Technical Note – Demand Uncertainty Reduction in Decentralized Supply Chains

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This note analyzes the effects associated with reducing demand uncertainty in a decentralized supply chain comprising one manufacturer, one retailer, and a wholesale price contract that governs the transactions between them. The demand uncertainty level is parameterized through a mean-preserving spread, and the manufacturer's and the retailer's equilibrium decisions are solved accordingly. We consider the case of an exogenous retail price as well as the case of an endogenous retail price, and we find in both cases that the manufacturer's and the retailer's expected profits in equilibrium are not necessarily monotone decreasing in the uncertainty level. Thus, we find that, even if the cost of reducing demand uncertainty is zero, uncertainty reduction can hurt rather than benefit either or both members of the supply chain.

Key words: newsvendor model, demand variance reduction, pricing, wholesale price contract

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1. Introduction

This note derives technical properties associated with uncertainty reduction (UR) in a decentralized supply chain (DC) by analyzing a stylized model in which a manufacturer leads by announcing a per-unit wholesale price and its newsvendor retailer follows by choosing the quantity to purchase accordingly (Lariviere and Porteus 2001). Within this model, we assess the would-be effects of UR on the optimal outcomes of the DC by asking the following questions: (i) Which firm(s) would benefit from UR, under what circumstances, and at what cost within the DC? (ii) Under what circumstances, if any, would both firms benefit from UR? (iii) More interestingly, under what circumstances, if any, would *neither* firm benefit from UR?

To answer these questions, we adopt a definition of UR that hinges on the notion of mean preservation, which is a frequently applied convex-ordering stipulation for studying UR (Rothschild and Stiglitz 1970, Gerchak and Mossman 1992, Xu et al. 2010), and we study two fundamental cases, defined by whether the retailer is a price-taker (the *exogenous retail price* case in §2) or a price-setter (the *endogenous retail price* case in §3). Overall, we find that the manufacturer's and retailer's optimal expected profits are not necessarily monotone decreasing in the uncertainty level, which underpins three novel technical results: (i) UR has the potential to hurt the manufacturer when the demand uncertainty is high; (ii) UR has the potential to hurt the retailer when the demand uncertainty is low; and (iii) UR has the potential to hurt both the retailer and the manufacturer when the demand uncertainty is moderate.

The analytical study of UR has focused largely on centralized models (Leland 1972, Eppen 1979, Corbett and Rajaram 2006), whereas the study of UR in DC models is comparatively recent and blossom (Iyer and Bergen 1997, Lau and Lau 2001, Dong and Rudi 2004, Miyaoka and Hausman 2008, Taylor and Xiao 2010, Cavusoglu et al. 2012, Ebrahim-Khanjari et al. 2013). This literature, which has focused on models that incorporate specific demand distributions, has found UR to benefit the manufacturer in a DC in which the retail price is exogenous (Shi et al. 2013, page 1245). In similar vein, it has illustrated numerically that UR can benefit both the manufacturer and the retailer in a DC in which the retail price is endogenous (Lau and Lau 2002). In contrast, we consider a general demand distribution and show that UR does not necessarily benefit the manufacturer or the retailer in either case. More broadly, we are the first to show analytically how UR could result in a mutually beneficial, a unilaterally beneficial, or even a non-beneficial outcome for the manufacturer and retailer in a bilateral DC.

2. Exogenous Retail Price Case

A manufacturer produces a single product that it sells to a retailer that, in turn, sells to end consumers during a single period. Each unit of the product is produced by the manufacturer at a unit production cost $c(\geq 0)$ and is sold to consumers by the retailer for an exogenous unit retail price $r(\geq c)$. The manufacturer first decides the unit wholesale price w to charge the retailer. The retailer then decides the order quantity q before consumer demand is realized. See Figure 1 for a graphical illustration of the setting and the key parameters.

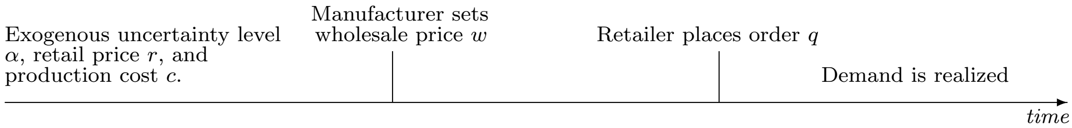

Figure 1 The model setting and the sequence of events for the exogenous retail price case.

The demand is $D(\alpha) = \alpha X + (1-\alpha)\mu$, where $\alpha \in [0,1]$ and $\mu = \mathrm{E}[X]$ (Rothschild and Stiglitz 1970, Gerchak and Mossman 1992). Thus, $\mathrm{E}[D(\alpha)] = \mathrm{E}[X]$, but $Var[D(\alpha)] = \alpha^2 Var[X] \leq Var[X]$. Demand X is a continuous random variable with a distribution function $\Phi(\cdot)$, a complementary distribution function $\bar{\Phi}(\cdot) = 1 - \Phi(\cdot)$, and a probability density function $\phi(\cdot)$. We assume that demand X is IFR (increasing failure rate), i.e., that $h(x) := \phi(x)/\bar{\Phi}(x)$ is a non-decreasing function. IFR is an assumption that is widely applied in the operations management literature because, among other benefits, it imposes mild restrictions on demand models (Wang et al. 2004, Kocabıyıkoğlu and Popescu 2011). Examples of IFR distributions prevalent in the related literature include uniform, truncated exponential and truncated normal, as well as the power, gamma and Weibull distributions, subject to parameter restrictions (Barlow et al. 1996).

For a given wholesale price w, the expected profit of the retailer is $r\mathrm{E}[D(\alpha) \wedge q] - wq$, where $x \wedge y = \min\{x, y\}$. Accordingly, define a variable z such that $q = \alpha z + (1-\alpha)\mu$ so that the retailer's problem reduces to $\pi_R(\alpha) = (1-\alpha)(r-w)\mu + \alpha \sup_{z \geq 0}[r\mathrm{E}(z \wedge X) - wz]$. Then, for a given w, the retailer's optimal z is independent of α and the correspondingly optimal order quantity is $q = \alpha z + (1-\alpha)\mu = \alpha \bar{\Phi}^{-1}(w/r) + (1-\alpha)\mu$. Thus, the manufacturer's problem can be written as

$$\pi_M(\alpha) := \sup_{z \in [0, \bar{\Phi}^{-1}(k)]} \pi_M(\alpha, z) = \sup_{z \in [0, \bar{\Phi}^{-1}(k)]} r\left(\bar{\Phi}(z) - k\right)[\alpha z + (1-\alpha)\mu], \tag{1}$$

where $k := c/r$ is the system fractile (ratio of production cost to retail price).

