Entanglement of levitated nanoparticles by wave-packet dispersion

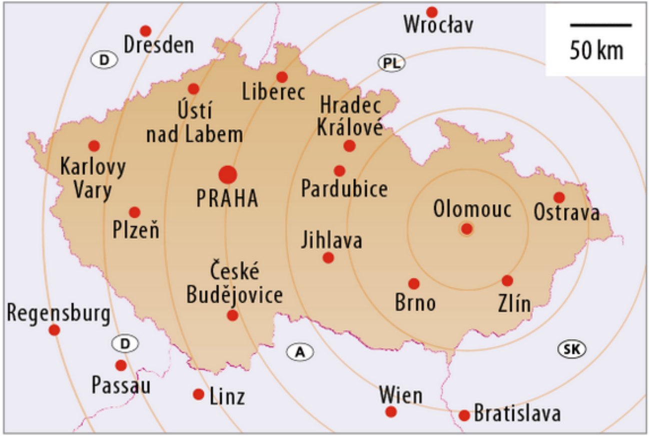
Andrey A. Rakhubovsky, Radim Filip

Department of Optics, Palacký University, Czech Republic

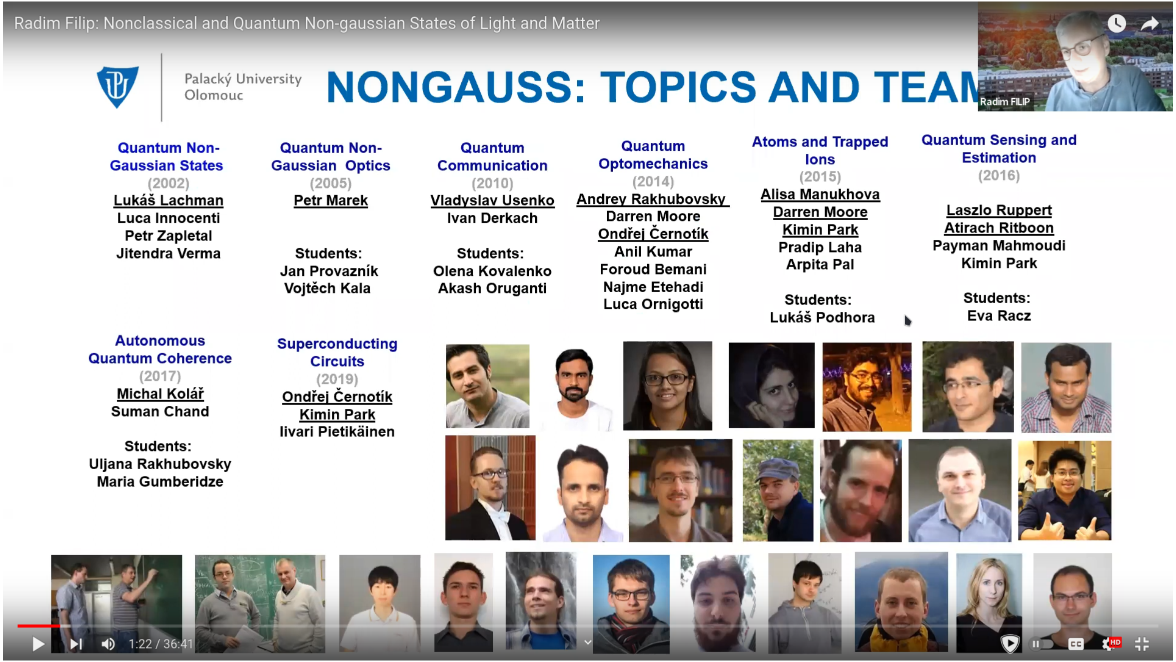
LPHYS'25
Szeged
July 2, 2025

ESTONIA
LATVIA
LITHUANIA
DENMARK
UNITED KINGDOM
IRELAND
NETHERLANDS
GERMANY
BELARUS
4.5
Olomouc
CZECHIA
SLOVAKIA
UKRAINE
KAZAKHSTAN
AUSTRIA
FRANCE
SWITZERLAND
HUNGARY
ROMANIA
CROATIA
SERBIA
ITALY
BULGARIA
GEORGIA
UZBEKISTAN
AZERBAIJAN
PORTUGAL
SPAIN
GREECE
TURKEY
TURKMENISTAN
TAJIKISTAN
SYRIA
TUNISIA
IRAQ
AFGHANISTAN
IRAN
MOROCCO
JORDAN

100 km
Amsterdam
NL
Berlin
Warszawa
PL
Bruxelles
D
Luxembourg
L
UA
PRAHA
CZ
SK
Wien
Bratislava
Budapest
Bern
Vaduz
A
CH
FL
H
RO
Ljubljana
SLO
Zagreb
I
YU
HR
BIH
Beograd


Wrocław
50 km
Dresden
D
PL
Liberec
Ústí
nad Labem
Hradec
Králové
Karlovy
Vary
PRAHA
Pardubice
Olomouc
Ostrava
Plzeň
Jihlava
České
Budějovice
Brno
Zlín
Regensburg
D
A
SK
Passau
Linz
Wien
Bratislava

Radim Filip's Group in Olomouc [from 2022 video]



The Optomechanics Group in Olomouc [within R. Filip's group]

Prof. Radim Filip

Dr. Alisa Manukhova

Dr. Foroud Bemani

Dr. Surabhi Yadav

Dr. Najmeh Etehadi

Now @KIT

Shaoni Datta

Dr. Lewis Clark

Now @A*STAR

Ph.D. and Postdoc positions available

Quantum non-Gaussian Everything

ELSEVIER
Progress in Quantum Electronics
Volume 93, January 2024, 100495

Quantum non-Gaussian optomechanics and electromechanics

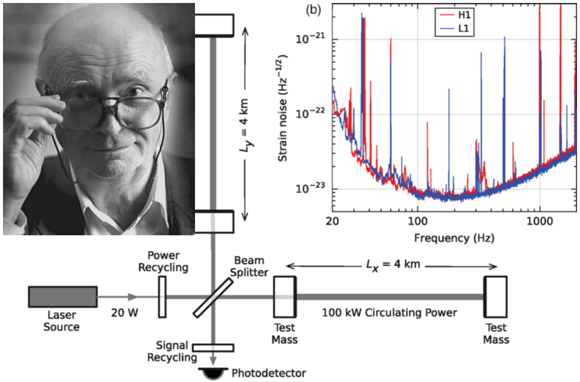
Andrey A. Rakhubovsky, Darren W. Moore, Radim Filip

