Entanglement of levitated nanoparticles by wave-packet dispersion

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LPHYS'25
Szeged
July 2, 2025


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TURKEY
TURKMENISTAN
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AFGHANISTAN
IRAN
JORDAN
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100 km
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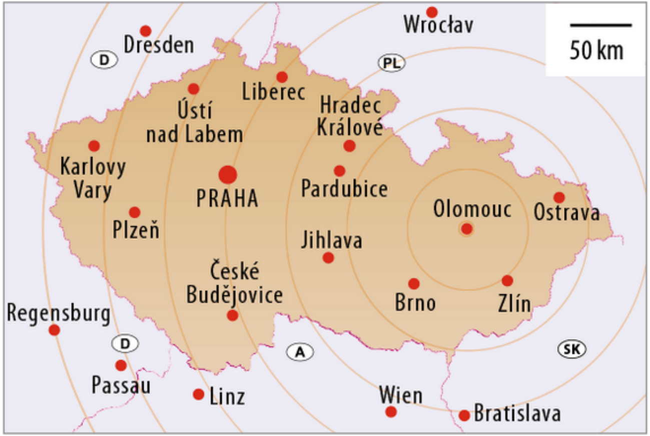
Wrocław
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Plzeň
Olomouc
Ostrava
Jihlava
České
Budějovice
Brno
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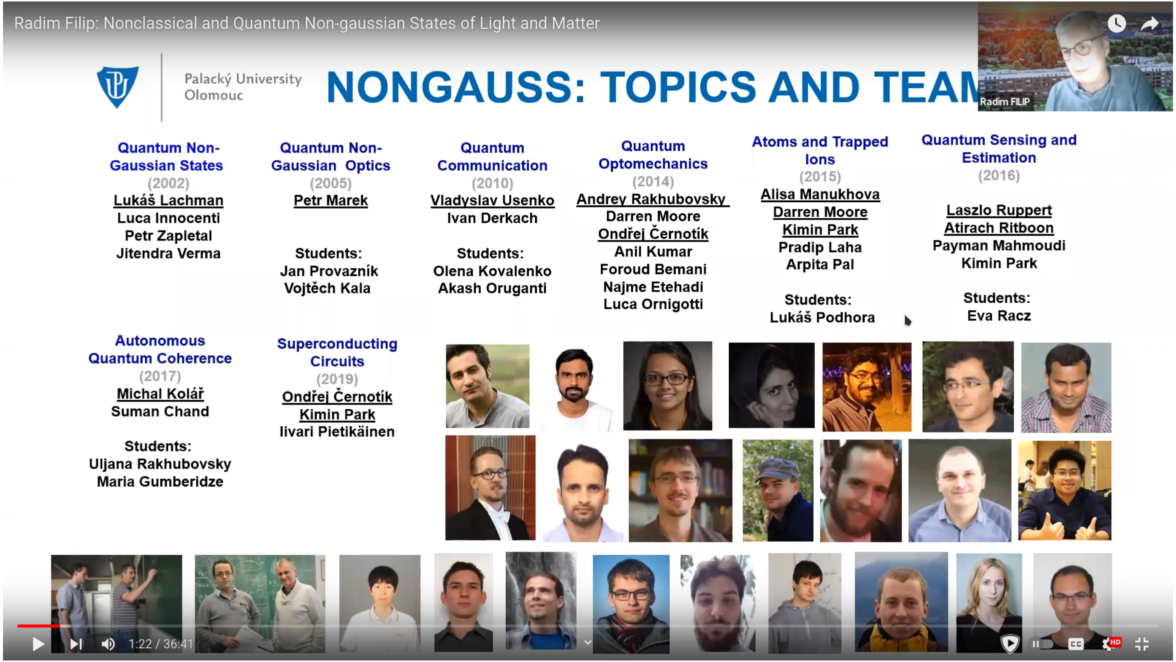
Bratislava

Radim Filip's Group in Olomouc [from 2022 video]



The Optomechanics Group in Olomouc [within R. Filip's group]

Prof. Radim Filip

Dr. Alisa Manukhova

Dr. Foroud Bemani

Dr. Surabhi Yadav

Dr. Najmeh Etehadi

Now @KIT

Shaoni Datta

Dr. Lewis Clark

Now @A*STAR

Ph.D. and Postdoc positions available

Quantum non-Gaussian Everything

Progress in Quantum Electronics
Volume 93, January 2024, 100495

Quantum non-Gaussian optomechanics and electromechanics

Andrey A. Rakhubovsky, Darren W. Moore, Radim Filip

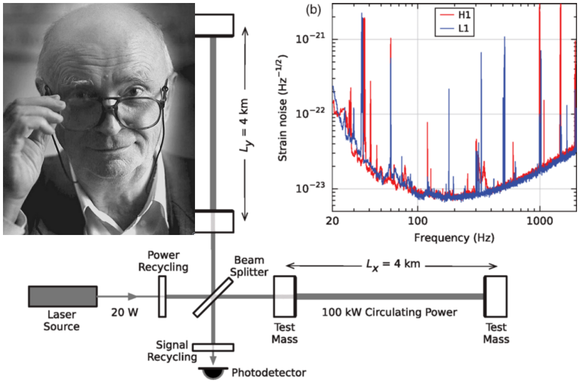
40 pages, 487 references

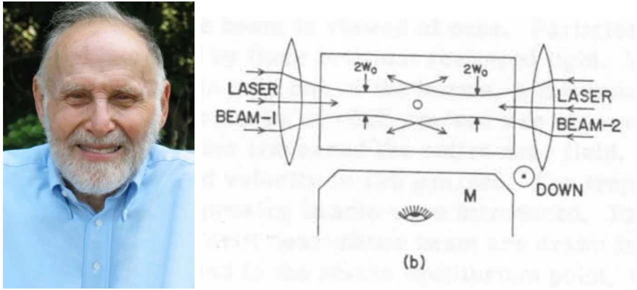
Introduction

- Quantum Optomechanics
- Levitated Optomechanics

Entanglement of Levitated Nanoparticles

Quantum Optomechanics



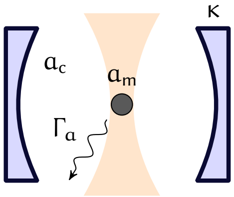


Braginsky & Manukin, Soviet JETP **25**, 653 (1967)
Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)

