Quantum Non-Gaussian Optomechanics

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Prague


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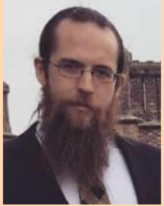
The Optomechanics Group in Olomouc [within R. Filip's group]

Radim Filip

Foroud Bemani

Darren Moore



Alisa Manukhova

Najmeh Etehadi-Abari

(Now @KIT, Germany)

Surabhi Yadav

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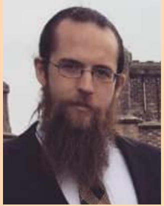

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?

PhD and Postdoc positions available

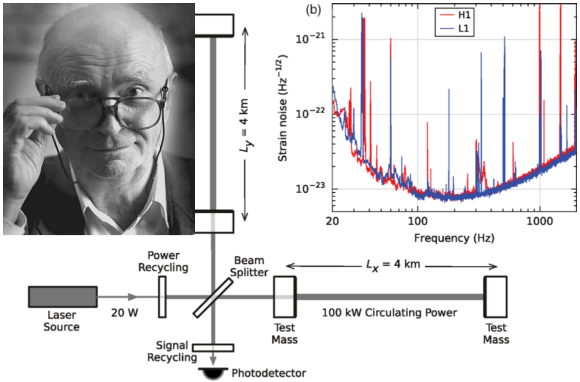
Introduction

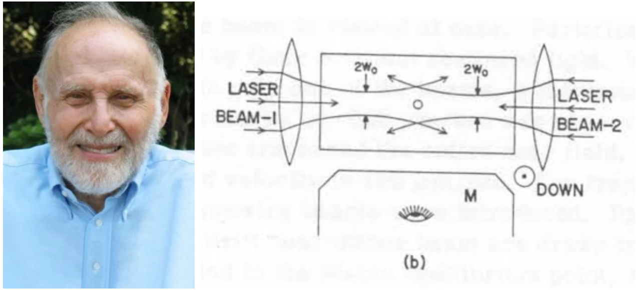
- Quantum Optomechanics
- Quantum non-Gaussianity
- Verification of quantum non-Gaussianity

Motional Nonlinearities

Single-Phonon Addition/Subtraction

Quantum Optomechanics





Braginsky & Manukin, Soviet JETP **25**, 653 (1967)

Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)

A. Ashkin, PRL **24**, 156 (1970)

$$H = \omega_{mech} a_m^\dagger a_m + \omega_{cav} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

Gaussian vs Quantum non-Gaussian states

Gaussian states

Squeezed displaced state

$$\hat{\rho}_x = \hat{S}(r)\hat{D}(\alpha)\left|0\right\rangle.$$

Convex mixtures thereof

$$\hat{\rho}_G = \int p(x)\hat{\rho}_x dx.$$

$p(x)$ is a probability density:

- $p(x) > 0$
- “not more singular” than Dirac δ.

Gaussian vs Quantum non-Gaussian states

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Quantum non-Gaussian states

Cannot be represented as convex mixtures of squeezed displaced states

$$\hat{\rho}_{NG} \neq \int p(x)\rho_x dx .$$

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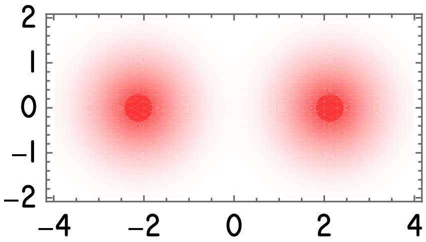
Examples

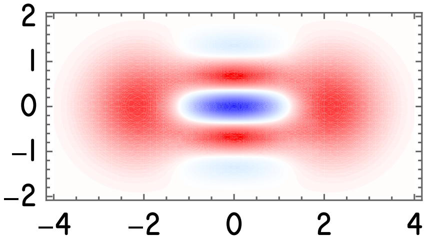
Classically NG state

$$\hat{\rho} \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$



Quantum NG state

$$|\psi\rangle \propto |\alpha\rangle + |-\alpha\rangle$$



Gaussian vs Quantum non-Gaussian states

Advantages of QNG states

- Universal quantum computing
- Quantum sensing
- Fundamental studies

QNG is a resource

F. Albarelli *et al.*, Phys. Rev. A **98**, 052350 (2018)

M. Walschaers, PRX Quantum **2**, 030204 (2021)

Quantum non-Gaussian states

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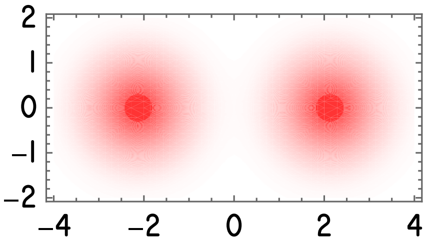