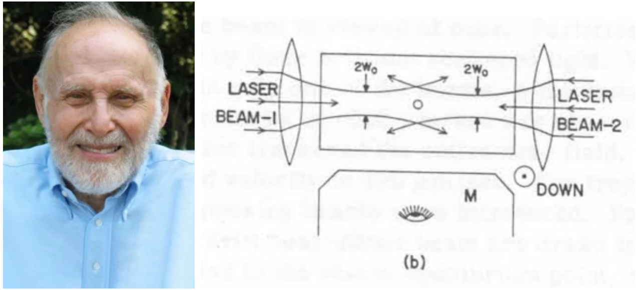
40 pages, 487 references

Introduction
- Quantum Optomechanics
- Levitated Optomechanics

Entanglement of Levitated Nanoparticles

Quantum Optomechanics



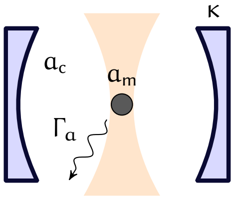


Braginsky & Manukin, Soviet JETP **25**, 653 (1967)
Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)

A. Ashkin, PRL **24**, 156 (1970)

$$H = \omega_{mech} a_m^\dagger a_m + \omega_{cav} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

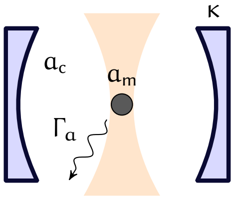
Levitated Optomechanics

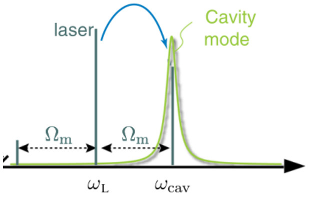


The Hamiltonian

$$H = \frac{p_m^2}{2m} + V(q_m) + \omega_{cav} a_c^\dagger a_c + g q_m (a_c^\dagger + a_c).$$

Levitated Optomechanics





M. Aspelmeyer *et al.*, Rev. Mod. Phys. **86**, 1391 (2014)

The Hamiltonian

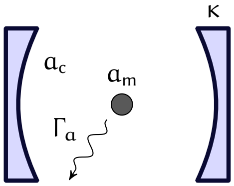
$$H = \frac{p_m^2}{2m} + V(q_m) + \omega_{cav} a_c^\dagger a_c + g q_m (a_c^\dagger + a_c).$$

Conventional Interaction (Cooling)

$$H_{int} = g(a_c^\dagger a_m + a_c a_m^\dagger)$$

- Cooling
- Beam-splitter / State swap

Levitated Optomechanics



The Hamiltonian

$$H = \frac{p_m^2}{2m} + \qquad \omega_{cav} a_c^\dagger a_c \qquad .$$

Free Fall

$$q \mapsto q + p\tau,$$
$$p \mapsto p.$$

Squeezing!

