

Quantum Non-Gaussian Optomechanics

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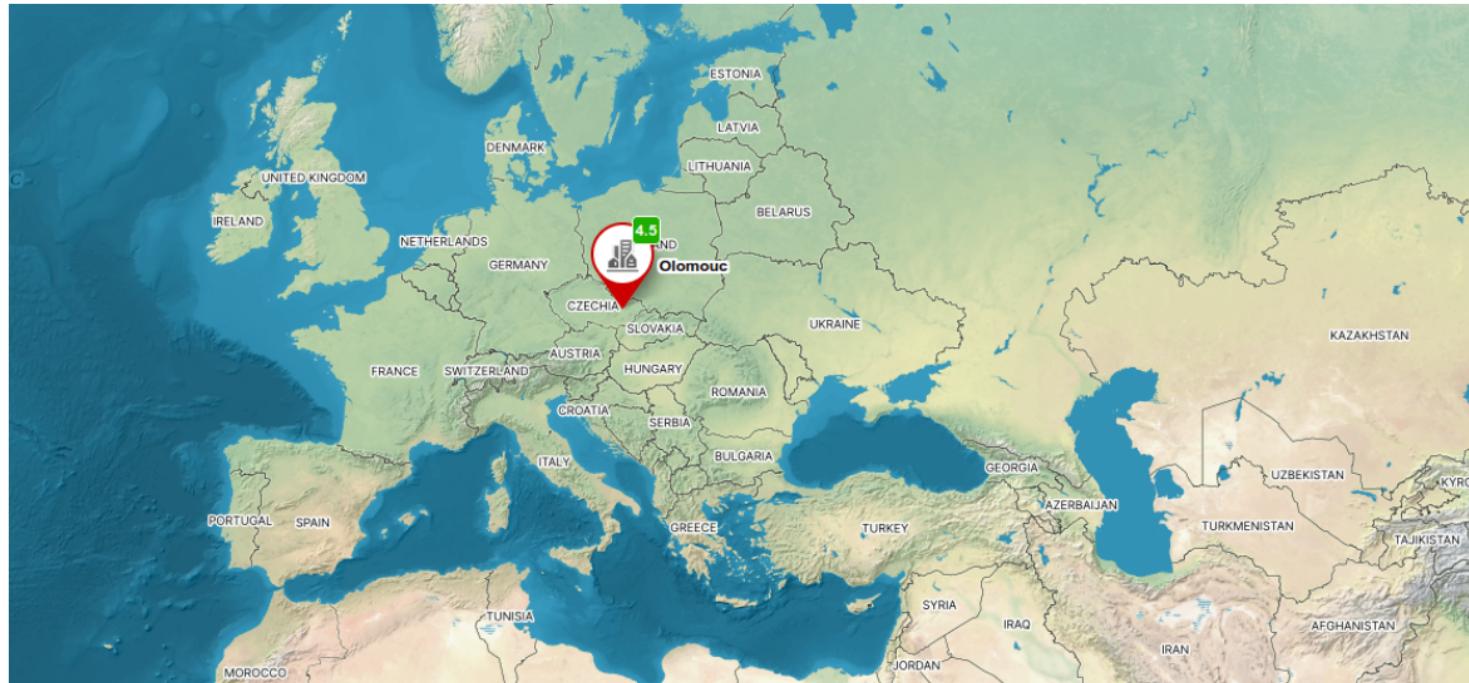
New Trends in Levitodynamics:
From Atoms to Nanostructures,
June 04, 2025,
Třešť



Spolufinancováno
Evropskou unií

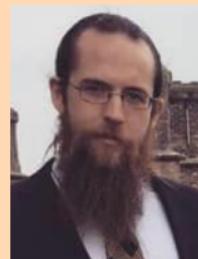


MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



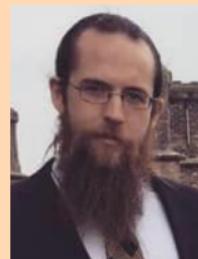


The People [within R. Filip's group]

Radim Filip**Foroud Bemani****Darren Moore****Alisa Manukhova****Najmeh Etehadi Abari****Surabhi Yadav****Shaoni Datta**

(Now @KIT, Germany)

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Introduction

Quantum Optomechanics

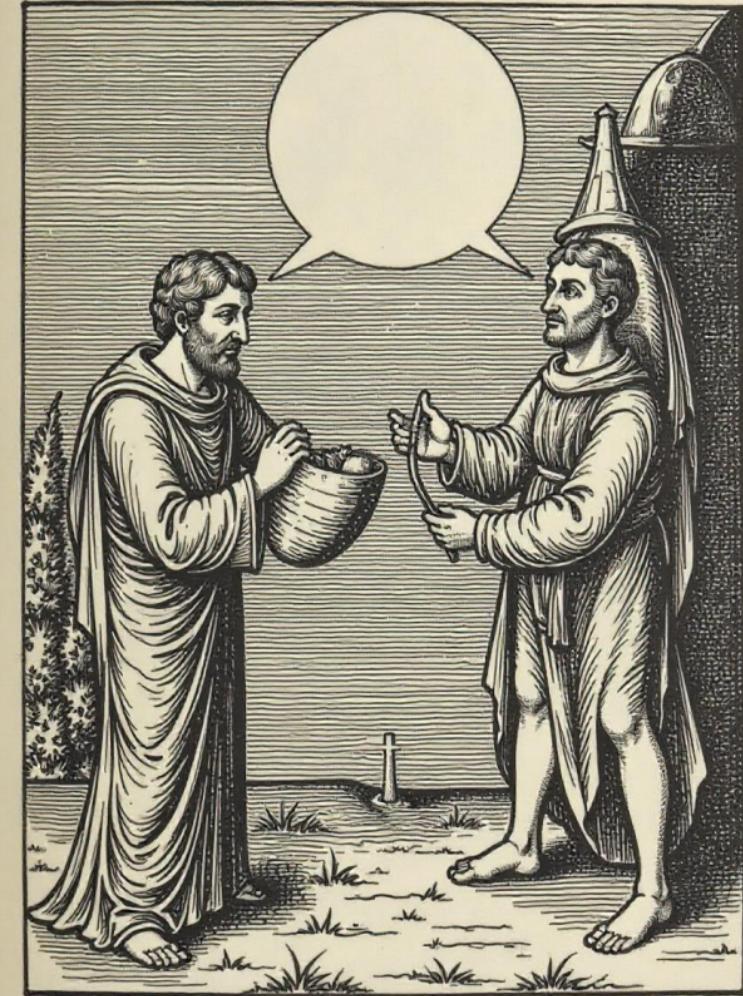
Quantum non-Gaussianity

Verification of quantum non-Gaussianity

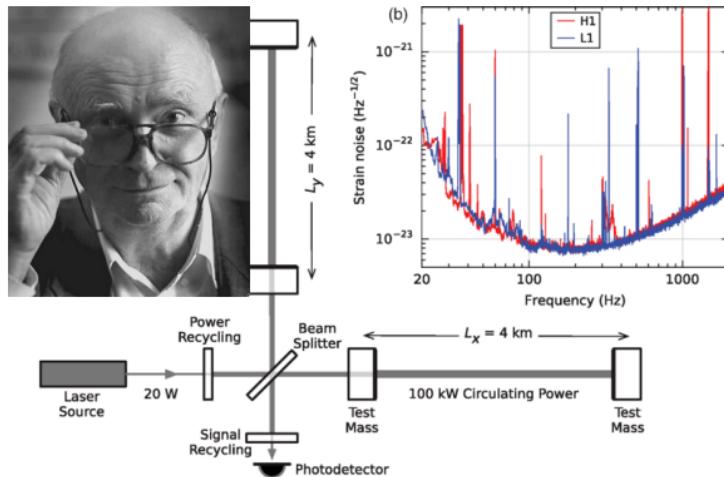
Motional Nonlinearities

Broadcasting the nonlinearity

Single-Phonon Addition/Subtraction

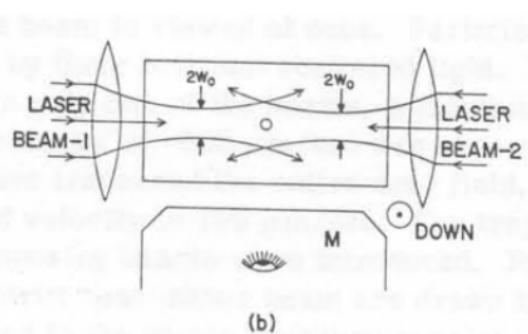
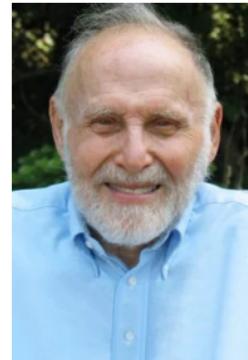


Quantum Optomechanics



Braginsky & Manukin, Soviet JETP **25**, 653 (1967)

Right img: B. P. Abbott et al., PRL **116**, 061102 (2016)



A. Ashkin, PRL **24**, 156 (1970)

$$\mathcal{H} = \omega_{\text{mech}} a_m^\dagger a_m + \omega_{\text{cav}} a_L^\dagger a_L + g(a_m + a_m^\dagger)(a_L + a_L^\dagger).$$

Gaussian vs Quantum non-Gaussian states

Gaussian states

Squeezed displaced state

$$\hat{\rho}_x = \hat{S}(r)\hat{D}(\alpha)|0\rangle.$$

Convex mixtures thereof

$$\hat{\rho}_G = \int p(x)\hat{\rho}_x dx.$$

$p(x)$ is a probability density:

- ★ $p(x) > 0$
- ★ “not more singular” than Dirac δ .

