Levitated Optomechanics for Quantum Information

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MSU
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Iraq
Tunisia
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Israel
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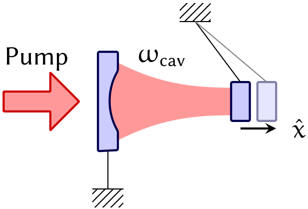
Cavity Optomechanics



- Radiation pressure drives mechanical motion
- Mechanical displacement changes optical length of the cavity

Experimental Realizations

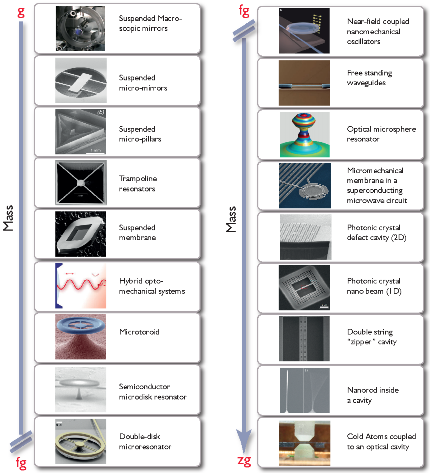
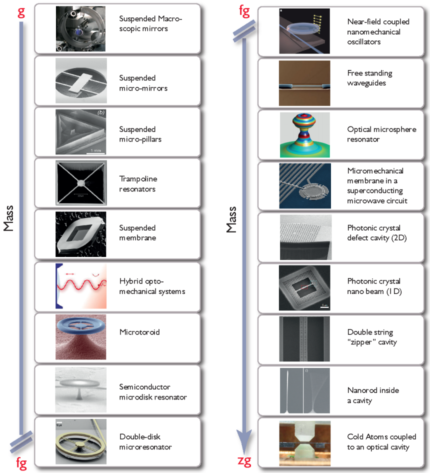


Figure source: [1]

[1] Aspelmeyer, Kippenberg and Marquardt, RMP **86**, 1391 (2014)

[2] Kashkanova *et al.*, Nat. Phys. **13**, 74 (2017)

Experimental Realizations



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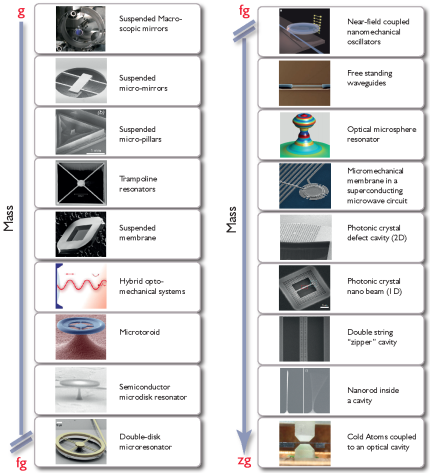

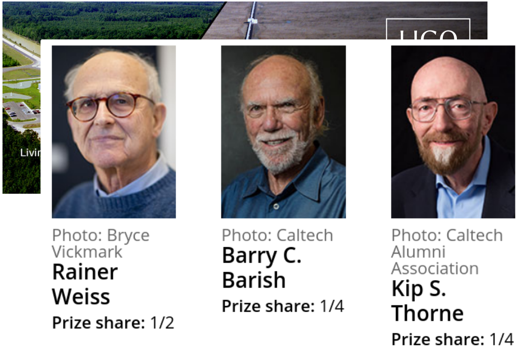

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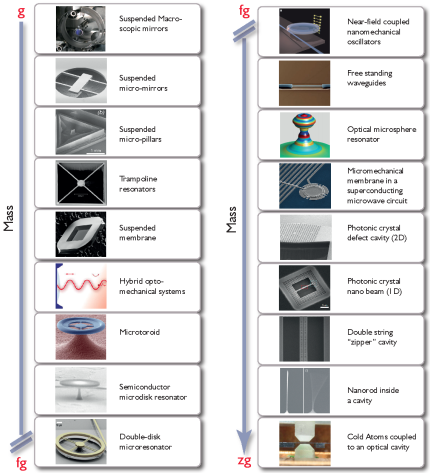

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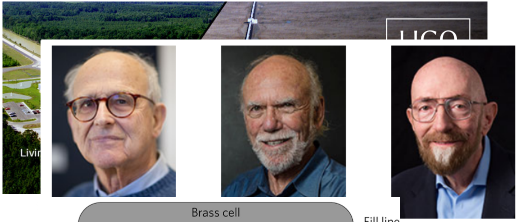


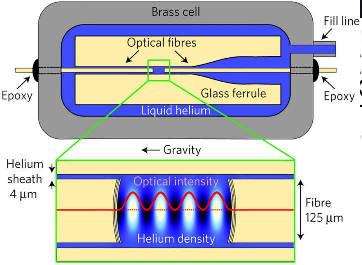


Figure source: [2]

[1] Aspelmeyer, Kippenberg and Marquardt, RMP **86**, 1391 (2014)

[2] Kashkanova *et al.*, Nat. Phys. **13**, 74 (2017)

Levitated Optomechanics

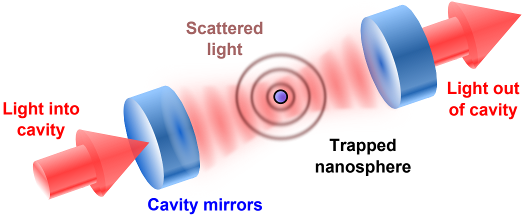


Figure from [3]

[3] Genoni *et al.*, NJP **17**, 073019 (2015)

Advantages of Optomechanics for Quantum Information

Uniform Type of Radiation Pressure

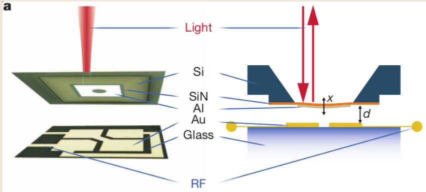


Figure source [4]

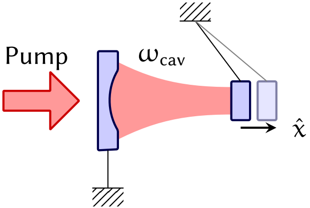
Nonlinear Mechanical Potential

Strong Coupling (High Cooperativity)

High Q-factors

[4] Bagci *et al.*, Nature **507**, 81 (2014)

The Optomechanical systems



Radiation

Standard quantization of the cavity field

$$\hat{E}(\mathbf{r}, t) = \sum_{p} \sum_{k} \mathbf{e}_p u_k(\mathbf{r}) \hat{a}_k(t)$$

Mechanics

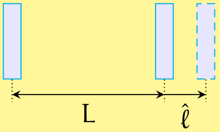
Displacement field

$$\hat{\mathbf{v}}(\mathbf{r}, t) = \sum_{n} v_n(\mathbf{r}) \hat{x}_n(t)$$

Only one field mode a and one mechanical x_n are considered.