Introduction

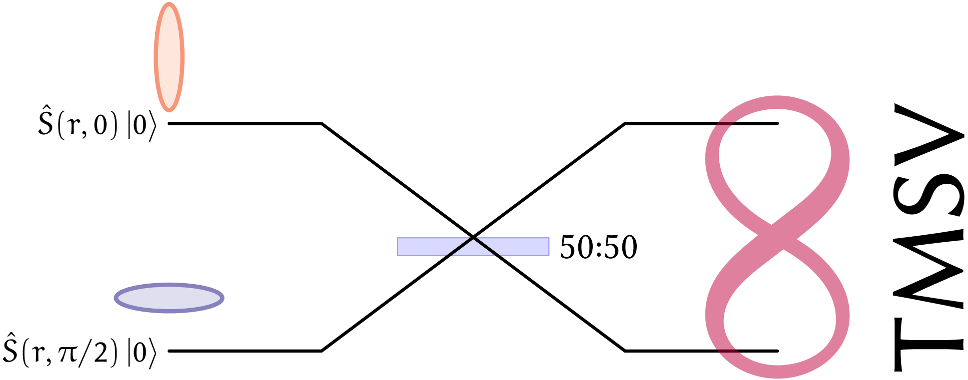
Quantum Optomechanics

Levitated Optomechanics

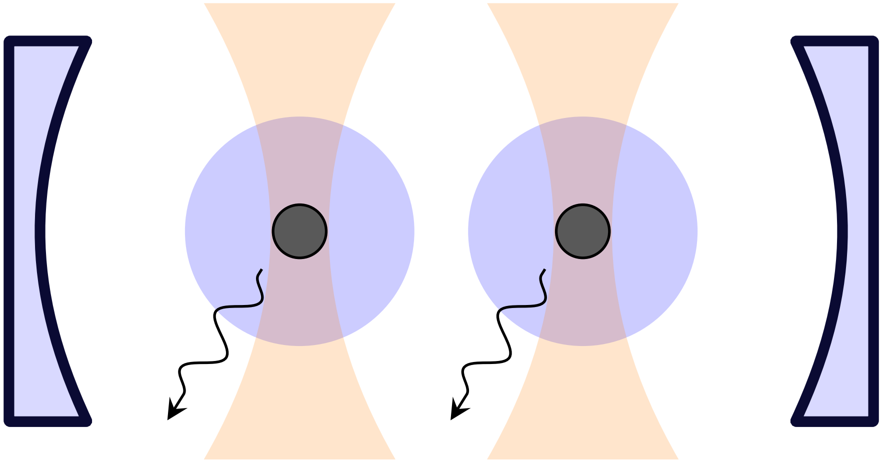
Entanglement of Levitated Nanoparticles

The Recipe

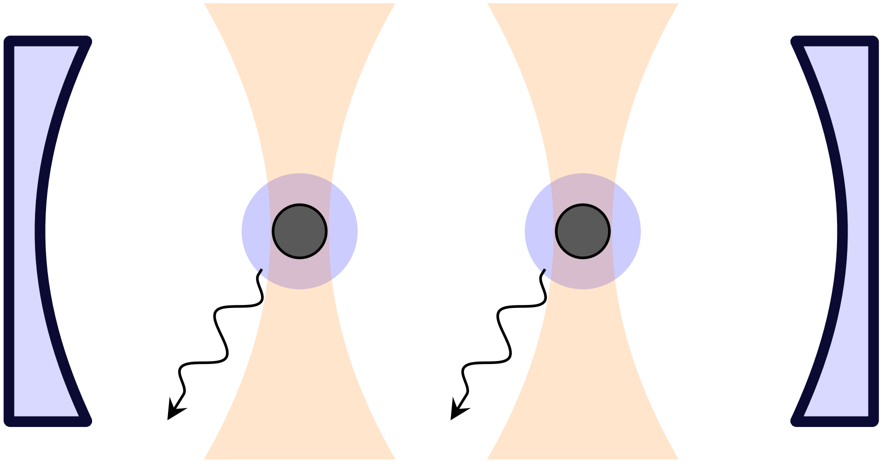
Quantum optics analogy



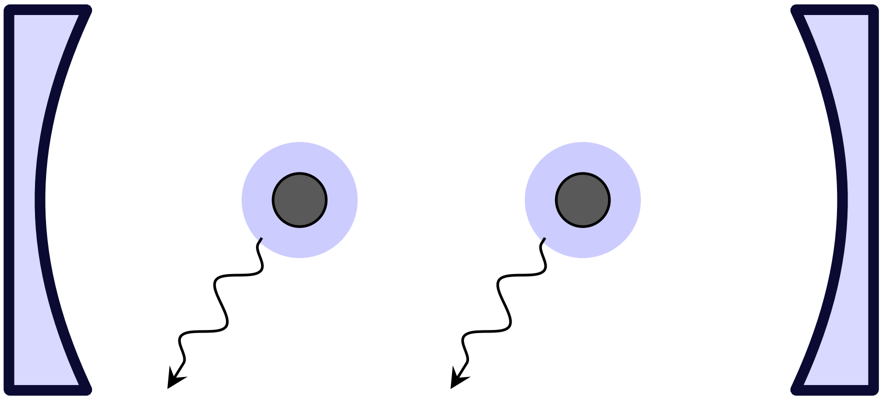
The Protocol



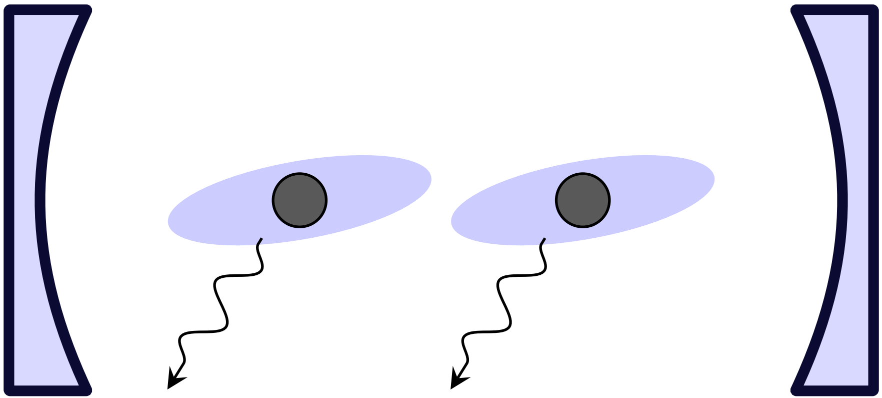
The Protocol



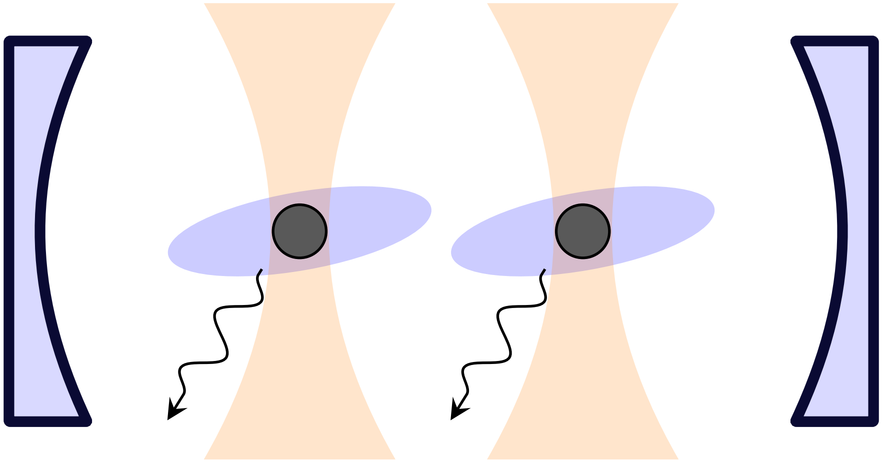
The Protocol



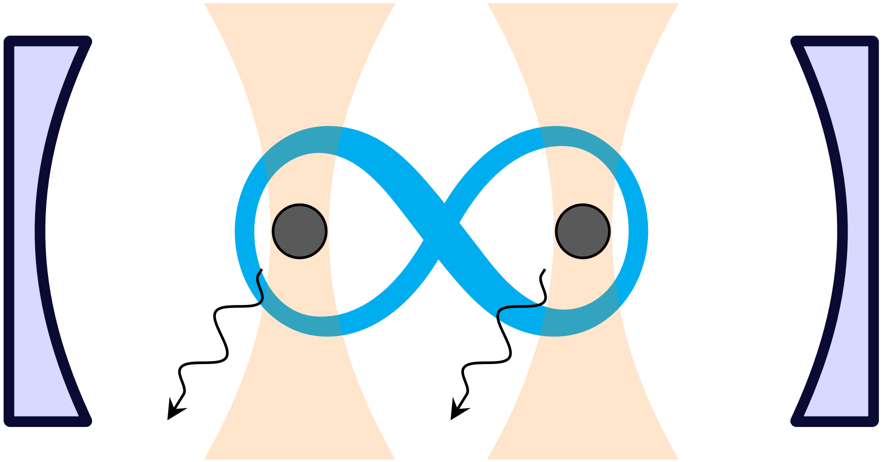
The Protocol



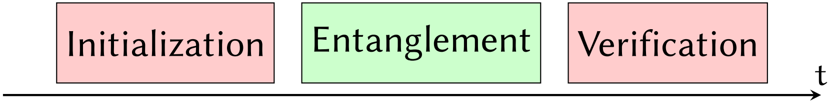
The Protocol



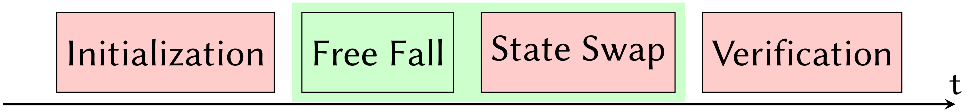
The Protocol



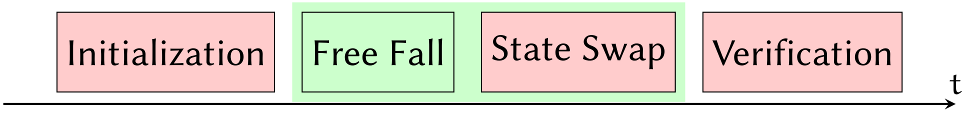
The Protocol



The Protocol

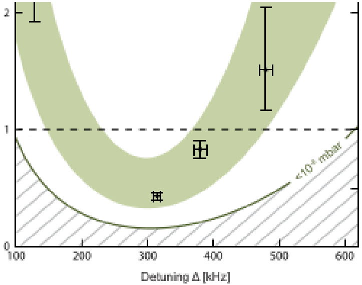


The Protocol



Initialization

Cooling to the ground state ($\bar{n} = 0.43$)



U. Delić, Science **367**, 892 (2020)

Avoid dark modes

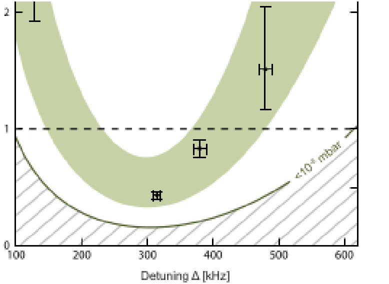
The Protocol

Initialization | Free Fall | State Swap | Verification