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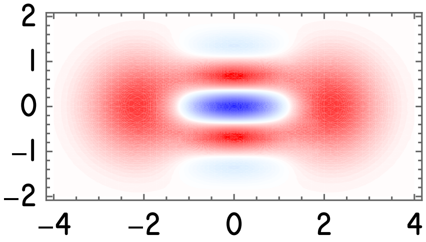
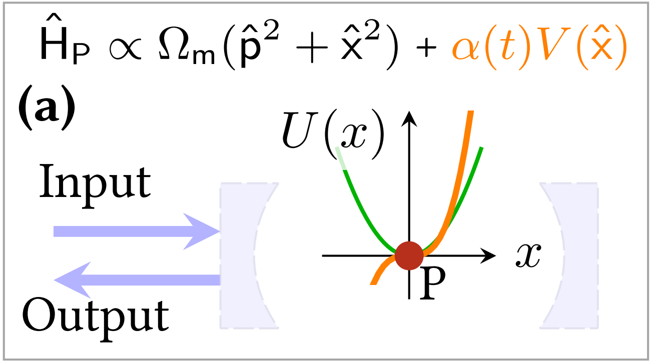


Routes to quantum non-Gaussianity in optomechanics

Add a nonlinear element

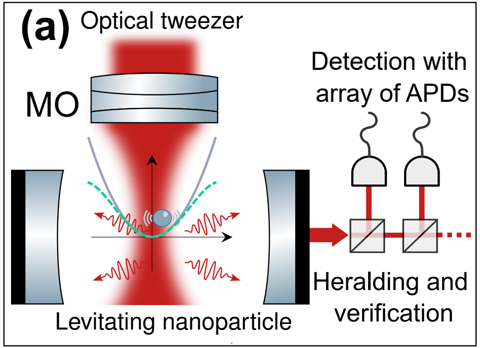


Nonlinear potential of mechanical motion

AR, R. Filip, Npj Quantum Inf **7**, 120 (2021)

D.W. Moore, **AR**, R. Filip, NJP **21**, 113050 (2019)

Use non-linear detection



Counting photons

AR, R. Filip, Quantum Sci. Technol. **10**, 015014 (2024)

F. Bemani, **AR**, R. Filip, Submitted , (2024)

We don't consider here upload of QNG states

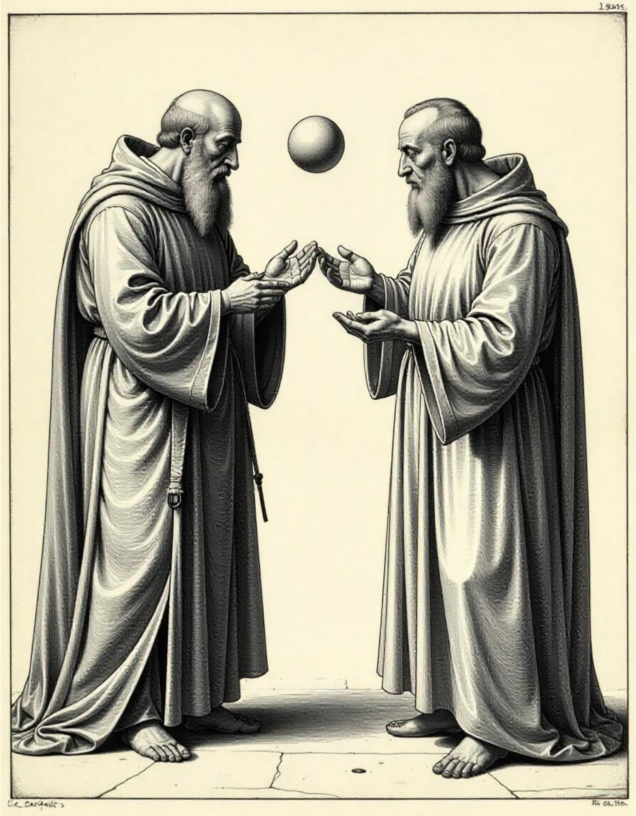
AR, R. Filip, Sci. Rep. **7**, 46764 (2017)

Verification of quantum non-Gaussianity (QNG)

