Nonclassicality and Higher-Order Nonlinearity in Levitated Optomechanics

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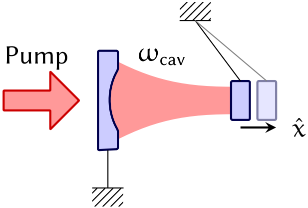
15.05.2019

Cavity Optomechanics



- Radiation pressure drives mechanical motion
- Mechanical displacement changes optical length of the cavity

Experimental Realizations

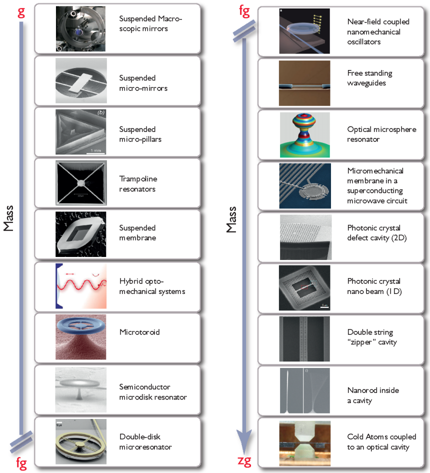


Figure source: [1]

[1] Aspelmeyer, Kippenberg and Marquardt, RMP **86**, 1391 (2014)

[2] Kashkanova *et al.*, Nat. Phys. **13**, 74 (2017)

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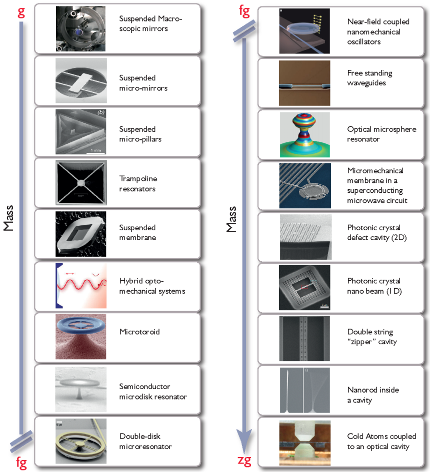

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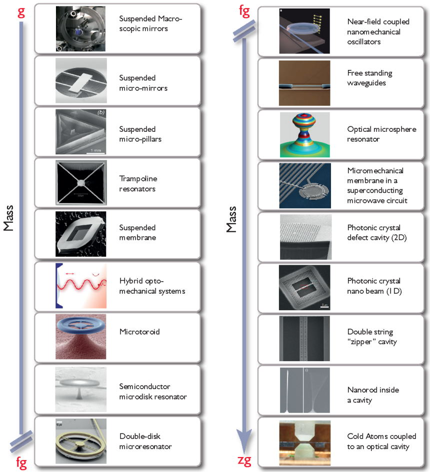

Figure source: [1]

Photo: Bryce Vickmark
Rainer Weiss
Prize share: 1/2

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Barry C. Barish
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Photo: Caltech Alumni Association
Kip S. Thorne
Prize share: 1/4

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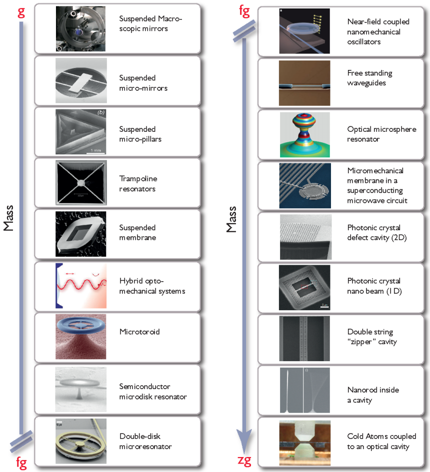

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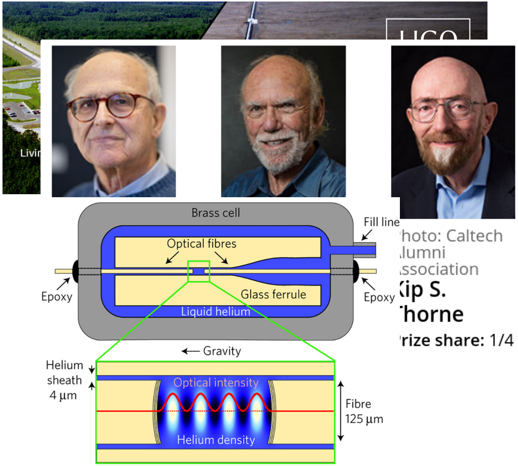

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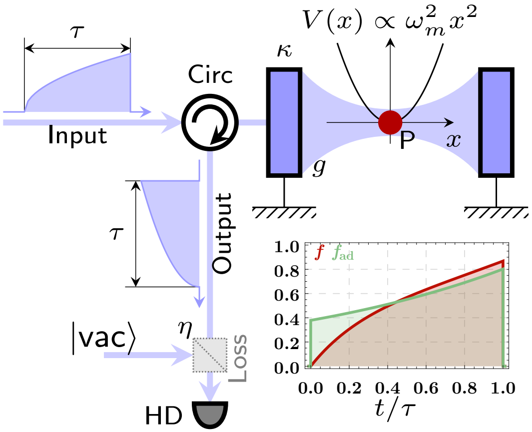
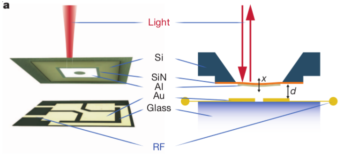
Levitated Optomechanics



