Overconfident Competing Newsvendors

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Overconfidence is one of the most consistent, powerful, and widespread cognitive biases affecting decision making in situations characterized by random outcomes. In this paper, we study the effects and implications of overconfidence in a competitive newsvendor setting. In this context, overconfidence is defined as a cognitive bias in which decision makers behave as though the outcome of an uncertain event is less risky than it really is. This bias unequivocally leads to a lower expected profit for a newsvendor that does not compete on inventory availability. Nevertheless, it can be a positive force for competing newsvendors. Indeed, we find that when the product's profit margin is high, overconfidence can lead to a first-best outcome. In a similar vein, we also show that the more biased of two competing newsvendors is not necessarily destined to a smaller expected profit than its less biased competitor.

Key words: overconfidence, decision analysis, competitive newsvendors

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No problem in judgement and decision making is more prevalent and more potentially catastrophic than overconfidence. ——Page 217, Plous (1993)

1. Introduction

People, even experts, generally are *overconfident* in their estimations of random outcomes in the sense that they behave as though the expected outcome of an uncertain event is more precise than can be theoretically inferred from its context (Oskamp 1965, Wallsten et al. 1993, Fischhoff et al. 1977, Moore and Healy 2008, Van den Steen 2011). Overconfidence is one of the most consistent, powerful, and widespread cognitive biases, and it has been observed in a wide range of fields from psychology (Lichtenstein et al. 1982) to economics (Grubb 2009) to operations management (Ren and Croson 2013), both in field studies and in controlled experiments. Such a bias also has been observed anecdotally in practice. For example, "[a] leading U.S. manufacturer, planning production capacity for a new factory, solicited a projected range of sales from its marketing staff. The range turned out to be much too narrow and, consequently, the factory could not adjust to unexpected demand" (Russo and Schoemaker 1992).

This bias even occurs when trained decision makers are presented with an unambiguous characterization of the random event. For example, in a survey involving a large number of undergraduate and graduate students, Croson et al. (2011) provided the following specific instructions:

> Consider a uniform distribution with a lower bound of 1 and an upper bound of 300. This means that any number between 1 and 300 is equally likely to be drawn.
>
> A. What is your estimate of the 25th percentile of this distribution? The 25th percentile is a number such that 25% of the time, a random draw from this distribution is less than this number.
>
> B. What is your estimate of the 75th percentile of this distribution? The 75th percentile is a number such that 75% of the time, a random draw from this distribution is less than this number.

They found that the experimental difference between the Question B and Question A responses is 126.24 rather than the theoretical difference of 150. In essence, the participants behaved as though a "random draw" is more likely to derive from the center of the distribution than is true.

In the context of operations management, overconfidence has been used to provide a plausible explanation (Ren and Croson 2013) for the well-known pull-to-center effect in newsvendor experiments comprising undergraduate or MBA students as subjects (Schweitzer and Cachon 2000, Bostian et al. 2008, Bolton and Katok 2008, Katok and Wu 2009, Ho et al. 2010). Interestingly, this pull-to-center effect also has been observed in similar experiments comprising purchasing managers who range in experience from one to thirty years (Bolton et al. 2012). Thus, even senior managers can be affected by overconfidence when placing orders in laboratory-controlled environments. Note, however, that all of the above referenced experiments are designed to test decision making under uncertainty in non-competitive settings. Nevertheless, Ovchinnikov et al. (2015) more recently have designed and conducted similar laboratory-controlled experiments to study newsvendor decision making by trained human subjects in simulated competitive settings, and they have similarly observed a detectable pull-to-center effect. Thus, it seems, overconfidence remains a plausible explanation of cognitive bias under non-competitive and competitive newsvendor environments alike.

We thus explore analytically in this paper how overconfidence would affect the ordering decisions and expected profits of newsvendors that compete on inventory availability. To that end, we develop a model that incorporates the notion of overconfidence into the standard newsvendor duopoly pioneered by Lippman and McCardle (1997). As validation for our model, we show analytically not only that the pull-to-center effect observed in the non-competitive newsvendor setting also would exist in its competitive analog, but also that overconfidence would drive a newsvendor to order more in a competitive environment than in a non-competitive environment. More interestingly, we then find that, unlike in a non-competitive setting, overconfidence in a competitive setting can lead to *increased* expected profits for rival newsvendors. In particular, we show that if profit margins are relatively high, then low levels of overconfidence can benefit newsvendors that compete on inventory availability. Intuitively, under such conditions, overconfidence drives the newsvendors to order less, thus providing a counterbalance to competition's drive to order more. Sometimes, the net effect is beneficial to the newsvendors.

To help put this counterbalance into perspective, we then consider a hypothetical baseline system consisting of two unbiased newsvendors managed by an unbiased central planner. We show that the total expected profit of such a system decreases if the unbiased central planner is replaced by an overconfident central planner; likewise, the total expected profit would decrease if the central planner were eliminated so that the two unbiased newsvendors were left to compete on inventory availability. However, if the central planner were eliminated *and* the two newsvendors were replaced with overconfident competitors, then the total expected profit of the system could potentially equal that of the hypothetical baseline. In this sense, overconfidence could serve as a force that coordinates a newsvendor duopoly with its centralized benchmark.

Digging deeper, we then demonstrate that the net effect of overconfidence on the overall system performance of the newsvendor duopoly is driven not by the individual bias levels of the two newsvendors but rather by the arithmetic average of these bias levels. Moreover, we show that although a comparatively lower bias level generally would be advantageous in the duopoly, it does not provide a performance guarantee: Depending on the profit margin, the more biased of the two competing newsvendors can earn a higher profit than its less biased rival, even if the less biased newsvendor is altogether unbiased.

Overall, our analysis contributes to the general realm of behavioral operations management (see, for example, Su (2008), Katok (2011), Chen et al. (2012), Gurnani et al. (2013), Lau et al. (2014), Kumar et al. (2014), Zhao and Zhao (2015), Quiroga et al. (2015), and Ovchinnikov et al. (2015)). This emerging literature identifies and clarifies fundamental inconsistencies between empirical observations and theoretical prescriptions in a variety of operations management settings, thus underscoring a practical need to reconcile the two. We follow suit by adopting an enduring theory of cognitive bias in information processing and decision making, namely overconfidence, as one plausible explanation for reconciling newsvendor prescription with practical applications thereof, and we analytically explore the significance and implications of that explanation for firms that compete for customers on inventory availability.

Previous research has appealed to overconfidence in its various forms to explain and describe a variety of decision making behavior. For example, Camerer and Lovallo (1999) argue that overoptimism about one's level of ability can explain supraoptimal rates of market entry and entrepreneurship. Malmendier and Tate (2005) show that CEOs who suffer from overplacement are more likely to engage in (unprofitable) acquisitions. Haran et al. (2010) argue that overprecision is the most robust type of overconfidence. In the context of Operations Management, Croson et al. (2011) and Ren and Croson (2013) are the first to observe overconfidence in newsvendors. They find that overconfidence is robust across different controlled experiments, and interestingly, overconfidence can reasonably explain the famous pull-to-center effect that has been observed by Schweitzer and Cachon (2000), Bostian et al. (2008), Bolton and Katok (2008), Katok and Wu (2009) and Özer et al. (2014). They focus on reducing overconfidence by applying techniques developed by Haran et al. (2010) to non-competitive newsvendors. In contrast, we investigate the theoretical impact of overconfidence in a competitive marketplace, and find that overconfidence can potentially be a positive force.

There is also an extensive literature that analytically studies competition between newsvendors (see, for example, Parlar 1988, Lippman and McCardle 1997, Netessine and Rudi 2003). A main focus of this literature is to examine the effects of inventory competition on inventory levels at Nash equilibrium. We extend this literature by incorporating the prevalent notion of overconfidence as a cognitive bias into a competitive newsvendor model to investigate its theoretical impact and implications. In doing so, we make two key contributions.

First, we find that overconfidence can benefit newsvendors that compete against one another on inventory availability, even though that bias never benefits newsvendors that do not face competition. Indeed, we demonstrate that overconfidence is a bias that can coordinate a newsvendor duopoly with its system-wide first-best benchmark. Thus, we tangentially add to the body of literature in operations management that focuses on contracts and mechanisms for decentralized or uncooperative systems to achieve first-best outcomes (see, for example, Cachon (2003), Cui et al. (2007) and Su and Zhang (2008)) by showing that a newsvendor's overconfidence potentially can mitigate the need for full-scale system coordination mechanisms. Second, and counter to intuition, the relatively more biased of two newsvendors in direct competition can have an advantage over its relatively less biased rival, provided that the product they sell has a low profit margin. Furthermore, not even unbiased decision making coupled with full knowledge of a rival's overconfidence is necessarily sufficient to neutralize a direct competitor's overconfidence advantage.

2. Overconfident Newsvendors

In the classic newsvendor problem, a decision maker must commit to an order quantity at the beginning of a single selling season before observing the realization of random demand X. The well-known prescriptive solution is as follows. Given a cost c (≥ 0) of acquiring each unit and a price p $(\geq c)$ of selling each unit, the order quantity q^* that maximizes the expected profit $\pi := p\mathrm{E}[X \wedge q] - cq$, where $x \wedge y = \min\{x, y\}$, is characterized by $F_X(q^*) = \beta$, where $\beta := (p-c)/p$ is the critical fractile and $F_X(\cdot)$ is the distribution function (CDF) of X.[1] Thus, an *unbiased* newsvendor's optimal order quantity is $q^* = F_X^{-1}(\beta)$, and its correspondingly optimal expected profit is $\pi^* = p\mathrm{E}[X \wedge q^*] - cq^*$.

Within this construct, an overconfident newsvendor is a newsvendor that exhibits cognitive bias by instead placing an order *as though* the demand distribution were $F_D(\cdot)$ rather than the given

[1] Without loss of generality, the shortage penalty cost for unsatisfied demands and salvage value for leftover inventory are normalized to be zero.

$F_X(\cdot)$, where

$$D := \alpha\mu + (1-\alpha)X, \tag{1}$$

for $\alpha \in [0,1]$ and $\mathrm{E}(X) = \mu$. In other words, an overconfident newsvendor is a newsvendor that *behaves* as though it were maximizing $p\mathrm{E}[D \wedge q] - cq$ rather than $p\mathrm{E}[X \wedge q] - cq$.

In (1), the parameter α is our measure of overconfidence. If $\alpha > 0$, then the newsvendor is overconfident, behaving as though demand is less variable than X. In the extreme, $\alpha = 1$ means that the newsvendor behaves as though demand is constant and equal to its mean. At the other extreme, $\alpha = 0$ denotes a newsvendor that is unbiased.

Note that (1) has been used widely to model overconfidence. For example, it has been used to study situations in which agents tend to be overoptimistic about their chances of success (Van den Steen 2004), to study the optimal tariff design when consumers are overconfident (Grubb 2009), and to explain the pull-to-the-center phenomenon in a non-competitive newsvendor setting (Ren and Croson 2013). Indeed, Croson et al. (2011) and Ren and Croson (2013), who use data from newsvendor experiments to estimate the average level of overconfidence exhibited by participants, find that the overconfidence level α can vary from 0.52 to 0.83. Please refer to Croson et al. (2011) for further discussion on the efficacy of (1) for modeling the overconfidence observed in newsvendor experiments.

Given (1), Lemma 1 summarizes a newsvendor's ordering bias and resulting expected profit as functions of the overconfidence level (α). These results were documented in Croson et al. (2011) and Ren and Croson (2013).

Lemma 1 **(Croson et al. (2011), Ren and Croson (2013))**. *Given $F_X(\cdot)$:*
a) A newsvendor described by the overconfidence level α orders a quantity $\hat{q}(\alpha) = \alpha\mu + (1-\alpha)q^*$. Hence, $\hat{q}'(\alpha) < 0$ if and only if $\beta > \hat{m}$, where $\hat{m} := F_X(\mu)$.
b) Accordingly, the newsvendor's resulting expected profit $\hat{\pi}(\alpha)$ is decreasing and concave in α, i.e., $\hat{\pi}'(\alpha) \leq 0$ and $\hat{\pi}''(\alpha) \leq 0$.

