

Entangling Levitated Nanoparticles by Wave-Packet Dispersion

Andrey A. Rakhubovsky, and Radim Filip

Department of Optics, Palacký University, 17. Listopadu 12, 771 46 Olomouc, Czech Republic
rakhubovsky@optics.upol.cz



Palacký University
Olomouc

INTRODUCTION

A crucial difference of the nanoparticles from the traditional clamped mechanical resonators is the possibility to adjust the potential of the mechanical motion as the potential itself is defined by highly controllable tweezer field. In particular, it is possible to alternate harmonic-oscillator motion with the free motion, using the latter for nearly unitary expansion of the nanoparticles' wave packet which results in quantum squeezing of the mechanical motion. Here, we show that combining the free-fall-induced squeezing with the coupling of multiple nanoparticles can lead to the quantum entanglement between the nanoparticles. We evaluate the attainable entanglement in the presence of the relevant sources of decoherence and prove the feasibility of generating entanglement by passive operations.

METHODS

We use conventions $[x, p] = 2i$, consequently the ground-state variance $\langle 0|x^2|0 \rangle = 1$. Effective dynamics is described by quantum Langevin equations:

$$\frac{d\mathbf{r}}{dt} = \mathbb{A}\mathbf{r} + \mathbf{v}.$$

Solution

$$\mathbf{r}(\tau) = \mathbb{M}(\tau)\mathbf{r}(0) + \int_0^\tau ds \mathbb{M}(\tau-s)\mathbf{v}(s),$$

with $\mathbb{M}(t) = e^{\mathbb{A}t}$.

The quantum state of the nanoparticles + intracavity light is zero-mean Gaussian, with covariance matrix

$$\mathbb{V}_{\text{om}}(\tau) = \mathbb{M}(\tau)\mathbb{V}_{\text{om}}(0)\mathbb{M}^T(\tau) + \int_0^\tau ds \mathbb{M}(\tau-s)\mathbb{D}\mathbb{M}^T(\tau-s),$$

with \mathbb{D} containing covariances of noises (assuming Markovian). Keeping only the relevant entries (removing the cavity mode), gives the CM of the two NPs:

$$\mathbb{V} = \begin{pmatrix} \mathbb{V}_a & \mathbb{V}_{ab} \\ \mathbb{V}_{ab}^T & \mathbb{V}_b \end{pmatrix}.$$

Entanglement is measured by *logarithmic negativity* E_N :

$$E_N = \max(0, -\log_2 \nu_-(\mathbb{V})),$$

where ν_- is the smallest symplectic eigenvalue of \mathbb{V} :

$$2\nu_-^2 = \Sigma(\mathbb{V}) - \sqrt{\Sigma^2(\mathbb{V}) - 4\det \mathbb{V}},$$

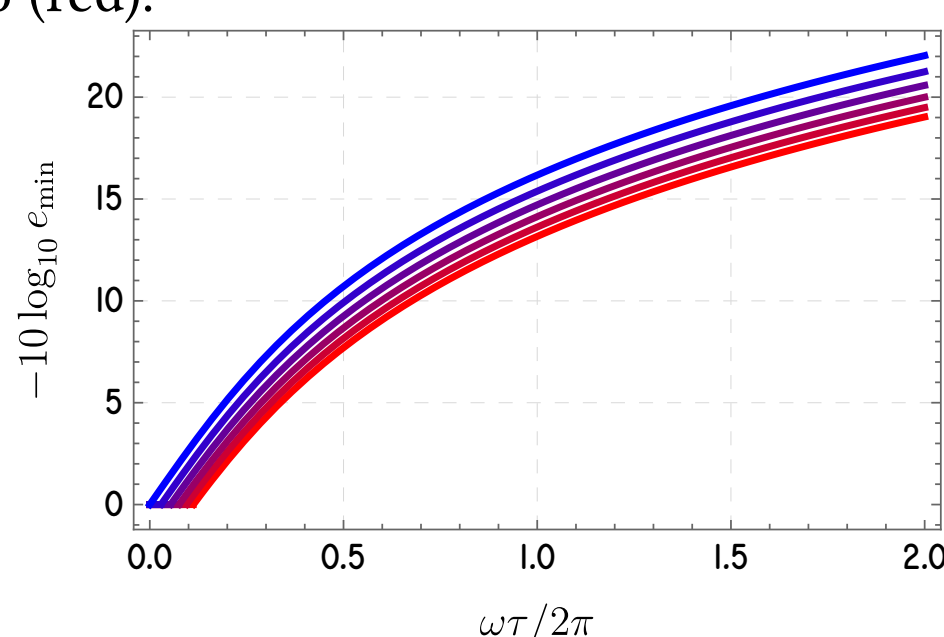
$$\Sigma(\mathbb{V}) = \det(\mathbb{V}_a) + \det(\mathbb{V}_b) - 2\det(\mathbb{V}_{ab}).$$