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Quantum non-Gaussian states

Cannot be represented as convex mixtures of squeezed displaced states

$$\hat{\rho}_{NG} \neq \int p(x)\rho_x dx.$$

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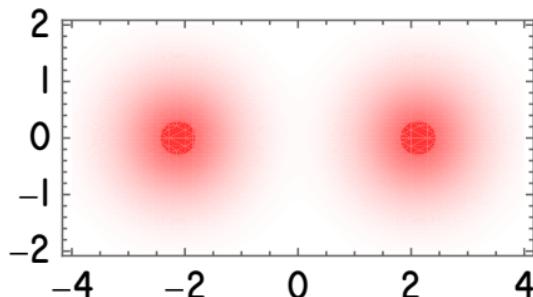
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Examples in the phase space

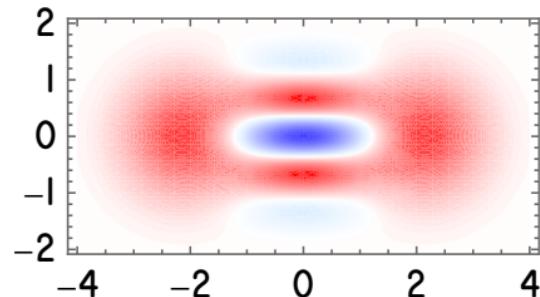
Classically NG state

$$\hat{\rho} \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$



Quantum NG state

$$|\psi\rangle \propto |\alpha\rangle - |-\alpha\rangle$$



Gaussian vs Quantum non-Gaussian states

Advantages of QNG states

- ★ Universal quantum computing
- ★ Quantum sensing
- ★ Fundamental studies

QNG is a resource

F. Albarelli *et al.*, Phys. Rev. A **98**, 052350 (2018)

M. Walschaers, PRX Quantum **2**, 030204 (2021)

Quantum non-Gaussian states

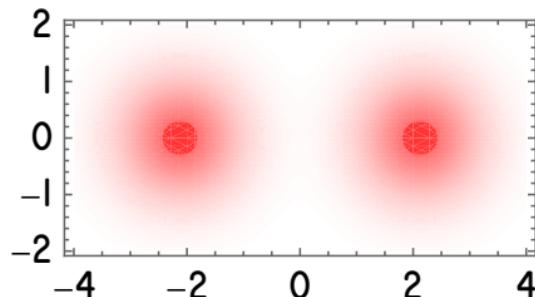
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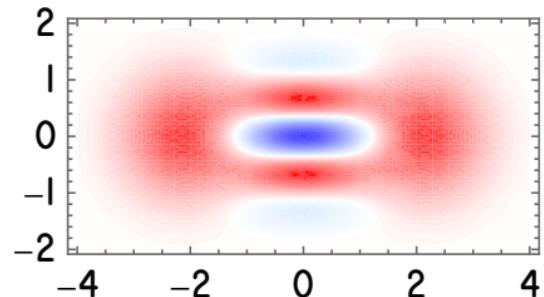
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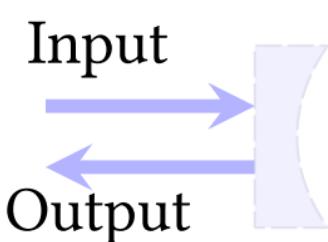


Routes to quantum non-Gaussianity in optomechanics

Add a nonlinear element

$$\hat{H}_P \propto \Omega_m (\hat{p}^2 + \hat{x}^2) + \alpha(t) V(\hat{x})$$

(a)



Nonlinear potential of mechanical motion

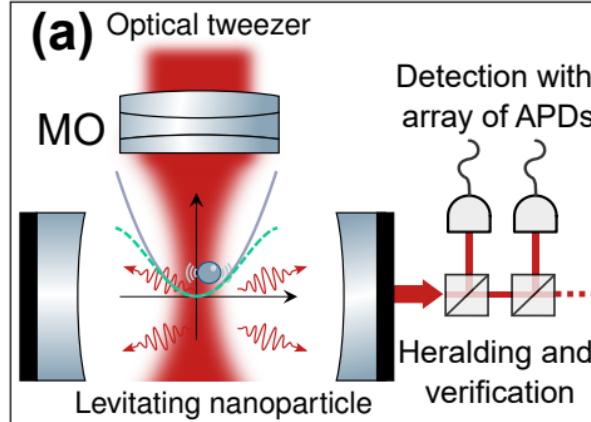
AR, R. Filip, Npj Quantum Inf 7, 120 (2021)

D.W. Moore, **AR**, R. Filip, NJP 21, 113050 (2019)

We don't consider here upload of QNG states

AR, R. Filip, Sci. Rep. 7, 46764 (2017)

Use non-linear detection



Counting photons

AR, R. Filip, Quantum Sci. Technol. 10, 015014 (2024)

F. Bemani, **AR**, R. Filip, Submitted , (2024)

Verification of quantum non-Gaussianity (QNG)

QNG
→

Linear Gaussian Dynamics



Intermediate control



Universal Quantum Control



We need better figures of merit than fidelity

Introduction

Quantum Optomechanics

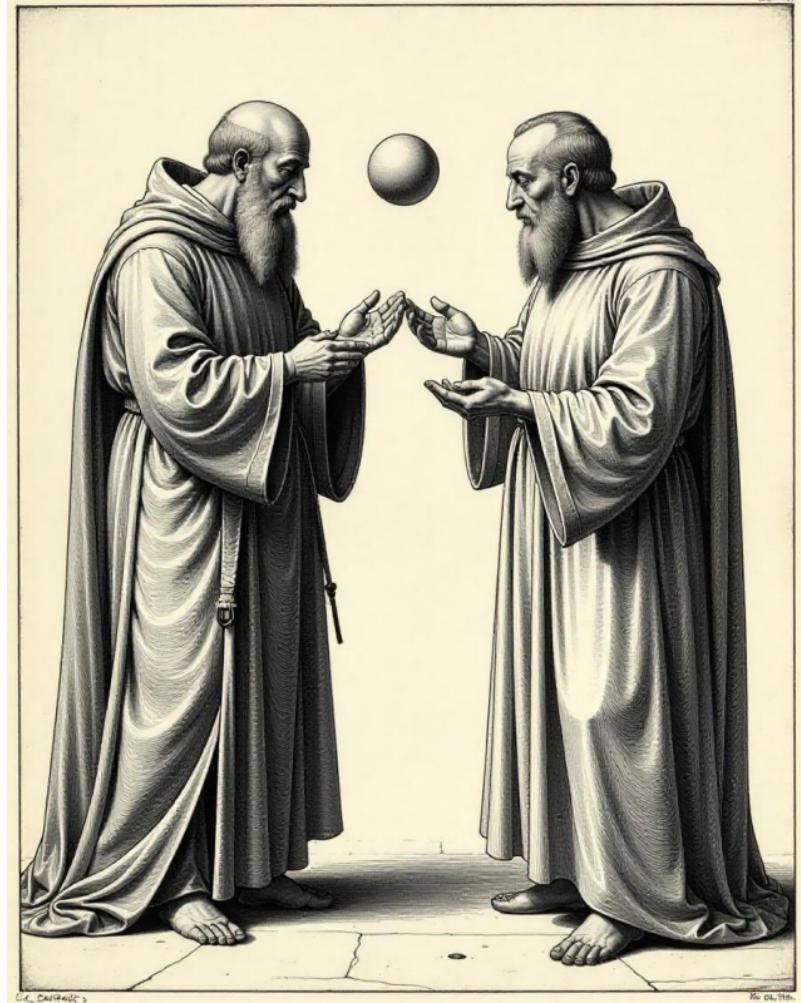
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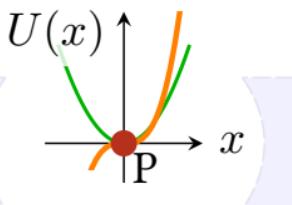
Nonlinear potential for a levitated nanoparticle

$$\hat{H}_P \propto \Omega_m(\hat{p}^2 + \hat{x}^2) + \alpha(t)V(\hat{x})$$

(a)

Input

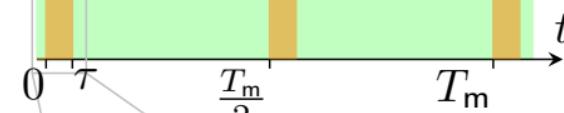
Output



Periodic Temporal Dynamics

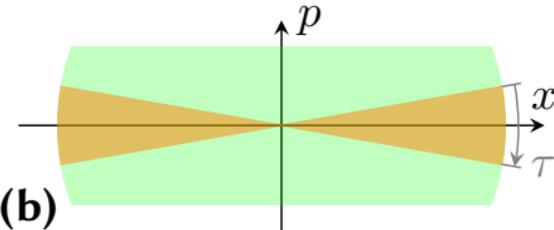
$$\alpha(t)$$

(c)



Phase Space Evolution

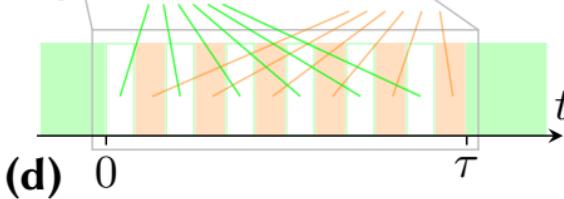
(b)



Suzuki-Trotter Simulation

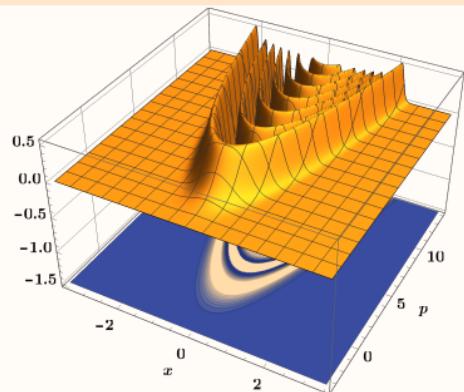
$$e^{-i\hat{H}_{HO}\tau/N} \quad e^{-iV(\hat{x})\tau/N}$$

(d)



Figures of merit

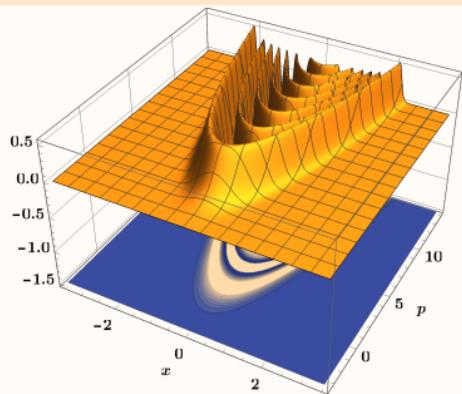
Wigner function



$$W_{\text{CPS}}(x, p) \propto \text{Ai}[p - \gamma x^2]$$

Figures of merit

Wigner function



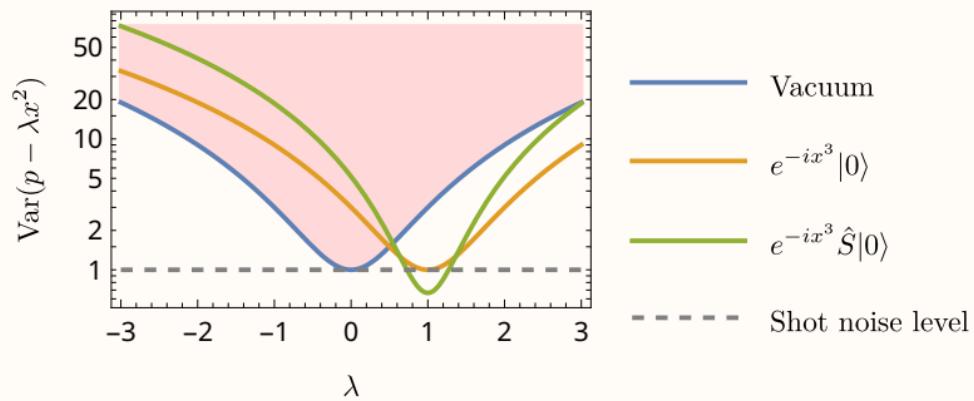
$$W_{\text{CPS}}(x, p) \propto \text{Ai}[p - \gamma x^2]$$

