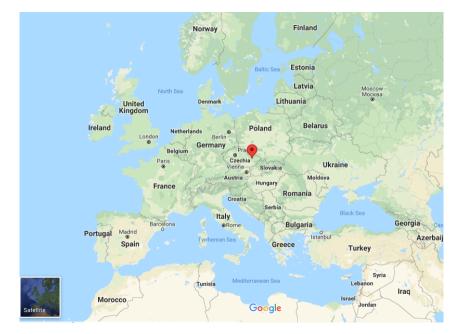
Robust Entanglement With A Thermal Mechanical Oscillator

Andrey A. Rakhubovsky Radim Filip

Department of Optics, Palacký University, Czech Republic

Phys. Rev. A 91, 062317 (2015).

Quantum Optics Lab S-Pb, 13.04.2018



Pulsed Entanglement Introduction 1/32

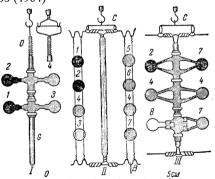
Pressure of Light I

1619 J. Kepler De Cometis Libelli Tres

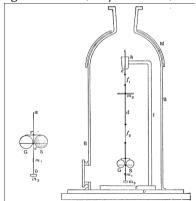
1862 J.C. Maxwell

1901

P.N. Lebedev; "Untersuchungen über die Druckkräfte des Lichtes", Annalen der Physik **6**,433 (1901)



E.F. Nichols and G.F. Hull "A preliminary communication on the pressure of heat and light radiation", Phys. Rev. **13**, 307 (1901)



Pulsed Entanglement Introduction 2/33

Pressure of Light II

1964 V.B. Braginsky, I.I. Minakova, MSU Bulletin 1, 83 (1964)

1970 V.B. Braginsky, <u>Investigation of dissipative ponderomotive effects of electromagnetic</u> radiation Soviet Physics JETP **31**, 5 (1970)

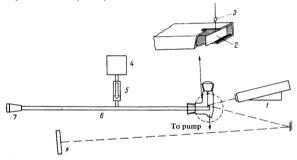
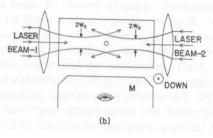


FIG. 1. Schematic diagram of the experimental arrangement: 1–laser, 2–plate-oscillator, 3–mirror, 4–magnetron, 5–ferrite valve, 6–resonator, 7–mobile piston, 8–photographic film.

Pulsed Entanglement Introduction 3/32

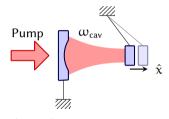
Pressure of Light III

1970 A. Ashkin, Acceleration and Trapping of Particles by Radiation Pressure. Phys. Rev. Lett. **24**, 156–159 (1970).



Pulsed Entanglement Introduction 4/3

Cavity Optomechanics



- * Radiation pressure drives mechanical motion
- ★ Mechanical displacement changes optical length of the cavity

Pulsed Entanglement Introduction 5/32

Experimental Realizations

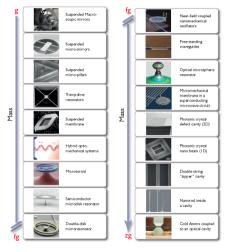


Figure source:

¹Aspelmeyer, Kippenberg and Marquardt, RMP 86, 1391 (2014)

²Kashkanova et al., Nat. Phys. 13, 74 (2017)

Pulsed Entanglement Introduction 5 / 32

Experimental Realizations

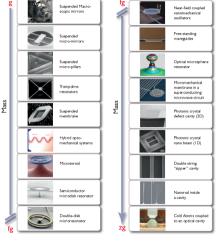




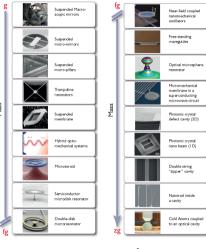
Figure source:

¹Aspelmeyer, Kippenberg and Marquardt, RMP 86, 1391 (2014)

²Kashkanova et al., Nat. Phys. 13, 74 (2017)

Pulsed Entanglement Introduction 5/32

Experimental Realizations



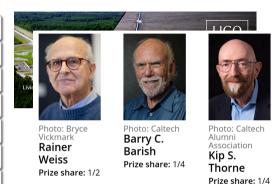


Figure source:

¹Aspelmeyer, Kippenberg and Marquardt, RMP 86, 1391 (2014)

²Kashkanova et al., Nat. Phys. 13, 74 (2017)

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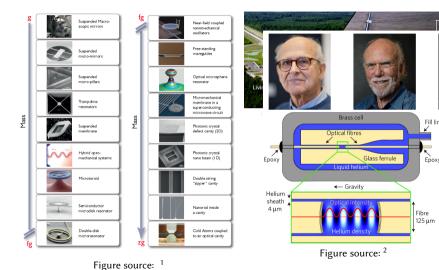
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Experimental Realizations



¹Aspelmeyer, Kippenberg and Marquardt, RMP 86, 1391 (2014)

²Kashkanova et al., Nat. Phys. 13, 74 (2017)

Pulsed Entanglement Introduction 6 / 32

Advantages of Optomechanics for Quantum Information

Uniform Type of Radiation Pressure

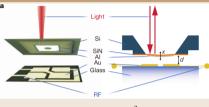


Figure source 3

Nonlinear Mechanical Potential

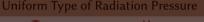
Strong Coupling (High Cooperativity)

Long Coherence Time

³Bagci et al., Nature 507, 81 (2014)

Pulsed Entanglement Introduction 6 / 32

Advantages of Optomechanics for Quantum Information





Can Work at the Quantum Level

- ★ Can capture quantum signals
- ★ Can transduce quantum signals

Nonlinear Mechanical Potentia

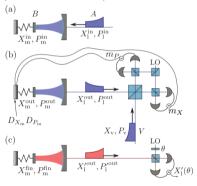
Strong Coupling (High Cooperativity)

Long Coherence Time

Pulsed Entanglement Introduction 7/32

Pulsed Optomechanical Entanglement I

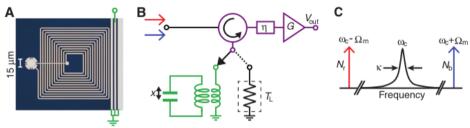
2011 S. G. Hofer, W. Wieczorek, M. Aspelmeyer, K. Hammerer, Quantum entanglement and teleportation in pulsed cavity optomechanics. Phys. Rev. A **84**, 052327 (2011).



Pulsed Entanglement Introduction 8 / 32

Pulsed Optomechanical Entanglement II

2013 T. A. Palomaki, J. D. Teufel, R. W. Simmonds, K. W. Lehnert, Entangling Mechanical Motion with Microwave Fields. Science **342**, 710–713 (2013).