A. Ashkin, PRL **24**, 156 (1970)

$$H = \omega_{mech} a_m^\dagger a_m + \omega_{cav} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

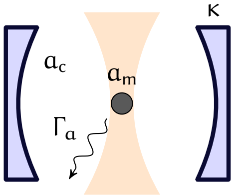
Levitated Optomechanics

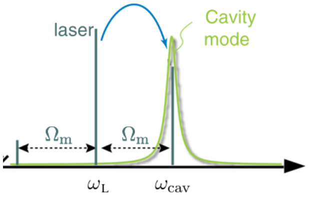


The Hamiltonian

$$H = \frac{p_m^2}{2m} + V(q_m) + \omega_{cav} a_c^\dagger a_c + g q_m (a_c^\dagger + a_c).$$

Levitated Optomechanics





M. Aspelmeyer *et al.*, Rev. Mod. Phys. **86**, 1391 (2014)

The Hamiltonian

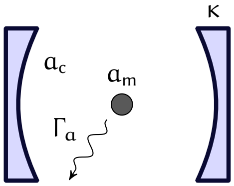
$$H = \frac{p_m^2}{2m} + V(q_m) + \omega_{cav} a_c^\dagger a_c + g q_m (a_c^\dagger + a_c).$$

Conventional Interaction (Cooling)

$$H_{int} = g(a_c^\dagger a_m + a_c a_m^\dagger)$$

- Cooling
- Beam-splitter / State swap

Levitated Optomechanics



The Hamiltonian

$$H = \frac{p_m^2}{2m} + \qquad \omega_{cav} a_c^\dagger a_c \qquad .$$

Free Fall

$$q \mapsto q + p\tau,$$
$$p \mapsto p.$$

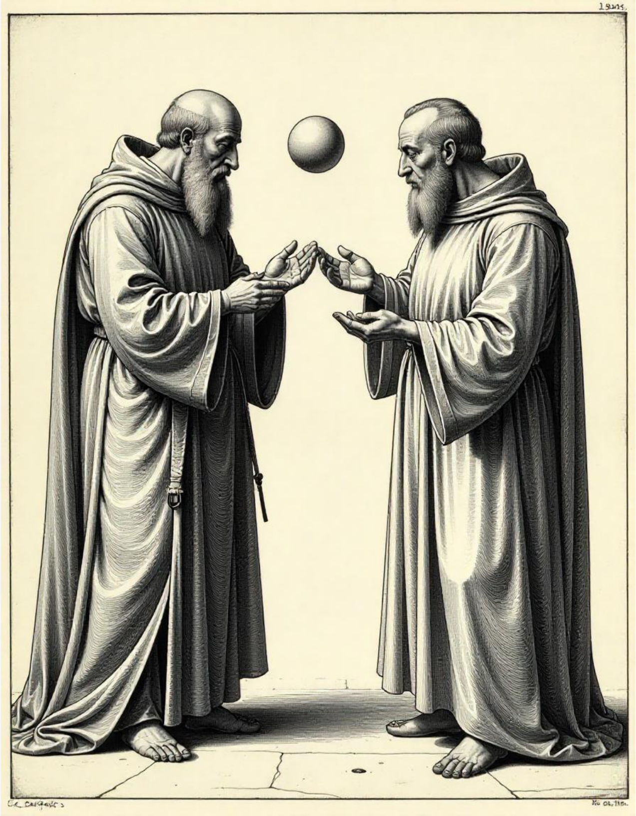
Squeezing!