t

Initialization

Cooling to the ground state ($\bar{n} = 0.43$)



U. Delić, Science **367**, 892 (2020)

Avoid dark modes

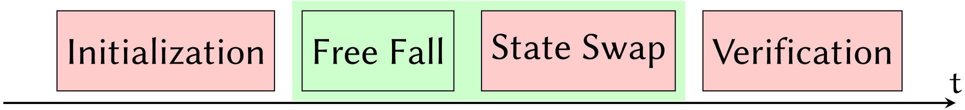
Free Fall

Squeezing

State Swap

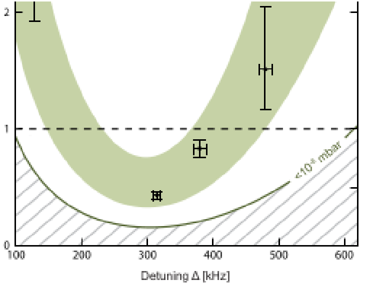
Via interacting with the same cavity mode

The Protocol



Initialization

Cooling to the ground state ($\bar{n} = 0.43$)



U. Delić, Science **367**, 892 (2020)
Avoid dark modes

Free Fall

Squeezing

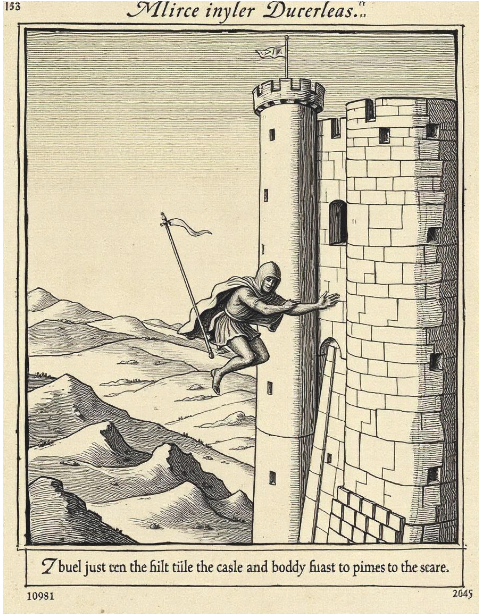
State Swap

Via interacting with the same cavity mode

Readout

Via state swap to leaking light and homodyne

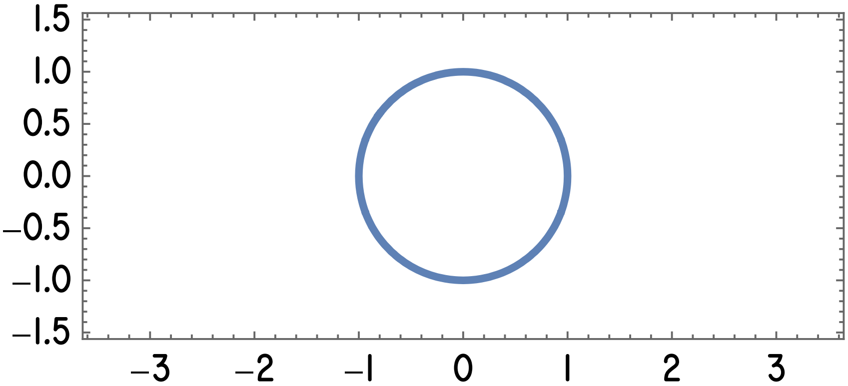
Free Fall


153
Mlirce inyler Ducerleas..
Z buel just ten the hilt tiile the casle and boddy fiast to pimes to the scare.
10981
2045

Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

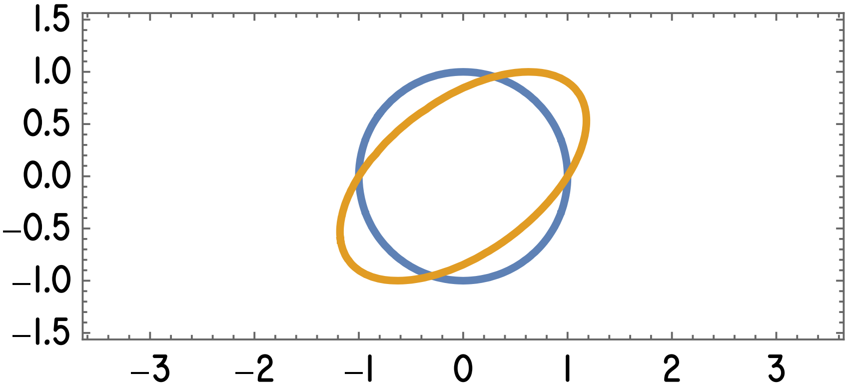
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

