Quantum Non-Gaussian Optomechanics

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New Trends in Levitodynamics:
From Atoms to Nanostructures,
June 04, 2025,
Třešť

Spolufinancováno
Evropskou unií

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MLÁDEŽE A TĚLOVÝCHOVY


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Olomouc
ESTONIA
LATVIA
LITHUANIA
DENMARK
UNITED KINGDOM
IRELAND
NETHERLANDS
GERMANY
BELARUS
CZECHIA
SLOVAKIA
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AZERBAIJAN
PORTUGAL
SPAIN
GREECE
TURKEY
TURKMENISTAN
TAJIKISTAN
SYRIA
IRAQ
AFGHANISTAN
TUNISIA
IRAN
JORDAN
MOROCCO

100 km
Amsterdam
NL
Berlin
Warszawa
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Bruxelles
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Budapest
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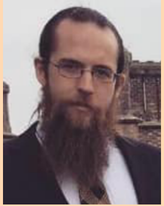
CH
Vaduz
FL
A
H
RO
Ljubljana
SLO
Zagreb
I
YU
HR
BIH
Beograd

The People [within R. Filip's group]

Radim Filip

Foroud Bemani

Darren Moore



Alisa Manukhova

Najmeh Etehadi Abari

(Now @KIT, Germany)

Surabhi Yadav

Shaoni Datta

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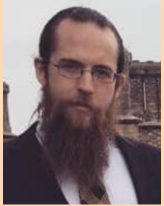

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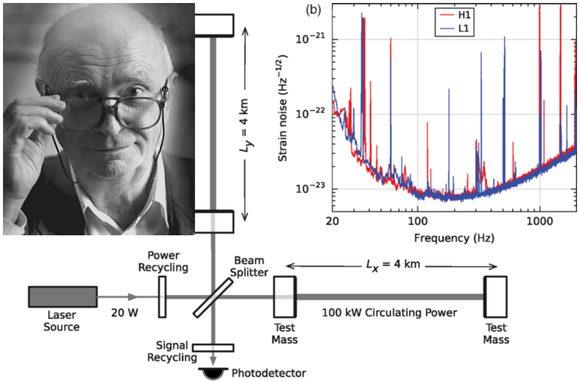
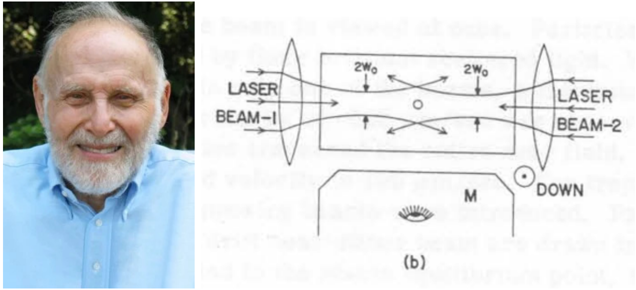
Quantum Optomechanics





Braginsky & Manukin, Soviet JETP **25**, 653 (1967)

A. Ashkin, PRL **24**, 156 (1970)

Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)

$$H = \omega_{mech} a_m^\dagger a_m + \omega_{cav} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

Gaussian vs Quantum non-Gaussian states

Gaussian states

Squeezed displaced state

$$\hat{\rho}_x = \hat{S}(r)\hat{D}(\alpha)\left|0\right\rangle .$$

Convex mixtures thereof

$$\hat{\rho}_G = \int p(x)\hat{\rho}_x dx .$$

$p(x)$ is a probability density:

- ★ $p(x) > 0$
- ★ “not more singular” than Dirac δ.

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Quantum non-Gaussian states

Cannot be represented as convex mixtures of squeezed displaced states

$$\hat{\rho}_{NG} \neq \int p(x)\rho_x dx .$$

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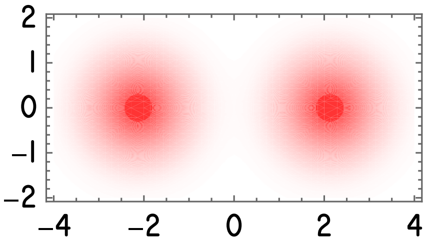
$$\hat{\rho}_{NG} \neq \int p(x)\rho_x dx .$$

Examples in the phase space

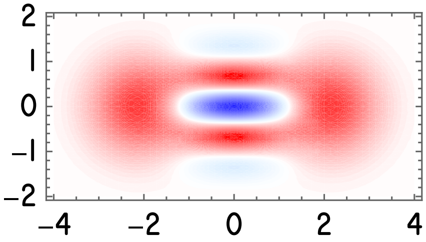
Classically NG state

$$\hat{\rho} \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$



Quantum NG state

$$|\psi\rangle \propto |\alpha\rangle - |-\alpha\rangle$$



Gaussian vs Quantum non-Gaussian states

Advantages of QNG states

- Universal quantum computing
- Quantum sensing
- Fundamental studies

QNG is a resource

F. Albarelli *et al.*, Phys. Rev. A **98**, 052350 (2018)

M. Walschaers, PRX Quantum **2**, 030204 (2021)

Quantum non-Gaussian states

Cannot be represented as convex mixtures of squeezed displaced states

$$\hat{\rho}_{NG} \neq \int p(x) \rho_x dx \,.$$

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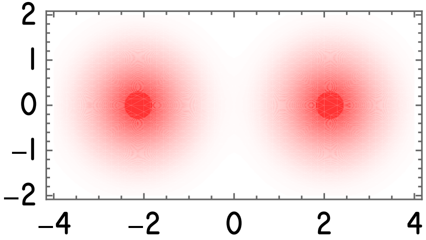

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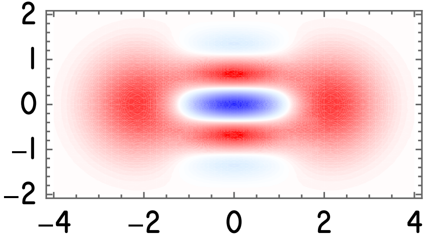
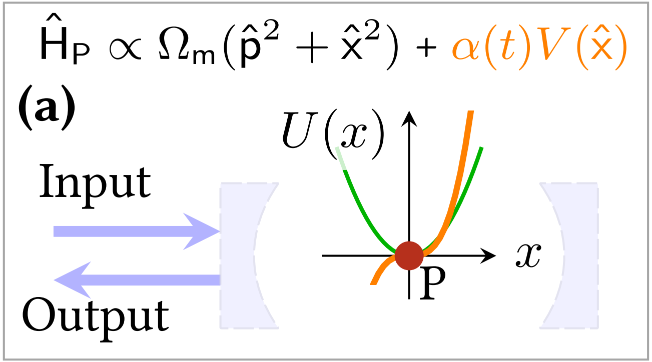


Routes to quantum non-Gaussianity in optomechanics

Add a nonlinear element

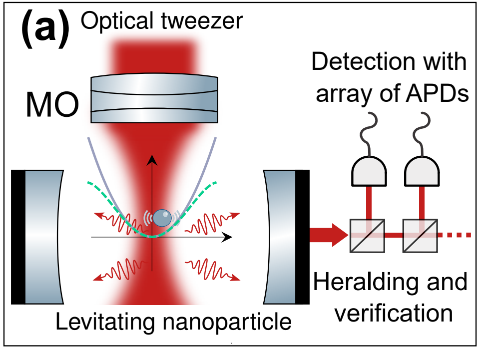


Nonlinear potential of mechanical motion

AR, R. Filip, Npj Quantum Inf **7**, 120 (2021)

D.W. Moore, **AR**, R. Filip, NJP **21**, 113050 (2019)

Use non-linear detection



Counting photons

AR, R. Filip, Quantum Sci. Technol. **10**, 015014 (2024)