Lemma 1a indicates that a newsvendor's order quantity is a linear function of its overconfidence level α and it is either decreasing or increasing in α depending on the relationship between the fractile value β and the pull-to-center threshold value $\hat{m}$. Thus, under a high-profit condition ($\beta > \hat{m}$), an overconfident newsvendor underorders relative to an unbiased newsvendor, whereas under a low-profit condition ($\beta < \hat{m}$), the overconfident newsvendor overorders. This behavior is observed in laboratory settings (Schweitzer and Cachon 2000, Bostian et al. 2008, Bolton and Katok 2008, Katok and Wu 2009). Moreover, Lemma 1b asserts that the newsvendor's expected profit decreases in its overconfidence level α.

3. Competitive Overconfident Newsvendors

We derive our notion of competitive newsvendors from Lippman and McCardle (1997). Two firms (indexed 1 and 2) compete on the availability of an identical product that sells for one selling season. Firm i, $i = 1, 2$, chooses order quantity q_i at the beginning of the selling season in anticipation of random demand X_i for the product. Customers who initially visit one firm but find the product unavailable visit the competing firm and attempt to purchase there. Given demands (X_1, X_2) and order quantities (q_1, q_2), the expected profit for Firm i is, for $i = 1, 2$,

$$\pi_i(q_1, q_2) = p\mathrm{E}[(X_i + (X_{3-i} - q_{3-i})^+) \wedge q_i] - cq_i, \tag{2}$$

where $x^+ = \max\{0, x\}$ and $X_i + (X_{3-i} - q_{3-i})^+$ is Firm i's effective demand. Note that the effective demand of Firm i includes two parts: the initial demand X_i and the spillover demand unmet by its competitor $(X_{3-i} - q_{3-i})^+$. Given (2), Lippman and McCardle (1997) establish that the (unbiased) equilibrium order quantities (q_1^c, q_2^c) uniquely satisfy

$$\mathrm{P}\left(X_i + (X_{3-i} - q_{3-i}^c)^+ \leq q_i^c\right) = \beta \tag{3}$$

for $i = 1, 2$. Moreover, $q_1^c = q_2^c$ and $\pi_1^c(q_1^c, q_2^c) = \pi_2^c(q_1^c, q_2^c)$ if X_1 and X_2 are identically distributed (although not necessarily independent). Accordingly, let $q_n = q_1^c = q_2^c$ and $\pi_n = \pi_1(q_1^c, q_2^c) = \pi_2(q_1^c, q_2^c)$ denote the equilibrium order quantities and expected profits, respectively, for two competing unbiased newsvendors given that $X_i \sim F_X(\cdot)$ for $i = 1, 2$.

We now incorporate the notion of overconfidence (§2) into this competitive newsvendor model. We focus here on the case of symmetric overconfidence by assuming a common overconfidence level α for the two competing newsvendors, and we extend our analysis to the case of asymmetric overconfindence in §4. Given overconfidence level α, Newsvendor i behaves as though random demands X_1 and X_2 are instead $D_1(\alpha)$ and $D_2(\alpha)$, respectively, where $D_i(\alpha)=\alpha\mu+(1-\alpha)X_i$ and $\mu=\mathrm{E}[X_1]=\mathrm{E}[X_2]$. Newsvendor i behaves as though its game with its rival satisfies[2]

$$\max_{q_i} \pi_i(\alpha)=p\mathrm{E}[(D_i(\alpha)+(D_{3-i}(\alpha)-q_{3-i})^+)\wedge q_i]-cq_i, \tag{4}$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha)=p\mathrm{E}[(D_{3-i}(\alpha)+(D_i(\alpha)-q_i)^+)\wedge q_{3-i}]-cq_{3-i}. \tag{5}$$

Lemma 2. *The equilibrium ordering quantities* $(\hat{q}_1^c,\hat{q}_2^c)$ *for the overconfident newsvendor system (4)-(5) exist and are unique. Moreover, they are identical, and* $\hat{q}^c:=\hat{q}_1^c=\hat{q}_2^c$ *satisfies* $P(D_1(\alpha)+(D_2(\alpha)-\hat{q}^c)^+\le\hat{q}^c)=P(D_2(\alpha)+(D_1(\alpha)-\hat{q}^c)^+\le\hat{q}^c)=\beta$.

One direct implication of Lemma 2 is that inventory spillover induces the newsvendors to order more than they otherwise would if competition were absent. In light of Lemma 2, define $\hat{z}^c$ by

$$\hat{q}^c(\alpha)=\alpha\mu+(1-\alpha)\hat{z}^c$$

so that an overconfident newsvendor's order quantity has two parts: a part allocated to mean demand (namely, $\alpha\mu$) and a part allocated to demand variance (namely, $(1-\alpha)\hat{z}^c$). Let

$$g(\cdot):=\frac{F_{\bar{X}}(\cdot)+F_{X,X}(\cdot,\cdot)}{2},$$

where $F_{\bar{X}}$ and $F_{X,X}(\cdot,\cdot)$ denote the CDFs of $\bar{X}=(X_1+X_2)/2$ and (X_1,X_2), respectively.

Proposition 1. *The equilibrium* $\hat{z}^c$ *for the overconfident newsvendor system (4)-(5) is* $\hat{z}^c=g^{-1}(\beta)=q_n$. *Accordingly,* $d\hat{q}^c(\alpha)/d\alpha<0$ *if and only if* $\beta>\hat{m}^c$ *for* $i=1,2$, *where* $\hat{m}^c:=g(\mu)<\hat{m}$.

On the one hand, Proposition 1 shows that the impact of overconfidence in the competitive setting is similar to its impact in the non-competitive setting in the sense that the equilibrium order quantity boils down to a weighted average of the unbiased equilibrium order quantity (q_n) and

[2] In defining the game between the two overconfident newsvendors, we borrow the concept of *naive realism* from psychology (Pronin et al. 2004) and implicitly assume that each newsvendor places an order as if its competitor behaves the same way. Many empirical studies have demonstrated that people believe and act as naive realists in that people believe they see things as they are, objectively, and believe other reasonable people will share their responses (Ross and Ward 1996).

the demand mean (μ). As a result, we should expect to see the same "pull-to-center" effect from overconfidence in a competitive newsvendor setting that has been observed in the non-competitive setting. And, indeed, this is the case. In particular, in their controlled laboratory experiment designed to emulate the Lippman and McCardle setting, Ovchinnikov et al. (2015) found the mean order quantity (67.62) among the participants to be less than the equilibrium order quantity (72) prescribed by the Lippman and McCardle solution. In their experiments, $\hat{m}^c = 3/8 < 2/3 = \beta$. Thus, Ovchinnikov et al. (2015) provides one test case that helps validate Proposition 1.

On the other hand, the specific pull-to-center threshold value in Proposition 1 ($\hat{m}^c$) is strictly less than its analog in Lemma 1 ($\hat{m}$). Thus, when competing on inventory availability against a rival, an overconfident newsvendor is more prone to underorder, and less prone to overorder. This is interesting because inventory competition is a force that drives the newsvendor to order more; hence overconfidence serves as a counterweight to that force.

To develop insight into the net effects of these two forces, we next investigate the impact of overconfidence on the expected profits of the two competing newsvendors. Recall from Lemma 1 that overconfidence reduces a newsvendor's expected profit in the non-competitive setting. Proposition 2 establishes that this need not be true in the competitive setting.

Proposition 2. *Let $\hat{\pi}^c(\alpha) = \hat{\pi}_1(\alpha) = \hat{\pi}_2(\alpha)$ denote the equilibrium expected profits for the overconfident newsvendor system (4)-(5) and let $m := F_{\bar{X}}(\mu)$. Then, $\hat{\pi}^c(\alpha)$ is concave in α. Moreover: (a) if $\beta \leq \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ decreases in α; (b) if $\hat{m}^c < \beta < m$, then $\hat{\pi}^c(\alpha)$ increases in α; (c) if $m \leq \beta$, then, $\hat{\pi}^c(\alpha)$ increases in α for $\alpha \in (0, \hat{\alpha}]$ and then decreases in α for $\alpha \in [\hat{\alpha}, 1]$, where $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \mu)$.*

According to Proposition 2, the bottom-line impact of overconfidence on a newsvendor that competes on inventory availability depends on how the profit margin β of the product compares to the thresholds $\hat{m}^c$ and m. If the profit margin is low ($\beta \leq \hat{m}^c$), then overconfidence hurts a competing newsvendor just as it hurts a non-competing newsvendor. However, if the profit margin is intermediate ($\hat{m}^c < \beta < m$), then overconfidence actually boosts a competing newsvendor's performance. Similarly, if the profit margin is high ($\beta \geq m$), then a high level of overconfidence potentially boosts the competing newsvendor's performance. These three cases are illustrated in Figure 1.

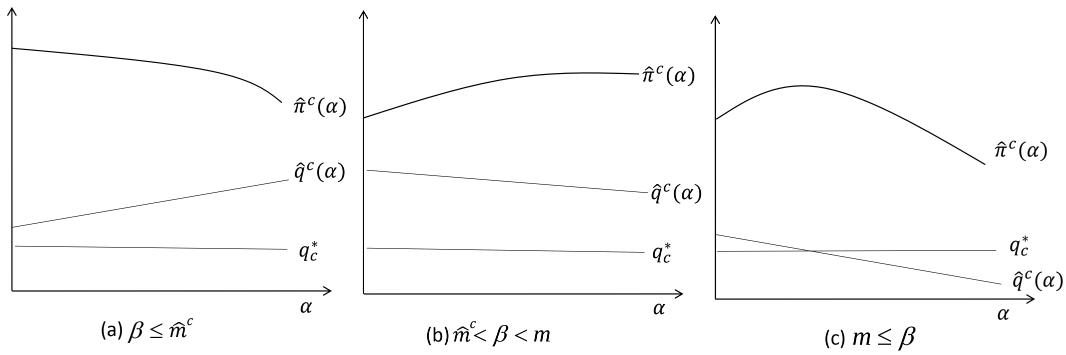

Figure 1 An illustration of Proposition 2.

To understand and explore the intuition behind Proposition 2, consider the following hypothetical construct. Suppose an unbiased central planner maximizes the combined expected profits of the two newsvendors to determine the first-best order quantities for the system in which spillover demand from one newsvendor can be satisfied by the other newsvendor. The joint expected profit of such a system is $p\mathrm{E}[(X_1 + (X_2 - q_2)^+) \wedge q_1 + (X_2 + (X_1 - q_1)^+) \wedge q_2] - c(q_1 + q_2) = p\mathrm{E}[(X_1 + X_2) \wedge (q_1 + q_2)] - c(q_1 + q_2)$. Therefore, the unbiased central planner's first-best order quantity q_c^* for each newsvendor is defined by $\mathrm{P}(X_1 + X_2 \le 2q_c^*) = \beta$ or $q_c^* = F_{\bar{X}}^{-1}(\beta)$.

Intuitively, the performance of competing newsvendors depends on the relationship between their equilibrium order quantities $\hat{q}^c(\alpha)$ and the system-wide first-best order quantity q_c^*. The closer is $\hat{q}^c(\alpha)$ to q_c^*, the better is the equilibrium performance of the competing newsvendors. From Figure 1, note that, at one extreme, q_c^* is less than the equilibrium order quantity $\hat{q}^c(\alpha)$ that would result if the two competing newsvendors were unbiased ($\alpha = 0$), but at the other extreme, q_c^* could be either less than or greater than the equilibrium order quantity $\hat{q}^c(\alpha)$ that would result if the two competing newsvendors were infinitely overconfident ($\alpha = 1$). We formalize this observation with Lemma 3.

LEMMA 3. *(a) An (unbiased) centralized planner orders less than unbiased competing newsvendors. That is,* $q_c^* < \hat{q}^c(\alpha = 0)$.

(b) Suppose $\alpha = 1$. *An (unbiased) centralized planner orders less than the overconfident competing*

newsvendors if $\beta < m$ but orders more than the competing overconfident newsvendors if $\beta > m$. That is, $q_c^* < \hat{q}(\alpha = 1) \Longleftrightarrow \beta < m$.

