Stroboscopic High-Order Nonlinearity in Optomechanics

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Quantum Engineering of Levitated Systems
Benasque 2022


Norway
Finland
Baltic Sea
Estonia
Latvia
Lithuania
North Sea
Denmark
United Kingdom
Ireland
Netherlands
Berlin
Poland
Belarus
Moscow
Москва
London
Belgium
Germany
Czechia
Paris
Vienna
Slovakia
Ukraine
Austria
Hungary
Moldova
France
Romania
Croatia
Serbia
Black Sea
Italy
Bulgaria
Georgia
Barcelona
Rome
Portugal
Madrid
Spain
Istanbul
Tyrrhenian Sea
Greece
Turkey
Syria
Lebanon
Tunisia
Mediterranean Sea
Iraq
Israel
Jordan
Morocco
Satellite
Google

Main Results

Levitated Nanoparticle

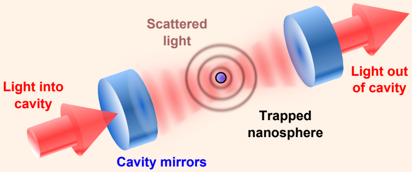
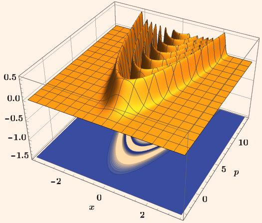


Fig. from Genoni et al. NJP **17**, 073019 (2015).

Approximate Cubic Phase State



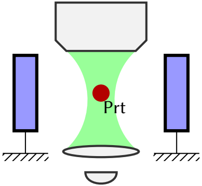
$$|\gamma\rangle = e^{i\gamma x^3}|p\rangle \approx e^{i\gamma x^3}\hat{S}\,|0\rangle$$

Levitated Nanoparticles in Engineered Potentials

$$H_{\mathrm{trap}} = -\frac{1}{2}\int_{\mathrm{Vol}} d\mathbf{r}\, \mathbf{P}(\mathbf{r})\mathbf{E}(\mathbf{r}) \propto -\int d\mathbf{r}\, \mathbf{E}^2(\mathbf{r}),$$

$$\mathbf{P} \propto \mathbf{E},$$

$$\text{Equiv. potential: } V(\mathbf{r};t) \propto -I(\mathbf{r};t),$$



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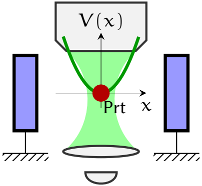

Gaussian intensity profile

$$I(x) \propto \exp\left[-\frac{x^2}{2x_0^2}\right] \approx 1 - \frac{x^2}{2x_0^2}.$$

$$V(x) \propto \omega_m^2 x^2$$

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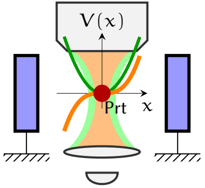


Gaussian intensity profile

$$I(x) \propto \exp\left[-\frac{x^2}{2x_0^2}\right] \approx 1 - \frac{x^2}{2x_0^2}.$$

$$V(x) \propto \omega_m^2 x^2 + k_3 x^3$$

Can engineer complicated $I(x)$, particularly cubic $I \propto x^3$

Requirements

Ability to swich the nonlinear contribution faster than ω_m.

Cubic phase state

Devised by Gottesman, Kitaev and Preskill [1]

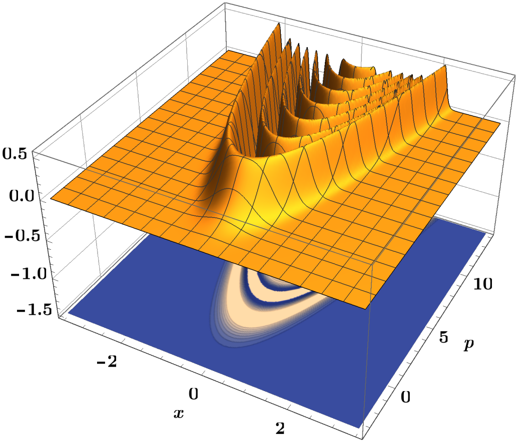
$$|\gamma_{GKP}\rangle \propto e^{-i\gamma x^3} |p = 0\rangle ,$$