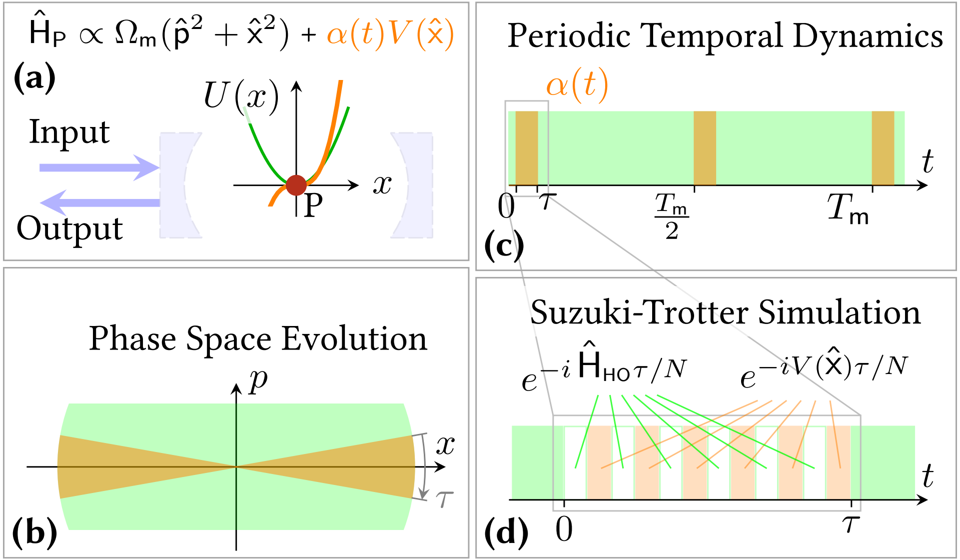
Linear Gaussian Dynamics

Universal Quantum Control

We need better figures of merit than fidelity

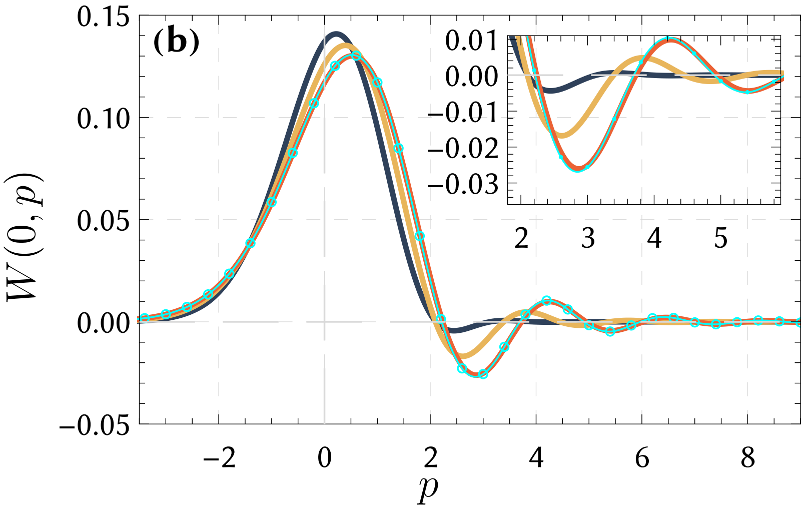


Nonlinear potential for a levitated nanopatricle



AR, R. Filip, Npj Quantum Inf **7**, 120 (2021)

Nonlinear potential for a levitated nanopatricle

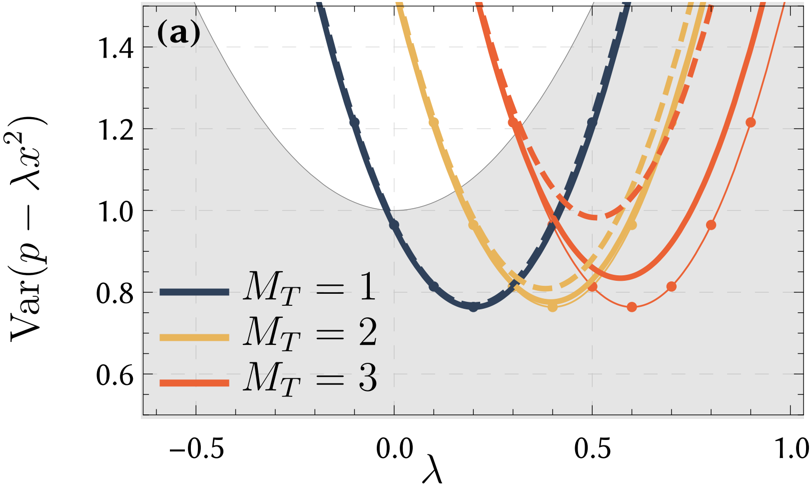


Wigner functions

Poor-man's fidelity (red & cyan)

$$4\pi \int \mathrm{d}x\mathrm{d}y W_{\mathrm{red}}(x,y) W_{\mathrm{cyan}}(x,y) = 0.9877.$$

Nonlinear potential for a levitated nanopatricle



Nonlinear variance

$$v_3 \equiv \mathrm{Var}(\hat{p} - \lambda \hat{x}^2)$$

Compare with conventional squeezing:

$$\begin{aligned} v_2 &\equiv \mathrm{Var}(\hat{x}\cos\theta + \hat{p}\sin\theta) \\ &= \sin^2\theta \cdot \mathrm{Var}(\hat{p} + \lambda\hat{x}), \end{aligned}$$

with $\lambda = \cot\theta$.