Nonlinear squeezing

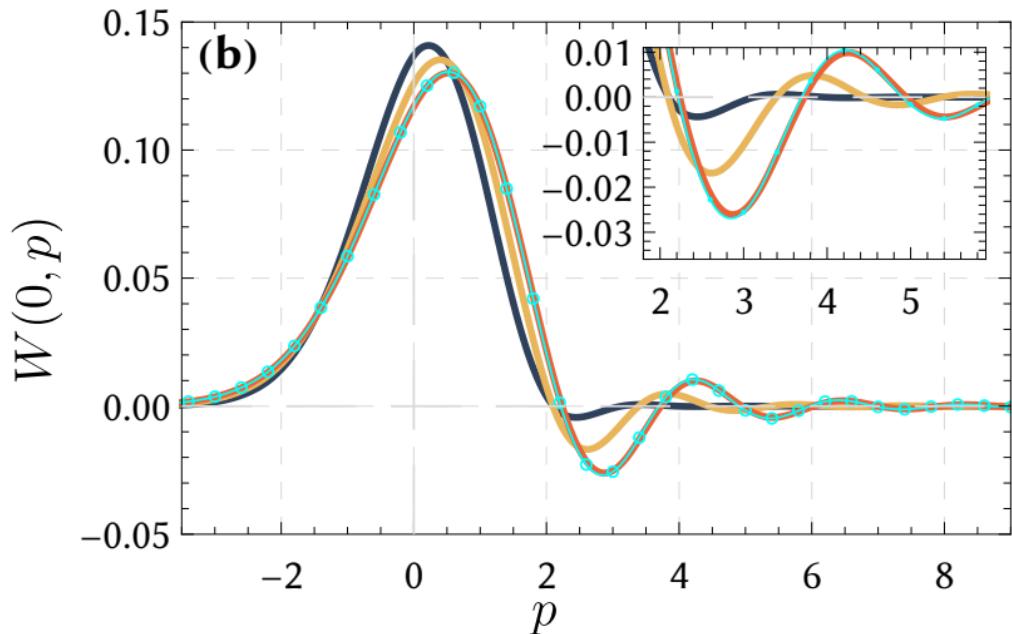
$$\begin{pmatrix} x \\ p \end{pmatrix} \xrightarrow{\exp[-i\gamma x^3]} \begin{pmatrix} x \\ p + \gamma x^2 \end{pmatrix}$$

Nonlinear squeezing

$$\sigma_3(\lambda; \rho) = \text{Var}_\rho(\hat{p} - \lambda \hat{x}^2).$$



Nonlinear potential for a levitated nanoparticle

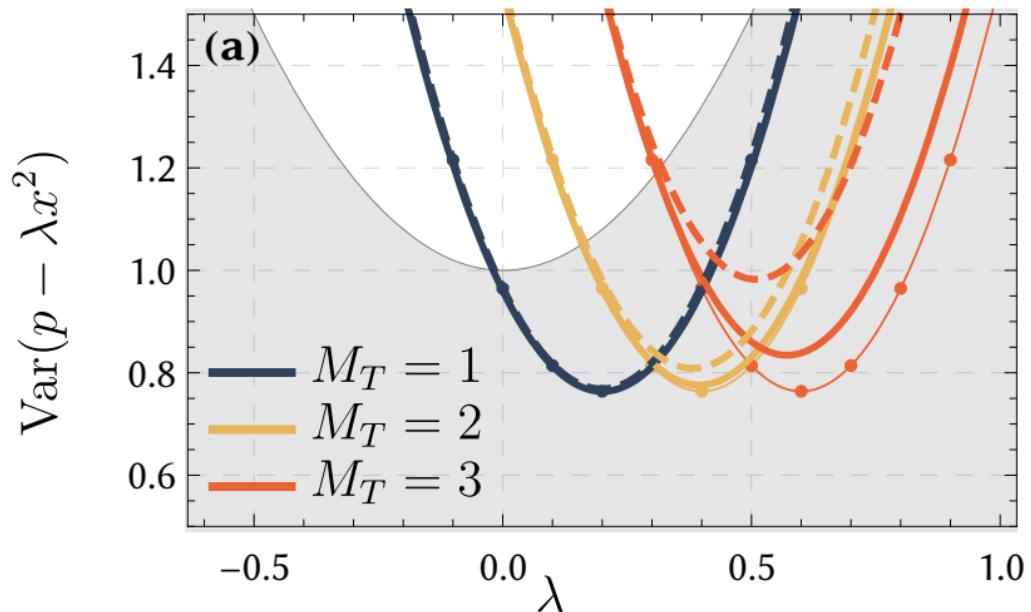


Wigner functions

Poor-man's fidelity (red & cyan)

$$4\pi \int dx dy W_{\text{red}}(x, y) W_{\text{cyan}}(x, y) = 0.9877.$$

Nonlinear potential for a levitated nanoparticle



Nonlinear variance

$$v_3 \equiv \text{Var}(\hat{p} - \lambda \hat{x}^2)$$

Compare with conventional squeezing:

$$\begin{aligned} v_2 &\equiv \text{Var}(\hat{x} \cos \theta + \hat{p} \sin \theta) \\ &= \sin^2 \theta \cdot \text{Var}(\hat{p} + \lambda \hat{x}), \end{aligned}$$

with $\lambda = \cot \theta$.

Related works about levitated NPs in nonlinear potentials (currently all theory)

Palacký University, cubic potential, multiple periods

AR, R. Filip, Npj Quantum Inf 7, 120 (2021) (arxiv 2019)

University of Vienna: "Super Mario", cubic potential, short pulse

L. Neumeier *et al.*, PNAS 121, e2306953121 (2024) (arxiv 2022)

University of Innsbruck, more complex potentials

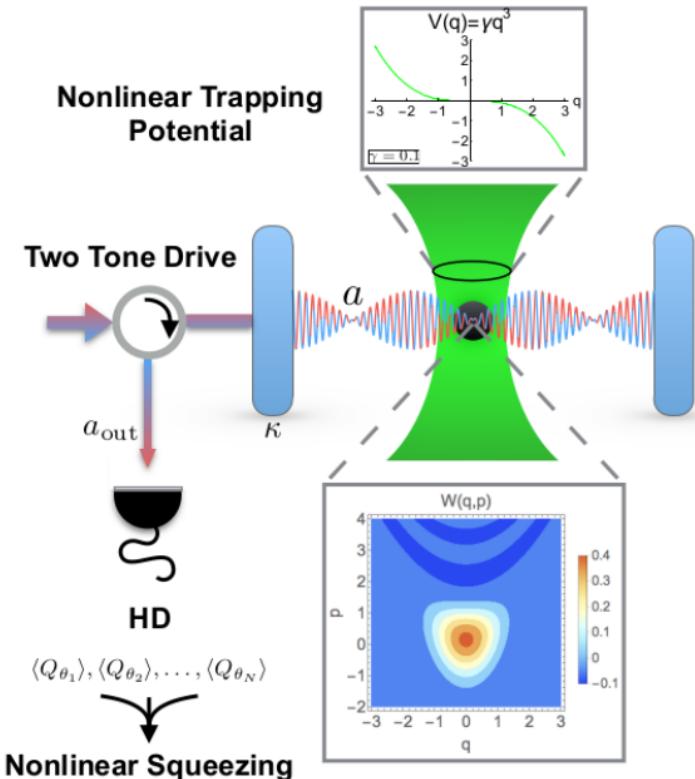
M. Roda-Llordes *et al.*, Phys. Rev. Res. 6, 013262 (2024),

M. Roda-Llordes *et al.*, Phys. Rev. Lett. 132, 023601 (2024),

A. Riera-Campeny *et al.*, Quantum 8, 1393 (2024)



Detecting Nonlinear Squeezing



Approximate cubic phase state in mechanics

$$e^{i\gamma q^3} \hat{S} |0\rangle .$$

Pulsed QND interaction

$$H_{int} \propto x_{light}(q \cos \phi + p \sin \phi).$$

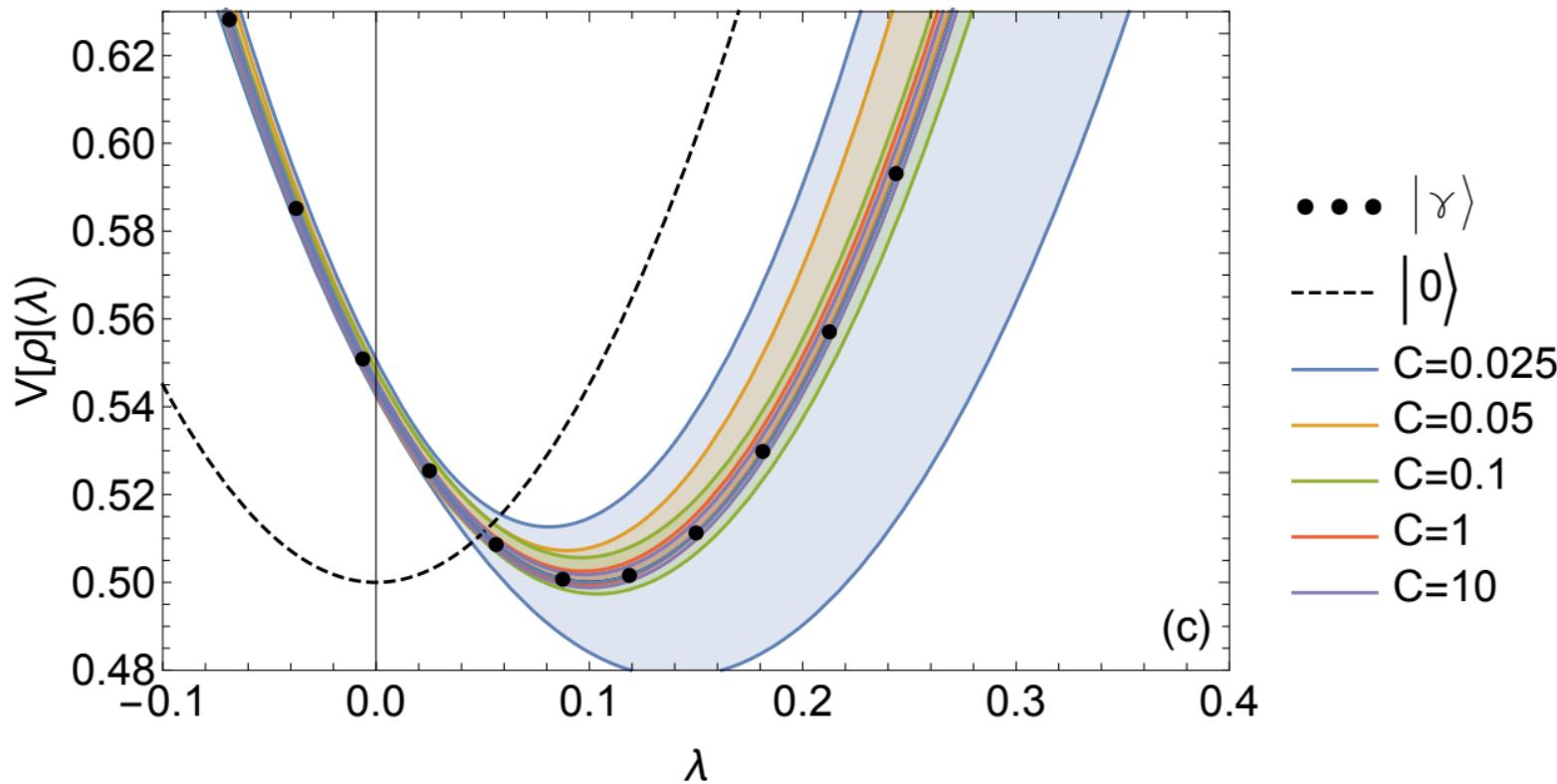
Detect leaking light

Estimate nonlinear variance

$$q_{NL} = p - \lambda x^2 \Rightarrow \text{Var}[q_{NL}]$$

D.W. Moore, **AR**, R. Filip, NJP **21**, 113050 (2019)

$$\text{Evaluation } V[\rho](\lambda) = \text{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$$