Proposition 1. *Let $\bar{\alpha} := \frac{\mu\phi(0)}{1-k+\mu\phi(0)}$. Moreover, let $z(\alpha)$ denote the optimal solution to* (1) *for any given α. Then, $z(\alpha)$ exists, is unique, and is non-decreasing in α. In particular, $z(\alpha) = 0$ for $\alpha \leq \bar{\alpha}$, and $z(\alpha)$ is the unique solution to the following equation for $\alpha > \bar{\alpha}$: $\bar{\Phi}(z) - \frac{(1-\alpha)\mu + \alpha z}{\alpha}\phi(z) = k$.*

Proposition 1 shows that there exists a threshold level of uncertainty (namely, $\bar{\alpha}$) below which $z(\alpha) = 0$. As a result, if $\alpha \leq \bar{\alpha}$, then the manufacturer sets its wholesale price equal to r, thus, converting, in effect, the DC into a centralized one and capturing all of the surplus from the system. If $\alpha > \bar{\alpha}$, then the manufacturer's resulting optimal wholesale price $w(\alpha) = r\bar{\Phi}(z(\alpha))$ decreases in the uncertainty level. Intuitively, this is because the retailer will order a purchase quantity that is closer to the demand mean the less is the uncertainty surrounding its random demand (Gerchak and Mossman 1992). As a result, the lower is the market uncertainty, the higher is the manufacturer's pricing power (over the retailer). Ultimately, this means that UR in the decentralized system stimulates an intra-chain tradeoff that leads to the following proposition.

PROPOSITION 2. *Let* $G := \bar{\Phi}(\mu) - \mu\phi(\mu)$ *and let* $\alpha_M := \frac{\mu\phi(\mu)}{\bar{\Phi}(\mu) - k}$. *If* $G < k$, *then* $\pi_M(\alpha)$ *is decreasing for all* α. *However, if* $G \geq k$, *then* $\pi_M(\alpha)$ *is decreasing for* $\alpha < \alpha_M$, *but it is increasing for* $\alpha \geq \alpha_M$.

Thus, the qualitative impact of UR on the manufacturer's optimal profit boils down to an unambiguous, but distribution-specific, critical fractile test: If $G < k$ is true, then it means that the manufacturer always would benefit from UR. However, if $G < k$ is *not* true, then it means that the manufacturer would not necessarily benefit from UR. In particular, if the system fractile is relatively low ($k < G$) while the demand uncertainty is relatively high ($\alpha > \alpha_M$), then UR will lead the retailer to reduce its safety stock, which has a negative effect on the manufacturer's profit, ceteris paribus.

Given Proposition 1, the retailer's equilibrium expected profit is

$$\begin{aligned}\pi_R(\alpha) &= (1-\alpha)r\mu\Phi(z(\alpha)) + \alpha r\mathrm{E}[(z(\alpha) \wedge X) - z(\alpha)\Phi^{-1}(z(\alpha))] \\ &= r\left[(1-\alpha)\mu\Phi(z(\alpha)) + \alpha\int_0^{z(\alpha)} x\phi(x)dx\right], \end{aligned} \tag{2}$$

which implies the following key technical result.

PROPOSITION 3. *Let* $H(\cdot) := \frac{\phi^2(\cdot)(\bar{\Phi}(\cdot)-k)}{2\phi^2(\cdot)+(\bar{\Phi}(\cdot)-k)\phi'(\cdot)} - \Phi(\cdot)$. *If* $H'(\cdot)|_{H(\cdot)\leq 0} \leq 0$, *then* $\pi_R(\alpha)$ *is either increasing or increasing-decreasing in* α.

Qualitatively, an increasing $\pi_R(\alpha)$ means that UR always would hurt the retailer. However, an increasing-decreasing $\pi_R(\alpha)$ means that UR would hurt the retailer only if α is less than a unique threshold value of α, namely α_R, implicitly defined by $\alpha_R := \arg\max_{\alpha\in[0,1]} \pi_R(\alpha)$.

Together Propositions 2 and 3 imply the following, depending on the distribution $\Phi(\cdot)$. If $\pi_M(\alpha)$ and $\pi_R(\alpha)$ both are monotone in α, then the manufacturer would unilaterally benefit from UR at the retailer's expense. The power distribution $\phi(x) = ax^{a-1}$ for $x \in [0,1]$ with scale parameter a satisfying $1-(a+1)^{(1-a)} \geq k$ defines one example in which this possible outcome would prevail (see §EC.2.1 of Electronic Companion for details). However, if $\pi_M(\alpha)$ is monotone while $\pi_R(\alpha)$ is non-monotone, then UR could result in either a unilaterally beneficial situation for the manufacturer, or a mutually beneficial situation for both the manufacturer and the retailer, depending on the uncertainty level α. The power distribution with scale parameter a satisfying $1-(a+1)^{(1-a)} < k$ provides one example of this possible outcome (§EC.2.1). More interestingly, if $\pi_M(\alpha)$ is *non*-monotone while $\pi_R(\alpha)$ is monotone, then not only is a unilaterally beneficial situation possible, but also is a *no-win* situation possible. We attribute this somewhat ironic situation in which neither the manufacturer nor the retailer has incentive to improve to an overall shrunken pie stemming from the amplified double marginalization effects that would be produced by the manufacturer's exertion of increased monopoly pricing power in response to the retailer's increased operational efficiency. The truncated exponential distribution $\phi(x) = \frac{\lambda e^{\lambda(a-(x-A))}}{e^{a\lambda}-1}$ for $x \in [A, A+a]$ provides one representative example in which such a no-win situation is possible (§EC.2.2). Similarly, if $\pi_M(\alpha)$ and $\pi_R(\alpha)$ both are non-monotone, then multiple permutations are possible, depending on the relative magnitudes of α_M and α_R: If $\alpha_R < \alpha_M$, then UR would be mutually beneficial (see Figure 2a). However if $\alpha_M < \alpha_R$, then the band of uncertainty levels between the two thresholds only serves to provide the negative reinforcement that fuels the status quo (see Figure 2b). See §EC.2.2 for an example of each of these cases.

3. Endogenous Retail Price Case

In §2, we demonstrated, for the case of exogenous retail price case, that UR can hurt rather than benefit either of both members of the DC even if the cost of UR is zero. In this section, we next

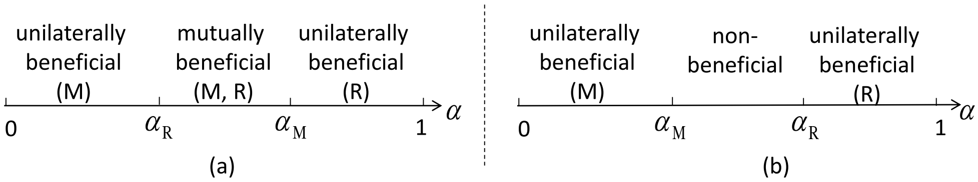

Figure 2 The would-be benefit of UR on the manufacturer (M) and retailer (R).

consider the case of an endogenous retail price to demonstrate that these results extend qualitatively if retail price effects also are taken into account. Toward that end, let random demand be redefined as follows (Li and Atkins 2005, Song et al. 2008): $D(\alpha, r) = \epsilon(\alpha) - br$, where $b > 0$ and $\epsilon(\alpha) = \alpha X + (1-\alpha)\mu$. We assume that $X \geq A$, where A is sufficiently large so that $D(\alpha, r) \geq 0$. Given $D(\alpha, r)$, we redefine z from §2 such that, in this section, $q = \alpha z + (1-\alpha)\mu - br$. Correspondingly, the retailer's problem is $\pi_R(\alpha) = \max_{r,z} \alpha r \mathrm{E}[X \wedge z] - \alpha w z + (r-w)[(1-\alpha)\mu - br]$. Thus, for any given wholesale price w, the retailer's optimal z can be characterized implicitly as $\bar{\Phi}(z) v(\alpha, z)/b = w$, where $v(\alpha, z) := \frac{\alpha E[X \wedge z] + (1-\alpha)\mu}{1 + \Phi(z)}$, and its optimal retail price can be written explictly as $r(\alpha, z) = v(\alpha, z)/b$.