The Hamiltonian

$$H = \hbar\omega_{cav}(\hat{\ell})a^\dagger a + \hbar\omega_m b^\dagger b$$

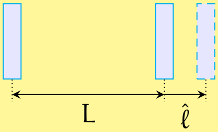


a — optical mode

b — mechanical mode

The Hamiltonian

$$
\begin{aligned}
H &= \hbar\omega_{cav}(\hat{\ell})a^\dagger a + \hbar\omega_m b^\dagger b \\
&= \hbar\omega_{cav} a^\dagger a - \hbar\frac{\omega_{cav}}{L}\hat{\ell} a^\dagger a + \hbar\omega_m b^\dagger b
\end{aligned}
$$



a — optical mode

b — mechanical mode

Modulation of the cavity frequency

$$
\omega_{cav}(\hat{\ell}) = \frac{\pi n c}{L + \hat{\ell}} \approx \frac{\pi n c}{L}\left[1 - \frac{\hat{\ell}}{L}\right] = \omega_{cav} - \hat{\ell}\frac{\omega_{cav}}{L}.
$$

The Hamiltonian

$$
\begin{aligned}
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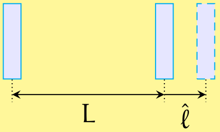

a — optical mode
b — mechanical mode

$$\hat{\ell} = \sqrt{\frac{\hbar}{2m\omega_m}}(b + b^\dagger)$$

$$\hat{\Phi} = \sqrt{\frac{\hbar m\omega_m}{2}}(b - b^\dagger)/i$$

$$[\hat{\ell}, \hat{\Phi}] = i\hbar$$

The Hamiltonian

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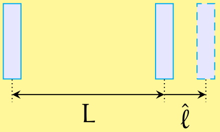
In dimensionless units

$$H_{int} = -\hbar\omega_{cav}\frac{x_{zpf}}{L}(b+b^\dagger)a^\dagger a = -\hbar g_0 (b+b^\dagger)a^\dagger a.$$

With the single-photon coupling strength

$$g_0 = \omega_{cav}\frac{x_{zpf}}{L} = \omega_{cav}\sqrt{\frac{\hbar}{2m\omega_m L^2}}$$



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$$\hat{\ell} = \sqrt{\frac{\hbar}{2m\omega_m}}(b+b^\dagger) = x_{zpf}x$$

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$$[\hat{\ell},\hat{\Phi}] = i\hbar$$

$$x = b+b^\dagger; \quad p = (b-b^\dagger)/i,$$

$$[x,p] = 2i.$$

$$\mathrm{Var}[x]_{|0\rangle} \equiv \langle 0|(x-\bar{x})^2|0\rangle = 1.$$

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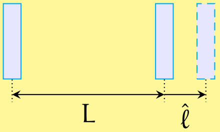
With $m = 10$ ng, $\omega_m = 1$ MHz, $L = 10$ mm,

$x_{\mathrm{zpf}} \sim 0.1$ fm, $g_0 \sim 10$ Hz.



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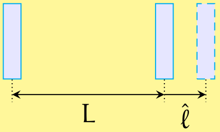
$x_{zpf} \sim 0.1$ fm, $g_0 \sim 10$ Hz.

Too weak $\Rightarrow$ enhance by strong pump and linearize.



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Assume strong classical driving of the cavity @ω_p

$\epsilon \propto$ power of the pump

$$H = \hbar\omega_{cav} a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g_0 a^\dagger a (b^\dagger + b) - \hbar\epsilon\left(a^\dagger e^{-i\omega_p t} + \text{h.c.}\right)$$

$\Delta \equiv \omega_{cav} - \omega_p$ – detuning

At the frame defined by $H = \hbar\omega_p a^\dagger a$:

$$H = \hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g_0 a^\dagger a (b^\dagger + b) - \hbar\epsilon(a^\dagger + a).$$

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After substitutions

$a \to \alpha + \delta a$
$b \to \beta + \delta b$

$$\alpha = \frac{\epsilon}{\Delta + 2\beta g_0}, \ \beta = \text{Homework.}$$

$$H = \hbar \underbrace{\left[\Delta - \frac{2\alpha^2 g_0^2}{\omega_m}\right]}_{\Delta} \delta a^\dagger \delta a + \hbar\omega_m \delta b^\dagger \delta b \\ - \hbar g_0 \left[\alpha(\delta a^\dagger + \delta a) + \cancel{\delta a^\dagger \delta a}\right](\delta b^\dagger + \delta b).$$

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$\alpha = \dfrac{\epsilon}{\Delta + 2\beta g_0}$, $\beta =$ Homework.

$$H = \hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g(a^\dagger + a)(b^\dagger + b)$$

$g \equiv g_0\alpha = g_0\sqrt{n_p}$

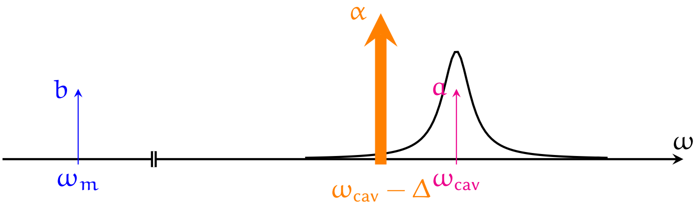
Linearized Optomechanics

The Hamiltonian

$$H = \hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b - \hbar g(a^\dagger + a)(b^\dagger + b)$$