Wigner Function

$$W_{GKP}(x, p) \propto \mathrm{Ai}\left[\left(\frac{4}{3\gamma}\right)^{1/3} (3\gamma x^2 - p)\right] .$$

More physical is an approximation

$$|\gamma\rangle = e^{-i\gamma x^3} \hat{S} |0\rangle .$$



[1] Gottesman, Kitaev, Preskill, PRA **64**, 012310 (2001)

Motivation: Measurement-based computing

PHYSICAL REVIEW A **97**, 022329 (2018)

General implementation of arbitrary nonlinear quadrature phase gates

Petr Marek,[1,*] Radim Filip,[1] Hisashi Ogawa,[2] Atsushi Sakaguchi,[2] Shuntaro Takeda,[2] Jun-ichi Yoshikawa,[2,3] and Akira Furusawa[2,†]

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[2]*Department of Applied Physics, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*
[3]*Quantum-Phase Electronics Center, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

(Received 10 August 2017; published 20 February 2018)

We propose general methodology of deterministic single-mode quantum interaction nonlinearly modifying single quadrature variable of a continuous-variable system. The methodology is based on linear coupling of the system to ancillary systems subsequently measured by quadrature detectors. The nonlinear interaction is obtained by using the data from the quadrature detection for dynamical manipulation of the coupling parameters. This measurement-induced methodology enables direct realization of arbitrary nonlinear quadrature interactions without the need to construct them from the lowest-order gates. Such nonlinear interactions are crucial for more practical and efficient manipulation of continuous quadrature variables as well as qubits encoded in continuous-variable systems.

DOI: 10.1103/PhysRevA.97.022329

Motivation: Measurement-based computing

PHYSICAL REVIEW A **97**, 022329 (2018)

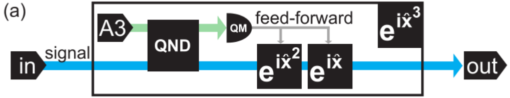

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FIG. 1. Schematic circuits for various implementations of nonlinear gates. QND: quantum nondemolition interaction; QM: quadrature measurement; Ak: ancillary state of the kth order squeezed in $\hat{p} - N\chi_N\hat{x}^{N-1}$. $e^{i\hat{x}^k}$: unitary realization of kth-order nonlinear gate with arbitrary strength. (a) Cubic gate with $N = 3$; (b) $(N+1)$th-

Required $\mathrm{Var}(p - \gamma x^2) \to 0$ for ancilla

Motivation: Measurement-based computing

$$\text{Noise} \propto \text{Var}_{|\text{Ancilla}\rangle}(p - \gamma x^2)$$

For a CPS $|\text{Ancilla}\rangle = |\gamma_{\text{GKP}}\rangle := e^{i\gamma x^3}|p = 0\rangle$,

the variance vanishes $\text{Var}_{|\gamma_{\text{GKP}}\rangle}(p - \gamma x^2)$.

