Optimal Generation and Detection of Nonclassical Correlations

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Introduction

We propose to use two-mode-squeezing operation in optomechanical system to generate non-classical correlations between light and mechanics. An optimization routine allows maximization of attainable correlations and an effective detection protocol allows their detection without full tomography.

Toolbox and Notations

Linearized Optomechanical Interaction

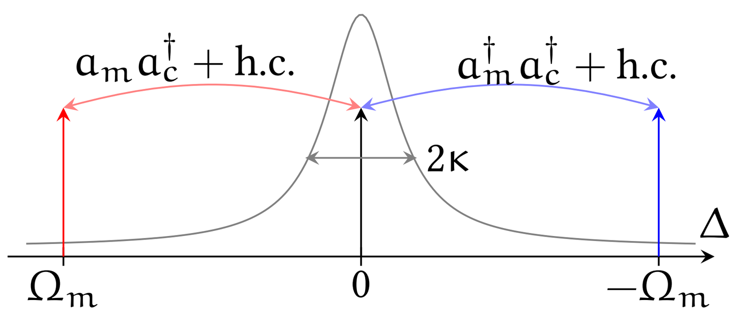
We consider a levitated nanoparticle trapped in a harmonic potential $\frac{1}{2}m\Omega_m^2 x^2$. The particle is linearly coupled at rate g to a single mode of cavity with (amplitude) decay rate κ. The particle is heated by a Markovian bath at rate Γ. The Hamiltonian of the system (in a suitable rotating frame)

$$H = \Delta a_c^\dagger a_c + \Omega_m a_m^\dagger a_m - g(a_c + a_c^\dagger)(a_m + a_m^\dagger),$$

where $\Delta = \omega_{cav} - \omega_{drive}$ is the detuning between the drive (tweezer, in case of coherent scattering coupling) and the cavity resonance.

Resolved-sideband Optomechanics

In the resolved sideband $\kappa \ll \Omega_m$, resonant detuning $\Delta = \pm\Omega_m$ allows resonant enhancement of a certain parametric interaction (see figure).



$$\Delta = \Omega_m, \Rightarrow H_{eff} \propto a_c a_m^\dagger + \text{h.c. [Beamsplitter]},$$
$$\Delta = -\Omega_m, \Rightarrow H_{eff} \propto a_c^\dagger a_m^\dagger + \text{h.c. [Amp.]}.$$

These Hamiltonians generate linear dynamics and preserve Gaussianity of initial states.

Pulsed Optomechanics

Instead of steady states, we consider the quantum states of radiation defined in pulses. By advantage of an input-output relation

$$a^{out}(t) = -a^{in}(t) + \sqrt{2\kappa}a_c(t),$$

we express the leaking field via the incoming and the intracavity ones.

The states of the pulses are then defined via bosonic operators

$$\mathcal{A}^{in} = \int_0^\tau dt\, f^{in}(t)a^{in}(t), \quad \mathcal{A}^{out} = \int_0^\tau dt\, f^{out}(t)a^{out}(t),$$

where the functions $f(\cdot)$ are normalized to ensure proper commutations like $[\mathcal{A}^{in}, \mathcal{A}^{in\dagger}] = 1$.

Quantum States and Detection

The system is described by a vector of quadratures $\boldsymbol{r} = (\mathcal{X}^{out}, \mathcal{Y}^{out}, x_m, p_m)$. At each moment the quantum state of the system is Gaussian, and fully described by the matrix of second moments (covariance matrix [CM]):

$$\mathbb{V}_{ij} = \langle \boldsymbol{r}_i \circ \boldsymbol{r}_j \rangle = \begin{pmatrix} \mathbb{V}^L & \mathbb{V}^C \\ \mathbb{V}^{CT} & \mathbb{V}^M \end{pmatrix}. \qquad (1)$$