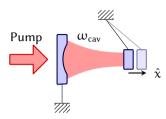


Introduction **Optomechanics Pulsed Optomechanics** Gaussian Entanglement

Gaussian Entanglement with Pulsed Optomechanics

Pulsed Entanglement Optomechanics 9 / 32

The Optomechanical systems



Radiation

Standard quantization of the cavity field

$$\hat{E}(\mathbf{r},t) = \sum_{p} \sum_{k} e_{p} u_{k}(\mathbf{r}) \hat{a}_{k}(t)$$

Mechanics

Displacement field

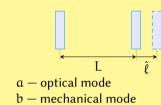
$$\hat{\mathbf{u}}(\mathbf{r},t) = \sum_{n} \mathbf{u}_{n}(\mathbf{r}) \hat{\mathbf{x}}_{n}(t)$$

Only one field mode α and one mechanical x_n are considered.

Pulsed Entanglement Optomechanics 10 / 32

The Hamiltonian

$$H=\hbar\omega_{cav}(\hat{\ell})a^{\dagger}a+\hbar\omega_{\mathfrak{m}}b^{\dagger}b$$



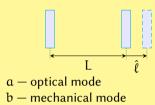
Pulsed Entanglement Optomechanics 10 / 32

The Hamiltonian

$$\begin{split} H &= \hbar \omega_{cav}(\hat{\ell}) \alpha^{\dagger} \alpha + \hbar \omega_{m} b^{\dagger} b \\ &= \hbar \omega_{cav} \alpha^{\dagger} \alpha - \hbar \frac{\omega_{cav}}{I} \hat{\ell} \alpha^{\dagger} \alpha + \hbar \omega_{m} b^{\dagger} b \end{split}$$

Modulation of the cavity frequency

$$\omega_{cav}(\hat{\ell}) = \frac{\pi nc}{L + \hat{\ell}} \approx \frac{\pi nc}{L} \left[1 - \frac{\hat{\ell}}{L} \right] = \omega_{cav} - \hat{\ell} \frac{\omega_{cav}}{L}.$$



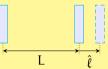
Pulsed Entanglement Optomechanics 10 / 32

The Hamiltonian

$$\begin{split} H &= \hbar \omega_{cav}(\hat{\ell}) \alpha^{\dagger} \alpha + \hbar \omega_{m} b^{\dagger} b \\ &= \hbar \omega_{cav} \alpha^{\dagger} \alpha - \hbar \frac{\omega_{cav}}{I} \hat{\ell} \alpha^{\dagger} \alpha + \hbar \omega_{m} b^{\dagger} b \end{split}$$

Modulation of the cavity frequency

$$\omega_{cav}(\hat{\ell}) = \frac{\pi nc}{L + \hat{\ell}} \approx \frac{\pi nc}{L} \left[1 - \frac{\hat{\ell}}{L} \right] = \omega_{cav} - \hat{\ell} \frac{\omega_{cav}}{L}.$$



a — optical modeb — mechanical mode

$$\begin{split} \hat{\ell} &= \sqrt{\frac{\hbar}{2 m \omega_m}} (b + b^\dagger) \\ \hat{\Phi} &= \sqrt{\frac{\hbar m \omega_m}{2}} (b - b^\dagger) / i \\ \hat{[\ell, \hat{\Phi}]} &= i \hbar \end{split}$$

The Hamiltonian

$$\begin{split} H &= \hbar \omega_{cav}(\hat{\ell}) \alpha^{\dagger} \alpha + \hbar \omega_{m} b^{\dagger} b \\ &= \hbar \omega_{cav} \alpha^{\dagger} \alpha - \hbar \frac{\omega_{cav}}{I} \hat{\ell} \alpha^{\dagger} \alpha + \hbar \omega_{m} b^{\dagger} b \end{split}$$

Modulation of the cavity frequency

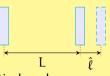
$$\omega_{cav}(\hat{\ell}) = \frac{\pi nc}{L + \hat{\ell}} \approx \frac{\pi nc}{L} \left[1 - \frac{\hat{\ell}}{L} \right] = \omega_{cav} - \hat{\ell} \frac{\omega_{cav}}{L}.$$

In dimensionless units

$$H_{int} = -\hbar\omega_{cav}\frac{x_{zpf}}{I}(b+b^{\dagger})\alpha^{\dagger}\alpha = -\hbar g_{0}(b+b^{\dagger})\alpha^{\dagger}\alpha.$$

With the single-photon coupling strength

$$g_0 = \omega_{cav} \frac{x_{zpf}}{L} = \omega_{cav} \sqrt{\frac{\hbar}{2m\omega_m L^2}}$$



a — optical mode b — mechanical mode

$$\begin{split} \hat{\ell} &= \sqrt{\frac{\hbar}{2 m \omega_m}} (b + b^\dagger) = x_{zpf} x \\ \hat{\Phi} &= \sqrt{\frac{\hbar m \omega_m}{2}} (b - b^\dagger) / i \end{split}$$

$$\hat{\ell}$$
, $\hat{\Phi}$ = ih

$$[\ell, \Psi] = in$$

[x, y] = 2i.

$$Var[x]_{|0\rangle} \equiv \langle 0|(x-\bar{x})^2|0\rangle = 1.$$

 $x = b + b^{\dagger}$; $p = (b - b^{\dagger})/i$,

Modulation of the cavity frequency

$$\omega_{cav}(\hat{\ell}) = \frac{\pi nc}{L + \hat{\ell}} \approx \frac{\pi nc}{L} \left[1 - \frac{\hat{\ell}}{L} \right] = \omega_{cav} - \hat{\ell} \frac{\omega_{cav}}{L}.$$

In dimensionless units

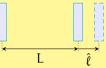
$$H_{int} = -\hbar\omega_{cav}\frac{x_{zpf}}{L}(b+b^{\dagger})\alpha^{\dagger}\alpha = -\hbar g_{0}(b+b^{\dagger})\alpha^{\dagger}\alpha.$$

With the single-photon coupling strength

$$g_0 = \omega_{cav} \frac{x_{zpf}}{L} = \omega_{cav} \sqrt{\frac{\hbar}{2m\omega_m L^2}}$$

With m = 10 ng, $\omega_m = 1$ MHz, L = 10 mm,

$$x_{zpf}\sim 0.1$$
 fm, $g_0\sim 10$ Hz.