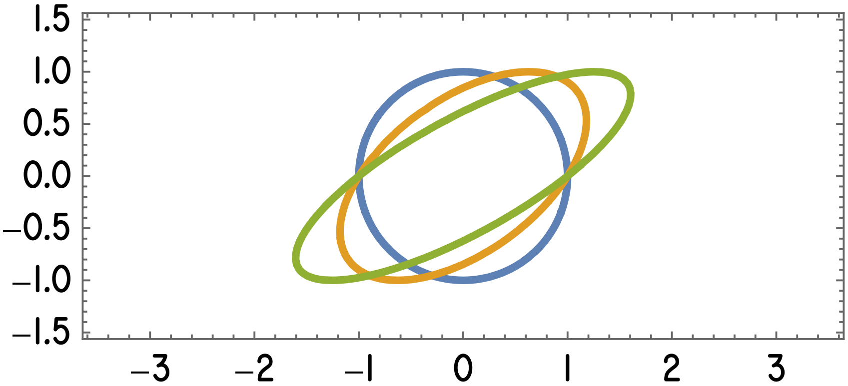
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

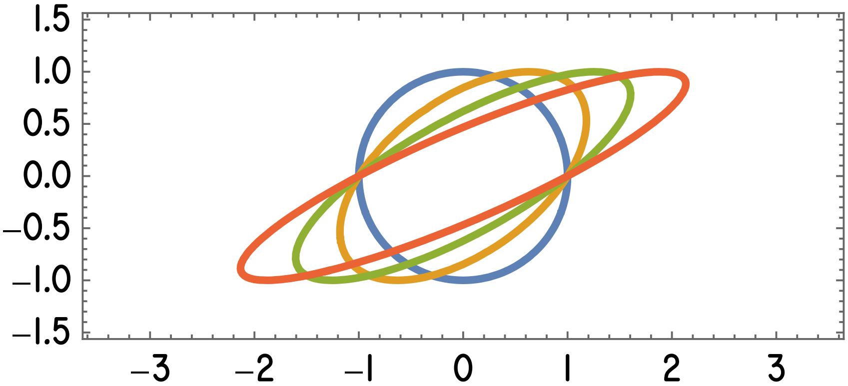
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

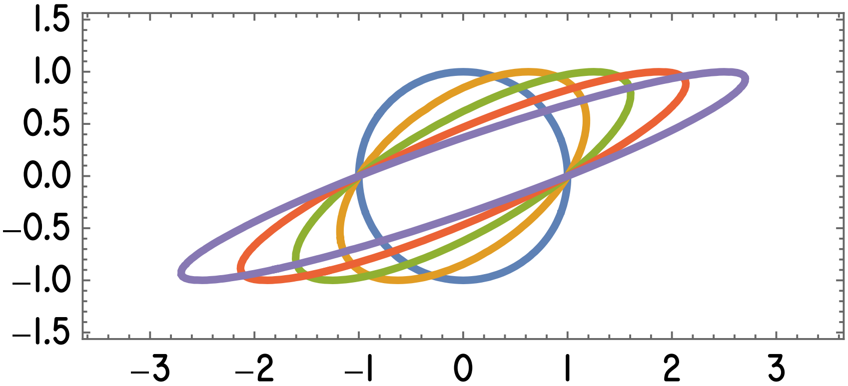
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

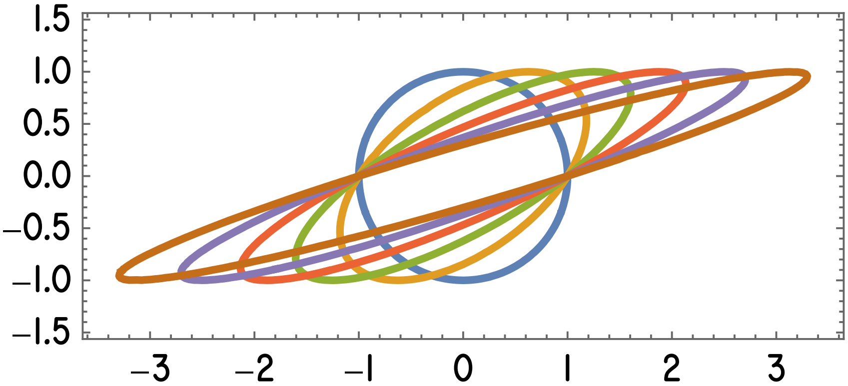
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

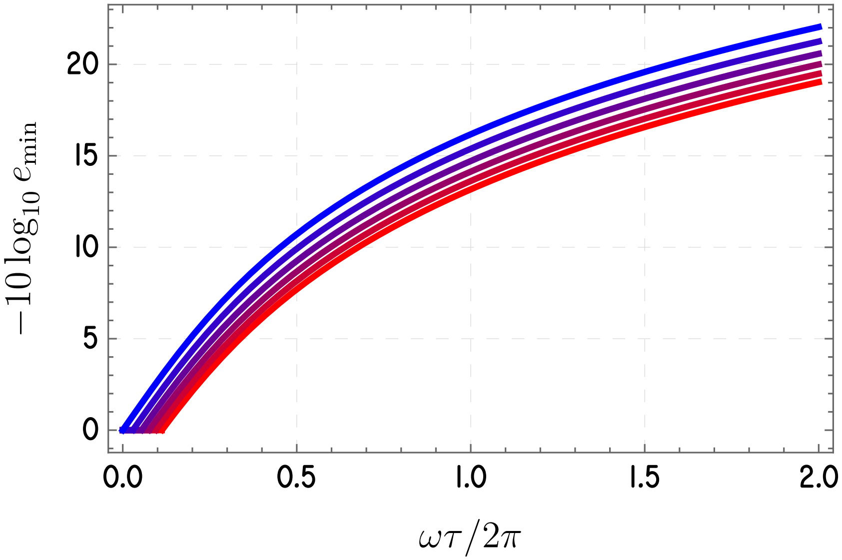
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



