# Stroboscopic High-Order Nonlinearity in Optomechanics

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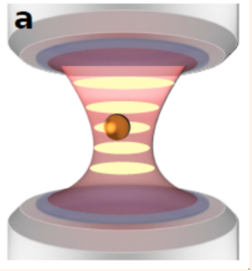
New J. Phys. **21** 113050 & npj Quantum Inf. **7**, 120

Quantum Engineering of Levitated Systems
Benasque 2022


Norway
Finland
Baltic Sea
Estonia
Latvia
Moscow
Москва
North Sea
Denmark
Lithuania
United Kingdom
Ireland
Netherlands
Berlin
Poland
Belarus
London
Germany
Belgium
Czechia
Paris
Vienna
Slovakia
Ukraine
Austria
Moldova
Hungary
France
Romania
Croatia
Serbia
Black Sea
Italy
Rome
Barcelona
Bulgaria
Georgia
Madrid
Portugal
Istanbul
Spain
Tyrrhenian Sea
Greece
Turkey
Syria
Lebanon
Tunisia
Mediterranean Sea
Iraq
Morocco
Israel
Jordan
Google
Satellite

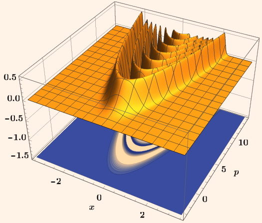
# Main Results

## Levitated Nanoparticle




Requirement: a nonlinear trapping potential $V(x)$.

## Approximate Cubic Phase State




$$|\gamma\rangle = e^{i\gamma x^3}|p\rangle \approx e^{i\gamma x^3}\hat{S}|0\rangle$$

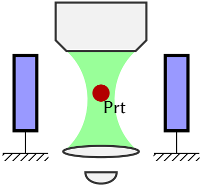
Applies to any regular nonlinear potential, will be illustrated for the cubic one

# Levitated Nanoparticles in Engineered Potentials

$$H_{\text{trap}} = -\frac{1}{2}\int_{\text{Vol}} d\mathbf{r}\, \mathbf{P}(\mathbf{r})\mathbf{E}(\mathbf{r}) \propto -\int d\mathbf{r}\, \mathbf{E}^2(\mathbf{r}),$$

$$\mathbf{P} \propto \mathbf{E},$$

Equiv. potential: $V(\mathbf{r};t) \propto -I(\mathbf{r};t)$,



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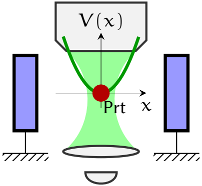


Gaussian intensity profile

$$I(x) \propto \exp\left[-\frac{x^2}{2x_0^2}\right] \approx 1 - \frac{x^2}{2x_0^2}.$$

$$V(x) \propto \omega_m^2 x^2$$

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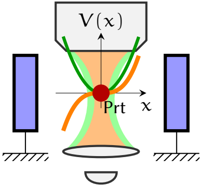



Gaussian intensity profile

$$I(x) \propto \exp\left[-\frac{x^2}{2x_0^2}\right] \approx 1 - \frac{x^2}{2x_0^2}.$$

$$V(x) \propto \omega_m^2 x^2 + k_3 x^3$$

Can engineer complicated $I(x)$, particularly cubic $I \propto x^3$

**Requirements**

Ability to switch the nonlinear contribution faster than $\omega_m$.

# Cubic phase state

Devised by Gottesman, Kitaev and Preskill [1]

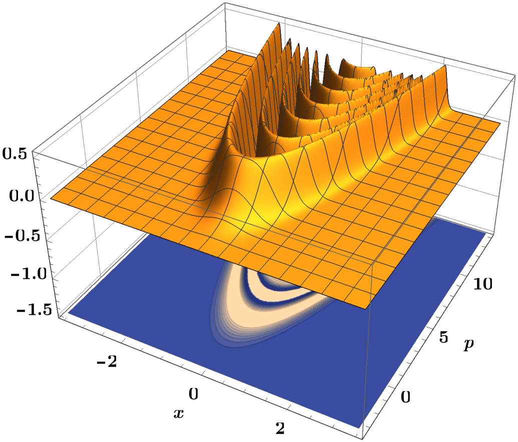
$$|\gamma_{\mathrm{GKP}}\rangle \propto e^{-i\gamma x^3} |p = 0\rangle \,,$$

