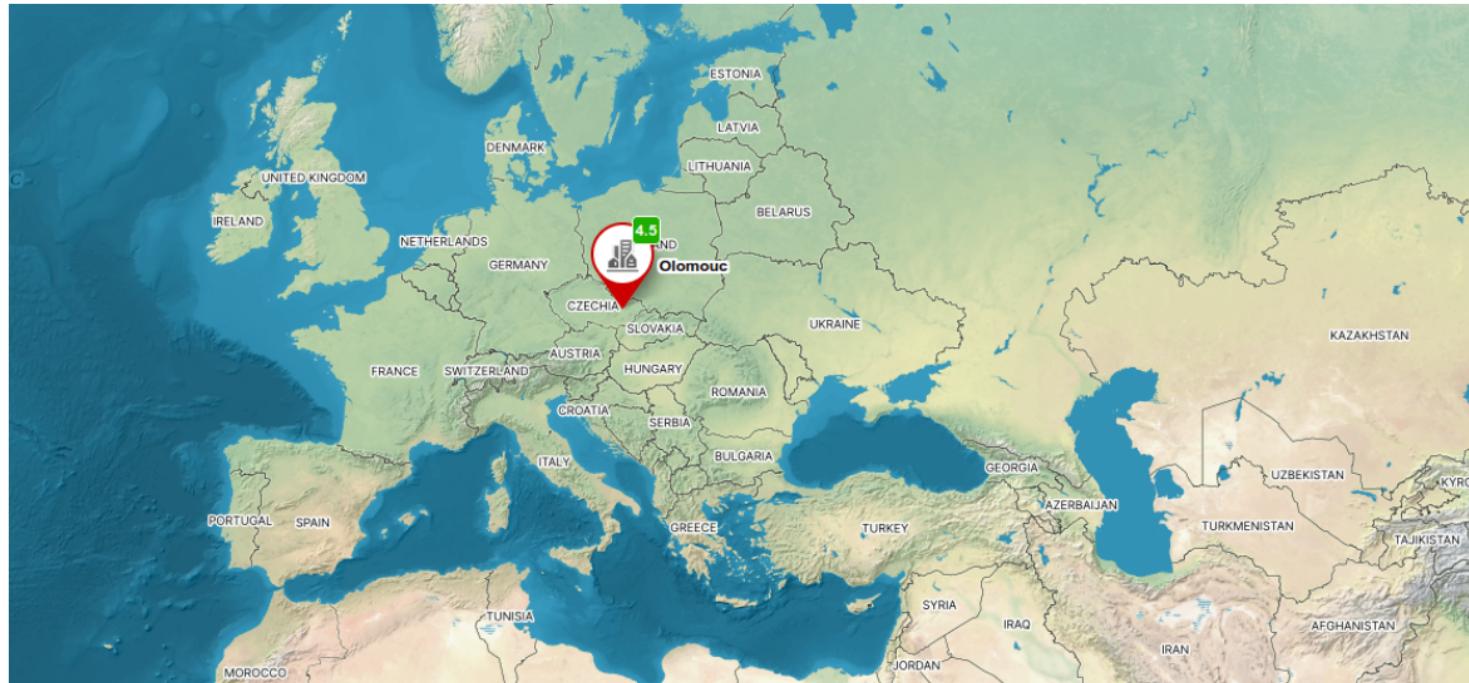


Quantum Non-Gaussian Optomechanics

Andrey A. Rakhubovsky,
Foroud Bemani, Darren W. Moore, Radim Filip

Department of Optics, Palacký University, Czech Republic

MOST 2024,
October 9, 2024,
Samarkand





The Optomechanics Group in Olomouc [within R. Filip's group]

Radim Filip



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Darren Moore



Alisa Manukhova



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PhD and Postdoc
positions available

Introduction

Quantum Optomechanics

Quantum non-Gaussianity

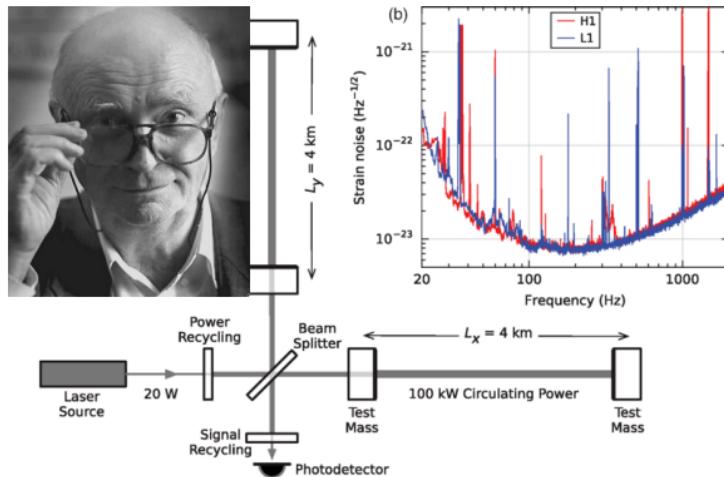
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Motional Nonlinearities

Single-Phonon Addition/Subtraction

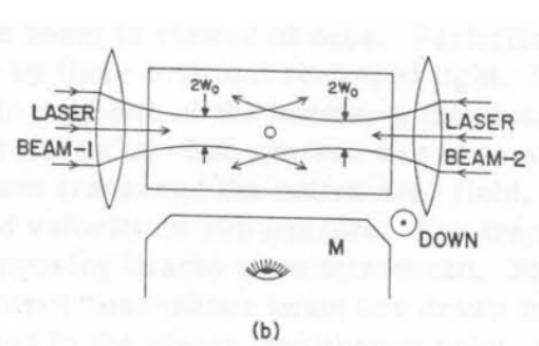
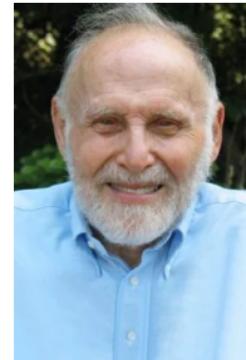


Quantum Optomechanics



Braginsky & Manukin, Soviet JETP **25**, 653 (1967)

Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)



A. Ashkin, PRL **24**, 156 (1970)

$$\mathcal{H} = \omega_{\text{mech}} a_m^\dagger a_m + \omega_{\text{cav}} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

Gaussian vs Quantum non-Gaussian states

Gaussian states

Squeezed displaced state

$$\hat{\rho}_x = \hat{S}(r)\hat{D}(\alpha)|0\rangle\langle 0|.$$

Convex mixtures thereof

$$\hat{\rho}_G = \int p(x)\hat{\rho}_x dx.$$

$p(x)$ is a probability density:

- ★ $p(x) > 0$
- ★ “not more singular” than Dirac δ .