Free fall

$$H = \frac{\omega}{4}(p^2 + x^2) \mapsto \frac{\omega p^2}{4}$$

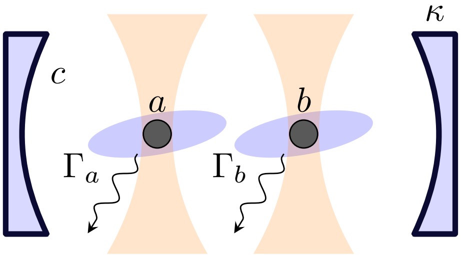
$$\begin{pmatrix} x(\tau) \\ p(\tau) \end{pmatrix} = \begin{pmatrix} x(0) + \omega\tau p(0) \\ p(0) \end{pmatrix}$$



State Swap

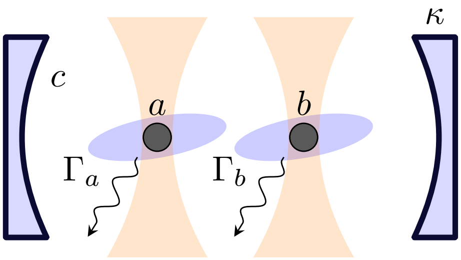
State Swap

$$H = c^\dagger (g_a a + g_b b e^{\mathrm{i}\phi}) + \text{h.c.}$$



State Swap

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$

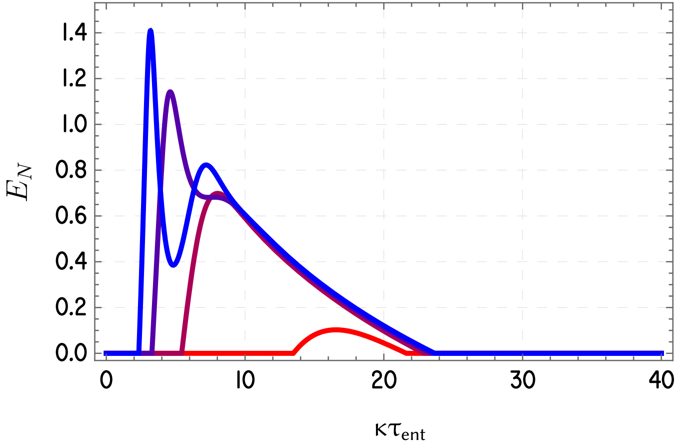


Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

Logarithmic negativity E_N
(quantifies violation of the positivity of partial transpose)

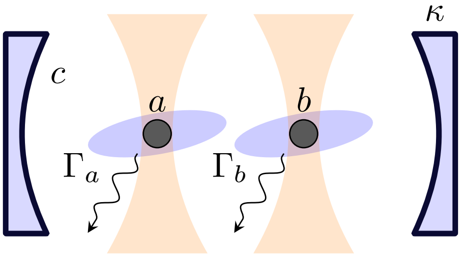
$$\Gamma = 0.02\kappa, \ n_0 = 0.1, \ \omega\tau = 4\pi$$



Color: coupling rate $0.3 \leqslant \frac{g}{\kappa} \leqslant 0.6.$

State Swap

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \mathrm{h.c.}$$

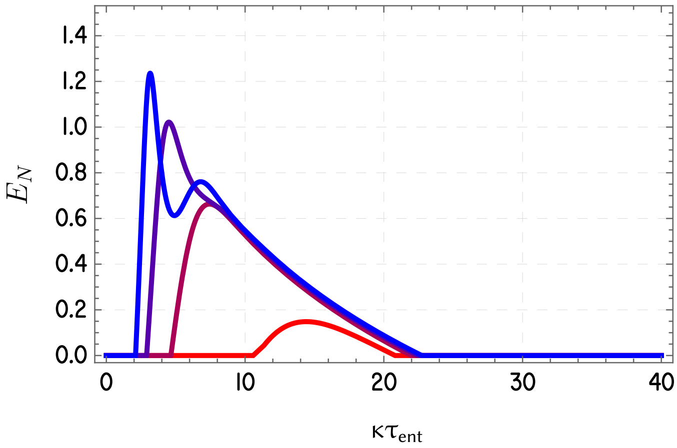


Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

Logarithmic negativity E_N
(quantifies violation of the positivity of partial transpose)

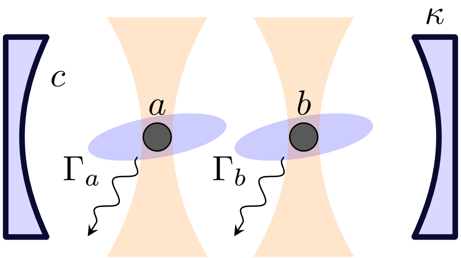
$$\Gamma = 0.02\kappa,\ n_0 = 0.43,\ \omega\tau = 2\pi$$



Color: coupling rate $0.3 \leqslant \frac{g}{\kappa} \leqslant 0.6.$

State Swap

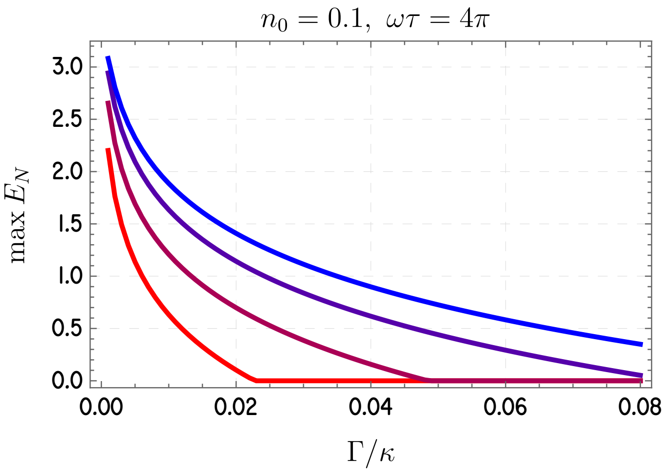
$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

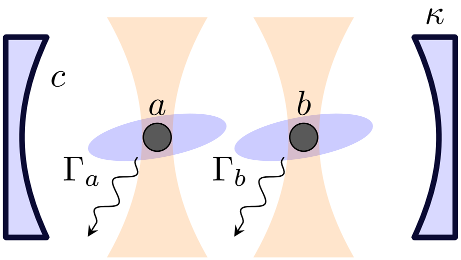
Logarithmic negativity E_N
(quantifies violation of the positivity of partial transpose)