Advantages of Optomechanics for Quantum Technology

Quantum Communication

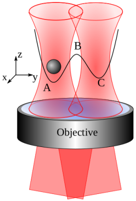


Bagci *et al.*, Nature **507**, 81 (2014)

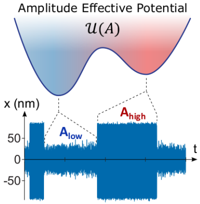
Quantum Metrology

- High-Q oscillators
- Squeezed states

Quantum Computation and Simulation: Nonlinear Potentials



Rondin *et al.*, Nat. Nano **12**, 1130 (2017)



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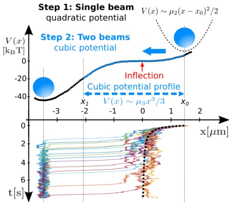

Šiler *et al.*, Sci Rep **7**, 1697 (2017)

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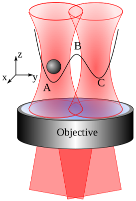

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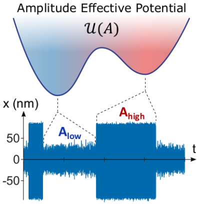

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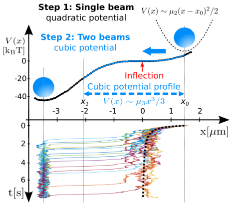

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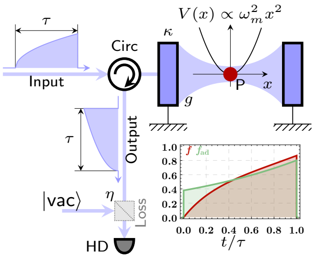
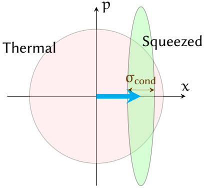
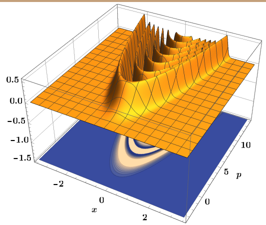
Main Results

Squeezing Mechanical Motion



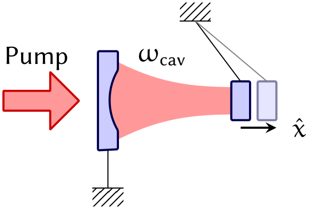


Approximate Cubic Phase State



$$|\gamma\rangle = e^{i\gamma x^3}|p\rangle \approx e^{i\gamma x^3}\hat{S}|0\rangle$$

The Optomechanical systems



Radiation

Standard quantization of the cavity field

$$\hat{E}(\mathbf{r},t) = \sum_p \sum_k \mathbf{e}_p u_k(\mathbf{r}) \hat{a}_k(t)$$

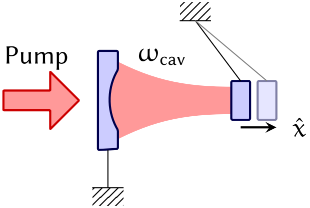
Mechanics

Displacement field

$$\hat{\mathbf{v}}(\mathbf{r},t) = \sum_n \mathbf{v}_n(\mathbf{r}) \hat{x}_n(t)$$

Only one field mode a and one mechanical x are considered.

The Optomechanical interaction

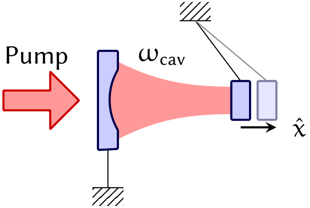


The single-mode cavity Hamiltonian

$$H_{tot} = \hbar\omega_{cav}[x]a^{\dagger}a + H_m$$

g_0 — single-photon optomechanical coupling, typically small

The Optomechanical interaction



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$$H_{tot} = \hbar\omega_{cav}[x]a^\dagger a + H_m = \hbar\left[\omega_{cav} + \left.\frac{\partial\omega_{cav}}{\partial x}\right|_{x=0} \cdot x\right] a^\dagger a + H_m$$

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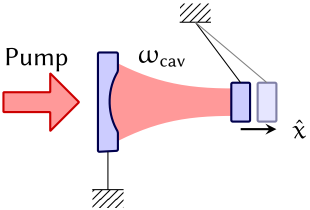

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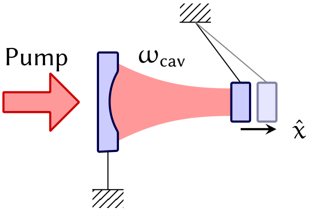