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Quantum non-Gaussian states

Cannot be represented as convex mixtures of squeezed displaced states

$$\hat{\rho}_{NG} \neq \int p(x)\hat{\rho}_x dx.$$

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Examples

Classically NG state

$$\hat{\rho} \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$

Quantum NG state

$$|\psi\rangle \propto |\alpha\rangle + |-\alpha\rangle$$

Gaussian vs Quantum non-Gaussian states

Advantages of QNG states

- ★ Universal quantum computing
- ★ Quantum sensing
- ★ Fundamental studies

QNG is a resource

F. Albarelli *et al.*, Phys. Rev. A **98**, 052350 (2018)

M. Walschaers, PRX Quantum **2**, 030204 (2021)

Quantum non-Gaussian states

Cannot be represented as convex mixtures of squeezed displaced states

$$\hat{\rho}_{\text{NG}} \neq \int p(x) \rho_x dx .$$

Examples

Classically NG state

$$\hat{\rho} \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$

Quantum NG state

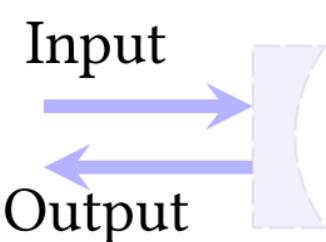
$$|\psi\rangle \propto |\alpha\rangle + |-\alpha\rangle$$

Routes to quantum non-Gaussianity in optomechanics

Add a nonlinear element

$$\hat{H}_P \propto \Omega_m (\hat{p}^2 + \hat{x}^2) + \alpha(t) V(\hat{x})$$

(a)



Nonlinear potential of mechanical motion

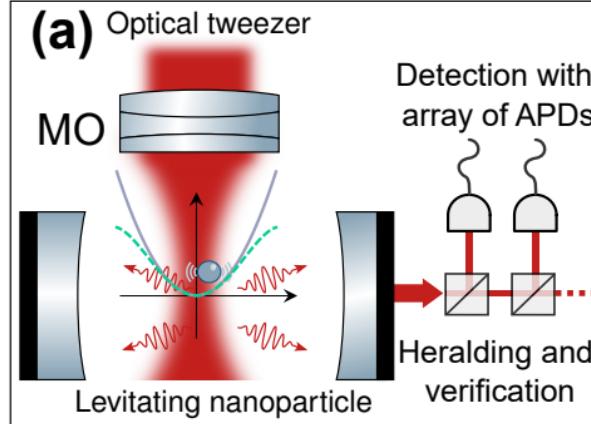
AR, R. Filip, Npj Quantum Inf 7, 120 (2021)

D.W. Moore, **AR**, R. Filip, NJP 21, 113050 (2019)

We don't consider here upload of QNG states

AR, R. Filip, Sci. Rep. 7, 46764 (2017)

Use non-linear detection

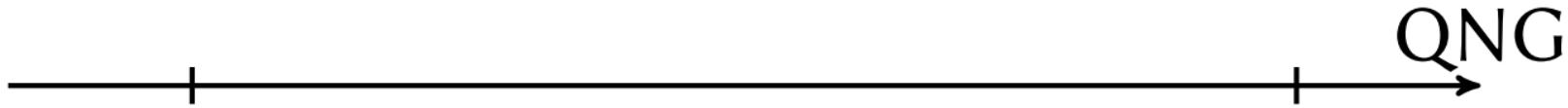


Counting photons

AR, R. Filip, Accepted in QST , doi:10.1088/2058-9565/ad8304 (2024)

F. Bemani, **AR**, R. Filip, Submitted , (2024)

Verification of quantum non-Gaussianity (QNG)



Linear Gaussian Dynamics

Universal Quantum Control

We need better verification tools than fidelity

Introduction

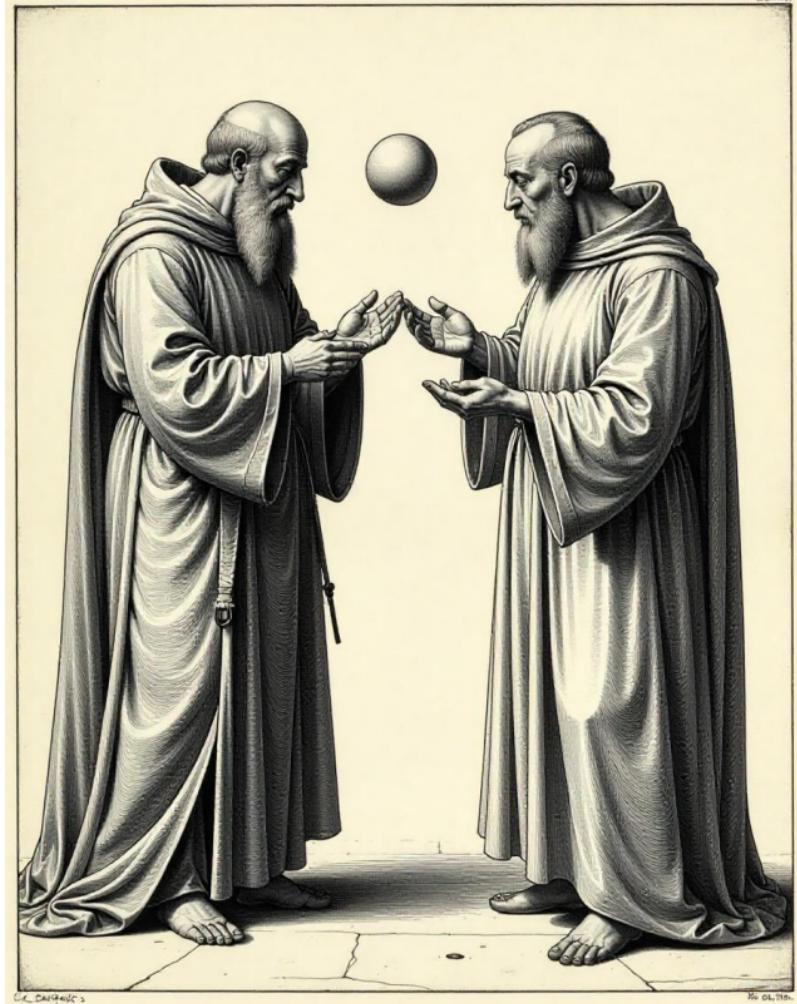
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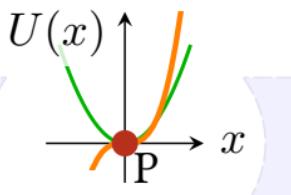
Nonlinear potential for a levitated nanoparticle

$$\hat{H}_P \propto \Omega_m(\hat{p}^2 + \hat{x}^2) + \alpha(t)V(\hat{x})$$

(a)