F. Bemani, **AR**, R. Filip, Submitted , (2024)

We don't consider here upload of QNG states

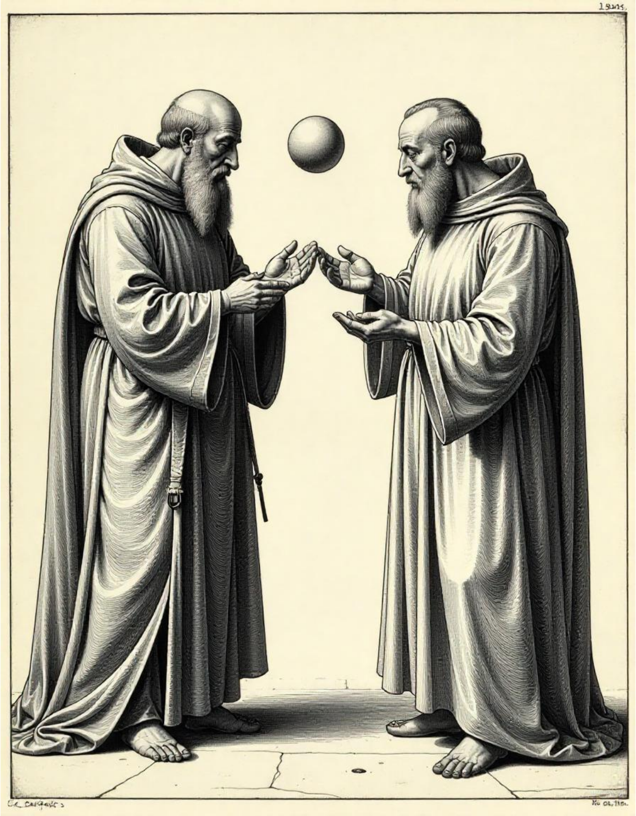
AR, R. Filip, Sci. Rep. **7**, 46764 (2017)

Verification of quantum non-Gaussianity (QNG)

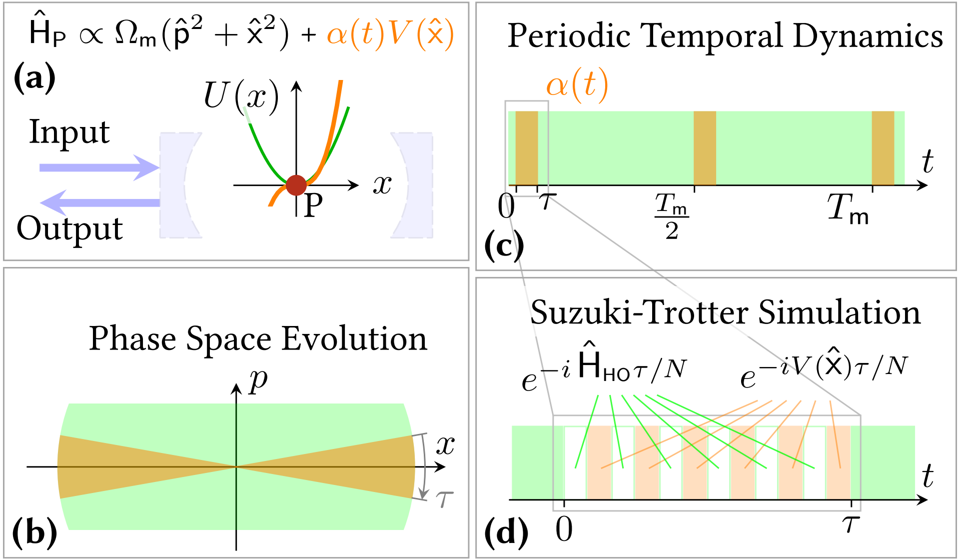
QNG

Linear Gaussian Dynamics	Intermediate control	Universal Quantum Control
		

We need better figures of merit than fidelity



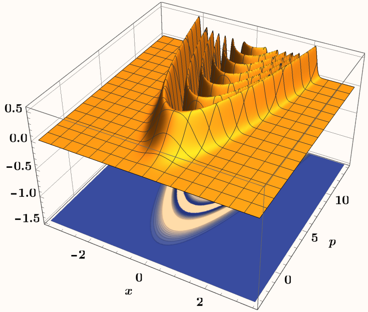
Nonlinear potential for a levitated nanopatricle



AR, R. Filip, Npj Quantum Inf **7**, 120 (2021)

Figures of merit

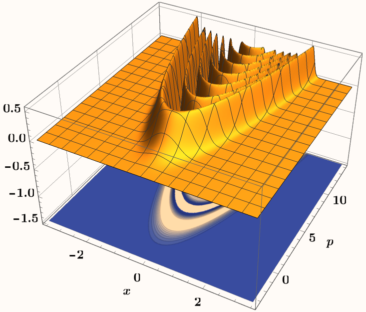
Wigner function



$$W_{\mathrm{CPS}}(x,p) \propto \mathrm{Ai}[p - \gamma x^2]$$

Figures of merit

Wigner function



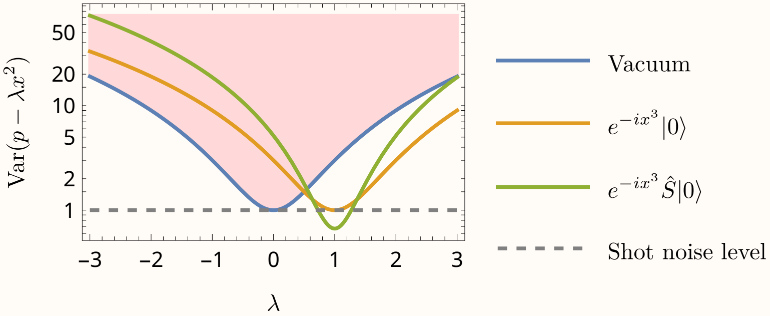
$$W_{\text{CPS}}(x,p) \propto \text{Ai}[p - \gamma x^2]$$

Nonlinear squeezing

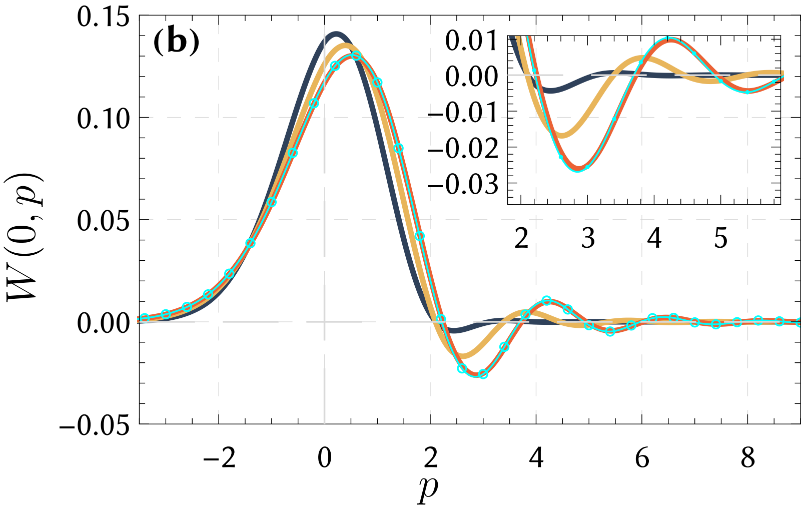
$$\begin{pmatrix} x \\ p \end{pmatrix} \xrightarrow{\exp[-i\gamma x^3]} \begin{pmatrix} x \\ p + \gamma x^2 \end{pmatrix}$$

