   (d) there is a Web page that includes links to both Web page a and Web page b.

3. (Problem 45 on page 583 from Rosen)

   (a) How many relations are there on the set $\{a, b, c, d\}$?

   (b) How many relations are there on the set $\{a, b, c, d\}$ that contain the pair (a, a)?

4. (Problem 1 on page 596 from Rosen) Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order).

   (a) $\{(1, 1), (1, 2), (1, 3)\}$

   (b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$

   (c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

   (d) $\{(1, 3), (3, 1)\}$

5. (Problem 3 on page 596 from Rosen) List all the ordered pairs in the relation on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

   (a)

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

   (b)

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

6. (Problem 1 on page 615 from Rosen) Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

   (a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
   (b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
   (c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
   (d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
   (e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

7. (Problem 9 on page 615 from Rosen) Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that $f(x) = f(y)$.

   (a) Show that R is an equivalence relation on A.
   (b) What are the equivalence classes of R?

8. (Problem 25 on page 616 from Rosen) Show that the relation R on the set of all bit strings such that sRt if and only if s and t contain the same number of 1s is an equivalence relation.

9. (Problem 45 on page 617 from Rosen) Which of these are partitions of the set $\mathbf{Z} \times \mathbf{Z}$ of ordered pairs of integers?

   (a) the set of pairs (x, y), where x or y is odd; the set of pairs (x, y), where x is even; the set of pairs (x, y), where y is even
   (b) the set of pairs (x, y), where both x and y are odd; the set of pairs (x, y), where exactly one of x and y is odd; the set of pairs (x, y), where both x and y is even
   (c) the set of pairs (x, y), where x is positive; the set of pairs (x, y), where y is positive; the set of pairs (x, y), where both x and y are negative
   (d) the set of pairs (x, y), where $3|x$ and $3|y$; the set of pairs (x, y), where $3|x$ and $3 \not| y$; the set of pairs (x, y), where $3 \not| x$ and $3|y$; the set of pairs (x, y), where $3 \not| x$ and $3 \not| y$ is even
   (e) the set of pairs (x, y), where $x > 0$ and $y > 0$; the set of pairs (x, y), where $x > 0$ and $y \leq 0$; the set of pairs (x, y), where $x \leq 0$ and $y \leq 0$
   (f) the set of pairs (x, y), where $x \neq 0$ and $y \neq 0$; the set of pairs (x, y), where $x = 0$ and $y \neq 0$; the set of pairs (x, y), where $x \neq 0$ and $y = 0$

10. (Problem 57 on page 617 from Rosen) Consider the equivalence relation $R = \{(x, y) | x - y \text{ is an integer}\}$.

    (a) What is the equivalence class of 1 for this equivalence relation?
    (b) What is the equivalence class of $1/2$ for this equivalence relation?