Introduction

Quantum Optomechanics

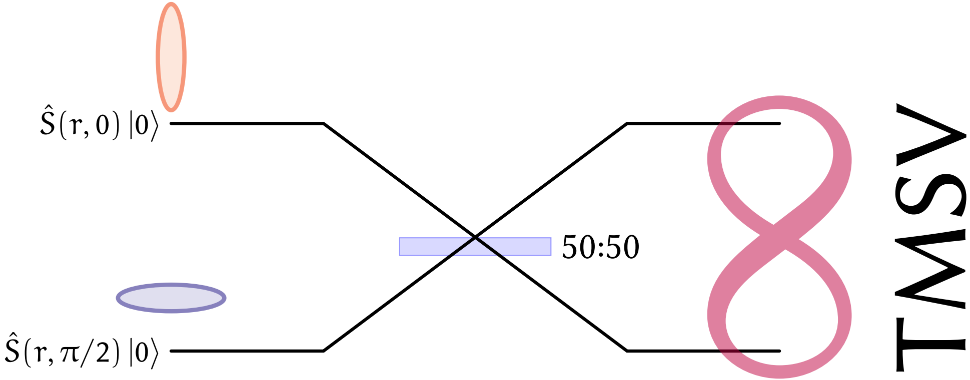
Levitated Optomechanics

Entanglement of Levitated Nanoparticles

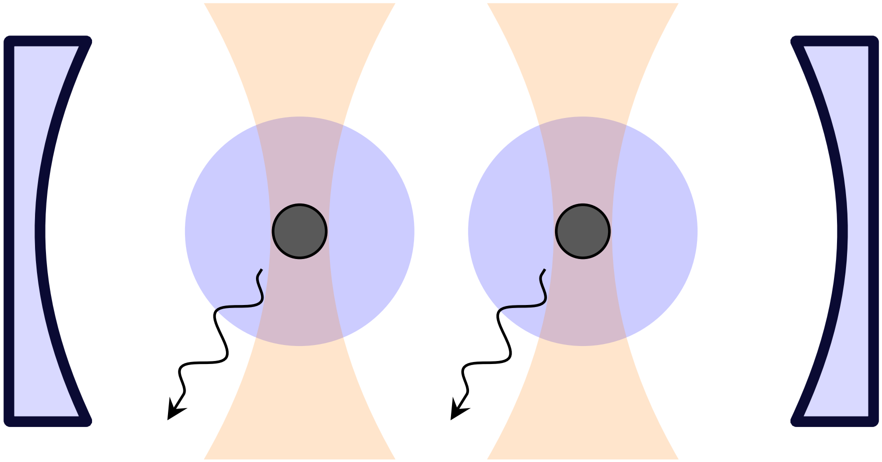


The Recipe

Quantum optics analogy



The Protocol



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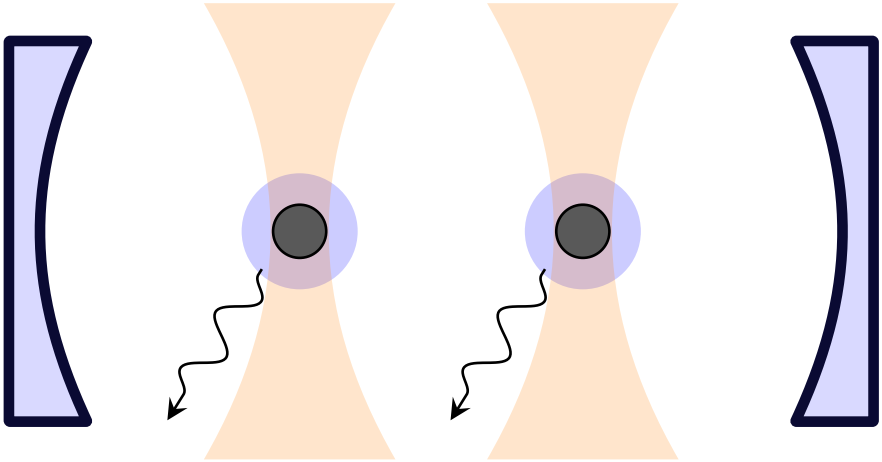

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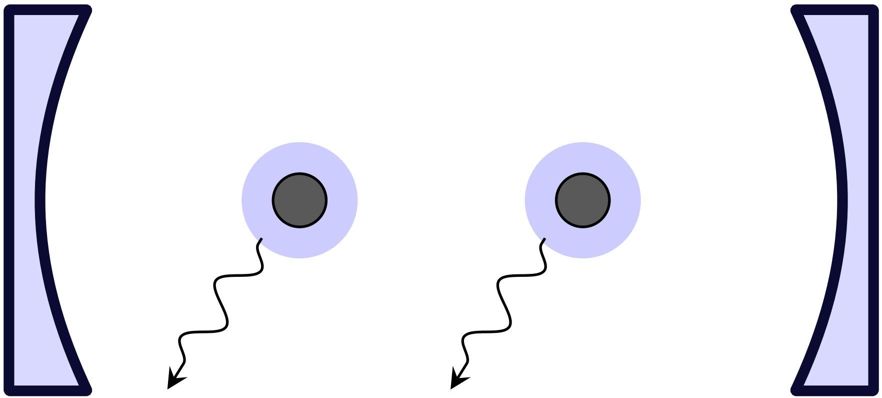

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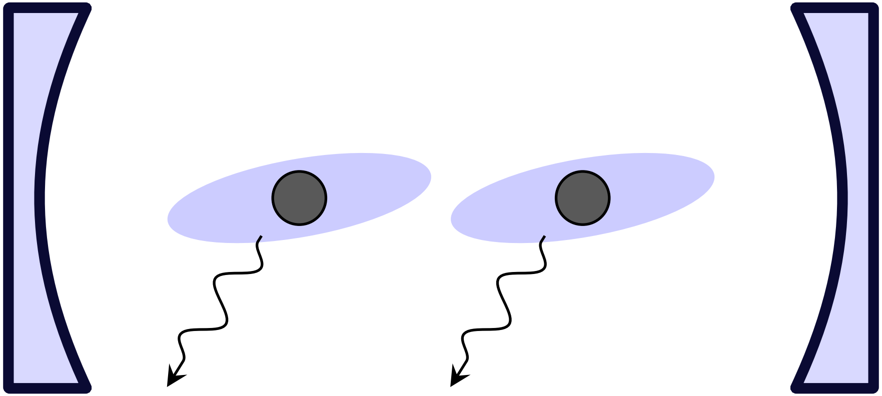

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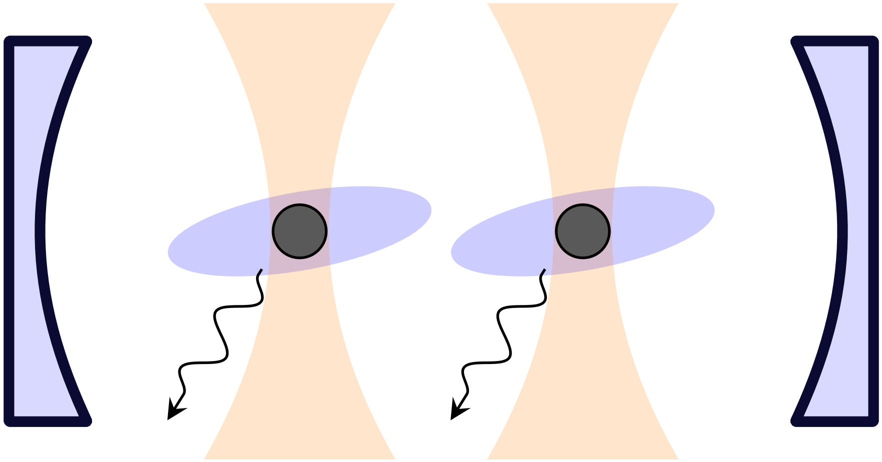

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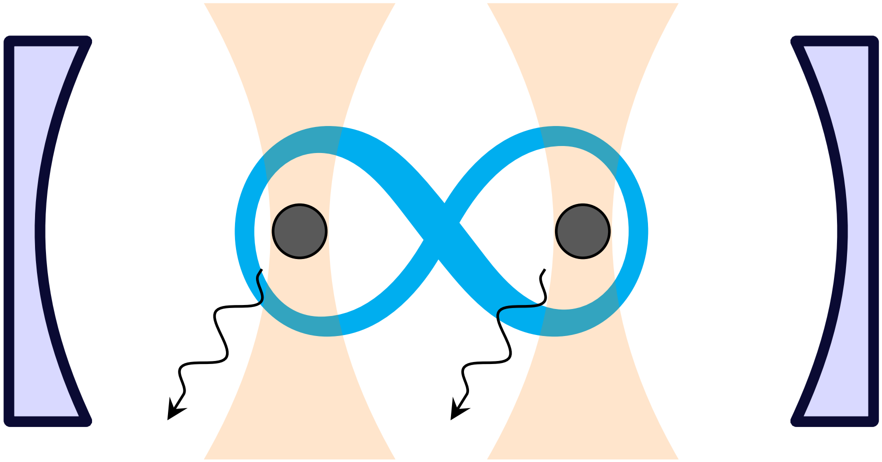