Related works about levitated NPs in nonlinear potentials (currently all theory)

Palacký University, cubic potential, multiple periods

AR, R. Filip, Npj Quantum Inf **7**, 120 (2021) (arxiv 2019)

University of Vienna: “Super Mario”, cubic potential, short pulse

L. Neumeier *et al.*, PNAS **121**, e2306953121 (2024) (arxiv 2022)

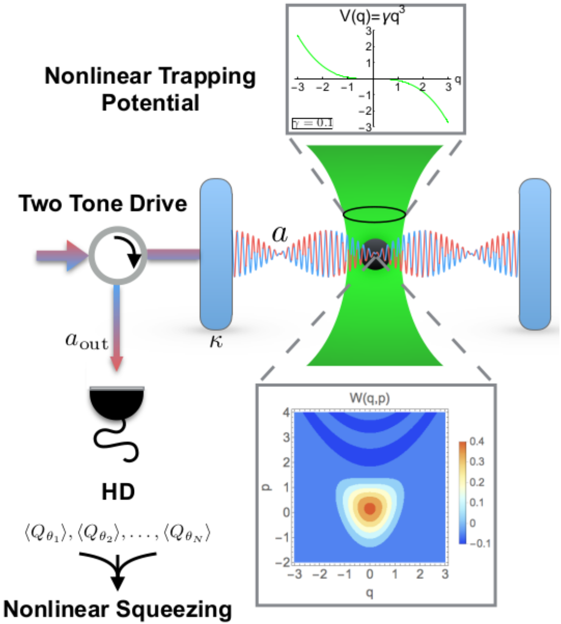
University of Innsbruck, more complex potentials

M. Roda-Llordes *et al.*, Phys. Rev. Res. **6**, 013262 (2024),

M. Roda-Llordes *et al.*, Phys. Rev. Lett. **132**, 023601 (2024),

A. Riera-Campeny *et al.*, Quantum **8**, 1393 (2024)

Detecting Nonlinear squeezing



Approximate cubic phase state in mechanics

$$e^{i\gamma q^3}\hat{S}\,|0\rangle\,.$$

Pulsed QND interaction

$$H_{\rm int} \propto x_{\rm light}(q\cos\phi + p\sin\phi).$$

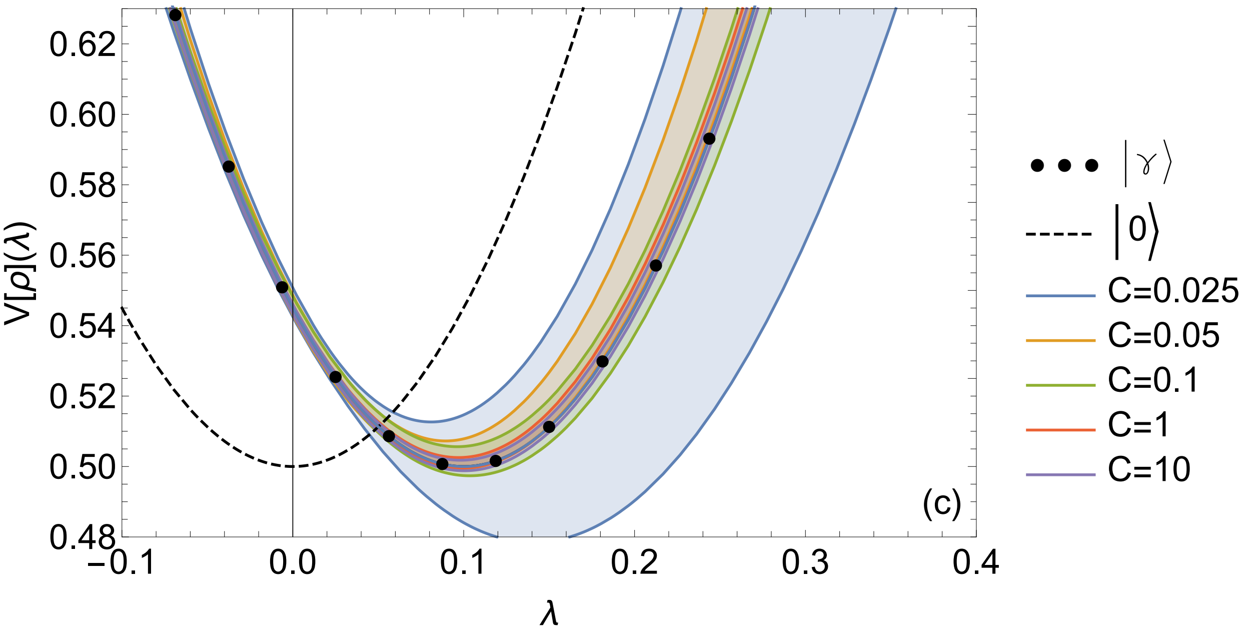
Detect leaking light

Estimate nonlinear variance

$$q_{\rm NL} = p - \lambda x^2 \Rightarrow \mathrm{Var}[q_{\rm NL}]$$

D.W. Moore, **AR**, R. Filip, NJP **21**, 113050 (2019)

Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Introduction

Quantum Optomechanics

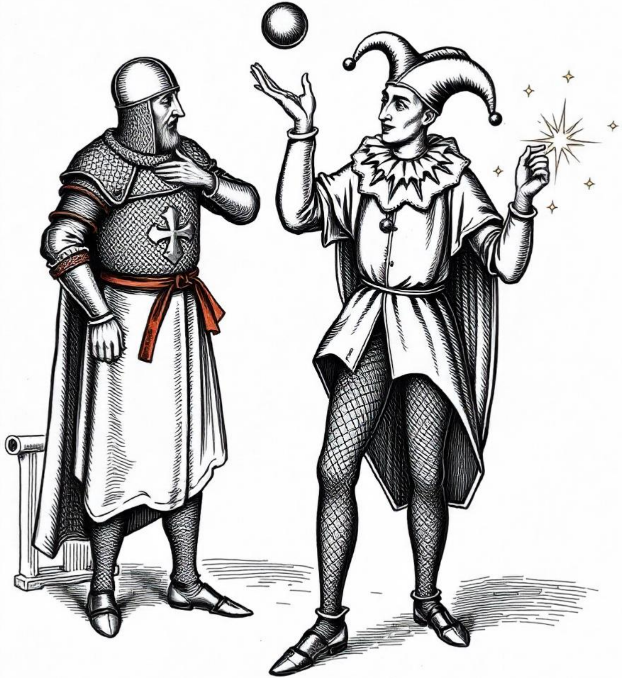
Quantum non-Gaussianity

Verification of quantum non-Gaussianity

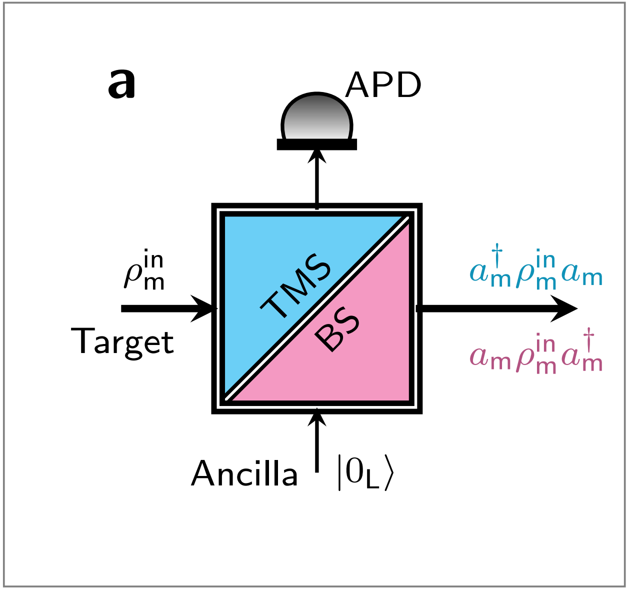
Motional Nonlinearities

Single-Phonon Addition/Subtraction

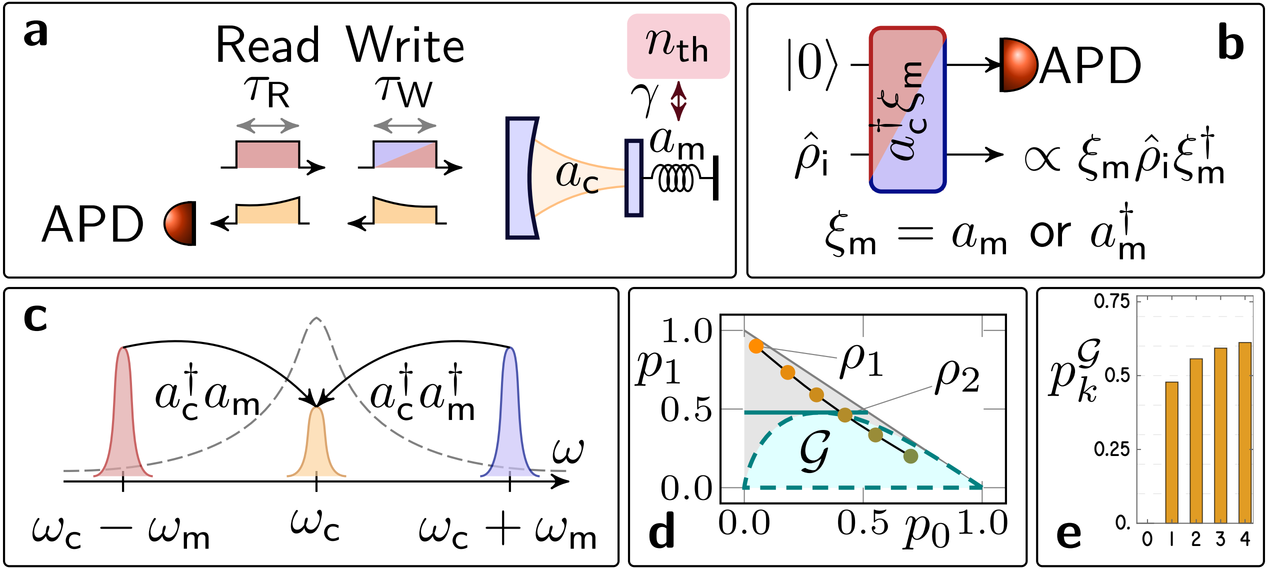
Poster by
Foroud Bemani



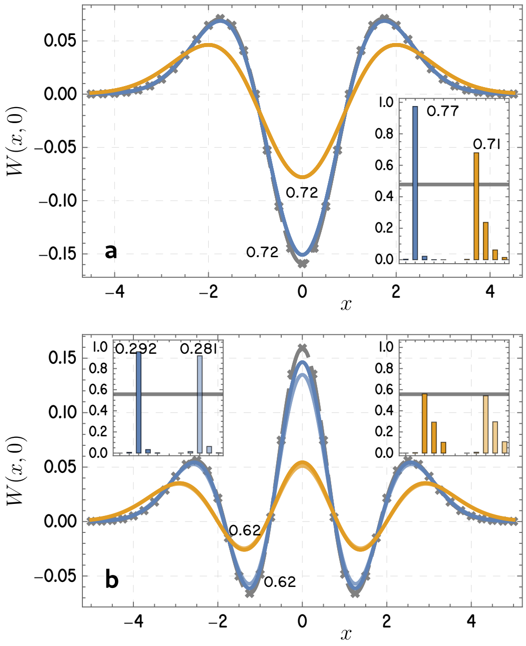
Single-phonon addition or subtraction in optomechanics



Single-phonon addition or subtraction in optomechanics



Evaluation of multiphonon quantum non-Gaussianity (superfluid He)



Multiphonon probabilities

$$p_k = \langle k|\rho|k\rangle$$