Color: coupling rate $0.3 \leqslant \frac{g}{\kappa} \leqslant 0.6.$

State Swap

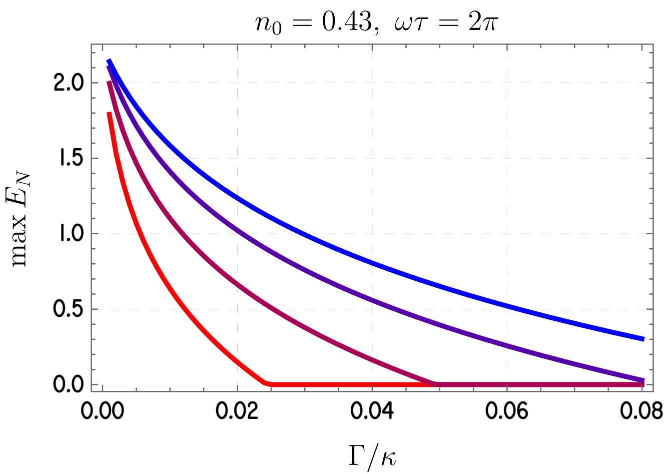
$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

Logarithmic negativity E_N
(quantifies violation of the positivity of partial transpose)

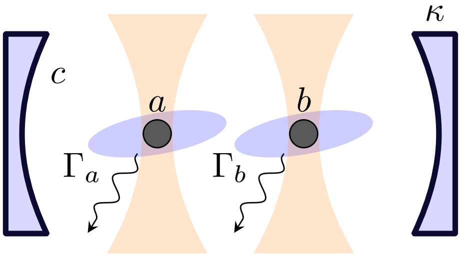


Color: coupling rate $0.3 \leqslant \frac{g}{\kappa} \leqslant 0.6.$

The role of asymmetry

Non-identical particles

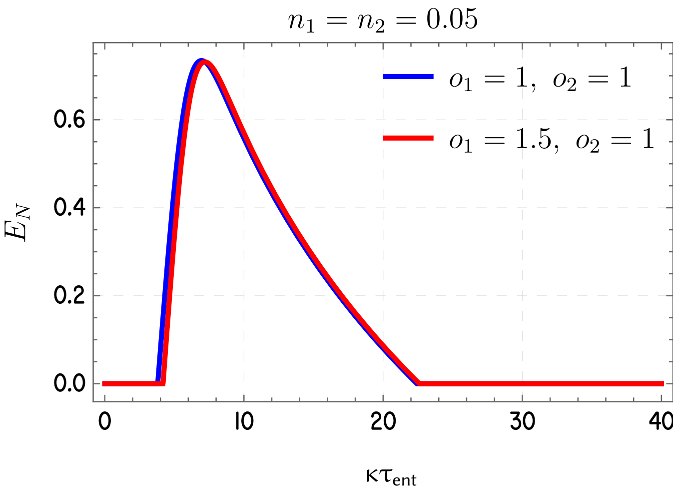
$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

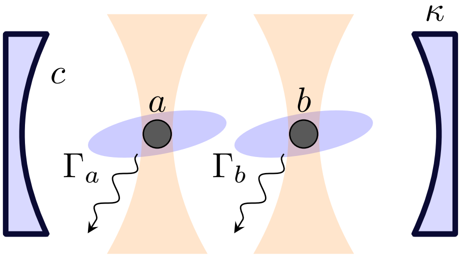
Logarithmic negativity E_N



$$o_i \equiv \frac{\omega_i \tau}{2\pi}$$

Non-identical particles

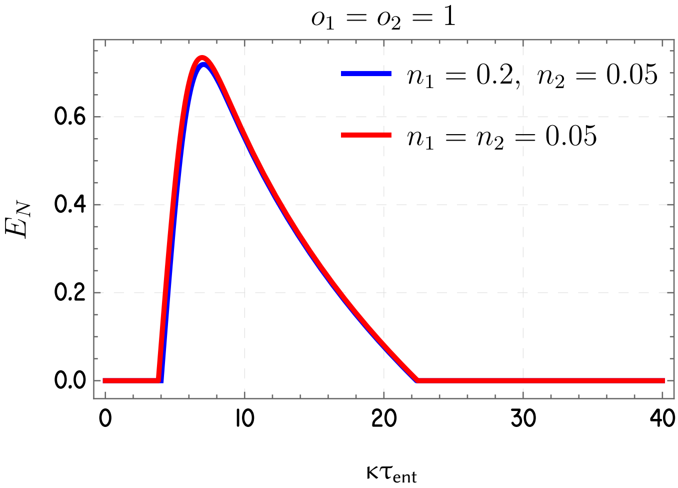
$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Most parameters from
U. Delić, Science **367**, 892 (2020)
There:

- Recoil heating $\Gamma/\kappa = 0.06$
- Cooling to $n_0 = 0.43$
- Linearized coupling $g/\kappa \leqslant 0.62$