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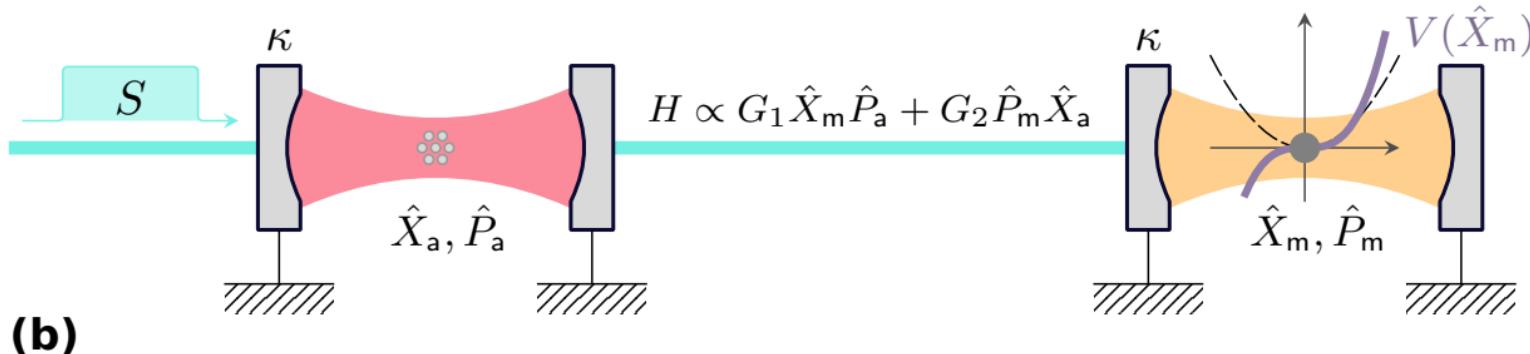
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Single-Phonon Addition/Subtraction



Broadcasting Protocol



We have

- ★ Gaussian interactions $\hat{H} \propto \hat{Q}_a \hat{Q}_m$
- ★ Nonlinearity in mechanics

$$\exp[-i\gamma \hat{X}_m^3]$$

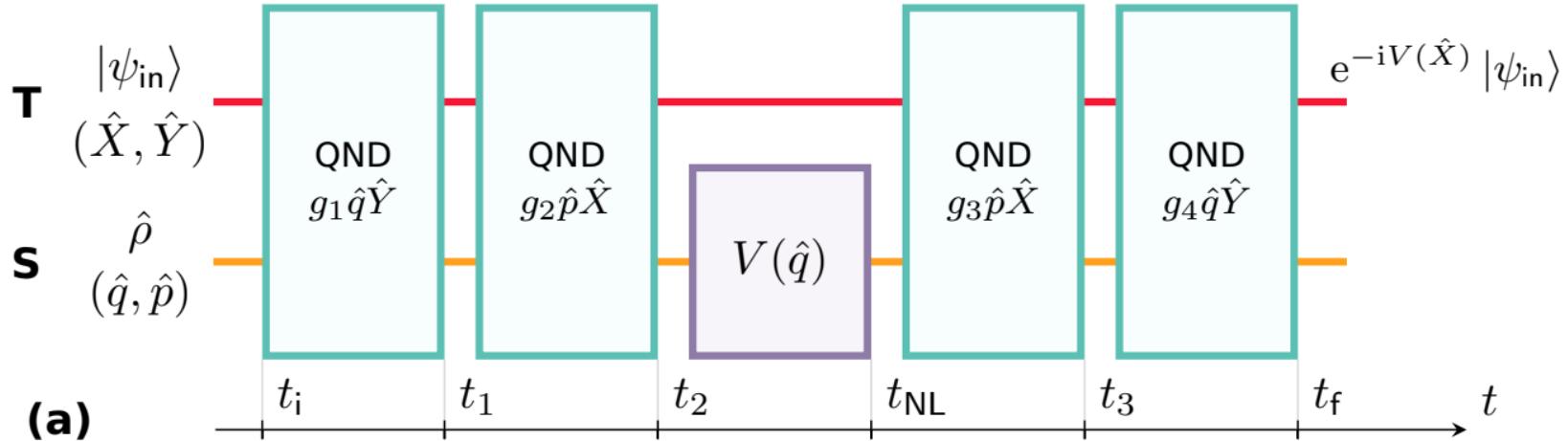
$$\hat{X}_m \mapsto \hat{X}_m, \quad \hat{Y}_m \mapsto \hat{Y}_m + \gamma \hat{X}_m^2.$$

We want

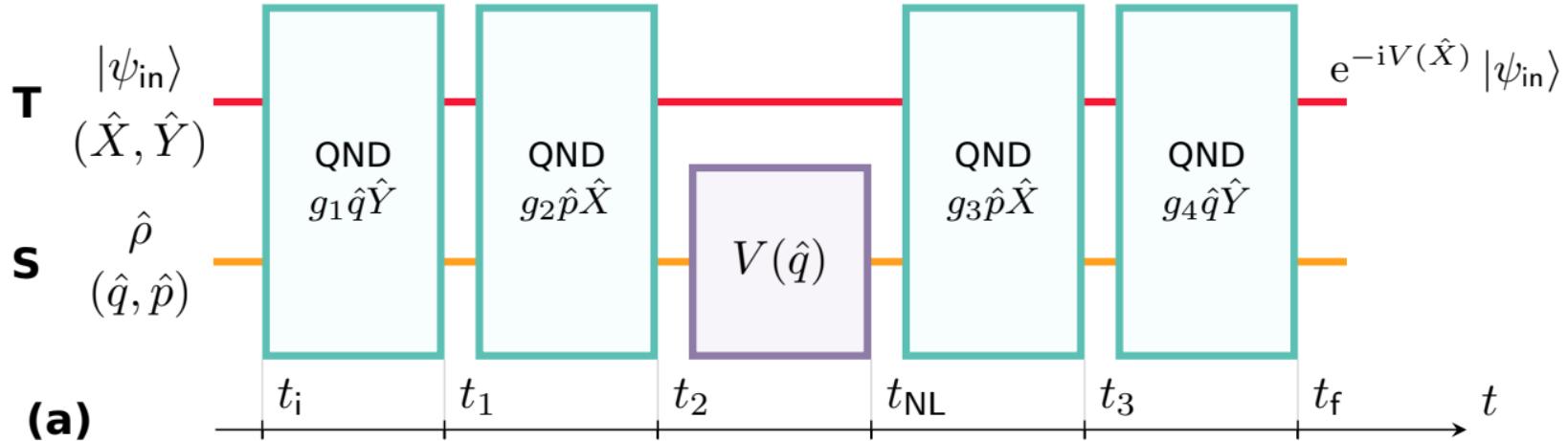
Nonlinearity in Atoms

$$\hat{X}_a \mapsto \hat{X}_a, \quad \hat{Y}_a \mapsto \hat{Y}_a + \tilde{\gamma} \hat{X}_a^2.$$

Broadcasting Protocol



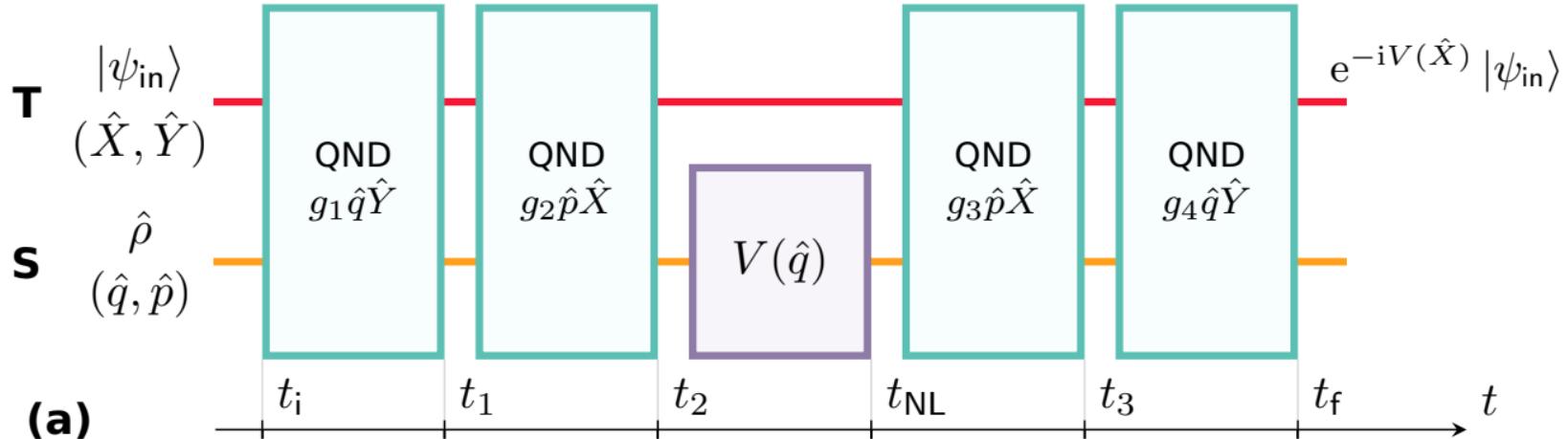
Broadcasting Protocol



$$\hat{\mathbf{X}} = (1 + g_4(g_2 + g_3))\hat{\mathbf{X}} + (g_4 + g_1(1 + g_4(g_2 + g_3)))\hat{\mathbf{q}},$$

$$\hat{\mathbf{Y}} = (1 + g_1(g_2 + g_3))\hat{\mathbf{Y}} - (g_2 + g_3)\hat{\mathbf{p}} + g_3\gamma V' ((1 + g_1g_2)\hat{\mathbf{q}} + g_2\hat{\mathbf{X}}).$$