Criteria of absolute n-phonon quantum non-Gaussianity

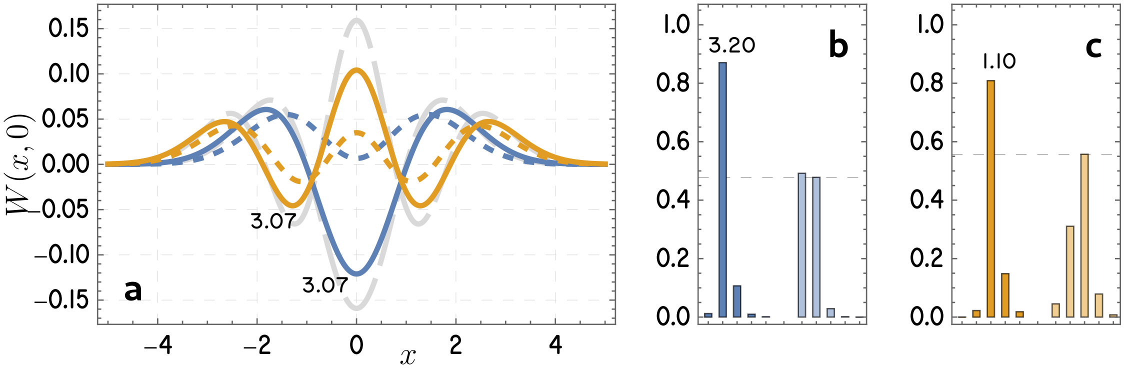
$$p_k^{\mathcal{G}} = \max_{\alpha, r, \{c_i\}} \left| \left\langle k \middle| \hat{D}(\alpha)\hat{S}(r) \sum_{i=0}^{k-1} c_i |i\rangle \right\rangle \right|^2 .$$

$$p_2^{\mathcal{G}} = \max_{\alpha, r, c_0, c_1} \left| \left\langle 2 \middle| \hat{D}(\alpha)\hat{S}(r) \Big(c_0 |0\rangle + c_1 |1\rangle \Big) \right\rangle \right|^2 .$$

k	1	2	3
$p_k^{\mathcal{G}}$	0.478	0.557	0.593

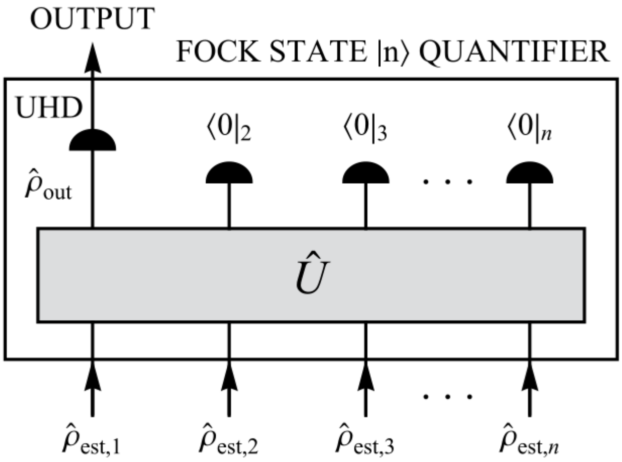
AR, R. Filip, Quantum Sci. Technol. **10**, 015014 (2024)

Readout and verification



Inset numbers show QNG depth: loss (in dB) to lose QNG.

