Bounding Treatment Effects in Experimental Studies with Non-Compliance

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Randomized Trials and Identifying the Impact of Treatment

- Suppose we want to understand if a treatment or intervention is actually helpful.
  - Does a job training program help people improve their employment outcomes?
- Randomized trials are a popular method to evaluate the impact of any such treatment.
  - Randomly assign participants of study to treatment or control arm.
  - Suppose all participants comply with their assigned treatment arm.
  - Compare outcomes in treatment arm to those of agents in control arm.
- In many randomized trials non-compliance with assigned treatment is common.

	Trained	Not Trained	Total
Assigned to Treatment	4804	2683	7487
Assigned to Control	54	3663	3717
Total	4868	6346	11204

Table 1: Non-compliance in the JTPA Study.

The Problem of Non-Compliance

- What are the consequences of non-compliance?
- Suppose we compare outcomes in treatment arm to those in control arm.
  - $\Rightarrow$ Measure impact of an offer of training, not training itself.
- Suppose we compare those who accept training to those who did not.
  - $\Rightarrow$ Biased results since complying agents are self selecting into treatment.
- Some agents can be encouraged to accept treatment by extending an offer.
  - $\Rightarrow$ Proportion of agents who accept offer of treatment can be identified.
  - $\Rightarrow$ Impact of actual treatment received for this sub-group can be identified.
- Can't identify impact of actual treatment received on full population of agents.

Understanding Non-Compliance

- We don't know why only some agents assigned to treatment actually accept.
- Some trials have additional information which might help understand this behavior.

	Reason	% of respondents
treatment not beneficial?	Took Job	18
	Changed Mind	7
	Needed Job	4
unexpected shock	Transport Problem	3
	Health Problem	2

Table 2: Reasons for non-compliance recorded in JTPA Study.

return

- This information can be used to understand non-compliance.

Using Follow up Surveys to Bound Treatment Effects

- I use reasons for non-compliance to bound the impact of treatment received.
  - $\Rightarrow$ This information is often present but never used when analyzing treatment effects
  - $\Rightarrow$ JTPA Study – a very popular dataset – collected this information.
- First define a model which rationalizes non-compliance.
  - $\Rightarrow$ Quantifies qualitative information in reasons for non-compliance.
  - $\Rightarrow$ Will require assumptions on agent beliefs about treatment efficacy.
- Then use model to define set of treatment effects consistent with observed data.
  - $\Rightarrow$ Treatment effect will be partially identified.

Importance of Following Up with Non-Compliers

- Model implied bounds will be much tighter than bounds without reasons information.
  - ⋆ Following up with non-compliers is important!

Outcome	Worst Case Bounds	Model Bounds	Improvement
Found job post treatment	[-0.25 , 0.10]	[-0.03 , 0.09]	66%
Months Employed post treatment	[-4.81 , 5.71]	[-0.48 , 5.11]	47%
Earnings post treatment (in '000s)	[-3.58 , 24.47]	[0.09 , 21.16]	25%

Table 3: Estimated identified sets for treatment effects of JTPA for different outcomes.

Setup I

- There are two potential outcomes taking on values in $\{0, 1\}$.

$$\text{Potential Outcomes: } \begin{cases} \mathrm{Y_i}(0) & \text{if not treated,} \\ \mathrm{Y_i}(1) & \text{if treated.} \end{cases}$$

- I am interested in the average treatment effect:

$$\text{ATE} = \mathbb{E}[Y_i(1) - Y_i(0)].$$

- D_i denotes if agent i received treatment $\Rightarrow$ $\mathrm{Y_i} = D_i \cdot \mathrm{Y_i}(1) + (1 - D_i) \cdot \mathrm{Y_i}(0)$.
- Z_i denotes if agent was assigned to treatment.

Setup II - Non-Compliance Again

- Assignment is independent of potential outcomes i.e. $Z_i \perp (Y_i(1), Y_i(0))$.
- Non-assigned cannot receive treatment i.e. $Z_i = 0 \Rightarrow D_i = 0$.
- There is non-compliance i.e. not everyone assigned to treatment accepted the offer:

$$0 < \mathbb{P}(D_i = 1 \mid Z_i = 1) < 1.$$

The Problem of Non-Compliance

$$\underbrace{\mathbb{E}[Y_i \mid Z_i = 1] - \mathbb{E}[Y_i \mid Z_i = 0]}_{\text{Intention to Treat Effect } (\theta^{ITT})} = \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i = 1, Z_i = 1] \cdot \underbrace{\mathbb{P}(D_i = 1 \mid Z_i = 1)}_{\text{No problem if compliance perfect.}}$$

- Why? Treatment effect for non-compliers is zero.