a — optical modeb — mechanical mode

$$\hat{\ell} = \sqrt{\frac{\hbar}{2m\omega_{\,\mathrm{m}}}}(b+b^{\dagger}) = x_{zpf}x$$

$$\hat{\Phi} = \sqrt{\frac{\hbar m \omega_m}{2}} (b - b^\dagger)/i$$

$$[\hat{\ell},\hat{\Phi}]=\mathrm{i}\hbar$$

$$x = b + b^{\dagger}; \quad p = (b - b^{\dagger})/i,$$

$$[x, p] = 2i.$$

$$\operatorname{Var}[\mathbf{x}]_{|0\rangle} \equiv \langle 0|(\mathbf{x} - \bar{\mathbf{x}})^2|0\rangle = 1.$$

Modulation of the cavity frequency

$$\omega_{cav}(\hat{\ell}) = \frac{\pi nc}{L + \hat{\ell}} \approx \frac{\pi nc}{L} \left[1 - \frac{\hat{\ell}}{L} \right] = \omega_{cav} - \hat{\ell} \frac{\omega_{cav}}{L}.$$

In dimensionless units

$$H_{int} = -\hbar\omega_{cav}\frac{x_{zpf}}{L}(b+b^{\dagger})\alpha^{\dagger}\alpha = -\hbar g_{0}(b+b^{\dagger})\alpha^{\dagger}\alpha.$$

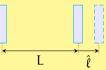
With the single-photon coupling strength

$$g_0 = \omega_{cav} \frac{x_{zpf}}{L} = \omega_{cav} \sqrt{\frac{\hbar}{2m\omega_m L^2}}$$

With m = 10 ng, $\omega_m = 1$ MHz, L = 10 mm,

$$x_{zpf}\sim 0.1$$
 fm, $g_0\sim 10$ Hz.

Too weak \Rightarrow enhance by strong pump and linearize.



a — optical modeb — mechanical mode

$$\hat{\ell} = \sqrt{\frac{\hbar}{2m\omega_m}}(b + b^\dagger) = x_{zpf}x$$

$$\hat{\Phi} = \sqrt{\frac{\hbar m \omega_m}{2}} (b - b^\dagger)/i$$

$$\left[\hat{\ell},\hat{\Phi}
ight]=\mathrm{i}\hbar$$

$$x = b + b^{\dagger}; \quad p = (b - b^{\dagger})/i,$$

$$[x,p]=2i.$$

$$Var[x]_{|0\rangle} \equiv \langle 0|(x-\bar{x})^2|0\rangle = 1.$$

Assume strong classical driving of the cavity $@\omega_p$

 $\varepsilon \propto$ power of the pump

$$H = \hbar \omega_{cav} a^{\dagger} a + \hbar \omega_{m} b^{\dagger} b - \hbar g_{0} a^{\dagger} a (b^{\dagger} + b) - \hbar \varepsilon \Big(a^{\dagger} e^{-i \omega_{p} t} + \text{h.c.} \Big)$$

 $\Delta \equiv \omega_{\text{cav}} - \omega_{p} - \text{detuning}$

At frame defined by $H = \hbar \omega_p a^{\dagger} a$:

$$H=\hbar\Delta a^{\dagger}a+\hbar\omega_{\mathfrak{m}}b^{\dagger}b-\hbar g_{0}a^{\dagger}a(b^{\dagger}+b)-\hbar\varepsilon(a^{\dagger}+a).$$

Assume strong classical driving of the cavity $@\omega_p$

 $\epsilon \propto$ power of the pump

$$H=\hbar\omega_{cav}\alpha^{\dagger}\alpha+\hbar\omega_{m}b^{\dagger}b-\hbar g_{0}\alpha^{\dagger}\alpha(b^{\dagger}+b)-\hbar\varepsilon\Big(\alpha^{\dagger}e^{-i\omega_{p}t}+h.c.\Big)$$

 $\Delta \equiv \omega_{\mathsf{cav}} - \omega_{\mathfrak{p}} - \mathsf{detuning}$

At frame defined by $H = \hbar \omega_p a^{\dagger} a$:

$$H = \hbar \Delta a^{\dagger} a + \hbar \omega_{m} b^{\dagger} b - \hbar g_{0} a^{\dagger} a (b^{\dagger} + b) - \hbar \varepsilon (a^{\dagger} + a).$$

After substitutions

$$\begin{split} \mathsf{H} &= \hbar \underbrace{\left[\Delta - \frac{2\alpha^2 g_0^2}{\omega_{\mathrm{m}}}\right]}_{\Delta} \delta a^{\dagger} \delta a + \hbar \omega_{\mathrm{m}} \delta b^{\dagger} \delta b \\ &- \hbar g_0 \left[\alpha (\delta a^{\dagger} + \delta a) + \underbrace{\delta a^{\dagger} \delta a}_{}\right] (\delta b^{\dagger} + \delta b). \end{split}$$

$$a \rightarrow \alpha + \delta a b \rightarrow \beta + \delta b$$

$$\alpha = \frac{\epsilon}{\Lambda + 2\beta q_0}, \beta = \text{Homework}.$$

Assume strong classical driving of the cavity $@\omega_p$

 $\epsilon \propto$ power of the pump

$$H=\hbar\omega_{cav}\alpha^{\dagger}\alpha+\hbar\omega_{m}b^{\dagger}b-\hbar g_{0}\alpha^{\dagger}\alpha(b^{\dagger}+b)-\hbar\varepsilon\Big(\alpha^{\dagger}e^{-i\omega_{p}t}+h.c.\Big)$$

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After substitutions

$$\begin{split} H = \hbar \underbrace{\left[\Delta - \frac{2\alpha^2 g_0^2}{\omega_m}\right]}_{\Delta} \delta a^{\dagger} \delta a + \hbar \omega_m \delta b^{\dagger} \delta b \\ - \hbar g_0 \left[\alpha (\delta a^{\dagger} + \delta a) + \delta a^{\dagger} \delta a\right] (\delta b^{\dagger} + \delta b). \end{split}$$

$$H = \hbar \Delta a^{\dagger} a + \hbar \omega_{m} b^{\dagger} b - \hbar g (a^{\dagger} + a) (b^{\dagger} + b)$$

$$a \rightarrow \alpha + \delta a$$

 $b \rightarrow \beta + \delta b$

$$\alpha = \frac{\epsilon}{\Delta + 2\beta q_0}, \beta = \text{Homework}.$$

$$g\equiv g_0\alpha=g_0\sqrt{n_p}$$

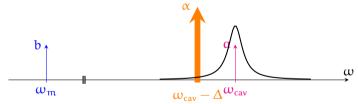
Linearized Optomechanics

The Hamiltonian

$$H = \hbar \Delta a^{\dagger} a + \hbar \omega_{m} b^{\dagger} b - \hbar g (a^{\dagger} + a) (b^{\dagger} + b)$$