Broadcasting Protocol

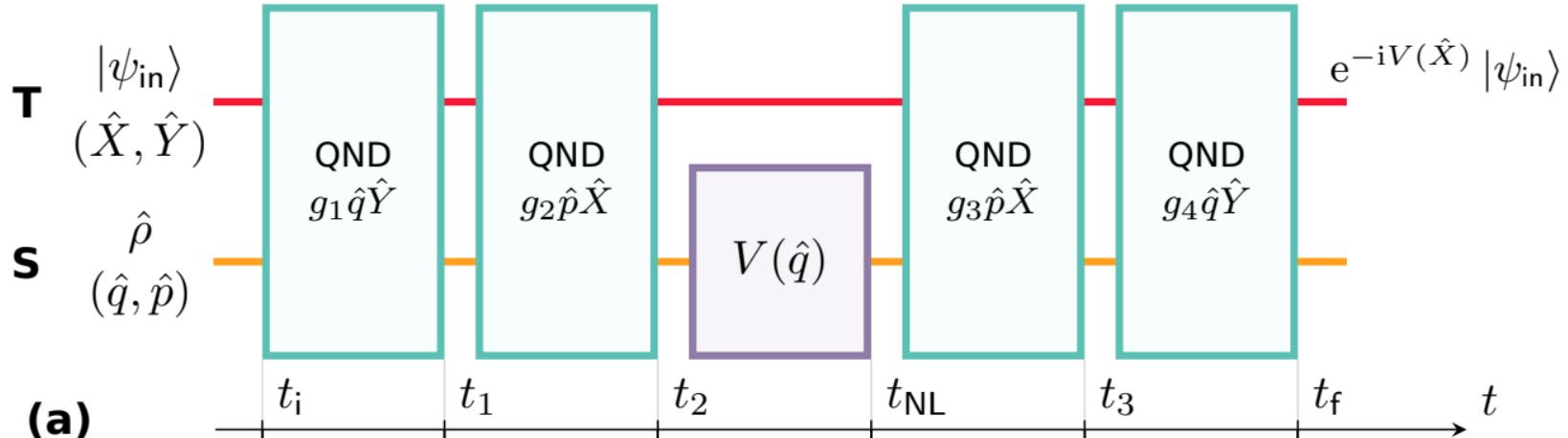


$$g_2 = -g_3$$

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Broadcasting Protocol



$$-1/g_1 = g_2 = -g_3 = 1/g_4$$

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Regimes of operation

Broadcasting of the cubic nonlinearity

$$-1/g_1 = g_2 = -g_3 = 1/g_4 = g$$

$$\hat{\mathbf{X}} \mapsto \hat{\mathbf{X}},$$

$$\hat{\mathbf{Y}} \mapsto \hat{\mathbf{Y}} + \gamma g^3 \hat{\mathbf{X}}^2.$$

- ★ Valid for arbitrary state of mechanics
- ★ Implements the operation $\exp[-i\gamma g^3 \hat{\mathbf{X}}^3]$.

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Figure of merit: Nonlinear squeezing

$$\text{Var}(\hat{Y} - \lambda \hat{X}^2) = 1 + 2(\lambda - \gamma g^3)^2 \geq 1$$

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Generation of nonlinear squeezing

$$g_2 = g - 1/g_1, \quad g_3 = -g, \quad g_4 = 1/g$$

$$\begin{aligned}\hat{\mathbf{X}} &\mapsto \hat{\mathbf{X}}_f = \left(1 - \frac{1}{gg_1}\right) \hat{\mathbf{X}} + g_1 \hat{\mathbf{q}} \\ \hat{\mathbf{Y}} &\mapsto \frac{1}{g_1} \hat{\mathbf{p}} + \gamma g(g \hat{\mathbf{X}}_f)^2.\end{aligned}$$

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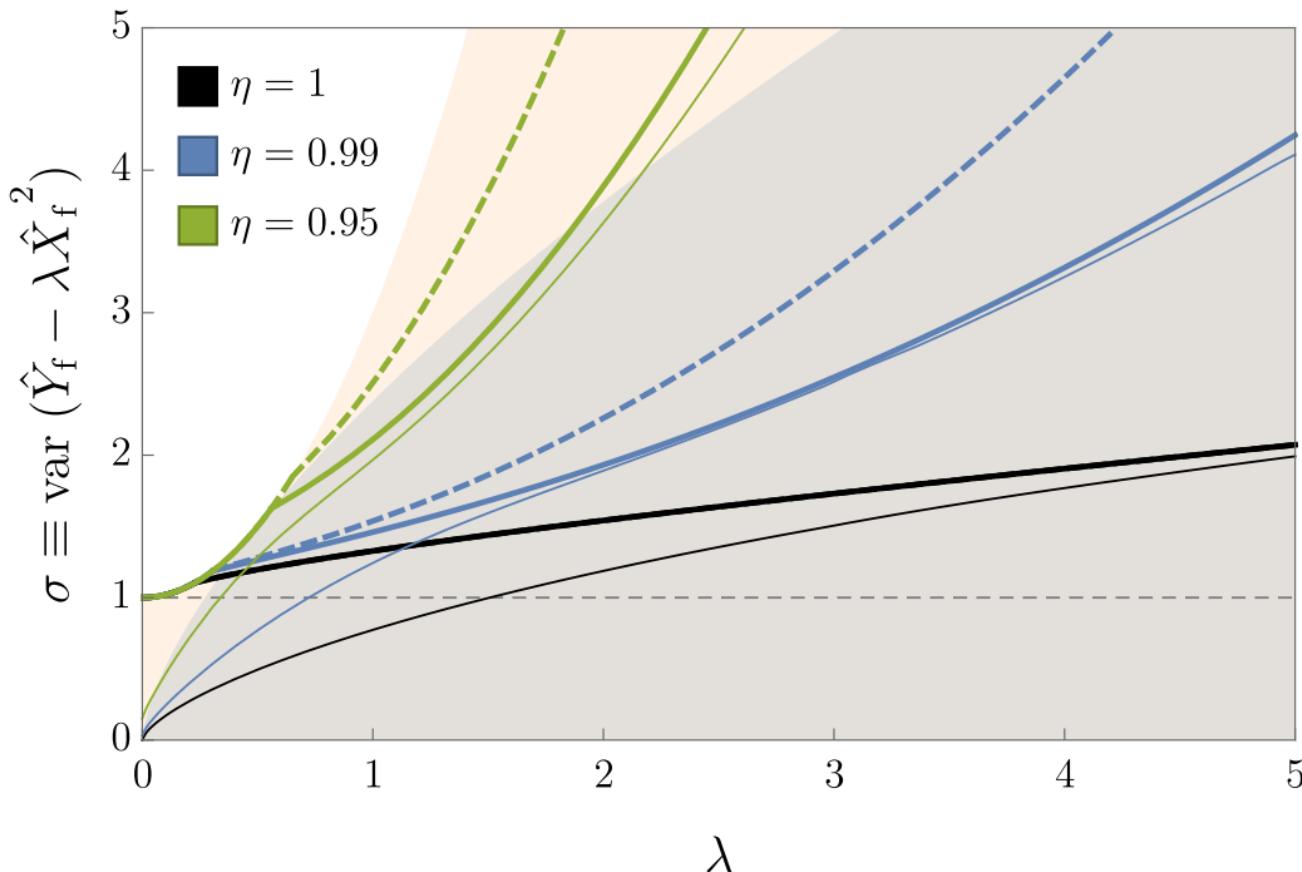
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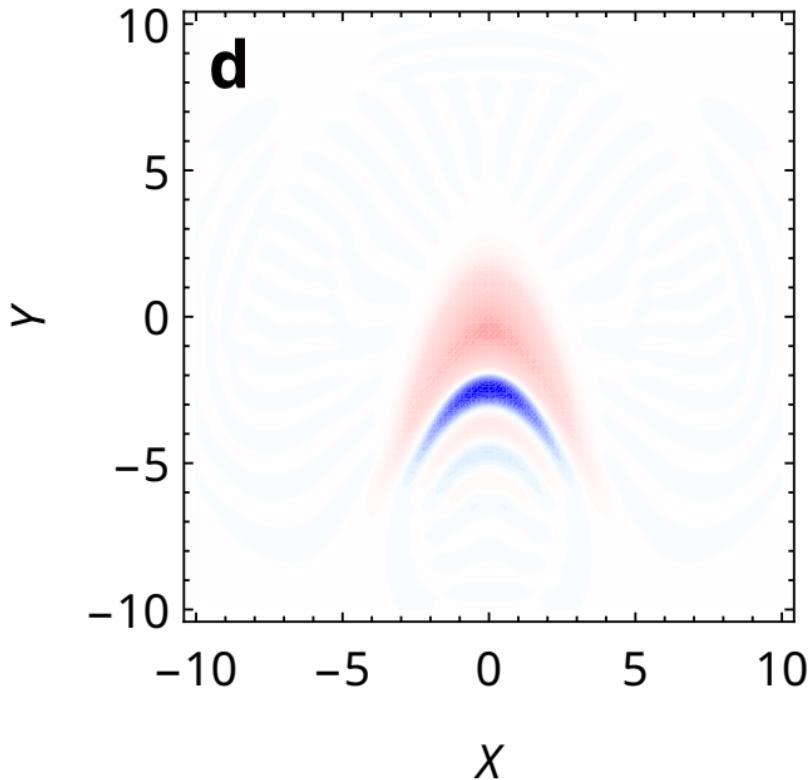
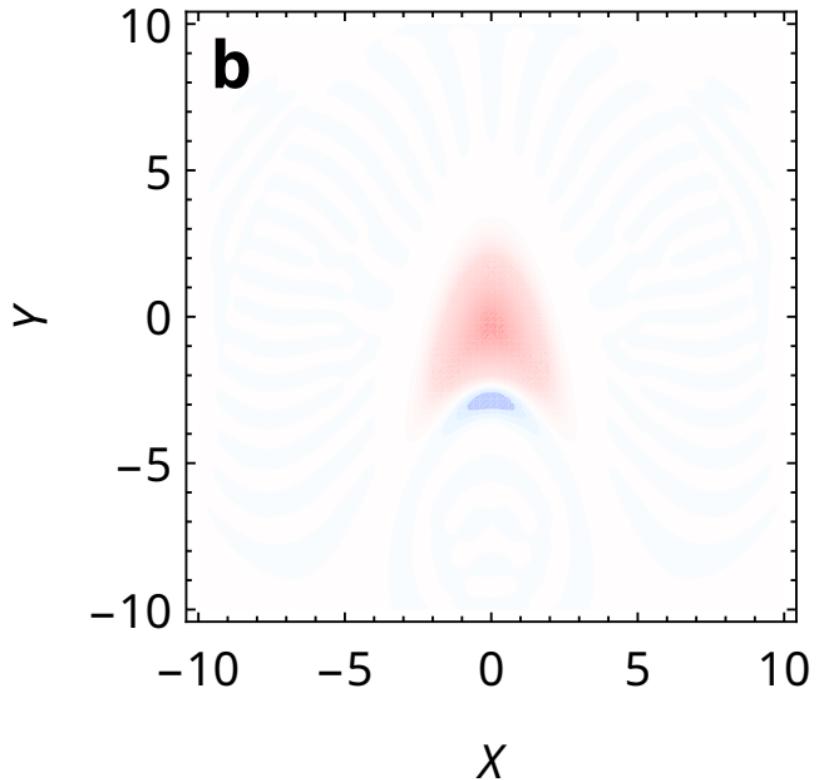
Nonlinear squeezing

$$\text{Var}(\hat{\mathbf{Y}}_f - \lambda \hat{\mathbf{X}}_f^2) = \frac{1}{g_1^2} \text{Var}(\hat{\mathbf{p}}) + 2(\lambda - \gamma g^3)^2 \text{Var}(\hat{\mathbf{X}}^2)$$