The main participants

- a quantum optical mode at ω_{cav}
- α strong classical pump at $\omega_{cav} - \Delta$
- b quantized mechanical motion at ω_m



$$H = \underbrace{\hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b}_{H_{RF}} - \hbar g(a^\dagger + a)(b^\dagger + b)$$

$$a \to a e^{-i\Delta t}; \quad b \to b e^{-i\omega_m t};$$

$$H = \underbrace{\hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b}_{H_{RF}} - \hbar g(a^\dagger + a)(b^\dagger + b)$$

$$a \to a e^{-i\Delta t}; \quad b \to b e^{-i\omega_m t};$$

$$H = -\hbar g(a b e^{-i(\Delta+\omega_m)} + \text{h.c.}) \\ - \hbar g(a b^\dagger e^{-i(\Delta-\omega_m)} + \text{h.c.})$$

$$\Delta = \omega_{cav} - \omega_p$$

$$H = \underbrace{\hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b}_{H_{RF}} - \hbar g(a^\dagger + a)(b^\dagger + b)$$

$$a \to a e^{-i\Delta t}; \quad b \to b e^{-i\omega_m t};$$

$$H = -\hbar g(ab e^{-i(\Delta+\omega_m)} + \text{h.c.}) - \hbar g(ab^\dagger e^{-i(\Delta-\omega_m)} + \text{h.c.})$$

$$\Delta = \omega_{\text{cav}} - \omega_p$$

In the Rotating Wave Approximation (RWA) (assuming g small)

Lower sideband pump $\Delta = +\omega_m$

$$H = -\hbar g(ab^\dagger + ab e^{-2i\omega_m t}) + \text{h.c.}$$
$$\approx -\hbar g\left[ab^\dagger + a^\dagger b\right]$$

$$H = \underbrace{\hbar\Delta a^\dagger a + \hbar\omega_m b^\dagger b}_{H_{RF}} - \hbar g(a^\dagger + a)(b^\dagger + b)$$

$$a \to a e^{-i\Delta t}; \quad b \to b e^{-i\omega_m t};$$

$$H = -\hbar g(ab e^{-i(\Delta+\omega_m)} + \text{h.c.}) - \hbar g(ab^\dagger e^{-i(\Delta-\omega_m)} + \text{h.c.})$$

$$\Delta = \omega_{cav} - \omega_p$$

In the Rotating Wave Approximation (RWA) (assuming g small)

Lower sideband pump $\Delta = +\omega_m$

$$H = -\hbar g(ab^\dagger + ab e^{-2i\omega_m t}) + \text{h.c.} \approx -\hbar g\left[ab^\dagger + a^\dagger b\right]$$

Upper sideband pump $\Delta = -\omega_m$

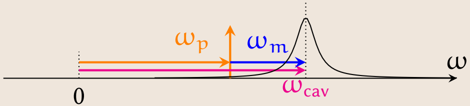
$$H = -\hbar g(ab^\dagger e^{2i\omega_m t} + ab) + \text{h.c.} \approx -\hbar g\left[ab + a^\dagger b^\dagger\right].$$

Resonantly detuned optomechanics

Lower Mechanical Sideband

Pump at the difference frequency

$$\omega_p = \omega_{cav} - \omega_m$$



$$H \propto ab^\dagger + a^\dagger b$$

- Parametric Converter / Beam-splitter
- State swap / Cooling

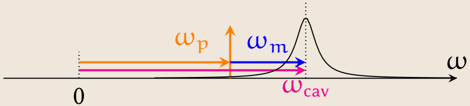
Resolved sideband $\kappa \ll \omega_m$

Resonantly detuned optomechanics

Lower Mechanical Sideband

Pump at the difference frequency

$$\omega_p = \omega_{cav} - \omega_m$$



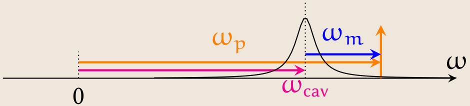
$$H \propto ab^\dagger + a^\dagger b$$

- ★ Parametric Converter / Beam-splitter
- ★ State swap / Cooling

Upper Mechanical Sideband

Pump at the sum frequency

$$\omega_p = \omega_{cav} + \omega_m,$$



$$H = ab + a^\dagger b^\dagger$$

- ★ Parametric Amp / Two-mode squeezing
- ★ Entanglement

Resolved sideband $\kappa \ll \omega_m$

Digression: Optical Spring

Radiation Pressure Force

$$F_{RP}(t) \propto P(x) = -Kx$$
$$= -Kx(t - \tau_*)$$
$$\approx -K \times (x - \tau_* \dot{x}) = -Kx + \Gamma \dot{x}$$

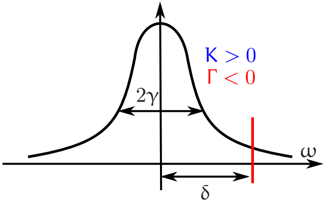
Digression: Optical Spring

Radiation Pressure Force

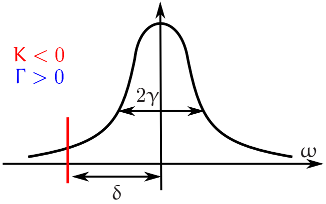
$$
\begin{aligned}
F_{RP}(t) \propto P(x) &= -Kx \\
&= -Kx(t-\tau_*) \\
\approx -K\times(x-\tau_*\dot{x}) &= -Kx + \Gamma\dot{x}
\end{aligned}
$$

Электромагнитная жесткость впервые наблюдалась В. Б. Брагинским [3]

Настройка на правый склон



Настройка на левый склон



Положительная жесткость и отрицательное затухание

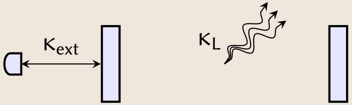
Отрицательная жесткость и положительное затухание

Назад

[3] V.B. Braginsky, I.I. Minakova Bull. MSU III 1, 83 (1964)

Environment

Optical Environment



κ_{ext} detection channel, κ_L losses

Interacts with the modes of travelling light, (almost) each in vacuum. Collective operator a_i

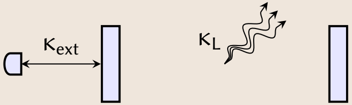
$$\left[a_i(t), a_i^\dagger(t')\right] = \delta(t-t');$$

$$\frac{1}{2}\left\langle a_i(t)a_i^\dagger(t') + a_i^\dagger(t')a_i(t)\right\rangle = \delta(t-t').$$

Typically the cavity is overcoupled with $\kappa_{ext} \gg \kappa_L$

Environment

Optical Environment



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Typically the cavity is overcoupled with $\kappa_{ext} \gg \kappa_L$

Mechanical Environment

Q-factor:

$$Q_{tot}^{-1} = Q_{clamp}^{-1} + Q_{mat}^{-1} + Q_{air}^{-1} + Q_{scat}^{-1} + \dots$$