The main participants

- α quantum optical mode at ω_{cav}
- $\alpha -$ strong classical pump at $\omega_{cav} \Delta$
- b quantized mechanical motion at ω_m



Pulsed Entanglement Optomechanics 13/

$$H = \underbrace{\hbar \Delta a^{\dagger} a + \hbar \omega_{\mathfrak{m}} b^{\dagger} b}_{H_{BF}} - \hbar g (a^{\dagger} + a) (b^{\dagger} + b)$$

$$a \rightarrow ae^{-i\Delta t}; \quad b \rightarrow be^{-i\omega_m t};$$

Pulsed Entanglement Optomechanics 13 /

$$H = \underbrace{\hbar \Delta a^{\dagger} a + \hbar \omega_{m} b^{\dagger} b}_{H_{or}} - \hbar g (a^{\dagger} + a) (b^{\dagger} + b)$$

$$a \rightarrow a e^{-i\Delta t}; \quad b \rightarrow b e^{-i\omega_{\mathfrak{m}}t};$$

$$\begin{split} H = -\hbar g (abe^{-\mathfrak{i}(\Delta + \omega_{\mathfrak{m}})} + \text{h.c.}) \\ &- \hbar g (ab^{\dagger} e^{-\mathfrak{i}(\Delta - \omega_{\mathfrak{m}})} + \text{h.c.}) \end{split}$$

$$\Delta = \omega_{cav} - \omega_{p}$$

$$H = \underbrace{\hbar \Delta a^{\dagger} a + \hbar \omega_{\mathfrak{m}} b^{\dagger} b}_{H_{DE}} - \hbar g (a^{\dagger} + a) (b^{\dagger} + b)$$

$$a \rightarrow a e^{-i\Delta t}; \quad b \rightarrow b e^{-i\omega_{\mathfrak{m}}t};$$

$$H = -\hbar g(abe^{-i(\Delta + \omega_{m})} + h.c.)$$
$$- \hbar g(ab^{\dagger}e^{-i(\Delta - \omega_{m})} + h.c.)$$

$$\Delta = \omega_{cav} - \omega_p$$

In the Rotating Wave Approximation (RWA) (assuming g small) Lower sideband pump $\Delta = +\omega_{\mathfrak{m}}$

$$\begin{split} H = -\hbar g (ab^\dagger + abe^{-2i\omega_m t}) + \text{h.c.} \\ \approx -\hbar g \left[ab^\dagger + a^\dagger b \right] \end{split}$$

Pulsed Entanglement Optomechanics 13/3

$$H = \underbrace{\hbar \Delta a^{\dagger} a + \hbar \omega_{m} b^{\dagger} b}_{H_{BF}} - \hbar g (a^{\dagger} + a) (b^{\dagger} + b)$$

$$a \to a e^{-i\Delta t}; \quad b \to b e^{-i\omega_\mathfrak{m} t};$$

$$\begin{split} \mathsf{H} &= -\hbar g (ab e^{-\mathfrak{i}(\Delta + \omega_{\mathfrak{m}})} + \mathrm{h.c.}) \\ &\qquad \qquad - \hbar g (ab^{\dagger} e^{-\mathfrak{i}(\Delta - \omega_{\mathfrak{m}})} + \mathrm{h.c.}) \end{split}$$

$$\Delta = \omega_{\rm cav} - \omega_{\rm p}$$

In the Rotating Wave Approximation (RWA) (assuming g small)

Lower sideband pump $\Delta = +\omega_m$

Upper sideband pump $\Delta = -\omega_{\mathfrak{m}}$

$$H = -\hbar g(ab^{\dagger} + abe^{-2i\omega_{m}t}) + h.c.$$

$$\approx -\hbar g \left[a b^{\dagger} + a^{\dagger} b \right]$$

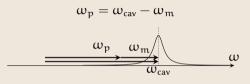
$$H = -\hbar g(ab^{\dagger}e^{2i\omega_{m}t} + ab) + h.c.$$

$$\approx -\hbar g \left[ab + a^{\dagger}b^{\dagger}\right].$$

Pulsed Entanglement Optomechanics 14/32

Resonantly detuned optomechanics

Lower Mechanical Sideband



$$H \propto ab^{\dagger} + a^{\dagger}b$$

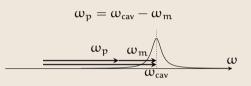
- * Parametric Converter / Beam-splitter
- * State swap / Cooling

Resolved sideband $\kappa \ll \omega_{\mathfrak{m}}$

Pulsed Entanglement Optomechanics 14/32

Resonantly detuned optomechanics

Lower Mechanical Sideband



$$H \propto ab^{\dagger} + a^{\dagger}b$$

- Parametric Converter / Beam-splitter
- * State swap / Cooling

Resolved sideband $\kappa \ll \omega_m$

Upper Mechanical Sideband

$$\omega_{p} = \omega_{cav} + \omega_{m},$$

$$\omega_{p} \qquad \omega_{m}$$

$$\omega_{cav} \qquad \omega_{m}$$

$$H = ab + a^{\dagger}b^{\dagger}$$

- ★ Parametric Amp / Two-mode squeezing
- * Entanglement

Pulsed Entanglement Optomechanics 15 / 3

Digression: Optical Spring

Radiation Pressure Force

$$\begin{split} F_{RP}(t) &\propto P(x) = -Kx \\ &= -Kx(t-\tau_*) \\ &\approx -K \times (x-\tau_*\dot{x}) = -Kx + \Gamma \dot{x} \end{split}$$

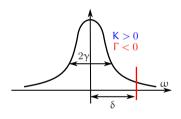
Digression: Optical Spring

Radiation Pressure Force

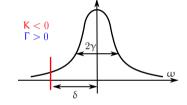
$$\begin{split} F_{RP}(t) &\propto P(x) = -Kx \\ &= -Kx(t-\tau_*) \\ &\approx -K \times (x-\tau_*\dot{x}) = -Kx + \Gamma \dot{x} \end{split}$$

Электромагнитная жесткость впервые наблюдалась В. Б. Брагинским 3

Настройка на правый склон



Настройка на левый склон



Положительная жесткость и отрицательное затухание

Отрицательная жесткость и положительное затухание

→ Назад

³V.B. Braginsky, I.I. Minakova Bull. MSU III 1, 83 (1964) A. Рахубовский (физсрак МГУ)