Input

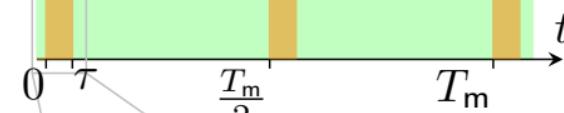
Output



Periodic Temporal Dynamics

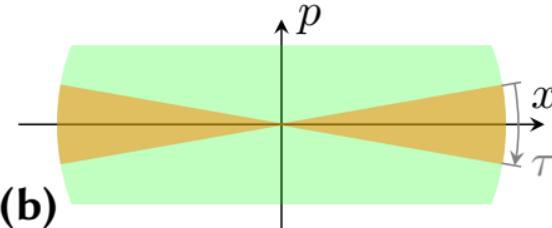
$$\alpha(t)$$

(c)



Phase Space Evolution

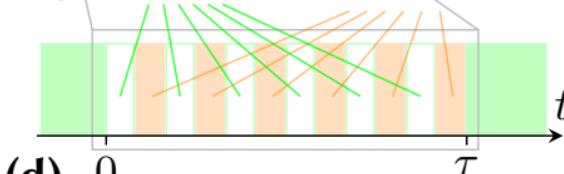
(b)



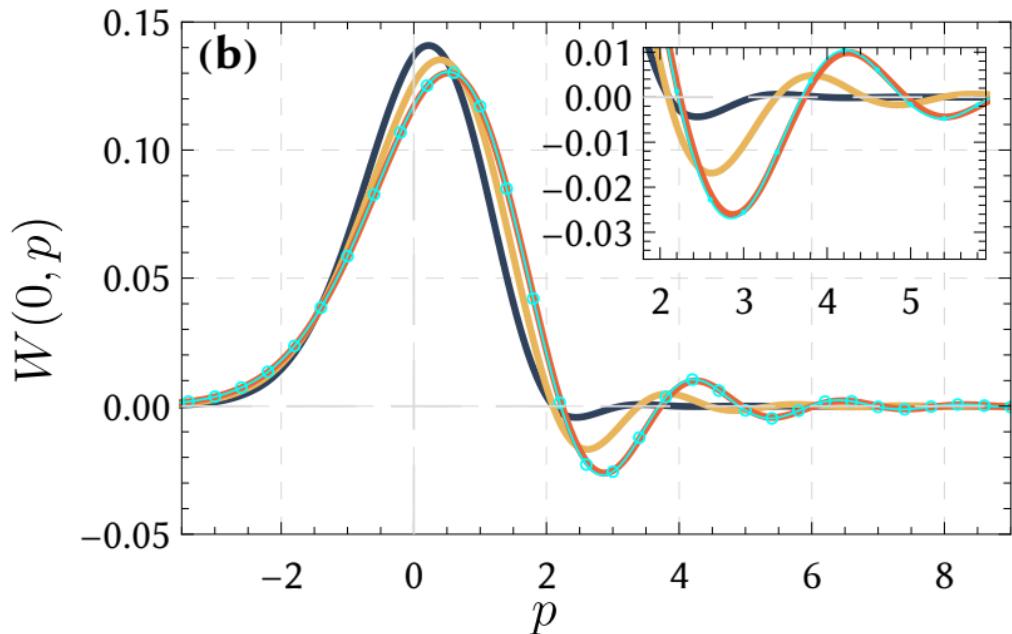
Suzuki-Trotter Simulation

$$e^{-i\hat{H}_{HO}\tau/N} e^{-iV(\hat{x})\tau/N}$$

(d)



Nonlinear potential for a levitated nanoparticle

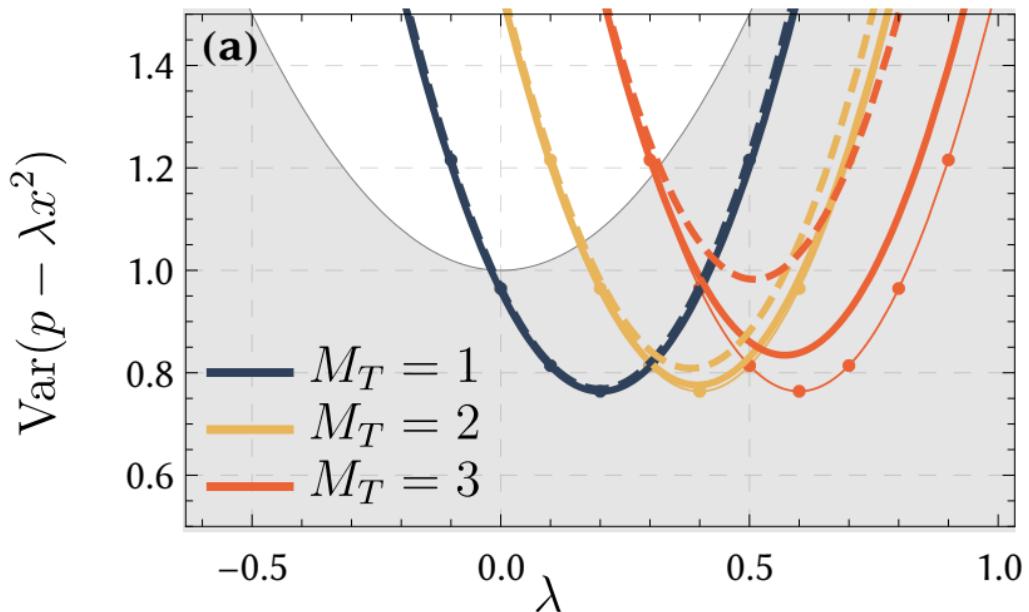


Wigner functions

Poor-man's fidelity (red & cyan)

$$4\pi \int dx dy W_{\text{red}}(x, y) W_{\text{cyan}}(x, y) = 0.9877.$$

Nonlinear potential for a levitated nanoparticle



Nonlinear variance

$$v_3 \equiv \text{Var}(\hat{p} - \lambda \hat{x}^2)$$

Compare with conventional squeezing:

$$v_2 \equiv \text{Var}(\hat{x} \cos \theta + \hat{p} \sin \theta)$$

$$= \sin^2 \theta \cdot \text{Var}(\hat{p} + \lambda \hat{x}),$$

with $\lambda = \cot \theta$.

