

# Entanglement of levitated nanoparticles by wave-packet dispersion

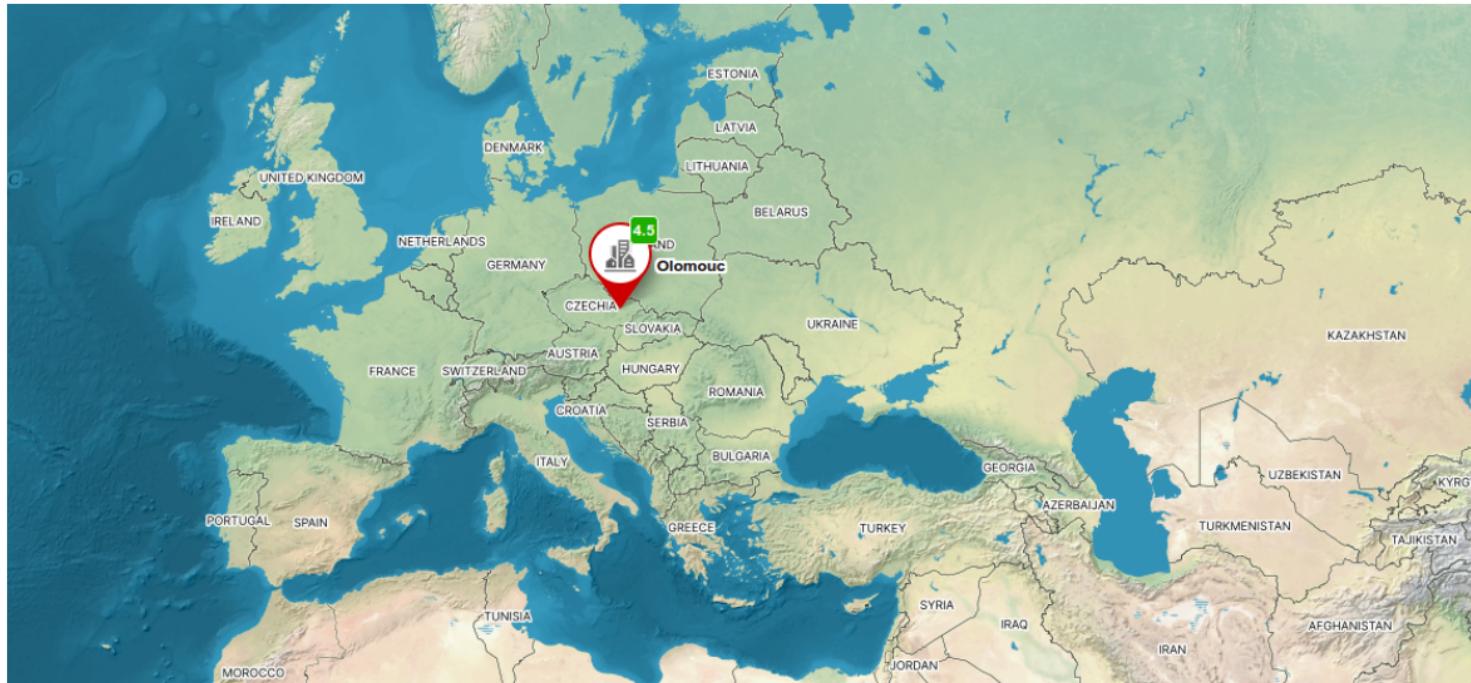
Andrey A. Rakhubovsky, Radim Filip

Department of Optics, Palacký University, Czech Republic

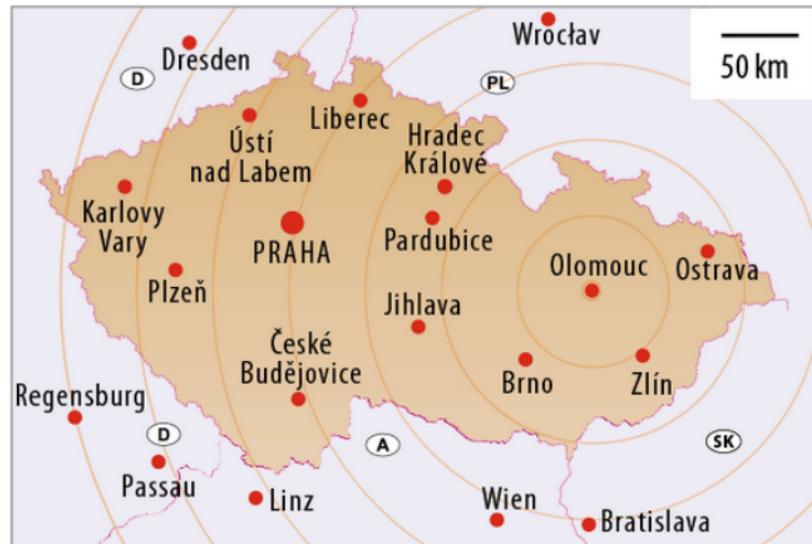
LPHYS'25

Szeged

July 2, 2025







# Radim Filip's Group in Olomouc [from 2022 video]

Radim Filip: Nonclassical and Quantum Non-gaussian States of Light and Matter



Palacký University  
Olomouc

## NONGAUSS: TOPICS AND TEAM



### Quantum Non-Gaussian States (2002)

Lukáš Lachman  
Luca Innocenti  
Petr Zapletal  
Jitendra Verma

### Quantum Non-Gaussian Optics (2005)

Petr Marek

Students:  
Jan Provazník  
Vojtěch Kala

### Quantum Communication (2010)

Vladyslav Usenko  
Ivan Derkach

Students:  
Olena Kovalenko  
Akash Oruganti

### Quantum Optomechanics (2014)

Andrey Rakhubovsky  
Darren Moore  
Ondřej Černotík

Anil Kumar  
Foroud Bemani  
Najme Etehadi  
Luca Ornigotti

### Atoms and Trapped Ions (2015)

Alisa Manukhova  
Darren Moore  
Kimin Park  
Pradip Laha  
Arpita Pal

Students:  
Lukáš Podhora

### Quantum Sensing and Estimation (2016)

Laszlo Ruppert  
Atirach Ritboon  
Payman Mahmoudi  
Kimin Park

Students:  
Eva Racz

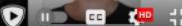
### Autonomous Quantum Coherence (2017)

Michal Kolář  
Suman Chand

Students:  
Uljana Rakhubovsky  
Maria Gumberidze

### Superconducting Circuits (2019)

Ondřej Černotík  
Kimin Park  
Iivari Pietikäinen



# The Group in Olomouc [within R. Filip's group]

Prof. Radim Filip



Dr. Alisa Manukhova



Dr. Foroud Bemani



Dr. Surabhi Yadav



Dr. Najmeh Etehadi



Now @KIT

Shaoni Datta



Dr. Lewis Clark



Now @A\*STAR

Ph.D. and Postdoc positions available

# Quantum non-Gaussian Everything



Progress in Quantum Electronics

Volume 93, January 2024, 100495



## Quantum non-Gaussian optomechanics and electromechanics

Andrey A. Rakhubovsky  Darren W. Moore  Radim Filip  

40 pages, 487 references

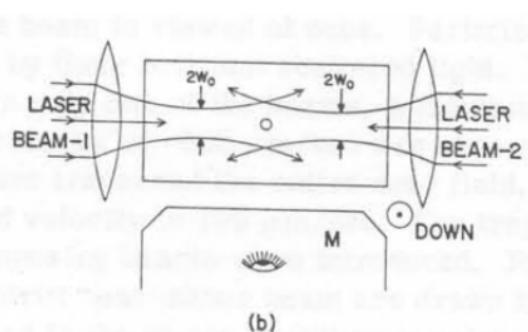
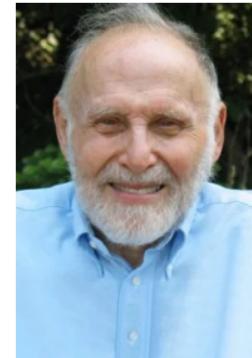
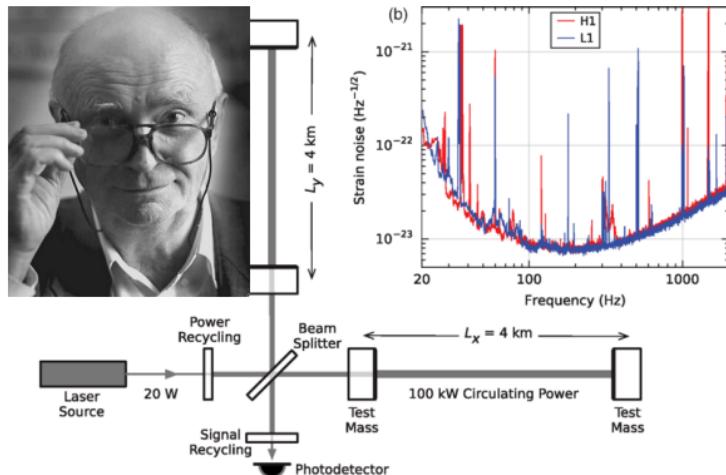
## Introduction