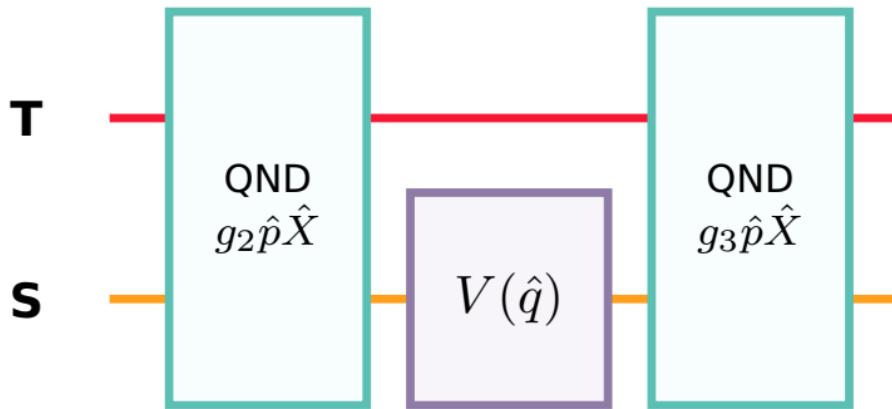
Results: Nonlinear squeezing



Results: Wigner functions



Simplified broadcasting protocol

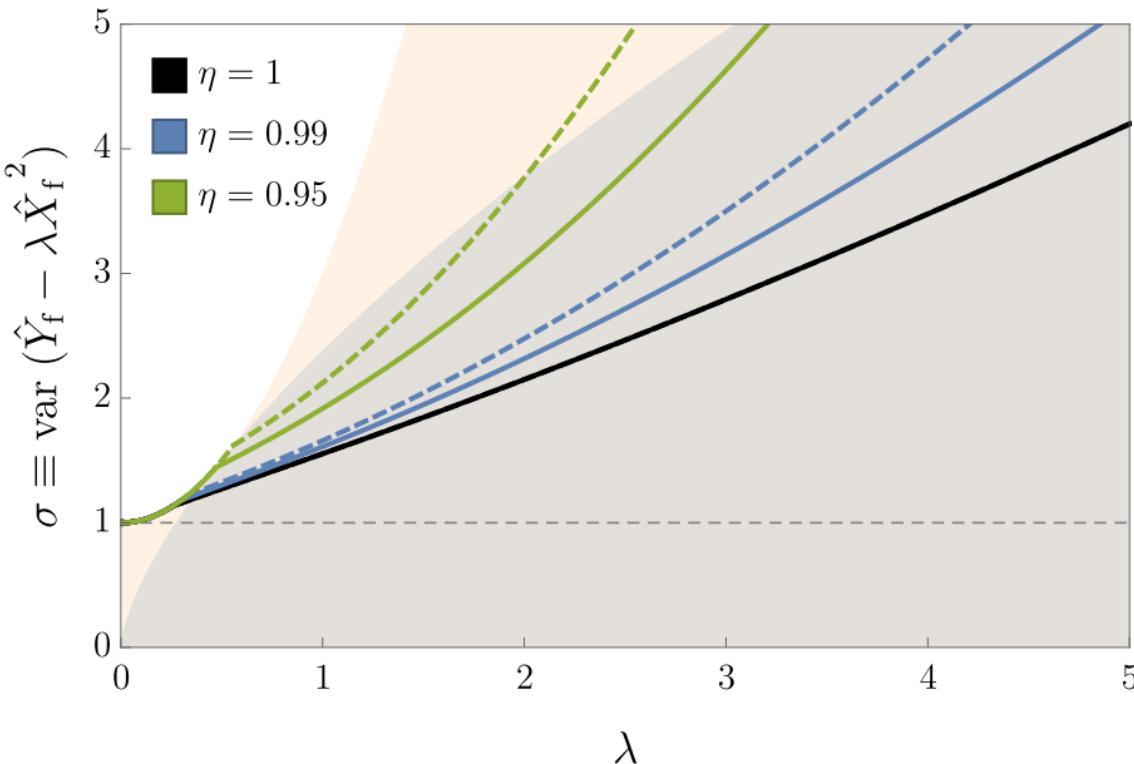


Input-output relations

$$\hat{\mathbf{X}} = \hat{\mathbf{X}},$$

$$\hat{\mathbf{Y}} = \hat{\mathbf{Y}} - g\gamma V'(\hat{\mathbf{q}} + g\hat{\mathbf{X}}).$$

Results: Simplified broadcasting



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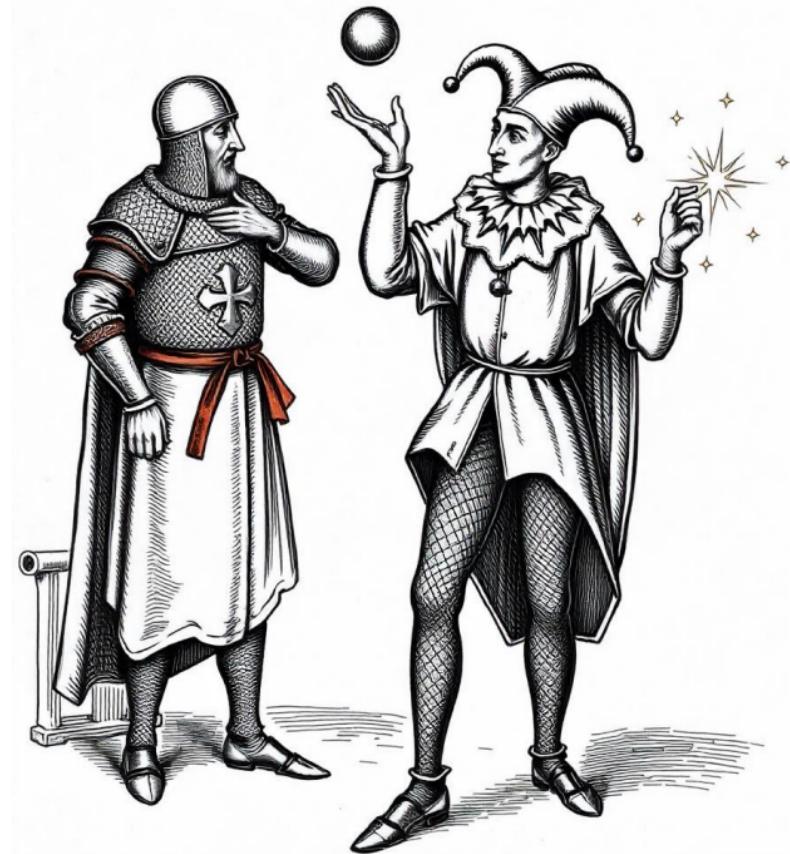
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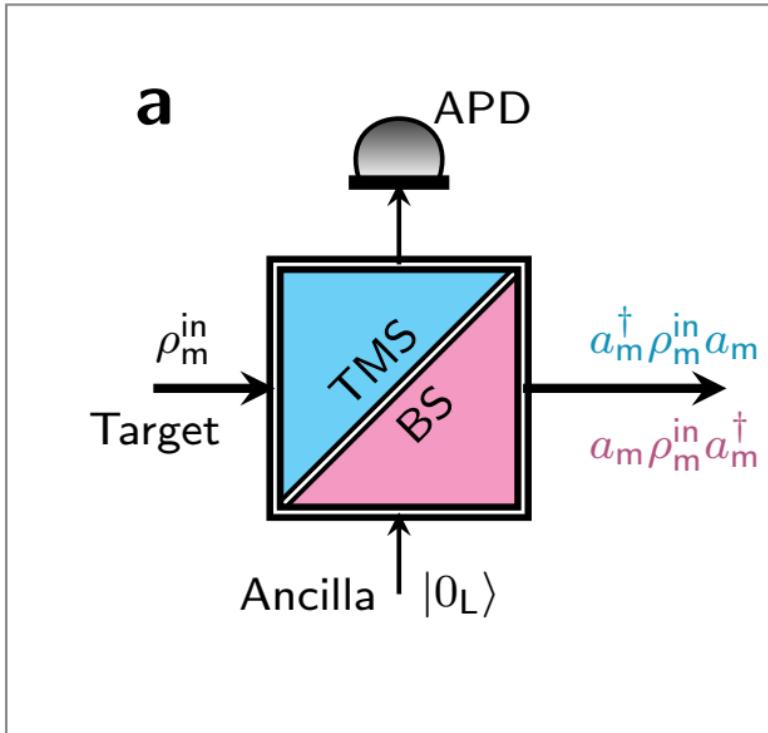
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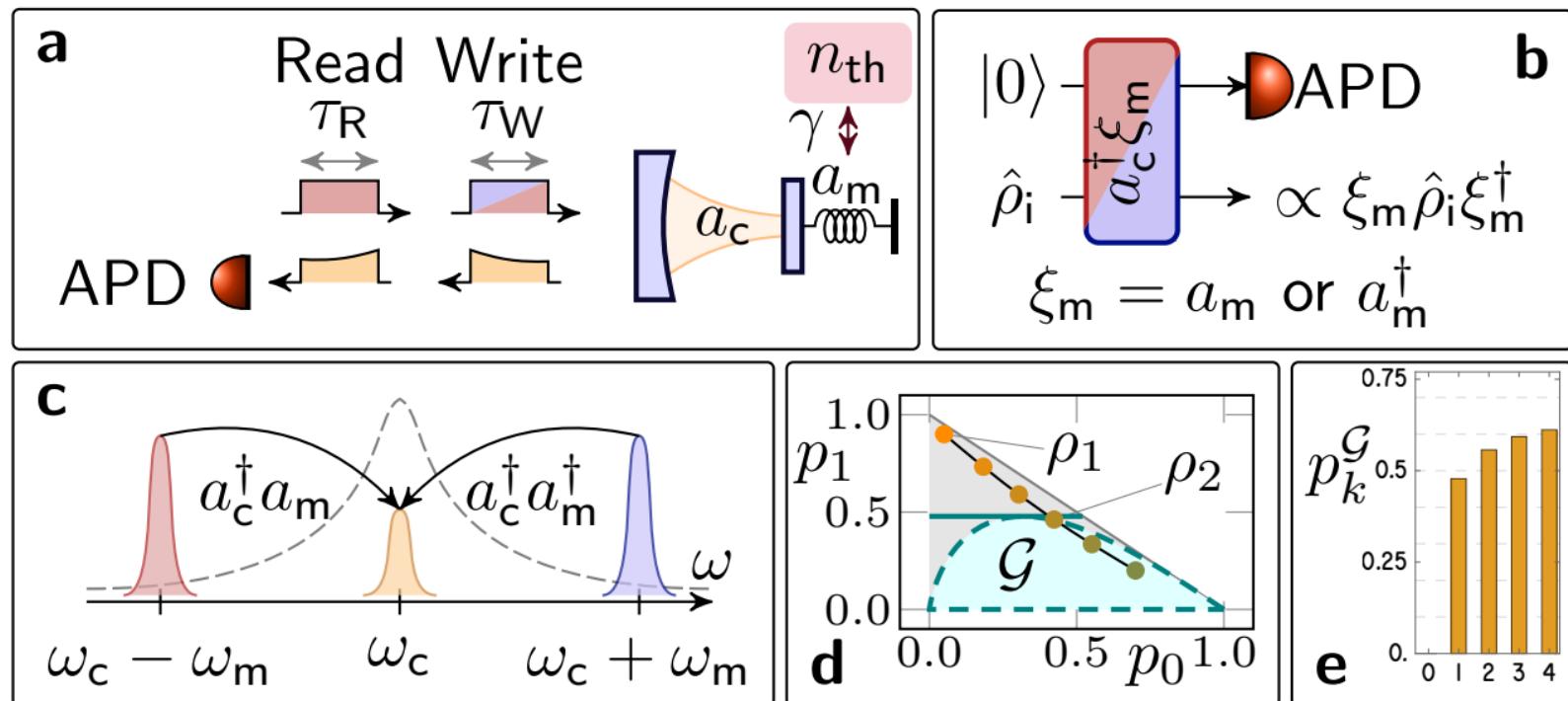
Single-Phonon Addition/Subtraction