Given the solution to the retailer's problem as a function of w, the manufacturer's problem of determining its optimal w can be rewritten so that the manufacturer instead maximizes its profit over z as follows: $\pi_M(\alpha) = \max_{w \in [c,r]} (w - c)[\alpha z + (1-\alpha)\mu - br] = \frac{1}{b} \max_{z \geq A, \bar{\Phi}(z) v(\alpha,z) \geq bc} \left(\bar{\Phi}(z) v(\alpha, z) - bc\right) [\alpha z + (1-\alpha)\mu - v(\alpha, z)]$, where $z \geq A$ and $\bar{\Phi}(z) v(\alpha, z) \geq bc$ correspond to $w \leq r$ and $w \geq c$, respectively. Because the manufacturer's objective function depends on c only through the product bc, we let $b = 1$ without loss of generality. Accordingly, let $z(\alpha)$ be defined as the value of z that maximizes the manufacturer's expected profit as a function of α. Then, the manufacturer's and the retailer's equilibrium expected profits are, respectively, $\pi_M(\alpha) = [\bar{\Phi}(z(\alpha)) v(z(\alpha), \alpha) - c][\alpha z(\alpha) + (1-\alpha)\mu - v(z(\alpha), \alpha)]$ and $\pi_R(\alpha) = v(z(\alpha), \alpha)[\alpha \int_A^{z(\alpha)} x \phi(x) dx + \Phi(z(\alpha))((1-\alpha)\mu - v(\alpha, z))]$. This leads to the following technical result:

Proposition 4. *If there exists any region of $\alpha \in [0, 1]$ such that $z(\alpha) \geq \mu$, then $\pi_M(\alpha)$ is increasing over that region. Regardless, if there exists an $\tilde{\alpha} \in [0, 1]$ such that $\pi_R(\tilde{\alpha}) > \frac{(\mu - c)^2}{16}$, then there exists some region of $\alpha \in [0, \tilde{\alpha}]$ such that $\pi_R(\alpha)$ is increasing in α.*

Note that $z(\alpha) \geq \mu$ indicates an equilibrium in which the retailer's safety stock is non-negative, and $\frac{(\mu-c)^2}{16}$ denotes the retailer's equilibrium expected profit for the extreme case in which there is no demand uncertainty (i.e., when $\alpha = 0$). Thus, Proposition 4 establishes that UR would not be in the manufacturer's best interest when demand uncertainty induces the retailer to carry a positive stock, and UR would not be in the retailer's best interest when demand uncertainty provides a tacit defense against the manufacturer's vertical control.

To demonstrate the applicability of Proposition 4, we turn again to the truncated exponential distribution: Let $\phi(x) = \frac{\lambda e^{\lambda(a-(x-A))}}{e^{a\lambda}-1}$ for $x \in [A, A+a]$, and suppose, for example, that $A = 5.2, \lambda = 0.1, a = 9, c = 0$. Then, $z(\alpha) \geq \mu$ for $0.9 < \alpha \leq 1$, and $\tilde{\alpha} = 1$ is one instance for which $\pi_R(\tilde{\alpha}) > \frac{(\mu-c)^2}{16}$ (see §EC.3 for details). Thus, analogues to the results of §2, Proposition 4 implies that either the manufacturer, the retailer, or potentially both could be hurt by UR. Figure 3 completes this illustration by providing a plot of $\pi_M(\alpha)$ as well as of $\pi_R(\alpha)$. Notice that $\pi_M(\alpha)$ first decreases and then increases in α, whereas $\pi_R(\alpha)$ first increases and then decreases in α. Thus, as in the case of Figure 2b, UR would be unilaterally beneficial to the manufacturer for relatively low values of α, it would be unilaterally beneficial to the retailer for relatively high values of α, and it would be beneficial to neither for some range of α in between.

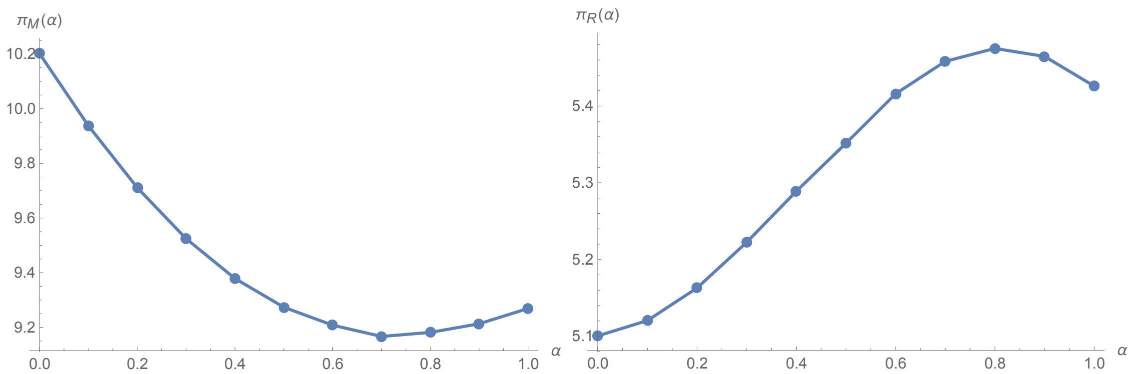

Figure 3 **Plots of $\pi_M(\alpha)$ and $\pi_R(\alpha)$ for the truncated exponential distribution ($\phi(x) = \frac{\lambda e^{\lambda(a-(x-A))}}{e^{a\lambda}-1}$) when $A = 5.2, \lambda = 0.1, a = 9, c = 0$.**

4. Concluding Remarks

We conclude with a brief discussion of two extensions. First, we defined UR to mean an incremental reduction in α. If we instead define it to mean a discrete reduction from α_1 to α_2, where

$\alpha_2 < \alpha_1$, then the spirit of our results would continue to hold because the functional shapes of $\pi_M(\alpha)$ and $\pi_R(\alpha)$ established by Propositions 1-4 would be unaffected. Second, we employed an additive demand model in our analysis of the endogenous retail price case. If we instead employ the multiplicative case $D(\alpha) = \epsilon(\alpha)e^{-br}$ (Song et al. 2008), then we again would find that UR can lead to a non-beneficial outcome for either the manufacturer, the retailer, or both if, for example, X is characterized by a truncated exponential distribution (see §EC.4 for details).