Nonlinear squeezing

$$\sigma_3(\lambda;\rho) = \text{Var}_\rho(\hat{p} - \lambda\hat{x}^2).$$



Nonlinear potential for a levitated nanopatricle

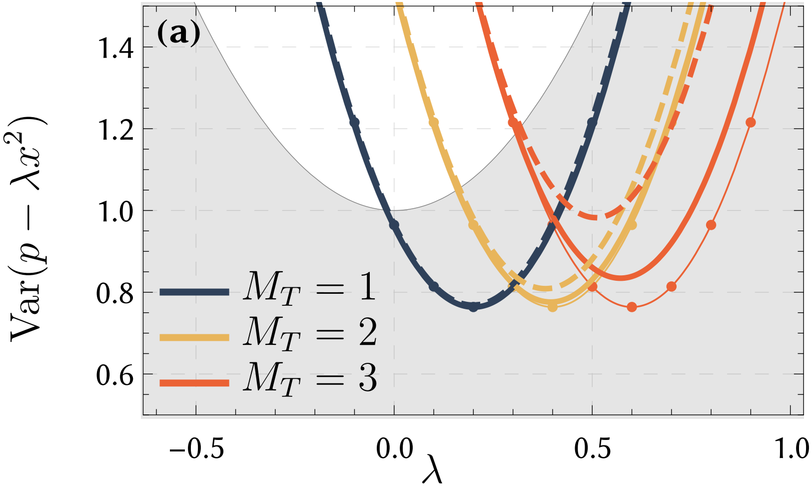


Wigner functions

Poor-man's fidelity (red & cyan)

$$4\pi \int \mathrm{d}x\mathrm{d}y W_{\text{red}}(x, y) W_{\text{cyan}}(x, y) = 0.9877.$$

Nonlinear potential for a levitated nanopatricle



Nonlinear variance

$$v_3 \equiv \mathrm{Var}(\hat{p} - \lambda \hat{x}^2)$$

Compare with conventional squeezing:

$$v_2 \equiv \mathrm{Var}(\hat{x}\cos\theta + \hat{p}\sin\theta)$$
$$= \sin^2\theta \cdot \mathrm{Var}(\hat{p} + \lambda\hat{x}),$$

with $\lambda = \cot\theta$.

Related works about levitated NPs in nonlinear potentials (currently all theory)

Palacký University, cubic potential, multiple periods

AR, R. Filip, Npj Quantum Inf **7**, 120 (2021) (arxiv 2019)

University of Vienna: “Super Mario”, cubic potential, short pulse

L. Neumeier *et al.*, PNAS **121**, e2306953121 (2024) (arxiv 2022)

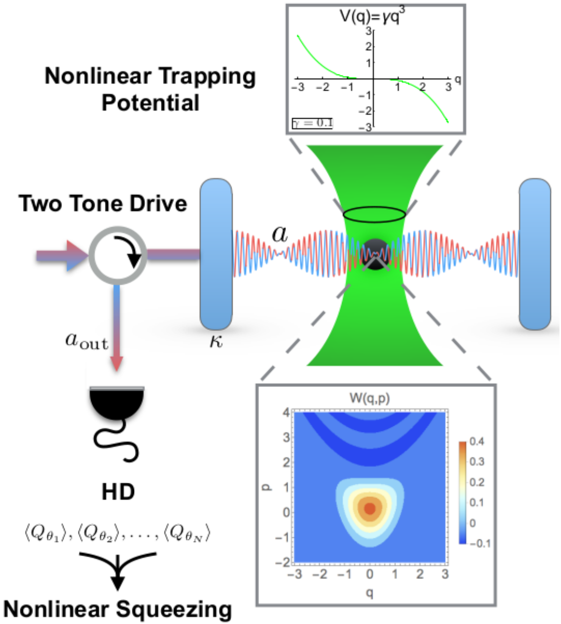
University of Innsbruck, more complex potentials

M. Roda-Llordes *et al.*, Phys. Rev. Res. **6**, 013262 (2024),

M. Roda-Llordes *et al.*, Phys. Rev. Lett. **132**, 023601 (2024),

A. Riera-Campeny *et al.*, Quantum **8**, 1393 (2024)

Detecting Nonlinear Squeezing



Approximate cubic phase state in mechanics

$$e^{i\gamma q^3}\hat{S}\left|0\right\rangle .$$

Pulsed QND interaction

$$H_{\text{int}} \propto x_{\text{light}}(q\cos\phi + p\sin\phi).$$

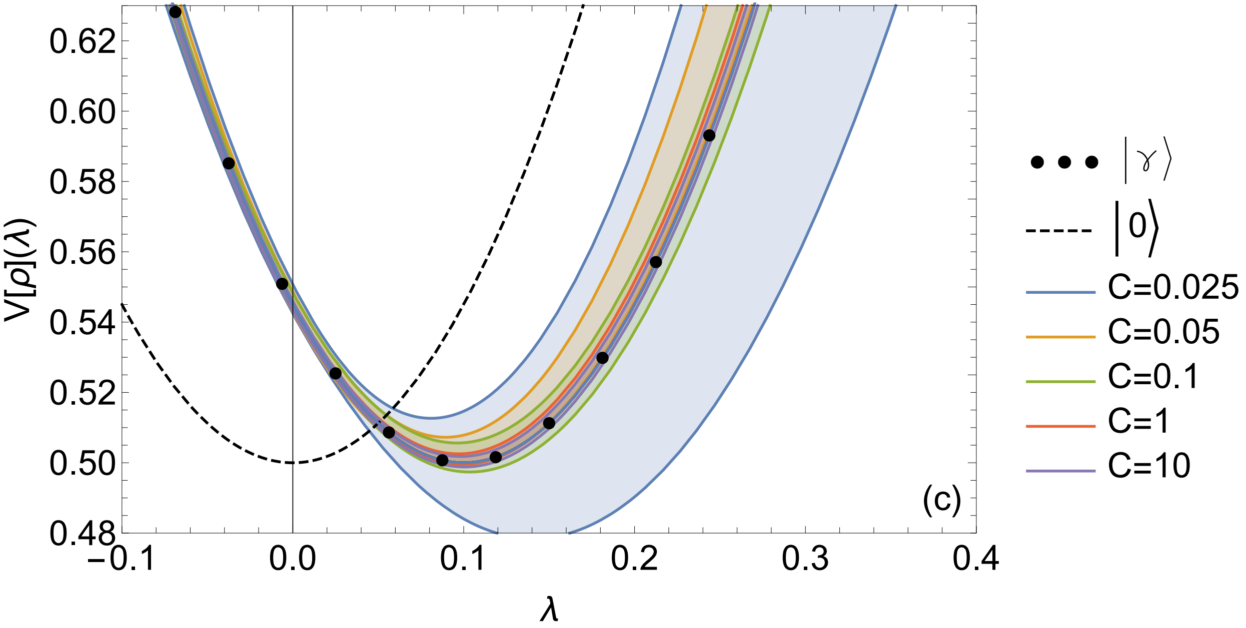
Detect leaking light

Estimate nonlinear variance

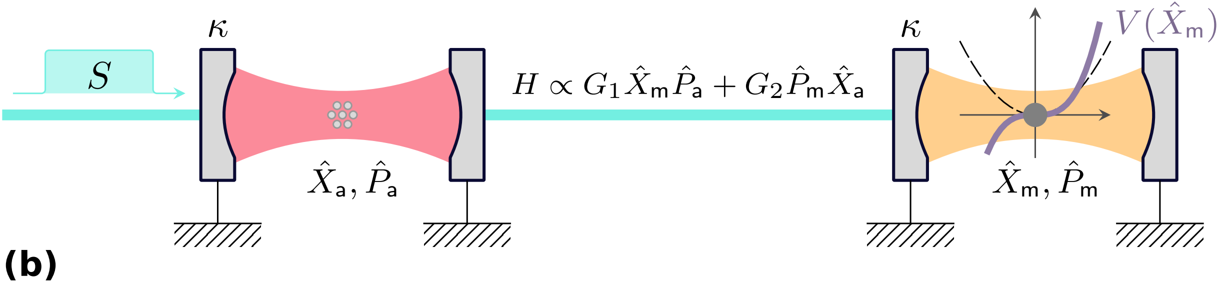
$$q_{\text{NL}} = p - \lambda x^2 \Rightarrow \text{Var}[q_{\text{NL}}]$$

D.W. Moore, **AR**, R. Filip, NJP **21**, 113050 (2019)

Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Broadcasting Protocol



We have

- Gaussian interactions $\hat{H} \propto \hat{Q}_\mathrm{a} \hat{Q}_\mathrm{m}$
- Nonlinearity in mechanics

$$\exp[-i\gamma \hat{X}_\mathrm{m}^3]$$

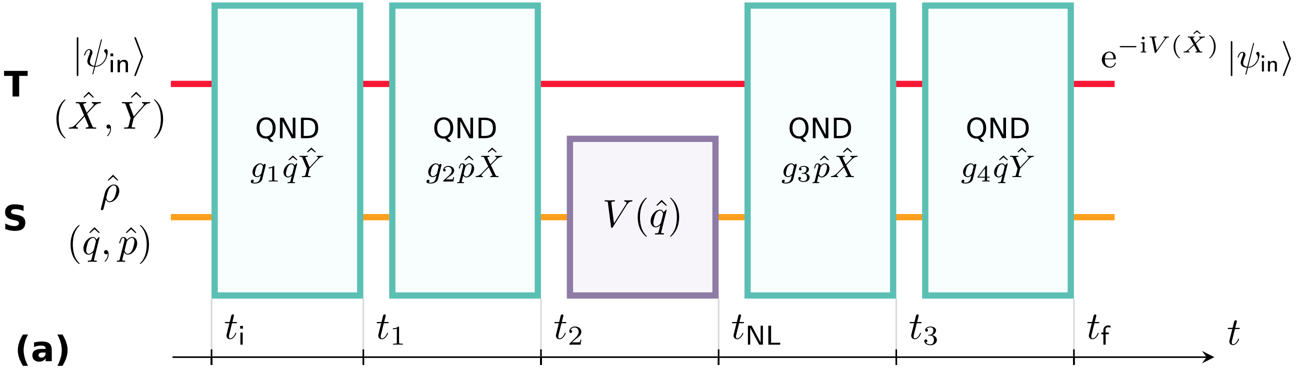
$$\hat{X}_\mathrm{m} \mapsto \hat{X}_\mathrm{m}, \quad \hat{Y}_\mathrm{m} \mapsto \hat{Y}_\mathrm{m} + \gamma \hat{X}_\mathrm{m}^2.$$

We want

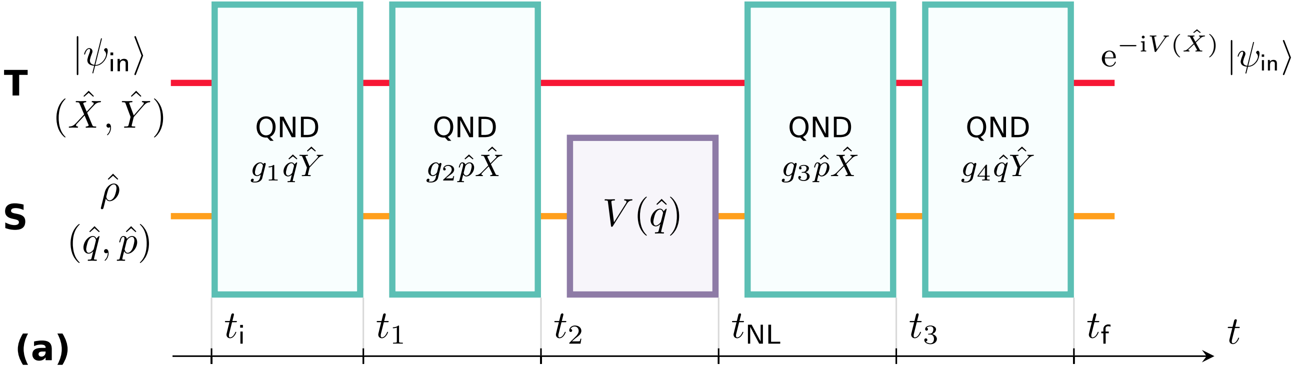
Nonlinearity in Atoms

$$\hat{X}_\mathrm{a} \mapsto \hat{X}_\mathrm{a}, \quad \hat{Y}_\mathrm{a} \mapsto \hat{Y}_\mathrm{a} + \tilde{\gamma} \hat{X}_\mathrm{a}^2.$$

Broadcasting Protocol

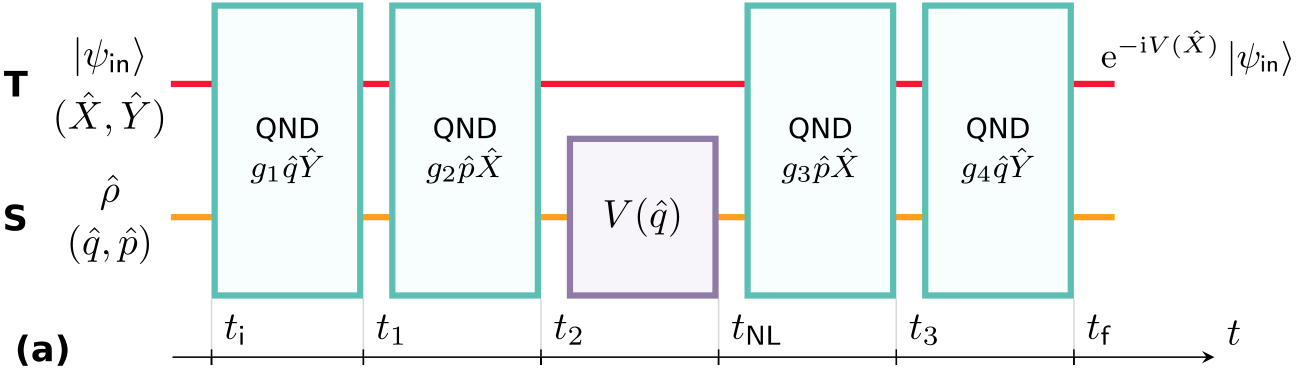


Broadcasting Protocol



$$\hat{X} = (1+g_4(g_2+g_3))\hat{X}+(g_4+g_1(1+g_4(g_2+g_3)))\hat{q},$$
$$\hat{Y} = (1+g_1(g_2+g_3))\hat{Y}-(g_2+g_3)\hat{p}+g_3\gamma V'\left((1+g_1 g_2)\hat{q}+g_2\hat{X}\right).$$

Broadcasting Protocol



$$g_2 = -g_3$$

$$\hat{X} = (1 + g_4(g_2 + g_3))\hat{X} + (g_4 + g_1(1 + g_4(g_2 + g_3)))\hat{q},$$

$$\hat{Y} = (1 + g_1(g_2 + g_3))\hat{Y} - (g_2 + g_3)\hat{p} + g_3 \gamma V' \left((1 + g_1 g_2)\hat{q} + g_2 \hat{X}\right).$$

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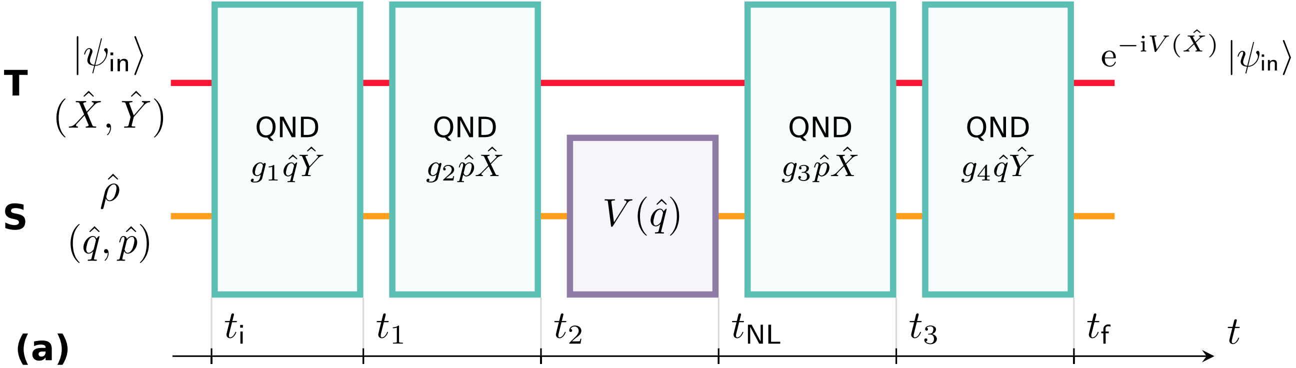

$$-1/g_1 = g_2 = -g_3 = 1/g_4$$

$$\hat{\mathbf{X}} = (1 + g_4(g_2 + g_3))\hat{\mathbf{X}} + (g_4 + g_1(1 + g_4(g_2 + g_3)))\hat{\mathbf{q}},$$

$$\hat{\mathbf{Y}} = (1 + g_1(g_2 + g_3))\hat{\mathbf{Y}} - (g_2 + g_3)\hat{\mathbf{p}} + g_3 \gamma V'\left((1 + g_1 g_2)\hat{\mathbf{q}} + g_2 \hat{\mathbf{X}}\right).$$