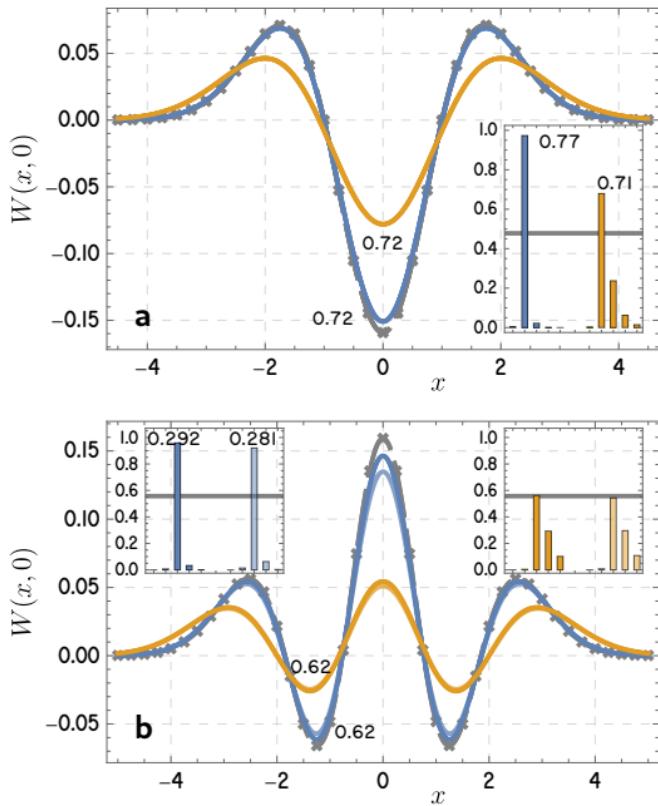
Single-phonon addition or subtraction in optomechanics



Single-phonon addition or subtraction in optomechanics



Evaluation of multiphonon quantum non-Gaussianity (superfluid He)



Multiphonon probabilities

$$p_k = \langle k | \rho | k \rangle$$

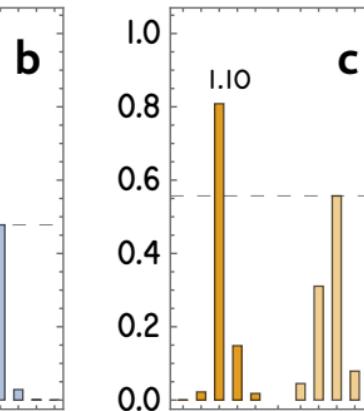
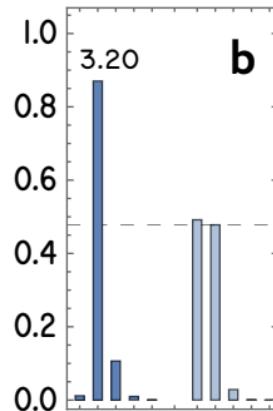
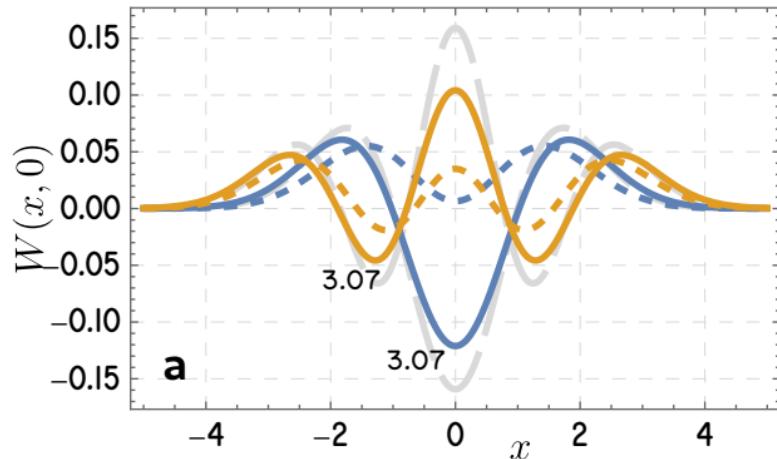
Criteria of absolute n -phonon quantum non-Gaussianity

$$p_k^G = \max_{\alpha, r, \{c_i\}} \left| \left\langle k \left| \hat{D}(\alpha) \hat{S}(r) \sum_{i=0}^{k-1} c_i |i\rangle \right. \right\rangle \right|^2.$$

$$p_2^G = \max_{\alpha, r, c_0, c_1} \left| \left\langle 2 \left| \hat{D}(\alpha) \hat{S}(r) \left(c_0 |0\rangle + c_1 |1\rangle \right) \right. \right\rangle \right|^2.$$

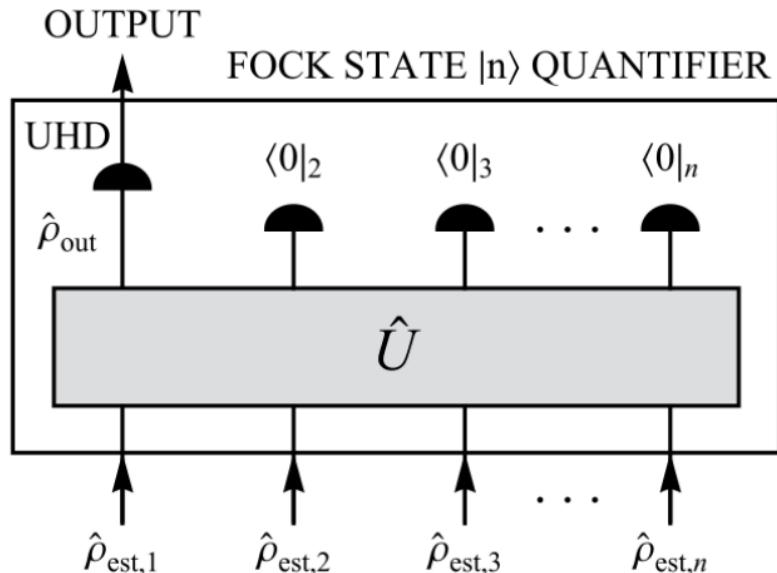
k	1	2	3
p_k^G	0.478	0.557	0.593

Readout and verification



Inset numbers show QNG depth: loss (in dB) to lose QNG.