21.02.2013

2013 27 / 25



Environment

Optical Environment



 $\kappa_{ext} \ detection \ channel, \ \kappa_L \ losses$ Interacts with the modes of travelling light, (almost) each in vacuum. Collective operator α_i

$$\begin{split} \left[\alpha_{i}(t),\alpha_{i}^{\dagger}(t')\right] &= \delta(t-t'); \\ \frac{1}{2}\left\langle\alpha_{i}(t)\alpha_{i}^{\dagger}(t') + \alpha_{i}^{\dagger}(t')\alpha_{i}(t)\right\rangle &= 1. \end{split}$$

Typically the cavity is overcoupled with $\kappa_{ext}\gg\kappa_L$

Environment

Optical Environment



 κ_{ext} detection channel, κ_L losses Interacts with the modes of travelling light, (almost) each in vacuum. Collective operator α_i

$$\begin{split} \left[\alpha_i(t),\alpha_i^\dagger(t')\right] &= \delta(t-t'); \\ \frac{1}{2}\left\langle\alpha_i(t)\alpha_i^\dagger(t') + \alpha_i^\dagger(t')\alpha_i(t)\right\rangle &= 1. \end{split}$$

Typically the cavity is overcoupled with $\kappa_{ext}\gg\kappa_L$

Mechanical Environment

O-factor:

$$Q_{tot}^{-1} = Q_{clamp}^{-1} + Q_{mat}^{-1} + Q_{air}^{-1} + Q_{scat}^{-1} + \dots$$

At rate $\gamma = \omega_m/Q$ coupled to a thermal bath with bosonic operator b^{th} :

$$\begin{split} \left[b^{th}(t),b^{th\dagger}(t')\right] &= \delta(t-t'),\\ \frac{1}{2}\left\langle\left\{b^{th}(t),b^{th\dagger}(t')\right\}\right\rangle &= (2n_{th}+1)\delta(t-t'). \end{split}$$

$$n_{th} = \frac{1}{\exp[\hbar \omega_m / k_B t] - 1} \approx k_B T / \hbar \omega_m$$

Assume red detuning $\omega_p = \omega_{cav} - \omega_m$, therefore $H = -\hbar g (ab^\dagger + a^\dagger b)$.

$$\dot{a} = igb - \kappa a + \sqrt{2\kappa}a^{in},$$

$$\dot{b}=ig\alpha-\frac{\gamma}{2}b+\sqrt{\gamma}b^{th}$$

Input-output relation for optics $\alpha^{out} = -\alpha^{in} + \sqrt{2\kappa}\alpha$

Assume red detuning $\omega_p = \omega_{cav} - \omega_m$, therefore $H = -\hbar g (ab^\dagger + a^\dagger b)$.

$$\begin{split} \dot{a} &= igb - \kappa a + \sqrt{2\kappa} a^{in}, \\ \dot{b} &= iga - \frac{\gamma}{2} b + \sqrt{\gamma} b^{th} \end{split}$$

Input-output relation for optics $a^{out} = -a^{in} + \sqrt{2\kappa}a$

Define
$$\alpha=(\alpha,b), \mathbb{A}, f=(\sqrt{2\kappa}\alpha^{in},\sqrt{\gamma}b^{th}),$$
 then

$$\dot{\mathfrak{a}} = \mathbb{A}.\mathfrak{a} + \mathfrak{f}$$

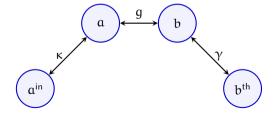
Formal solution (with $M(s) = \exp[-As]$)

$$\mathbf{a}(\mathbf{t}) = \mathbb{M}(\mathbf{t})\mathbf{a}(0) + \int_0^{\mathbf{t}} d\mathbf{s} \, \mathbb{M}(\mathbf{t} - \mathbf{s}).\mathbf{f}(\mathbf{s}).$$

Assume red detuning $\omega_\mathfrak{p}=\omega_{cav}-\omega_\mathfrak{m},$ therefore $H=-\hbar g(\mathfrak{a}\mathfrak{b}^\dagger+\mathfrak{a}^\dagger\mathfrak{b}).$

$$\begin{split} \dot{a} &= igb - \kappa a + \sqrt{2\kappa} \alpha^{in}, \\ \dot{b} &= iga - \frac{\gamma}{2} b + \sqrt{\gamma} b^{th} \end{split}$$

Input-output relation for optics $\alpha^{out} = -\alpha^{in} + \sqrt{2\kappa}\alpha$



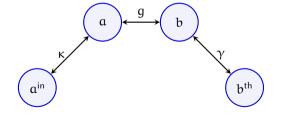
Parameters:

- \star Resolved sideband κ $\ll ω_m$
- ★ Weak coupling $g \sim 10^{-3 \div -1} \kappa$
- \bigstar Slow mechanical decay $\gamma \sim 10^{-7 \div -4} \kappa$
- \star Not too hot bath $\gamma n_{th} \leq \{g, \kappa\}$

Assume red detuning $\omega_p = \omega_{cav} - \omega_m$, therefore $H = -\hbar g (ab^\dagger + a^\dagger b)$.

$$\begin{split} \dot{a} &= igb - \kappa a + \sqrt{2\kappa}\alpha^{\text{in}}, \\ \dot{b} &= iga \end{split}$$

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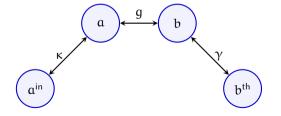
That is,

mechanical decay can be approximately ignored

Assume red detuning $\omega_p = \omega_{cav} - \omega_m$, therefore $H = -\hbar g (ab^\dagger + a^\dagger b)$.

$$\begin{split} 0 &= \text{ig} b - \kappa \alpha + \sqrt{2\kappa} \alpha^{\text{in}}, \\ \dot{b} &= \text{ig} \alpha \end{split}$$

Input-output relation for optics $\alpha^{out} = -\alpha^{in} + \sqrt{2\kappa}\alpha$



Parameters:

- \star Resolved sideband $\kappa \ll \omega_{\mathrm{m}}$
- ★ Weak coupling $g \sim 10^{-3 \div -1} \kappa$
- \star Slow mechanical decay $\gamma \sim 10^{-7 \div -4} \kappa$
- \star Not too hot bath $\gamma n_{th} \leq \{g, \kappa\}$

That is,

- mechanical decay can be approximately ignored
- * cavity mode can be adiabatically eliminated

$$\begin{aligned} 0 &= igb - \kappa \alpha + \sqrt{2\kappa}\alpha^{in}, \\ \dot{b} &= ig\alpha. \end{aligned}$$

$$0 = igb - \kappa a + \sqrt{2\kappa}a^{in},$$

$$\dot{b} = iga.$$

$$\begin{split} \alpha &= i\frac{g}{\kappa}b + \sqrt{\frac{2}{\kappa}}\alpha^{in},\\ \dot{b} &= -Gb + i\sqrt{2G}\alpha^{in}, \quad G \equiv g^2/\kappa \end{split}$$

$$\begin{aligned} 0 &= igb - \kappa a + \sqrt{2\kappa} a^{in}, \\ \dot{b} &= iga. \end{aligned}$$