Two-mode squeezing

State is squeezed if the minimal eigenvalue of the corresponding covariance matrix is smaller than shot noise:

$$S_{dB} = -10\log_{10}\frac{\min[\text{Eig}(\mathbb{V})]}{\sigma_{vac}}.$$

Logarithmic negativity

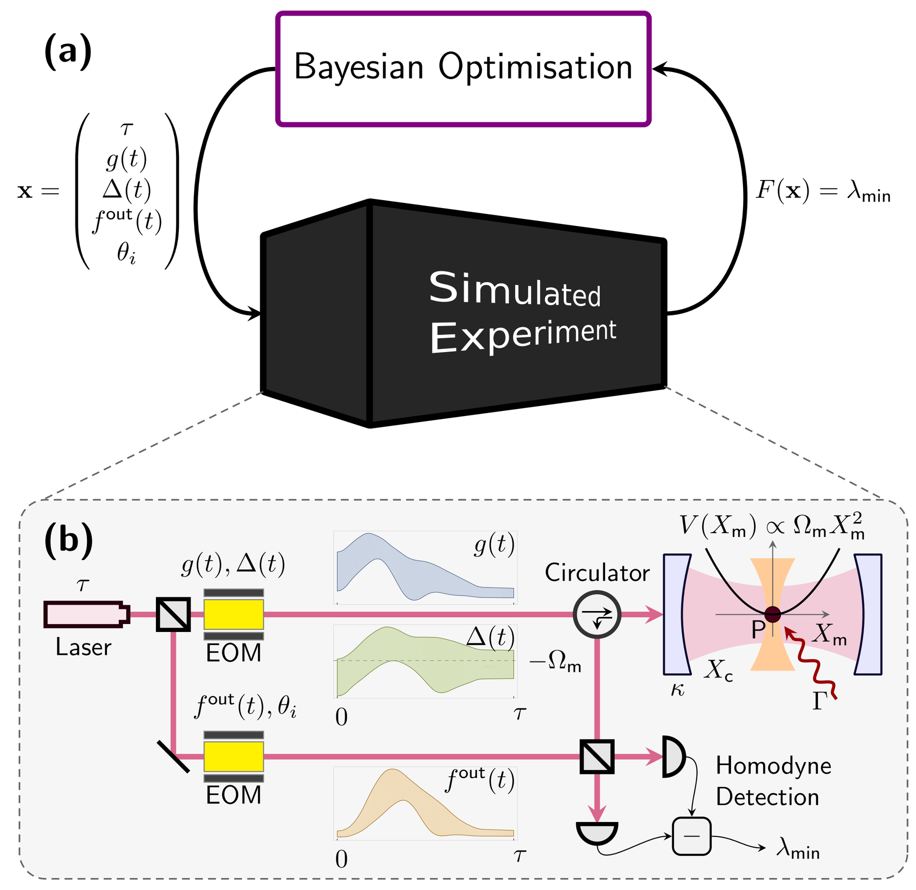
For a bipartite state with CM as in Eq. (1) the *Logarithmic Negativity* can be computed as $E_N = \max[0, -\log\nu_-]$, where

$$\nu_- = \frac{1}{\sqrt{2}}\sqrt{\Sigma_V - \sqrt{\Sigma_V^2 - 4\det\mathbb{V}}},$$

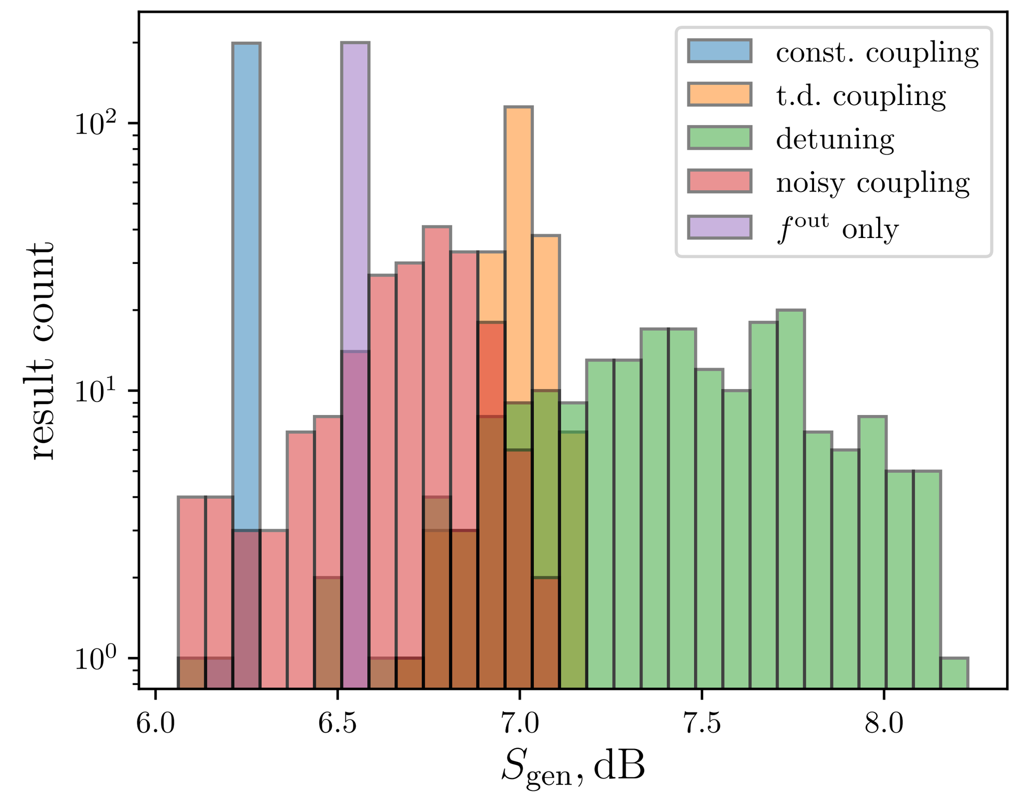
is the smaller symplectic eigenvalue of the covariance matrix of the partially transposed quantum state, with $\Sigma_V = \det V_m + \det V_L - 2\det V_c$.