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g_0 — single-photon optomechanical coupling, typically small
In presence of strong classical pump the interaction is linearized

$$H = \hbar\omega_{cav}a^\dagger a - \hbar g(a^\dagger + a)(b_m^\dagger + b_m) + \hbar\omega_m b_m^\dagger b_m.$$

Linearized Optomechanics

The Hamiltonian

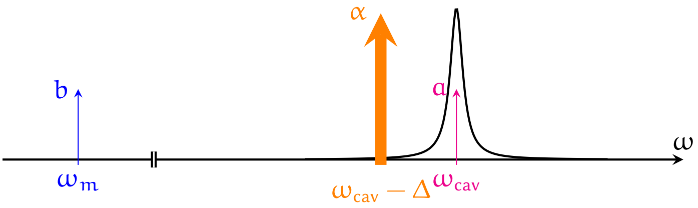
$$H = \hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g(a^\dagger + a)(b^\dagger + b)$$

The main participants

a quantum optical mode at ω_{cav}

α strong classical pump at $\omega_{cav} - \Delta$

b quantized mechanical motion at ω_m

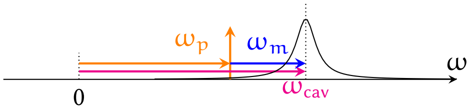


Resolved-sideband optomechanics

Lower Mechanical Sideband

Pump at the difference frequency

$$\omega_p = \omega_{cav} - \omega_m$$



$$H \propto a b^\dagger + a^\dagger b$$

- Parametric Converter / Beam-splitter
- State swap / Cooling

Required: resolved sideband $\kappa \ll \omega_m$

Resolved-sideband optomechanics

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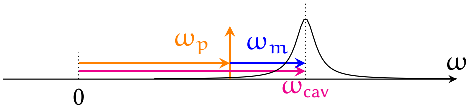


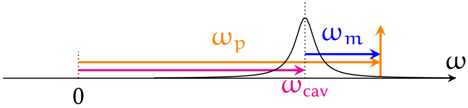
$$H \propto ab^\dagger + a^\dagger b$$

- Parametric Converter / Beam-splitter
- State swap / Cooling

Upper Mechanical Sideband

Pump at the sum frequency

$$\omega_p = \omega_{cav} + \omega_m,$$



$$H = ab + a^\dagger b^\dagger$$

- Parametric Amp / Two-mode squeezing
- Entanglement

Required: resolved sideband $\kappa \ll \omega_m$

Digression: Optical Spring

Radiation Pressure Force

$$
\begin{aligned}
F_{RP}(t) \propto P(x) &= -Kx \\
&= -Kx(t-\tau_*) \\
\approx -K\times(x - \tau_*\dot{x}) &= -Kx + \Gamma\dot{x}
\end{aligned}
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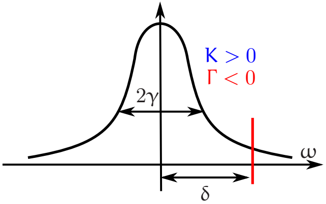
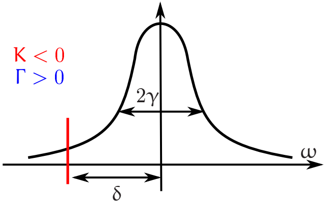
Электромагнитная жесткость впервые наблюдалась В. Б. Брагинским [3]

Настройка на правый склон



Положительная жесткость и отрицательное затухание

Настройка на левый склон



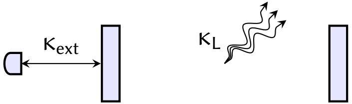
Отрицательная жесткость и положительное затухание

Назад

[3] V.B. Braginsky, I.I. Minakova Bull. MSU III 1, 83 (1964)

Environment

Optical Environment



κ_{ext} detection channel, κ_L losses
Interacts with the modes of travelling light, (almost) each in vacuum. Collective operator a_i

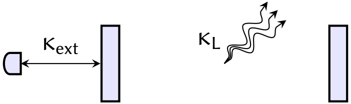
$$\left[a_i(t), a_i^\dagger(t')\right] = \delta(t - t');$$

$$\frac{1}{2}\left\langle a_i(t)a_i^\dagger(t') + a_i^\dagger(t')a_i(t)\right\rangle = \delta(t - t').$$

Typically the cavity is overcoupled with $\kappa_{ext} \gg \kappa_L$

Environment

Optical Environment



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Mechanical Environment

Q-factor:

$$Q_{tot}^{-1} = Q_{clamp}^{-1} + Q_{mat}^{-1} + Q_{air}^{-1} + Q_{scat}^{-1} + \dots$$

At rate $\gamma = \omega_m/Q$ coupled to a thermal bath with bosonic operator b^{th}:

$$\left[b^{th}(t), b^{th\dagger}(t')\right] = \delta(t-t'),$$

$$\frac{1}{2}\left\langle\left\{b^{th}(t), b^{th\dagger}(t')\right\}\right\rangle = (2n_{th}+1)\delta(t-t').$$

$$n_{th} = \frac{1}{\exp[\hbar\omega_m/k_B t] - 1} \approx k_B T/\hbar\omega_m$$

Equations of motion

Assume blue detuning $\omega_p = \omega_{cav} + \omega_m$, therefore $H = -\hbar g(a^\dagger b^\dagger + ab)$.