Bunching capability

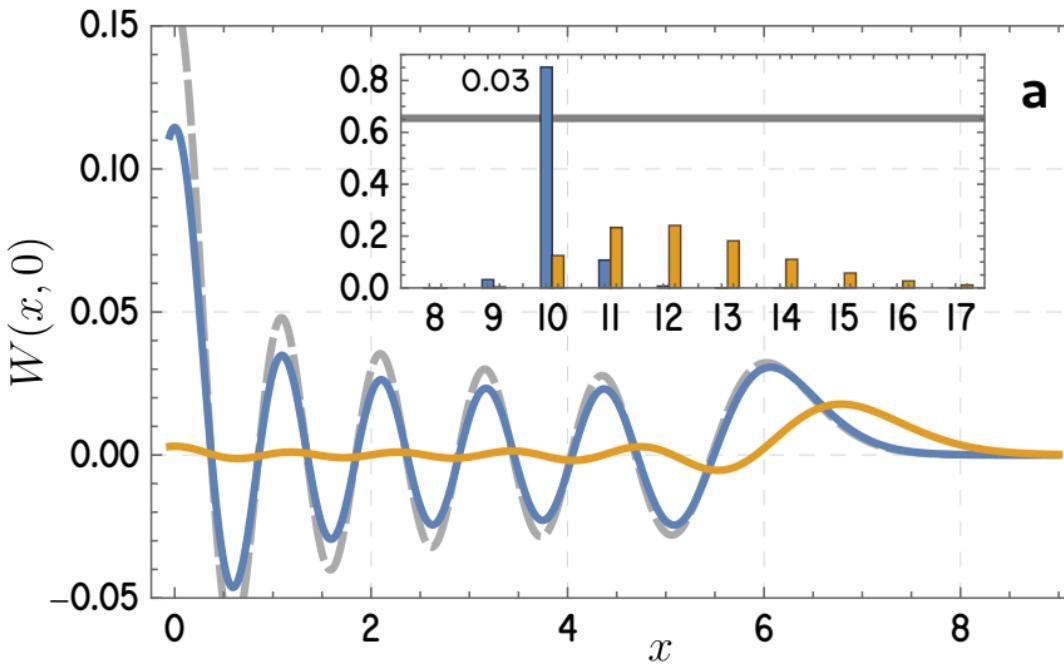


The recipe

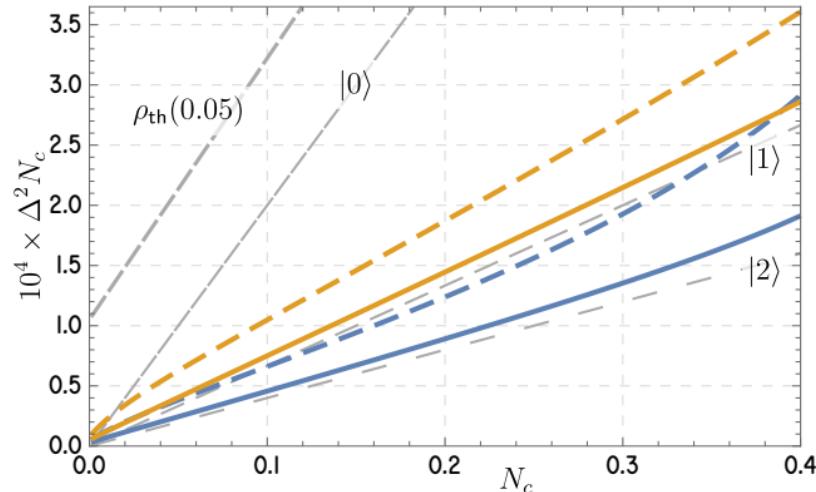
- ★ measure the statistics $\langle k|\hat{\rho}_{\text{est}}|k\rangle$
- ★ compute hypothetical bunching state

Original proposal P. Zapletal and R. Filip, Sci Rep 7, 1 (2017)
Implementations with OPA: P. Zapletal *et al.*, OPTICA 8, 743 (2021).

Bunching capability



Application: detection of phase-randomized displacement



Phase-randomized displacement

$$\rho_{\text{in}} \mapsto \int_0^{2\pi} \frac{d\phi}{2\pi} \hat{D}(\sqrt{N_c} e^{i\phi}) \rho_{\text{in}} \hat{D}^\dagger(\sqrt{N_c} e^{i\phi})$$

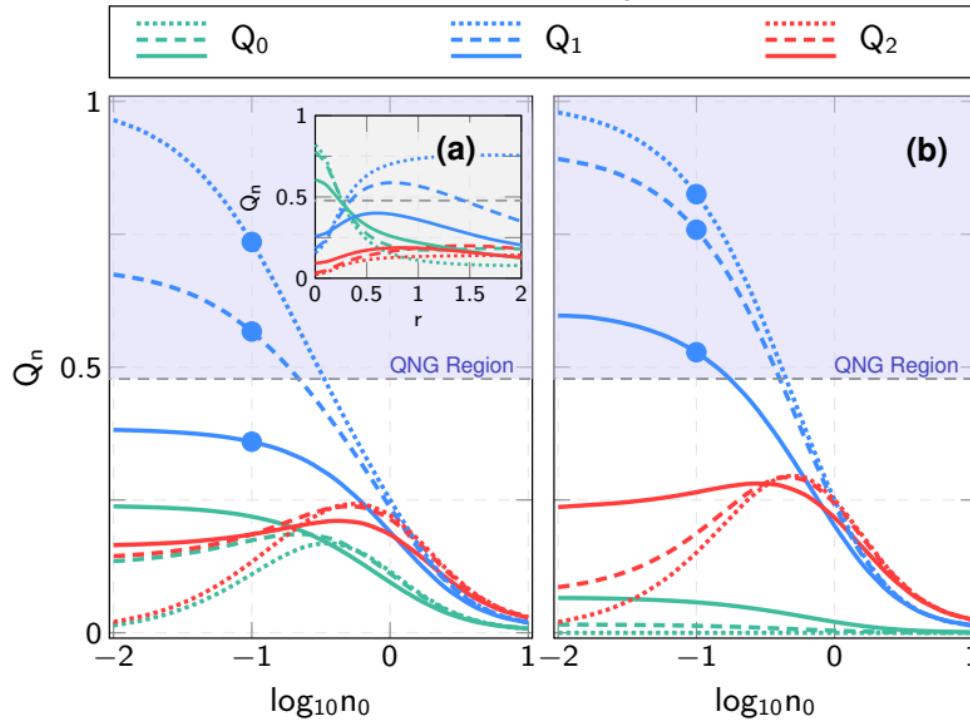
Cramér-Rao bound

$$\Delta^2 N_c \geq \frac{1}{M \cdot F(N_c)},$$

M – number of copies, F – quantum Fisher information

In levitated optomechanics

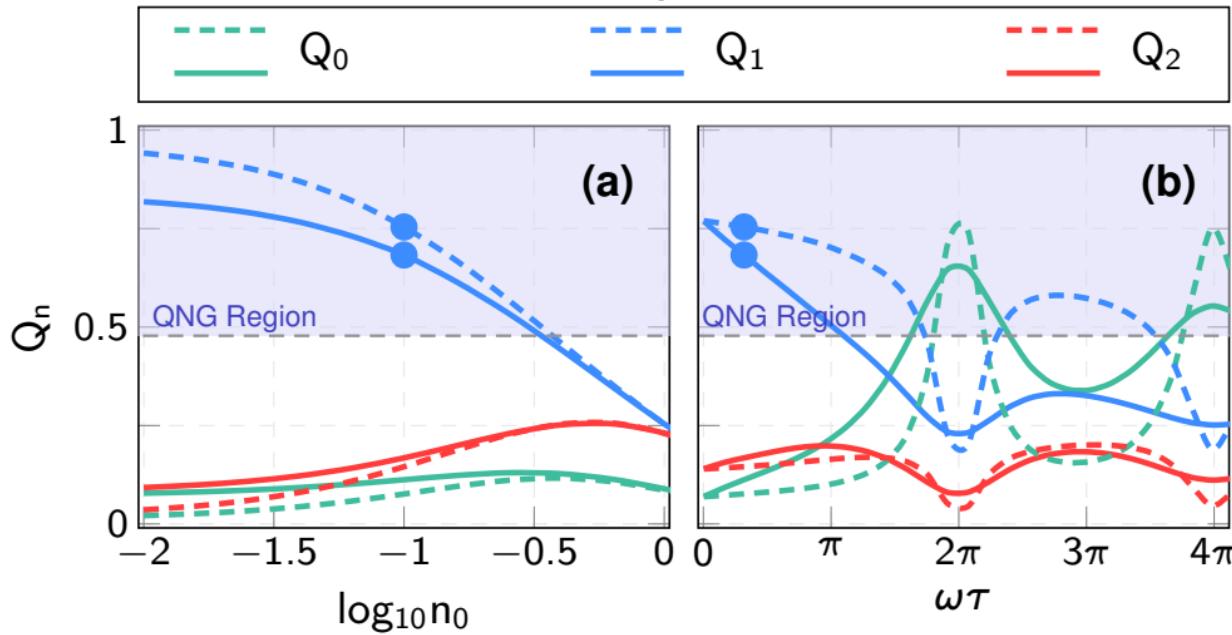
Inside a cavity



Parameters U. Delić, Science 367, 892 (2020)

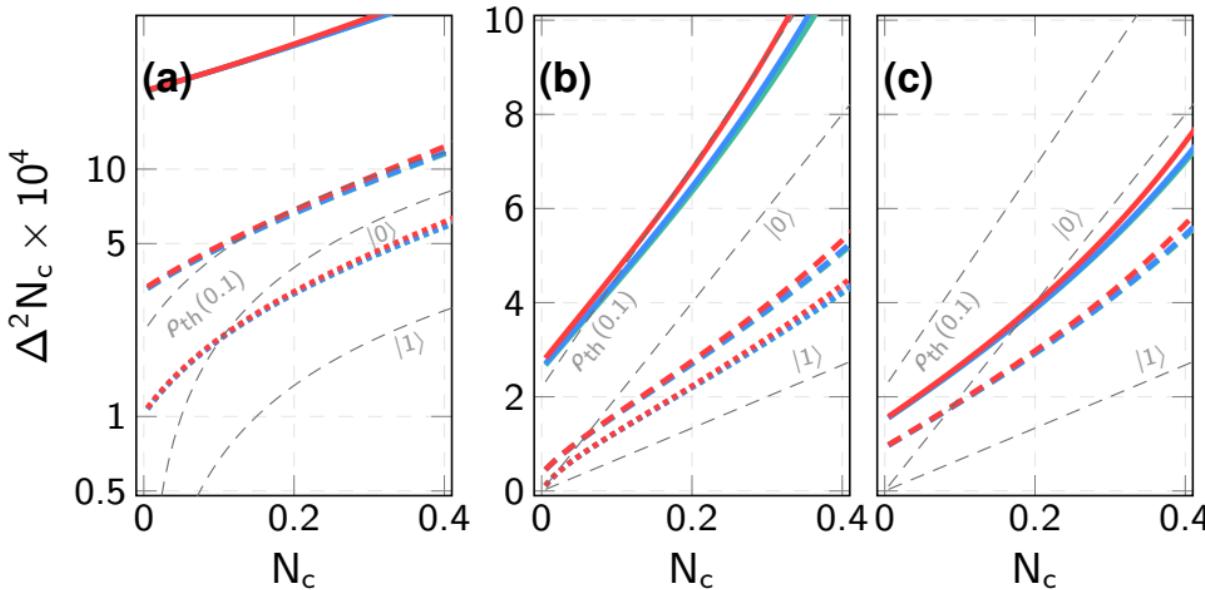
In levitated optomechanics

In free space

Parameters: L. Magrini, Phys. Rev. Lett **129**, 053601 (2022)

In levitated optomechanics

Phase-randomized displacement sensitivity



Conclusions

- ★ Quantum non-Gaussianity is possible in optomechanics and outside
- ★ both continuous-variable [CV] (nonlinear potentials) and discrete-variable [DV] (photon counting) regimes
- ★ Nonlinearity can be broadcast to linear systems
- ★ single-phonon-added states are helpful for phase-randomized force detection

Thank You!



These slides
<https://bit.ly/andrey-trest-25>



Beware of the appendix slide!

Effective classical simulation

Consider the setup:

- ★ n quantum subsystems
- ★ t operations
- ★ local measurements performed on individual subsystems

The measurement produces one list of outcomes:

$$\mathbf{k} \equiv \{k_1, k_2, \dots, k_n\},$$

one outcome per system.

Probability of the outcomes

$$P(k_1, \dots, k_n) \quad (1)$$

A classical algorithm is said to capable of effective sampling, if it

- ★ is efficient (polynomial) in t and n
- ★ provides outcomes \mathbf{k} draws from the same probability as (1)

The very last frame which is empty