Optimal Generation of Correlations [1]

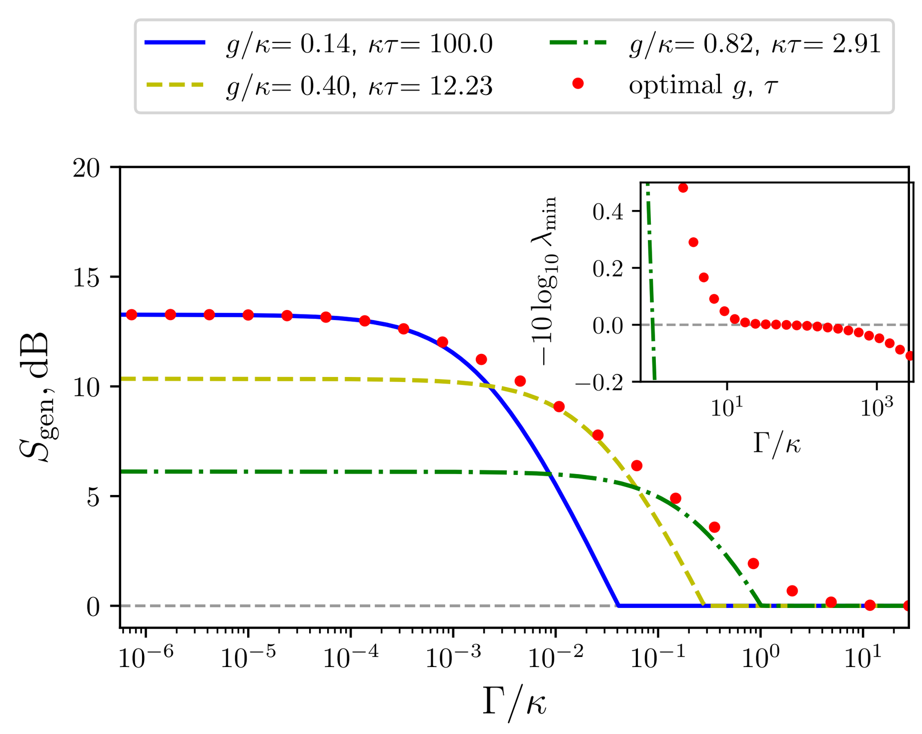
A blue-detuned driving pulse creates two-mode squeezing between mechanical motion and leaking light. The pulse is defined by the drive strength, detuning, and duration, and during the detection one has to specify the temporal mode f^{out} being detected.