This therefore suggests that, to the extent that overconfidence could exact a balanced (i.e., symmetric) distortion of both the overage and underage costs that the central planner would assess, the equilibrium order quantity would correspond to q_c^*. In essence, $\hat{\alpha}$ from Proposition 2, which can exist if and only if $\beta > m$, denotes that very balance. This is illustrated in Figure 1(c) and is highlighted by Proposition 3.

PROPOSITION 3. *If $\beta > m$, then competing newsvendors with overconfidence level $\alpha = \hat{\alpha}$ order the same quantity that an (unbiased) central planner orders, i.e., $\hat{q}^c(\hat{\alpha}) = q_c^*$.*

Proposition 3 illustrates that if $\alpha = \hat{\alpha}$, then overconfidence induces system coordination when $\beta > m$. In particular, if $\beta > m$, then there exists a level of overconfidence at which two identical competing newsvendors converge on the system-wide first-best order quantities. Thus, overconfidence is a bias that can offset the effect of competition such that a system actually is equivalent with its centralized benchmark. An example of a coordinating overconfidence level arises from the experimental specification from Ovchinnikov et al. (2015): $p = 3$, $c = 1$, and $X_i \sim U[1, 100]$. In their setting, $\beta = 2/3$, $\mu = 101/2$ and $m = 1/2 \leq \beta$. In addition, because $F_{\bar{X}}(z) = \frac{2(z-1)^2}{9801}$ and $g(z) = \frac{(1-z)^2}{6534}$ for $z \leq \frac{101}{2}$ whereas $F_{\bar{X}}(z) = 1 - \frac{2(100-z)^2}{9801}$ and $g(z) = \frac{(398-z)z-10198}{19602}$ for $z > \frac{101}{2}$, $\beta = \frac{2}{3}$ implies that $F_{\bar{X}}^{-1}(\beta) = \frac{1}{2}(200 - 33\sqrt{6})$ and $g^{-1}(\beta) = 199 - 33\sqrt{15}$. Hence, an overconfidence level of $\hat{\alpha} = \frac{199-33\sqrt{15}-\frac{1}{2}(200-33\sqrt{6})}{199-33\sqrt{15}-\frac{101}{2}} = \frac{2\sqrt{15}-\sqrt{6}-6}{2\sqrt{15}-9}(\sim 0.56)$ effectively coordinates this competitive newsvendor system. This overconfidence level falls within the range experimentally observed in the laboratory (Ren and Croson 2013).

4. Asymmetric Overconfidence

In this section, we extend the competitive newsvendor system studied in §3 to the case in which the overconfidence biases of the newsvendors are not symmetric by assuming, without loss of generality, that $\alpha_1 < \alpha_2$. Analogous to (4)-(5), Newsvendor i behaves as though its game with its rival were described by

$$\max_{q_i} \pi_i(\alpha_i) = p\mathrm{E}[(D_i(\alpha_i) + (D_{3-i}(\alpha_i) - q_{3-i})^+) \wedge q_i] - cq_i, \tag{6}$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha_i) = p\mathrm{E}[(D_{3-i}(\alpha_i) + (D_i(\alpha_i) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}. \tag{7}$$

The equilibrium solution to (6)-(7) is given by $\mathrm{P}(D_i(\alpha_i) + (D_{3-i}(\alpha_i) - q_{3-i})^+) \le q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + (D_i(\alpha_i) - q_i)^+ \le q_{3-i}) = \beta$. Thus, Newsvendor 1 behaves as though it solves (6)-(7) instead of (2), thereby deriving the equilibrium order quantities $(\hat{q}_1^c(\alpha_1), \hat{q}_2^c(\alpha_1))$. Similarly, Newsvendor 2 employs the equilibrium order quantities $(\hat{q}_1^c(\alpha_2), \hat{q}_2^c(\alpha_2))$. Accordingly, Newsvendor 1 orders $\hat{q}_1(\alpha_1)$ and anticipates Newsvendor 2 to order $\hat{q}_2(\alpha_1)$, whereas Newsvendor 2 orders $\hat{q}_2(\alpha_2)$ and anticipates Newsvendor 1 to order $\hat{q}_1(\alpha_2)$. The ensuing expected profits are

$$\hat{\pi}_i^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_i + (X_{3-i} - \hat{q}_{3-i}^c(\alpha_{3-i}))^+ \wedge \hat{q}_i^c(\alpha_i)] - c\hat{q}_i^c(\alpha_i). \tag{8}$$

Lemma 4. *Associated with the overconfidence levels α_1 and α_2 are the order quantities $\hat{q}_i^c(\alpha_i) = q_n - \alpha_i(q_n - \mu)$, where $q_n = g^{-1}(\beta)$ as defined in Proposition* 1.

The significance of Lemma 4 is that, consistent with Proposition 1, it indicates that a newsvendor's overconfidence essentially distorts the magnitudes of its safety stock $|\hat{q}_i^c(\alpha_i) - \mu|$ by virtue of discounting the absolute value of the unbiased safety stock $|q_n - \mu|$ by its overconfidence level α_i, thereby pulling its actual order quantity toward the demand mean. As a result, although the direction of pull is the same for two competing newsvendors regardless of whether their overconfidence biases are symmetric (Proposition 1) or asymmetric (Lemma 4), the final order quantities of the two competing newsvendors will deviate from one another depending on the asymmetry of their biases. In particular, $\Delta(\alpha_1, \alpha_2) := |\hat{q}_1^c(\alpha_1) - \hat{q}_2^c(\alpha_2)| = (\alpha_2 - \alpha_1)|q_n - \mu| > 0$ so that $\Delta(\alpha_1, \alpha_2)$ is linear in $\alpha_2 - \alpha_1$.

Given $\Delta(\alpha_1, \alpha_2)$, we next explore whether there exist asymmetric overconfidence combinations that would effectively coordinate the system with the decision of the unbiased central planner from §3. Toward that end, recall that the unbiased central planner from §3 would order $2q_c^* = 2F_X^{-1}(\beta)$ for the system, whereas the total order quantity for the two competing, asymmetrically biased, newsvendors is $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\mu + (2 - \alpha_1 - \alpha_2)q_n$.

Proposition 4. *Suppose* $\beta > m$ *and that* α_1 *and* α_2 *satisfy* $(\alpha_1+\alpha_2)/2 = \hat{\alpha}$, *where* $\hat{\alpha} = (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \mu)$ *as defined in Proposition 2. Then the sum of the newsvendors' orders equals that of a central planner:* $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$.

Proposition 4 not only extends Proposition 3 to the asymmetric bias case, but also verifies that $\hat{\alpha}$ represents the level of bias distortion that effectively counterbalances the impact of competition. Proposition 5 reveals that a less biased newsvendor does not necessarily earn a higher expected profit than its more biased rival.

Proposition 5. *If* $\beta > \hat{m}^c$, *then the less biased newsvendor earns a higher expected profit than the more biased newsvendor, i.e., if* $\beta > \hat{m}^c$, *then* $\alpha_1 < \alpha_2 \Longrightarrow \hat{\pi}_1^c(\alpha_1, \alpha_2) > \hat{\pi}_2^c(\alpha_1, \alpha_2)$. *However, if* $\beta < \hat{m}^c$, *then the more biased newsvendor can earn a higher expected profit than the less biased newsvendor.*

For insight, consider the extreme in which $\alpha_1 = \alpha_2 = 0$. In this case, both newsvendors have the same order quantity and they each extract fifty percent of the system-wide expected profit. However, if $\alpha_2 > 0$, then the order quantity of Newsvendor 2 would be closer in magnitude to the mean demand than the unbiased newsvendor's order quantity. Thus, if a competitive situation were such that the unbiased newsvendor's equilibrium order quantity is less than the demand mean, as is the case when $\beta < \hat{m}^c$, then overconfidence leads Newsvendor 2 to a higher order quantity. As a result, the effective demand for Newsvendor 2 is stochastically larger and leads to a larger expected profit relative to its competitor.

5. Sophisticated Newsvendor

Next, we demonstrate that the Proposition 5 advantage attributed to overconfidence is robust; it remains even if the biased newsvendor's competitor is sophisticated, where we define a sophisticated newsvendor as one who is *unbiased* as well as *fully cognizant* of its rival's overconfidence. If Newsvendor 1 not only is unbiased but also knows that Newsvendor 2 is overconfident, then Newsvendor 1 solves $\max_{q_1} \pi_1(\alpha_1 = 0)$ from (6) while understanding that Newsvendor 2 behaves as though it were solving $\max_{q_2} \pi_2(\alpha_2)$ from (7); Newsvendor 2 believes its game with its rival is described directly by (6)-(7). In this case, it follows from Lemma 4 that Newsvendor 2's order quantity satisfies $\hat{q}_2^c(\alpha_2) = q_n - \alpha_2(q_n - \mu)$, where $q_n = g^{-1}(\beta)$. Accordingly, from (3), Newsvendor 1's order quantity $q_1^{fc}(\alpha_2)$ satisfies $\mathrm{P}(X_1 + (X_2 - \hat{q}_2^c(\alpha_2))^+ \leq q_1^{fc}(\alpha_2)) = \beta$. Note that $q_1^{fc}(\alpha_2) \neq q_n$.

PROPOSITION 6. *A sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased competitor.*

From Propositions 5-6, we see that overconfidence can lead to a comparative advantage against a rival regardless of whether the rival is aware of that cognitive behavior. Because the sophisticated newsvendor can foresee the biased newsvendor's decision, at the least, the unbiased newsvendor can mimic the biased newsvendor's decision and earn the same expected profit as the biased newsvendor. Yet, ironically, Proposition 6 indicates that although the sophisticated newsvendor can increase its own expected profit by reacting optimally, its optimal reaction nevertheless can result in increasing its competitor's expected profit even *more*, depending on the profit margin of the product on which the newsvendors compete. Thus, consistent with Propositions 2-4, a decrease in one newsvendor's bias can increase the size of the overall competitive system pie, but consistent with Propositions 5-6, that decrease in bias nevertheless can increase its competitor's good fortunes more than its own.

6. Concluding Remarks

We study the effects and implications of overconfidence in a traditional competitive-newsvendor setting. In this context, overconfidence is defined as a cognitive bias that essentially describes decision makers who behave as though the expected outcome of an uncertain event is more precise than theoretically can be inferred from an unambiguous characterization of the events. Although this bias always leads to a lower expected profit in a non-competitive setting, this need not be the case for competing newsvendors. Indeed, depending on the specific profit margin associated with the product on which the newsvendors compete, overconfidence is a cognitive bias that can lead to a first-best outcome. In a similar vein, we also show that the more biased of two competing newsvendors is not necessarily destined to receive a smaller expected profit than its less biased competitor: It depends on the profit margin. If the profit margin is small enough that a non-competing *unbiased* newsvendor should order less than the mean demand, then a comparatively higher level of overconfidence between two competing newsvendors can distort the system equilibrium such that net spillover demand flows from the less biased to the more biased competitor to the extent that the expected profit of the more biased newsvendor exceeds the expected profit of its less biased rival. Moreover, this particular overconfidence advantage remains in favor of the more biased of two competing newsvendors even in the extreme case in which the less biased newsvendor not only is *unbiased* but also is *fully cognizant* of its competitor's overconfidence. Thus, to the extent that relative performance between firms is an important practical concern (Lichtendahl and Winkler 2007), overconfidence can potentially serve as a comparative advantage.

In our analysis, we adopted the standard newsvendor duopoly pioneered by Lippman and McCardle (1997) as our baseline model of competition. Nevertheless, to explore the scope and applicability of our key results, we extended our analysis by considering in the electronic companion to this paper the effects of three key variants to this model of competition: partial spillover (§EC.1), inventory-dependent demand rationing (§EC.2), and random demand splitting (§EC.3). In the partial spillover extension, only a portion of customers who initially visit one newsvendor but find the product unavailable visit the competing newsvendor to attempt to purchase there, while the remaining portion of customers results in immediate lost sales for the system. In the inventory-dependent demand rationing extension, the fraction of random market demand that initially visits each newsvendor is proportional to each newsvendor's order size, whereas in the random demand splitting extension, a random fraction of random market demand initially visits each newsvendor. In each of these extensions, we find that although quantitative details differ depending on the modeling parameters, our key qualitative results remain true, namely that overconfidence can be a positive force (Proposition 2) and that the less biased of the two newsvendors can be destined for a lower expected profit than its rival (Proposition 5) even if the newsvendor is altogether unbiased and fully cognizant of its rival's overconfidence (Proposition 6).