Related works about levitated NPs in nonlinear potentials (currently all theory)

Palacký University, cubic potential, multiple periods

AR, R. Filip, *Npj Quantum Inf* **7**, 120 (2021) (arxiv 2019)

University of Vienna: “Super Mario”, cubic potential, short pulse

L. Neumeier *et al.*, *PNAS* **121**, e2306953121 (2024) (arxiv 2022)

University of Innsbruck, more complex potentials

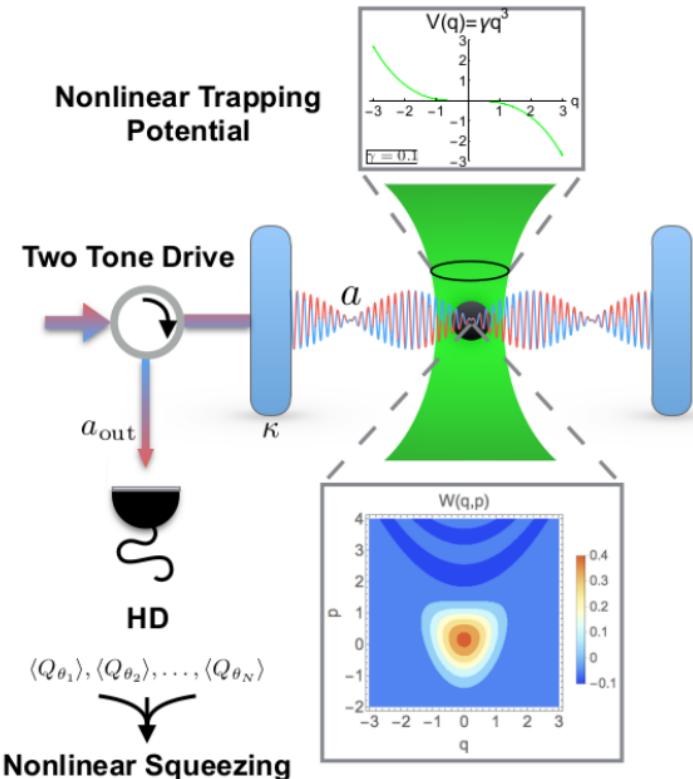
M. Roda-Llordes *et al.*, *Phys. Rev. Res.* **6**, 013262 (2024),

M. Roda-Llordes *et al.*, *Phys. Rev. Lett.* **132**, 023601 (2024),

A. Riera-Campeny *et al.*, arXiv:2307.14106 (2023)



Detecting Nonlinear squeezing



Approximate cubic phase state in mechanics

$$e^{i\gamma q^3} \hat{S} |0\rangle .$$

Pulsed QND interaction

$$H_{int} \propto x_{light}(q \cos \phi + p \sin \phi).$$

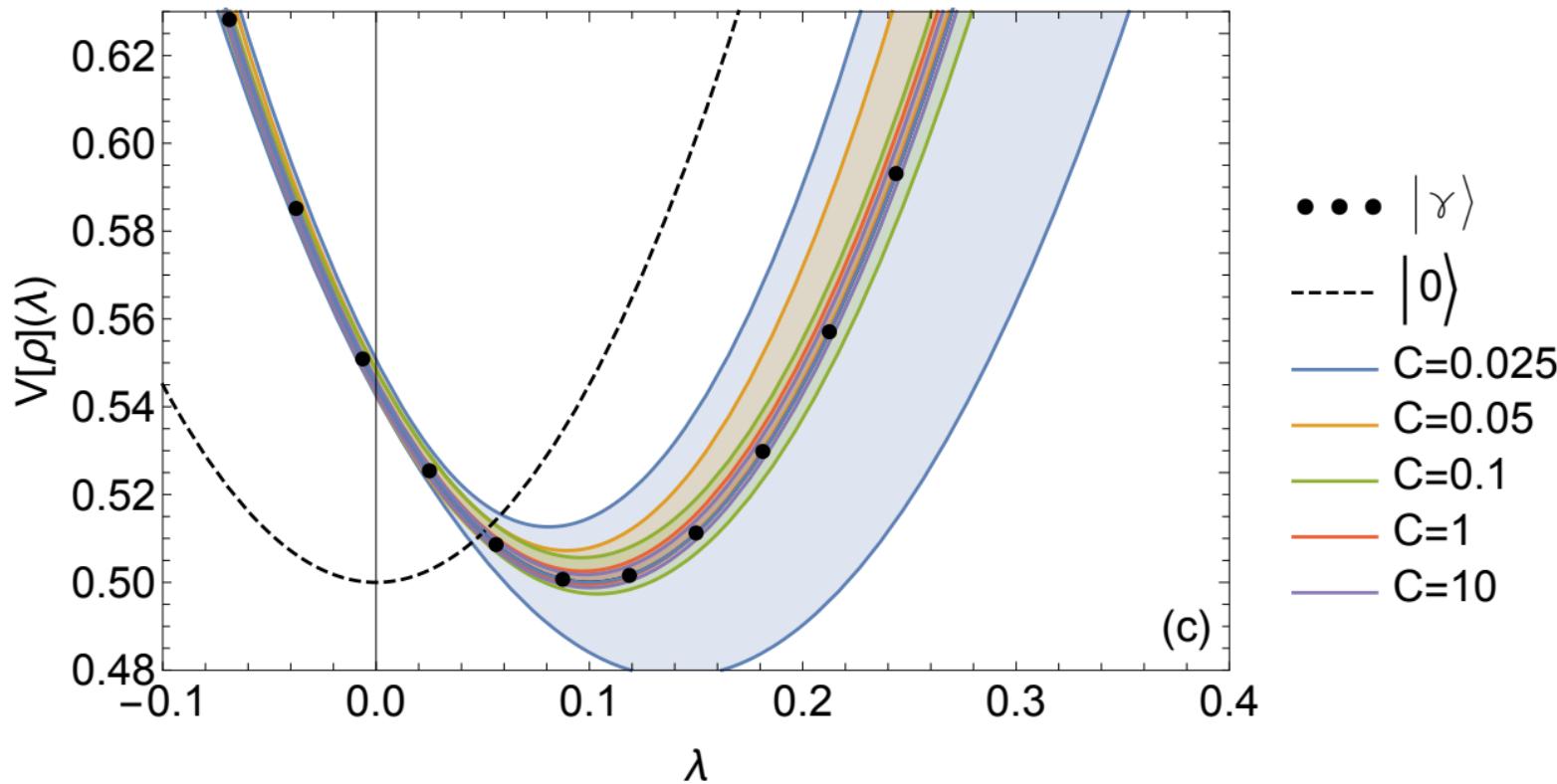
Detect leaking light

Estimate nonlinear variance

$$q_{NL} = p - \lambda x^2 \Rightarrow \text{Var}[q_{NL}]$$

D.W. Moore, AR, R. Filip, NJP **21**, 113050 (2019)

$$\text{Evaluation } V[\rho](\lambda) = \text{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$$