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References

Barlow, R. E., F. Proschan, L. C. Hunter. 1996. *Mathematical Theory of Reliability*. Society for Industrial and Applied Mathematics, Philadelphia, PA.

Cavusoglu, H., H. Cavusoglu, S. Raghunathan. 2012. Value of and interaction between production postponement and information sharing strategies for supply chain firms. *Production and Operations Management* **21**(3) 470–488.

Corbett, C.J., K. Rajaram. 2006. A generalization of the inventory pooling effect to nonnormal dependent demand. *Manufacturing & Service Operations Management* **8**(4) 351–358.

Dong, L., N. Rudi. 2004. Who benefits from transshipment? Exogenous vs. endogenous wholesale prices. *Management Science* **50**(5) 645–657.

Ebrahim-Khanjari, N., S. Iravani, H. Shin. 2013. The impact of the manufacturer-hired sales agent on a supply chain with information asymmetry. *Manufacturing & Service Operations Management* **16**(1) 76–88.

Eppen, G. D. 1979. Effects of centralization on expected costs in a multi-location newsboy problem. *Management Science* **25**(5) 498–501.

Gerchak, Y., D. Mossman. 1992. On the effect of demand randomness on inventories and costs. *Operations Research* **40**(4) 804–807.

Iyer, A.V., M. E. Bergen. 1997. Quick response in manufacturer-retailer channels. *Management Science* **43**(4) 559–570.

Kocabıyıkoğlu, A., I. Popescu. 2011. An elasticity approach to the newsvendor with price-sensitive demand. *Operations Research* **59**(2) 301–312.

Lariviere, M. A., E. L. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management* **3**(4) 293–305.

Lau, H-L., H-S. Lau. 2001. Some two-echelon style-goods inventory models with asymmetric market information. *European Journal of Operational Research* **134**(1) 29–42.

Lau, H-L., H-S. Lau. 2002. The effects of reducing demand uncertainty in a manufacturer–retailer channel for single-period products. *Computers & Operations Research* **29**(11) 1583–1602.

Leland, H. E. 1972. Theory of the firm facing uncertain demand. *The American Economic Review* **62**(3) 278–291.

Li, Q., D. Atkins. 2005. On the effect of demand randomness on a price/quantity setting firm. *IIE Transactions* **37**(12) 1143–1153.

Miyaoka, J., W. H. Hausman. 2008. How improved forecasts can degrade decentralized supply chains. *Manufacturing & Service Operations Management* **10**(3) 547–562.

Rothschild, M., J. E. Stiglitz. 1970. Increasing risk: I. a definition. *Journal of Economic theory* **2**(3) 225–243.

Shi, R., J. Zhang, J. Ru. 2013. Impacts of power structure on supply chains with uncertain demand. *Production and Operations Management* **22**(5) 1232–1249.

Song, Y., S. Ray, S. Li. 2008. Structural properties of buyback contracts for price-setting newsvendors. *Manufacturing & Service Operations Management* **10**(1) 1–18.

Taylor, T. A., W. Xiao. 2010. Does a manufacturer benefit from selling to a better-forecasting retailer? *Management Science* **56**(9) 1584–1598.

Wang, Y., L. Jiang, Z. M. Shen. 2004. Channel performance under consignment contract with revenue sharing. *Management Science* **50**(1) 34–47.

Xu, M., Y. F. Chen, X. Xu. 2010. The effect of demand uncertainty in a price-setting newsvendor model. *European Journal of Operational Research* **207**(2) 946–957.

Appendix

Proof of Proposition 1. We first show that $\pi_M(\alpha, z) := r\left(\bar{\Phi}(z) - k\right)[\alpha z + (1-\alpha)\mu]$ is either decreasing in z or increasing-decreasing in z for any given α. We make two observations. First, $\frac{\partial \pi_M(\alpha,z)}{\partial z}\Big|_{z=\Phi^{-1}(k)} = -r[\alpha z + (1-\alpha)\mu]\phi(z) \leq 0$ because $\frac{\partial \pi_M(\alpha,z)}{\partial z} = \alpha r[\bar{\Phi}(z) - k] - r[\alpha z + (1-\alpha)\mu]\phi(z)$. Thus, for any given α, $\pi_M(\alpha, z)$ "ends up" decreasing in z. Second, $\frac{\partial \pi_M(\alpha,z)}{\partial z} = r\bar{\Phi}(z)\left[\alpha - [\alpha z + (1-\alpha)\mu]\frac{\phi(z)}{\bar{\Phi}(z)}\right] - \alpha r k$ and $\frac{1}{r}\frac{\partial^2 \pi_M(\alpha,z)}{\partial z^2}|_{\frac{\partial \pi_M(\alpha,z)}{dz}=0} = \frac{-\phi(z)}{\bar{\Phi}(z)}[\frac{d\pi_M(\alpha,z)}{dz} + \alpha k] - [\alpha z + (1 -$

$\alpha)\mu]\bar{\Phi}(z)h'(z) - \alpha\phi(z) = \frac{-\phi(z)}{\bar{\Phi}(z)}\alpha k - [\alpha z + (1-\alpha)\mu]\bar{\Phi}(z)h'(z) - \alpha\phi(z) \leq 0$, where the inequality is from the IFR assumption. From the above two observations, we conclude that $\pi_M(\alpha, z)$ is either decreasing or increasing-decreasing in z for any given α. Consequently, $\frac{d\pi_M(\alpha,z)}{dz} \leq 0$ for all $z \geq 0$ if and only if $\frac{\partial \pi_M(\alpha,z)}{\partial z}\Big|_{z=0} = \alpha(1-k) - (1-\alpha)\mu\phi(0) \leq 0$. Accordingly, if $\alpha > \bar{\alpha}$, then $z(\alpha)$ exists as the unique solution to

$$\bar{\Phi}(z) - \frac{(1-\alpha)\mu + \alpha z}{\alpha}\phi(z) = k. \tag{3}$$

Otherwise, if $\alpha \leq \bar{\alpha}$, then $z(\alpha) = 0$. Next, by taking the derivative with respect to α on both sides of (3), we have $[-2\phi(z) - z\phi'(z) + \mu\phi'(z)]z'(\alpha) - \frac{\mu}{\alpha}\phi'(z)z'(\alpha) + \frac{\mu\phi(z)}{\alpha^2} = 0$. This implies that $z'(\alpha) = \frac{\phi(z)\mu}{\alpha + \phi(z)\left[(\alpha - h(z)(\mu - \alpha\mu + \alpha z)) + \bar{\Phi}(z)[\alpha h(z) + (\mu - \alpha\mu + \alpha z)h'(z)]\right]} \geq 0$ where the inequality follows because $1 - h(z)\left(\frac{\mu - \alpha\mu + \alpha z}{\alpha}\right) = \frac{k}{\bar{\Phi}(z)} \geq 0$ from (3). Q.E.D.