Wigner Function

$$W_{\mathrm{GKP}}(x,p) \propto \mathrm{Ai}\left[\left(\frac{4}{3\gamma}\right)^{1/3}(3\gamma x^2 - p)\right].$$

More physical is an approximation

$$|\gamma\rangle = e^{-i\gamma x^3}\hat{S}\,|0\rangle \,.$$




[1] Gottesman, Kitaev, Preskill, PRA **64**, 012310 (2001)

# Motivation: Measurement-based computing

PHYSICAL REVIEW A **97**, 022329 (2018)

## General implementation of arbitrary nonlinear quadrature phase gates

Petr Marek,[1,*] Radim Filip,[1] Hisashi Ogawa,[2] Atsushi Sakaguchi,[2] Shuntaro Takeda,[2] Jun-ichi Yoshikawa,[2,3] and Akira Furusawa[2,†]

[1]*Department of Optics, Palacký University, 17. listopadu 1192/12, 77146 Olomouc, Czech Republic*
[2]*Department of Applied Physics, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*
[3]*Quantum-Phase Electronics Center, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

(Received 10 August 2017; published 20 February 2018)

We propose general methodology of deterministic single-mode quantum interaction nonlinearly modifying single quadrature variable of a continuous-variable system. The methodology is based on linear coupling of the system to ancillary systems subsequently measured by quadrature detectors. The nonlinear interaction is obtained by using the data from the quadrature detection for dynamical manipulation of the coupling parameters. This measurement-induced methodology enables direct realization of arbitrary nonlinear quadrature interactions without the need to construct them from the lowest-order gates. Such nonlinear interactions are crucial for more practical and efficient manipulation of continuous quadrature variables as well as qubits encoded in continuous-variable systems.

DOI: 10.1103/PhysRevA.97.022329

# Motivation: Measurement-based computing

PHYSICAL REVIEW A 97, 022329 (2018)

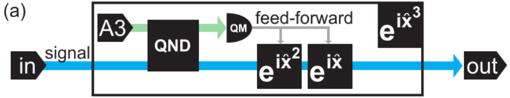



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FIG. 1. Schematic circuits for various implementations of nonlinear gates. QND: quantum nondemolition interaction; QM: quadrature measurement; $Ak$: ancillary state of the $k$th order squeezed in $\hat{p} - N\chi_N\hat{x}^{N-1}$. $e^{i\hat{x}^k}$: unitary realization of $k$th-order nonlinear gate with arbitrary strength. (a) Cubic gate with $N = 3$; (b) $(N + 1)$th-

Required $\mathrm{Var}(p - \gamma x^2) \to 0$ for ancilla

# Motivation: Measurement-based computing


$$\text{Noise} \propto \text{Var}_{|\text{Ancilla}\rangle}(p - \gamma x^2)$$

For a CPS $|\text{Ancilla}\rangle = |\gamma_{\text{GKP}}\rangle := e^{i\gamma x^3}|p = 0\rangle$,

the variance vanishes $\text{Var}_{|\gamma_{\text{GKP}}\rangle}(p - \gamma x^2) = 0$.

## Figure of Merit: Nonlinear Variance

The Nonlinear Variance for the implementation of $\exp[-i\gamma x^k]$ is

$$\sigma_k(\lambda) = \mathrm{Var}(p - \lambda x^{k-1}).$$

For a cubic gate $\exp[-i\gamma x^3]$,

$$\sigma_3(\lambda) = \mathrm{Var}(p - \lambda x^2)$$

Evaluated on vacuum state

$$\sigma_3^{\mathrm{vac}}(\lambda) = (1 + 2\lambda^2)\sigma^{\mathrm{vac}}.$$

# Analogy with quadratic squeezing

A quantum state is squeezed when for some $\theta$

$$\mathrm{Var}(p\cos\theta - x\sin\theta) < \sigma_{\mathrm{vac}}$$

Equivalent to

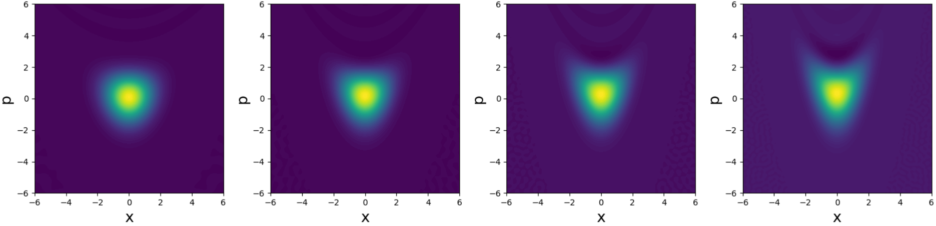
$$\sigma_2(\lambda) = \mathrm{Var}(p - \lambda x) < (1+\lambda^2)\sigma_{\mathrm{vac}}, \text{ with } \lambda = \tan\theta.$$