Partially Identified Treatment Effects

- If there is non-compliance, part of *ATE* is unobserved in data:

$$\text{ATE} = \theta^{ITT} + \mathbb{P}(D_i = 0 \mid Z_i = 1) \cdot \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i = 0, Z_i = 1]$$

- Recall outcomes are binary so it must be:

$$\underbrace{\theta^{ITT} - \mathbb{P}(D_i = 0 \mid Z_i = 1)}_{\textit{identified}} \leqslant \text{ATE} \leqslant \underbrace{\theta^{ITT} + \mathbb{P}(D_i = 0 \mid Z_i = 1)}_{\textit{identified}}$$

- Can we do better? Yes.
  - ⋆ Following up with non-compliers $\Rightarrow$ narrower identified set.

A Compliance Model

- When assigned to treatment agent does the following:
  - ⋆ Form expectation of treatment effect:

$$\theta_i^e = \mathbb{E}[Y_i(1) - Y_i(0) \mid \mathcal{I}_i]$$

  - ⋆ Observe cost of accepting treatment: C_i.
    - Not related to treatment but may discourage/encourage acceptance of offer.
- Agent accepts offer of treatment if net benefit is non-negative.

$$D_i = \mathbb{1}\left\{\theta_i^e - C_i \geqslant 0\right\}$$

A Compliance Model - Reasons for Non-Compliance

- Suppose for those who do not accept treatment we see a binary variable R_i such that:

$$R_i = \begin{cases} 1 & \Rightarrow \text{Did not think treatment would help.} \\ 0 & \Rightarrow \text{Would have accepted if not for unrelated cost shock.} \end{cases}$$

OR

$$R_i = \begin{cases} 1 & \Rightarrow \theta_i^e < 0, \quad \theta_i^e < C_i. \\ 0 & \Rightarrow \theta_i^e \geqslant 0, \quad \theta_i^e < C_i. \end{cases}$$

reasons in JTPA

- Agents know θ_i^e AND C_i when assigned to treatment/control.
  - ⋆ I'm assuming R_i correctly tells me the sign of θ_i^e.

How Does this Help?

- How does agent's treatment effect relate to θ_i^e?

$$\theta_i \equiv Y_i(1) - Y_i(0) = \underbrace{\mathbb{E}[\,Y_i(1) - Y_i(0) \mid \mathcal{I}_i\,]}_{\theta_i^e} \quad + \quad \underbrace{\nu_i}_{\mathbb{E}[\nu_i|\mathcal{I}_i]=0}$$

- Consider agents who "changed their mind" after being assigned to treatment.

$$\mathbb{E}[\theta_i \mid R_i = 1, D_i = 0, Z_i = 1] = \mathbb{E}[\theta_i^e + \nu_i \mid R_i = 1, D_i = 0, Z_i = 1]$$

$$= \mathbb{E}[\theta_i^e + \nu_i \mid \underbrace{\theta_i^e < 0}_{\text{defn. of } R_i}, \underbrace{\theta_i^e < C_i}_{D_i = 0}]$$

$$= \mathbb{E}[\theta_i^e \mid \theta_i^e < 0, \theta_i^e < C_i] < 0.$$

How Does this Help? II

- For non-compliers the following is true:

$$\mathbb{E}[\theta_i \mid D_i = 0, Z_i = 1] = \mathbb{P}(R_i = 1, D_i = 0 \mid Z_i = 1) \cdot \underbrace{\mathbb{E}[\theta_i \mid R_i = 1, D_i = 0, Z_i = 1]}_{<0}$$

$$+ \mathbb{P}(R_i = 0, D_i = 0 \mid Z_i = 1) \cdot \underbrace{\mathbb{E}[\theta_i \mid R_i = 0, D_i = 0, Z_i = 1]}_{\geqslant 0}$$

- What is lowest value for average treatment effect for non-compliers?

$$\text{With model + } R_i \text{ variable} : -\mathbb{P}(R_i = 1, D_i = 0 \mid Z_i = 1)$$

$$\text{Without } R_i \text{ variable} : -\mathbb{P}(D_i = 0 \mid Z_i = 1)$$

$$\Rightarrow \text{Gain from } R_i : \quad \mathbb{P}(R_i = 0, D_i = 0 \mid Z_i = 1)$$

Model Implied Bounds

- We can now compare bounds on the ATE with/without the model.

	Description	Bound	Difference
Lower	Without model	$\theta^{ITT} - \mathbb{P}(D_i = 0 \mid Z_i = 1)$	$\mathbb{P}(R_i = 0, D_i = 0 \mid Z_i = 1)$
	With model	$\theta^{ITT} - \mathbb{P}(R_i = 1, D_i = 0 \mid Z_i = 1)$	
Upper	Without model	$\theta^{ITT} + \mathbb{P}(D_i = 0 \mid Z_i = 1)$	$-\mathbb{P}(R_i = 1, D_i = 0 \mid Z_i = 1)$
	With model	$\theta^{ITT} + \mathbb{P}(R_i = 0, D_i = 0 \mid Z_i = 1)$	

– Proportion of non-compliers who felt program would've helped $\uparrow$ $\Rightarrow$ lower bound $\uparrow$.