Introduction

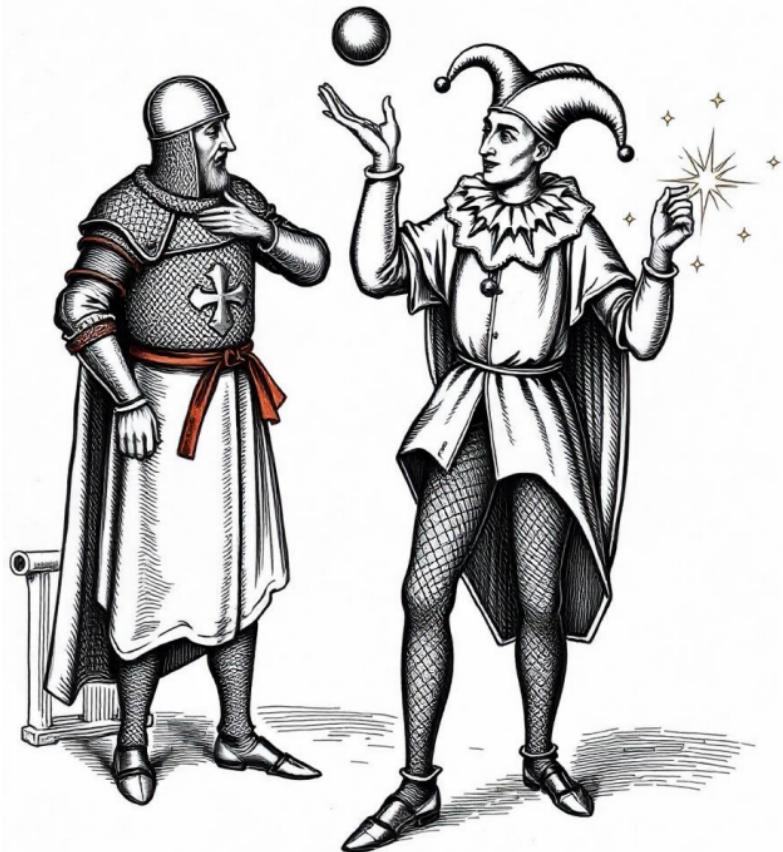
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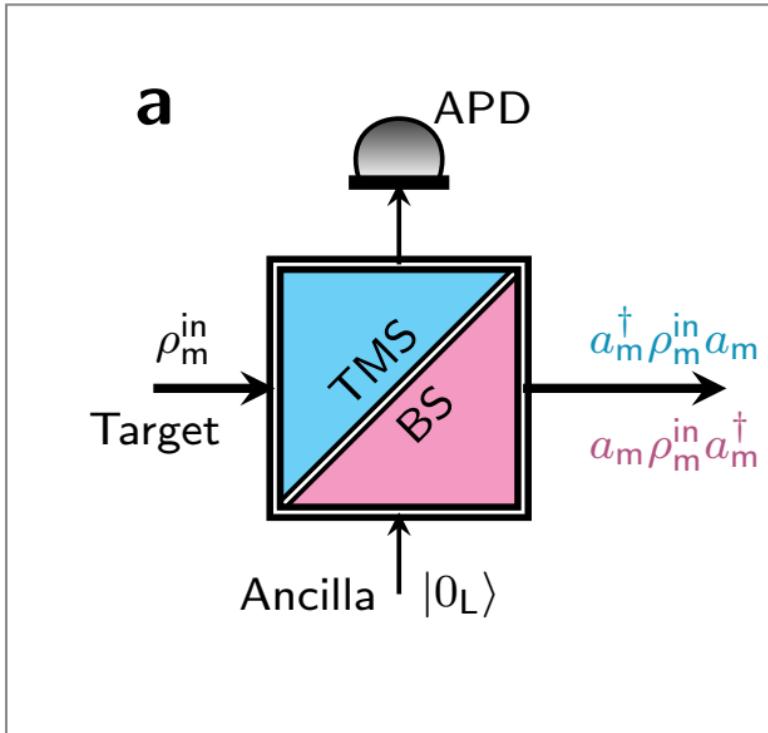
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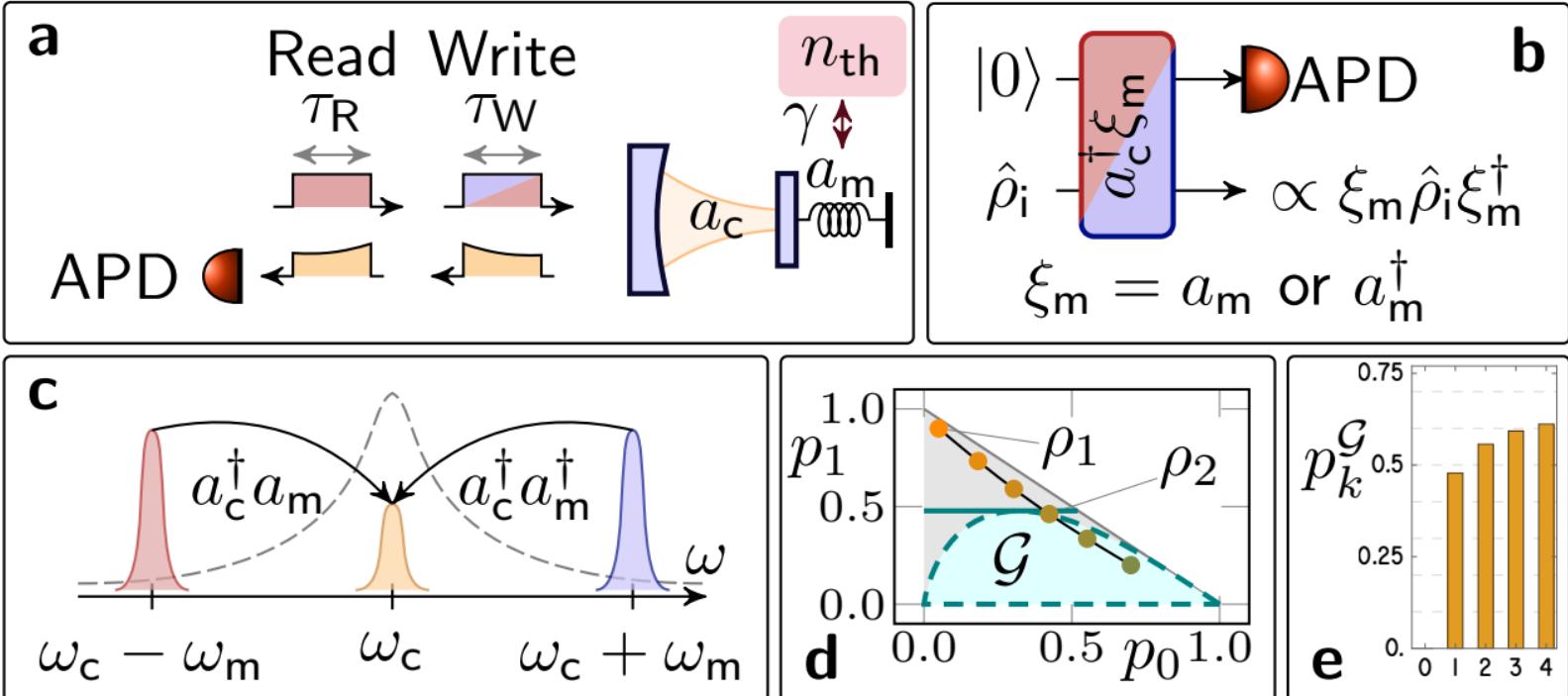
Single-Phonon Addition/Subtraction