Regimes of operation

Broadcasting of the cubic nonlinearity

$$-1/g_1 = g_2 = -g_3 = 1/g_4 = g$$

$$\begin{aligned} \hat{\mathbf{X}} &\mapsto \hat{\mathbf{X}}, \\ \hat{\mathbf{Y}} &\mapsto \hat{\mathbf{Y}} + \gamma g^3 \hat{\mathbf{X}}^2. \end{aligned}$$

- ★ Valid for arbitrary state of mechanics
- ★ Implements the operation $\exp[-i\gamma g^3 \hat{X}^3]$.

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Figure of merit: Nonlinear squeezing

$$\mathrm{Var}(\hat{Y} - \lambda\hat{X}^2) = 1 + 2(\lambda - \gamma g^3)^2 \geqslant 1$$

- ★ Valid for arbitrary state of mechanics
- ★ Implements the <u>operation</u> $\exp[-i\gamma g^3 \hat{X}^3]$.

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- Valid for arbitrary state of mechanics
- Implements the operation $\exp[-i\gamma g^3 \hat{X}^3]$.

Generation of nonlinear squeezing

$$g_2 = g - 1/g_1,\ g_3 = -g,\ g_4 = 1/g$$

$$\hat{\mathbf{X}} \mapsto \hat{\mathbf{X}}_f = \left(1 - \frac{1}{g g_1}\right) \hat{\mathbf{X}} + g_1 \hat{\mathbf{q}}$$
$$\hat{\mathbf{Y}} \mapsto \frac{1}{g_1} \hat{\mathbf{p}} + \gamma g (g \hat{\mathbf{X}}_f)^2.$$

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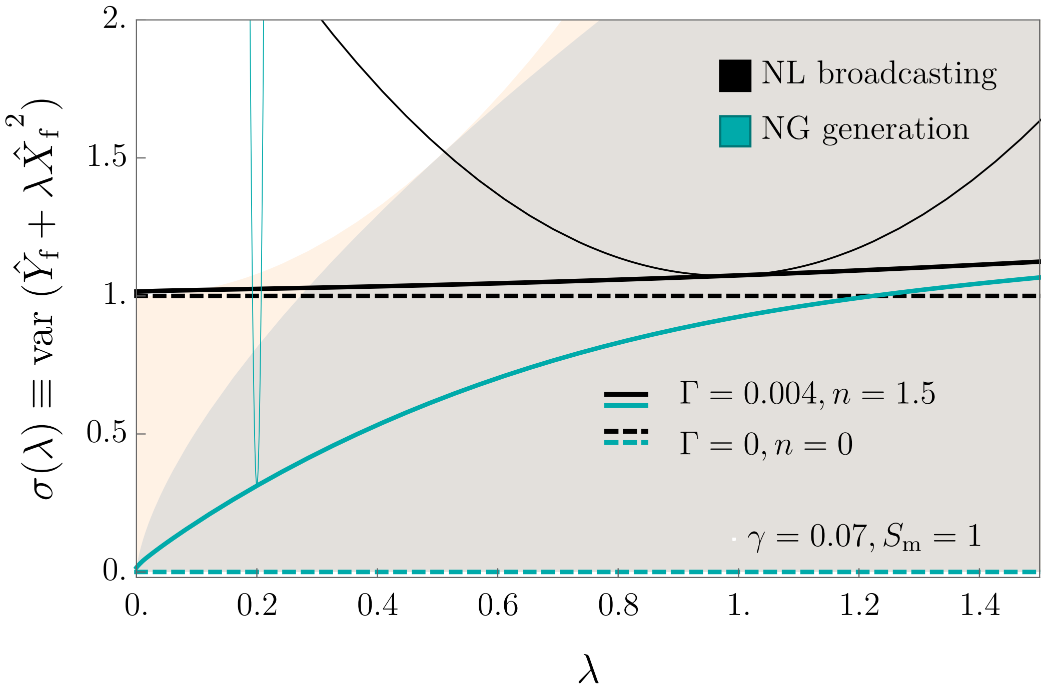
$$\hat{\mathbf{Y}} \mapsto \frac{1}{g_1} \hat{\mathbf{p}} + \gamma g (g \hat{\mathbf{X}}_f)^2.$$

Nonlinear squeezing

$$\mathrm{Var}(\hat{Y}_f - \lambda \hat{X}_f^2) = \frac{1}{g_1^2} \mathrm{Var}(\hat{p}) + 2(\lambda - \gamma g^3)^2 \mathrm{Var}(\hat{X}^2)$$