 $a = i \frac{g}{\kappa} b + \sqrt{\frac{2}{\kappa}} a^{in},$

$$\dot{b} = -Gb + i\sqrt{2G}\alpha^{in}, \quad G \equiv g^2/\kappa$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau} \int_0^{\tau} dt \ \alpha^{\text{in}}(t)e^{Gt}.$$

$$\begin{split} \alpha &= i\frac{g}{\kappa}b + \sqrt{\frac{2}{\kappa}}\alpha^{in},\\ \dot{b} &= -Gb + i\sqrt{2G}\alpha^{in}, \quad G \equiv g^2/\kappa \end{split}$$

$$b(\tau) &= b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau}\int_0^{\tau}dt\;\alpha^{in}(t)e^{Gt}. \end{split}$$

 $a^{\text{out}}(t) = -a^{\text{in}}(t) + \sqrt{2\kappa}a(t)$.

$$a=i\frac{g}{\kappa}b+\sqrt{\frac{2}{\kappa}}\alpha^{in},$$

$$\dot{b}=-Gb+i\sqrt{2G}\alpha^{in},\quad G\equiv g^2/\kappa$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau} \int_0^{\tau} dt \, a^{in}(t)e^{Gt}.$$

$$a^{\text{out}}(t) = a^{\text{in}}(t) + i\sqrt{2G}b(t) = a^{\text{in}}(t) + i\sqrt{2G}\underline{b(0)}e^{-Gt} - 2Ge^{-Gt} \int_0^t d\xi \ a^{\text{in}}(\xi)e^{G\xi}.$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau} \int_0^{\tau} dt \, a^{in}(t)e^{Gt}.$$

$$\alpha^{out}(t) = \alpha^{in}(t) + i\sqrt{2G}b(t) = \alpha^{in}(t) + i\sqrt{2G}\underline{b(0)}e^{-Gt} - 2Ge^{-Gt}\int_0^t d\xi \; \alpha^{in}(\xi)e^{G\xi}.$$

$$A^{\text{out}} = \sqrt{\frac{2G}{1 - e^{-2G\tau}}} \int_{0}^{\tau} d\xi \, \alpha^{\text{out}}(\xi) e^{-G\xi}; \quad \left[A^{\text{out}}, A^{\text{out}\dagger}\right] = 1.$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau} \int_0^{\tau} dt \ a^{in}(t)e^{Gt}.$$

$$A^{\text{out}} = \sqrt{\frac{2G}{1 - e^{-2G\tau}}} \int_0^{\tau} d\xi \, \alpha^{\text{out}}(\xi) e^{-G\xi}; \quad \left[A^{\text{out}}, A^{\text{out}\dagger}\right] = 1.$$

$$\begin{split} \int_0^\tau dt \ \alpha^{out}(t) e^{-Gt} &= \int_0^\tau dt \ \alpha^{in}(t) e^{-Gt} + i \sqrt{2G} b(0) \int_0^\tau dt \ e^{-2Gt} \\ &- 2G \int_0^\tau dt \ e^{-2Gt} \int_0^t d\xi \ \alpha^{in}(\xi) e^{G\xi} \end{split}$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau} \int_0^{\tau} dt \ a^{in}(t)e^{Gt}.$$

$$A^{out} = \sqrt{\frac{2G}{1 - e^{-2G\tau}}} \int_0^{\tau} d\xi \, \alpha^{out}(\xi) e^{-G\xi}; \quad \left[A^{out}, A^{out\dagger}\right] = 1.$$

$$\begin{split} A^{out} &= \sqrt{\frac{2G}{1 - e^{-2G\tau}}} \int_0^\tau dt \; a^{out}(t) e^{-Gt} \\ &= i \sqrt{1 - e^{-2G\tau}} b(0) + \sqrt{\frac{2G}{1 - e^{-2G\tau}}} e^{-2G\tau} \int_0^\tau dt \; a^{in}(t) e^{Gt} \end{split}$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau} \int_0^{\tau} dt \ a^{in}(t)e^{Gt}.$$

$$A^{\text{out}} = \sqrt{\frac{2G}{1 - e^{-2G\tau}}} \int_0^{\tau} d\xi \, \alpha^{\text{out}}(\xi) e^{-G\xi}; \quad \left[A^{\text{out}}, A^{\text{out}\dagger} \right] = 1.$$

$$A^{\text{out}} = \sqrt{\frac{2G}{1 - e^{-2G\tau}}} \int_0^{\tau} dt \ a^{\text{out}}(t) e^{-Gt}$$

$$=i\sqrt{1-e^{-2G\tau}b(0)}+e^{-G\tau}A^{in}.$$

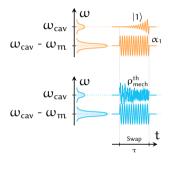
$$\begin{split} B^{out} &= \sqrt{T} B^{in} + i \sqrt{1-T} A^{in}, \\ A^{out} &= \sqrt{T} A^{in} + i \sqrt{1-T} B^{in}. \end{split}$$

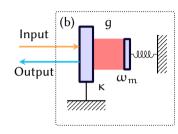
$$\begin{split} B^{out} &= \sqrt{T} B^{in} + i \sqrt{1 - T} A^{in}, \\ A^{out} &= \sqrt{T} A^{in} + i \sqrt{1 - T} B^{in}. \end{split}$$

$$B^{in} = b(0); \quad B^{out} = b(\tau),$$

$$\begin{split} A^{out} &= \sqrt{\frac{2G}{1-e^{-2G\tau}}} \int_0^\tau dt \; \alpha^{out}(t) e^{-Gt} \\ A^{in} &= \sqrt{\frac{2G}{e^{2G\tau}-1}} \int_0^\tau d\xi \; \alpha^{in}(\xi) e^{G\xi}, \\ T &\equiv e^{-2G\tau}, \quad G = q^2/\kappa. \end{split}$$

Pulsed State Swap





Pulsed Entanglement

Blue tuning (to the upper sideband, $\omega_p = \omega_{cav} + \omega_m$).