At rate $\gamma = \omega_m/Q$ coupled to a thermal bath with bosonic operator b^{th}:

$$\left[b^{th}(t), b^{th\dagger}(t')\right] = \delta(t-t'),$$

$$\frac{1}{2}\langle\{b^{th}(t), b^{th\dagger}(t')\}\rangle = (2n_{th}+1)\delta(t-t').$$

$$n_{th} = \frac{1}{\exp[\hbar\omega_m/k_B t] - 1} \approx k_B T/\hbar\omega_m$$

Equations of motion

Assume red detuning $\omega_p = \omega_{cav} - \omega_m$, therefore $H = -\hbar g(ab^\dagger + a^\dagger b)$.

$$\dot{a} = igb - \kappa a + \sqrt{2\kappa}a^{in},$$
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Input-output relation for optics $a^{out} = -a^{in} + \sqrt{2\kappa}a$

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Define $\mathbf{a} = (a, b)$, $\mathbb{A}$, $\mathbf{f} = (\sqrt{2\kappa}a^{in}, \sqrt{\gamma}b^{th})$, then

$$\dot{\mathbf{a}} = \mathbb{A}.\mathbf{a} + \mathbf{f}$$

Formal solution (with $\mathbb{M}(s) = \exp[-\mathbb{A}s]$)

$$\mathbf{a}(t) = \mathbb{M}(t)\mathbf{a}(0) + \int_0^t ds\, \mathbb{M}(t-s).\mathbf{f}(s).$$

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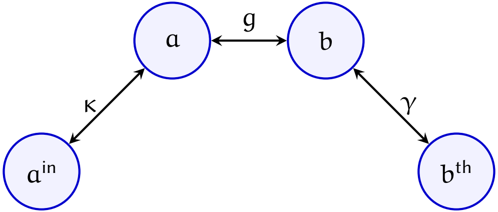

Parameters:

- Resolved sideband $\kappa \ll \omega_m$
- Weak coupling $g \sim 10^{-3 \div -1}\kappa$
- Slow mechanical decay $\gamma \sim 10^{-7 \div -4}\kappa$
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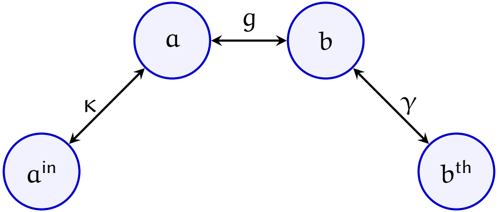

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That is,

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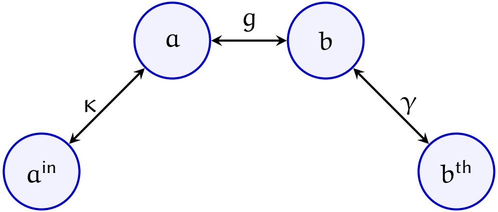

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That is,

- mechanical decay can be approximately ignored
- cavity mode can be adiabatically eliminated

$$0 = igb - \kappa a + \sqrt{2\kappa} a^{\text{in}},$$
$$\dot{b} = iga.$$

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\dot{b} &= iga.
\end{aligned}
$$

$$
\begin{aligned}
a &= i\frac{g}{\kappa}b + \sqrt{\frac{2}{\kappa}}a^{\text{in}}, \\
\dot{b} &= -Gb + i\sqrt{2G}a^{\text{in}}, \quad G \equiv g^2/\kappa
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$$A^{out} = \sqrt{\frac{2G}{1-e^{-2G\tau}}}\int_0^\tau d\xi\, a^{out}(\xi)e^{-G\xi}; \quad \left[A^{out}, A^{out\dagger}\right] = 1.$$

$$b(\tau) = b(0)e^{-G\tau} + i\sqrt{2G}e^{-G\tau}\int_0^\tau dt\; a^{in}(t)e^{Gt}.$$

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$$\int_0^\tau dt\; a^{out}(t)e^{-Gt} = \int_0^\tau dt\; a^{in}(t)e^{-Gt} + i\sqrt{2G}b(0)\int_0^\tau dt\; e^{-2Gt} - 2G\int_0^\tau dt\; e^{-2Gt}\int_0^t d\xi\; a^{in}(\xi)e^{G\xi}$$

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$$\begin{aligned} A^{out} &= \sqrt{\frac{2G}{1-e^{-2G\tau}}}\int_0^\tau dt\, a^{out}(t)e^{-Gt} \\ &= i\sqrt{1-e^{-2G\tau}}b(0) + \sqrt{\frac{2G}{1-e^{-2G\tau}}}e^{-2G\tau}\int_0^\tau dt\, a^{in}(t)e^{Gt} \end{aligned}$$

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$$= i\sqrt{1-e^{-2G\tau}}b(0) + e^{-G\tau}A^{in}.$$

$$
B^{\mathrm{out}} = \sqrt{T} B^{\mathrm{in}} + i\sqrt{1-T} A^{\mathrm{in}},
$$
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$$B^{\mathrm{in}} = b(0); \quad B^{\mathrm{out}} = b(\tau),$$

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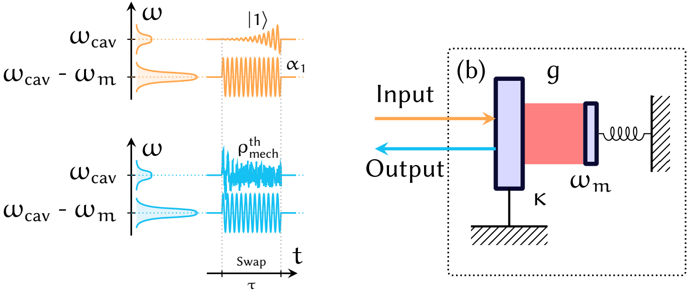
$$A^{\mathrm{in}} = \sqrt{\frac{2G}{e^{2G\tau}-1}} \int_0^\tau d\xi\, a^{\mathrm{in}}(\xi) e^{G\xi},$$

$$T \equiv e^{-2G\tau}, \quad G = g^2/\kappa.$$

Pulsed State Swap



Pulsed Entanglement

Blue tuning (to the upper sideband, $\omega_p = \omega_{cav} + \omega_m$).

$$H = -\hbar g(ab + a^\dagger b^\dagger)$$

In a similar fashion, assuming no thermal decoherence and adiabatic elimination of cavity mode,

$$A^{out} = \sqrt{K}A^{in} + i\sqrt{K-1}B^{in\dagger},$$
$$B^{out} = \sqrt{K}B^{in} + i\sqrt{K-1}A^{in\dagger},$$