In our analysis, we also adopted the standard notion that overconfidence biases a decision maker's cognitive processing of a random variable's variance but not its mean. In §EC.4 of the electronic companion, we expanded this definition to reflect a situation in which overconfidence essentially distorts both the mean and variance by considering the case in which, given random demand X, a decision maker behaves as though demand instead were characterized as $D = \alpha\rho + (1-\alpha)X$, where $\rho(>0)$ is an arbitrary constant. In this variant, ρ represents the *target* of overconfidence; the higher is α, the more "confident" is the decision maker that demand is equal to ρ. Thus, for example, $\rho = \mu$ denotes our baseline model of overconfidence whereas $\rho > \mu$ reflects the case in which a decision maker behaves as though demand were stochastically larger than it really is. In a similar spirit, $\rho = q^*$ would represent a newsvendor who behaves as though demand converges to its ordering decision. Given this expanded definition of overconfidence, we again find that the analogs of Propositions 2, 5, and 6 continue to hold.

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References

Bolton, Gary E, Elena Katok. 2008. Learning by doing in the newsvendor problem: A laboratory investigation of the role of experience and feedback. *Manufacturing & Service Operations Management* **10**(3) 519–538.

Bolton, Gary E, Axel Ockenfels, Ulrich W Thonemann. 2012. Managers and students as newsvendors. *Management Science* **58**(12) 2225–2233.

Bostian, AJ A, Charles A Holt, Angela M Smith. 2008. Newsvendor pull-to-center effect: Adaptive learning in a laboratory experiment. *Manufacturing & Service Operations Management* **10**(4) 590–608.

Cachon, Gérard P. 2003. Supply chain coordination with contracts. Elsevier Science Publisher, North–Holland, 227–339.

Camerer, Colin, Dan Lovallo. 1999. Overconfidence and excess entry: An experimental approach. *American Economic Review* **89**(1) 306–318.

Chen, Yefen, Xuanming Su, Xiaobo Zhao. 2012. Modeling bounded rationality in capacity allocation games with the quantal response equilibrium. *Management Science* **58**(10) 1952–1962.

Croson, D, Rachel Croson, Yufei Ren. 2011. The overconfident newsvendor, Working paper, University of Texas at Arlington, Arlington, TX.

Cui, Tony Haitao, Jagmohan S Raju, Z John Zhang. 2007. Fairness and channel coordination. *Management Science* **53**(8) 1303–1314.

Fischhoff, Baruch, Paul Slovic, Sarah Lichtenstein. 1977. Knowing with certainty: The appropriateness of extreme confidence. *Journal of Experimental Psychology: Human perception and performance* **3**(4) 552–564.

Grubb, Michael D. 2009. Selling to overconfident consumers. *The American Economic Review* **99**(5) 1770–1807.

Gurnani, Haresh, Karthik Ramachandran, Saibal Ray, Yusen Xia. 2013. Ordering behavior under supply risk: an experimental investigation. *Manufacturing & Service Operations Management* **16**(1) 61–75.

Haran, Uriel, Don A Moore, Carey K Morewedge. 2010. A simple remedy for overprecision in judgment. *Judgment and Decision Making* **5**(7) 467–476.

Ho, Teck-Hua, Noah Lim, Tony Haitao Cui. 2010. Reference dependence in multilocation newsvendor models: A structural analysis. *Management Science* **56**(11) 1891–1910.

Katok, Elena. 2011. *Using laboratory experiments to build better operations management models*. Foundations and Trends in Technology, Information, and Operations Management.

Katok, Elena, Diana Yan Wu. 2009. Contracting in supply chains: A laboratory investigation. *Management Science* **55**(12) 1953–1968.

Kumar, V, Xi Zhang, Anita Luo. 2014. Modeling customer opt-in and opt-out in a permission-based marketing context. *Journal of Marketing Research* **51**(4) 403–419.

Lau, Nelson, Sameer Hasija, J Neil Bearden. 2014. Newsvendor pull-to-center reconsidered. *Decision Support Systems* **58** 68–73.

Lichtendahl, Kenneth C, Robert L Winkler. 2007. Probability elicitation, scoring rules, and competition among forecasters. *Management Science* **53**(11) 1745–1755.

Lichtenstein, Sarah, Baruch Fischhoff, Lawrence D Phillips. 1982. *Calibration of probabilities: The state of the art to 1980*. In D. Kahneman, P. Slovic, and A. Tversky (Eds.), Judgment Under Uncertainty: Heuristics and Biases, Cambridge University Press, Cambridge, England.

Lippman, Steven A, Kevin F McCardle. 1997. The competitive newsboy. *Operations Research* **45**(1) 54–65.

Malmendier, Ulrike, Geoffrey Tate. 2005. CEO overconfidence and corporate investment. *The journal of finance* **60**(6) 2661–2700.

Moore, Don A, Paul J Healy. 2008. The trouble with overconfidence. *Psychological Review* **115**(2) 502–517.

Netessine, Serguei, Nils Rudi. 2003. Centralized and competitive inventory models with demand substitution. *Operations Research* **51**(2) 329–335.

Oskamp, Stuart. 1965. Overconfidence in case-study judgments. *Journal of consulting psychology* **29**(3) 261–265.

Ovchinnikov, Anton, Brent Moritz, Bernardo F Quiroga. 2015. How to compete against a behavioral newsvendor. *Production and Operations Management* Forthcoming.

Özer, Özalp, Yanchong Zheng, Yufei Ren. 2014. Trust, trustworthiness, and information sharing in supply chains bridging china and the US. *Management Science,* Forthcoming.

Parlar, Mahmut. 1988. Game theoretic analysis of the substitutable product inventory problem with random demands. *Naval Research Logistics* **35**(3) 397–409.

Plous, Scott. 1993. *The psychology of judgment and decision making*. McGraw-Hill, New York, NY.

Pronin, Emily, Thomas Gilovich, Lee Ross. 2004. Objectivity in the eye of the beholder: divergent perceptions of bias in self versus others. *Psychological review* **111**(3) 781.

Quiroga, Bernardo F, Brent Moritz, Anton Ovchinnikov. 2015. On the cost of behavioral ordering: Inventory, competition and policy policy Working paper, Clemson University, Clemson, SC.

Ren, Yufei, Rachel Croson. 2013. Overconfidence in newsvendor orders: An experimental study. *Management Science* **59**(11) 2502–2517.

Ross, Lee, Andrew Ward. 1996. Naive realism in everyday life: Implications for social conflict and misunderstanding. *In T. Brown, E. Reed, and E. Turiel (eds.), Values and knowledge* 103–135.

Russo, J Edward, Paul JH Schoemaker. 1992. Managing overconfidence. *Sloan Management Review* **33**(2) 7–17.

Schweitzer, Maurice E, Gérard P Cachon. 2000. Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Management Science* **46**(3) 404–420.

Su, Xuanming. 2008. Bounded rationality in newsvendor models. *Manufacturing & Service Operations Management* **10**(4) 566–589.

Su, Xuanming, Fuqiang Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science* **54**(10) 1759–1773.

Van den Steen, Eric. 2004. Rational overoptimism (and other biases). *The American Economic Review* **94**(4) 1141–1151.

Van den Steen, Eric. 2011. Overconfidence by bayesian-rational agents. *Management Science* **57**(5) 884–896.

Wallsten, Thomas S, David V Budescu, Rami Zwick. 1993. Comparing the calibration and coherence of numerical and verbal probability judgments. *Management Science* **39**(2) 176–190.

Zhao, Yingshuai, Xiaobo Zhao. 2015. How a competing environment influences newsvendor ordering decisions. *International Journal of Production Research* Forthcoming.

Appendix

Proof of Lemma 1 a) An overconfident newsvendor with overconfidence level α orders $\hat{q}(\alpha)$ such that $F_D(\hat{q}(\alpha)) = \beta$ when the demand is X. Thus, $F_D(\hat{q}(\alpha)) = F_X(q^*) = \beta$. According to (1), $\hat{q}(\alpha) = \alpha\mu + (1-\alpha)q^*$. As a result, $\hat{q}'(\alpha) = \mu - q^*$. Consequently, $\hat{q}'(\alpha) < 0$ if and only if $q^* > \mu$, which is identical to $\beta > \hat{m}$.

b) Because the newsvendor's expected profit is $\hat{\pi}(\alpha) = p\mathrm{E}[X \wedge \hat{q}] - c\hat{q}$,

$$\hat{\pi}'(\alpha) = [p\bar{F}_X(\alpha\mu + (1-\alpha)q^*) - c](\mu - q^*). \tag{9}$$

If $q^* > \mu$, then $\alpha\mu + (1-\alpha)q^* < q^*$, and as a result, $\bar{F}_X(\alpha\mu + (1-\alpha)q^*) > \bar{F}_X(q^*) = 1 - \beta$. Consequently, $\hat{\pi}'(\alpha) \leq 0$ if $q^* \geq \mu$. Similarly, $\hat{\pi}'(\alpha) \leq 0$ if $q^* < \mu$. Thus, by combining both cases, we conclude that $\hat{\pi}'(\alpha) \leq 0$ for all α. Moreover, from (9), $\hat{\pi}''(\alpha) = -pf_X(\alpha\mu + (1-\alpha)q^*)(q^* - \mu)^2 \leq 0$. Q.E.D.

Proof of Lemma 2 Following Lippman and McCardle (1997), the equilibrium ordering quantities $(\hat{q}_1^c, \hat{q}_2^c)$ exist and are unique. Moreover, from (3), they are identical because $\mathrm{P}(D_1(\alpha) + (D_2(\alpha) - \hat{q}^c)^+ < \hat{q}^c) = \mathrm{P}(D_2(\alpha) + (D_1(\alpha) - \hat{q}^c)^+ < \hat{q}^c) = \beta$, where $\hat{q}^c = \hat{q}_1^c = \hat{q}_2^c$. Q.E.D.

Proof of Proposition 1 We partition the demand space $\{(X_1, X_2) : X_1 + X_2 \leq q_1 + q_2\}$ given (q_1, q_2) into three domains as shown in Figure 2: Domain Ω_1 where $X_1 \leq q_1$ and $X_2 \leq q_2$, Ω_2 where $X_2 > q_2, X_1 + X_2 \leq q_1 + q_2$, and Ω_3 where $X_1 > q_1, X_1 + X_2 \leq q_1 + q_2$. It is easy to see that the total measure $M(\cdot)$ of domains Ω_1 and Ω_3 satisfies $M(\Omega_1 \cup \Omega_3) = \mathrm{P}\left[X_2 + (X_1 - q_1)^+ \leq q_2\right]$. Similarly, the total measure of domains Ω_1 and Ω_2 satisfies $M(\Omega_1 \cup \Omega_2) = \mathrm{P}\left[X_1 + (X_2 - q_2)^+ \leq q_1\right]$. From (3) and $\hat{q}^c = \alpha\mu + (1-\alpha)\hat{z}^c$, $M(\Omega_1 \cup \Omega_2) = M(\Omega_1 \cup \Omega_3) = \beta$ when $(q_1, q_2) = (\hat{z}^c, \hat{z}^c)$. As a result, $\hat{z}^c = q_n$. Because $M(\Omega_1 \cup \Omega_2) = M(\Omega_1 \cup \Omega_3)$, $M(\Omega_2) = M(\Omega_3)$. Consequently, $M(\Omega_2) = M(\Omega_3) = \frac{1}{2}\left[F_{X_1+X_2}(2q_n) - F_{X_1,X_2}(q_n, q_n)\right]$, and thus, $g(q_n) = \frac{1}{2}F_{X_1+X_2}(2q_n) + \frac{1}{2}F_{X_1,X_2}(q_n, q_n) = \beta$. A key observation here is that q_n is independent of α. Thus, $d\hat{q}^c(\alpha)/d\alpha < 0$ if and only if $q_n > \mu$, which is identical to $\beta > \hat{m}^c = g(\mu)$ for $i = 1, 2$. Moreover, $\hat{m}^c < \hat{m}$ because $\hat{m}^c = g(\mu) = F_{\bar{X}}(\mu)/2 + F_{X,X}(\mu, \mu)/2 = \mathrm{P}\left[D_2 + (D_1 - \mu)^+ \leq \mu\right] < \mathrm{P}(X_1 < \mu) = \hat{m}$. Q.E.D.