– Proportion of non-compliers who felt program wouldn't help $\uparrow$ $\Rightarrow$ upper bound $\downarrow$.

⋆ Why? Agents are correct about their assessment of the ATE on average.

Empirical Application: The JTPA Study

- Commissioned to measure benefits of training offered under the Job Training Partnership Act.
  - Meant to improve outcomes of economically, socially disadvantaged.
  - Treatments included classroom training, practical training, job search assistance.
- Eligible applicants were randomly assigned to a treatment or control group.
  - Treatment group offered training under JTPA.
  - Control group barred from JTPA for 18 months.
- Participants tracked for 30 months post treatment phase.

Empirical Application: Did JTPA Help Applicants Find Jobs?

- Look at whether applicants found employment within 12 months after treatment.

Description			N	ITT	LATE	Worst Case	Model Bounds
Overall			10826	0.01	0.02	[-0.25 , 0.10]	[-0.03 , 0.09]
				(0.00 , 0.03)	(0.00 , 0.05)	(-0.27 , 0.11)	(-0.05 , 0.10)
No income	Age $<$ 30	Did not graduate HS	506	-0.02	-0.04	[-0.20 , 0.11]	[-0.05 , 0.09]
				(-0.11 , 0.06)	(-0.16 , 0.09)	(-0.27 , 0.18)	(-0.12 , 0.16)
		Graduated HS	541	0.12	0.18	[-0.13 , 0.20]	[0.07 , 0.20]
				(0.04 , 0.20)	(0.05 , 0.30)	(-0.21 , 0.27)	(0.00 , 0.27)
	Age $\geqslant$ 30	Did not graduate HS	656	0.04	0.07	[-0.18 , 0.24]	[0.01 , 0.22]
				(-0.04 , 0.12)	(-0.07 , 0.20)	(-0.24 , 0.30)	(-0.06 , 0.28)
		Graduated HS	870	0.05	0.07	[-0.11 , 0.17]	[0.03 , 0.17]
				(-0.02 , 0.12)	(-0.02 , 0.17)	(-0.17 , 0.23)	(-0.03 , 0.23)

Table 4: ATE on whether applicant found employment within 12 months of treatment phase.

Distribution Treatment Effects

- Treatment effects can be identified for many other outcomes.
- Consider an agent's aggregate earnings over post treatment phase E_i and define:

$$Y_i = \mathbb{1}\{E_i \leqslant y\}, \quad \text{for } y \geqslant 0.$$

  Then we can define distribution treatment effects:

$$\mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{P}(E_i(1) \leqslant y) - \mathbb{P}(E_i(0) \leqslant y).$$

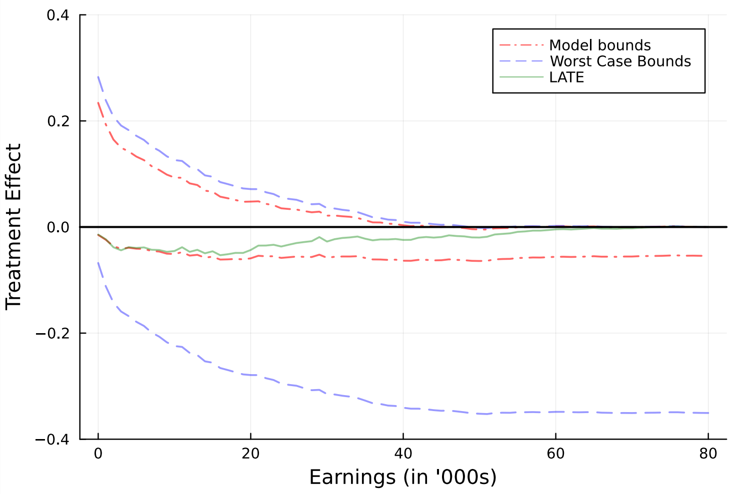
- Key distinction:

Looking at avg. earnings → Positive treatment effect is good.

Looking at distribution of earnings → Negative treatment effect is good.

Empirical Application: Did JTPA Improve Applicant Earnings?

- Now look at the distribution of total earnings in post treatment phase.



Conclusion

- I propose a model to rationalize non-compliance behavior in randomized trials.
- Bounds on ATE produced by model can greatly improve identified sets for ATE.
- Illustrates the importance of following up with non-compliers.
  - Collecting reasons for non-compliance is easy.
  - Phrasing of questions asking for this information is important - remove ambiguity.