Figure of Merit: Nonlinear Variance

The Nonlinear Variance for the implementation of $\exp[-i\gamma x^k]$ is

$$\sigma_k(\lambda) = \mathrm{Var}(p - \lambda x^{k-1}).$$

For a cubic gate $\exp[-i\gamma x^3]$,

$$\sigma_3(\lambda) = \mathrm{Var}(p - \lambda x^2)$$

Evaluated on vacuum state

$$\sigma_3^{\mathrm{vac}} = 1 + 2\lambda^2$$

Analogy with quadratic squeezing

A quantum state is squeezed when for some θ

$$\mathrm{Var}(p\cos\theta + x\sin\theta) < \sigma_{\mathrm{vac}}$$

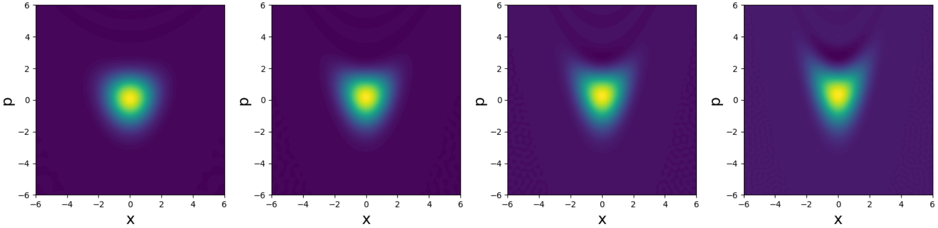
Equivalent to

$$\sigma_2(\lambda) = \mathrm{Var}(p + \lambda x) < \sigma_{\mathrm{vac}}(1+\lambda^2), \text{ with } \lambda = \tan\theta.$$

- Introduction
  - Prerequisites: Nonlinear Potential
  - Cubic States
- Cubic Phase State in Levitated Optomechanics
  - CPS Preparation
  - CPS Evaluation

Why bother with protocols

Wigner functions from $\exp[-i(\gamma\hat{x}^3)\tau]$:



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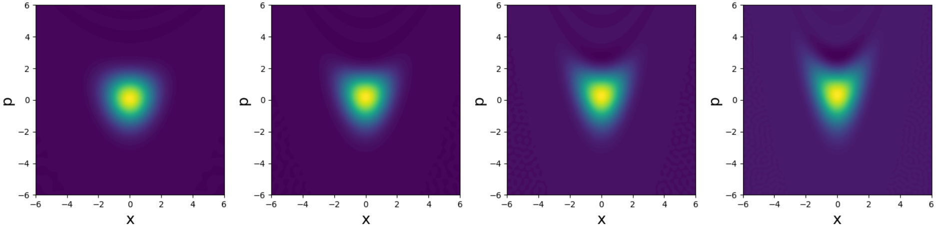
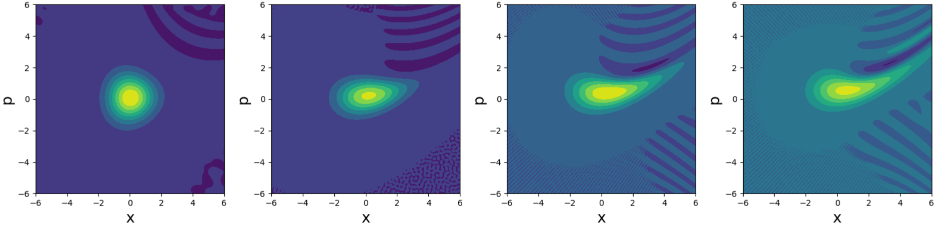


Wigner functions from $\exp[-i(H_{HO}(\hat{p}) + \gamma\hat{x}^3)\tau]$:



The Model

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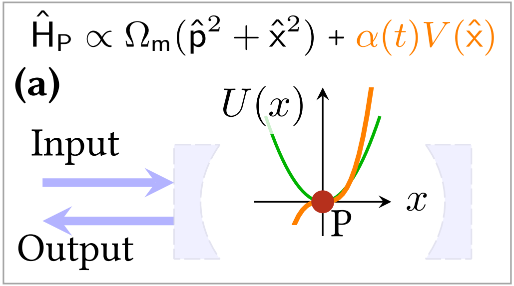
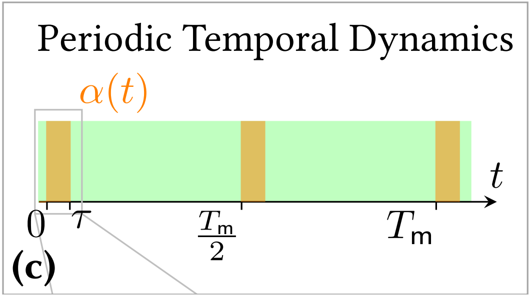
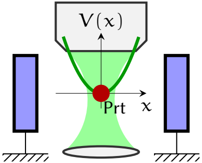
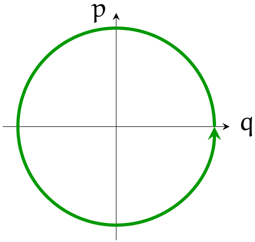