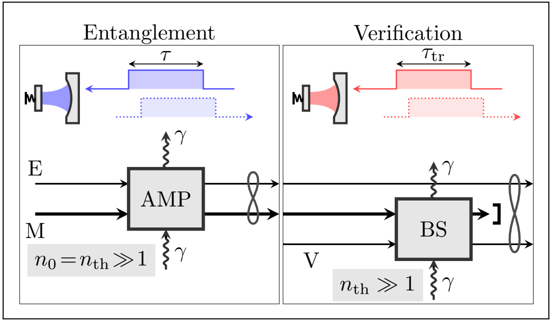
Two-Mode Squeezed State (ideally, vacuum: TMSV).

$$A^{in} = \sqrt{\frac{2G}{1-e^{-2G\tau}}}\int_0^\tau dt\, a^{in}(t)e^{-Gt},$$
$$A^{out} = \sqrt{\frac{2G}{e^{2G\tau}-1}}\int_0^\tau dt\, a^{out}(t)e^{Gt}.$$

The Protocol

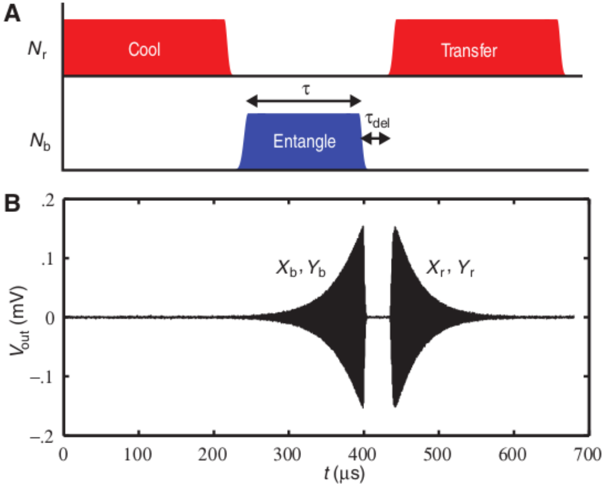
Classical pump $@\omega_p$, quantum cavity mode $@\omega_{cav}$, mechanical mode $@\omega_m$.
In the rotating frame we deal with slow amplitudes
Pump power $\mapsto g(t) \mapsto$ Temporal mode profile
For nice exponential envelopes assume constant pump

Blue detuning $\omega_p = \omega_{cav} + \omega_m$	Red detuning $\omega_p = \omega_{cav} - \omega_m$
Two-Mode squeezing interaction	State swap interaction



Evaluation

[Palomaki,2013] following [Hofer, 2011] evaluate the Duan's variance:



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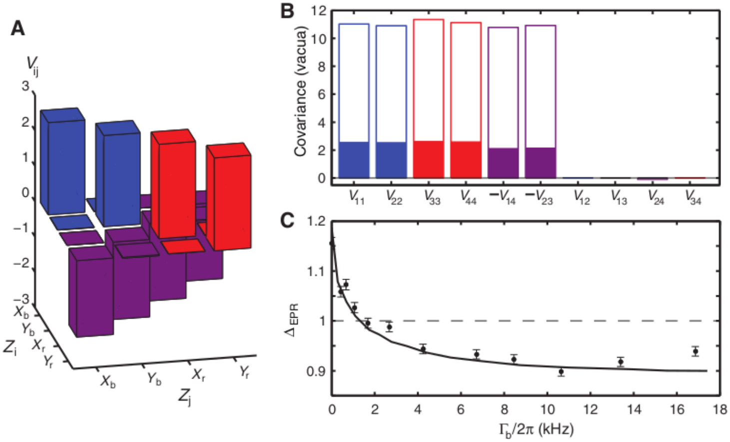

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By expressing Eqs. (7) in terms of quadratures $X_{\mathrm{m}}^{i} = (B_i + B_i^{\dagger})/\sqrt{2}$ and $X_{\mathrm{l}}^{i} = (A_i + A_i^{\dagger})/\sqrt{2}$, where $i \in \{\mathrm{in}, \mathrm{out}\}$, and their corresponding conjugate variables, we can calculate the so-called EPR variance Δ_{EPR} of the state after the interaction. For light initially in vacuum $(\Delta X_{\mathrm{l}}^{\mathrm{in}})^2 = (\Delta P_{\mathrm{l}}^{\mathrm{in}})^2 = \frac{1}{2}$ and the mirror in a thermal state $(\Delta X_{\mathrm{m}}^{\mathrm{in}})^2 = (\Delta P_{\mathrm{m}}^{\mathrm{in}})^2 = n_0 + \frac{1}{2}$, the state is entangled iff [52]

$$\begin{aligned} \Delta_{\mathrm{EPR}} &= \left[\Delta\left(X_{\mathrm{m}}^{\mathrm{out}} + P_{\mathrm{l}}^{\mathrm{out}}\right)\right]^2 + \left[\Delta\left(P_{\mathrm{m}}^{\mathrm{out}} + X_{\mathrm{l}}^{\mathrm{out}}\right)\right]^2 \\ &= 2(n_0 + 1)\left(e^r - \sqrt{e^{2r} - 1}\right)^2 < 2, \end{aligned} \tag{8}$$

Evaluation

[Palomaki,2013] following [Hofer, 2011] evaluate the Duan's variance:

$$\hat{u} = |a|\hat{x}_1 + \frac{1}{a}\hat{x}_2 ,$$

$$\hat{v} = |a|\hat{p}_1 - \frac{1}{a}\hat{p}_2 ,$$

Theorem 1.–Sufficient criterion for inseparability: For any separable quantum state ρ, the total variance of a pair of EPR-like operators defined by Eqs. (2a) and (2b) with the commutators $[\hat{x}_j, \hat{p}_{j'}] = i\delta_{jj'}$ $(j, j' = 1, 2)$ satisfies the inequality

$$\langle(\Delta\hat{u})^2\rangle_\rho + \langle(\Delta\hat{v})^2\rangle_\rho \geq a^2 + \frac{1}{a^2} . \qquad (3)$$

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A proper criterion has to be applied!