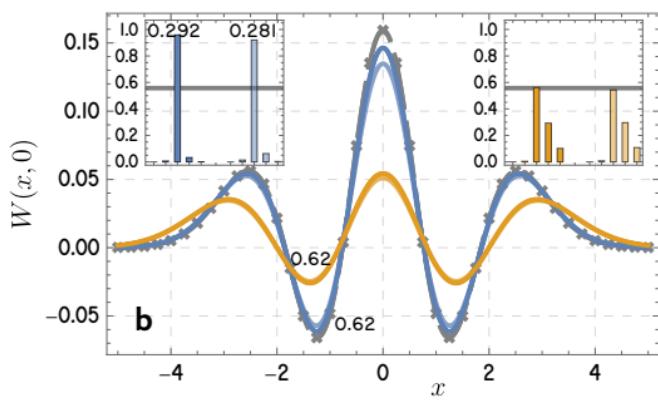
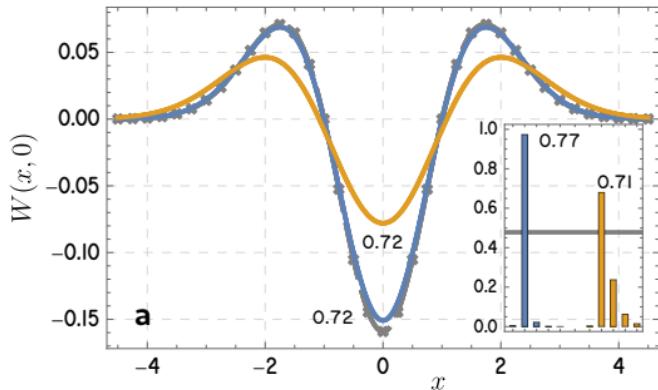
Single-phonon addition or subtraction in optomechanics



Single-phonon addition or subtraction in optomechanics



Evaluation of multiphonon quantum non-Gaussianity (superfluid He)

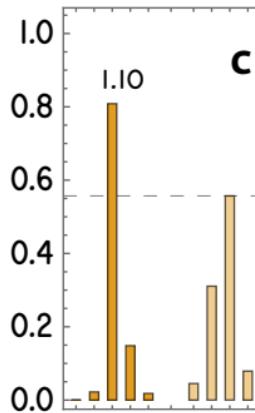
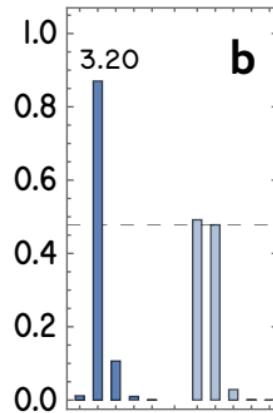
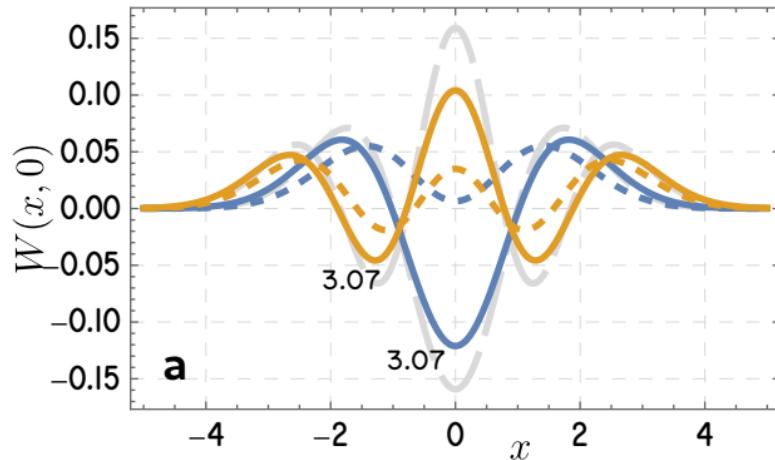


Criteria of absolute n -phonon quantum non-Gaussianity

$$p_k^G = \max_{\alpha, r, \{c_i\}} \left| \left\langle k \left| \hat{D}(\alpha) \hat{S}(r) \sum_{i=0}^{k-1} c_i |i\rangle \right| \right\rangle \right|^2.$$

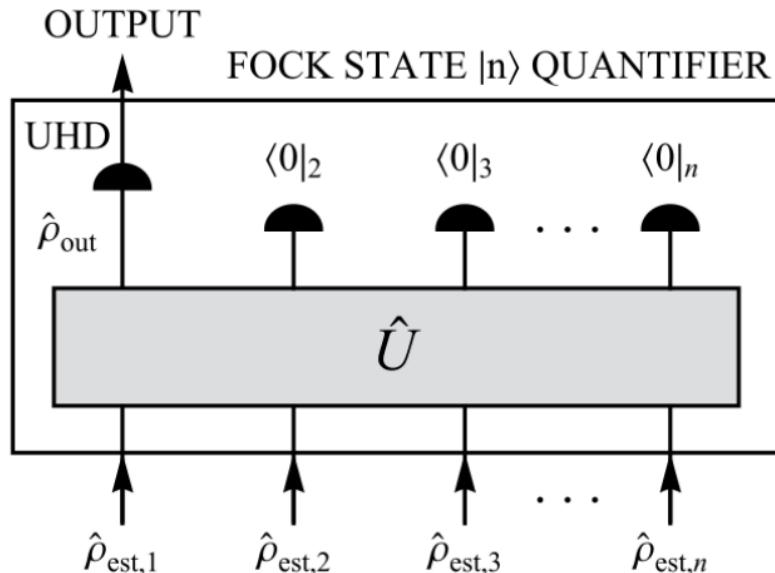
$$p_2^G = \frac{\max_{\alpha, r, c_0, c_1} \left| \left\langle 2 \left| \hat{D}(\alpha) \hat{S}(r) \left(c_0 |0\rangle + c_1 |1\rangle \right) \right| \right\rangle \right|^2}{\begin{array}{cccc} k & 1 & 2 & 3 \\ p_k^G & 0.478 & 0.557 & 0.593 \end{array}}$$

Readout and verification



Inset numbers show QNG depth: loss (in dB) to lose QNG.