Compare to

$$\sigma_3(\lambda) = \mathrm{Var}(p - \lambda x^2) < (1+2\lambda^2)\sigma_{\mathrm{vac}}.$$

# Why bother with protocols

Wigner functions from $\exp[-i(\gamma\hat{x}^3)\tau]$:



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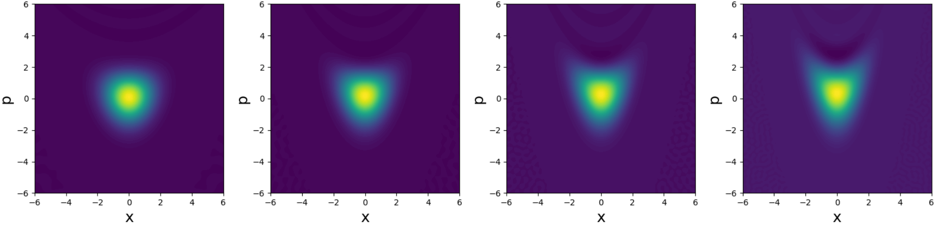
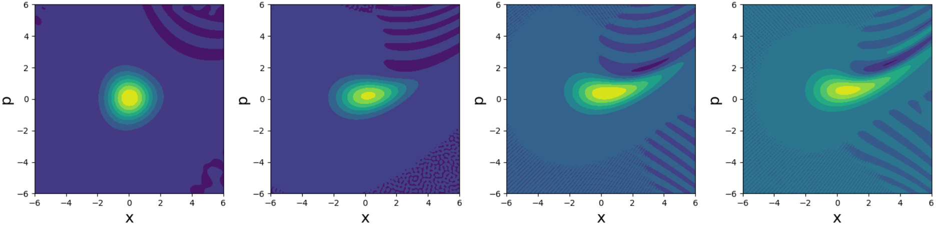
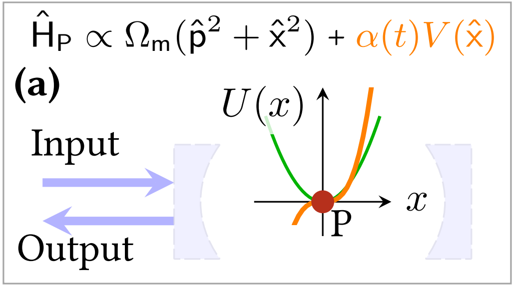


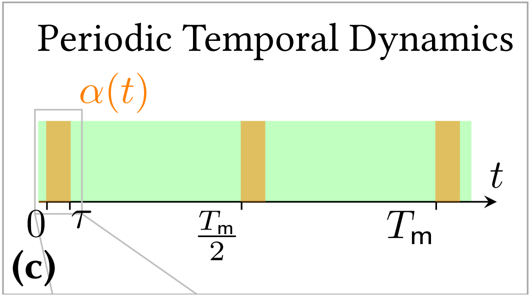

Wigner functions from $\exp[-i(H_{HO}(\hat{p}) + \gamma\hat{x}^3)\tau]$:



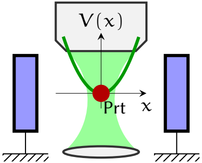
# The Model

Rakhubovsky and Filip, npj Quantum Information **7**, 120 (2021)

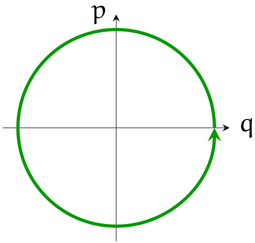


# The Protocol: Stroboscopic Application of Nonlinear Potential




Phase space picture




- ★ Cool the particle close to the ground state

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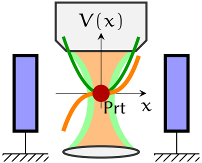
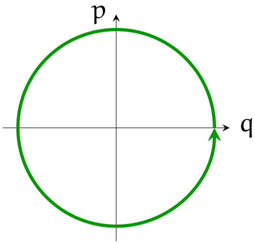