Either the properly generalized Duan variance or logarighmic negativity

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Continuous Variables Systems

Each mode is described by annihilation operator a_k:

$$\left[a_i, a_j^\dagger\right] = \delta_{ij}; \quad [a_i, a_j] = \left[a_i^\dagger, a_j^\dagger\right] = 0.$$

Or quadratures

$$x_k = a_k + a_k^\dagger; \quad p_k = (a_k - a_k^\dagger)/i,$$

which form the vector

$$\mathbf{R} = \{x_1, p_1, x_2, p_2, \ldots, x_N, p_N\}.$$

$$[R_i, R_j] = 2i\Omega_{ij}; \quad \Omega_{ij} = \oplus_{k=1}^{N}\omega; \quad \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Gaussian states: $\langle \mathbf{R} \rangle$ and covariance matrix

$$\mathbb{V}_{ij} = \frac{1}{2}\left\langle\{(R_i - \langle R_i\rangle), (R_j - \langle R_j\rangle)\}\right\rangle \mapsto \frac{1}{2}\left\langle R_i R_j + R_j R_i\right\rangle, \text{ if } \mathbf{R} = 0.$$

Bipartite Gaussian System

$$\mathbf{R} = (x_1, p_1, x_2, p_2), \quad [x_1, p_1] = 2i; \quad [x_2, p_2] = 2i;$$

Assuming $\langle \mathbf{R} \rangle = 0$,

$$\mathbb{V} = \left(\begin{array}{cc|cc} \langle x_1^2 \rangle & \langle x_1 \circ p_1 \rangle & \langle x_1 \circ x_2 \rangle & \langle x_1 \circ p_2 \rangle \\ \langle p_1 \circ x_1 \rangle & \langle p_1^2 \rangle & \langle p_1 \circ x_2 \rangle & \langle p_1 \circ p_2 \rangle \\ \hline \langle x_2 \circ x_1 \rangle & \langle x_2 \circ p_1 \rangle & \langle x_2^2 \rangle & \langle x_2 \circ p_2 \rangle \\ \langle p_2 \circ x_1 \rangle & \langle p_2 \circ p_1 \rangle & \langle p_2 \circ x_2 \rangle & \langle p_2^2 \rangle \end{array}\right), \qquad \text{where } a \circ b \equiv \tfrac{1}{2}(ab + ba).$$

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Symplectic Transformations

Transformation $\mathbf{R} \mapsto S\mathbf{R}$ is symplectic, if

$$S^{\mathsf{T}} \Omega S = \Omega$$

Important symplectic transformation diagonalizes the CM

$$\mathbb{V} = S^{\mathsf{T}} \mathbb{N} S; \quad \mathbb{N} = \mathrm{diag}(\nu_1, \nu_1, \nu_2, \nu_2, \dots).$$

ν_k: symplectic eigenvalues
A physical state has all $\nu_k \geqslant \sigma_{vac}$ (shot-noise variance).

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$$\sigma_{\mathrm{vac}} \equiv \left\langle 0 \middle| x^2 \middle| 0 \right\rangle = \left\langle (a + a^\dagger)^2 \right\rangle_{|0\rangle} = 1.$$

If e.g. define $x = (a + a^\dagger)/\sqrt{2}$, then $\sigma_{\mathrm{vac}} = 1/2$.

Entanglement and Partial Transposition

A bipartite state ρ is separable if (otherwise, entangled)

$$\hat{\rho} = \sum_i p_i \hat{\rho}_i^A \otimes \hat{\rho}_i^B$$

Entanglement is a resource etc.

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Partial Transposition

If $\hat{\rho}$ is physical, so is $\hat{\rho}^T$
Idea: check transposition of a subsystem (Peres-Horodecki)

$$\hat{\rho}^{T_B} = \sum_i p_i \hat{\rho}_i^A \otimes (\hat{\rho}_i^B)^T$$

If $\hat{\rho}^{T_B}$ is physical, the state is separable

Partial Transposition and CM

In a CV system partial transposition is equivalent to reversing time:

$$x \to x; \quad p \to -p$$

Criterion of physicality: all symplectic eigenvalues $\nu_k \geqslant 1$.

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$$\Sigma(\mathbb{V}) \equiv \det V_1 + \det V_2 - 2 \det V_c .$$

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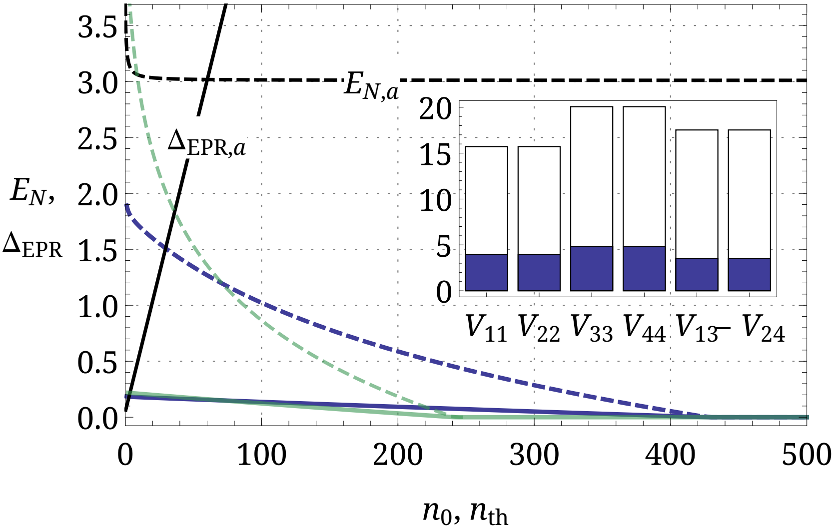
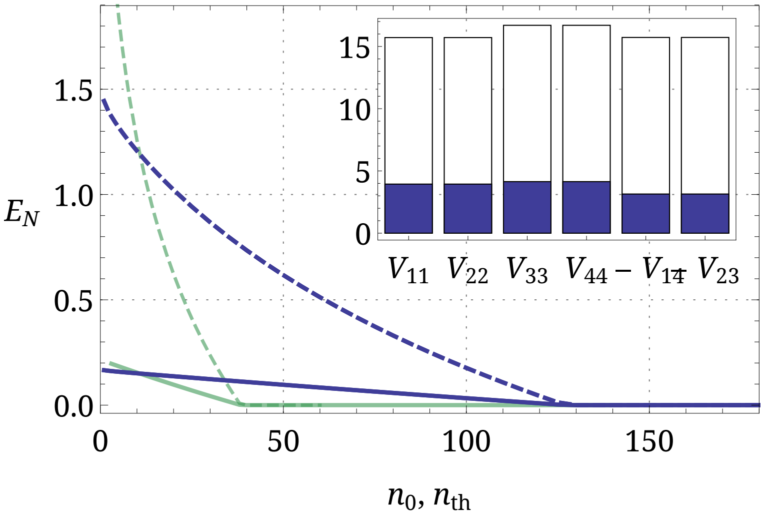
$$E_N = \max[-\log \nu_- / \sigma_{vac}], 0].$$