$$\dot{a} = igb^\dagger - \kappa a + \sqrt{2\kappa}a^{in},$$
$$\dot{b} = iga^\dagger - \frac{\gamma}{2}b + \sqrt{\gamma}b^{th}$$

Input-output relation for optics $a^{out} = -a^{in} + \sqrt{2\kappa}a$

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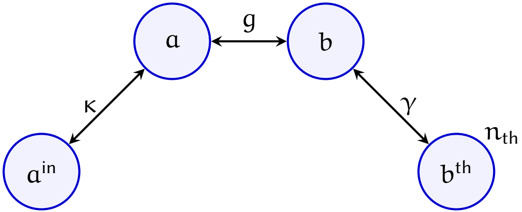

Parameters:

- ★ Resolved sideband $\kappa \ll \omega_m$
- ★ Weak coupling $g \sim 10^{-3 \div -1}\kappa$
- ★ Slow mechanical decay $\gamma \sim 10^{-7 \div -4}\kappa$
- ★ Not too hot bath $\gamma n_{th} \leqslant \{g, \kappa\}$

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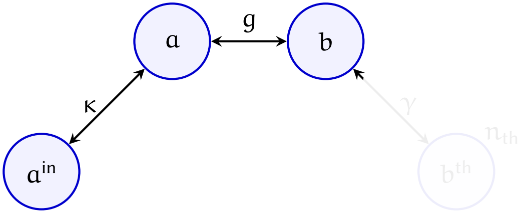

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That is,

- mechanical decay can be approximately ignored

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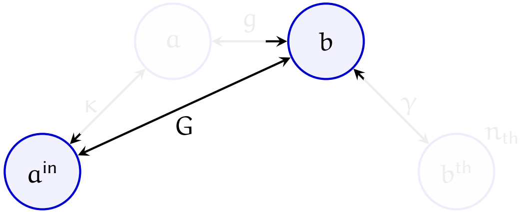

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$$A^{out} = \sqrt{\mathfrak{G}} A^{in} + \sqrt{\mathfrak{G} - 1} b^\dagger(0),$$
$$b(\tau) = \sqrt{\mathfrak{G}} b(0) + \sqrt{\mathfrak{G} - 1} A^{in,\dagger}.$$

$$A^{in} \propto \int_0^\tau dt\, a^{in}(t) e^{Gt}, \quad A^{out} \propto \int_0^\tau dt\, a^{out}(t) e^{-Gt}.$$

$$\left[a^k(t), a^{k',\dagger}(t')\right] = \delta(t - t')\delta_{kk'}.$$

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Two-mode squeezing

In general case (when adiabatic model does not hold, $g \propto \kappa$)
Define $\mathbf{a} = (a, b)$, $\mathbb{A}$, $\mathbf{f} = (\sqrt{2\kappa}a^{\text{in}}, \sqrt{\gamma}b^{\text{th}})$, then

$$\dot{\mathbf{a}} = \mathbb{A}.\mathbf{a} + \mathbf{f}$$

Formal solution (with $\mathbb{M}(s) = \exp[-\mathbb{A}s]$)

$$\mathbf{a}(t) = \mathbb{M}(t)\mathbf{a}(0) + \int_0^t ds\, \mathbb{M}(t-s).\mathbf{f}(s).$$

Input-output transformations

$$A^{\text{out}} = \sqrt{\mathfrak{G}}A^{\text{in}} + \sqrt{\mathfrak{G}-1}b^\dagger(0) + \text{Noise},$$
$$b(\tau) = \sqrt{\mathfrak{G}}b(0) + \sqrt{\mathfrak{G}-1}A^{\text{in},\dagger} + \text{Noise}.$$

Two-mode squeezing gain

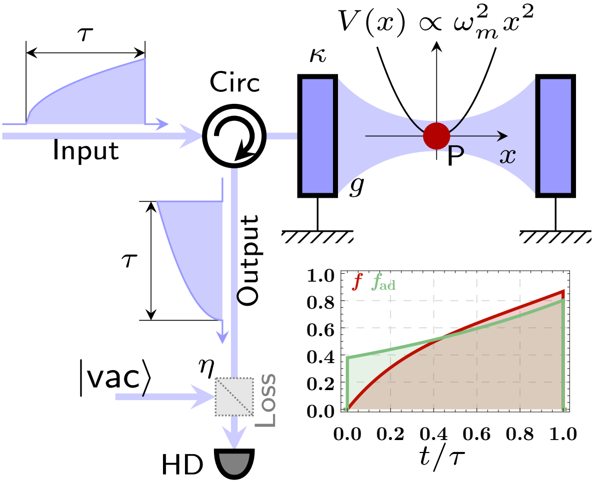
$$\mathfrak{G} = \mathfrak{G}(\kappa, g, \gamma_m, \tau),$$