Quantum Optomechanics  
Levitated Optomechanics

Entanglement of Levitated Nanoparticles



# Quantum Optomechanics

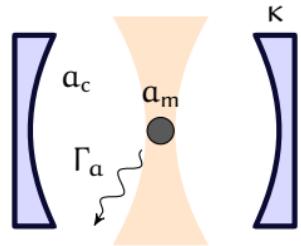


Braginsky & Manukin, Soviet JETP **25**, 653 (1967)  
Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)

A. Ashkin, PRL **24**, 156 (1970)

$$H = \omega_{\text{mech}} a_m^\dagger a_m + \omega_{\text{cav}} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

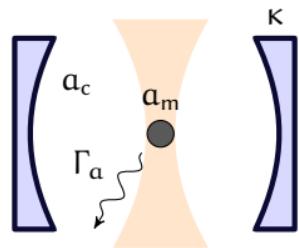
# Levitated Optomechanics



## The Hamiltonian

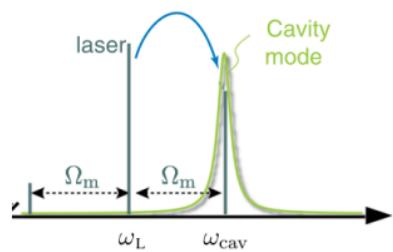
$$H = \frac{p_m^2}{2m} + V(q_m) + \omega_{\text{cav}} a_c^\dagger a_c + g q_m (a_c^\dagger + a_c).$$

# Levitated Optomechanics



## The Hamiltonian

$$H = \frac{p_m^2}{2m} + V(q_m) + \omega_{cav} a_c^\dagger a_c + g q_m (a_c^\dagger + a_c).$$



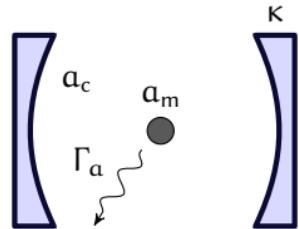
## Conventional Interaction (Cooling)

$$H_{int} = g(a_c^\dagger a_m + a_c a_m^\dagger)$$

- ★ Cooling
- ★ Beam-splitter / State swap

M. Aspelmeyer *et al.*, Rev. Mod. Phys. **86**, 1391 (2014)

# Levitated Optomechanics



## The Hamiltonian

$$H = \frac{p_m^2}{2m} + \omega_{cav} a_c^\dagger a_c .$$

## Free Fall

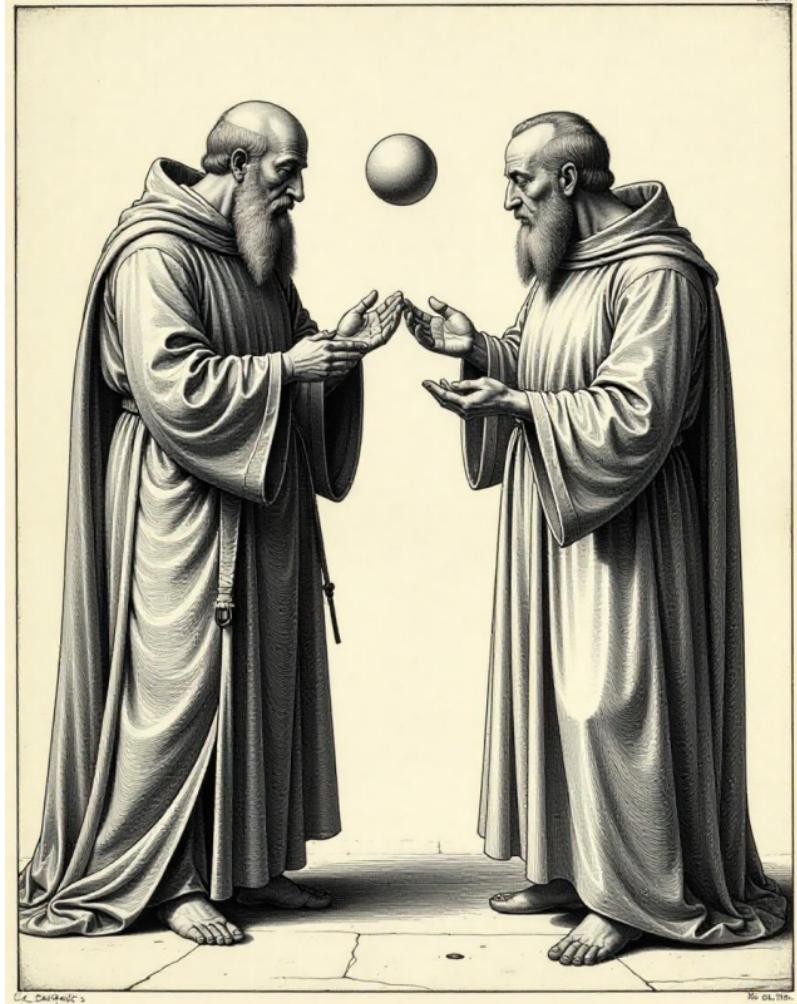
$$\begin{aligned} q &\mapsto q + p\tau, \\ p &\mapsto p. \end{aligned}$$

Squeezing!