Squeezing by wave-packet dispersion (free fall)

Squeezing is evaluated from the minimal value of the covariance matrix e_{\min} :

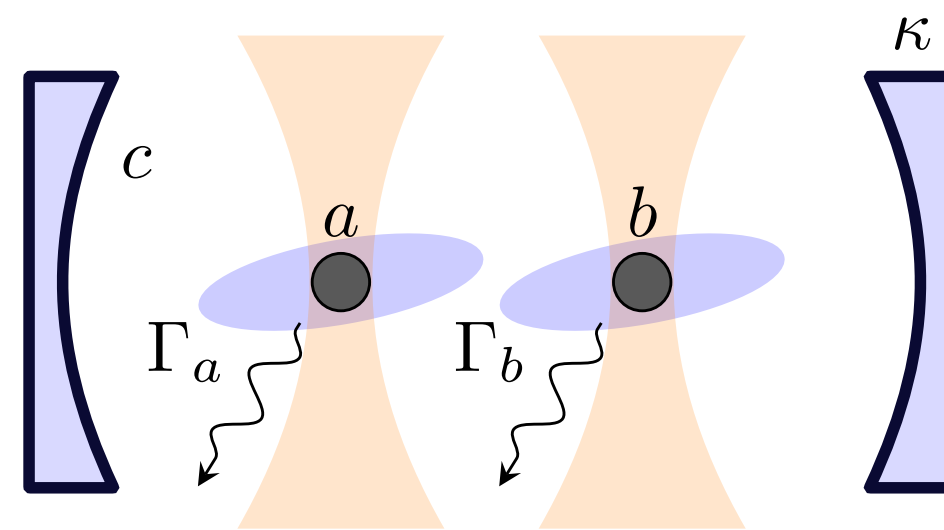
$$S[\text{dB}] = \max(0, -10 \log_{10} e_{\min}).$$

Squeezing from the unitary evolution (Hamiltonian $H_{\text{ff}} = \omega p^2/4$ for initial occupations between $n_0 = 0$ (blue) and $n_0 = 0.5$ (red).