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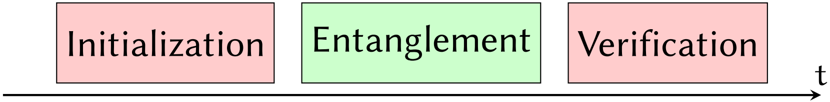

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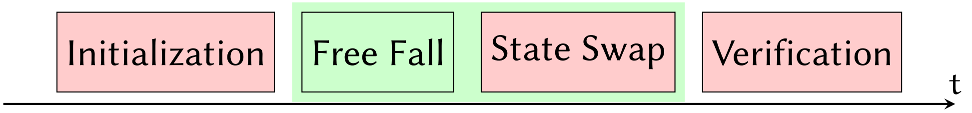

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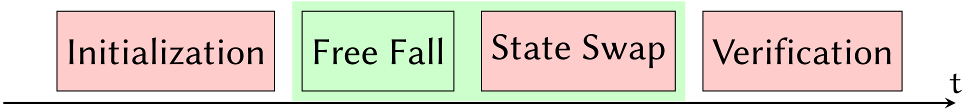
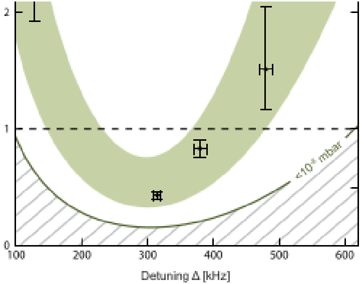


Initialization

Cooling to the ground state ($\bar{n} = 0.43$)



U. Delić, Science **367**, 892 (2020)

Avoid dark modes

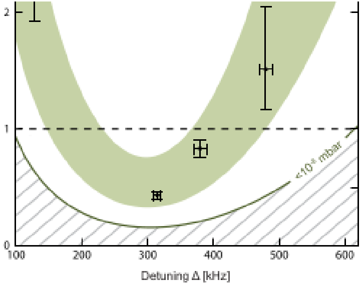
The Protocol

Initialization | Free Fall | State Swap | Verification

t

Initialization

Cooling to the ground state ($\bar{n} = 0.43$)



U. Delić, Science **367**, 892 (2020)

Avoid dark modes

Free Fall

Squeezing

State Swap

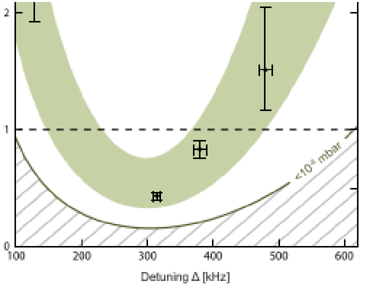
Via interacting with the same cavity mode

The Protocol

Initialization | Free Fall | State Swap | Verification → t

Initialization

Cooling to the ground state ($\bar{n} = 0.43$)



U. Delić, Science **367**, 892 (2020)

Avoid dark modes

Free Fall

Squeezing

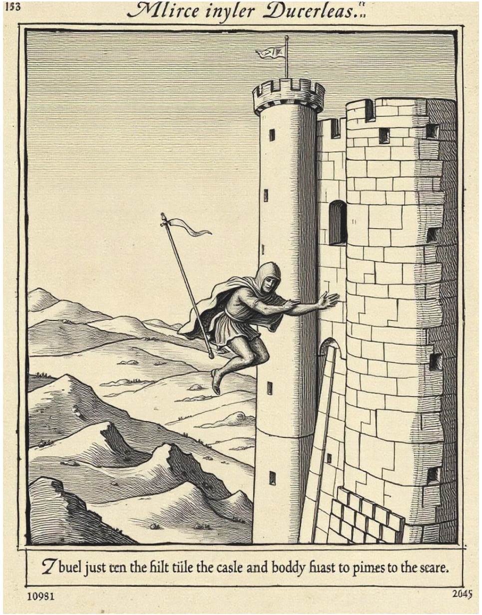
State Swap

Via interacting with the same cavity mode

Readout

Via state swap to leaking light and homodyne

Free Fall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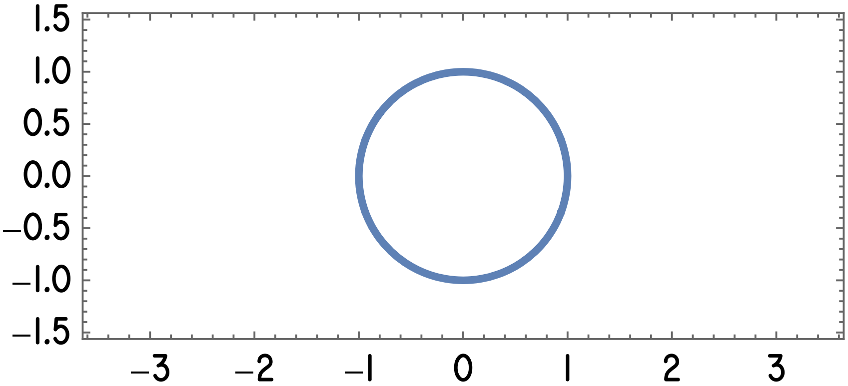
Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