Bunching capability

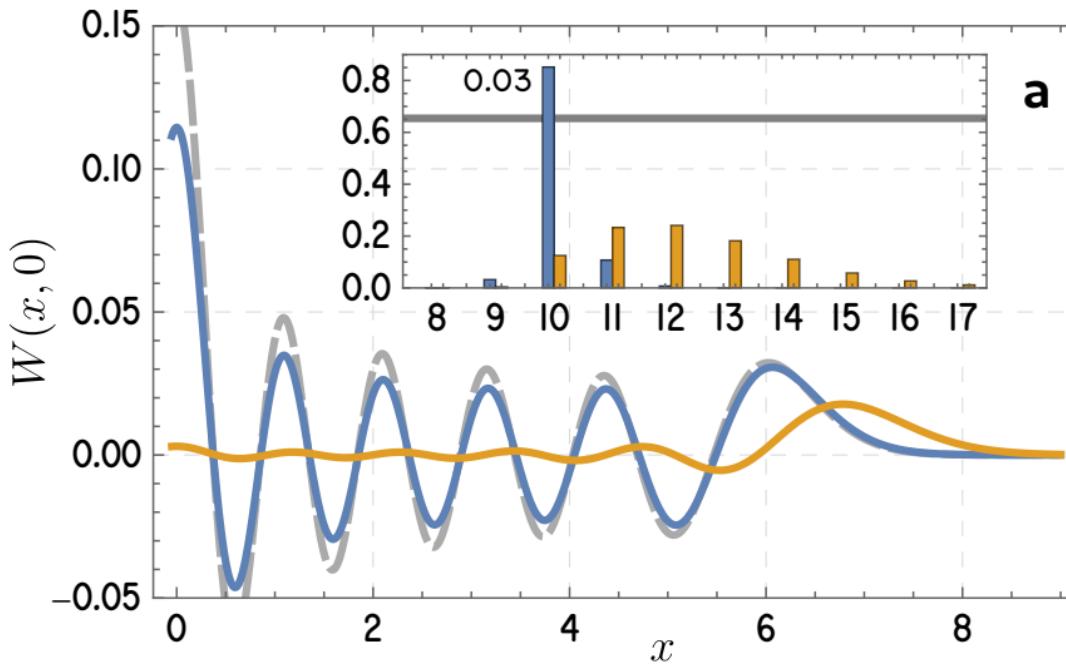


The recipe

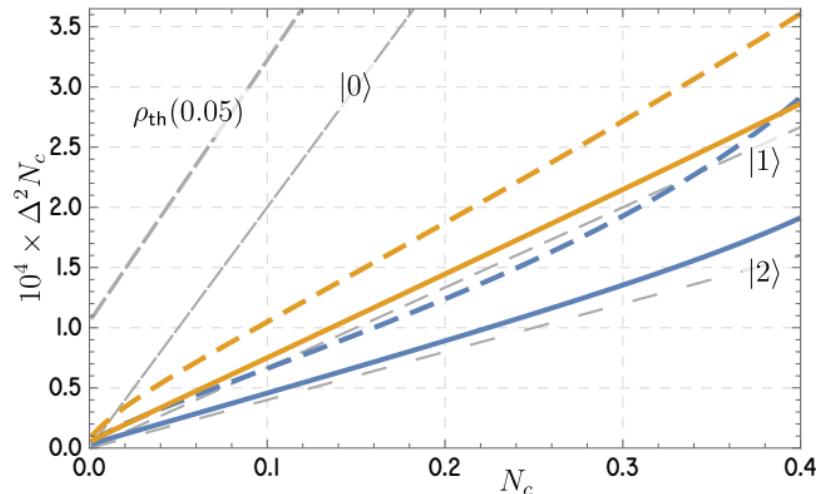
- ★ measure the statistics $\langle k|\hat{\rho}_{\text{est}}|k\rangle$
- ★ compute hypothetical bunching state

Original proposal P. Zapletal and R. Filip, Sci Rep 7, 1 (2017)
Implementations with OPA: P. Zapletal *et al.*, OPTICA 8, 743 (2021).

Bunching capability



Application: detection of phase-randomized displacement



Phase-randomized displacement

$$\rho_{\text{in}} \mapsto \int_0^{2\pi} \frac{d\phi}{2\pi} \hat{D}(\sqrt{N_c} e^{i\phi}) \rho_{\text{in}} \hat{D}^\dagger(\sqrt{N_c} e^{i\phi})$$

