Quantum stroboscopic nonlinearity from optomechanics for linear systems

Andrey A. Rakhubovsky <rakhubovsky@optics.upol.cz>

Department of Optics, Palacký University, 17. Listopadu 12, 771 46 Olomouc, Czech Republic

Palacký University Olomouc

Quantum Stroboscopic Nonlinearity in Levitated Optomechanics

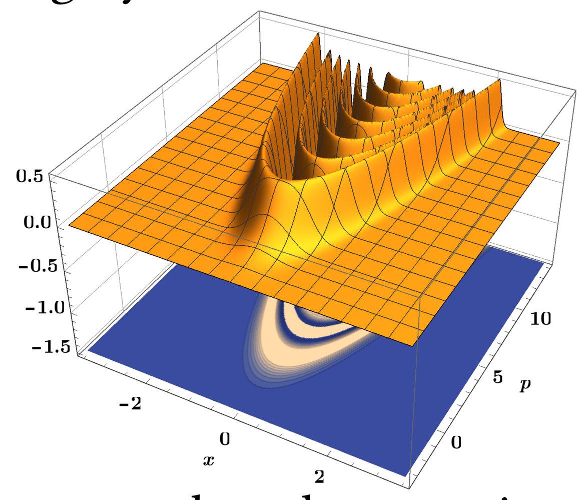
Introduction and Motivation

Recently reported ground state cooling of levitated nanoparticles (NP) combined with the ability to engineer nonlinear motional potential of these systems makes them a good candidate for truly quantum applications.

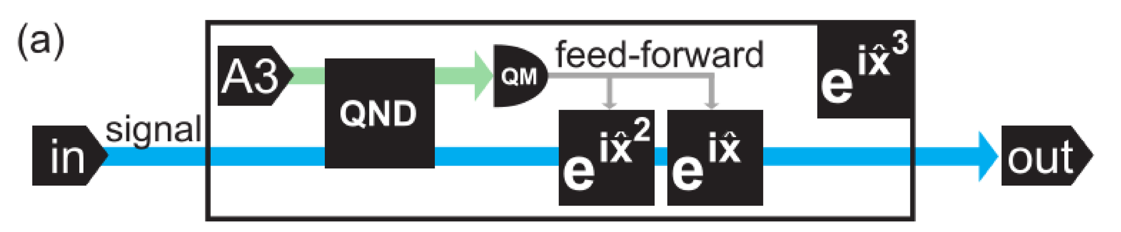
> We propose to engineer an approximate motional cubic phase state (CPS) of a levitated nanoparticle

Cubic phase space is a highly-nonclassical state:

$$|\gamma\rangle = e^{-i\gamma\hat{x}^3}|p=0\rangle \approx e^{-i\gamma\hat{x}^3}\hat{S}\,|0\rangle\,.$$



Has applications in measurement-based computing (Fig. from [1])



Figures of merit: negativity of Wigner function and *nonlinear squeezing.*

$$\exists\lambda\in\mathbb{R}:\sigma^{(3)}(\lambda)\equiv \operatorname*{Var}_{\rho}(p-\lambda x^2)<\sigma_{\mathrm{vac}}$$