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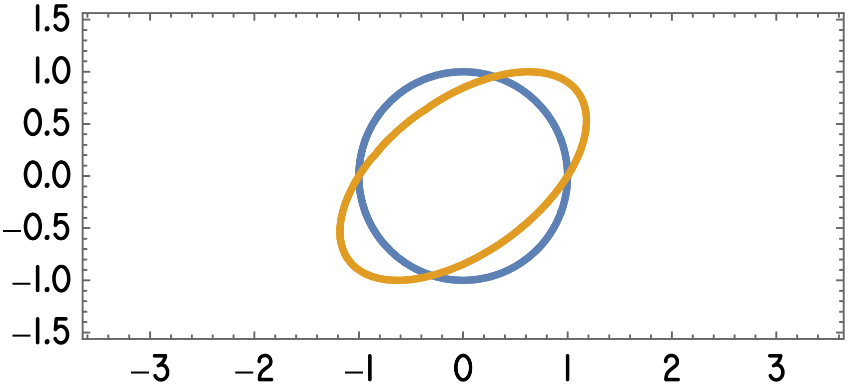

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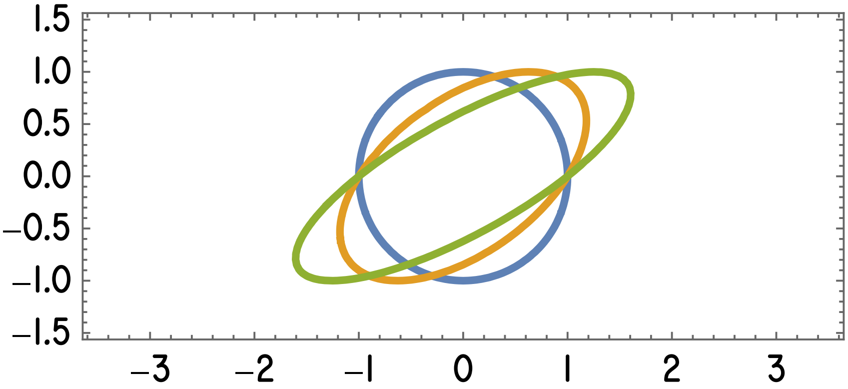

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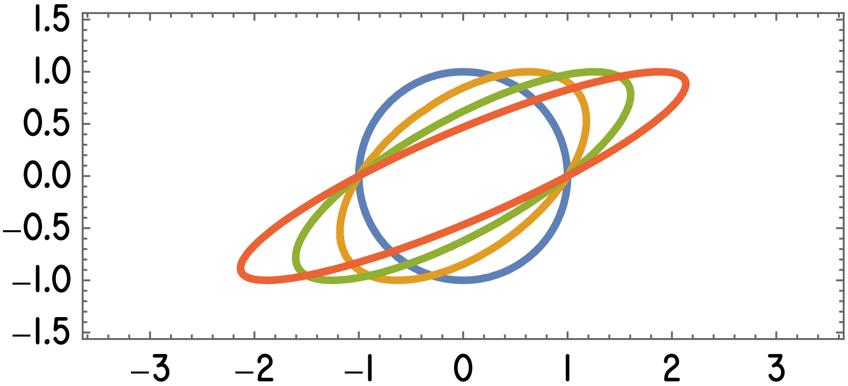

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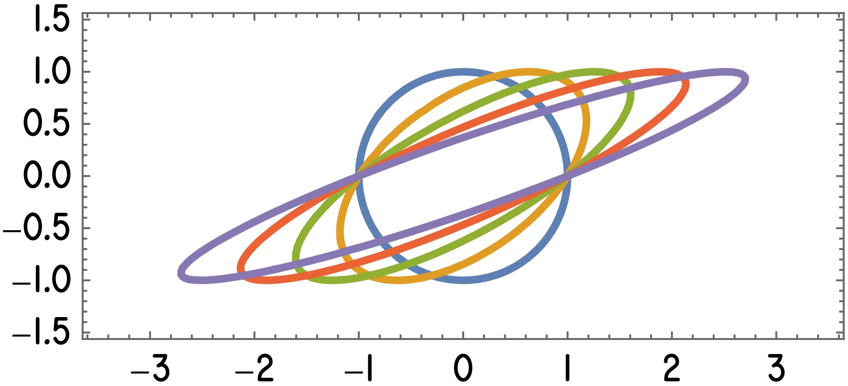

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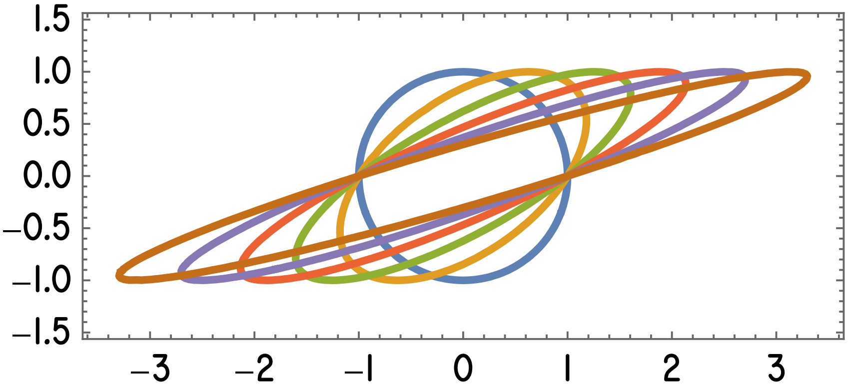

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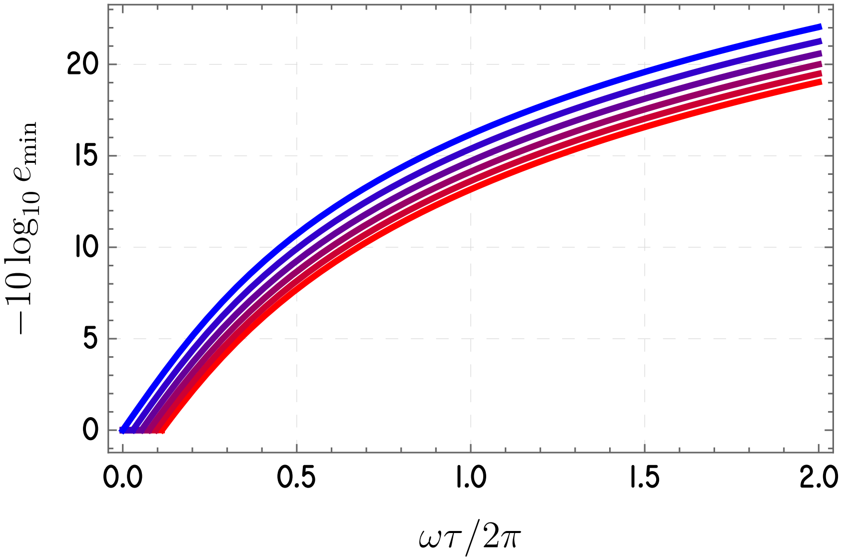
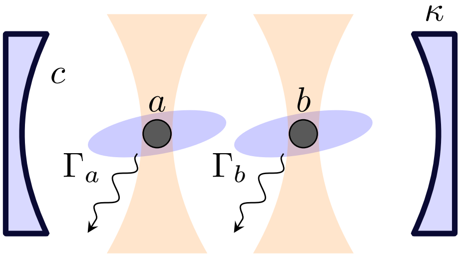