$$H = -\hbar g(ab + a^{\dagger}b^{\dagger})$$

In a similar fashion, assuming no thermal decoherence and adiabatic elimination of cavity mode,

$$\begin{split} A^{out} &= \sqrt{K} A^{in} + i \sqrt{K-1} B^{in\dagger}, \\ B^{out} &= \sqrt{K} B^{in} + i \sqrt{K-1} A^{in\dagger}, \end{split}$$

Two-Mode Squeezed State (ideally, vacuum: TMSV).

$$\begin{split} A^{\text{in}} &= \sqrt{\frac{2G}{1-e^{-2G\tau}}} \int_0^{\tau} dt \; \alpha^{\text{in}}(t) e^{-Gt}, \\ A^{\text{out}} &= \sqrt{\frac{2G}{e^{2G\tau}-1}} \int_0^{\tau} dt \; \alpha^{\text{out}}(t) e^{Gt}. \end{split}$$

The Protocol

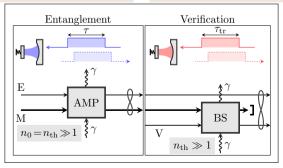
Classical pump $@\omega_p$, quantum cavity mode $@\omega_{cav}$, mechanical mode $@\omega_m$. In the rotating frame we deal with slow amplitudes Pump power $\mapsto g(t) \mapsto$ Temporal mode profile For nice exponential envelopes assume constant pump

Blue detuning $\omega_p = \omega_{cav} + \omega_m$

Red detuning $\omega_{\mathfrak{p}} = \omega_{\mathsf{cav}} - \omega_{\mathfrak{m}}$

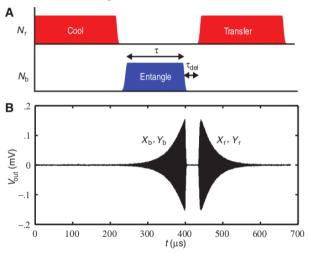
Two-Mode squeezing interaction

State swap interaction



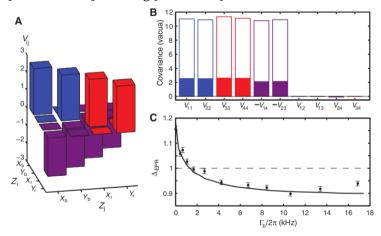
Evaluation

[Palomaki,2013] following [Hofer, 2011] evaluate the Duan's variance:



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By expressing Eqs. (7) in terms of quadratures $X_{\rm m}^i = (B_i + B_i^\dagger)/\sqrt{2}$ and $X_{\rm l}^i = (A_i + A_i^\dagger)/\sqrt{2}$, where $i \in \{\rm in, out\}$, and their corresponding conjugate variables, we can calculate the so-called EPR variance $\Delta_{\rm EPR}$ of the state after the interaction. For light initially in vacuum $(\Delta X_{\rm l}^{\rm in})^2 = (\Delta P_{\rm l}^{\rm in})^2 = \frac{1}{2}$ and the mirror in a thermal state $(\Delta X_{\rm m}^{\rm in})^2 = (\Delta P_{\rm m}^{\rm in})^2 = n_0 + \frac{1}{2}$, the state is entangled iff [52]

$$\Delta_{\text{EPR}} = \left[\Delta \left(X_{\text{m}}^{\text{out}} + P_{1}^{\text{out}} \right) \right]^{2} + \left[\Delta \left(P_{\text{m}}^{\text{out}} + X_{1}^{\text{out}} \right) \right]^{2}$$

$$= 2(n_{0} + 1) \left(e^{r} - \sqrt{e^{2r} - 1} \right)^{2} < 2, \tag{8}$$

Evaluation

[Palomaki,2013] following [Hofer, 2011] evaluate the Duan's variance:

$$\hat{u} = |a|\hat{x}_1 + \frac{1}{a}\hat{x}_2,$$

$$\hat{\boldsymbol{v}} = |a|\hat{p}_1 - \frac{1}{a}\,\hat{p}_2\,,$$

Theorem 1.—Sufficient criterion for inseparability: For any separable quantum state ρ , the total variance of a pair of EPR-like operators defined by Eqs. (2a) and (2b) with the commutators $[\hat{x}_j, \hat{p}_{j'}] = i \delta_{jj'}$ (j, j' = 1, 2) satisfies the inequality

$$\langle (\Delta \hat{u})^2 \rangle_{\rho} + \langle (\Delta \hat{v})^2 \rangle_{\rho} \ge a^2 + \frac{1}{a^2}.$$
 (3)

Evaluation

[Palomaki,2013] following [Hofer, 2011] evaluate the Duan's variance:

$$\hat{u} = |a|\hat{x}_1 + \frac{1}{a}\hat{x}_2,$$

A proper criterion has to be applied!

Either the properly generalized Duan variance or logarighmic negativity

Theorem 1.—Sufficient criterion for inseparability: For any separable quantum state ρ , the total variance of a pair of EPR-like operators defined by Eqs. (2a) and (2b) with the commutators $[\hat{x}_j, \hat{p}_{j'}] = i \, \delta_{jj'} \, (j, j' = 1, 2)$ satisfies the inequality

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 (3)

Gaussian Entanglement

Pulsed Entanglement Gaussian Entanglement 23 / 33

Continuous Variables Systems

Each mode is described by annihilation operator a_k :

$$\begin{bmatrix} \alpha_i, \alpha_j^{\dagger} \end{bmatrix} = \delta_{ij}; \quad [\alpha_i, \alpha_j] = \begin{bmatrix} \alpha_i^{\dagger}, \alpha_j^{\dagger} \end{bmatrix} = 0.$$

Or quadratures

$$x_k = a_k + a_k^{\dagger}; \quad p_k = (a_k - a_k^{\dagger})/i,$$

which form the vector

$$\mathbf{R} = \{x_1, p_1, x_2, p_2, \dots, x_N, p_N\}.$$

$$[R_i,R_j]=2i\Omega_{ij};\quad \Omega_{ij}=\oplus_{k=1}^N\omega;\quad \omega=\begin{pmatrix}0&1\\-1&0\end{pmatrix}.$$

Gaussian states: $\langle \mathbf{R} \rangle$ and covariance matrix

$$\mathbb{V}_{ij} = \frac{1}{2} \left\langle \{ (R_i - \langle R_i \rangle), (R_j - \langle R_j \rangle) \} \right\rangle \mapsto \frac{1}{2} \left\langle R_i R_j + R_j R_i \right\rangle, \text{ if } \mathbf{R} = 0.$$

Pulsed Entanglement Gaussian Entanglement 24/3

Bipartite Gaussian System

$$\mathbf{R} = (x_1, p_1, x_2, p_2), \quad [x_1, p_1] = 2i; \quad [x_2, p_2] = 2i;$$