Compare with ordinary (linear) squeezing

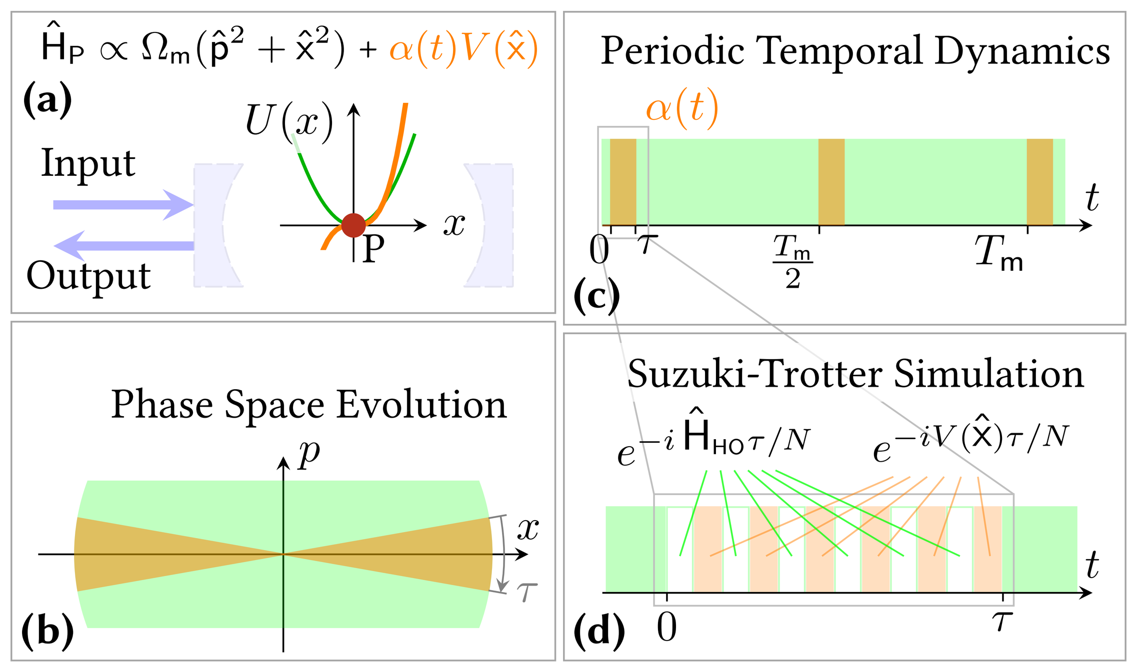
$$\operatorname{Var}(x\cos\theta - p\sin\theta) = (1+\lambda^2)^{-1}\operatorname{Var}(p-\lambda x)\propto\sigma^{(2)},$$

with $\lambda=\tan^{-1}\theta$. Squeezing condition: $\sigma^{(2)}(\lambda)<1+\lambda^2$.

Important thresholds for classical states and Gaussian states:

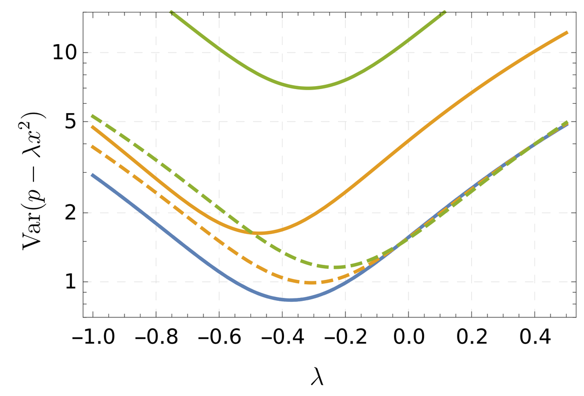
$$\sigma_{\mathrm{NC}}(\lambda)=1+2\lambda^2,\quad \sigma_{\mathrm{NG}}(\lambda)=3\lambda^{2/3}/2^{1/3}.$$