State Swap

State Swap

$$H = c^\dagger (g_a a + g_b b \mathrm{e}^{\mathrm{i}\phi}) + \mathrm{h.c.}$$



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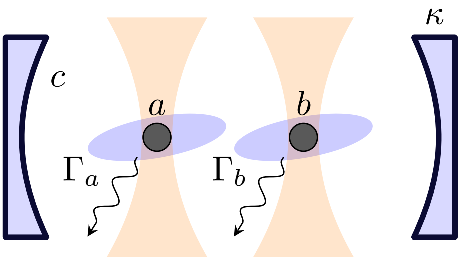
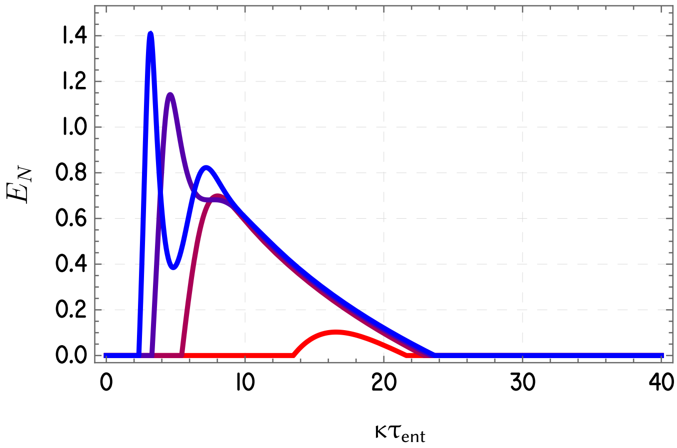


Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

Logarithmic negativity E_N
(quantifies violation of the positivity of partial transpose)

$$\Gamma = 0.02\kappa,\ n_0 = 0.1,\ \omega\tau = 4\pi$$



Color: coupling rate $0.3 \leqslant \frac{g}{\kappa} \leqslant 0.6$.

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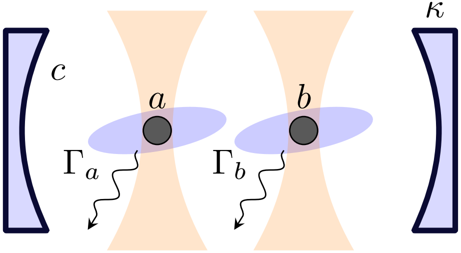

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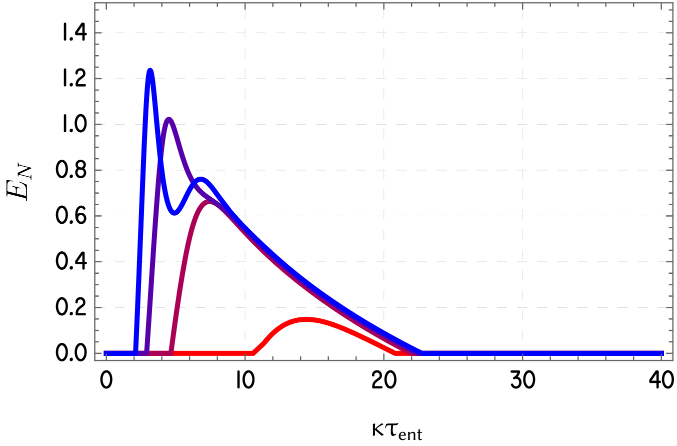

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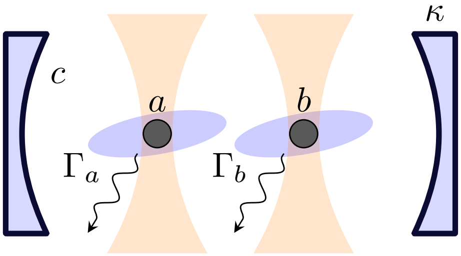

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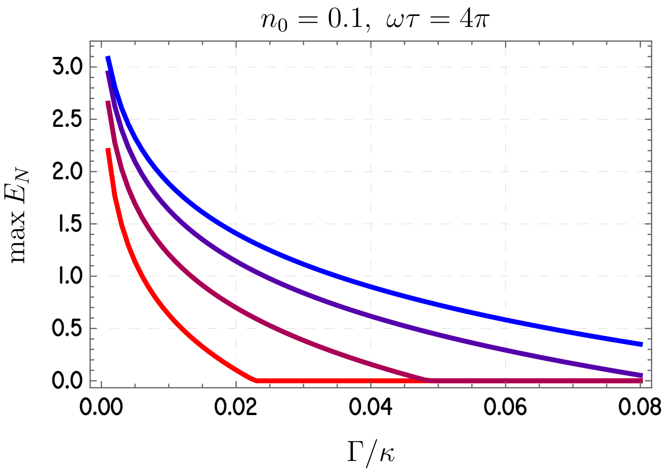

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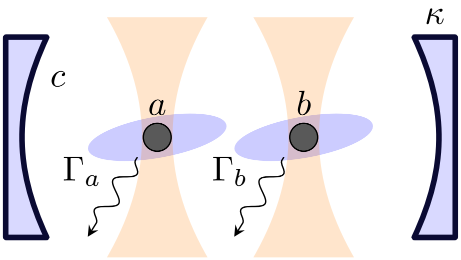