Input-output pulse bosonic operators

$$A^{\text{in}} = \int_0^\tau dt\; a^{\text{in}}(t) f^{\text{in}}(\tau - t),$$
$$A^{\text{out}} = \int_0^\tau dt\; a^{\text{out}}(t) f^{\text{out}}(\tau - t).$$

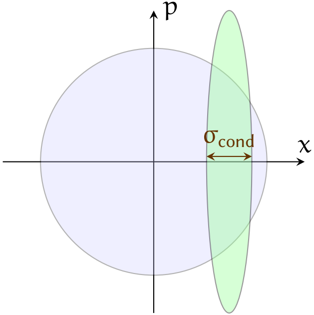
$$\left[a^k(t), a^{k',\dagger}(t')\right] = \delta(t-t')\delta_{kk'}.$$

The Protocol for Squeezing [Quant. Sci. Technol. 4, 024006 (2019)]

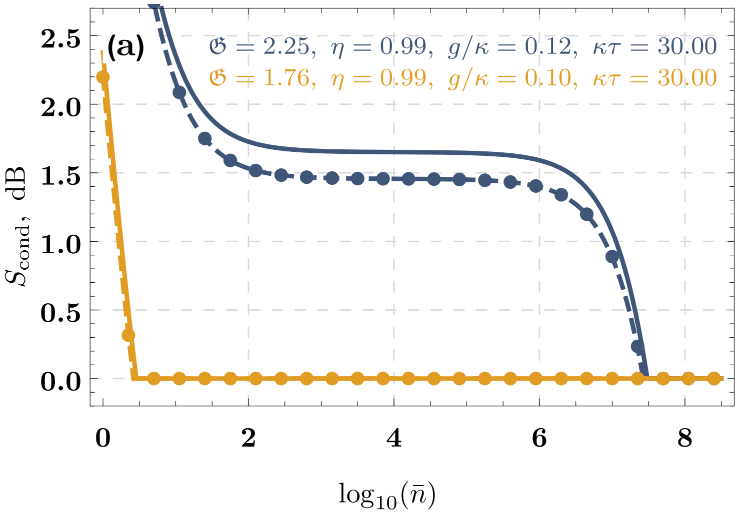


$$A^{\mathrm{out}} = \int_0^{\tau} dt\, a^{\mathrm{out}}(\tau - t) f^{\mathrm{out}}(t).$$

$f^{\mathrm{out}}(t)$ depends on the interaction type. Detecting proper A^{out} projects mechanical mode on a displaced squeezed state.

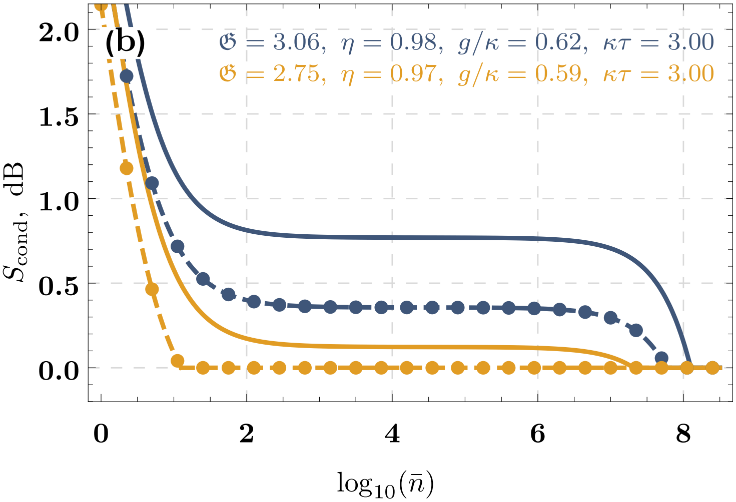


Squeezing in Adiabatic Regime



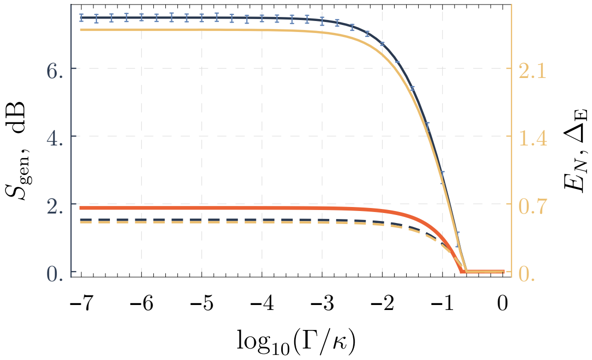
$$S_{\mathrm{cond}} = -10\log_{10}\sigma_{\mathrm{cond}}.$$

Squeezing in Non-adiabatic Regime



$$S_{\mathrm{cond}} = -10 \log_{10} \sigma_{\mathrm{cond}}.$$

Generalized Squeezing and Entanglement



$$S_{\mathrm{gen}} = -10\log_{10}\min \mathrm{EigenVal}[V_{\mathrm{cov}}].$$

Δ_{E} — Duan's variance criterion violation.