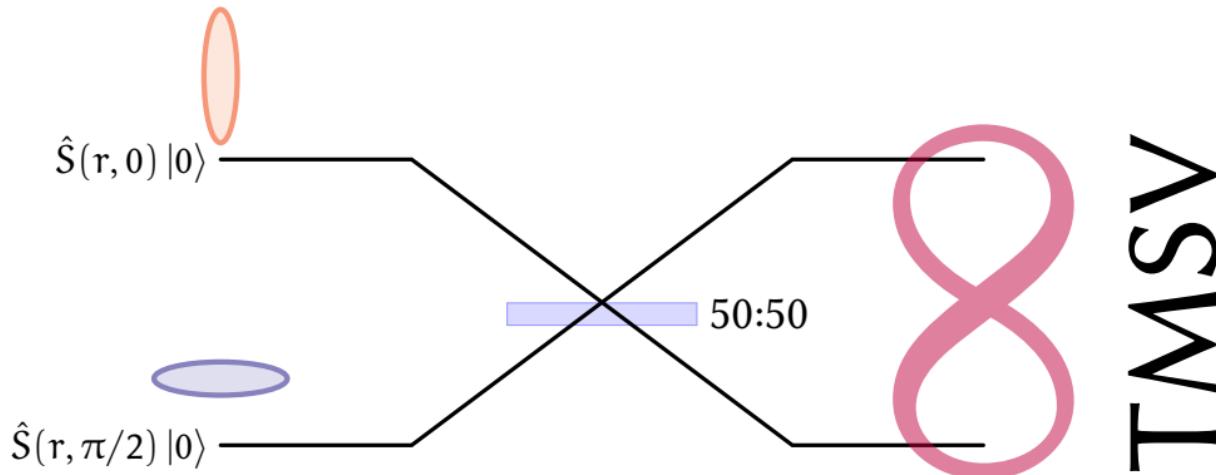
## Introduction

Quantum Optomechanics  
Levitated Optomechanics

## Entanglement of Levitated Nanoparticles



## Quantum optics analogy

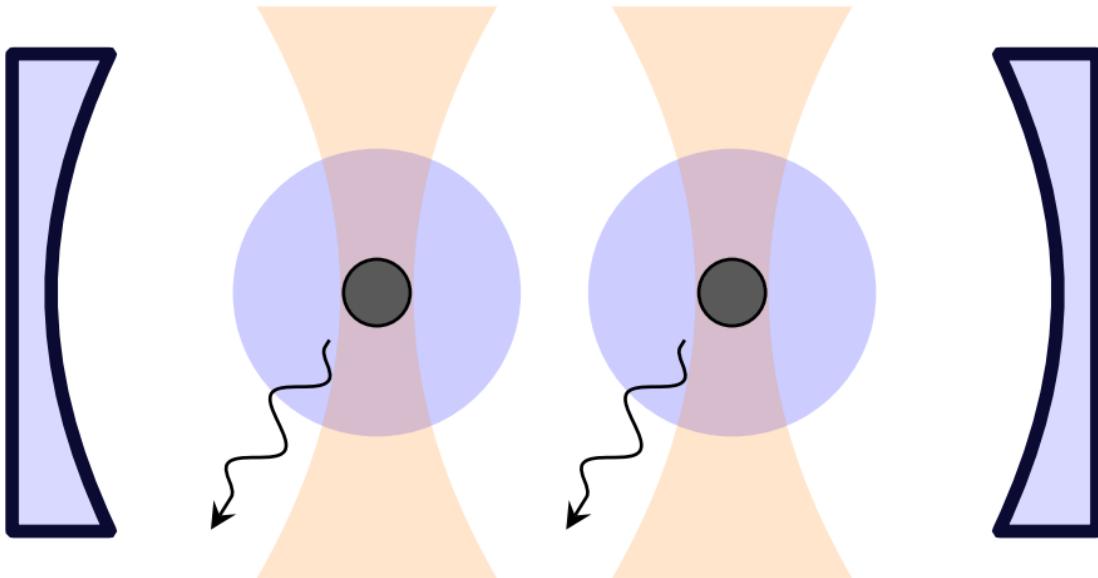


# The Recipe

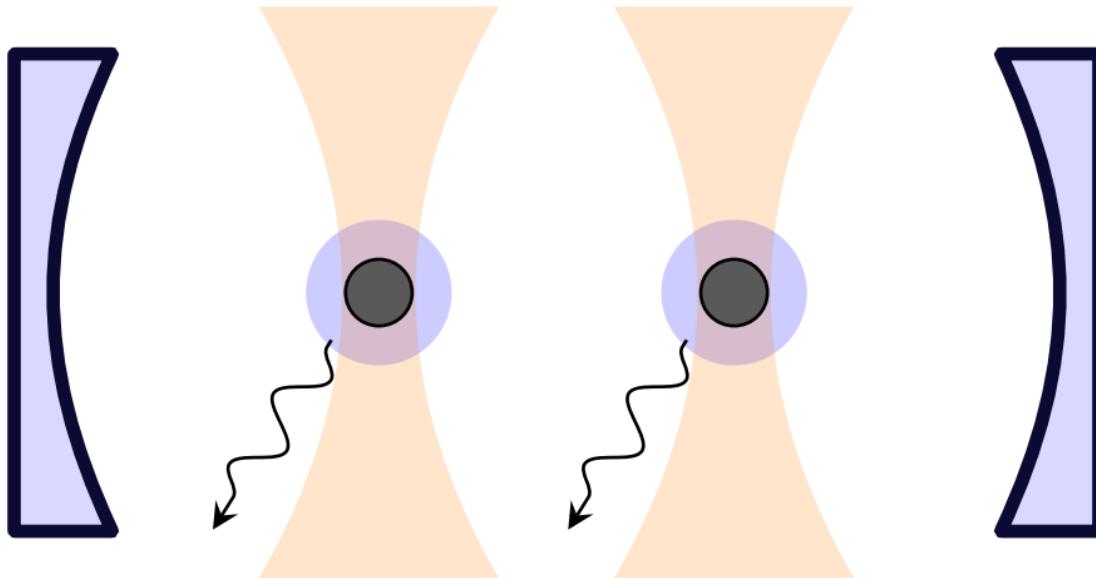


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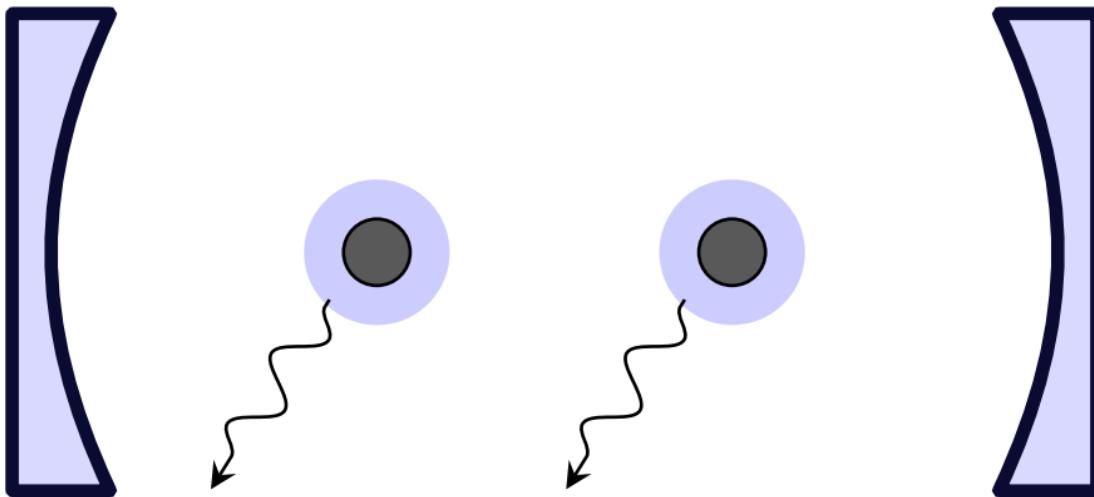
# The Protocol



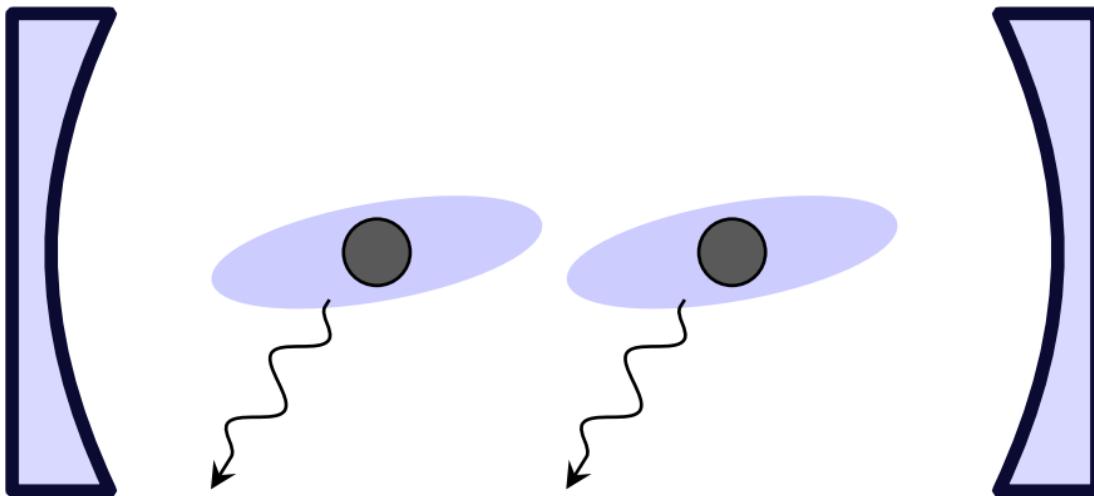
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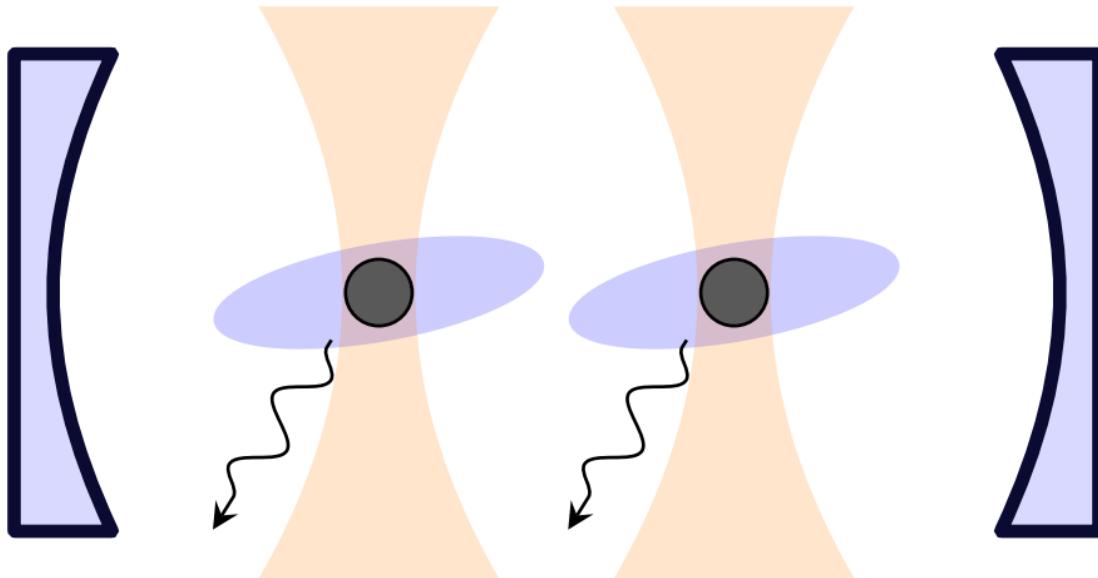
# The Protocol



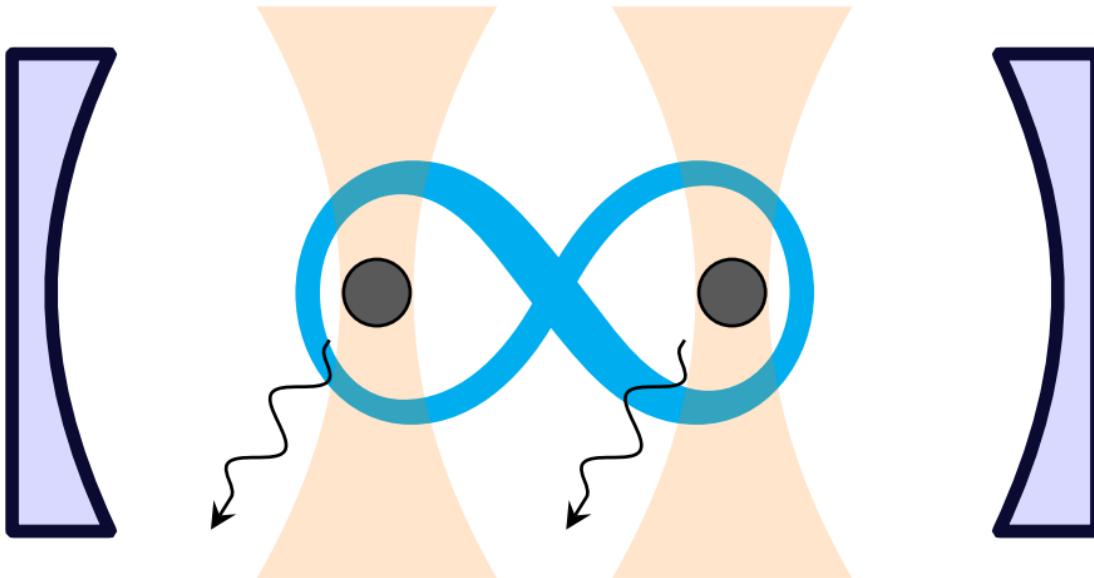
# The Protocol



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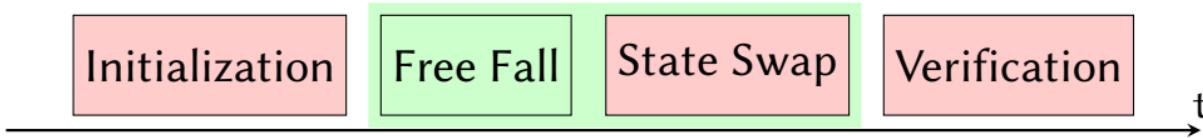
## The Protocol