Proof of Proposition 2. From the envelope theorem, $\frac{d\pi_M(\alpha)}{d\alpha} = \frac{\partial \pi_M(\alpha,z)}{\partial \alpha}|_{z=z(\alpha)} = r[\bar{\Phi}(z(\alpha)) - k](z(\alpha) - \mu)$. Because $\bar{\Phi}(z(\alpha)) \geq k$, the sign of $\frac{d\pi_M(\alpha)}{d\alpha}$ is solely determined by $z(\alpha) - \mu$. When $\alpha = 0$, the manufacturer's profit is $(r-c)\mu$, which is the upper bound of the manufacturer's profit. Thus, $\pi_M(\alpha)$ starts out decreasing in α. Because $z(\alpha)$ is an increasing function from Proposition 1, the sign of $\pi'_M(\alpha)$ changes at most once, from negative to positive. Therefore, $\pi_M(\alpha)$ either is decreasing or is decreasing-increasing in α. Moreover, $\pi_M(\alpha)$ is decreasing-increasing in α if and only if $\hat{z} := z(\alpha)|_{\alpha=1} > \mu$, where the equilibrium $\hat{z}$ satisfies $\mathcal{G}(z) := \bar{\Phi}(z) - z\phi(z) = k$ from (3).

We now study the case $\alpha = 1$ and show an *equivalence characterization* of the relationship between the equilibrium $\hat{z}$ and any nonnegative real number: For any $x \geq 0$, $x < \hat{z} \Longleftrightarrow \mathcal{G}(x) > k$. Let $g(z) := zh(z)$ and notice that $\mathcal{G}'(z) = -g'(z)\bar{\Phi}(z) - (1-g(z))\phi(z) = -g'(z)\bar{\Phi}(z) - \frac{\bar{\Phi}(z) - z\phi(z)}{\bar{\Phi}(z)}\phi(z)$. Therefore, if $\mathcal{G}(z) = \bar{\Phi}(z) - z\phi(z) \geq 0$, then $\mathcal{G}'(z) \leq 0$ because $h'(z) \geq 0$, $\phi(z) \geq 0$ and $\bar{\Phi}(z) \geq 0$. Moreover, because $\mathcal{G}(\hat{z}) = k \geq 0$, it follows that $\hat{z} < \tilde{z}$ where $\tilde{z}$ is the smallest z such that $\mathcal{G}(z) = 0$, i.e., $\tilde{z} := \inf_{z \geq 0}\{\mathcal{G}(z) = 0\}$. From the above property of $\mathcal{G}(z)$, $\hat{z} > x$ is equivalent to $\mathcal{G}(\hat{z}) = k < \mathcal{G}(x)$ if $\hat{z} < \tilde{z}$. We next show that $\hat{z} < x \Longrightarrow k > \mathcal{G}(x)$. There are two cases: $x < \tilde{z}$ and $x \geq \tilde{z}$. If $x < \tilde{z}$, then $\mathcal{G}(\hat{z}) > \mathcal{G}(x)$ holds from the property of $\mathcal{G}(z)$ as $z \geq 0$. If $x \geq \tilde{z}$, then $\mathcal{G}(x) \leq 0$; thus, $\mathcal{G}(\hat{z}) = k > \mathcal{G}(x)$. We now prove that $k > \mathcal{G}(x) \Longrightarrow \hat{z} < x$. Suppose that $\hat{z} \geq x$. Then $\mathcal{G}(\hat{z}) \leq \mathcal{G}(x)$ contradicts the fact that $\mathcal{G}(\hat{z}) = k > \mathcal{G}(x)$; thus, $\hat{z} < x$ holds. From the above equivalence result, $\hat{z} > \mu \Longleftrightarrow k < \mathcal{G}(\mu) = \bar{\Phi}(\mu) - \mu\phi(\mu)$. We then can conclude. Q.E.D.

Proof of Proposition 3. In this proof, without a qualifier, z means the equilibrium $z(\alpha)$. From

(2), $\frac{\pi_R(\alpha)}{r} = \alpha[\frac{(1-\alpha)\mu+\alpha z}{\alpha}\Phi(z) - \int_0^z (z-x)\phi(x)dx]$. Hence, $\frac{\pi'_R(\alpha)}{r} = \frac{(1-\alpha)\mu+\alpha z}{\alpha}\Phi(z) - \int_0^z (z-x)\phi(x)dx - \frac{\mu\Phi(z)}{\alpha} + \alpha z'(\alpha)\frac{(1-\alpha)\mu+\alpha z}{\alpha}\phi(z) = \frac{\pi_R(\alpha)}{r\alpha} + \frac{\phi(z)\alpha^2 z'(\alpha)\frac{(1-\alpha)\mu+\alpha z}{\alpha} - \mu\Phi(z)}{\alpha} = \frac{\pi_R(\alpha)}{r\alpha} + \frac{\alpha^2 z'(\alpha)(\bar{\Phi}(z)-k) - \mu\Phi(z)}{\alpha}$, where the second equality is from (3). So, $\frac{\pi''_R(\alpha)}{r}|_{\pi'_R(\alpha)=0} = \frac{1}{\alpha}\frac{d}{d\alpha}[\alpha^2 z'(\alpha)(\bar{\Phi}(z) - k) - \mu\Phi(z)] = \frac{\mu}{\alpha}\frac{d}{d\alpha}[\frac{\phi^2(z)(\bar{\Phi}(z)-k)}{2\phi^2(z)+(\bar{\Phi}(z)-k)\phi'(z)} - \Phi(z)] = \frac{\mu}{\alpha}\frac{dH(z)}{d\alpha}$. Note that $z'(\alpha) \geq 0$ from Proposition 1. Hence, $H'(z) \leq 0 \Longleftrightarrow \pi''_R(\alpha)|_{\pi'_R(\alpha)=0} \leq 0$. If $\pi'_R(\alpha) = 0$, then $H(z) \leq 0$ because $\frac{\pi_R(\alpha)}{r\alpha} \geq 0$. Thus, if $H'(\cdot)|_{H(\cdot)\leq 0} \leq 0$, then $\pi_R(\alpha)$ is either increasing or increasing-decreasing in α. Q.E.D.

Proof of Proposition 4. For the manufacturer's profit, $\pi_M(\alpha) = \max_{w\in[c,r]} \pi_M(\alpha, w) = \max_{w\in[c,r]}(w-c)[\alpha z + (1-\alpha)\mu - r] = \max_{w\in[c,r]}(w-c)[\alpha z + (1-\alpha)\mu - \frac{\alpha E[X\wedge z]+(1-\alpha)\mu+w}{2}]$. Thus, from the envelope theorem, $\frac{d\pi_M(\alpha)}{d\alpha} = \frac{\partial \pi_M(\alpha,z)}{\partial\alpha} = (w-c)(z - \mu - \frac{E[X\wedge z]-\mu}{2})|_{w=\arg\max_{w\in[c,r]}\pi_M(\alpha,z)}$. Consequently, if $z(\alpha) \geq \mu$, then $\frac{d\pi_M(\alpha)}{d\alpha} > 0$ because $w > c$ and $\frac{E[X\wedge z]-\mu}{2} \leq 0$.