CPS state

Devised by Gottesman, Kitaev and Preskill [3]

$$|\gamma_{\mathrm{GKP}}\rangle \propto \int dx\, e^{i\gamma x^3}|x\rangle = e^{i\gamma x^3}|p=0\rangle\,,$$

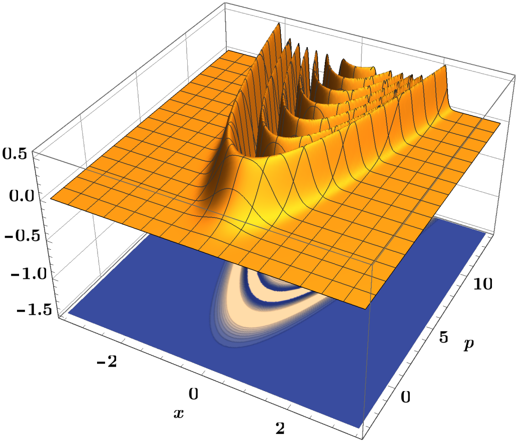
Wigner Function

$$W_{\mathrm{GKP}}(x,p) \propto \mathrm{Ai}\left[\left(\frac{4}{3\gamma}\right)^{1/3}(3\gamma x^2 - p)\right].$$

Nonlinear variance

$$(x,p) \to (x, p+\gamma x^2) \Rightarrow \left\langle \delta(p-\lambda x^2)^2 \right\rangle \to 0.$$

Required for (in particular) the measurement-based quantum computing.



[3] Gottesman, Kitaev, Preskill, PRA **64**, 012310 (2001)

Measurement-based computing

PHYSICAL REVIEW A **97**, 022329 (2018)

General implementation of arbitrary nonlinear quadrature phase gates

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We propose general methodology of deterministic single-mode quantum interaction nonlinearly modifying single quadrature variable of a continuous-variable system. The methodology is based on linear coupling of the system to ancillary systems subsequently measured by quadrature detectors. The nonlinear interaction is obtained by using the data from the quadrature detection for dynamical manipulation of the coupling parameters. This measurement-induced methodology enables direct realization of arbitrary nonlinear quadrature interactions without the need to construct them from the lowest-order gates. Such nonlinear interactions are crucial for more practical and efficient manipulation of continuous quadrature variables as well as qubits encoded in continuous-variable systems.

DOI: 10.1103/PhysRevA.97.022329

Measurement-based computing

PHYSICAL REVIEW A 97, 022329 (2018)

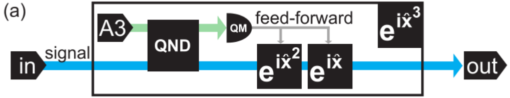


FIG. 1. Schematic circuits for various implementations of nonlinear gates. QND: quantum nondemolition interaction; QM: quadrature measurement; Ak: ancillary state of the kth order squeezed in $\hat{p} - N\chi_N\hat{x}^{N-1}$. $e^{i\hat{x}^k}$: unitary realization of kth-order nonlinear gate with arbitrary strength. (a) Cubic gate with $N = 3$; (b) $(N+1)$th-

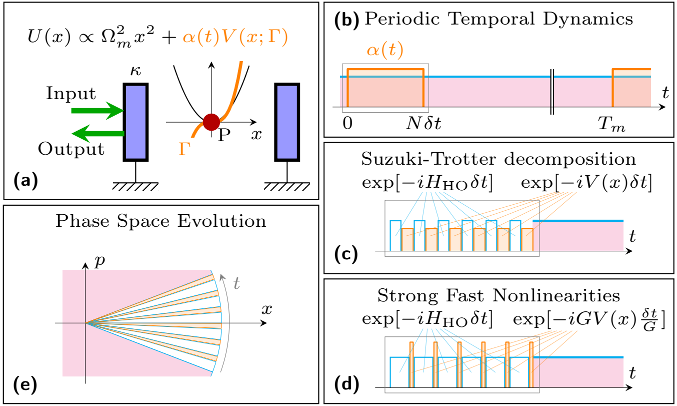
(Received 10 August 2017; published 20 February 2018)

We propose general methodology of deterministic single-mode quantum interaction nonlinearly modifying single quadrature variable of a continuous-variable system. The methodology is based on linear coupling of

Required $\mathrm{Var}(p - \lambda x^2) \to 0$

The Model

Rakhubovsky and Filip [arxiv:1904.00773]



$$\left[\exp[-i(H_{HO} + V(x))\delta t]\right]^N \approx \left[\mathcal{U}_{HO}(\delta t)\mathcal{U}_{NL}(\delta t) + O(\delta t^2)\right]^N,$$