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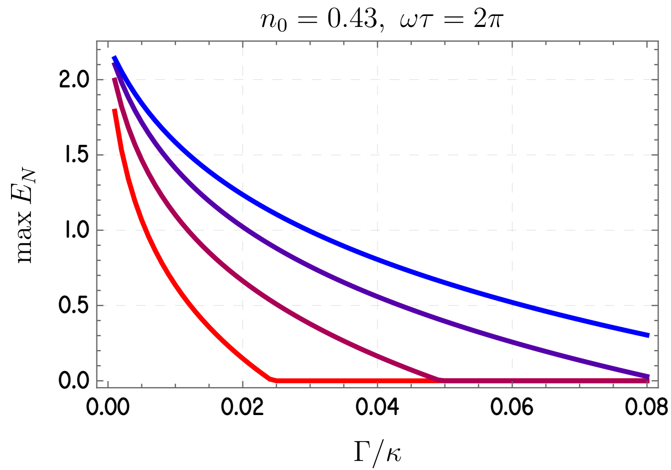
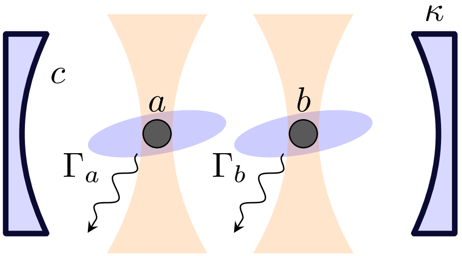


Color: coupling rate $0.3 \leqslant \frac{g}{\kappa} \leqslant 0.6$.

The role of asymmetry

Non-identical particles

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



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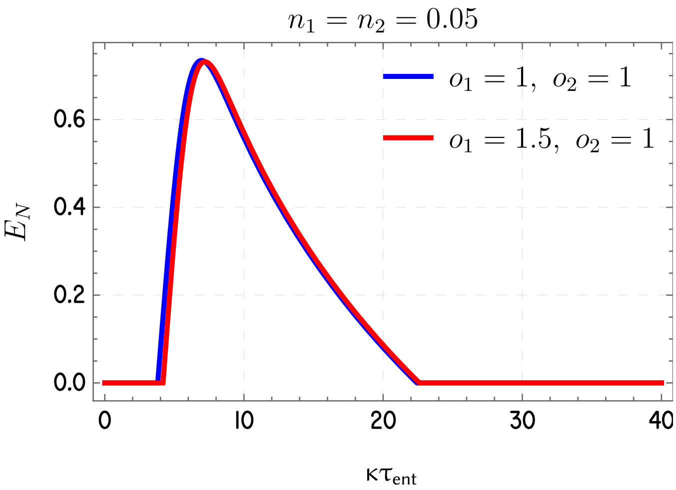
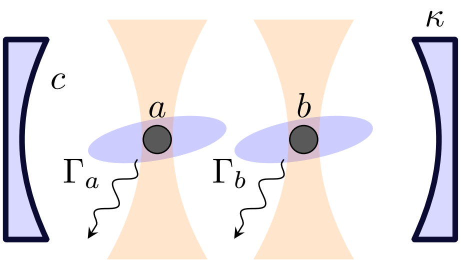
Logarithmic negativity E_N



$$o_i \equiv \frac{\omega_i \tau}{2\pi}$$

Non-identical particles

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



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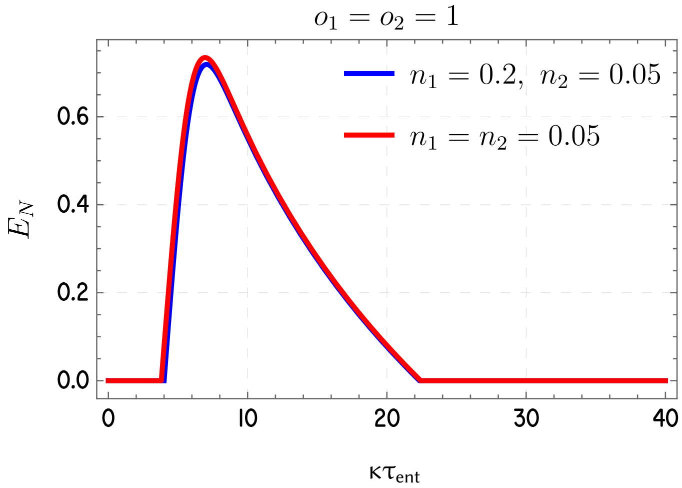

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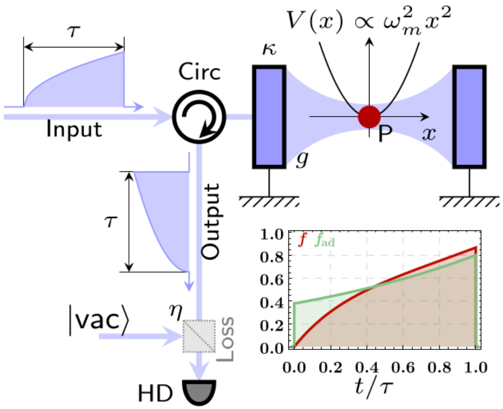
Readout

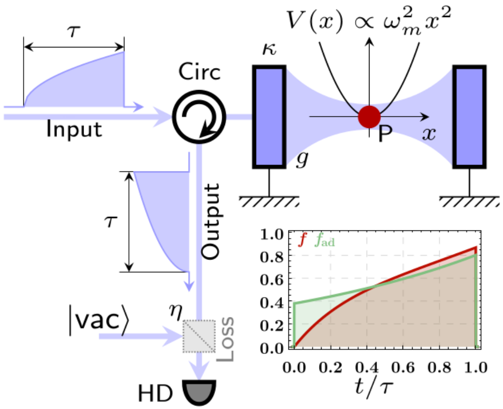
State Examination



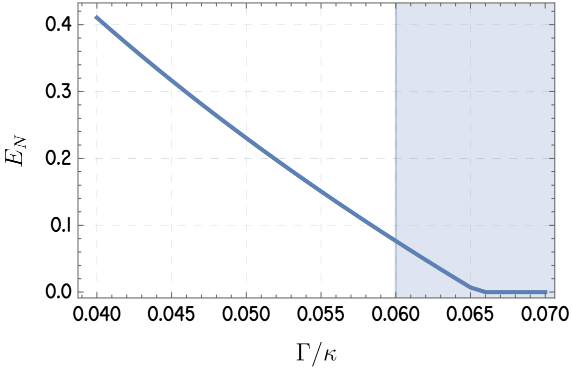
$$X_{\mathrm{out}} = \sqrt{\mathcal{T}}\, X_{\mathrm{mech}} + \sqrt{1-\mathcal{T}}\, X_{\mathrm{Noise}}.$$