We now prove the sufficiency test for the retailer. When $\alpha = 0$, the manufacturer's problem is $\pi_M(\alpha) = \max_w (w-c)(\mu - r) = \max_w (w-c)(\mu - \frac{\mu+w}{2})$. As a result, the manufacturer's equilibrium profit is $\pi_M(\alpha = 0) = \frac{(\mu-c)^2}{8}$ with the associated optimal wholesale price $w = \frac{c+\mu}{2}$. Furthermore, the retailer's profit is $\pi_R(\alpha = 0) = (r-w)(\mu - r) = \frac{(\mu-c)^2}{16}$. Thus, from the continuity of $\pi_R(\alpha)$, if there exists $\tilde{\alpha}$ such that $\pi(\tilde{\alpha}) < \frac{(u-c)^2}{16}$, then $\frac{d\pi_R(\alpha)}{d\alpha} > 0$ for some region of α such that $\alpha \in [0, \tilde{\alpha}]$. Q.E.D.

Electronic Companion To "Technical Note – Demand Uncertainty Reduction in Decentralized Supply Chains"

This technical companion provides a summary of notation in EC.1, followed by referenced examples of the exogenous retail price case in §EC.2, the referenced example of the endogenous retail price case in §EC.3, and the referenced extension in §EC.4.

EC.1. Summary of Notation

c, r, k: The unit production cost, unit retail price, and system fractile (c/r), respectively.

w, q: The manufacturer's unit wholesale price, and the retailer's order quantity, respectively.

$\alpha \in [0,1]$: The uncertainty level.

$X, \mu = \mathrm{E}[X]$: A random variable, and its expected value, respectively.

$\phi(\cdot), \Phi(\cdot), \bar{\Phi}(\cdot)$: The pdf, CDF, and complementary CDF of X, respectively.

$h(x) = \phi(x)/\bar{\Phi}(x)$: The failure rate of X.

$\pi_M(\alpha), \pi_R(\alpha)$: The manufacturer's and the retailer's equilibrium profit, respectively, for a given α.

α_M, α_R: $\alpha_M := \arg\min_{\alpha\in[0,1]} \pi_M(\alpha)$, $\alpha_R := \arg\max_{\alpha\in[0,1]} \pi_R(\alpha)$.

$z, G, H(\cdot)$: $z = \frac{q-(1-\alpha)\mu}{\alpha}$, $G := \bar{\Phi}(\mu) - \mu\phi(\mu), H(\cdot) := \frac{\phi^2(\cdot)(\bar{\Phi}(\cdot)-k)}{2\phi^2(\cdot)+(\bar{\Phi}(\cdot)-k)\phi'(\cdot)} - \Phi(\cdot)$.

EC.2. Referenced Examples of Exogenous Retail Price Case

EC.2.1. Power Distribution

Let $\phi(x) = ax^{a-1}$ for $x \geq 0$ and $a \geq 1$. Then, $G = \bar{\Phi}(\mu) - \mu\phi(\mu) \leq 0$, which, from Proposition 2, implies that $\pi_M(\alpha)$ is decreasing in α. Moreover, $H(z) = -\frac{x^a(k-1+z^a+2az^a)}{a+k-ak+z^a+az^a-1}$, which implies that $H(z) \leq 0$ if and only if $z^a + 2az^a \geq 1-k$. Thus, $H'(z)|_{H(z)\leq 0} = -\frac{az^{-1+a}[2(a-1)(1+2a)(1-k)z^a+(1+a)(1+2a)z^{2a}-(a-1)(1-k)^2]}{[k-1+z^a+a(1-k+z^a)]^2}\Big|_{(1+2a)z^a\geq 1-k} \leq -\frac{az^{-1+a}[(a-1)(1-k)^2]}{[k-1+z^a+a(1-k+z^a)]^2} \leq 0$. As a result, from Proposition 3, $\pi_R(\alpha)$ is increasing in α if $\pi_R'(\alpha)|_{\alpha=1} \geq 0$ and it is increasing-decreasing in α if $\pi_R'(\alpha)|_{\alpha=1} < 0$. Accordingly, note that $\pi_R'(\alpha)|_{\alpha=1} = \frac{a(kz)^a[(1+a)kz-1]}{(1+a)^2k} \geq 0$ if and only if $z|_{\alpha=1} \geq \frac{1}{(1+a)k}$, which holds, from the equivalence property in the proof of Proposition 2, if and only if $k \leq G(\frac{1}{(1+a)k}) = 1-(1+a)^{1-a}$. Thus, if $1-(a+1)^{(1-a)} \geq k$, then $\pi_R(\alpha)$ is increasing in α, and if $k > 1-(1+a)^{1-a}$, then $\pi_R(\alpha)$ is increasing-decreasing in α.

EC.2.2. Truncated Exponential Distribution

Let $\phi(x)=\frac{\lambda e^{\lambda(a-x)}}{e^{a\lambda}-1}$ for $x\in[0,a]$ and $\lambda\geq 0$. Then, $G=\bar{\Phi}(\mu)-\mu\phi(\mu)>0$ if, for example, $\theta:=a\lambda=2$. Thus, $G<k$ is not necessarily true. Hence, from Proposition 2, $\pi_M(\alpha)$ is decreasing-increasing in α for any $G\geq k$. Moreover, $H(z)=\frac{2e^{2a\lambda-z\lambda}+e^{(a+z)\lambda}(k-1)-e^{(2a+z)\lambda}k-e^{2a\lambda}}{(e^{a\lambda}-1)(e^{a\lambda}+e^{z\lambda}(1-k)+e^{(a+z)\lambda}k)}$, which implies that $H'(z)=-\frac{2e^{2a\lambda}[2(1-k)+e^{(a-z)\lambda}+2e^{a\lambda}k]\lambda}{(e^{a\lambda}-1)[e^{a\lambda}+e^{z\lambda}(1-k)+e^{(a+z)\lambda}k]^2}\leq 0$ for any z. Thus, from Proposition 3, $\pi_R(\alpha)$ is either increasing or increasing-decreasing in α. Accordingly, from (2),

$$\begin{aligned}
L(k):=\left.\frac{\pi_R'(\alpha)}{r}\right|_{\alpha=\alpha_M} &= \int_0^\mu x\phi(x)dx-\mu\Phi(\mu)+\phi(\mu)\mu z'(\alpha)\\
&= \int_0^\mu (x-\mu)\phi(x)dx+\frac{\phi^2(\mu)\mu^2}{\alpha_M^2[2\phi(\mu)+\mu\phi'(\mu)]+(1-\alpha_M)\mu\phi'(\mu)\alpha_M}\\
&= \int_0^\mu (x-\mu)\phi(x)dx+\frac{(\bar{\Phi}(\mu)-k)^2\phi(\mu)}{2\phi^2(\mu)+\phi'(\mu)(\bar{\Phi}(\mu)-k)},
\end{aligned}$$

where the second and third equalities are from Proposition 1 and Proposition 2, respectively. Moreover,