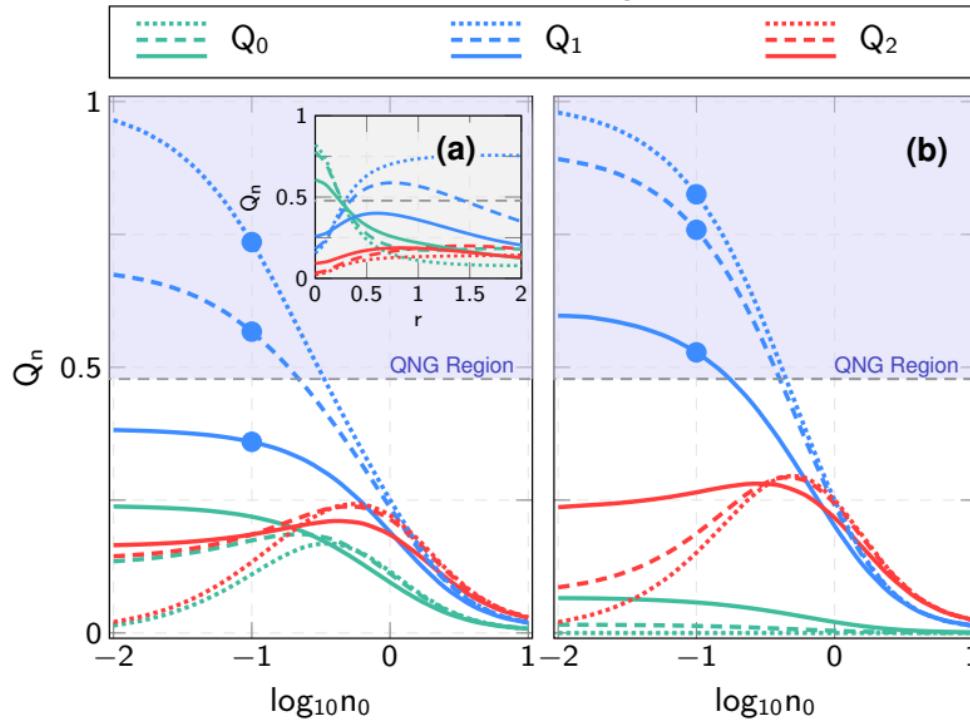
Cramér-Rao bound

$$\Delta^2 N_c \geq \frac{1}{M \cdot F(N_c)},$$

M – number of copies, F – quantum Fisher information

In levitated optomechanics

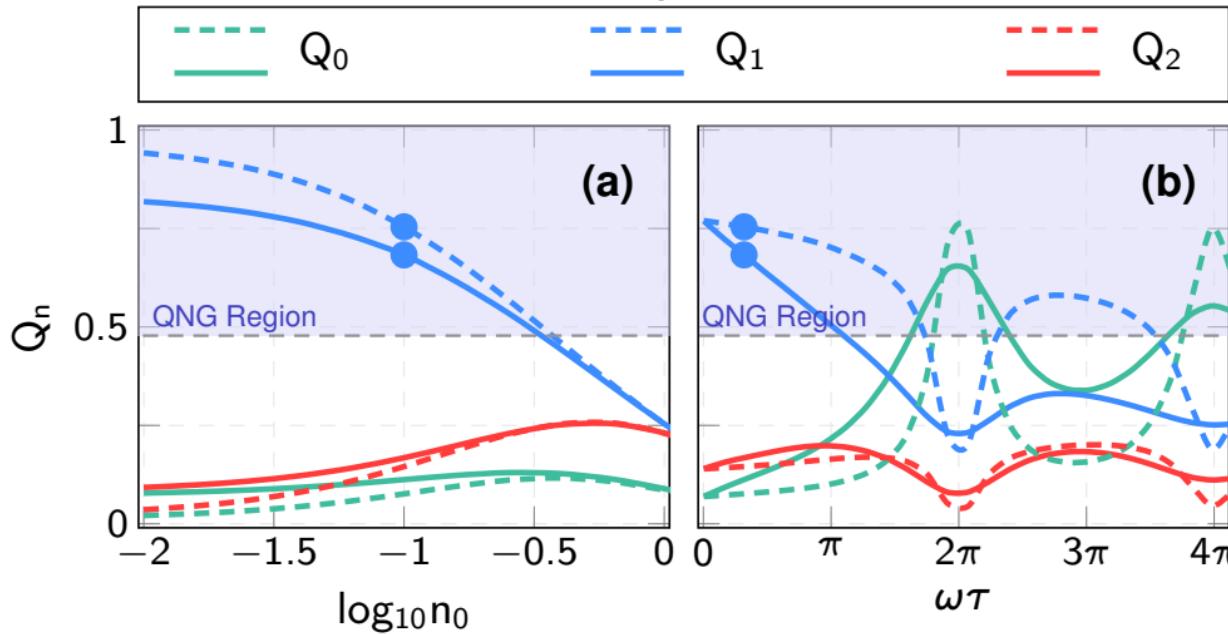
Inside a cavity



Parameters U. Delić, Science 367, 892 (2020)

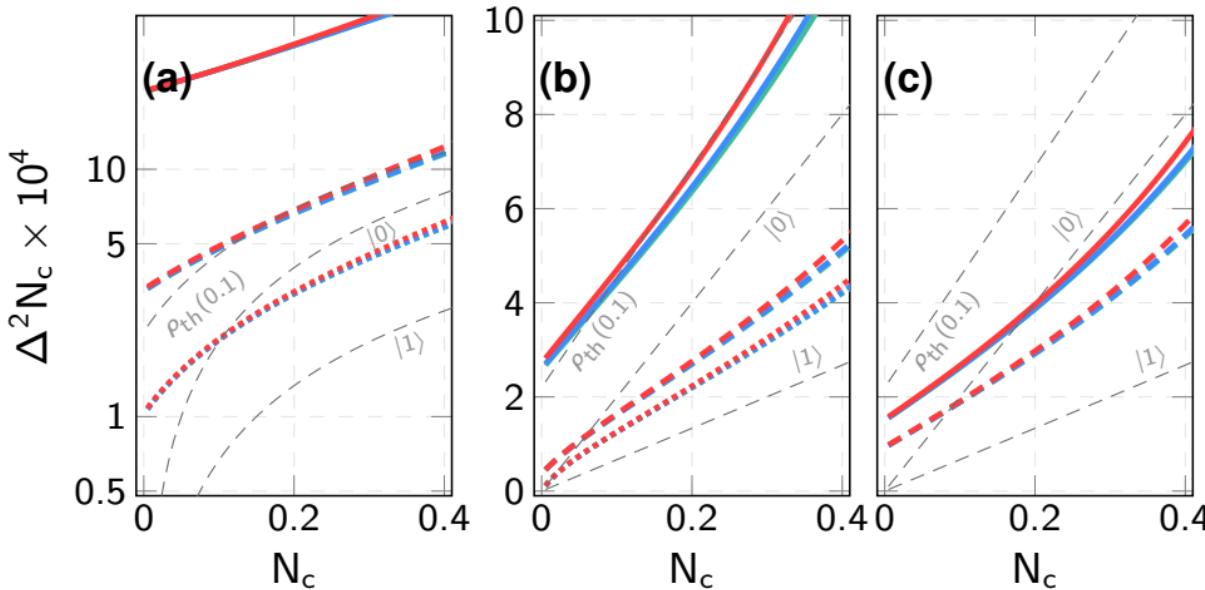
In levitated optomechanics

In free space

Parameters: L. Magrini, Phys. Rev. Lett **129**, 053601 (2022)

In levitated optomechanics

Phase-randomized displacement sensitivity



Thank You!

Ph.D. and postdoc positions available



Effective classical simulation

Consider the setup:

- ★ n quantum subsystems
- ★ t operations
- ★ local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \dots, k_n\},$$

one outcome per system.

Probability of the outcomes

$$P(k_1, \dots, k_n) \quad (1)$$

A classical algorithm is said to capable of effective sampling, if it

- ★ is efficient (polynomial) in t and n
- ★ provides outcomes \mathbf{k} draws from the same probability as (1)

The very last frame which is empty