Stroboscopic Application of Nonlinear Potential



Phase space picture



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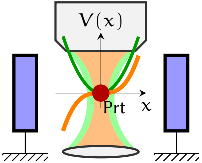

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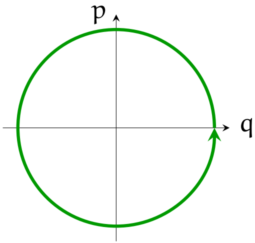

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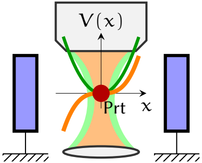
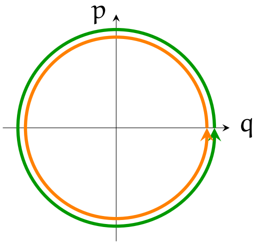
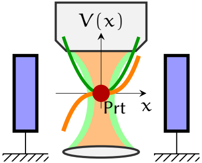


Phase space picture

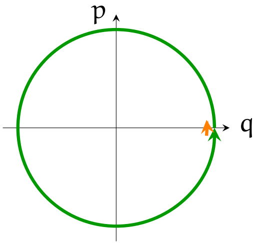


- Continuous application of cubic is smeared out by harmonic evolution

Stroboscopic Application of Nonlinear Potential



Phase space picture



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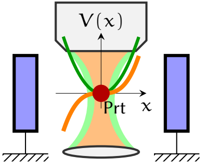
- Apply insantaneous nonlinearity at certain phases of oscillations $t = 0, T_m, 2T_m \ldots.$

Stroboscopic Application of Nonlinear Potential



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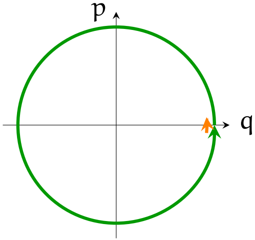

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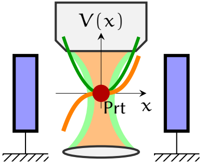
- Mechanical decoherence kicks in after $t = \Gamma_m^{-1}$

Stroboscopic Application of Nonlinear Potential



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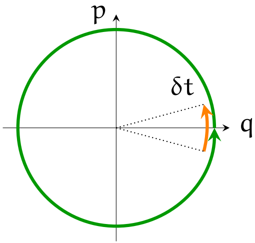

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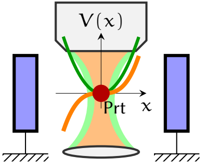
Stroboscopic Application of Nonlinear Potential



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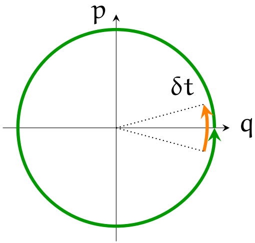

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Tradeoff between the number of periods $M_T < \Gamma_m^{-1}/T_m$ and duration of application within a certain period δt.

M_T vs δt.