Proof of Proposition 2 Because the equilibrium profits of the two newsvendors are identical, we have both newsvendors' expected equilibrium profits as

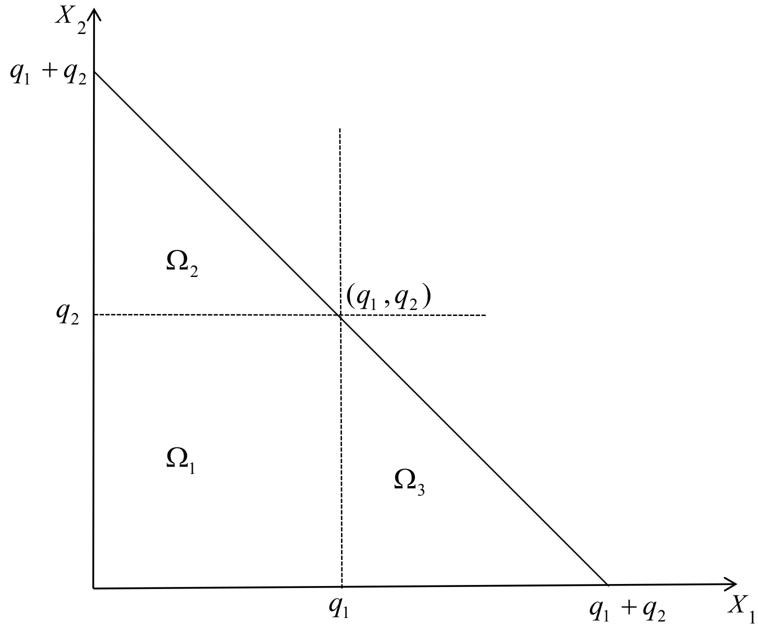

Figure 2 **A partition of the demand space** $\{(X_1, X_2) : X_1 + X_2 \le q_1 + q_2\}$.

$$\begin{aligned}\hat{\pi}^c(\alpha) &= \mu(p-c) - \frac{p-c}{2}\mathrm{E}[X_1 + X_2 - 2\hat{q}^c(\alpha)]^+ - \frac{c}{2}\mathrm{E}[2\hat{q}^c(\alpha) - X_1 - X_2]^+ \\ &= \mu(p-c) - (p-c)\mathrm{E}[\bar{X} - \alpha\mu - (1-\alpha)\hat{z}^c]^+ - c\mathrm{E}[\alpha\mu + (1-\alpha)\hat{z}^c - \bar{X}]^+.\end{aligned}$$

Thus,

$$\begin{aligned}\frac{d\hat{\pi}^c(\alpha)}{d\alpha} &= (p-c)\bar{F}_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)(\mu - \hat{z}^c) + cF_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)(\hat{z}^c - \mu) \qquad (10) \\ &= (\mu - \hat{z}^c)[(p-c)\bar{F}_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c) - cF_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)] \\ &= (\mu - \hat{z}^c)[(p-c) - pF_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)] \\ &= p[\beta - F_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)](\mu - \hat{z}^c),\end{aligned}$$

and $\frac{d^2\hat{\pi}^c(\alpha)}{d\alpha^2} = -p(\hat{z}^c - \mu)^2 f_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c) < 0$. As a result, $\hat{\pi}^c(\alpha)$ is concave in α. Accordingly, there are three cases to consider.

Case (a): If $\beta \le \hat{m}^c$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = p(\beta - F_{\bar{X}}(\hat{z}^c))(\mu - \hat{z}^c) \le 0$ because $\hat{z}^c < \mu$ and $\beta = g(\hat{z}^c) < F_{\bar{X}}(\hat{z}^c)$. Therefore, if $\beta \le \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ is a decreasing function because of its concavity.

Case (b): If $\hat{m}^c < \beta \le m$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=1} = p(\beta - m)(\mu - \hat{z}^c) > 0$ because $\hat{z}^c > \mu \Longleftrightarrow \beta > \hat{m}^c$. Therefore, if $\hat{m}^c < \beta \le m$, then $\hat{\pi}^c(\alpha)$ is an increasing function because of its concavity.

Case (c): If $\beta > m$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = p(\beta - F_{\bar{X}}(\hat{z}^c))(\mu - \hat{z}^c) > 0$ because $g(\hat{z}^c) = \beta$ and $g(\cdot) \le F_{\bar{X}}(\cdot)$.

Furthermore, $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=1} = p(\beta - m)(\mu - \hat{z}^c) < 0$. Therefore, if $\beta > m$, then $\hat{\pi}^c(\alpha)$ is an increasing-decreasing function. In order to let $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$, we have $\beta - F_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c) = 0$. Since $g(\hat{z}^c) = \beta$, we can conclude that $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \mu)$. Q.E.D.

Proof of Lemma 3 (a) In the competition case, the equilibrium quantities $q_n = \hat{q}^c(\alpha = 0)$ satisfy (3). In this case, the order quantity q_n in the competitive environment is higher than the order quantity q_c^* in the centralized environment because $\mathrm{P}\left(X_i + (X_{3-i} - q_n)^+ \leq q_n\right) = F_{\bar{X}}(q_c^*) = \beta$ and $\mathrm{P}\left(X_i + (X_{3-i} - x)^+ \leq x\right) \leq F_{\bar{X}}(x)$ for any x.

(b) In the competition case, the infinitely overconfident newsvendors order μ, i.e., $\hat{q}(\alpha = 1) = \mu$. In the centralized system, $F_{\bar{X}}(q_c^*) = \beta$. Thus, $q_c^* < \hat{q}(\alpha = 1) = \mu$ if and only if $\beta < m$, where $F_{\bar{X}}(q_c^*) = \beta$ and $F_{\bar{X}}(\mu) = m$. Q.E.D.

Proof of Proposition 3 If $\beta > m$, then $F_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c) = \beta$ when $\alpha = \hat{\alpha}$ from Proposition 2. Thus, when $\alpha = \hat{\alpha}$, the competing newsvendors would order the same quantity that the central planner would order. Q.E.D.

Proof of Lemma 4 Because $D_1(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X_1$, $D_2(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X_2$ and the equilibrium solution satisfies $\mathrm{P}(D_i(\alpha_i) + (D_{3-i}(\alpha_i) - q_{3-i})^+) \leq q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + (D_i(\alpha_i) - q_i)^+ \leq q_{3-i}) = \beta$, Newsvendor i solves $\mathrm{P}(D_i(\alpha_i) + (D_{3-i}(\alpha_i) - q_{3-i})^+) \leq q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + (D_i(\alpha_i) - q_i)^+ \leq q_{3-i}) = \beta$ to obtain the equilibrium order quantity $\hat{q}_i^c(\alpha_i) = \hat{q}_i^c(\alpha_i) = \alpha_i\mu + (1-\alpha_i)\hat{z}_i$. Thus, $\mathrm{P}(X_i + (X_{3-i} - \hat{z}_{3-i})^+) \leq \hat{z}_i) = \mathrm{P}(X_{3-i} + (X_i - \hat{z}_i)^+ \leq \hat{z}_{3-i}) = \beta$. As a result, from the proof of Proposition 1, $\hat{z}_1 = \hat{z}_2 = q_n$ and $g(q_n) = \beta$. Q.E.D.

Proof of Proposition 4 For the overconfident newsvendors, the order quantities satisfy $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\mu + (2 - \alpha_1 - \alpha_2)q_n$. In contrast, the central planner's equilibrium order quantity is $2F_{\bar{X}}^{-1}(\beta)$. Thus, if $\beta > m$, then the system can be coordinated as in Proposition 2. In particular, $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$ when $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \mu)$. Q.E.D.

Proof of Proposition 5 To prove Proposition 5, we first show the following two-part lemma. Lemma 5(a) Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial\hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial\alpha_i} \leq 0$, for $i = 1, 2$), and Lemma 5(b) Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial\hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial\alpha_{3-i}} < 0$ for $i = 1, 2$) if and only if $\beta < \hat{m}^c$. Toward that end, first note from (8) and Lemma 4 that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+ \wedge (\alpha_1\mu + (1-\alpha_1)q_n)] -$

$c(\alpha_1\mu+(1-\alpha_1)q_n)$. Thus,

$$\begin{aligned}\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_1} &= (\mu-q_n)p[\mathrm{P}(\alpha_1\mu+(1-\alpha_1)q_n<X_1+(X_2-\alpha_2\mu-(1-\alpha_2)q_n)^+)-(1-\beta)]\\ &= (\mu-q_n)p[\beta-\mathrm{P}(\alpha_1\mu+(1-\alpha_1)q_n>X_1+(X_2-\alpha_2\mu-(1-\alpha_2)q_n)^+)]. \qquad (11)\end{aligned}$$

Accordingly:

Proof of Lemma 5(a): Without loss of generality, we let $i=1$. First, if $q_n>\mu$, then

$$\begin{aligned}\mathrm{P}(\alpha_1\mu+(1-\alpha_1)q_n>X_1+(X_2-\alpha_2\mu-(1-\alpha_2)q_n)^+) &< \mathrm{P}(q_n>X_1+(X_2-\alpha_2\mu-(1-\alpha_2)q_n)^+)\\ &< \mathrm{P}(q_n>X_1+(X_2-q_n)^+)\\ &= \beta,\end{aligned}$$

where the first inequality is from $\alpha_1\mu+(1-\alpha_1)q_n<q_n$, the second inequality is from $\alpha_2\mu+(1-\alpha_2)q_n<q_n$, and the equality is from Lemma 4. As a result, $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_1}<0$ if $q_n>\mu$. Second, if $q_n\leq\mu$, then

$$\begin{aligned}\mathrm{P}(\alpha_1\mu+(1-\alpha_1)q_n\geq X_1+(X_2-\alpha_2\mu-(1-\alpha_2)q_n)^+) &\geq \mathrm{P}(q_n>X_1+(X_2-\alpha_2\mu-(1-\alpha_2)q_n)^+)\\ &\geq \mathrm{P}(q_n>X_1+(X_2-q_n)^+)\\ &= \beta,\end{aligned}$$

where the first inequality is from $\alpha_1\mu+(1-\alpha_1)q_n\geq q_n$, the second inequality is from $\alpha_2\mu+(1-\alpha_2)q_n\geq q_n$, and the equality is from Lemma 4. As a result, $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_1}\leq 0$ if $q_n\leq\mu$. This completes the proof of Lemma 5(a).

Proof of Lemma 5(b) Without loss of generality, we let $i=1$. Accordingly, note that

$$\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_2}=p(q_n-\mu)\mathrm{P}[\alpha_1\mu+(1-\alpha_1)q_n>X_1+X_2-\alpha_2\mu-(1-\alpha_2)q_n, X_2\geq\alpha_2\mu+(1-\alpha_2)q_n].$$

Because $\mathrm{P}[\alpha_1\mu+(1-\alpha_1)q_n>X_1+X_2-(1-\alpha_2)q_n-\alpha_2\mu, X_2\geq\alpha_2\mu+(1-\alpha_2)q_n]\geq 0$, the sign of $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_2}$ is determined by $q_n-\mu$. As a result, $\frac{\partial\hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial\alpha_2}<0\iff\mu>q_n$. Furthermore, from Proposition 1, $\mu>q_n\iff\hat{m}^c>\beta$. This completes the proof of Lemma 5(b).