Results: Nonlinear squeezing



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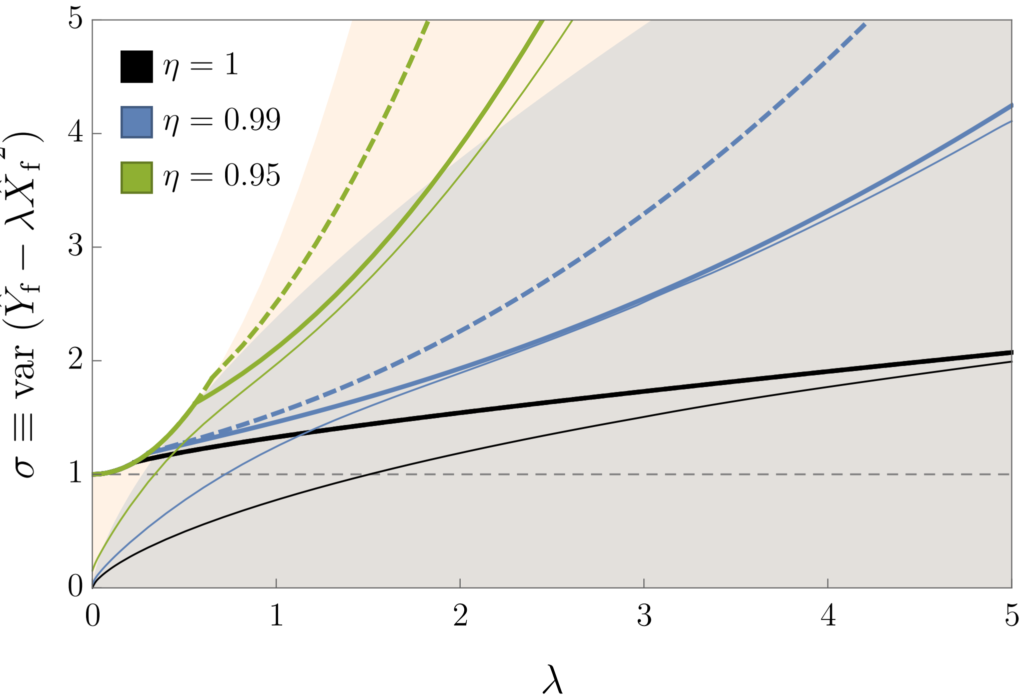
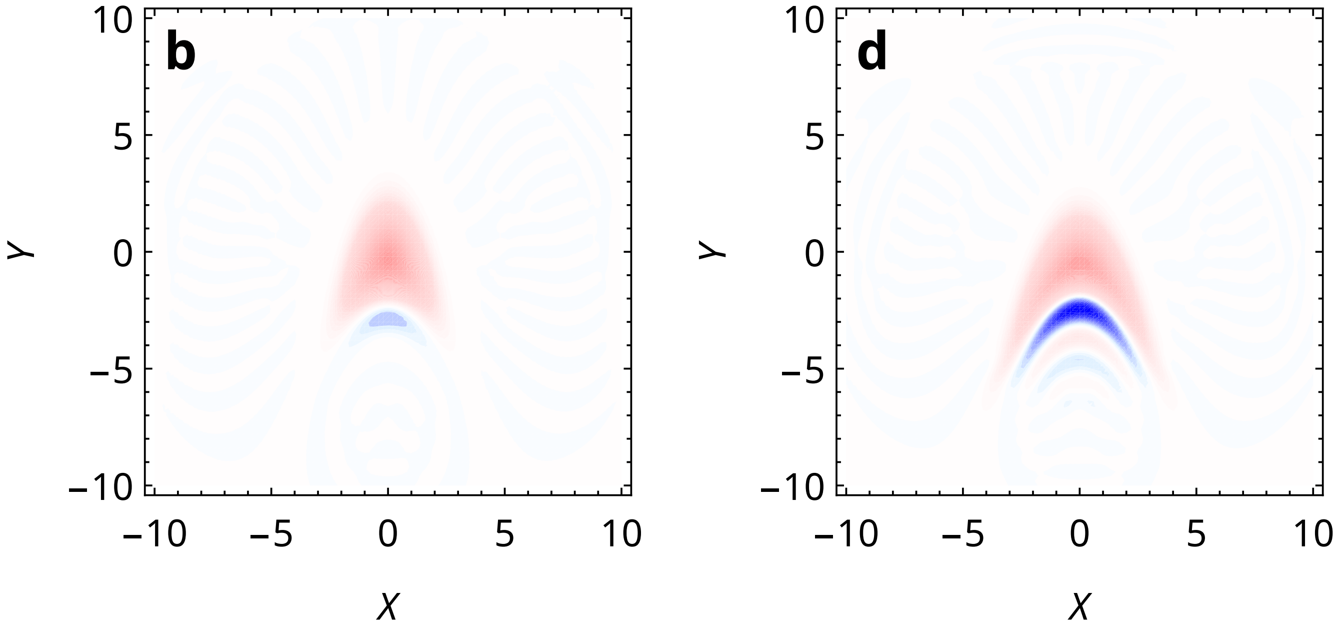
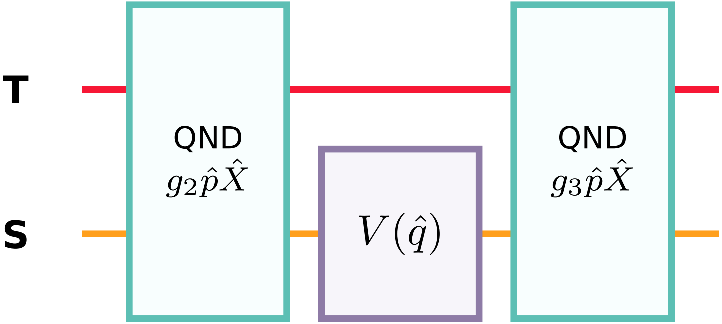


Results: Wigner functions



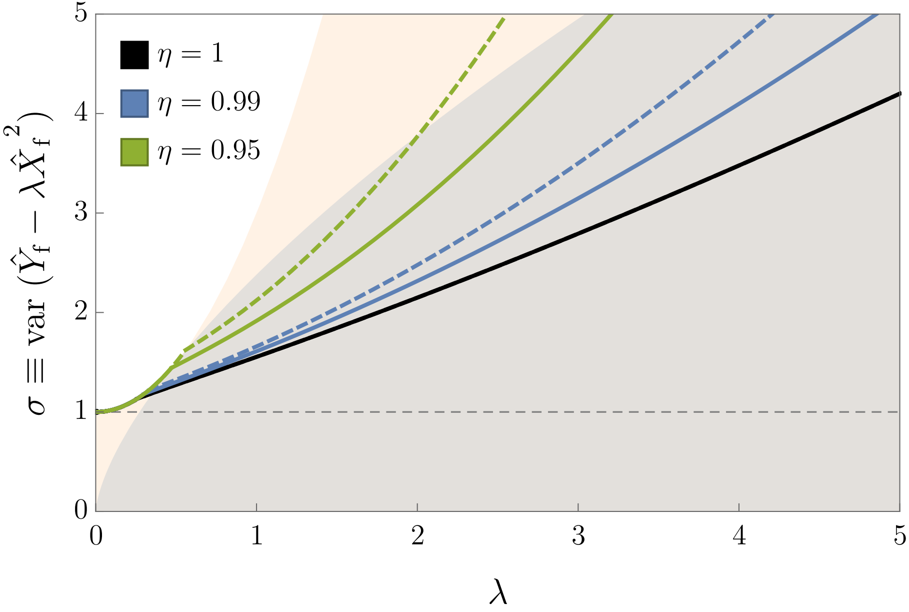
Simplified broadcasting protocol

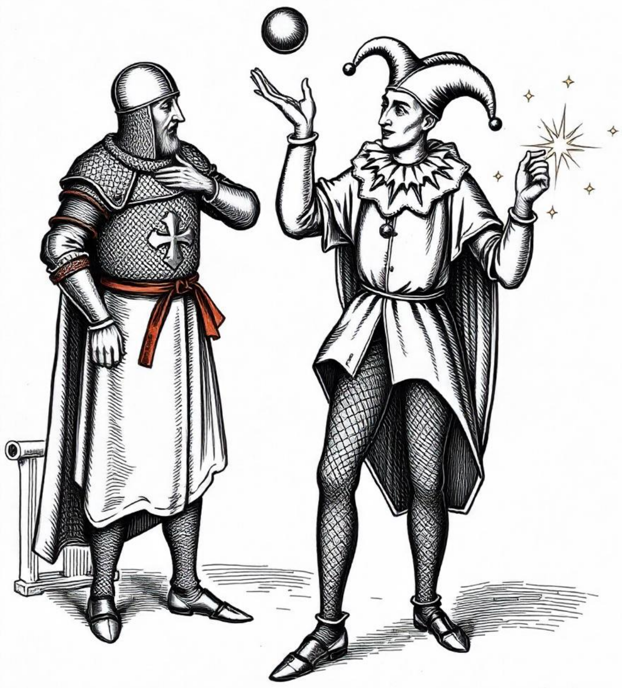


Input-output relations

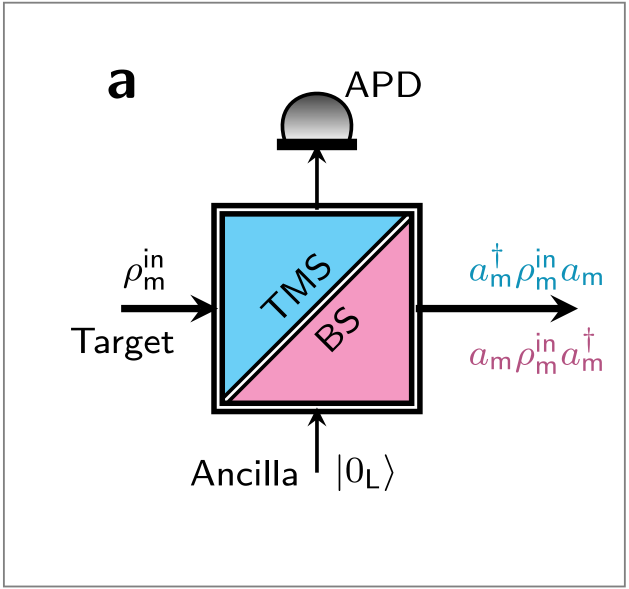
$$\hat{\mathbf{X}} = \hat{\mathbf{X}}, \qquad \hat{\mathbf{Y}} = \hat{\mathbf{Y}} - g\gamma V'(\hat{\mathbf{q}} + g\hat{\mathbf{X}}).$$