Stroboscopic QND measurement of mechanical motion

Quantum Non-Demolition measurement of mechanical position

$$H_{OM} = \Delta a_L^\dagger a_L + \Omega_m a_M^\dagger a_M + g X_L X_M$$

In order to realize a true QND coupling, get rid of the first two terms:
Tune on resonance $\Delta = 0$ and

Modulate the coupling rate

$$g \mapsto g(t) = g_0 \cos 2\Omega_m t$$

Hamiltonian in rotating frame

$$H_{OM} \mapsto \propto g_0 \tilde{X}_L \tilde{X}_M$$

Use short pulses

$$\Omega_m \tau \ll 1$$

Effective Hamiltonian

$$H_{OM} \mapsto g X_L X_M.$$

The Model

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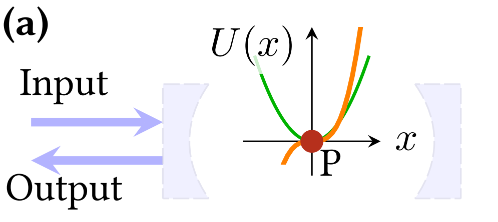
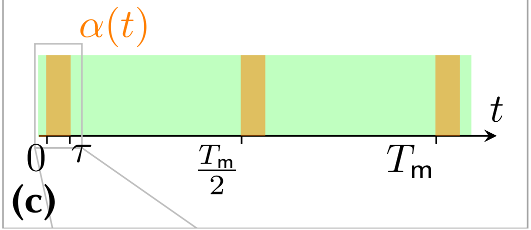
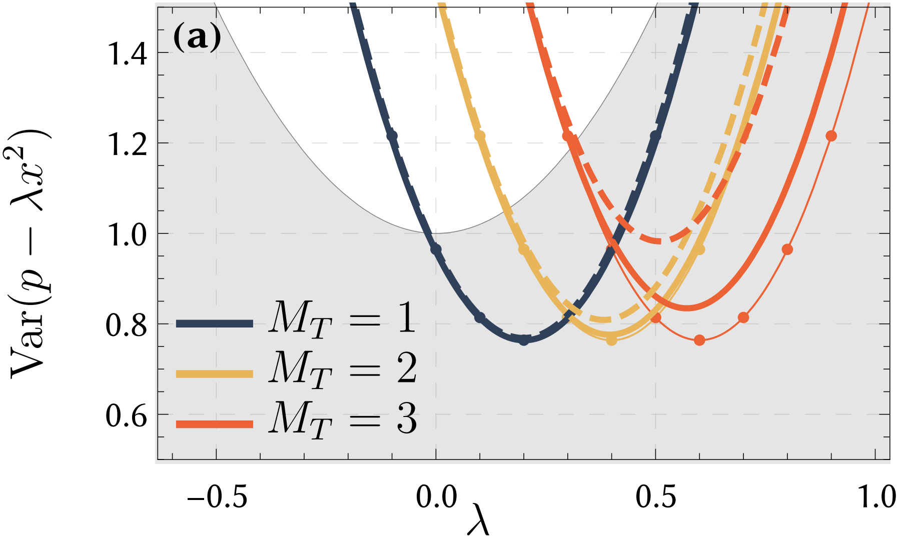
$\hat{H}_P \propto \Omega_m(\hat{p}^2 + \hat{x}^2) + \alpha(t)V(\hat{x})$