Opto- (Electro-) Mechanical Entanglement



Pulses Entanglement



GKP state

Devised by Gottesman, Kitaev and Preskill [5]

$$|\gamma_{\mathrm{GKP}}\rangle \propto \int dx\, e^{i\gamma x^3} |x\rangle\,,$$

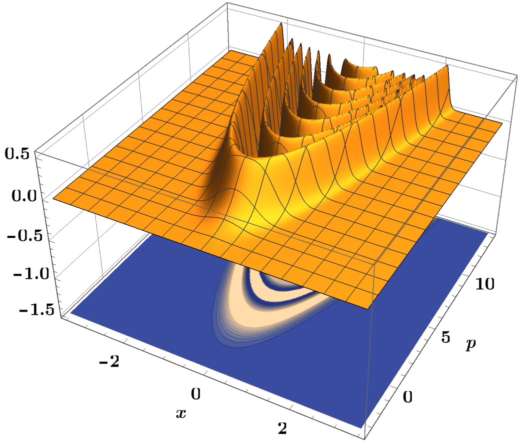
Wigner Function

$$W_{\mathrm{GKP}}(x,p) \propto \mathrm{Ai}\left[\left(\frac{4}{3\gamma}\right)^{1/3} (3\gamma x^2 - p)\right].$$

Nonlinear variance

$$(x,p) \to (x, p+\gamma x^2) \Rightarrow \langle \delta(p-\lambda x^2)^2 \rangle \to 0.$$

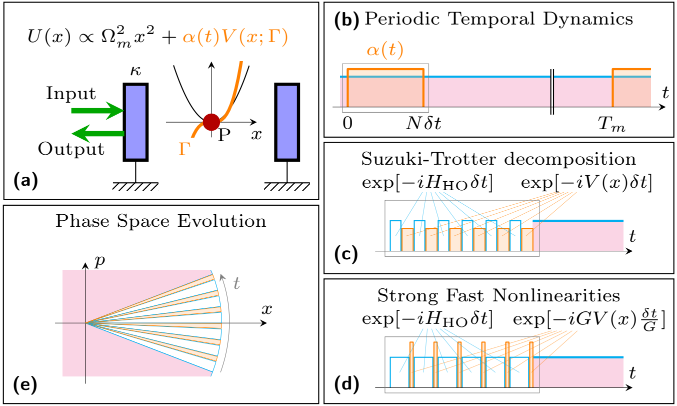
Required for the measurement-based quantum computing.



[5] Gottesman, Kitaev, Preskill, PRA **64**, 012310 (2001)

The Model

Rakhubovsky and Filip [arxiv:1904.00773]



$$\Big[\exp[-i(H_{HO} + V(x))\delta t]\Big]^N \approx \Big[\mathcal{U}_{HO}(\delta t)\mathcal{U}_{NL}(\delta t) + O(\delta t^2)\Big]^N,$$

Numerical evaluation

Harmonic evolution

Rotation in phase space

$$W(x,y) \to W(x\cos\delta + y\sin\delta, x\sin\delta + y\cos\delta).$$

Cubic evolution

In coordinate basis

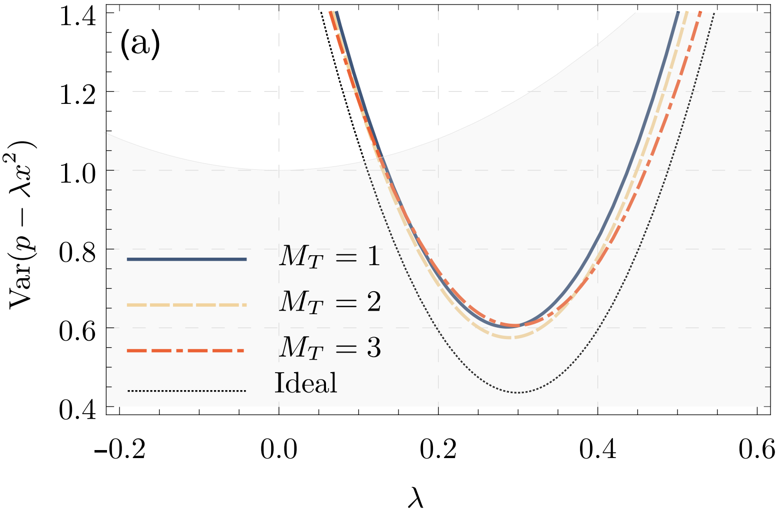
$$\rho(x,x') \to \rho(x,x')e^{-i\gamma(x^3 - x'^3)}.$$

Decoherence

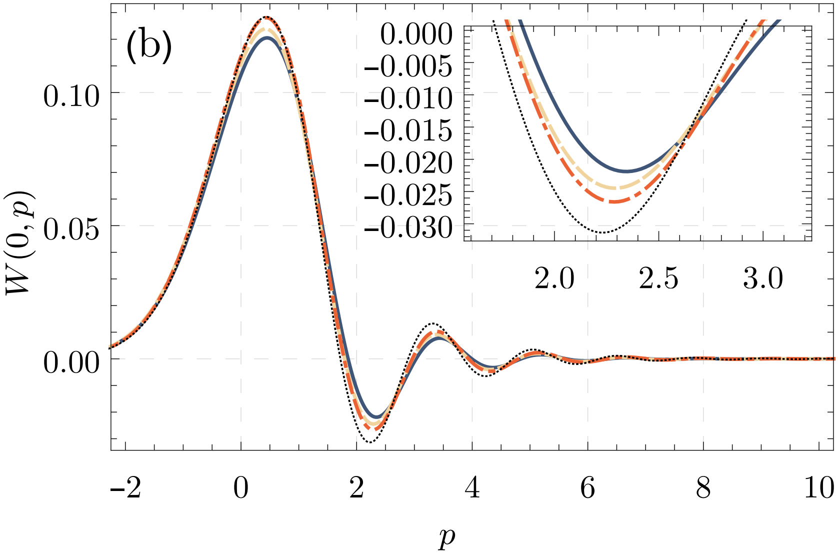
Convolution in phase space with a Gaussian kernel