Numerical evaluation

Cubic evolution

In coordinate basis

$$\rho(x, x') \to \rho(x, x') e^{-i\gamma(x^3 - x'^3)}.$$

Harmonic evolution

Rotation in phase space

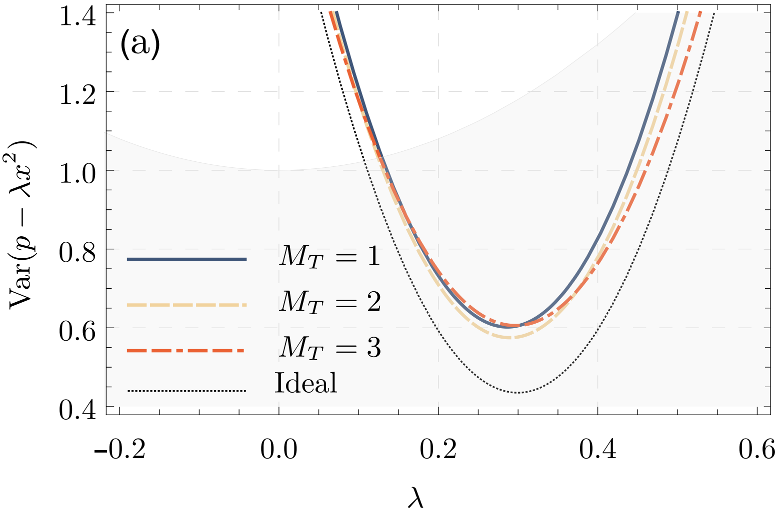
$$W(x, y) \to W(x \cos\delta + y \sin\delta, x \sin\delta + y \cos\delta).$$

Decoherence

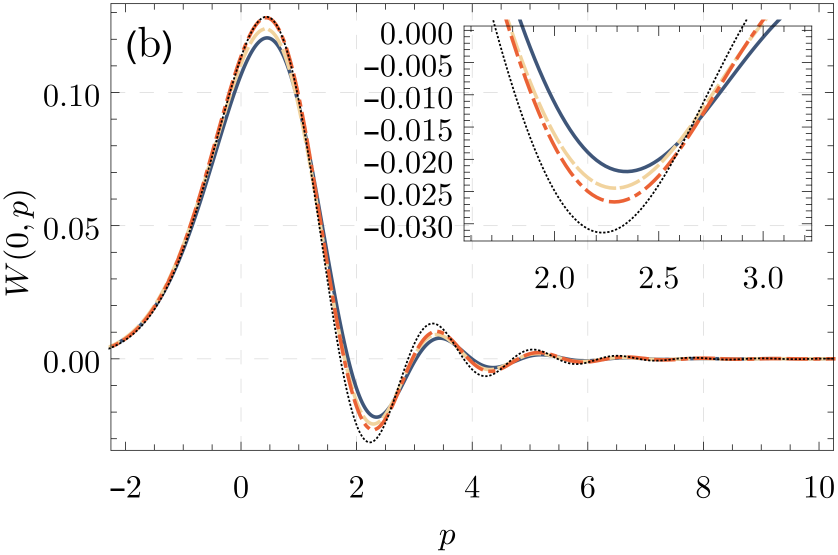
Convolution in phase space with a Gaussian kernel

$$W(x, y) \to \iint d\xi\, d\eta\, W(x - \xi, y - \eta) W_{\text{th}}(\xi, \eta).$$

Nonlinear Variance



Wigner Function Cuts

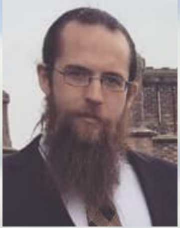


Introduction

Linear Optomechanics

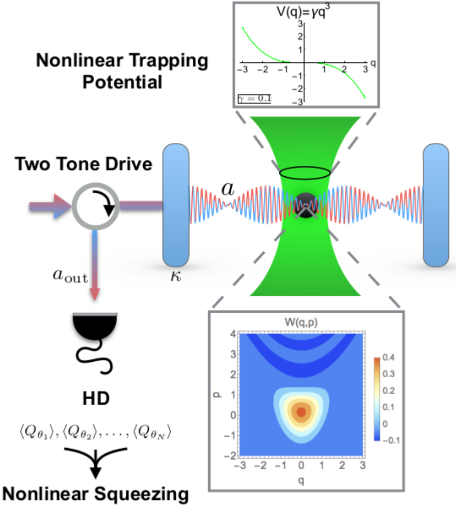
Quantum Squeezing and Entanglement
- Pulsed Optomechanics
- Mechanical Squeezing

Cubic Phase State in Levitated Optomechanics
- Introduction to Cubic Phase States
- CPS Preparation
- CPS Evaluation