Bunching capability

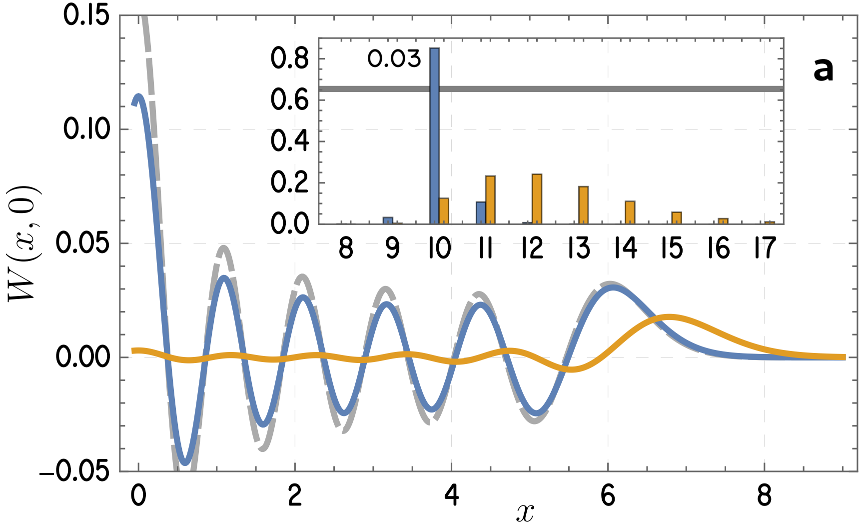


The recipe

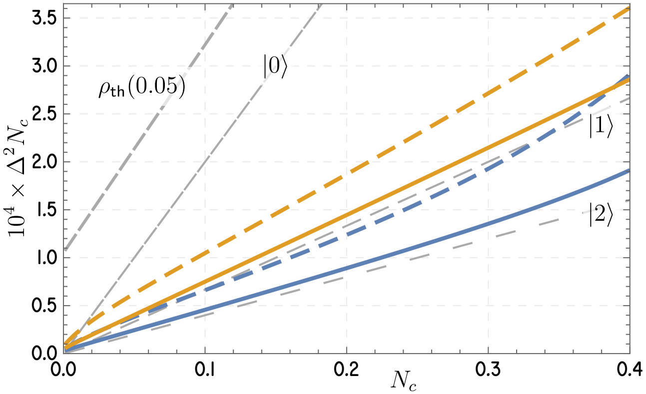
- measure the statistics $\langle k|\hat{\rho}_{\mathrm{est}}|k\rangle$
- compute hypothetical bunching state

Original proposal P. Zapletal and R. Filip, Sci Rep **7**, 1 (2017)
Implementations with OPA: P. Zapletal *et al.*, OPTICA **8**, 743 (2021).

Bunching capability



Application: detection of phase-randomized displacement



Phase-randomized displacement

$$\rho_{\mathrm{in}} \mapsto \int_0^{2\pi} \frac{d\phi}{2\pi} \hat{D}(\sqrt{N_c}e^{i\phi})\rho_{\mathrm{in}}\hat{D}^\dagger(\sqrt{N_c}e^{i\phi})$$