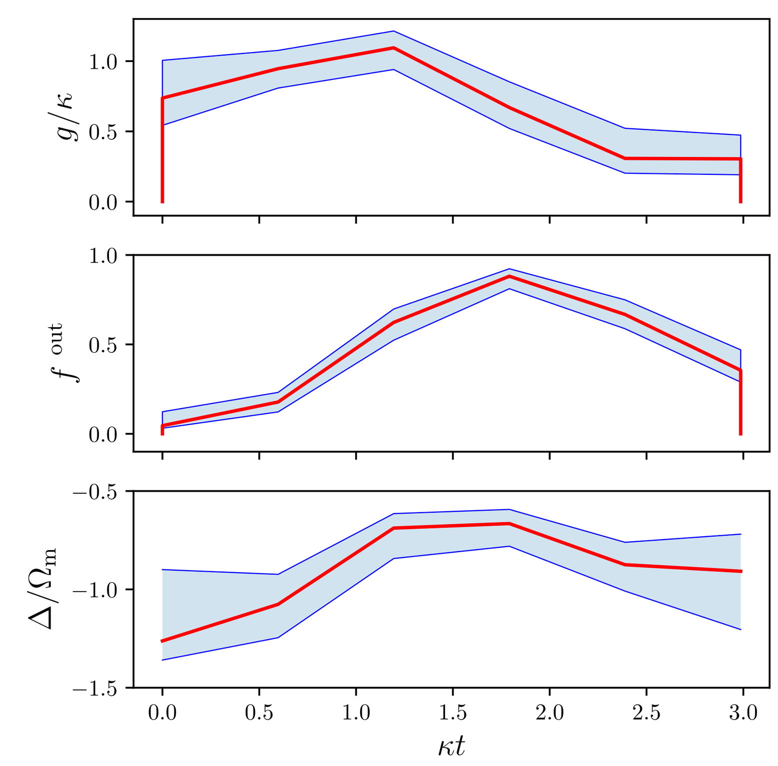
Expanding the optimization parameter space allows obtaining steadily increasing values of squeezing. The histogram shows results of 200 best optimization runs.



Optimization of rectangular pulses (g = const) allows to generate correlations at strong heating $\Gamma = \gamma_m n_{th}$.

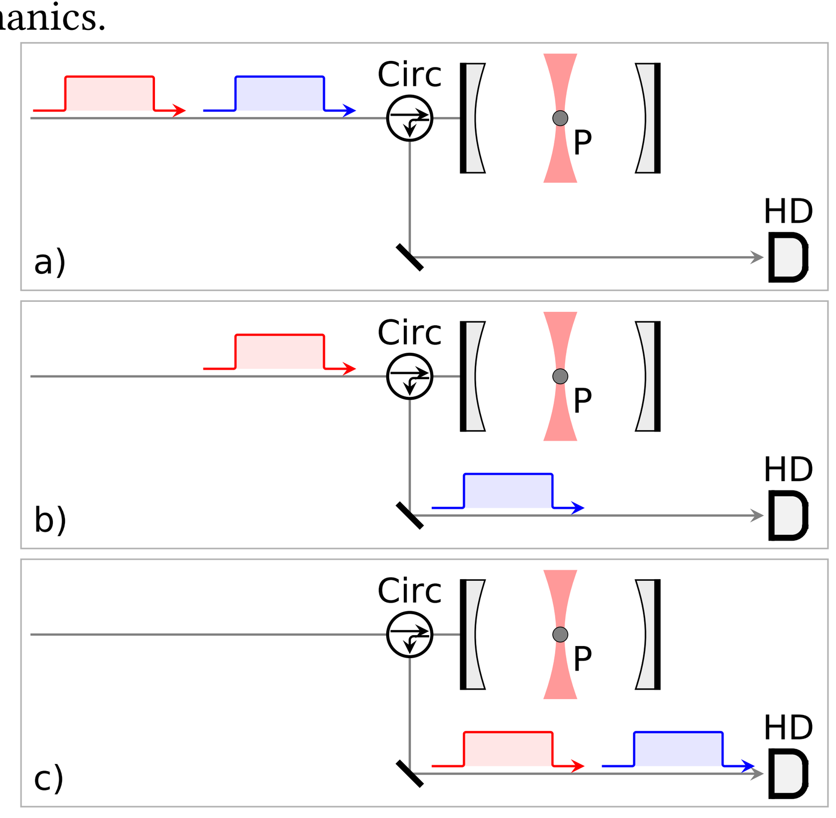


Examples of the optimal coupling rate (g), detection profile (f^{out}) and detuning (Δ) dependence on time. Red lines show mean value of the 200 best optimization repetitions. Blue area bounds 5 times standard deviation in pulse parameters.



Optimal Detection of Correlations [2]

A red-detuned pulse swaps the mechanical state to the leaking field. Probing this field allows indirect probing the mechanics.



Two-mode squeezing manifests in suppression of the smallest eigenvalue of the covariance matrix. In the eigenbasis, this eigenvalue is the variance of a quadrature.