Given that Lemma 5(a) and Lemma 5(b) both are true: If $\alpha_1=\alpha_2$, then the newsvendors have identical expected profits. If $\alpha_1<\alpha_2$ and $\beta>\hat{m}^c$, then Lemma 5(a) and Lemma 5(b) together imply that the less biased newsvendor has a higher expected profit. However, if $\alpha_1 < \alpha_2$ and $\beta < \hat{m}^c$, then we show by example that the less biased newsvendor can have a lower expected profit. To that end, suppose $\alpha_1 = 0$. Then, from (11), and Lemma 5 (b),

$$\left.\frac{\partial \hat{\pi}_2^c(\alpha_1 = 0, \alpha_2)}{\partial \alpha_2} - \frac{\partial \hat{\pi}_1^c(\alpha_1 = 0, \alpha_2)}{\partial \alpha_2}\right|_{\alpha_2 = 0} = (\mu - q_n)p[\beta - \mathrm{P}(q_n > X_1 + (X_2 - q_n)^+)] + p(\mu - q_n)\mathrm{P}(q_n > X_1 + X_2 - q_n, X_2 \geq q_n) > 0$$

because $\mathrm{P}(X_1 + (X_2 - q_n)^+ < q_n) = \beta$ and $\beta < \hat{m}^c \Longrightarrow \mu > q_n$ from Proposition 1. As a result, the more biased newsvendor (Newsvendor 2) can have a higher expected profit than the less biased newsvendor (Newsvendor 1). Q.E.D.

Proof of Proposition 6 Without loss of generality, define Newsvendor 2 as the biased newsvendor characterized by overconfidence level α_2 and define Newsvendor 1 as the unbiased newsvendor cognizant of its competitor's α_2. Accordingly, let $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ denote the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Correspondingly, let $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ denote the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^c(\alpha_2) = \alpha_2\mu + (1 - \alpha_2)q_n$ from Lemma 4, and $q_1^{fc}(\alpha_2)$ is defined implicitly by

$$\mathrm{P}(X_1 + (X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \leq q_1^{fc}(\alpha_2)) = \beta. \tag{12}$$

Accordingly, Newsvendor 2's equilibrium expected profit can be written as $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}[X_2 + (X_1 - q_1^{fc}(\alpha_2))^+ \wedge (\alpha_2\mu + (1 - \alpha_2)q_n] - c[\alpha_2\mu + (1 - \alpha_2)q_n]$, which implies that

$$\begin{aligned}\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2} &= (\mu - q_n)p[\mathrm{P}(\alpha_2\mu + (1 - \alpha_2)q_n < X_2 + (X_1 - q_1^{fc}(\alpha_2))^+) - (1 - \beta)] \\ &\quad - \mathrm{E}\left[\mathbb{1}_{X_2 + (X_1 - q_1^{fc}(\alpha_2))^+ < \alpha_2\mu + (1 - \alpha_2)q_n, X_1 > q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right] \\ &= (\mu - q_n)p[\beta - \mathrm{P}(\alpha_2\mu + (1 - \alpha_2)q_n > X_2 + (X_1 - q_1^{fc}(\alpha_2))^+)] \\ &\quad - \mathrm{E}\left[\mathbb{1}_{X_2 + X_1 - q_1^{fc}(\alpha_2) < \alpha_2\mu + (1 - \alpha_2)q_n, X_1 > q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right],\end{aligned}$$

where $\mathbb{1}$ is the indicator function. This, in turns, implies that

$$\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2 = 0} = (\mu - q_n)p[\beta - \mathrm{P}(q_n > X_2 + (X_1 - q_n)^+)] - \mathrm{E}\left[\mathbb{1}_{X_2 + X_1 - q_n < q_n, X_1 > q_n} \left.\frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2 = 0}\right]$$

$$= -\mathrm{E}\left[\mathbb{1}_{X_2+X_1-q_n<q_n, X_1>q_n} \left.\frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0}\right] > 0, \tag{13}$$

where the second equality follows because $\mathrm{P}(X_2 + (X_1 - q_n)^+ < q_n) = \beta$ from (3), and the inequality follows because $\beta < \hat{m}^c \Longrightarrow q_n < \mu$ from Proposition 1, $q_n < \mu \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^{c}(\alpha_2)}{d\alpha_2} = \mu - q_n < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$ from (12).

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+ \wedge q_1^{fc}(\alpha_2)] - cq_1^{fc}(\alpha_2)$. Thus, from the envelope theorem,

$$\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\mathrm{E}\left[\mathbb{1}_{X_1+X_2-\alpha_2\mu-(1-\alpha_2)q_n<q_1^{fc}(\alpha_2), X_2>\alpha_2\mu+(1-\alpha_2)q_n}(q_n - \mu)\right] < 0, \tag{14}$$

where the inequality is because, again, $\beta < \hat{m}^c \Longrightarrow q_n < \mu$ from Proposition 1.

Taken together, (13)-(14) mean that Newsvendor 2's equilibrium expected profit is strictly increasing in α_2 around $\alpha_2 = 0$, whereas Newsvendor 1's equilibrium expected profit is strictly decreasing in α_2 around $\alpha_2 = 0$. However, at $\alpha_2 = 0$, $\hat{\pi}_2^{fc}(\alpha_2) = \pi_1^{fc}(\alpha_2)$. Thus, all told, this means that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ around $\alpha_2 = 0$, which suffices to establish Proposition 6. Q.E.D.

Electronic Companion To "Overconfident Competing Newsvendors"

In this technical supplement, we verify the scope and applicability of our key results by exploring the effects of four modeling variations: partial spillover (§EC.1), inventory-dependent demand rationing (§EC.2), random demand splitting (§EC.3), and an expanded definition of overconfidence (§EC.4). The proofs for the technical results of this supplement are compiled in its appendix.

EC.1. Partial Spillover

In this extension, we consider the case in which only a fixed portion $\theta \in [0,1]$ of unsatisfied customers from one newsvendor is willing to visit the other newsvendor to assess the extent to which the qualitative results of §3-5 depend on the level of spillover demand between competing newsvendors. Note that when $\theta = 1$, this model reduces to the base model in §§3-5.

EC.1.1. Symmetric Overconfidence

Given demands (X_1, X_2) and order quantities (q_1, q_2), the expected profit for Newsvendor i is, for $i = 1, 2$,

$$\pi_i(q_1, q_2) = p\mathrm{E}[(X_i + \theta(X_{3-i} - q_{3-i})^+) \wedge q_i] - cq_i.$$

The (unbiased) equilibrium order quantities (q_1^c, q_2^c) satisfy

$$\mathrm{P}\left(X_i + \theta(X_{3-i} - q_{3-i}^c)^+ \leq q_i^c\right) = \beta, \tag{EC.1}$$

for $i = 1, 2$. Given overconfidence level α, Newsvendor i behaves as though random demands X_1 and X_2 are instead $D_1(\alpha)$ and $D_2(\alpha)$, respectively, where $D_i(\alpha) = \alpha\mu + (1-\alpha)X_i$ and $\mu = \mathrm{E}[X_1] = \mathrm{E}[X_2]$. Consequently, Newsvendor i behaves as though its game with its rival satisfies

$$\max_{q_i} \pi_i(\alpha) = p\mathrm{E}[(D_i(\alpha) + \theta(D_{3-i}(\alpha) - q_{3-i})^+) \wedge q_i] - cq_i, \tag{EC.2}$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha) = p\mathrm{E}[(D_{3-i}(\alpha) + \theta(D_i(\alpha) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}. \tag{EC.3}$$

As in §3, Lemma EC.1 characterizes the equilibrium solution to (EC.2)-(EC.3).

Lemma EC.1. *The equilibrium ordering quantities $(\hat{q}_1^c, \hat{q}_2^c)$ for the overconfident newsvendor system (EC.2)-(EC.3) exist and are unique. Moreover, they are identical, and $\hat{q}^c := \hat{q}_1^c = \hat{q}_2^c$ satisfies $P(D_1(\alpha) + \theta(D_2(\alpha) - \hat{q}^c)^+ \leq \hat{q}^c) = P(D_2(\alpha) + \theta(D_1(\alpha) - \hat{q}^c)^+ \leq \hat{q}^c) = \beta$.*

As in §3, define $\hat{z}^c$ by $\hat{q}^c(\alpha) = \alpha\mu + (1-\alpha)\hat{z}^c$. Proposition EC.1 next characterizes $\hat{z}^c$, and shows that "pull-to-center" effect in our full spillover setting (Proposition 1) also holds in a partial spillover setting.

Proposition EC.1. *The equilibrium $\hat{z}^c$ for the overconfident newsvendor system (EC.2)-(EC.3) is $\hat{z}^c = g^{-1}(\beta) = q_n$. Accordingly, $d\hat{q}^c(\alpha)/d\alpha < 0$ if and only if $\beta > \hat{m}^c$ for $i = 1, 2$, where $\hat{m}^c := g(\mu) < \hat{m}$ and $g(y) := P[X_1 + \theta(X_2 - y)^+ \leq y]$.*

As in the full spillover case (Proposition 2), the impact of overconfidence on the profits of newsvendors depends on the critical value β in Proposition EC.2.

Proposition EC.2. *Let $\hat{\pi}^c(\alpha) = \hat{\pi}_1(\alpha) = \hat{\pi}_2(\alpha)$ denote the equilibrium expected profits for the overconfident newsvendor system (EC.2)-(EC.3). Then, $\hat{\pi}^c(\alpha)$ is concave in α. Moreover: (a) if $\beta \leq \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ decreases in α; (b) if $\hat{m}^c < \beta \leq m$, then $\hat{\pi}^c(\alpha)$ increases in α, where $m := h(\mu)$ and $h(\cdot)$ is defined as*

$$h(x) := P[X_1 + \theta(X_2 - x)^+ \leq x] + \theta P[X_2 \geq x, X_1 + \theta X_2 \leq (1+\theta)x]; \tag{EC.4}$$

(c) if $\beta > m$, then, $\hat{\pi}^c(\alpha)$ increases in α for $\alpha \in (0, \hat{\alpha}]$ and then, decreases in α for $\alpha \in [\hat{\alpha}, 1]$, where $\hat{\alpha} := \frac{g^{-1}(\beta) - h^{-1}(\beta)}{g^{-1}(\beta) - \mu}$.

Proposition EC.2 thus shows that overconfidence remains a potential positive force in this extension. Indeed, because $\hat{m}^c = \mathrm{P}(X_1 + \theta(X_2 - \mu)^+ \leq \mu)$ decreases as θ increases, Proposition EC.2 implies that the greater is the spillover demand between competing newsvendors, the easier it is for overconfidence to yield a system-wide positive effect. Intuitively, this follows because the higher is the residual demand from its rival, the greater is the competitive force that derives a newsvendor to inflate its order quantity over the system optimal. And, the greater is that inflation, the more room is provided for the counterbalance benefit of overconfidence.

Lemma EC.2. *(a) An (unbiased) centralized planner orders less than unbiased competing newsvendors. That is,* $q_c^* < \hat{q}^c(\alpha = 0)$.

(b) Suppose $\alpha = 1$. *An (unbiased) centralized planner orders less than the overconfident competing newsvendors if* $\beta < m$ *but orders more than the competing overconfident newsvendors if* $\beta > m$. *That is,* $q_c^* < \hat{q}(\alpha = 1) \Longleftrightarrow \beta < m$.

Thus, similar to the full spillover case, overconfidence can coordinate the system.

Proposition EC.3. *If* $\beta > m$, *then competing newsvendors with overconfidence level* $\alpha = \hat{\alpha}$ *orders the same quantity that an (unbiased) central planner orders, i.e.,* $\hat{q}^c(\hat{\alpha}) = q_c^*$.