Results: Simplified broadcasting





Single-phonon addition or subtraction in optomechanics



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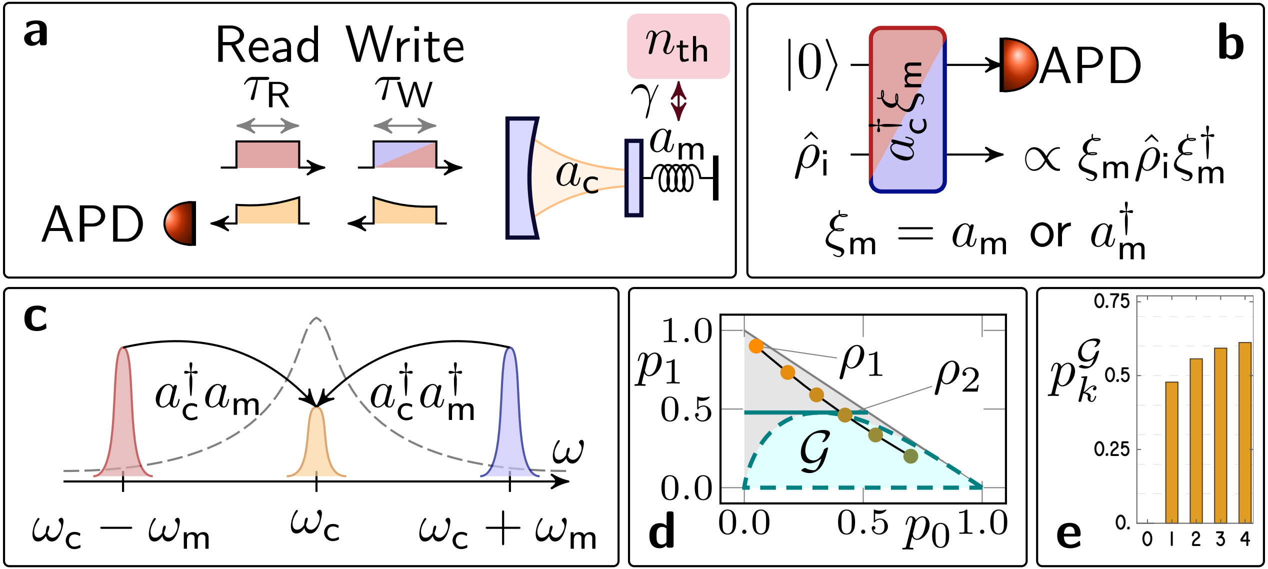
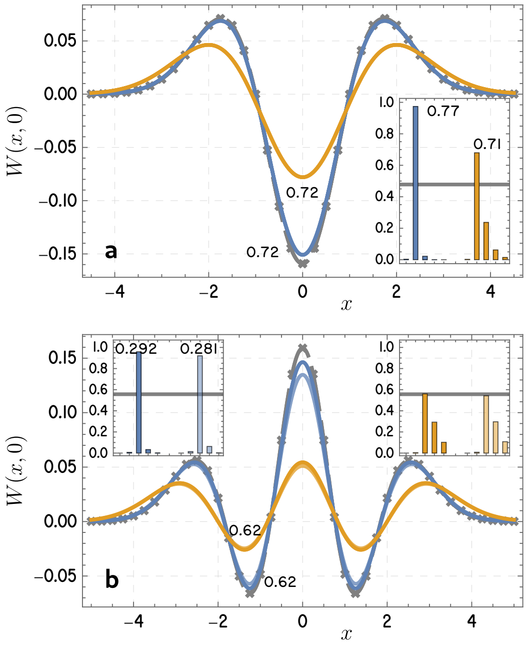


Evaluation of multiphonon quantum non-Gaussianity (superfluid He)



Multiphonon probabilities

$$p_k = \langle k|\rho|k\rangle$$

Criteria of absolute n-phonon quantum non-Gaussianity

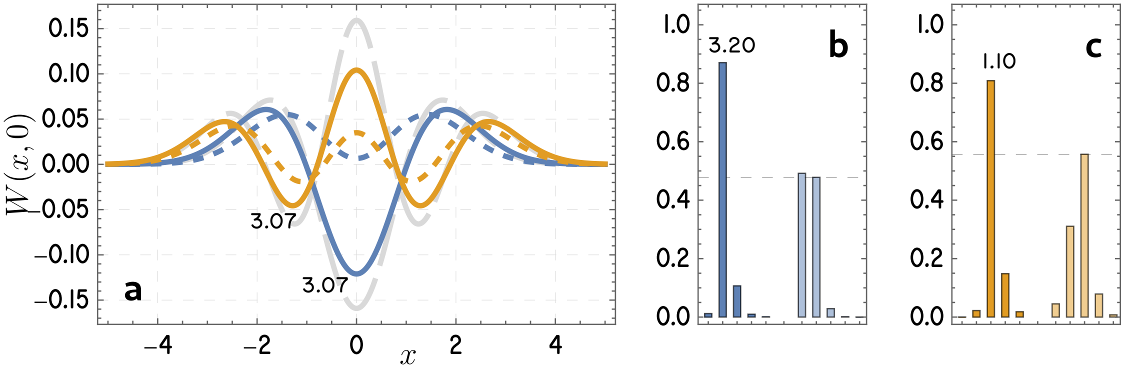
$$p_k^{\mathcal{G}} = \max_{\alpha,r,\{c_i\}} \left| \left\langle k \middle| \hat{D}(\alpha)\hat{S}(r) \sum_{i=0}^{k-1} c_i |i\rangle \right\rangle \right|^2 .$$

$$p_2^{\mathcal{G}} = \max_{\alpha,r,c_0,c_1} \left| \left\langle 2 \middle| \hat{D}(\alpha)\hat{S}(r) \Big(c_0 |0\rangle + c_1 |1\rangle \Big) \right\rangle \right|^2 .$$

k	1	2	3
$p_k^{\mathcal{G}}$	0.478	0.557	0.593

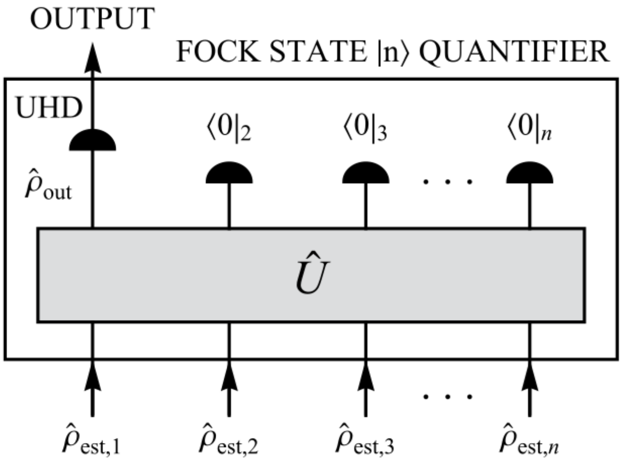
AR, R. Filip, Quantum Sci. Technol. **10**, 015014 (2024)

Readout and verification



Inset numbers show QNG depth: loss (in dB) to lose QNG.