Implementation with levitated optomechanics



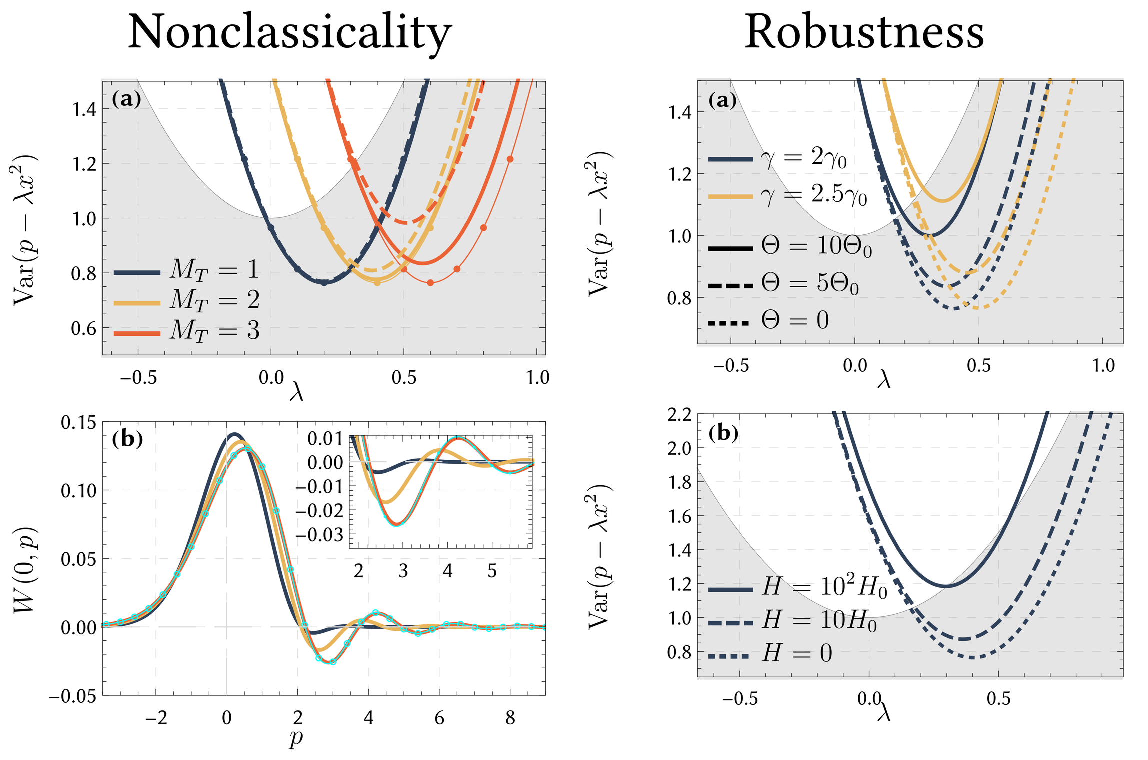
Stroboscopic dynamics allows to simulate the desired purely nonlinear dynamics from the full dynamics including free motion ($\propto p^2$):

$$e^{-i[(p^2+x^2)+\alpha x^3]} \rightsquigarrow e^{-i\alpha x^3}$$



Evolution by full Hamiltonian does not produce the needed correlations to have nonlinear squeezing.

Results [2]



Created state shows negativity of Wigner function, nonlinear squeezing, and some robustness to environmental heating. Base decoherence level H_0 corresponds to decreased by 100 heating from [3].

Broadcasting nonlinearity to a linear system [arXiv:230?.?????]

Motivation and preliminaries

Consider two harmonic oscillators that can have quantum non-demolition (QND) interaction (written as unitary with controllable gains $\chi_{1,2}$)

$$\hat{U}_{\mathrm{QND}}=\exp[-i(\chi_1 qY+\chi_2 pX)].$$

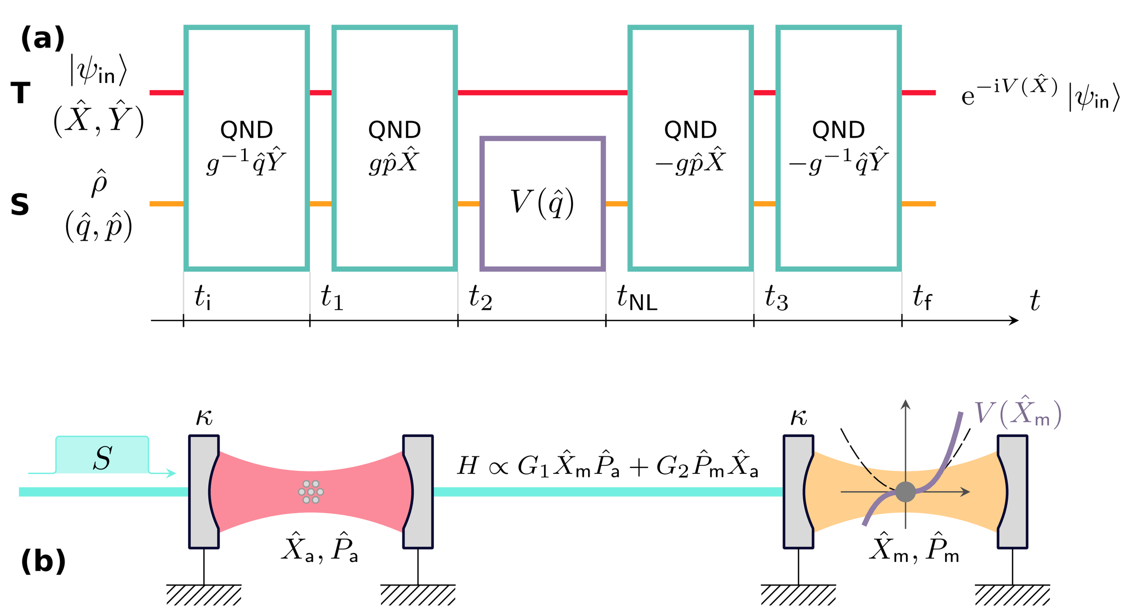
One of the oscillators (*source*) also has an access to a nonlinear transformation