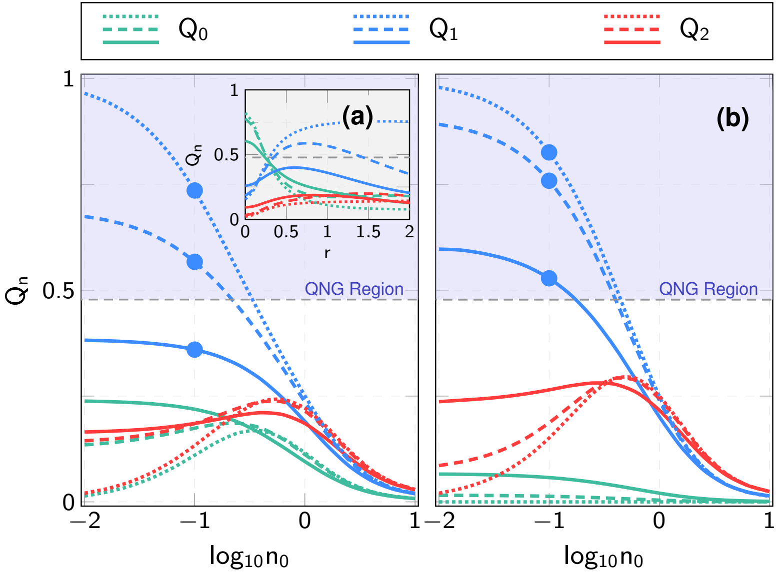
Cramér-Rao bound

$$\Delta^2 N_c \geqslant \frac{1}{M \cdot F(N_c)},$$

M — number of copies, F — quantum Fisher information

In levitated optomechanics

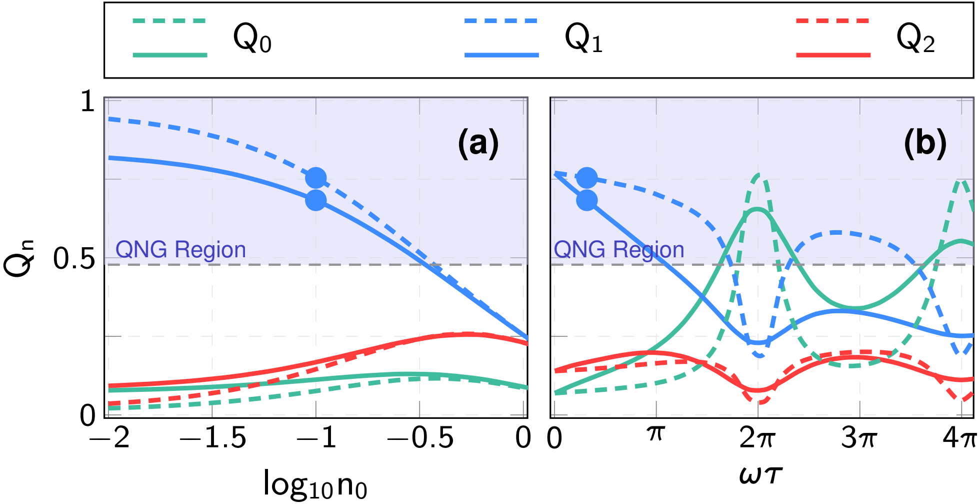
Inside a cavity



Parameters U. Delić, Science **367**, 892 (2020)

In levitated optomechanics

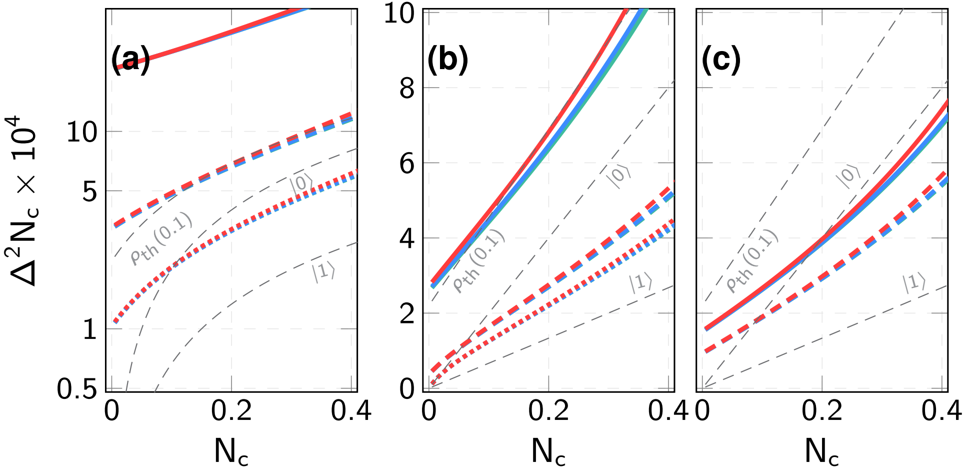
In free space



Parameters: L. Magrini, Phys. Rev. Lett **129**, 053601 (2022)

In levitated optomechanics

Phase-randomized displacement sensitivity



Conclusions

- Quantum non-Gaussianity is possible in optomechanics
- both continuous-variable [CV] (nonlinear potentials) and discrete-variable [DV] (photon counting) regimes
- CV non-Gaussianity can be effectively verified via non-linear variances $\mathrm{Var}(\hat{p} - \lambda\hat{x}^2)$
- DV non-Gaussianity is verified via multiphonon probabilities and bunching capability
- single-phonon-added states are helpful for phase-randomized force detection

Thank You!

These slides
https://bit.ly/andrey-most

Beware of the appendix slide!

Effective classical simulation

Consider the setup:

- n quantum subsystems
- t operations
- local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \dots, k_n\},$$

one outcome per system.
Probability of the outcomes

$$P(k_1, \dots, k_n) \tag{1}$$

A classical algorithm is said to capable of effective sampling, if it

- is efficient (polynomial) in t and n
- provides outcomes $\mathbf{k}$ draws from the same probability as (1)

The very last frame which is empty