Bunching capability

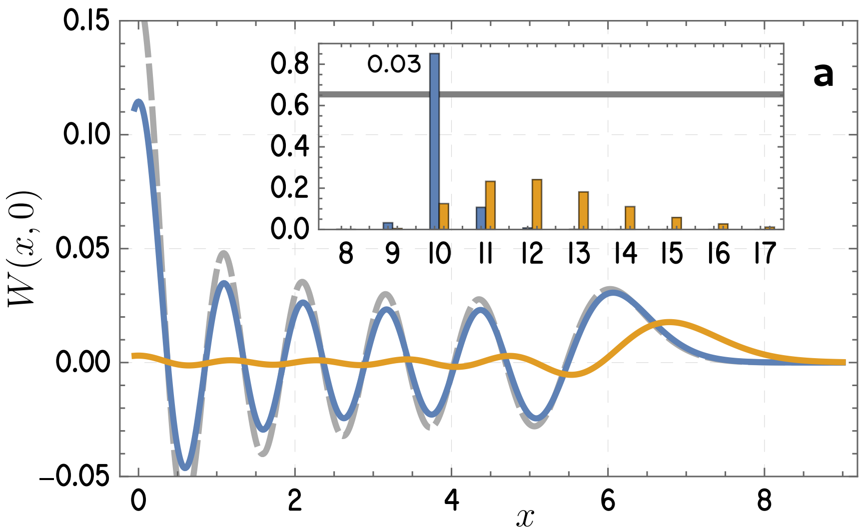


The recipe

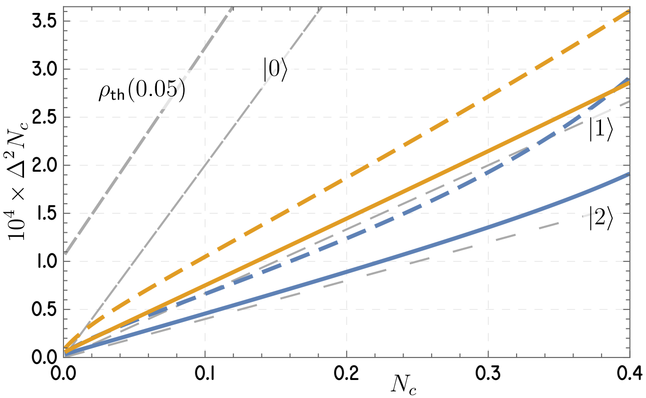
- ★ measure the statistics $\langle k|\hat\rho_{\mathrm{est}}|k\rangle$
- ★ compute hypothetical bunching state

Original proposal P. Zapletal and R. Filip, Sci Rep **7**, 1 (2017)
Implementations with OPA: P. Zapletal *et al.*, OPTICA **8**, 743 (2021).

Bunching capability



Application: detection of phase-randomized displacement



Phase-randomized displacement

$$\rho_{\text{in}} \mapsto \int_0^{2\pi} \frac{d\phi}{2\pi} \hat{D}(\sqrt{N_c}e^{i\phi})\rho_{\text{in}}\hat{D}^\dagger(\sqrt{N_c}e^{i\phi})$$

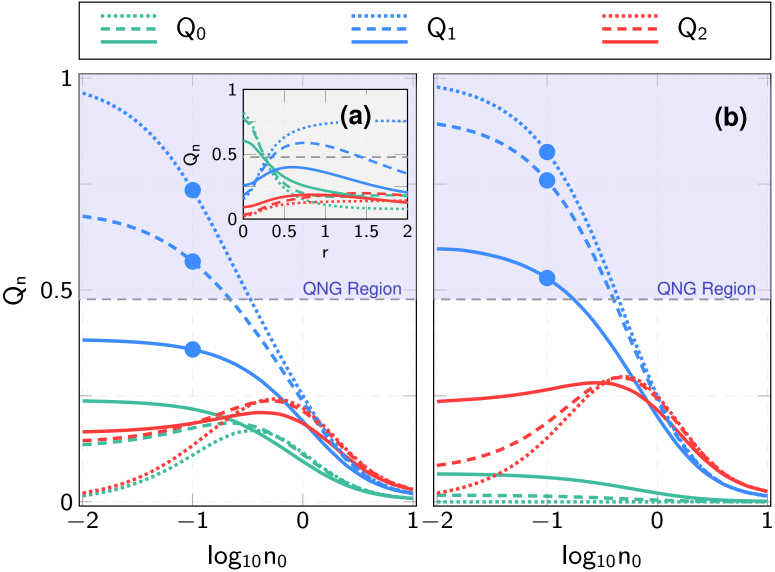
Cramér-Rao bound

$$\Delta^2 N_c \geqslant \frac{1}{M \cdot F(N_c)},$$

M — number of copies, F — quantum Fisher information

In levitated optomechanics

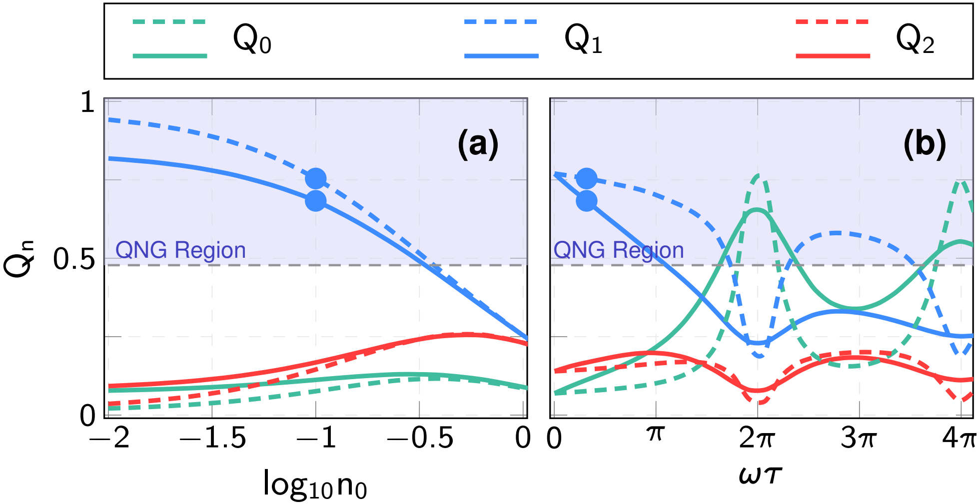
Inside a cavity



Parameters U. Delić, Science **367**, 892 (2020)

In levitated optomechanics

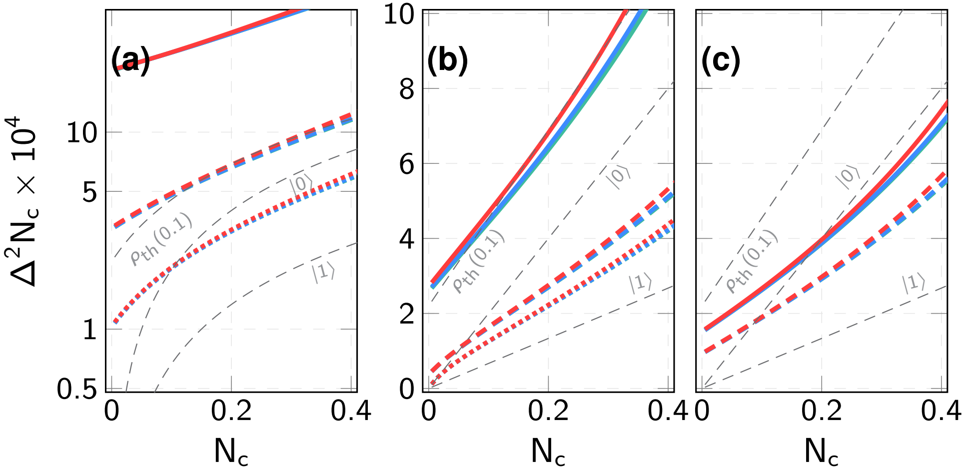
In free space



Parameters: L. Magrini, Phys. Rev. Lett **129**, 053601 (2022)

In levitated optomechanics

Phase-randomized displacement sensitivity



Conclusions

- Quantum non-Gaussianity is possible in optomechanics and outside
- both continuous-variable [CV] (nonlinear potentials) and discrete-variable [DV] (photon counting) regimes
- Nonlinearity can be broadcast to linear systems
- single-phonon-added states are helpful for phase-randomized force detection

Thank You!

These slides
`https://bit.ly/andrey-trest-25`

Beware of the appendix slide!

Effective classical simulation

Consider the setup:

- n quantum subsystems
- t operations
- local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \dots, k_n\},$$

one outcome per system.
Probability of the outcomes

$$P(k_1, \dots, k_n) \tag{1}$$

A classical algorithm is said to be capable of effective sampling, if it

- is efficient (polynomial) in t and n
- provides outcomes $\mathbf{k}$ drawn from the same probability as (1)

The very last frame which is empty