$$\begin{aligned}
\frac{d}{dk}\left[\frac{(\bar{\Phi}(\mu)-k)^2\phi(\mu)}{2\phi^2(\mu)+\phi'(\mu)(\bar{\Phi}(\mu)-k)}\right] &= \frac{-2(\bar{\Phi}(\mu)-k)\phi(\mu)[2\phi^2(\mu)+\phi'(\mu)(\bar{\Phi}(\mu)-k)]+\phi(\mu)\phi'(\mu)(\bar{\Phi}(\mu)-k)^2}{[\phi^2(\mu)+\phi'(\mu)(\bar{\Phi}(\mu)-k)]^2}\\
&= \frac{\phi(\mu)(\bar{\Phi}(\mu)-k)[-4\phi^2(\mu)-\phi'(\mu)(\bar{\Phi}(\mu)-k)]}{[\phi^2(\mu)+\phi'(\mu)(\bar{\Phi}(\mu)-k)]^2}<0,
\end{aligned}$$

where the inequality is from $2\phi^2(\mu)+\phi'(\mu)(\bar{\Phi}(\mu)-k)\geq 0$ as a result of $z'(\alpha)\geq 0$ (Proposition 1). This implies that the sign of $\left.\frac{\pi_R'(\alpha)}{r}\right|_{\alpha=\alpha_M}$ switches at most once from positive to negative for a value of $k\in[0,G]$. Note that $L(k=0)=\int_0^\mu(x-\mu)\phi(x)dx+\frac{\bar{\Phi}^2(\mu)\phi(\mu)}{2\phi^2(\mu)+\phi'(\mu)\bar{\Phi}(\mu)}$ and $L(k=G)=\int_0^\mu(x-\mu)\phi(x)dx+\frac{\mu^2\phi^2(\mu)}{2\phi(\mu)+\mu\phi'(\mu)}$. Suppose that $\theta=2$. Then, from Mathematica 9.0, $L(k=0)=\frac{-e+3e^{\theta(1+\frac{1}{e^\theta-1})}-e^{\theta(2+\frac{1}{e^\theta-1})}(3-\theta)+e^{1+\theta}(1+\theta)}{(e^\theta-1)^2(e+e^{\theta(1+\frac{1}{e^\theta-1})})\lambda}>0$ because $(e^\theta-1)^2(e+e^{\theta(1+\frac{1}{e^\theta-1})})\lambda>0$, $e^{1+\theta}(1+\theta)-e>0$, and $3e^{\theta(1+\frac{1}{e^\theta-1})}-e^{\theta(2+\frac{1}{e^\theta-1})}(3-\theta)=e^{\frac{\theta}{e^\theta-1}}e^\theta(3-2e^\theta+\theta e^\theta)>0$. Similarly, from Mathematica 9.0, $L(k=G)=\frac{ae^\theta(e^{1+\theta}-e+\theta e+(3+\theta)e^{\frac{\theta}{e^\theta-1}}-3e^{\theta+\frac{\theta}{e^\theta-1}})}{e(e^\theta-1)^2(e^\theta-1+\theta)}<0$ because $e^{1+\theta}-e+\theta e+(3+\theta)e^{\frac{\theta}{e^\theta-1}}-3e^{\theta+\frac{\theta}{e^\theta-1}}<0$ for $\theta>0$. Given that $L(k=G)<0<L(k=0)$, the sign of $L(k)$ switches from positive to negative at some k between $k=0$ and $k=G$, namely at $k=0.018$. Therefore, applying Propositions 2-3 to this example, $\pi_M(\alpha)$ is non-monotone if and only if $k\leq G=0.026$; and α_M and α_R are such that either one could be larger in magnitude than the other: if $k\in[0,0.018)$, then $\alpha_R<\alpha_M$ (Figure 2a); if $k\in[0.018,0.026]$, then $\alpha_R\geq\alpha_M$ (Figure 2b).

EC.3. Referenced Example of Endogenous Retail Price Case

Let $\phi(x) = \frac{\lambda e^{\lambda(a-(x-A))}}{e^{a\lambda}-1}$ for $x \in [A, A+a]$ and $\lambda \geq 0$. Consider the special case in which $A = 5.2, \lambda = 0.1, a = 9$ and $c = 0$. Then, from Mathematica 9.0, for any given α, the equilibrium z satisfies $2e^{\frac{z}{5}+\frac{116}{25}}[\alpha^2(25z^2-1235z+22496)+5\alpha(425z-16632)+36252]-4e^{\frac{3z}{10}+\frac{103}{25}}[\alpha^2(25z^2-1735z+25346)+\alpha(1555z-36906)+10640]-e^{\frac{z}{5}+\frac{187}{50}}[5\alpha^2(25z^2-1870z+36113)+4\alpha(1780z-67791)+99099]-e^{\frac{3z}{10}+\frac{71}{50}}(\alpha^2(25z^2-1960z+35291)+242\alpha(5z-196)+14641)+e^{\frac{z}{5}+\frac{71}{25}}(\alpha^2(75z^2-4880z+81673)+242\alpha(15z-488)+43923)+2e^{\frac{3z}{10}+\frac{161}{50}}(\alpha^2(100z^2-5165z+66929)+\alpha(3785z-70642)+20949)+e^{\frac{3z}{10}+\frac{58}{25}}[\alpha^2(1430z+11817-75z^2)-4\alpha(415z+6117)+3751]-8e^{\frac{z}{5}+\frac{277}{50}}[50\alpha^2(5z-101)+19\alpha(5z-176)+2166]+15200\alpha e^{\frac{2(z+9)}{5}}+50e^{142/25}\alpha[\alpha(5z-221)+121]+50\alpha e^{\frac{1}{50}(5z+213)}[\alpha(5z-221)+121]-50e^{329/50}\alpha[\alpha(5z-176)+76]+200\alpha e^{\frac{2z}{5}+\frac{9}{5}}[3\alpha(5z-106)+299]-200\alpha e^{\frac{2z}{5}+\frac{27}{10}}[\alpha(10z-197)+273]+100\alpha e^{\frac{1}{50}(5z+303)}[\alpha(15z-478)+190]-50\alpha e^{\frac{2z}{5}+\frac{9}{10}}[\alpha(20z-439)+363]-50\alpha e^{\frac{1}{50}(5z+258)}[\alpha(35z-1357)+681]+304e^{\frac{3z}{10}+\frac{251}{50}}[\alpha(5z-126)+38]=0$. This implies that $z > \mu = \frac{239}{45} - \frac{9}{e^{9/10}-1}$ for $\alpha \in [0.9, 1]$, and $z|_{\alpha=1} = 9.1784$. As a result, the retailer's equilibrium profit $\pi_R(\alpha = 1) = 5.415 > \frac{1}{16}(\frac{239}{45} - \frac{9}{e^{9/10}-1})^2 = \frac{(\mu-c)^2}{16}$.