Assuming $\langle \mathbf{R} \rangle = 0$,

$$\mathbb{V} = \begin{pmatrix} \langle x_1^2 \rangle & \langle x_1 \circ p_1 \rangle & \langle x_1 \circ x_2 \rangle & \langle x_1 \circ p_2 \rangle \\ \frac{\langle p_1 \circ x_1 \rangle}{\langle x_2 \circ x_1 \rangle} & \langle p_1^2 \rangle & \langle p_1 \circ x_2 \rangle & \langle p_1 \circ p_2 \rangle \\ \frac{\langle x_2 \circ x_1 \rangle}{\langle x_2 \circ x_1 \rangle} & \langle x_2 \circ p_1 \rangle & \langle x_2^2 \rangle & \langle x_2 \circ p_2 \rangle \\ \langle p_2 \circ x_1 \rangle & \langle p_2 \circ p_1 \rangle & \langle p_2 \circ x_2 \rangle & \langle p_2^2 \rangle \end{pmatrix}, \qquad \text{where } \alpha \circ b \equiv \frac{1}{2} (\alpha b + b \alpha).$$

Pulsed Entanglement Gaussian Entanglement 24/33

Bipartite Gaussian System

$$\mathbf{R} = (x_1, p_1, x_2, p_2), \quad [x_1, p_1] = 2i; \quad [x_2, p_2] = 2i;$$

Assuming $\langle \mathbf{R} \rangle = 0$,

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Pulsed Entanglement Gaussian Entanglement 25 / 3

Symplectic Transformations

Transformation $\mathbf{R} \mapsto S\mathbf{R}$ is symplectic, if

$$S^{\mathsf{T}}\Omega S = \Omega$$

Important symplectic transformation diagonalizes the CM

$$\mathbb{V} = S^T \mathbb{N} S; \quad \mathbb{N} = diag(\nu_1, \nu_1, \nu_2, \nu_2, \dots).$$

 ν_k : symplectic eigenvalues

A physical state has all $\nu_k \geqslant \sigma_{\text{vac}}$ (shot-noise variance).

Pulsed Entanglement Gaussian Entanglement 25/3

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 ν_k : symplectic eigenvalues

A physical state has all $\nu_k \geqslant \sigma_{vac}$ (shot-noise variance).

$$\sigma_{vac} \equiv \left<0 \middle| x^2 \middle| 0 \right> = \left<(\alpha + \alpha^\dagger)^2 \right>_{|0\rangle} = 1.$$

If e.g. define $x=(\alpha+\alpha^\dagger)/\sqrt{2}$, then $\sigma_{vac}=1/2$.

Pulsed Entanglement Gaussian Entanglement 26/32

Entanglement and Partial Transposition

A bipartite state ρ is separable if (otherwise, entangled)

$$\hat{\rho} = \sum_{i} p_{i} \hat{\rho}_{i}^{A} \otimes \hat{\rho}_{i}^{B}$$

Entanglement is a resource etc.

Pulsed Entanglement Gaussian Entanglement 26 / 3

Entanglement and Partial Transposition

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$$\hat{\rho} = \sum_{i} p_{i} \hat{\rho}_{i}^{A} \otimes \hat{\rho}_{i}^{B}$$

Partial Transposition

If $\hat{\rho}$ is physical, so is $\hat{\rho}^T$

Idea: check transposition of a subsystem (Peres-Horodecki)

$$\hat{\rho}^{\mathsf{T}_{\mathsf{B}}} = \sum_{\mathfrak{i}} \mathfrak{p}_{\mathfrak{i}} \hat{\rho}^{\mathsf{A}}_{\mathfrak{i}} \otimes (\hat{\rho}^{\mathsf{B}}_{\mathfrak{i}})^{\mathsf{T}}$$

If $\hat{\rho}^{T_B}$ is physical, the state is separable

Pulsed Entanglement Gaussian Entanglement 27 / 32

Partial Transposition and CM

In a CV system partial transposition is equivalent to reversing time:

$$x \rightarrow x$$
; $p \rightarrow -p$

Criterion of physicality: all symplectic eigenvalues $v_k \geqslant 1$.

$$\mathbb{V} = \begin{pmatrix} V_1 & V_c \\ V_c^\mathsf{T} & V_2 \end{pmatrix}$$

Pulsed Entanglement Gaussian Entanglement 27/3

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$$\begin{split} \mathbb{V} &= \begin{pmatrix} V_1 & V_c \\ V_c^\mathsf{T} & V_2 \end{pmatrix} \\ \nu_\pm &= \frac{1}{\sqrt{2}} \left[\Sigma(\mathbb{V}) - \sqrt{\Sigma(\mathbb{V})^2 - 4 \det \mathbb{V}} \right]^{1/2} \\ \Sigma(\mathbb{V}) &\equiv \det V_1 + \det V_2 - 2 \det V_c. \end{split}$$

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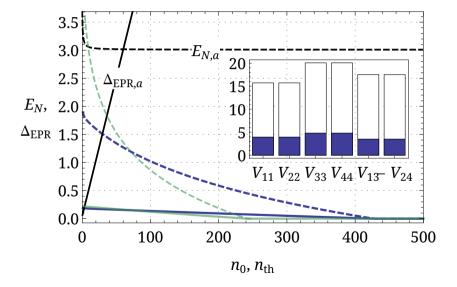
Criterion of physicality: all symplectic eigenvalues $v_k \ge 1$.

$$\begin{split} \mathbb{V} &= \begin{pmatrix} V_1 & V_c \\ V_c^\mathsf{T} & V_2 \end{pmatrix} \\ \nu_\pm &= \frac{1}{\sqrt{2}} \left[\Sigma(\mathbb{V}) - \sqrt{\Sigma(\mathbb{V})^2 - 4 \det \mathbb{V}} \right]^{1/2} \\ \Sigma(\mathbb{V}) &\equiv \det V_1 + \det V_2 - 2 \det V_c. \end{split}$$

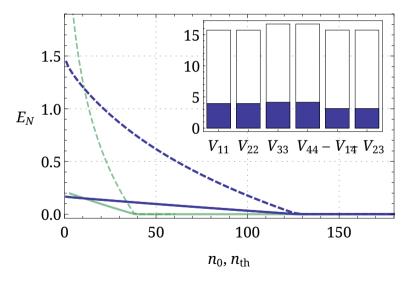
$$E_{N} = max[-\log \nu_{-}/\sigma_{vac}], 0].$$

Gaussian Entanglement with Pulsed Optomechanics

Opto- (Electro-) Mechanical Entanglement



Pulses Entanglement



Conclusion

Robust Entanglement Is Robust

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