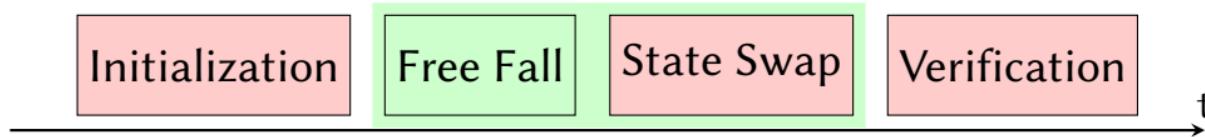
## The Protocol



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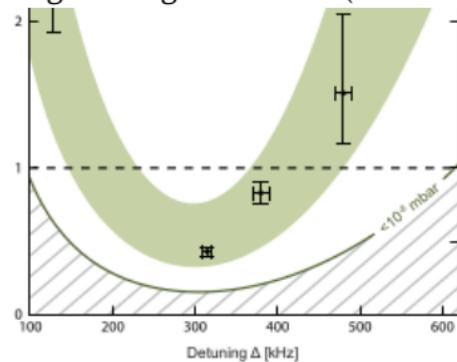


# The Protocol



## Initialization

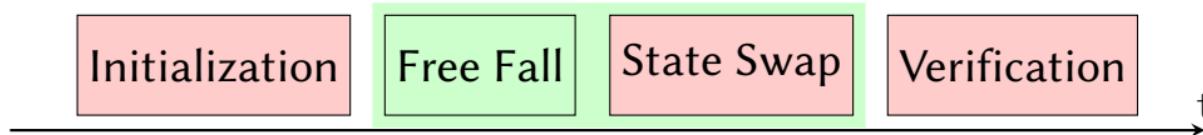
Cooling to the ground state ( $\bar{n} = 0.43$ )



U. Delić, Science 367, 892 (2020)

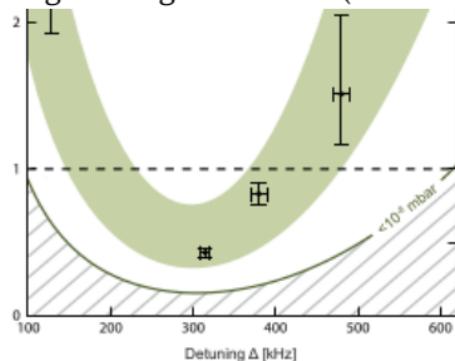
Avoid dark modes

# The Protocol



## Initialization

Cooling to the ground state ( $\bar{n} = 0.43$ )



## Free Fall

Squeezing

Bonvin (ETH) arXiv:2312.13111,  
Duchaň (ISI Brno) arXiv:2403.04302,  
Rossi (ETH) arXiv:2408.01264,  
Tomassi (ETH) arXiv:2503.20707

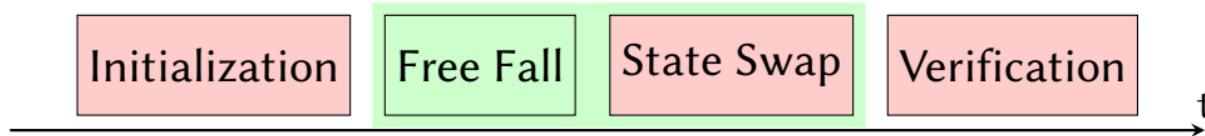
## State Swap

Via interacting with the same cavity mode

U. Delić, Science 367, 892 (2020)

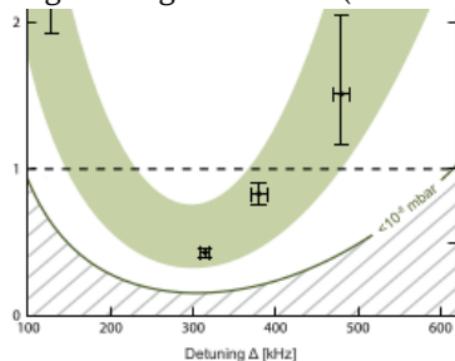
Avoid dark modes

# The Protocol



## Initialization

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Bonvin (ETH) arXiv:2312.13111,  
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## State Swap

Via interacting with the same cavity mode

## Readout

Via state swap to leaking light and homodyne

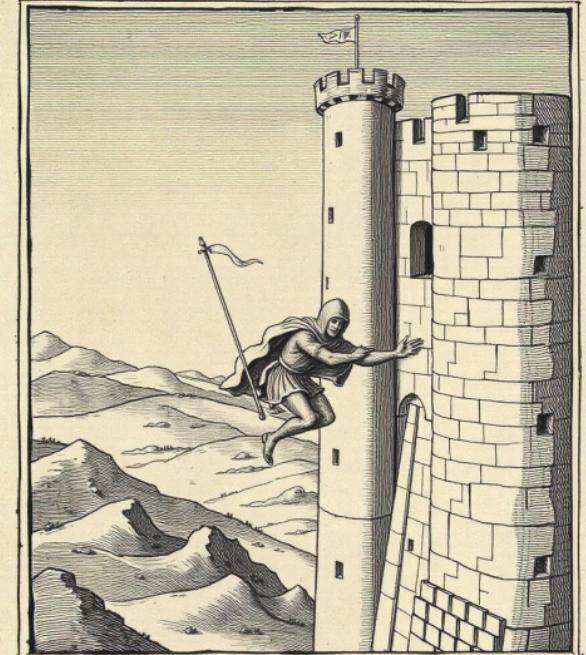
U. Delić, Science 367, 892 (2020)

Avoid dark modes

# Free Fall

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*Mirce inyler Ducerleas."*



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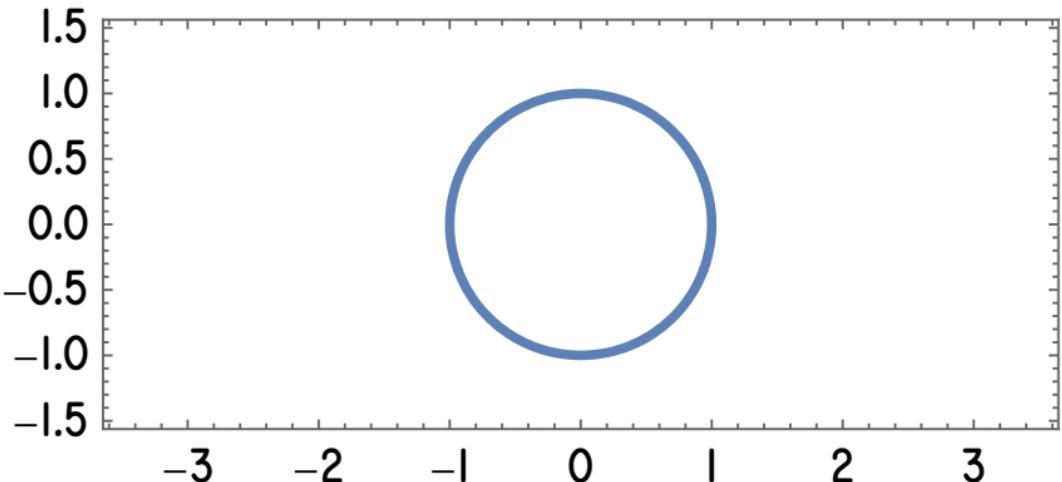
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## Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