$$\hat{U}_{\mathrm{NL}}=\exp[-i\alpha V(\hat{q})],$$

with $V(\bullet)$ being a regular nonlinear function [e.g., $V(\xi)\propto\xi^3$].

> We propose to implement an ideal unitary nonlinear transformation $e^{-iV(X)}|\psi_{\mathrm{in}}\rangle$ on the linear target system using *only linear* QND interactions with the source system.

Principal setup



Above: a sequence of optimally arranged QND interactions between source and target system. Below: an example implementation with spin ensemble and a levitated nanoparticle [4]. QND gate is implemented in a pulsed manner with help of squeezed light [5].

After the first two QND interactions (assuming arbitrary gains $g_{(1,2)}$ the input-output relations read:

$$\begin{aligned}\hat{q}_2&=g_2\hat{X}_i+(1-g_1g_2)\hat{q}_i,\\ \hat{p}_2&=\hat{p}_i+g_1\hat{Y}_i,\\ \hat{X}_2&=\hat{X}_i+g_1\hat{q}_i,\\ \hat{Y}_2&=g_2\hat{p}_i+(1-g_1g_2)\hat{Y}_i.\end{aligned}$$

Assuming $g_2=1/g_1=g$, the input $\hat{X}$ quadrature is mapped onto $\hat{q}$ and amplified: $\hat{q}_2=g\hat{X}_i$.

The nonlinear transformation maps $\hat{p}_3=\hat{p}_2+\alpha V'(\hat{q}_2)$. The two remaining QND (i) transfer the nonlinearity back to the target and (ii) cancel the effect of the source's initial state. Importantly, the nonlinearity is amplified:

$$\hat{X}_f=\hat{X}_i;\quad \hat{Y}_f=\hat{Y}_i-gV'(g\hat{X}_i).$$

Approximate Nonlinear Gate

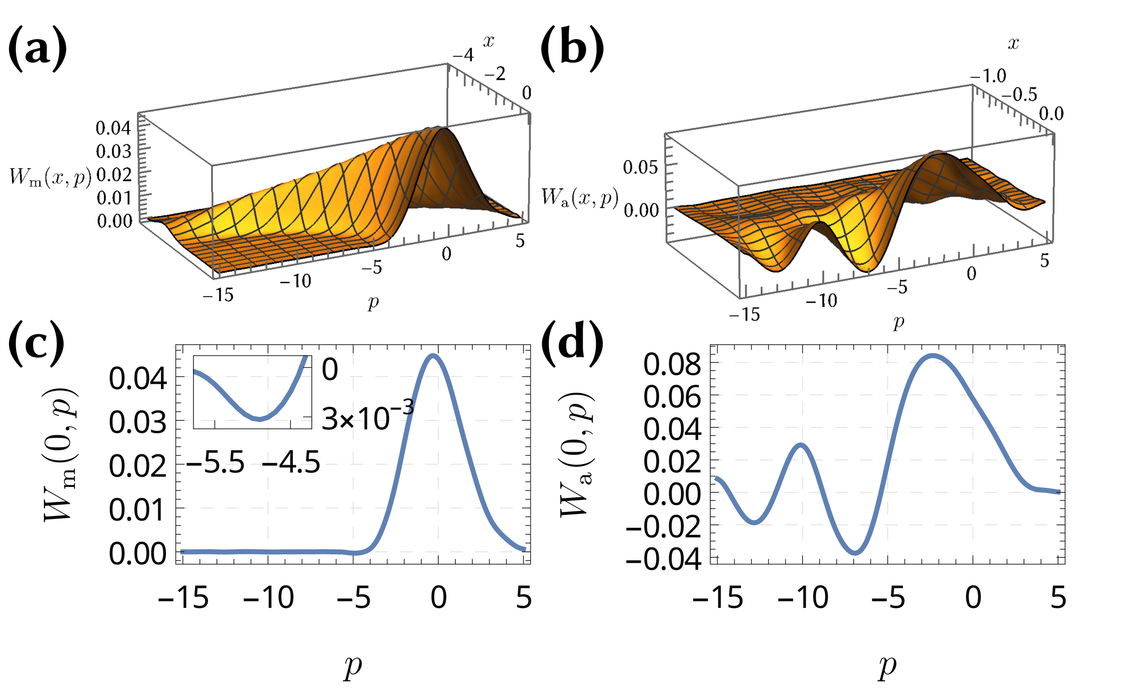
An approximate gate on the target is implemented by $e^{-ig_2pX}e^{-iV(q)}e^{+ig_1pX}$.