MODEL

$$H = c^\dagger (g_a a + g_b b e^{i\phi}) + \text{h.c.}$$

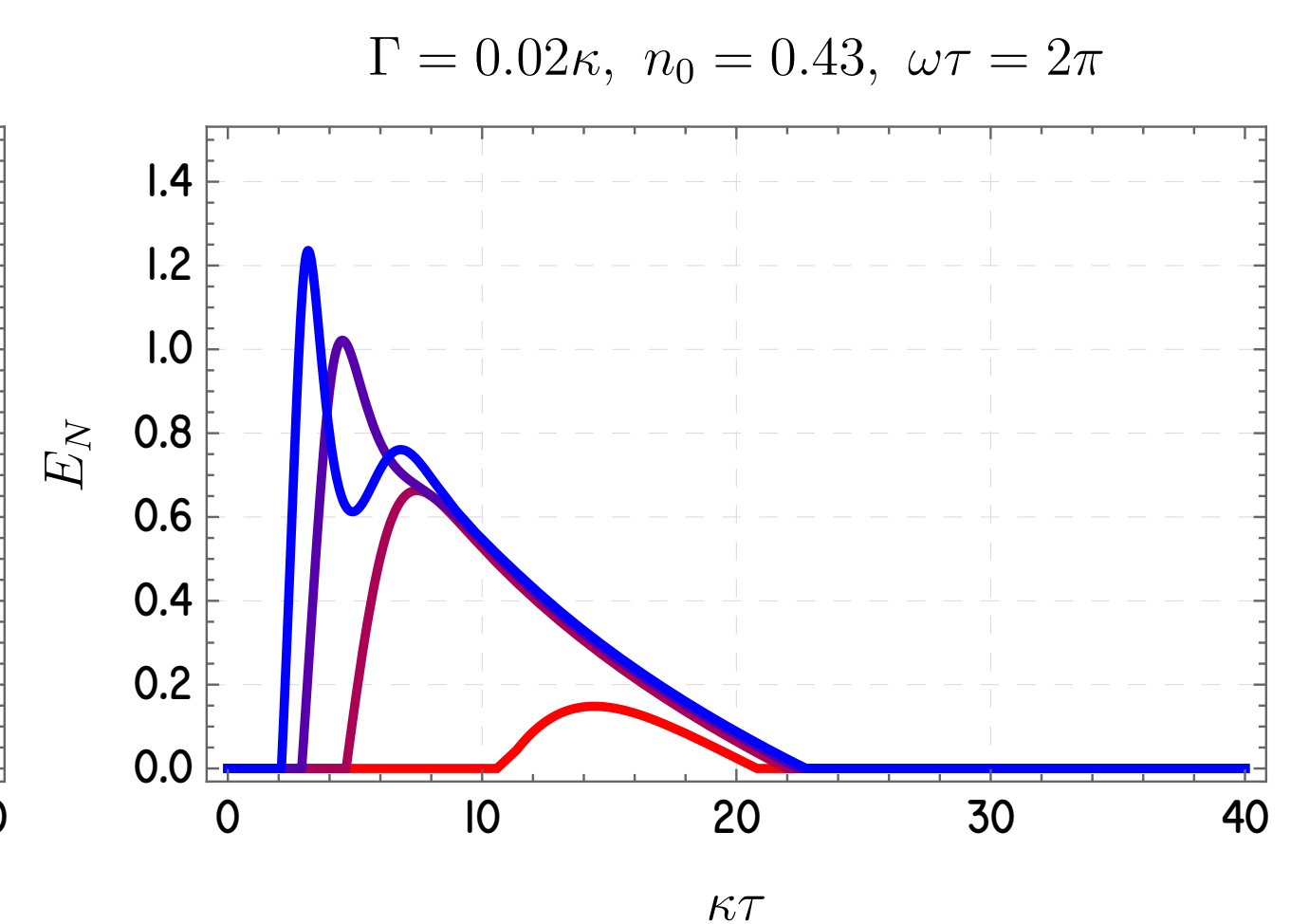
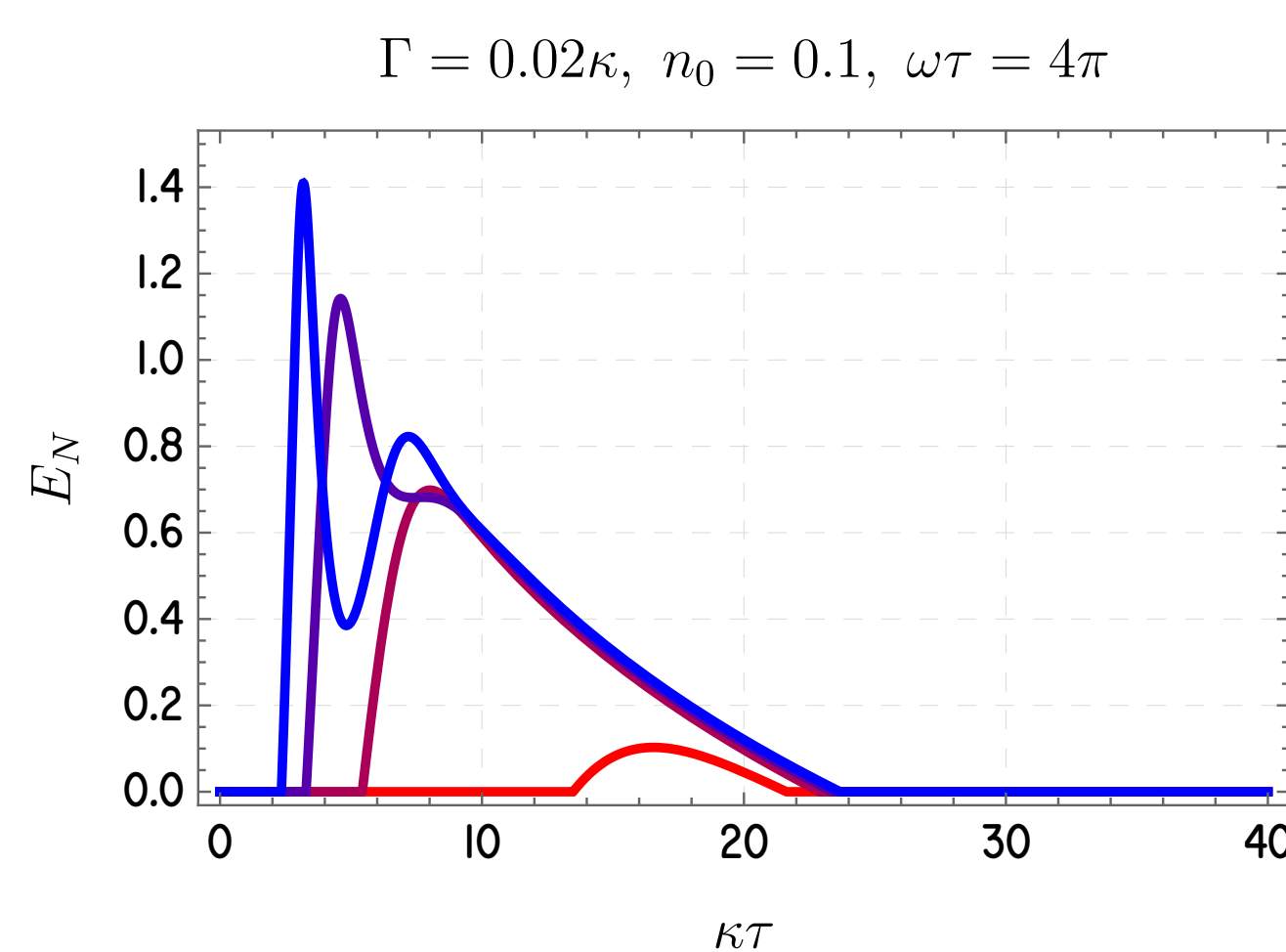