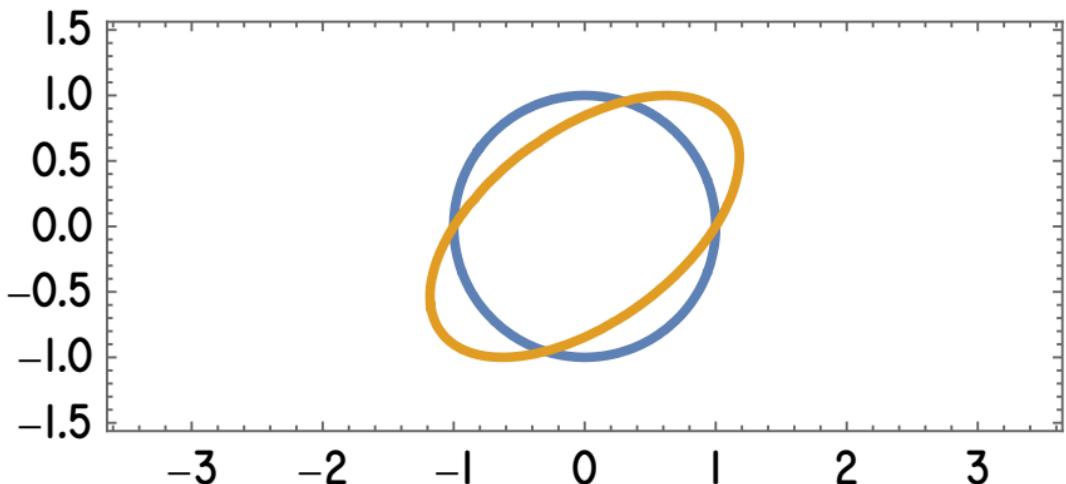
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



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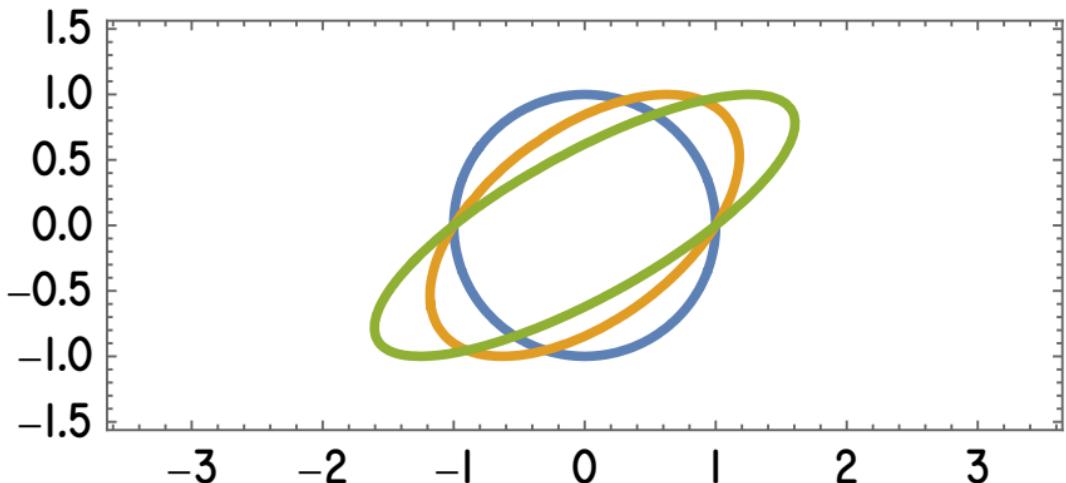
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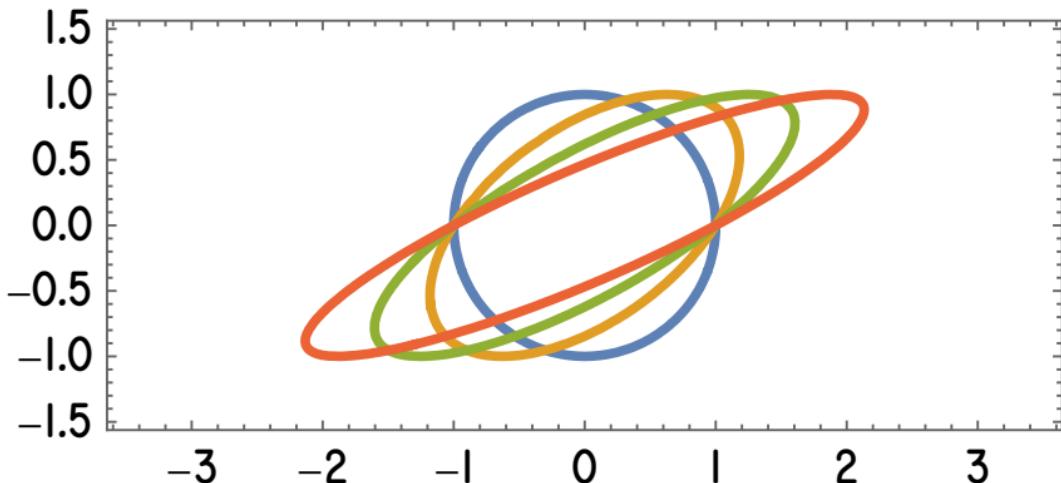
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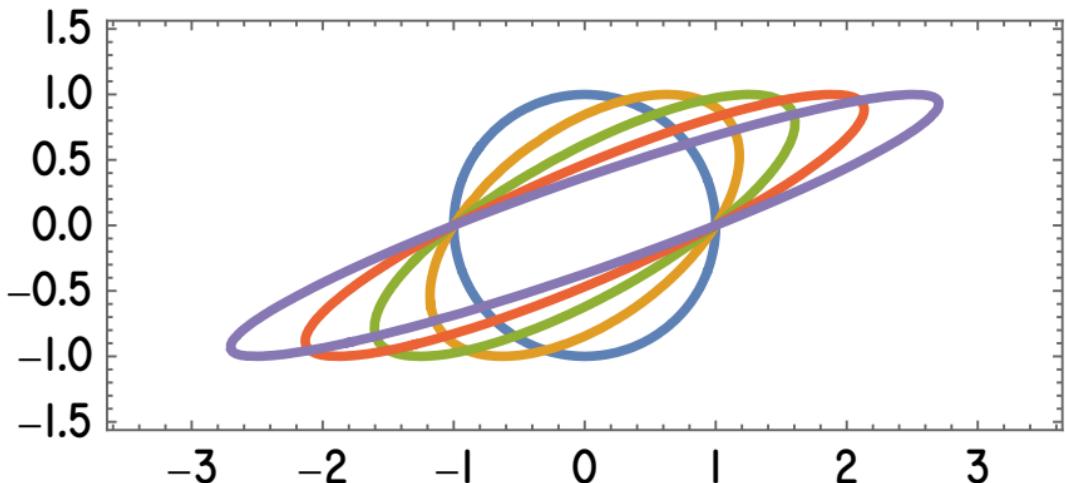
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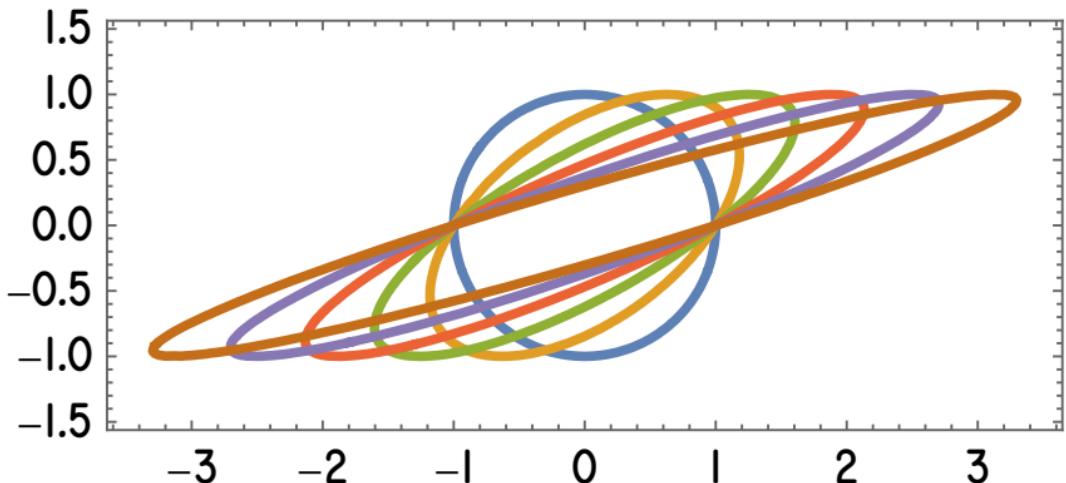
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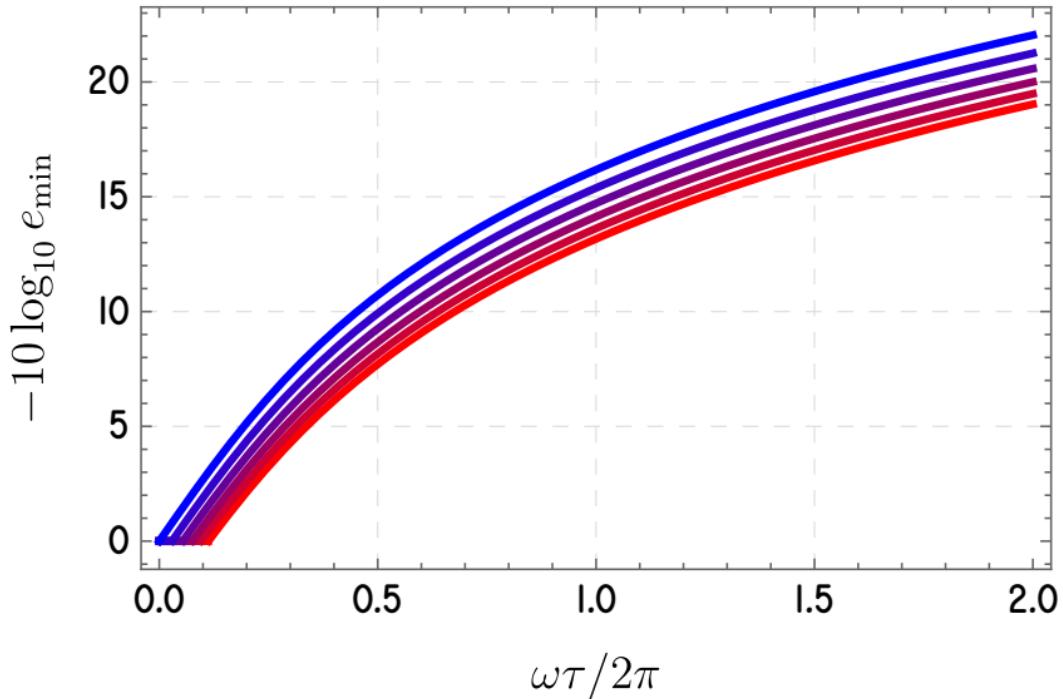
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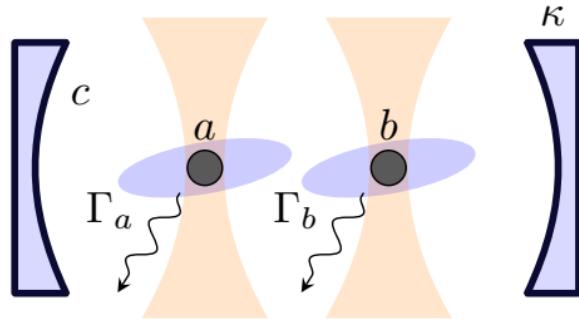