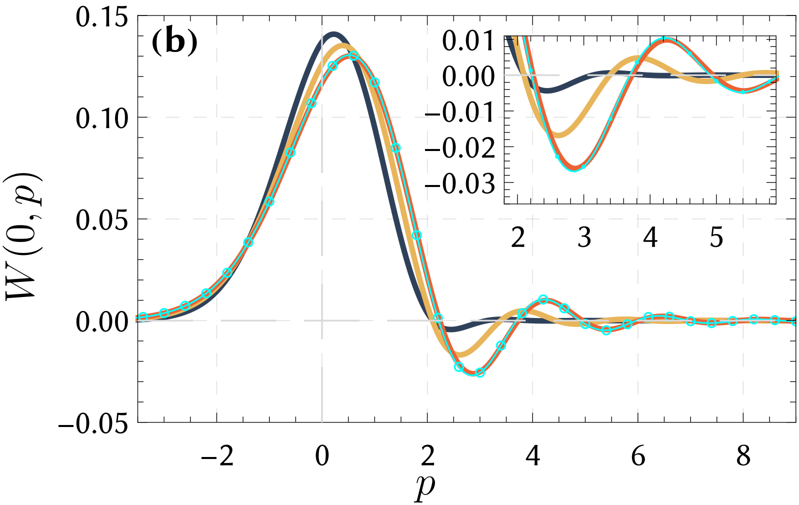




Nonlinear Variance



Wigner Function Cuts



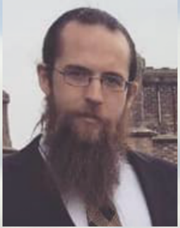
For red and cyan $\mathrm{Tr}[\rho_{\mathrm{red}}\rho_{\mathrm{cyan}}]/\mathrm{Tr}[\rho^2_{\mathrm{cyan}}] = 0.9877$

Introduction

- Prerequisites: Nonlinear Potential
- Cubic States

Cubic Phase State in Levitated Optomechanics

- CPS Preparation
- CPS Evaluation



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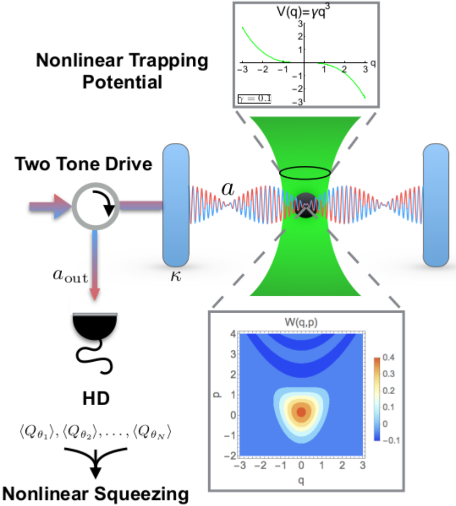
NJP **21** 113050

The Model



Approximate cubic phase state in mechanics

$$e^{i\gamma q^3}\hat{S}\,|0\rangle\,.$$

Pulsed QND interaction

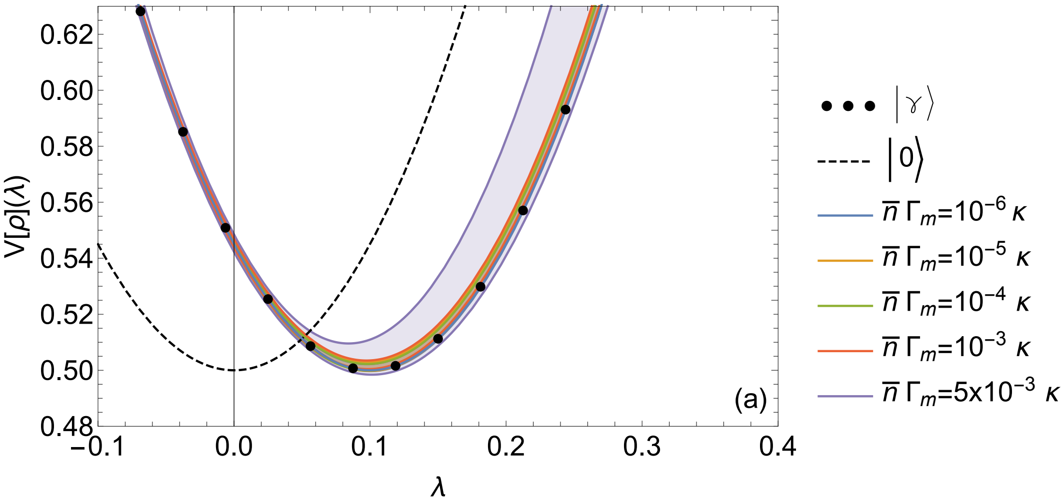
$$H_{\text{int}} \propto x_{\text{light}}(q\cos\phi + p\sin\phi).$$