Two nanoparticles are trapped inside a cavity and cooled by coherent scattering to the mean occupation $n_0 < 1$. During $-\tau_s < t < 0$, the NPs experience free fall with Hamiltonian $H_{\text{ff}} = \sum_i \omega p_i^2/4$ which squeezes the states of both NPs (p_i is the momentum of i -th NP). At $t = 0$, red-detuned tweezers re-trap the NPs and a beamsplitter-like interaction is established with effective Hamiltonian, see Fig. Here, g_i are the coupling rates, c corresponds to the cavity mode, a and b to the two NPs. Optional shift ϕ can be controlled by the tweezer phase. Base parameters from [1], assuming no gas heating.

EMERGENCE OF ENTANGLEMENT

Plots, assuming symmetric configuration (equal occupations, squeezing and coupling rates). As a function of time, the initial entanglement dissipates at the rate of optomechanical cooling $\propto g^2/\kappa$. Increasing the initial

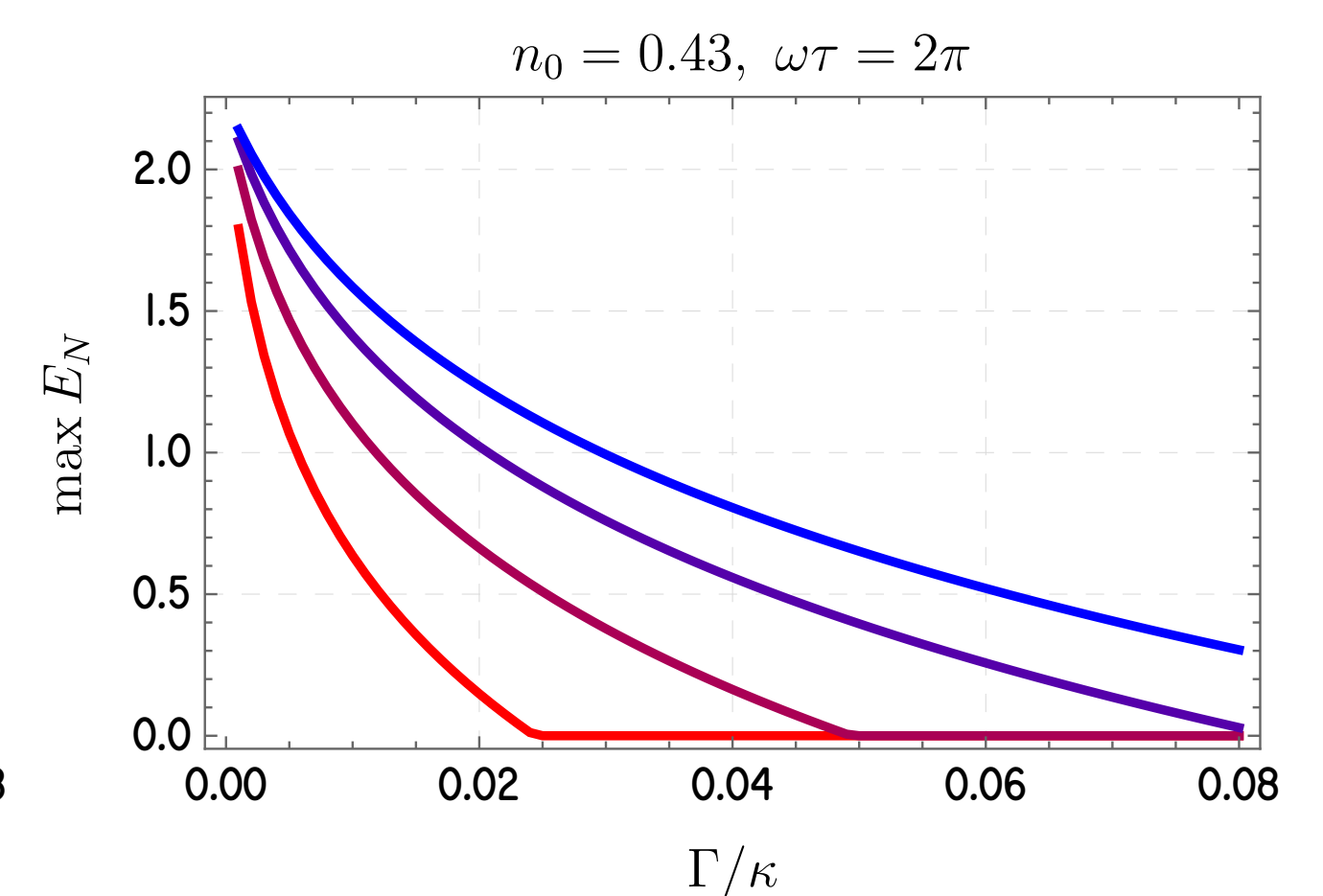
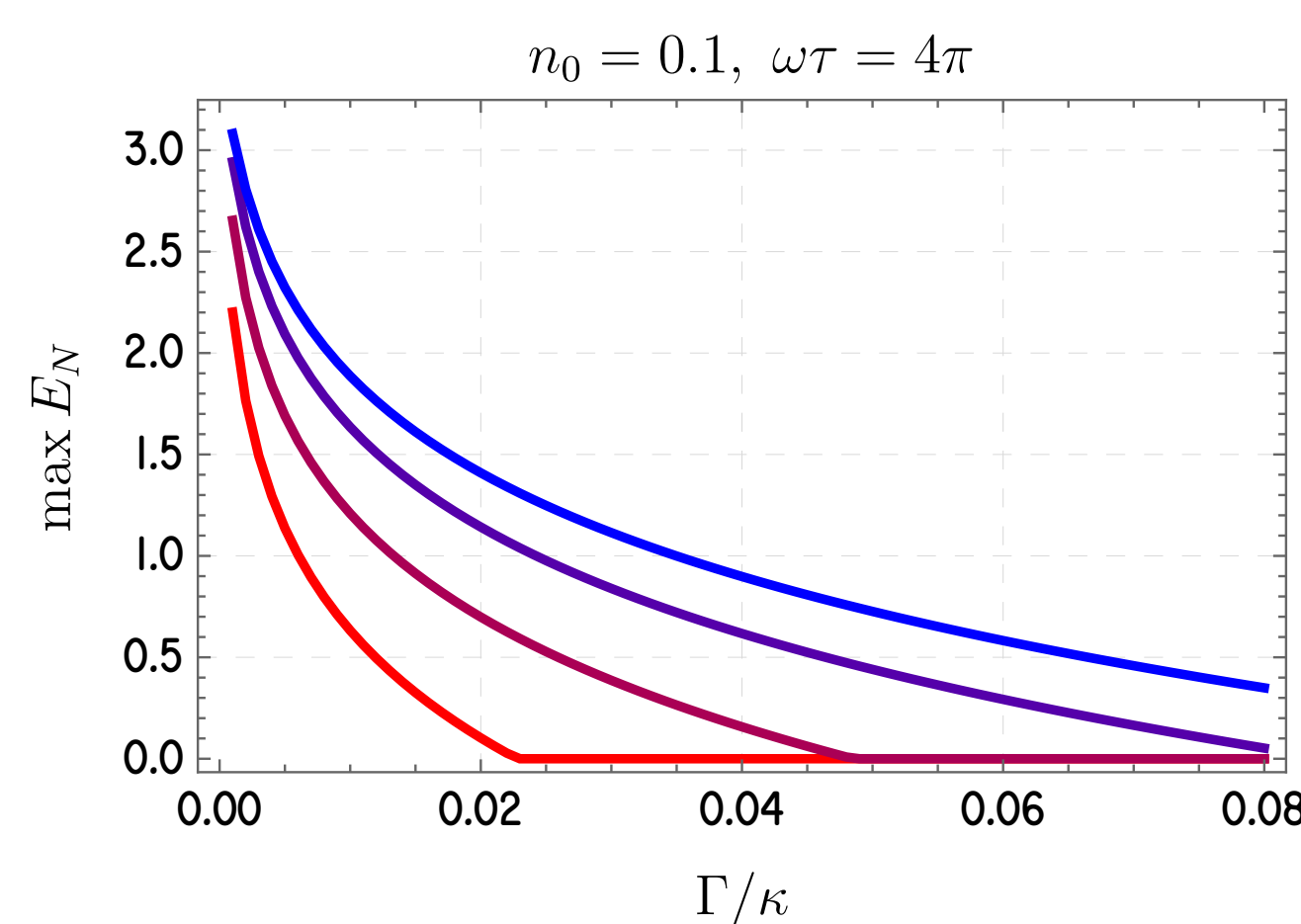
squeezing beyond $\omega\tau \geq 2\pi$ only marginally increases the entanglement. Different colors correspond to different coupling from $g = 0.3\kappa$ (red) to $g = 0.6\kappa$ (blue).