The corresponding input-output relations for the target are

$$\hat{X}_f=\hat{X}_i;\quad \hat{Y}_f=\hat{Y}_i+g_2V'(g_1\hat{X}_i+\hat{q}_i).$$

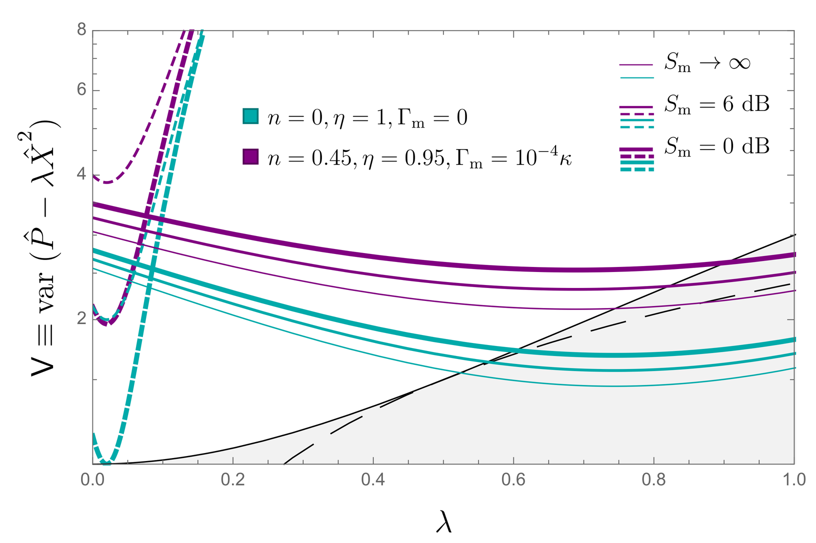
The term $\propto\hat{q}_i$ contributes noise which can be compensated by initial squeezing of mechanical oscillator (available for NPs).

Simulation of Wigner functions



Numerical simulation of Wigner functions of (a,c) mechanics at t_{NL}, and (b,d) atoms at t_f for idealized parameters. Numerical simulation assumes decoherence rates of atoms and mechanics [in units of cavity linewidth κ] $\gamma_a=10^{-7}\kappa$, $\gamma_m=10^{-10}\kappa$. Mechanical heating rate $\Gamma_m=10^{-5}\kappa$.

Simulation of nonlinear squeezing



Gray lines: thresholds of non-classical and non-Gaussian states. Dashed lines: $\sigma^{(3)}$ of mechanics at t_{NL}. n: initial mechanical occupation, η — optical loss of the mediator, S — optical squeezing of mediator.

References

1. Marek, P. et al. *Physical Review A* **97,** 022329 (Feb. 2018). 2. Rakhubovsky, A. A. & Filip, R. *npj Quantum Information* **7,** 120 (July 2021). 3. Delić, U. et al. *Science* **367,** 892–895 (Feb. 2020). 4. Thomas, R. A. et al. *Nature Physics,* 1–6 (Sept. 2020). 5. Manukhova, A. D. et al. *npj Quantum Information* **6,** 4 (1 Jan. 8, 2020).