EC.1.2. Asymmetric Overconfidence

In this case with asymmetric overconfidence, Newsvendor i behaves as though its game with its rival were described by

$$\max_{q_i} \pi_i(\alpha_i) = p\mathrm{E}[(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \wedge q_i] - cq_i,$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha_i) = p\mathrm{E}[(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}.$$

Thus, Newsvendor i, $i = 1, 2$, solves $\mathrm{P}(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \le q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) - q_i)^+ \le q_{3-i}) = \beta$ to derive the equilibrium order quantities $(\hat{q}_1^c(\alpha_i), \hat{q}_2^c(\alpha_i))$. Accordingly, Newsvendor 1 orders $\hat{q}_1(\alpha_1)$ and anticipates Newsvendor 2 to order $\hat{q}_2(\alpha_1)$, whereas Newsvendor 2 orders $\hat{q}_2(\alpha_2)$ and anticipates Newsvendor 1 to order $\hat{q}_1(\alpha_2)$. The ensuing expected profits are

$$\hat{\pi}_i^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_i + \theta(X_{3-i} - \hat{q}_{3-i}^c(\alpha_{3-i}))^+ \wedge \hat{q}_i^c(\alpha_i)] - c\hat{q}_i^c(\alpha_i). \tag{EC.5}$$

Lemma EC.3. *Associated with the overconfidence levels* α_1 *and* α_2 *are the order quantities* $\hat{q}_i^c(\alpha_i) = \alpha_i\mu + (1 - \alpha_i)q_n$, *where* $q_n = g^{-1}(\beta)$ *as defined in Proposition EC.1.*

We next explore whether there exist asymmetric overconfidence combinations that would effectively coordinate the system. Toward that end, note that the unbiased central planner would order $2q_c^* = 2h^{-1}(\beta)$ for the system, whereas the total order quantity for the two competing newsvendors is $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\mu + (2 - \alpha_1 - \alpha_2)q_n$.

Proposition EC.4. *Suppose $\beta > m$, and that α_1 and α_2 satisfy $(\alpha_1+\alpha_2)/2 = \hat{\alpha}$, where $\hat{\alpha} = (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta)-\mu)$ as defined in Proposition EC.2. Then the sum of the newsvendors' orders equals that of a central planner: $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$.*

Proposition EC.4 not only extends Proposition EC.3 to the asymmetric bias case, but also verifies that $\hat{\alpha}$ represents the level of bias distortion that effectively counterbalances the impact of competition. Proposition EC.5 next reveals that a less biased newsvendor does not necessarily earn a higher expected profit than its more biased competitor as in Proposition 5.

Proposition EC.5. *With a partial spillover, if $\beta > \hat{m}^c$, then the less biased newsvendor earns a higher equilibrium expected profit than the more biased newsvendor, i.e., if $\beta > \hat{m}^c$, then $\alpha_1 < \alpha_2 \Longrightarrow \hat{\pi}_1^c(\alpha_1, \alpha_2) > \hat{\pi}_2^c(\alpha_1, \alpha_2)$. However, if $\beta < \hat{m}^c$, then the more biased newsvendor can earn a higher equilibrium expected profit than the less biased newsvendor.*

Recall that $\hat{m}^c$ decreases in the spillover ratio θ. Thus, Proposition EC.5 further reveals that although a smaller spillover demand makes it harder for overconfidence to have an overall positive effect (as per Proposition EC.2), it nevertheless makes it easier for the more biased newsvendor to outperform its rival. Next, Proposition EC.6 demonstrates that this overconfidence advantage remains even if the biased newsvendor's competitor is a sophisticated newsvendor as in Proposition 6.

Proposition EC.6. *A sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased competitor.*

EC.2. Inventory-Dependent Demand Rationing

In this extension, we consider the case in which the total market demand is a random variable X with a support $[0,\infty)$ and a distribution function $F(\cdot)$, and the realized demand is initially split between two firms based on their order quantities q_1 and q_2. In particular, the initial demand for Newsvendor i is $\frac{q_i}{q_1+q_2}X$, for $i = 1, 2$.

We first consider the scenario in which both newsvendors are unbiased. This setting is an extension of the classic competitive newsvendor setting (Lippman and McCardle 1997). For order quantities (q_1, q_2), Newsvendor 1's effective demand is $\frac{q_1}{q_1+q_2}X + (\frac{q_2}{q_1+q_2}X - q_2)^+$, where the first term is Newsvendor 1's initial demand and the second term is the spillover demand from Newsvendor 2. It follows that Newsvendor 1's sales is

$$\left[\frac{q_1}{q_1+q_2}X + \left(\frac{q_2}{q_1+q_2}X - q_2\right)^+\right] \wedge q_1 = \begin{cases} \frac{q_1}{q_1+q_2}X & X \le q_1+q_2; \\ q_1 & X > q_1+q_2. \end{cases}$$

Similarly, we can obtain Newsvendor 2's sales for given order quantities (q_1, q_2). So, Newsvendor i's expected profit is

$$\pi_i(q_1,q_2) = -cq_i + p\int_{q_1+q_2} q_i dF(x) + p\int^{q_1+q_2} \frac{q_i}{q_1+q_2} x dF(x). \tag{EC.6}$$

It can be shown that $\pi_i(q_1,q_2)$ is strictly concave, and Nash equilibrium order quantities (q_1^c, q_2^c) exist and are unique with $q_1^c = q_2^c$. Let $q_n := q_1^c = q_2^c$ and $Q_n := 2q_n$. Then, Q_n satisfies

$$\frac{1}{2}F(Q_n) + \frac{1}{2Q_n}\int^{Q_n} F(x)dx = \beta. \tag{EC.7}$$

Next, we consider the case of overconfident newsvendors. In particular, Newsvendor i, with overconfidence level α_i, will act as if the total demand were $D(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X$. Then, $F_{D_i}(x) = F(\frac{x-\alpha_i\mu}{1-\alpha_i})$. It follows from (EC.7) that the equilibrium order quantities $(\hat{q}_1^c, \hat{q}_2^c)$ satisfy

$$\frac{1}{2}F_{D_i}(\hat{Q}^c) + \frac{1}{2\hat{Q}^c}\int_{\alpha\mu}^{\hat{Q}^c} F_{D_i}(x)dx = \beta, \tag{EC.8}$$

where $\hat{Q}^c = 2\hat{q}_c = 2\hat{q}_1^c = 2\hat{q}_2^c$. That is, $\frac{1}{2}F(\frac{\hat{Q}^c-\alpha_i\mu}{1-\alpha_i}) + \frac{1-\alpha}{2\hat{Q}^c}\int_0^{\frac{\hat{Q}^c-\alpha_i\mu}{1-\alpha_i}} F(x)dx = \beta$. To emphasize the dependence of $\hat{Q}^c$ on α, we write $\hat{Q}^c$ as $\hat{Q}^c(\alpha)$ when necessary.

Next, we compare the two newsvendors' profits when their overconfidence levels are different $(\alpha_1 < \alpha_2)$. Thus, Newsvendor i's order quantity is $\hat{q}_i^c = \hat{Q}^c(\alpha_i)/2$, for $i = 1, 2$. So, by (EC.6), Newsvendor i's expected profit is $\hat{\pi}_i^c = -c\hat{q}_i^c + p\int_{\hat{q}_1^c+\hat{q}_2^c} \hat{q}_i^c dF(x) + p\int^{\hat{q}_1^c+\hat{q}_2^c} \frac{\hat{q}_i^c}{\hat{q}_1^c+\hat{q}_2^c} x dF(x)$. It follows that

$$\hat{\pi}_2^c - \hat{\pi}_1^c = c\hat{q}_1^c - p\int_{\hat{q}_1^c+\hat{q}_2^c} \hat{q}_1^c dF(x) - p\int^{\hat{q}_1^c+\hat{q}_2^c} \frac{\hat{q}_1^c}{\hat{q}_1^c+\hat{q}_2^c} x dF(x) - c\hat{q}_2^c + p\int_{\hat{q}_1^c+\hat{q}_2^c} \hat{q}_2^c dF(x)$$

$$
\begin{aligned}
&+p\int^{\hat{q}_1^c+\hat{q}_2^c}\frac{\hat{q}_2^c}{\hat{q}_1^c+\hat{q}_2^c}xdF(x)\\
&=-c(\hat{q}_2^c-\hat{q}_1^c)+p(\hat{q}_2^c-\hat{q}_1^c)\bar{F}(\hat{q}_1^c+\hat{q}_2^c)+\frac{p(\hat{q}_2^c-\hat{q}_1^c)}{\hat{q}_1^c+\hat{q}_2^c}\int^{\hat{q}_1^c+\hat{q}_2^c}xdF(x)\\
&=(\hat{q}_2^c-\hat{q}_1^c)\left[-c+p\bar{F}(\hat{q}_1^c+\hat{q}_2^c)+\frac{p}{\hat{q}_1^c+\hat{q}_2^c}\int^{\hat{q}_1^c+\hat{q}_2^c}xdF(x)\right]\\
&=(\hat{q}_2^c-\hat{q}_1^c)\left[-c+p\bar{F}(\hat{q}_1^c+\hat{q}_2^c)+\frac{p}{\hat{q}_1^c+\hat{q}_2^c}\,xF(x)|_0^{\hat{q}_1^c+\hat{q}_2^c}-\frac{p}{\hat{q}_1^c+\hat{q}_2^c}\int^{\hat{q}_1^c+\hat{q}_2^c}F(x)dx\right]\\
&=p\frac{\hat{q}_2^c-\hat{q}_1^c}{\hat{q}_1^c+\hat{q}_2^c}\left(\beta(\hat{q}_1^c+\hat{q}_2^c)-\int^{\hat{q}_1^c+\hat{q}_2^c}F(x)dx\right). \qquad \text{(EC.9)}
\end{aligned}
$$

In order to develop analytical insights, we assume that $X\sim U[0,1]$ in the remainder of §EC.2. Hence, (EC.9) specializes to

$$
\hat{\pi}_2^c-\hat{\pi}_1^c=\begin{cases} p(\hat{q}_2^c-\hat{q}_1^c)(\beta-\frac{\hat{q}_1^c+\hat{q}_2^c}{2}) & \hat{q}_1^c+\hat{q}_2^c\le 1;\\ p\frac{\hat{q}_2^c-\hat{q}_1^c}{\hat{q}_1^c+\hat{q}_2^c}(\beta(\hat{q}_1^c+\hat{q}_2^c)-\frac{1}{2}) & \hat{q}_1^c+\hat{q}_2^c>1.\end{cases} \qquad \text{(EC.10)}
$$

Lemma EC.4 derives a closed-form solution for $\hat{q}_i^c(\alpha_i)$, the equilibrium order quantity of Newsvendor i with the overconfidence level α_i.

LEMMA EC.4. *Suppose $X\sim U[0,1]$. Then,*

$$
\hat{q}_i^c(\alpha_i)=\begin{cases}\frac{1}{6}\alpha_i+\frac{1}{3}\beta-\frac{1}{3}\alpha_i\beta+\frac{1}{6}h(\alpha_i,\beta) & \beta\le\frac{3-2\alpha_i}{2(2-\alpha_i)};\\ \frac{1}{8(1-\beta)} & \beta>\frac{3-2\alpha_i}{2(2-\alpha_i)},\end{cases}
$$

where

$$
h(\alpha_i,\beta)=\sqrt{4\alpha_i^2\beta^2-4\alpha_i^2\beta+\frac{1}{4}\alpha_i^2-8\alpha_i\beta^2+4\alpha_i\beta+4\beta^2}.
$$

EC.2.1. Symmetric Overconfidence

Assume $\alpha_1=a_2=\alpha$. Then, from Lemma EC.4, the equilibrium order quantity is

$$
\hat{q}^c(\alpha)=\hat{q}_i^c(\alpha)=\begin{cases}\frac{1}{6}\alpha+\frac{1}{3}\beta-\frac{1}{3}\alpha\beta+\frac{1}{6}h(\alpha,\beta) & \beta\le\frac{3-2\alpha}{2(2-\alpha)};\\ \frac{1}{8(1-\beta)} & \beta>\frac{3-2\alpha}{2(2-\alpha)}.\end{cases}
$$

And the equilibrium expected profit for each newsvendor is

$$
\hat{\pi}^c=\hat{\pi}_i^c=\begin{cases}-c\hat{q}^c+p\hat{q}^c(1-\hat{q}^c) & 2\hat{q}^c\le 1;\\ -c\hat{q}^c+\frac{1}{4}p & 2\hat{q}^c>1.\end{cases}
$$

Define β_c as the unique solution to the cubic equation $-16(1-\alpha)^2\beta^3+16\,(2\alpha-1)\,(\alpha-1)\,\beta^2+(16\alpha-17\alpha^2-4)\,\beta+\alpha^2=0$ in the interval $[\frac{1-2\alpha+\sqrt{3\alpha^2+1-3\alpha}}{4(1-\alpha)},\frac{1}{2}]$. Note that the existence and uniqueness of β_c are established in the proof of Proposition EC.7.