Phase space picture

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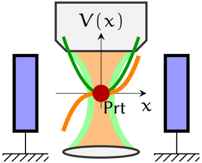


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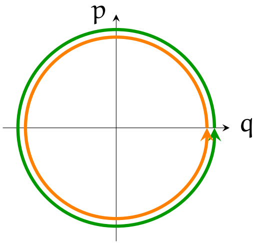


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- Continuous application of cubic is smeared out by harmonic evolution

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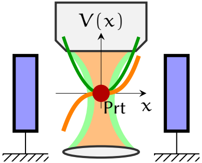


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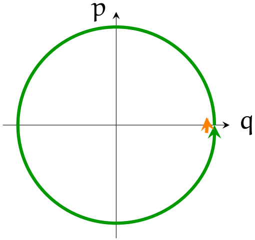


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- Apply instantaneous nonlinearity at certain phases of oscillations $t = 0, T_m, 2T_m \ldots.$

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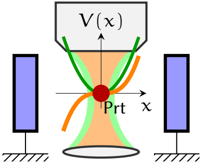


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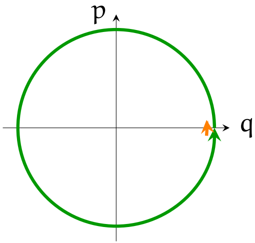


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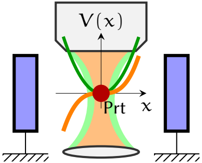


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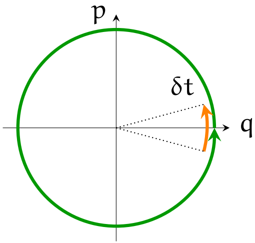


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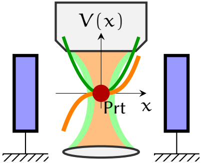
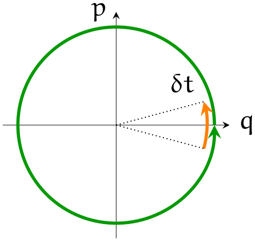



Phase space picture

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Optimization of the number of periods $M_T < \Gamma_m^{-1}/T_m$ and duration of application within a certain period $\delta t$.

$$M_T \ \& \ \delta t.$$

# Stroboscopic QND measurement of mechanical motion

Quantum Non-Demolition measurement of mechanical position

$$H_{OM} = \Delta a_L^\dagger a_L + \Omega_m a_M^\dagger a_M + g X_L X_M$$

In order to realize a true QND coupling, get rid of the first two terms:
Tune on resonance $\Delta = 0$ and