$$H = g_a a c_a^\dagger + g_b b c_b^\dagger + \mathrm{h.c.}$$

State Examination



$$X_{out} = \sqrt{\mathcal{T}}\, X_{mech} + \sqrt{1-\mathcal{T}}\, X_{Noise}.$$

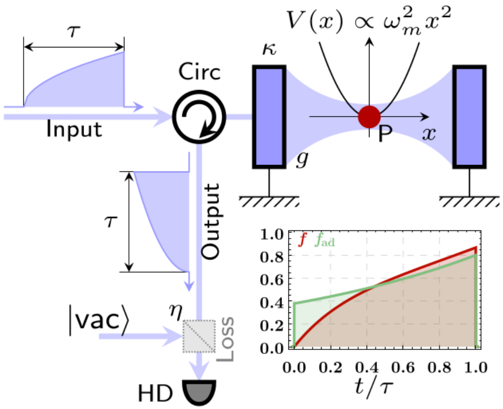


$$H = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$

- ★ Coupling rate $g = 0.6\kappa$
- ★ Initial occupation $n_0 = 0.43$

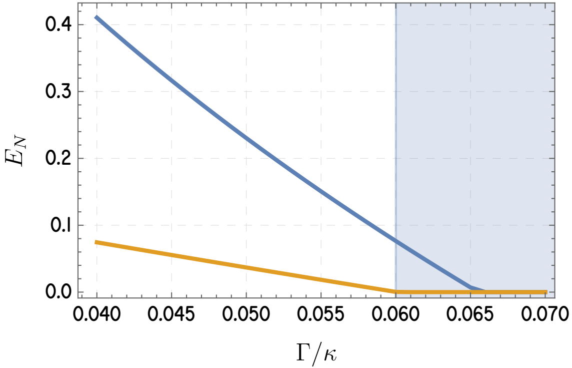
U. Delić, Science **367**, 892 (2020)

State Examination



$$\mathrm{H} = g_a a c_a^\dagger + g_b b c_b^\dagger + \mathrm{h.c.}$$

$$X_{out} = \sqrt{\mathcal{T}}\, X_{mech} + \sqrt{1-\mathcal{T}}\, X_{Noise}.$$



- Coupling rate $g = 0.6\kappa$
- Initial occupation $n_0 = 0.43$

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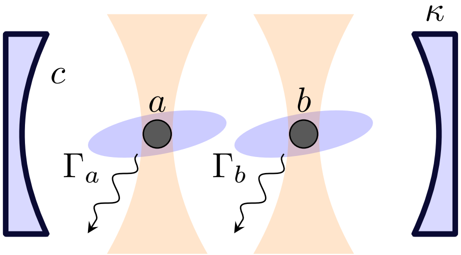
Conclusions

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Requirements for Entanglement

- ★ Beam-splitter interaction
- ★ Free-motion

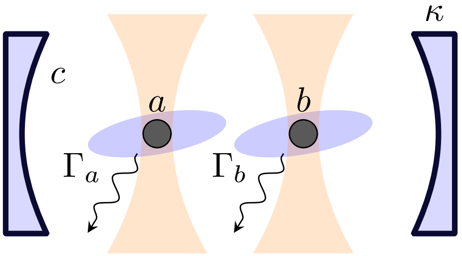
Requirements for Verification

- ★ Beam-splitter
- ★ Individual address

All operations are passive

Conclusions

$$H = c^\dagger (g_a a + g_b b e^{\mathrm{i}\phi}) + \text{h.c.}$$



Requirements for Entanglement

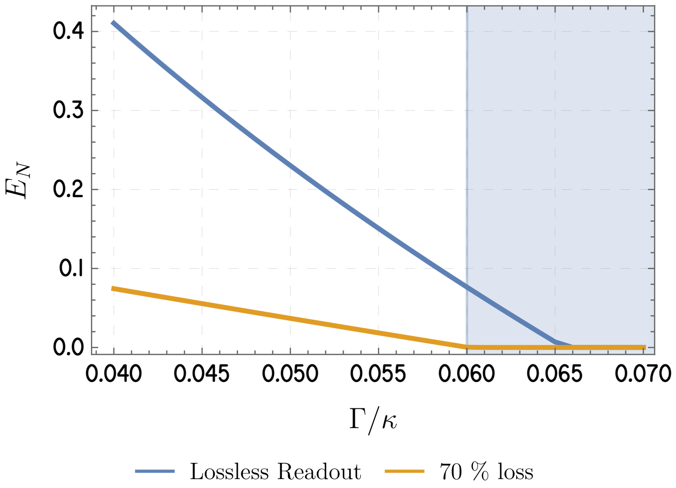
- Beam-splitter interaction
- Free-motion

Requirements for Verification

- Beam-splitter
- Individual address

All operations are passive

- Entanglement is possible
- Can persist recoil heating
- Survives asymmetries
- Does not require initial pure states (but doesn't mind)



Thank You!

These slides
https://bit.ly/andrey-lphys-2025

Phd and Postdoc positions available

Beware of the appendix slide!

Effective classical simulation

Consider the setup:

- n quantum subsystems
- t operations
- local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \ldots, k_n\},$$

one outcome per system.
Probability of the outcomes

$$P(k_1, \ldots, k_n) \tag{1}$$

A classical algorithm is said to capable of effective sampling, if it

- is efficient (polynomial) in t and n
- provides outcomes $\mathbf{k}$ draws from the same probability as (1)

The very last frame which is empty