Consider rotated bases for both light and mechanics

$$X^k(\theta) = Q^k\cos\theta + P^k\sin\theta, \quad \text{with } k = L, M$$

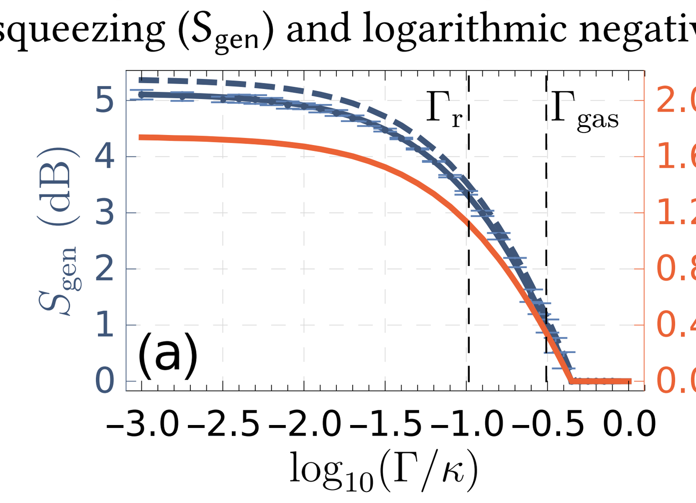
A generalized quadrature is

$$X_{gen} = X^L(\theta_L)\cos\phi + X^M(\theta_M)$$

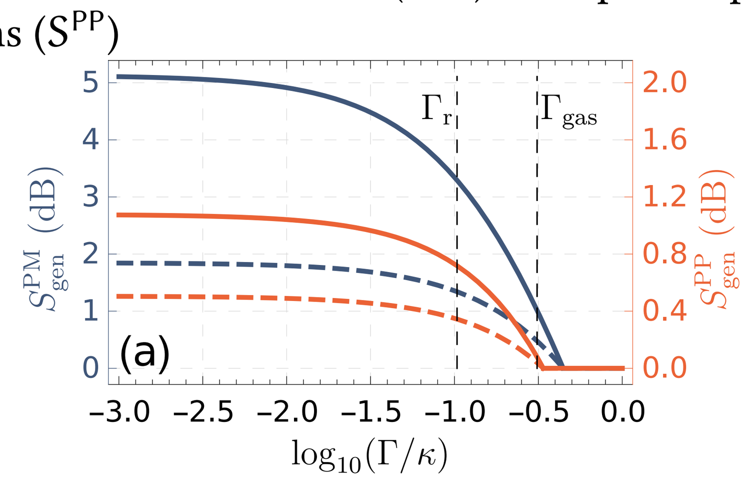
Provided optimal angles,

$$\text{Var}X_{gen}[\theta_L^{opt}, \theta_M^{opt}, \phi^{opt}] = \min[\text{ Eig }[\mathbb{V}]]$$

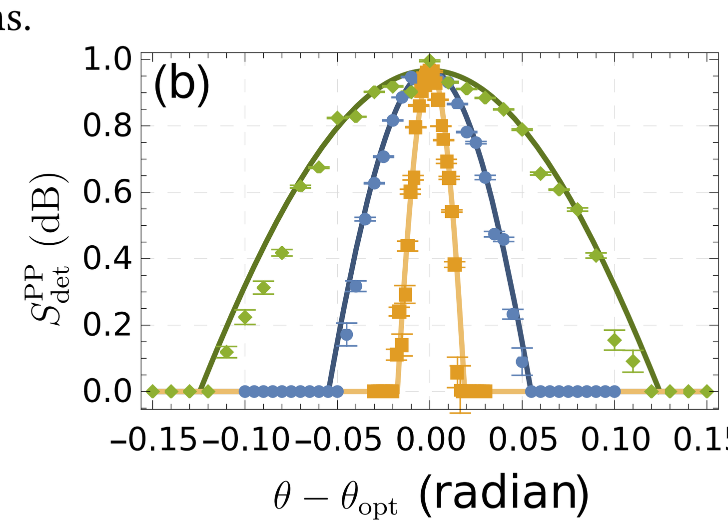
Correlations generated between light and mechanics: two-mode squeezing (S_{gen}) and logarithmic negativity (E_N)



Opto-mechanical correlations (S^{PM}) and Optical-optical correlations (S^{PP})



Detectable correlations as a function of imprecision in defining the optimal bases. Green, blue, yellow correspond to initial occupation of the mechanical oscillator $n_0 = 1, 10, 100$ phonons.



References

[1] Pitchford, A. et al. arXiv:2006.15050 [quant-ph] (2020). [2] Rakhubovsky, A. A. et al. Phys. Rev. Applied 14, 054052 (2020).