# State Swap



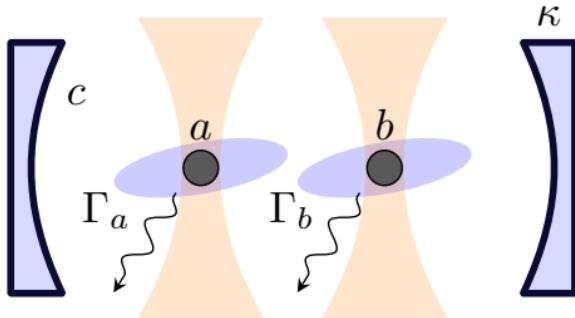
# State Swap

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



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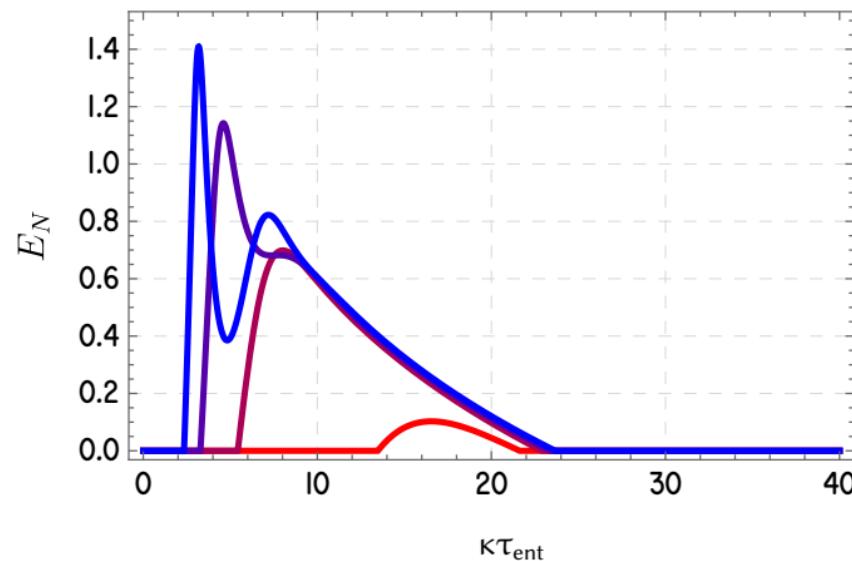
Most parameters from  
U. Delić, Science 367, 892 (2020)

There:

- ★ Recoil heating  $\Gamma/\kappa = 0.06$
- ★ Cooling to  $n_0 = 0.43$
- ★ Linearized coupling  $g/\kappa \leq 0.62$

Logarithmic negativity  $E_N$   
(quantifies violation of the positivity of  
partial transpose)

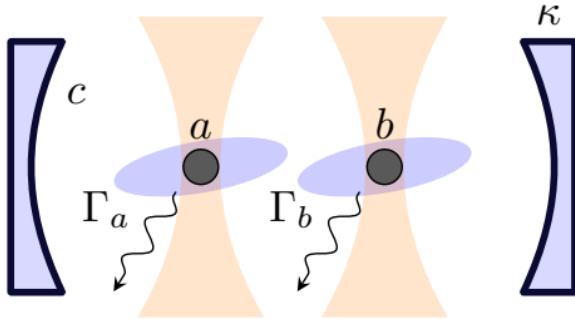
$$\Gamma = 0.02\kappa, n_0 = 0.1, \omega\tau = 4\pi$$



Color: coupling rate  $0.3 \leq \frac{g}{\kappa} \leq 0.6$ .

# State Swap

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



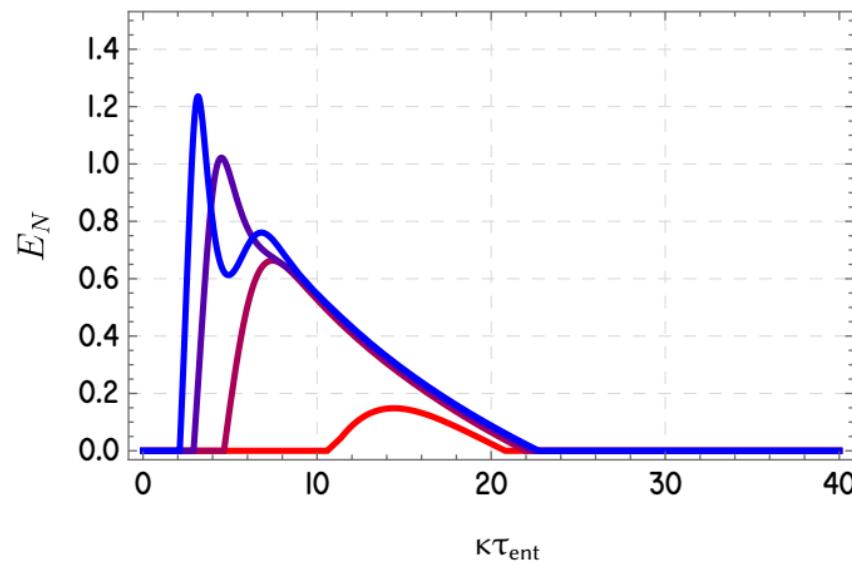
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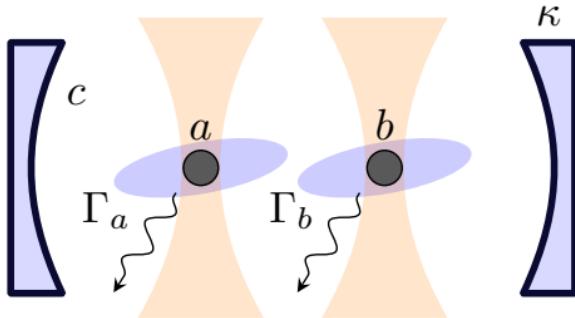
$$\Gamma = 0.02\kappa, n_0 = 0.43, \omega\tau = 2\pi$$



Color: coupling rate  $0.3 \leq \frac{g}{\kappa} \leq 0.6$ .