**Modulate the coupling rate**

$$g \mapsto g(t) = g_0 \cos 2\Omega_m t$$

Hamiltonian in rotating frame

$$H_{OM} \mapsto \propto g_0 \tilde{X}_L \tilde{X}_M$$

**Use short pulses**

$$\Omega_m \tau \ll 1$$

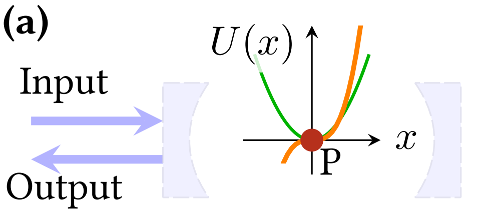
Effective Hamiltonian

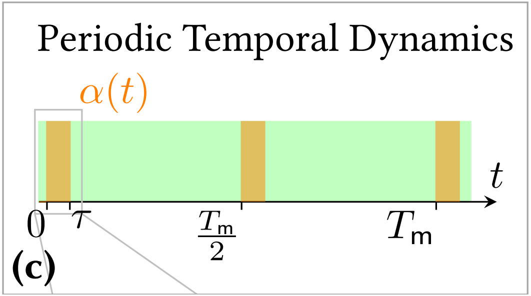
$$H_{OM} \mapsto g X_L X_M.$$

# The Model

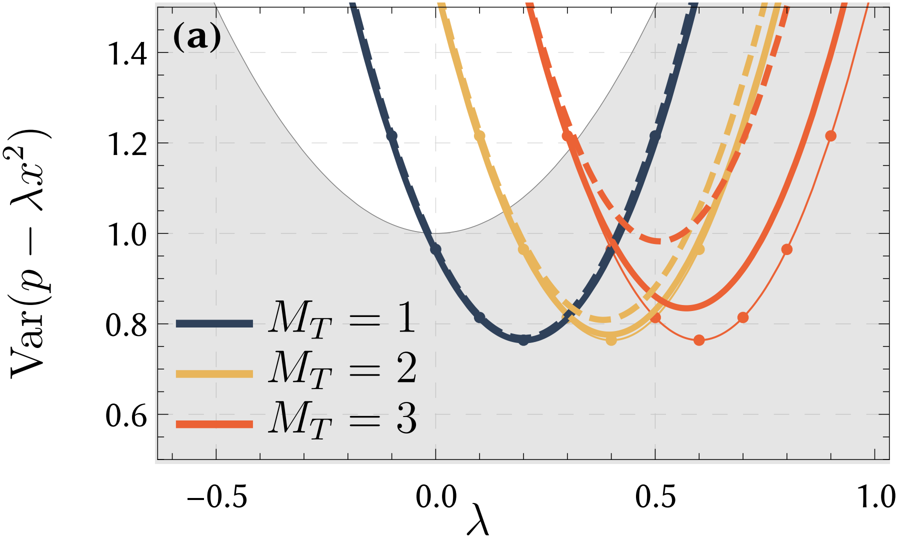
Rakhubovsky and Filip, npj Quantum Information **7**, 120 (2021)

$$\hat{\mathsf{H}}_{\mathsf{P}} \propto \Omega_{\mathsf{m}}(\hat{\mathsf{p}}^2 + \hat{\mathsf{x}}^2) + \alpha(t)V(\hat{\mathsf{x}})$$

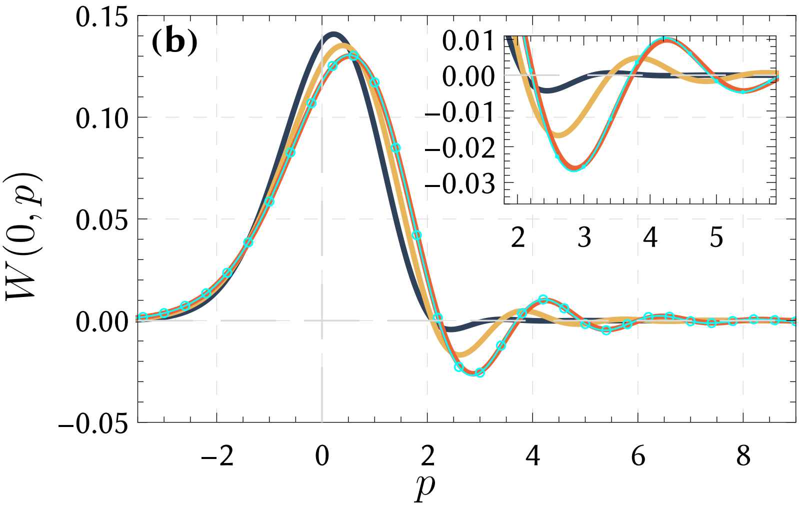


# Nonlinear Variance



## Wigner Function Cuts



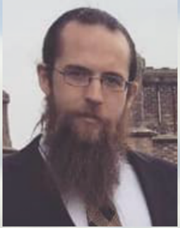

For red and cyan $\mathrm{Tr}[\rho_{red}\rho_{cyan}]/\,\mathrm{Tr}[\rho_{cyan}^2] = 0.9877$

Introduction

- Prerequisites: Nonlinear Potential
- Cubic States

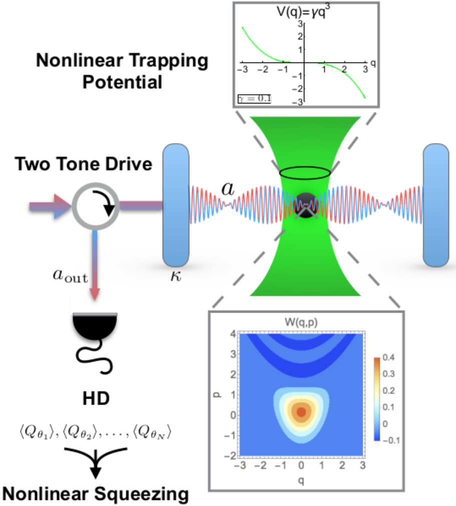
Cubic Phase State in Levitated Optomechanics