ROBUSTNESS TO HEATING

Entanglement at the instant of maximal entanglement for different coupling rates (same colors as above) as a

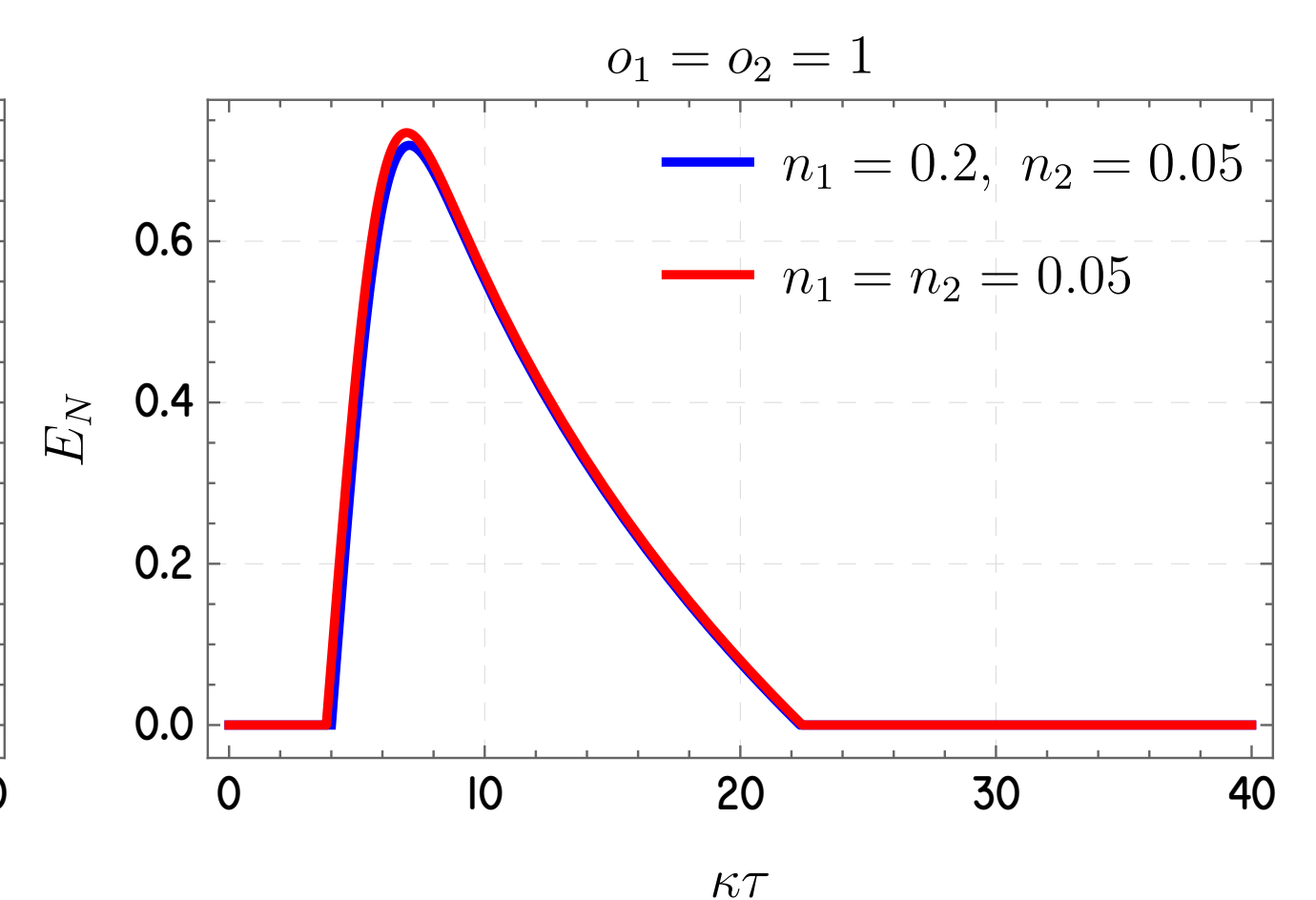
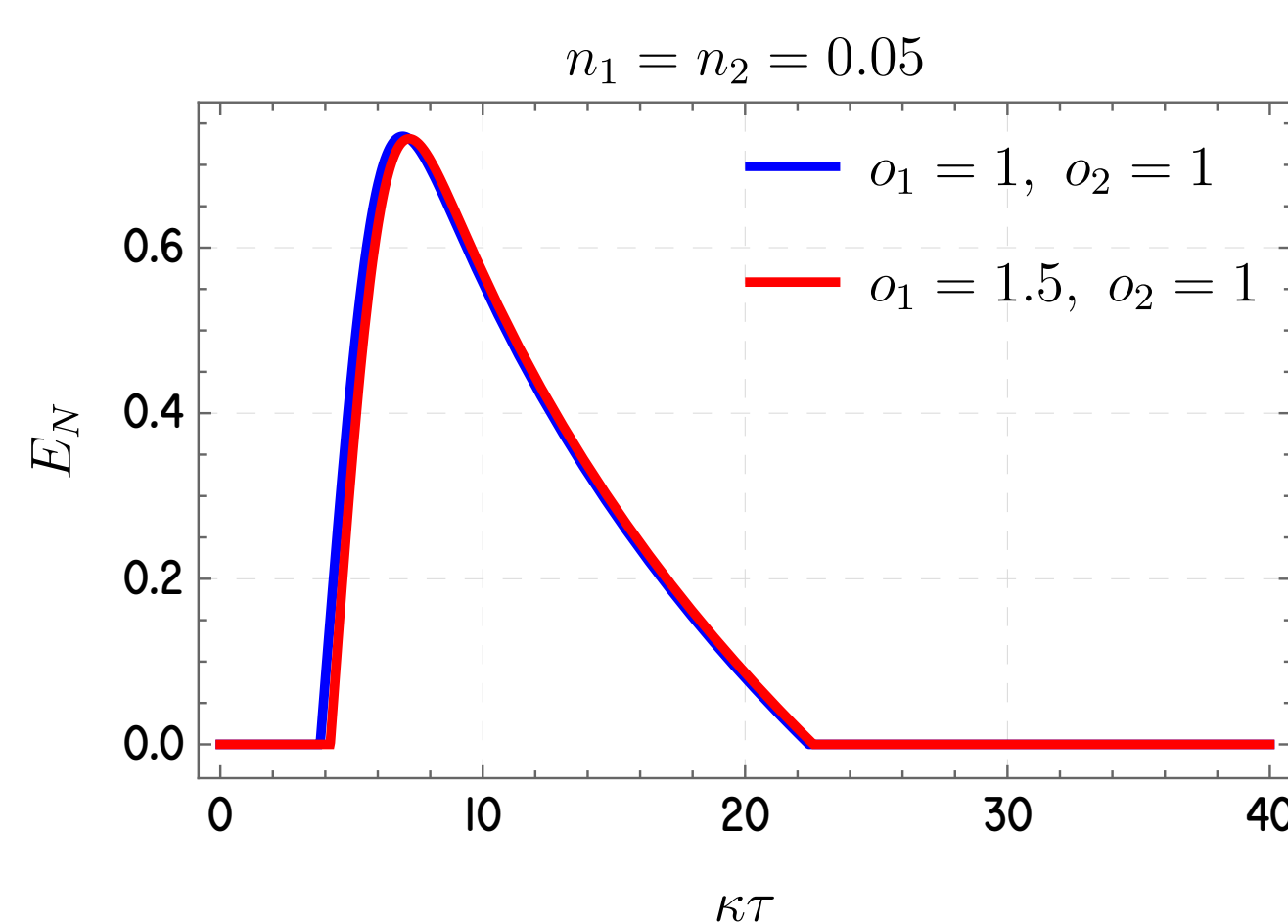
function of heating rate Γ/κ . In [delic_cooling_2020], recoil heating rate equals $\Gamma/\kappa = 0.06$.



ASYMMETRIC PARTICLES

At moderate squeezing, the system is insensitive to minor asymmetries, however, the phase of the coupling ϕ has to

be slightly adjusted. Below, $o_i = \omega\tau_i/(2\pi)$, $\Gamma/\kappa = 0.02$, $g/\kappa = 0.4$.



REFERENCES

- [1] U. Delić et al., Science **367**, 892 (2020), [2] A. Pitchford et al., Quantum Sci. Technol. **9**, 045044 (2024).