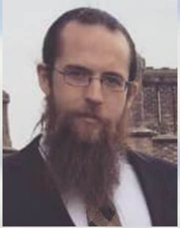
$$W(x,y) \to \iint d\xi\, d\eta\, W(x-\xi, y-\eta) W_{\mathrm{th}}(\xi,\eta).$$

Nonlinear Variance



Wigner Function Cuts





Darren Moore

The Model

Approximate cubic phase state in mechanics

$$e^{i\gamma x^3}\left|0\right\rangle .$$

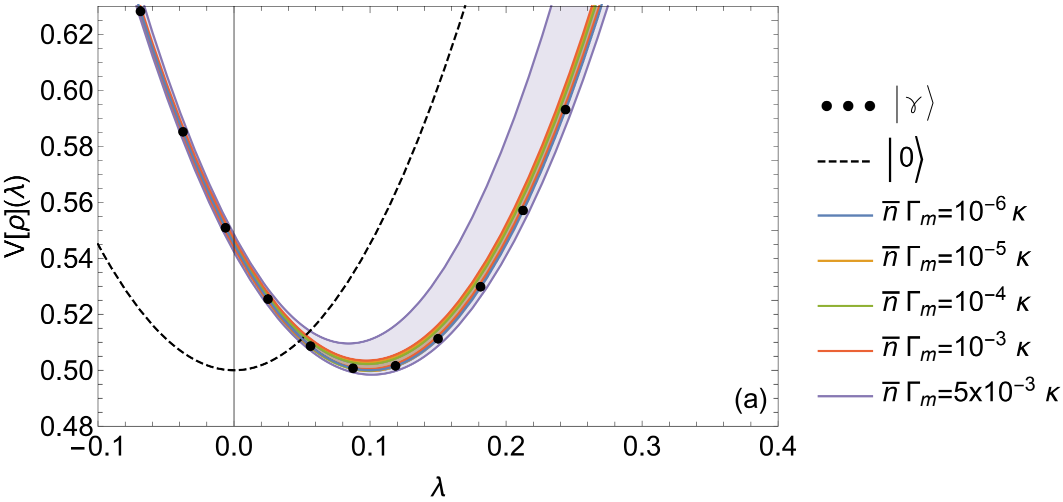
Pulsed QND interaction

$$H_{\mathrm{int}} \propto x_{\mathrm{light}}(x\cos\phi + p\sin\phi).$$

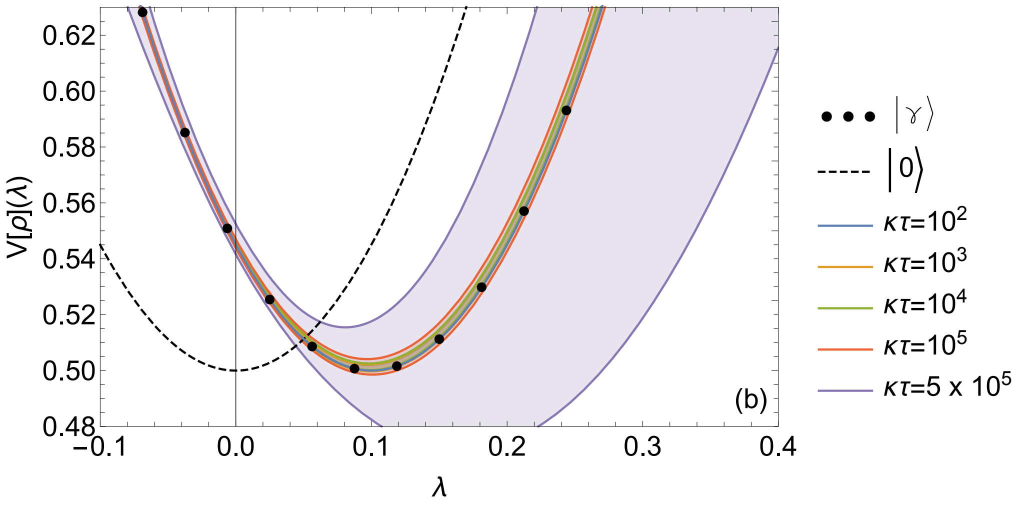
Estimate nonlinear variance

$$q_{\mathrm{NL}} = p - \lambda x^2 \Rightarrow \mathrm{Var}[q_{\mathrm{NL}}]$$

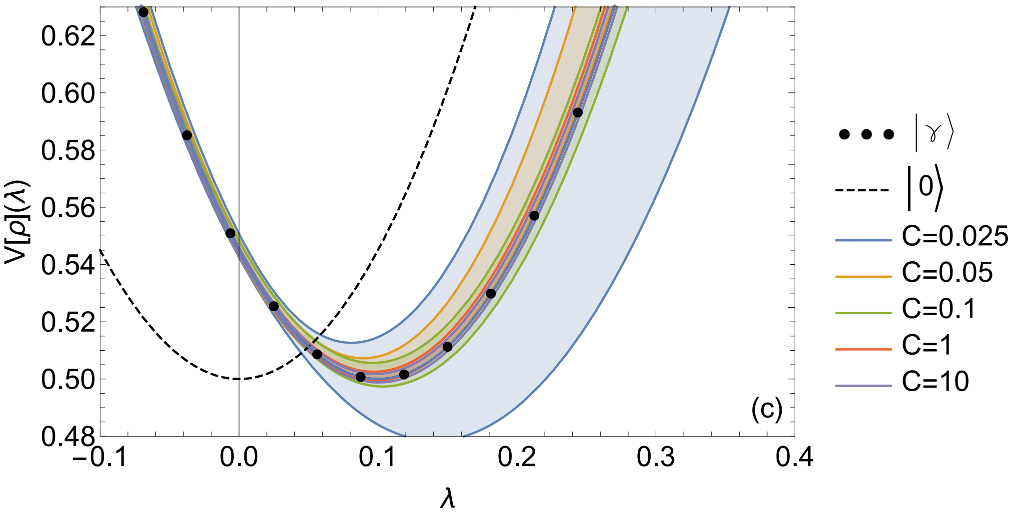
Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Evaluation $V[\rho](\lambda) = \mathrm{Tr}(\rho[\Delta(p - \lambda x^2)]^2)$



Conclusion

- ★ Levitated optomechanics allows production of mechanical squeeezed states
- ★ Entanglement
- ★ Approximate Cubic Phase State

Спасибо!

PhD positions available

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