# State Swap

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$

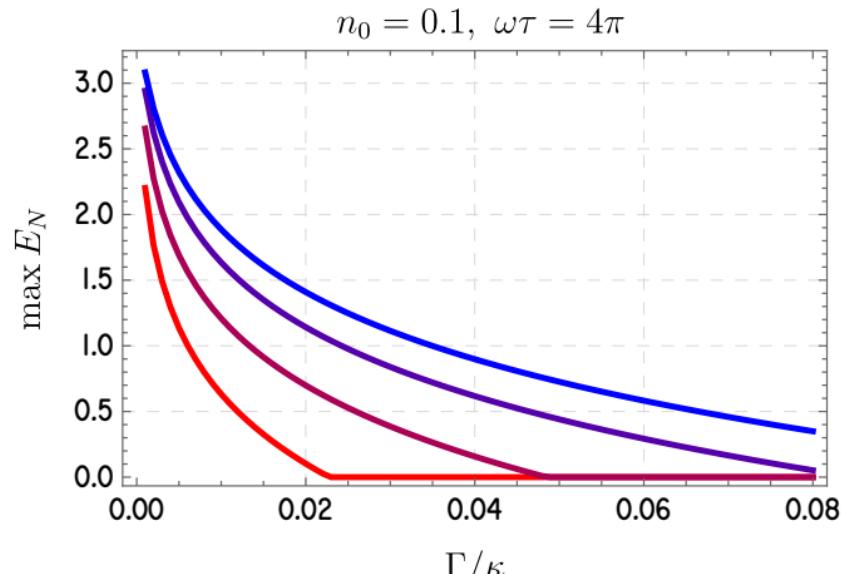


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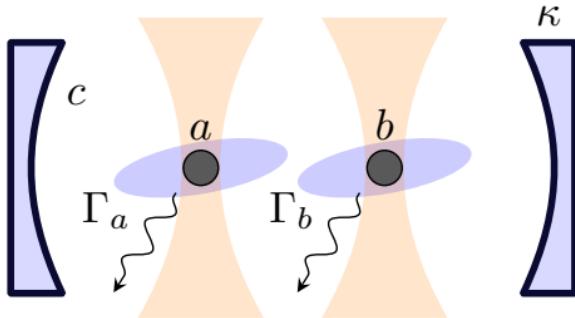
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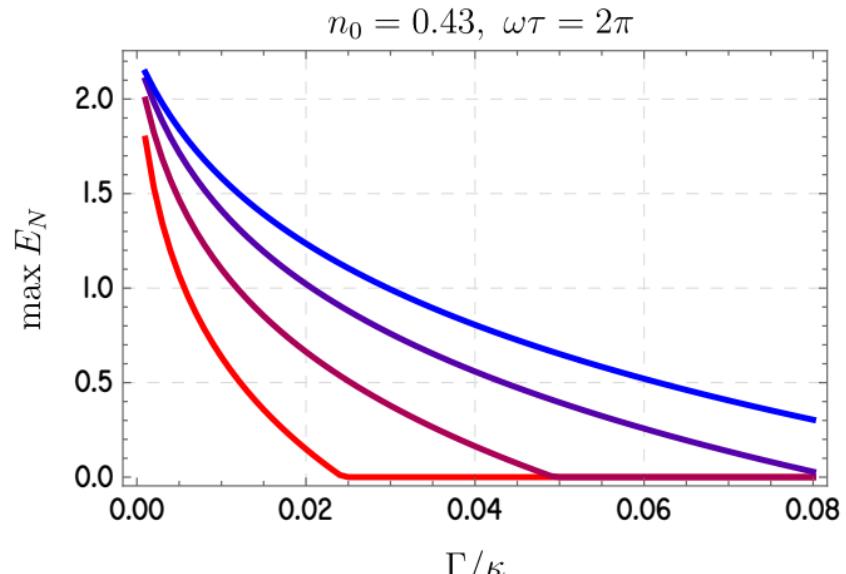


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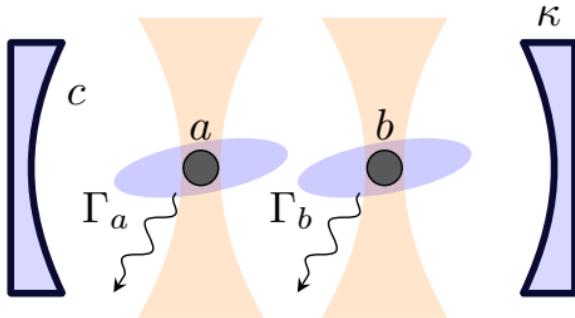
Color: coupling rate  $0.3 \leq \frac{g}{\kappa} \leq 0.6$ .

# The role of asymmetry



# Non-identical particles

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$

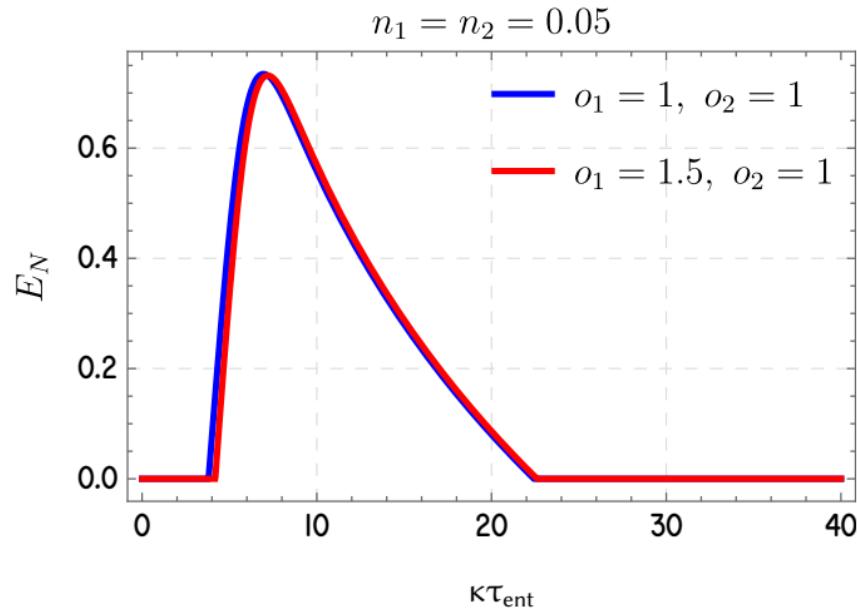


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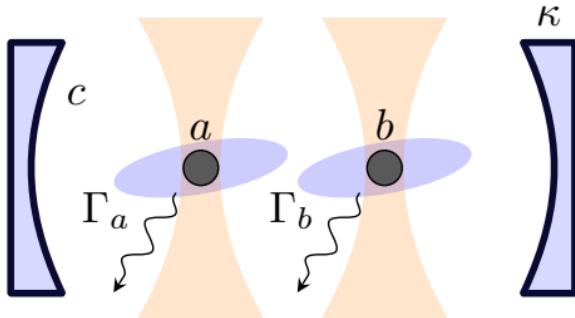
Logarithmic negativity  $E_N$



$$o_i \equiv \frac{\omega_i \tau}{2\pi}$$

# Non-identical particles

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$

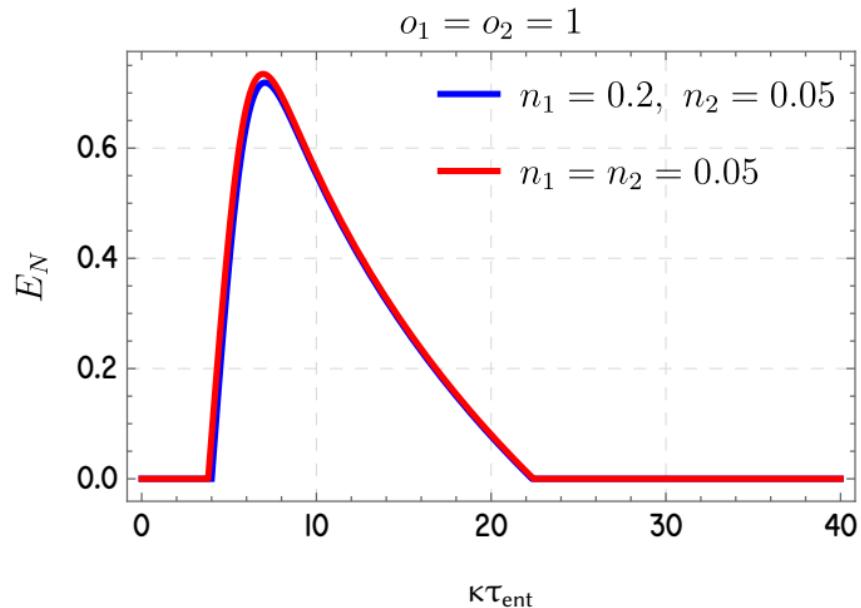


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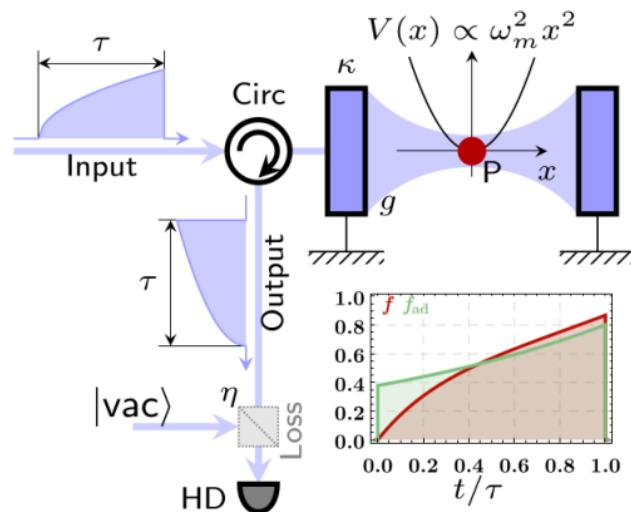