- CPS Preparation
- CPS Evaluation
- Outlook: Applications



Darren Moore
NJP **21** 113050

# The Model




**Approximate cubic phase state in mechanics**

$$e^{i\gamma q^3}\hat{S}\,|0\rangle\,.$$

**Pulsed QND interaction**

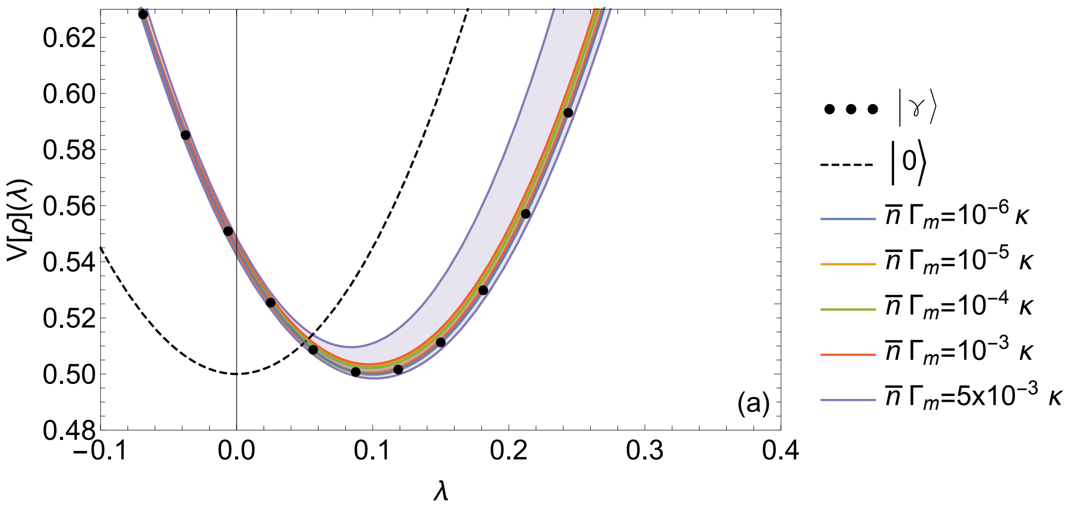
$$H_{\text{int}} \propto x_{\text{light}}(q\cos\phi + p\sin\phi).$$

Detect leaking light

**Estimate nonlinear variance**

$$q_{\text{NL}} = p - \lambda x^2 \Rightarrow \text{Var}[q_{\text{NL}}]$$

# Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



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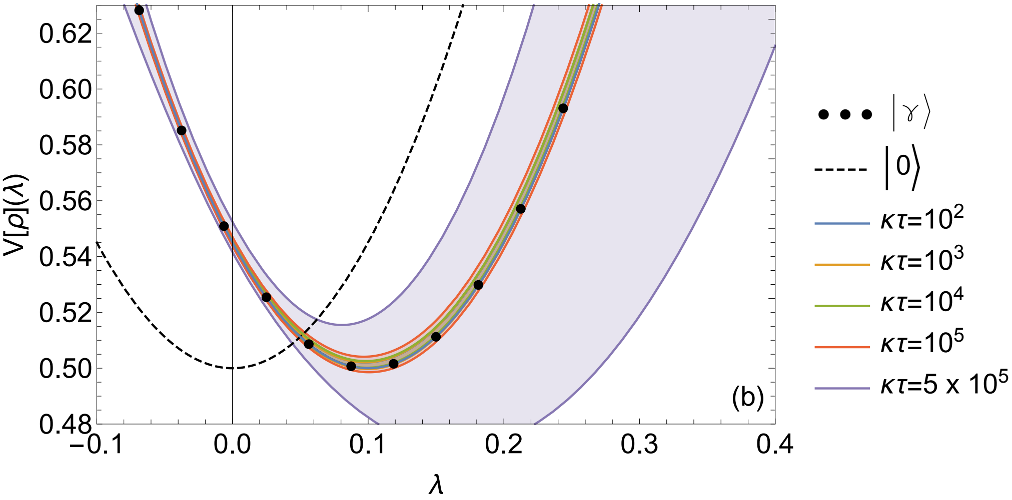