EC.4. Referenced Multiplicative Demand Extension

In this section, let demand be defined as follows: $D(\alpha) = \epsilon(\alpha)e^{-br}$, where $\epsilon(\alpha) = \alpha X + (1-\alpha)\mu$ and $X \geq 0$. Similar to §3, define z such that $q = [\alpha z + (1-\alpha)\mu]e^{-br}$. Then, for any w, the retailer's problem is

$$\pi_R(\alpha) = \max_{r,z} e^{-br}[\alpha[r\mathrm{E}(X \wedge z) - wz] + (1-\alpha)(r-w)\mu]. \tag{EC.1}$$

First, for any given w and r, the optimal z satisfies $e^{-br}\alpha(r\bar{\Phi}(z) - w) = 0$. Thus, $\bar{\Phi}(z) = w/r$. Second, for any given w and z, the optimal r is $r = \frac{1}{b} + \frac{w[\alpha z+(1-\alpha)\mu]}{\alpha E[X \wedge z]+(1-\alpha)\mu}(> w)$. Thus, from these two properties and some algebra, for any given w, the retailer's optimal z satisfies $\bar{\Phi}(z)\tau(\alpha,z)/b = w$, where $\tau(\alpha,z) := \frac{\alpha E[X\wedge z]+(1-\alpha)\mu}{\alpha\int_0^z x\phi(x)dx+(1-\alpha)\mu\Phi(z)}$, and the retailer's optimal r is $r(\alpha,z) = \tau(\alpha,z)/b$.

Consequently, the manufacturer's equilibrium profit is $\pi_M(\alpha) = \max_{w\in[c,r]}(w-c)[\alpha z + (1-\alpha)\mu]e^{-br} = \frac{1}{b}\max_{z\geq 0, \bar{\Phi}(z)\tau(\alpha,z)\geq bc}\left(\bar{\Phi}(z)\tau(\alpha,z) - bc\right)[\alpha z(\alpha) + (1-\alpha)\mu]e^{-\tau(\alpha,z)}$. Without loss of generality, let $b = 1$. Then, the manufacturer's and the retailer's equilibrium profits are as follows

$$\pi_M(\alpha) = \left[\bar{\Phi}(z(\alpha))\tau(z(\alpha),\alpha) - c\right] \left[\alpha z(\alpha) + (1-\alpha)\mu\right] e^{-\tau(z(\alpha),\alpha)}, \tag{EC.2}$$

$$\pi_R(\alpha) = [\alpha \mathrm{E}(X \wedge z(\alpha)) + (1-\alpha)\mu] e^{-\tau(z(\alpha),\alpha)}, \tag{EC.3}$$

where

$$z(\alpha) = \underset{z \geq 0; \bar{\Phi}(z)\tau(\alpha,z) \geq c}{\arg\max} \left(\bar{\Phi}(z)\tau(\alpha,z) - c\right) [\alpha z + (1-\alpha)\mu] e^{-\tau(\alpha,z)}. \tag{EC.4}$$

We next show that UR could hurt either the manufacturer, the retailer, or both of them. Toward that end, let $\phi(x) = \frac{\lambda e^{\lambda(a-x)}}{e^{a\lambda}-1}$ for $x \in [0,a]$. Then, from (EC.2)-(EC.4),

$$\begin{aligned}
\pi_M(\alpha) &= (1-\alpha+\alpha z)\left[\frac{1-e^{\lambda(a-z)}[e^{\lambda(z-a)}[\lambda(a+z)+1-\alpha(a\lambda+1)]+\alpha+(\alpha-1)\lambda z - e^{\lambda z}]}{\lambda(-\alpha a + a + \alpha z) + e^{\lambda z}(a(\alpha-1)\lambda - 1) - e^{a\lambda}(\alpha\lambda z + 1) + e^{\lambda(a+z)} + 1} - c\right] \\
&\quad \times \exp\left[\frac{(e^{a\lambda}-1)(e^{\lambda(z-a)}[\lambda(a+z)+1-\alpha(a\lambda+1)]+\alpha+(\alpha-1)\lambda z - e^{\lambda z})}{\lambda(-\alpha a + a + \alpha z) + e^{\lambda z}(a(\alpha-1)\lambda - 1) - e^{a\lambda}(\alpha\lambda z + 1) + e^{\lambda(a+z)} + 1}\right], \\
\pi_R(\alpha) &= \frac{e^{\lambda z}[(a\lambda-1)(\alpha-1) - \alpha\lambda z] + e^{\lambda(a+z)} - \alpha e^{a\lambda}}{\lambda(e^{a\lambda}-1)} \\
&\quad \times \exp\left[\frac{(e^{a\lambda}-1)e^{\lambda(z-a)}(-e^{a\lambda} + a\lambda + \alpha e^{\lambda(a-z)} + \alpha(\lambda z - 1 - a\lambda) + 1)}{\lambda(a - \alpha a + \alpha z) + e^{\lambda z}(a(\alpha-1)\lambda - 1) - e^{a\lambda}(\alpha\lambda z + 1) + e^{\lambda(a+z)} + 1} - \lambda z\right],
\end{aligned}$$

where $z = z(\alpha)$. For illustration, we plot $\pi_M(\alpha)$ and $\pi_R(\alpha)$ over $\alpha \in [0.2, 1]$ for the case of $c = 0, \lambda = 1, a = 2$ in Figure EC.1. Notice that $\pi_M(\alpha)$ first decreases for $\alpha \in [0.2, 0.9]$ and then increases in α for $\alpha \in [0.9, 1]$, whereas $\pi_R(\alpha)$ increases for $\alpha \in [0.2, 1]$. Thus, consistent with §§2-3, UR would be non-beneficial only for the retailer if $\alpha \in [0.2, 0.9)$, and it would be non-beneficial for both the manufacturer and the retailer if $\alpha \in [0.9, 1]$.

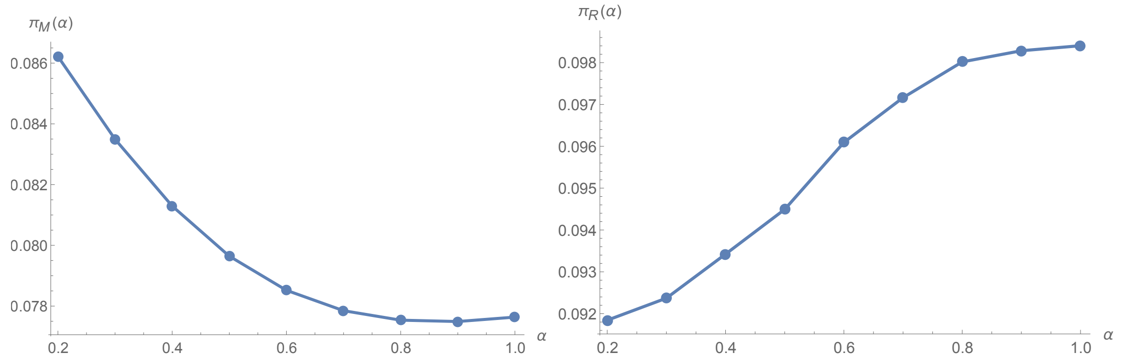

Figure EC.1 $\pi_M(\alpha)$ **and** $\pi_R(\alpha)$ **for the truncated exponential distribution (**$\phi(x) = \frac{\lambda e^{\lambda(a-(x-A))}}{e^{a\lambda}-1}$**) when** $c = 0, \lambda = 1, b = 1, a = 2$.