$$o_i \equiv \frac{\omega_i \tau}{2\pi}$$

# Readout



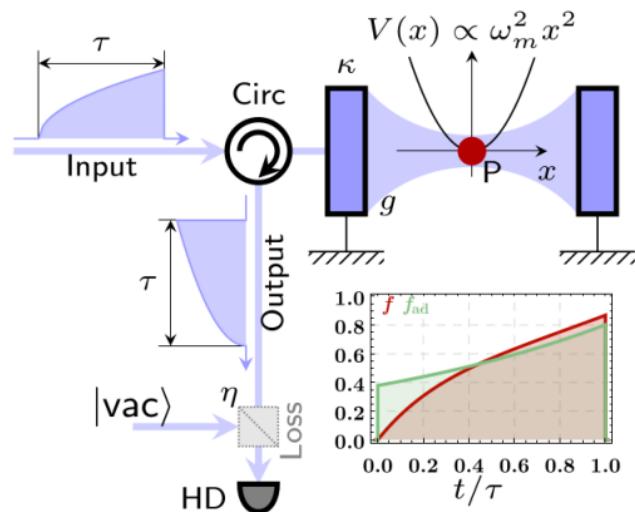
# State Examination



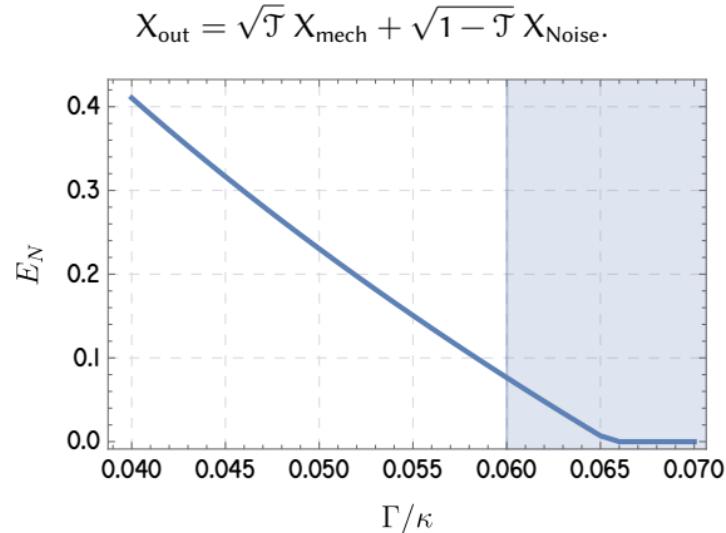
$$X_{\text{out}} = \sqrt{\mathcal{T}} X_{\text{mech}} + \sqrt{1-\mathcal{T}} X_{\text{Noise}}.$$

$$\mathcal{H} = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$

# State Examination



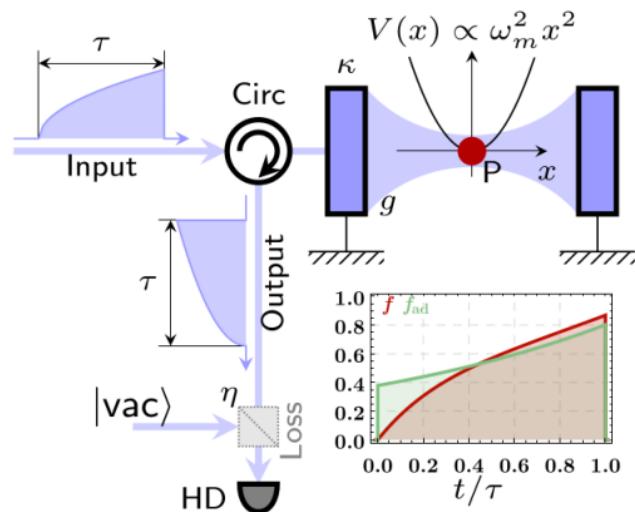
$$\mathcal{H} = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$



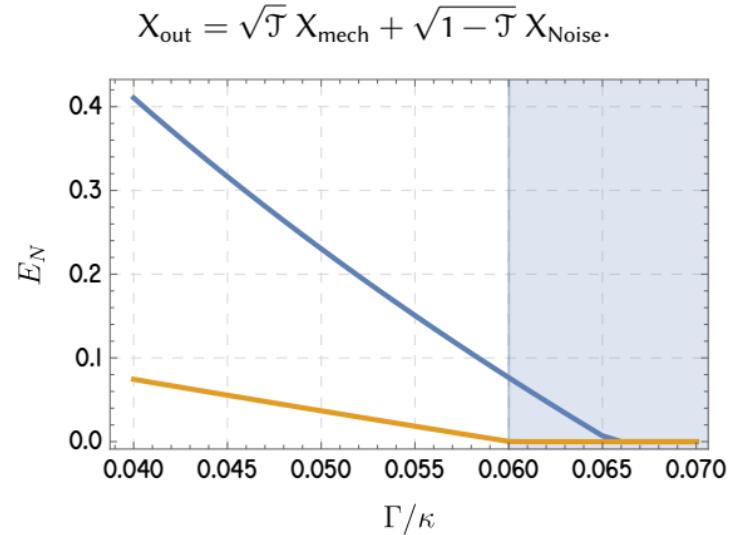
- ★ Coupling rate  $g = 0.6\kappa$
- ★ Initial occupation  $n_0 = 0.43$

U. Delić, Science 367, 892 (2020)

# State Examination



$$\mathcal{H} = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$



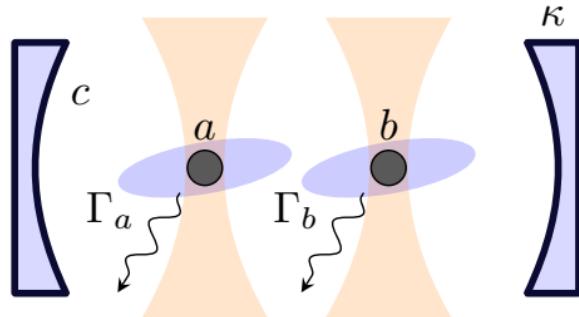
— Lossless Readout — 70 % loss

- ★ Coupling rate  $g = 0.6\kappa$
- ★ Initial occupation  $n_0 = 0.43$

U. Delić, Science 367, 892 (2020)

## Conclusions

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Requirements for Entanglement

- ★ Beam-splitter interaction
- ★ Free-motion

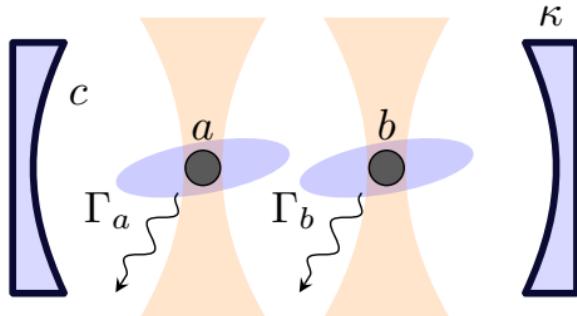
Requirements for Verification

- ★ Beam-splitter
- ★ Individual address

All operations are passive

# Conclusions

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Requirements for Entanglement

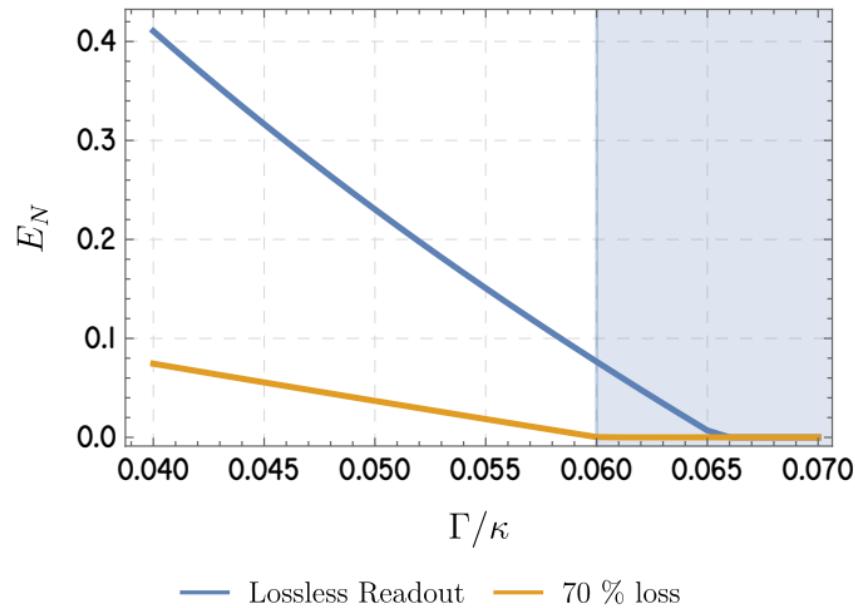
- ★ Beam-splitter interaction
- ★ Free-motion

Requirements for Verification

- ★ Beam-splitter
- ★ Individual address

All operations are passive

- ★ Entanglement is possible
- ★ Can persist recoil heating
- ★ Survives asymmetries
- ★ Does not require initial pure states (but doesn't mind)



# Thank You!



These slides  
<https://bit.ly/andrey-1phys-2025>

Phd and Postdoc positions available





# Beware of the appendix slide!

## Effective classical simulation

Consider the setup:

- ★  $n$  quantum subsystems
- ★  $t$  operations
- ★ local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \dots, k_n\},$$

one outcome per system.

Probability of the outcomes

$$P(k_1, \dots, k_n) \quad (1)$$

A classical algorithm is said to capable of effective sampling, if it

- ★ is efficient (polynomial) in  $t$  and  $n$
- ★ provides outcomes  $\mathbf{k}$  draws from the same probability as (1)

The very last frame which is empty