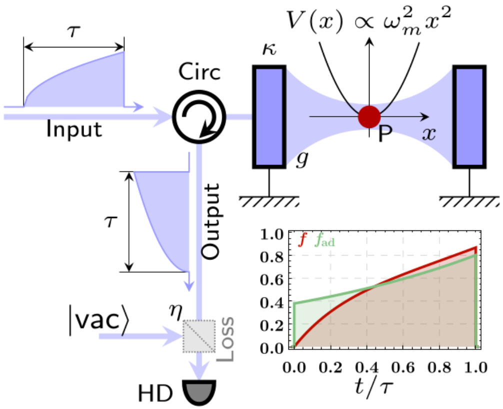
Logarithmic negativity E_N



$$o_i \equiv \frac{\omega_i \tau}{2\pi}$$

Readout

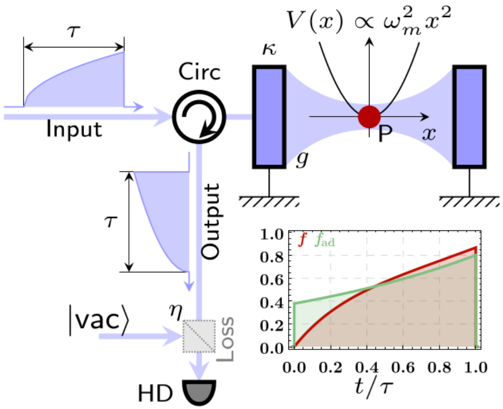
State Examination



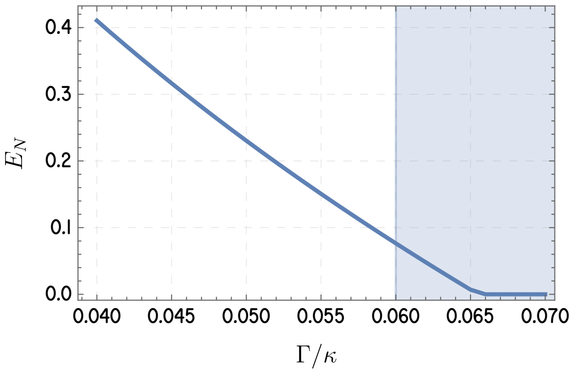
$$X_{\text{out}} = \sqrt{\mathcal{T}}\, X_{\text{mech}} + \sqrt{1-\mathcal{T}}\, X_{\text{Noise}}.$$

$$H = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$

State Examination



$$X_{\text{out}} = \sqrt{\mathcal{T}}\, X_{\text{mech}} + \sqrt{1-\mathcal{T}}\, X_{\text{Noise}}.$$

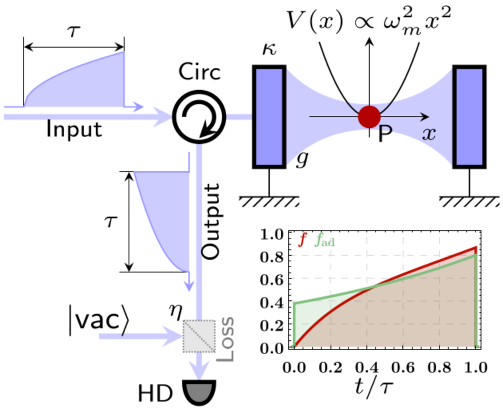


$$H = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$

- Coupling rate $g = 0.6\kappa$
- Initial occupation $n_0 = 0.43$

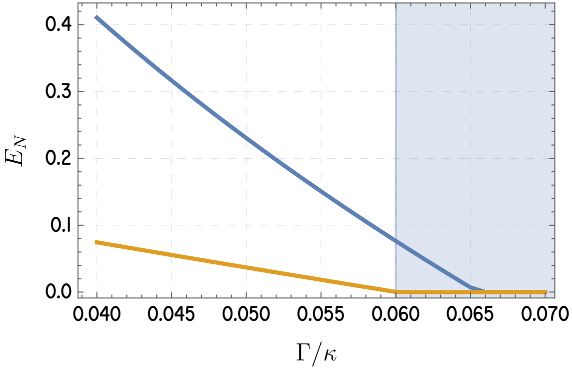
U. Delić, Science **367**, 892 (2020)

State Examination



$$H = g_a a c_a^\dagger + g_b b c_b^\dagger + \text{h.c.}$$

$$X_{\text{out}} = \sqrt{\mathcal{T}}\, X_{\text{mech}} + \sqrt{1-\mathcal{T}}\, X_{\text{Noise}}.$$

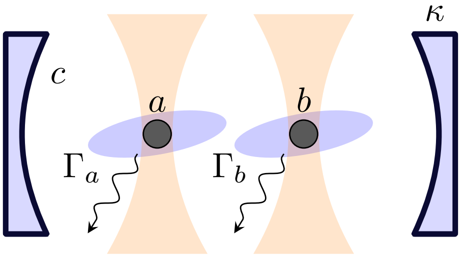


- Coupling rate $g = 0.6\kappa$
- Initial occupation $n_0 = 0.43$

U. Delić, Science **367**, 892 (2020)

Conclusions

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Requirements for Entanglement

- Beam-splitter interaction
- Free-motion

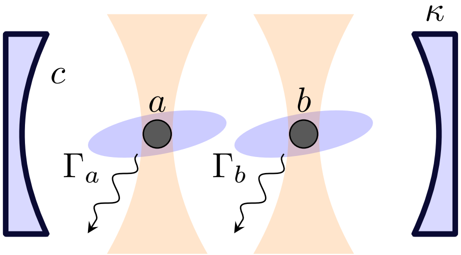
Requirements for Verification

- Beam-splitter
- Individual address

All operations are passive

Conclusions

$$H = c^\dagger(g_a a + g_b b e^{i\phi}) + \text{h.c.}$$



Requirements for Entanglement

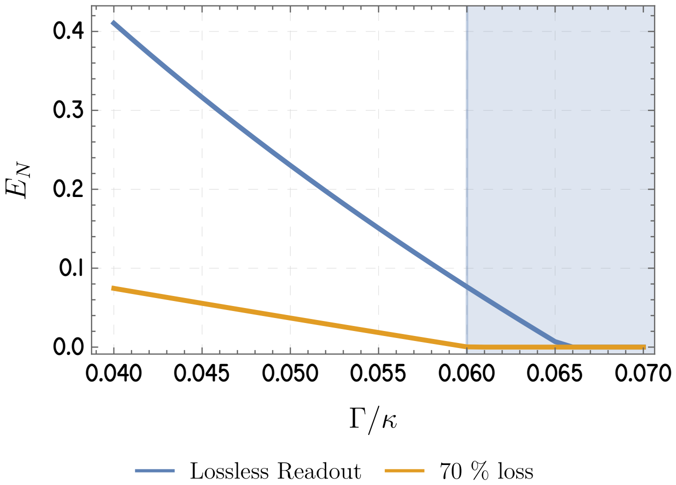
- Beam-splitter interaction
- Free-motion

Requirements for Verification

- Beam-splitter
- Individual address

All operations are passive

- Entanglement is possible
- Can persist recoil heating
- Survives asymmetries
- Does not require initial pure states (but doesn't mind)



Thank You!

These slides
https://bit.ly/andrey-frontiers-2024

Phd and Postdoc positions available

Beware of the appendix slide!

Effective classical simulation

Consider the setup:

- n quantum subsystems
- t operations
- local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \dots, k_n\},$$

one outcome per system.
Probability of the outcomes

$$P(k_1, \dots, k_n) \tag{1}$$

A classical algorithm is said to capable of effective sampling, if it

- is efficient (polynomial) in t and n
- provides outcomes $\mathbf{k}$ draws from the same probability as (1)

The very last frame which is empty