Mid term 2. p1

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1) a) $a_n = (a_{n-1}) - 2$ for $n = 1, 2, 3 \ldots$ and $a_0 = 2$

$$a_1 = a_0 - 2 = 2 - 2 = 0$$
$$a_2 = a_1 - 2 = (2-2) - 2 = -2$$
$$a_3 = a_2 - 2 = ((2-2) - 2) - 2 = -4$$

this is an Arithmetic sequence, which can be written as

$$a_n = 2 - 2(n)$$

2) a) $a = 18$, $m = 3$

Division algorithm = $a = dq + r$

$q = a \text{ div } d$

$r = a \bmod d$

$$a = 18 = (6)(3) + 0$$

$q = 18 \text{ div } 3 = 6$

$r = 18 \bmod 3 = 0$

b) $a = -88$, $m = 13$

$a = -88 = (-7)(13) + 3$

$q = -88 \text{ div } 13 = -7$

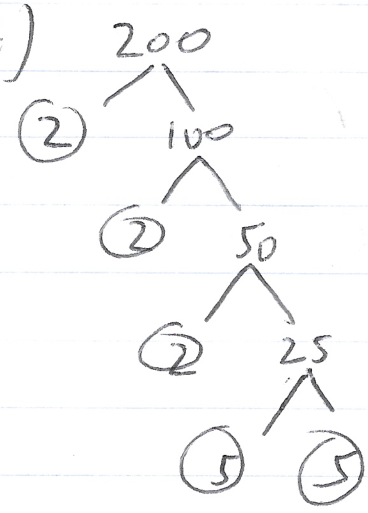
$r = -88 \text{ div } 13 = 3$

3) $a \equiv b \pmod{n}$ iff $n \mid (a-b)$

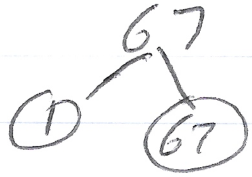
a) $\frac{(75-11)}{8} = 8$ with no remainder, therefore 75 is congruent to 11 modulo 8.

B) $\frac{(24-14)}{7} = \frac{10}{7}$

~~Since~~ Since there is a remainder, 24 is not congruent to 14 modulo 7.

4) 

Prime factorization of 200 is $(2^3)(5^2)$



Prime factorization of 67 is 67

5) a) GCD of 19, 3 is $(19^0)(3^0) = 1$

LCM of 19, 3 is $(19)(3) = 57$

b) GCD of $4^3 \times 7^2$, $2^3 \times 4^2 \times 7^5$
$= (2^0)(7^2)(4^2) = 784$

LCM of $4^3 \times 7^2$, $2^3 \times 4^2 \times 7^5$
$= (2^3)(4^3)(7^5) = 8,605,184$

6) Assuming the following

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

stop at noon
18 19 14 15 0 19 13 14 14 13

after inputting values in function $f(x) = (x+5) \bmod 26$
we get 23 24 19 20 5 24 18 19 19 18
which = X Y T U F Y S T T S

7) First, establishing the base case as per $n \geq 0$

$$3^0 = \frac{3^{(0+1)}-1}{2} = 1$$ So base case is true

Inductive hypothesis for $n \geq 0$

$P(n)$: $1+3+9+27+\dots+3^n = \cancel{3^{n+1}} \frac{3^{n+1}-1}{2}$

What do we need to prove in the inductive step?

let ~~n=k+1~~ $n=k$

we need to prove that

$P(k+1)$: $1+3+9+27+\dots+3^k+3^{k+1} = \frac{3^{(k+1)+1}-1}{2}$

Now we complete the inductive step.

$$1+3+9+27+\dots+3^k+3^{k+1} = \frac{3^{k+1}-1}{2}+3^{k+1}$$

$$= \frac{3^{k+1}-1+6^{k+1}}{2}$$

$$= \frac{3^{k+1}-1+2(3^{k+1})}{2}$$

$$= \frac{3\cdot 3^{k+1}-1}{2}$$

$$= \frac{3^{(k+1)+1}-1}{2}$$

This proves via mathematical induction that the result is true for $k+1$ and therefore for $n \geq 0$.