Darren Moore
arxiv:1904:00772

The Model



Approximate cubic phase state in mechanics

$$e^{i\gamma q^3}\,|p=0\rangle\,.$$

Pulsed QND interaction

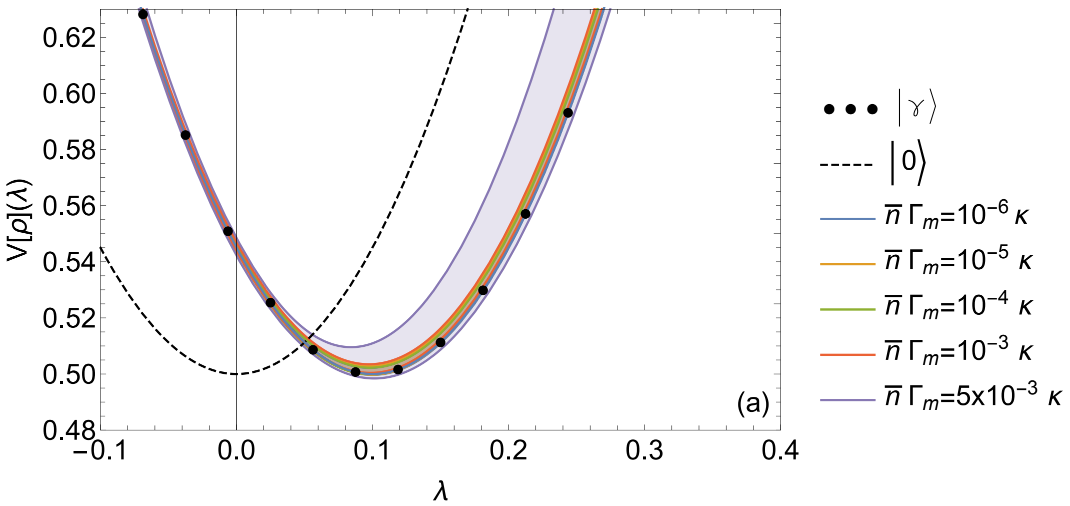
$$H_{\text{int}} \propto x_{\text{light}}(q\cos\phi + p\sin\phi).$$

Detect leaking light

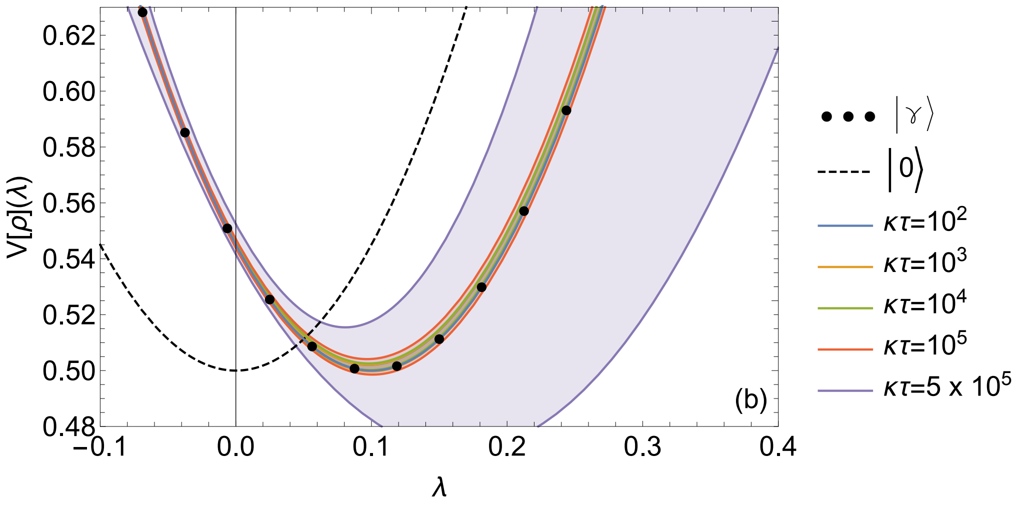
Estimate nonlinear variance

$$q_{\text{NL}} = p - \lambda x^2 \Rightarrow \text{Var}[q_{\text{NL}}]$$

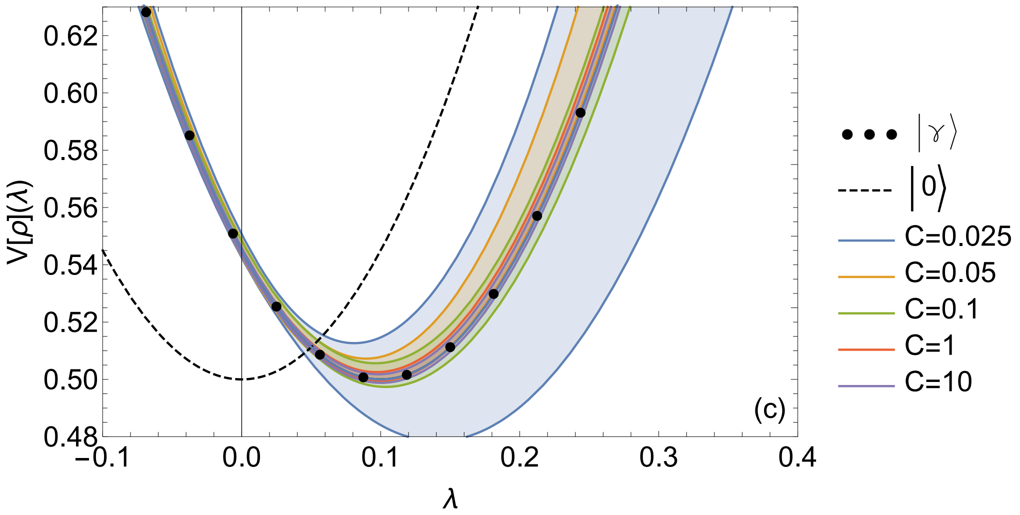
Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Conclusion

- Levitated optomechanics allows production of mechanical squeeezed states
- Entanglement
- Approximate Cubic Phase State

Thank You!