HW #2 Solution

1. a) This is false, since the empty set has no elements.
   b) This is false. The set on the right has only one element, namely the number 0, not the empty set.
   c) This is false. In fact, the empty set has no proper subsets.
   d) This is true. Every element of the set on the left is, vacuously, an element of the set on the right; and the
   set on the right contains an element, namely 0, that is not in the set on the left.
   e) This is false. The set on the right has only one element, namely the number 0, not the set containing the
   number 0.
   f) This is false. For one set to be a proper subset of another, the two sets cannot be equal.
   g) This is true. Every set is a subset of itself.

2.

a. T (in fact x is the only element)
b. T (every set is a subset of itself)
c. F (the only element of { x} is a letter, not a set)
d. T (in fact, { x} is the only element)
e. T (the empty set is a subset of every set)
f. F (the only element of { x} is a letter, not a set)

3. The cardinality of a set is the number of elements it has. The number of elements in its elements is irrelevant.

a. 1
b. 1
c. 2
d. 3

4.

a. Since the set we are working with has 3 elements, the power set has $2^3 = 8$ elements.
b. Since the set we are working with has 4 elements, the power set has $2^4 = 16$ elements.
c. The power set of the empty set has $2^\circ = 1$ element. The power set of this set therefore has $2^1 = 2$ elements. In particular, it is $\{ \emptyset, \{\emptyset\}\}$. (See Example 14.)

5.

a. We include all numbers that are in one or both of the sets, obtaining {0, 1, 2, 3, 4, 5, 6}.
b. There is only one number in both of these sets, so the answer is { 3}.
c. The set of numbers in A but not in B is {1, 2, 4, 5}.
d. The set of numbers in B but not in A is {0, 6}.

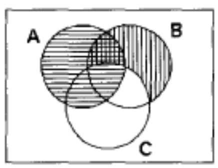
6.

a) In the figure we have shaded the A set with horizontal bars (including the double-shaded portion, which includes both horizontal and vertical bars), and we have shaded the set $B-C$ with vertical bars (that portion inside B but outside C. The intersection is where these overlap—the double-shaded portion (shaped like an arrowhead).

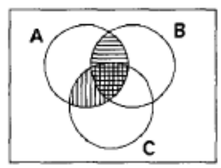
b) In the figure we have shaded the set $A \cap B$ with horizontal bars (including the double-shaded portion, which includes both horizontal and vertical bars), and we have shaded the set $A \cap C$ with vertical bars. The union is the entire region that has any shading at all (shaped like a tilted mustache).

c) In the figure we have shaded the set $A \cap \overline{B}$ with horizontal bars (including the double-shaded portion, which includes both horizontal and vertical bars), and we have shaded the set $A \cap \overline{C}$ with vertical bars. The union is the entire region that has any shading at all (everything inside A except the triangular middle portion where all three sets overlap) portion (shaped like an arrowhead).

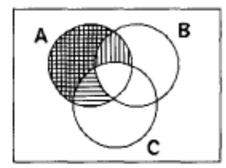
(a)



(b)



(c)



7.

a) If B adds nothing new to A, then we can conclude that all the elements of B were already in A. In other words, $B \subseteq A$.

b) In this case, all the elements of A are forced to be in B as well, so we conclude that $A \subseteq B$.

c) This equality holds precisely when none of the elements of A are in B (if there were any such elements, then $A-B$ would not contain all the elements of A). Thus we conclude that A and B are disjoint ($A \cap B = \emptyset$).

d) We can conclude nothing about A and B in this case, since this equality always holds.

e) Every element in $A-B$ must be in A, and every element in $B-A$ must not be in A. Since no item can be in A and not be in A at the same time, there are no elements in both $A-B$ and $B-A$. Thus the only way for these two sets to be equal is if both of them are the empty set. This means that every element of A must be in B, and every element of B must be in A. Thus we conclude that $A = B$.