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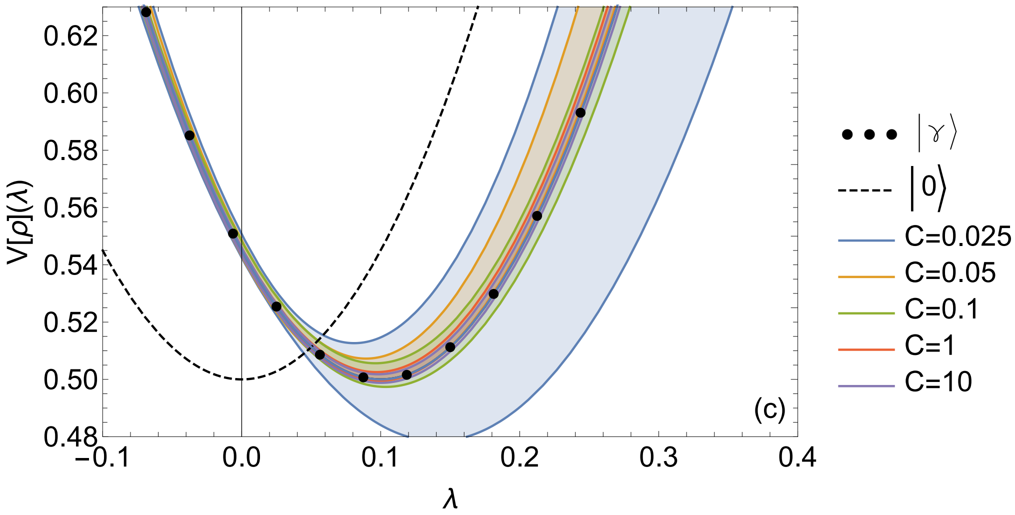
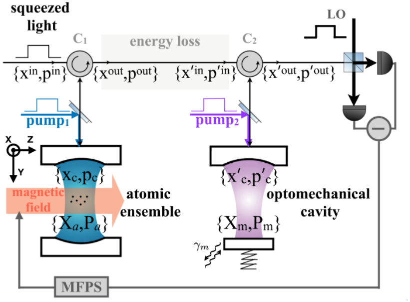


# Atom-Mechanical QND-gate

A. Manukhova [npj Quantum Inf. **6**, 4 (2020)]
(Largely inspired by [Hammerer PRL **102**, 020501 (2009)])




QND interaction between mechanics and atoms

$$H_{\text{atoms}-\text{mech}} \propto X_a P_m$$

Displaces mechanical quadrature by an amount proportional to the atomic one

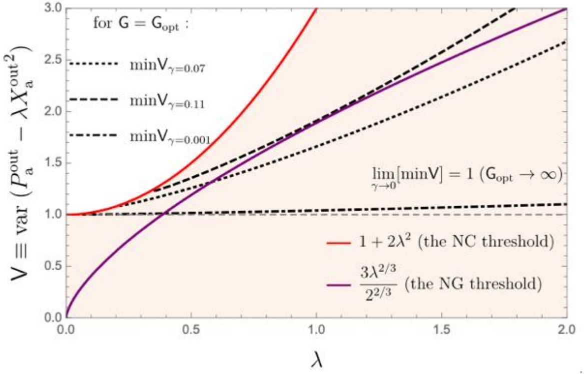
$$X_m(\text{final}) = X_m(\text{initial}) + \mathfrak{G} X_a(\text{initial}).$$

# Nonlinearity for Atoms (in Preparation)

Map $X_a$ on $X_m$
$X_m \propto X_a(0)$

Apply $e^{-i\gamma X_m^3}$
$P_m \propto X_a^2(0)$

Map back $P_m$ on $P_a$
$P_a \propto X_a^2(0)$




By Dr. Alisa Manukhova

# Conclusion

- ★ Optomechanics provides full linear control over a mechanical oscillator
- ★ Levitated nanoparticles combine advantages of linear optomechanics with possibilities to engineer nontrivial nonlinear potentials
- ★ Stroboscopic application of a cubic potential allows creation of approximate Cubic Phase States
- ★ With the toolbox of optomechanics these states can be read out, verified and used for quantum computation, or in other systems

# Thank You!

These slides: `https://bit.ly/ar-qels2022`

Ph.D. positions available