Detect leaking light

Estimate nonlinear variance

$$q_{\text{NL}} = p - \lambda x^2 \Rightarrow \text{Var}[q_{\text{NL}}]$$

Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



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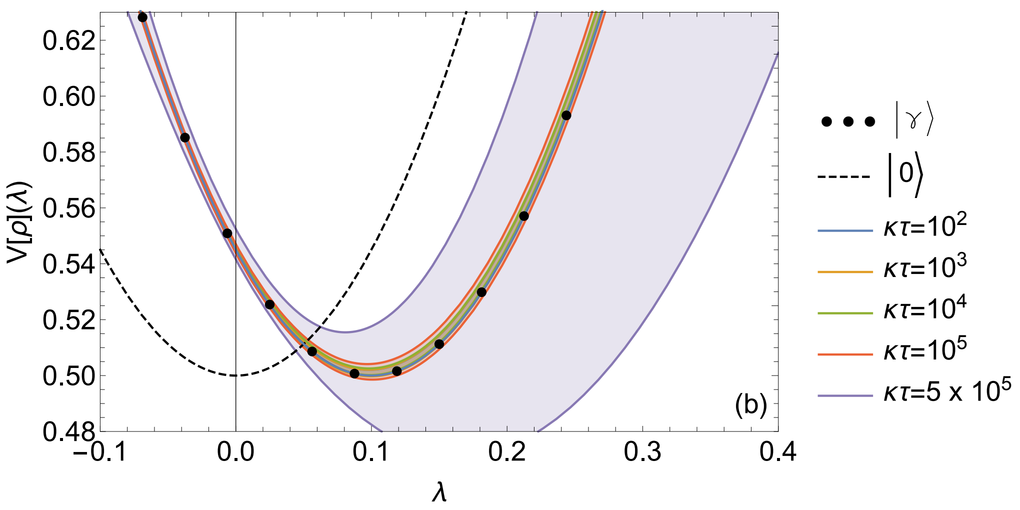

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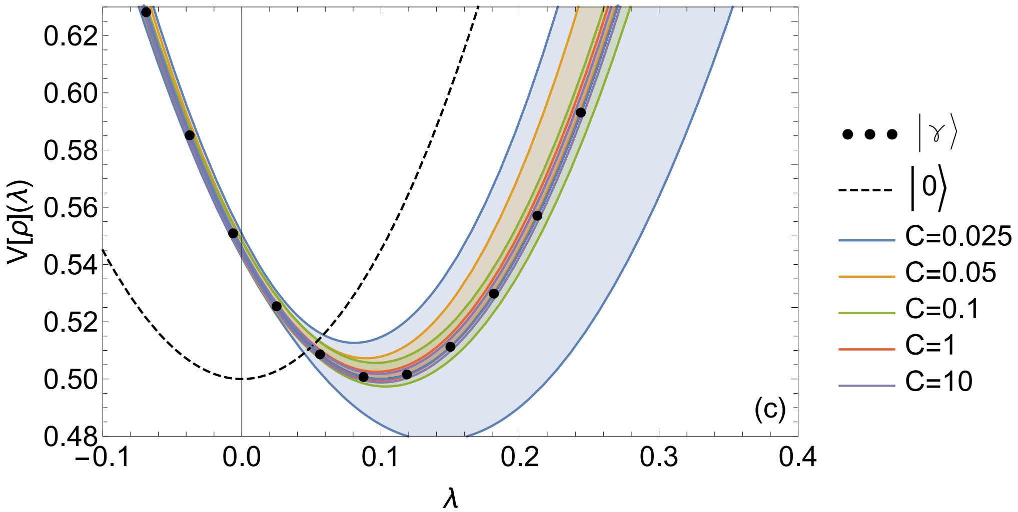

Conclusion

- Optomechanics provides full linear control over a mechanical oscillator
- Levitated nanoparticles combine advantages of linear optomechanics with possibilities to engineer nontrivial nonlinear potentials
- Stroboscopic application of a cubic potential allows creation of approximate Cubic Phase States
- With the toolbox of optomechanics these states can be read out, verified and used for quantum computation

Thank You!