Proposition EC.7. *$\frac{\partial \hat{q}^c}{\partial \alpha} \geq 0$ if $\beta \leq \beta_c$, $\frac{\partial \hat{q}^c}{\partial \alpha} \leq 0$ if $\beta_c < \beta \leq \frac{3-2\alpha}{2(2-\alpha)}$, and $\frac{\partial \hat{q}^c}{\partial \alpha} = 0$ for $\beta > \frac{3-2\alpha}{2(2-\alpha)}$.*

Proposition EC.8 is the counterpart of Proposition 2 in §3. It characterizes how the overconfidence level affects a firm's expected profit at equilibrium, and demonstrates that overconfidence can benefit the newsvendor if $\beta > \beta_c$.

Proposition EC.8. *$\frac{\partial \hat{\pi}^c}{\partial \alpha} \leq 0$ if $\beta \leq \beta_c$; $\frac{\partial \hat{\pi}^c}{\partial \alpha} \geq 0$ if $\beta_c < \beta \leq \frac{3-2\alpha}{4-2\alpha}$; $\frac{\partial \hat{\pi}^c}{\partial \alpha} = 0$ if $\beta > \frac{3-2\alpha}{4-2\alpha}$.*

In this inventory-dependent demand rationing case, the demands for the two competing newsvendors are positively correlated. Yet, if a similar scenario of perfectly correlated demands were applied in §3, then overconfidence would never be beneficial because, in that case, $\hat{m}^c = m$ and $\hat{\alpha} = 0$ from Proposition 2. Thus, similar to the effects and intuition associated with spillover demand in the first extension, demand-rationing in this extension essentially enhances the beneficial effects of overconfidence from §3. Note, however, from the proof of Proposition EC.8, that the sum of the order quantities of the two overconfident newsvendors is strictly greater than the optimal total order quantity for the two products in the centralized system. Thus, the two overconfident newsvendors cannot achieve the first-best outcome when $X \sim U[0,1]$. We attribute this technicality to the linearity of the uniform distribution.

EC.2.2. Asymmetric Overconfidence

Proposition EC.9 is the counterpart of Proposition 5 in §4, and shows that the more overconfident newsvendor can earn a higher expected profit than the less overconfident one.

Proposition EC.9. *Suppose $X \sim U[0,1]$ and $\alpha_1 = 0$. Then, (i) for $\beta \in [0, 3/16]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ if and only if $\alpha_2 \leq \frac{8}{3}\beta\left(4 - 8\beta - \sqrt{2}\sqrt{32\beta^2 - 32\beta + 5}\right)$; (ii) for $\beta \in (3/16, 3/8]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ for all α_2; (iii) for $\beta \in (3/8, 1/2]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ if and only if $\alpha_2 \leq 4/3 - (8\beta - 2)^2/3$; (iv) for $\beta \in (1/2, 3/4]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ if and only if $\alpha_2 \geq \frac{1}{2\beta - 2}(4\beta - 3)$; (v) for $\beta > 3/4$. $\hat{\pi}_2^c - \hat{\pi}_1^c = 0$ for all α_2.*

The intuition behind Proposition EC.9 is similar to that behind Proposition 5: The more overconfident newsvendor can earn a higher expected profit than the less overconfident one because the former may uncannily place a more proper order than its rival. Geometrically, the shape of $\hat{q}_2^c$ as a function of α_2 depends on the value of β. When the critical ratio β is low ($\beta \le 1/2$), $\hat{q}_2^c$ increases in α_2 when α_2 is low; consequently, $\hat{q}_2^c(\alpha_2) \ge \hat{q}_1^c(0)$ for low α_2. At the same time, both newsvendors tend to order less when β is small. These two factors imply that the more overconfident newsvendor (Newsvendor 2) earns a higher expected profit than the less overconfident one when the former's overconfidence level is low. When the critical ratio β is medium ($1/2 < \beta \le 3/4$), $\hat{q}_2^c$ decreases in α_2. Consequently, the overconfident newsvendor earns a higher expected profit than its rival when its overconfident level is high. When the critical ratio β is high ($\beta > 3/4$), both newsvendors earn the same expected profit because they order the same quantity regardless of the more biased newsvendor's overconfidence level.

Next, we show that an overconfident newsvendor's profit even can be greater than that of a sophisticated rival. Toward that end, we study a case where Newsvendor 1 is unbiased ($\alpha_1 = 0$), and is fully cognizant of Newsvendor 2's overconfidence level α_2. By (EC.6), for $(q_1, \hat{q}_2^c)$, Newsvendor 1's expected profit is

$$\begin{aligned}\pi_1(q_1, \hat{q}_2^c) &= -cq_1 + p\int_{q_1+\hat{q}_2^c} q_1 dx + p\int^{q_1+\hat{q}_2^c} \frac{q_1}{q_1+\hat{q}_2^c} x dx \\ &= \begin{cases} (p-c)q_1 - p\frac{q_1}{2}(q_1+\hat{q}_2^c) & q_1+\hat{q}_2^c \le 1; \\ \frac{1}{2}p\frac{q_1}{q_1+\hat{q}_2^c} - cq_1 & q_1+\hat{q}_2^c > 1. \end{cases}\end{aligned} \tag{EC.11}$$

Lemma EC.5. *Assume $X \sim U[0,1]$. Then, Newsvendor 1's equilibrium order quantity satisfies:*

$$\hat{q}_1^{fc} = \begin{cases} 0 & \beta \le \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ \beta - \frac{1}{2}\hat{q}_2^c & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}; \\ \frac{1}{2-2\beta}\left(\sqrt{2(1-\beta)\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right) & \beta > \frac{3}{4}. \end{cases} \tag{EC.12}$$

With (EC.11)-(EC.12), we have the two newsvendors' expected profits:

$$\hat{\pi}_1^{fc} = \begin{cases} 0 & \beta \le \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ (p-c)\hat{q}_1^{fc} - p\frac{\hat{q}_1^{fc}}{2}(\hat{q}_1^{fc} + \hat{q}_2^c) & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}; \\ -c\hat{q}_1^{fc} + \frac{1}{2}p\frac{\hat{q}_1^{fc}}{q_1+\hat{q}_2^c} & \beta > \frac{3}{4}. \end{cases}$$

$$\hat{\pi}_2^{fc} = \begin{cases} \hat{q}_2^c(p-c-p\hat{q}_2^c/2) & \beta \le \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ (p-c)\hat{q}_2^c - p\frac{\hat{q}_2^c}{2}(\hat{q}_1^{fc} + \hat{q}_2^c) & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}; \\ -c\hat{q}_2^c + \frac{1}{2}p\frac{\hat{q}_2^c}{\hat{q}_1^{fc}+\hat{q}_2^c} & \beta > \frac{3}{4}. \end{cases}$$

It follows that

$$\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \begin{cases} p\hat{q}_2^c(\beta - \hat{q}_2^c/2) & \beta \le \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16}; \\ p(\hat{q}_2^c - \hat{q}_1^{fc})(\beta - \frac{1}{2}(\hat{q}_1^{fc} + \hat{q}_2^c)) & \alpha_2 \frac{\sqrt{-\alpha_2+2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}; \\ p(\hat{q}_2^c - \hat{q}_1^{fc})(1 - \beta + \frac{1}{2}\frac{1}{\hat{q}_1^{fc}+\hat{q}_2^c}) & \beta > \frac{3}{4}. \end{cases} \tag{EC.13}$$

As a result, analogous to Proposition 6 in §5, Proposition EC.10 shows that the even a sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased rival.

Proposition EC.10. *Suppose $X \sim U[0,1]$ and Newsvendor 1 is sophisticated. Then,*

$$\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \begin{cases} \le 0 & 0 < \beta \le \frac{1}{8}\sqrt{4 - 3\alpha_2} + \frac{1}{4}; \\ \ge 0 & \frac{1}{8}\sqrt{4 - 3\alpha_2} + \frac{1}{4} < \beta \le \frac{3}{4}; \\ = 0 & \frac{3}{4} < \beta \le 1. \end{cases}$$

EC.3. Random Demand Splitting

Next, we consider a setting in which the market demand Y is split randomly between the two newsvendors. Specifically, Newsvendor i's demand is $X_i := r_i Y$, for $i = 1, 2$, where r_i is random with a support $[0,1]$ and $r_1 + r_2 = 1$ almost surely. Newsvendors are overconfident, and Newsvendor i behaves as though the demand is $D(\alpha_i) = \alpha_i \mu_i + (1 - \alpha_i) X_i$, where $\mu_i = \mathrm{E}[r_i Y]$.

EC.3.1. Symmetric Overconfidence

As in §3, assume that newsvendors are identical in their overconfidence biases, i.e., $\alpha_1 = \alpha_2 = \alpha$. Then, Newsvendor i behaves as though its game with its rival were described by $\max_{q_i} \pi_i(\alpha) = p\mathrm{E}[(D_i(\alpha) + (D_{3-i}(\alpha) - q_{3-i})^+) \wedge q_i] - cq_i$, and $\max_{q_{3-i}} \pi_{3-i}(\alpha) = p\mathrm{E}[(D_{3-i}(\alpha) + (D_i(\alpha) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}$, where $D_i(\alpha) = \alpha\mu_i + (1-\alpha)X_i$. Thus, the equilibrium order quantities satisfy $\mathrm{P}(D_1(\alpha) + (D_2(\alpha) - \hat{q}_2^c)^+ \le \hat{q}_1^c) = \mathrm{P}(D_2(\alpha) + (D_1(\alpha) - \hat{q}_1^c)^+ \le \hat{q}_2^c) = \beta$. Accordingly, define $\hat{z}_i^c$ such that $\hat{q}_i^c(\alpha) = \alpha\mu_i + (1-\alpha)\hat{z}_i^c$. Then, $\mathrm{P}(X_1 + (X_2 - \hat{z}_2^c)^+ \le \hat{z}_1^c) = \mathrm{P}(X_2 + (X_1 - \hat{z}_1^c)^+ \le \hat{z}_2^c) = \beta$.

Note that $\hat{z}_1^c$ and $\hat{z}_2^c$ both are independent of α. Thus, $dq_i^c(\alpha)/d\alpha < 0$ if and only if $\hat{z}_i^c > \mu$. Moreover, note that $\hat{z}_i^c$ is Newsvendor i's order quantity when $\alpha = 0$. Thus, the pull-to-center effect (Proposition 1) holds, however, unlike in Proposition 1, we cannot provide a closed-form characterization of the understocking vs. overstocking condition resulting from overconfidence.

Next, note that the equilibrium system profit is $\hat{\pi}_1^c(\alpha) + \hat{\pi}_2^c(\alpha) = 2\mu(p-c) - (p-c)\mathrm{E}[X_1 + X_2 - \hat{q}_1^c(\alpha) - \hat{q}_2^c(\alpha)]^+ - c\mathrm{E}[\hat{q}_1^c(\alpha) + \hat{q}_2^c(\alpha) - X_1 - X_2]^+$. Thus,

$$\frac{d[\hat{\pi}_1^c(\alpha) + \hat{\pi}_2^c(\alpha)]}{d\alpha} = 2(\bar{z} - \mu)[(c-p)\bar{F}_{\bar{X}}((1-\alpha)\bar{z} + \alpha\mu) + cF_{\bar{X}}((1-\alpha)\bar{z} + \alpha\mu)]$$

$$= 2(\bar{z}-\mu)[(c-p)+pF_{\bar{X}}((1-\alpha)\bar{z}+\alpha\mu)]$$

$$= 2p(\bar{z}-\mu)[-\beta+F_{\bar{X}}((1-\alpha)\bar{z}+\alpha\mu)],$$

where $\bar{X}=Y/2$ and $\bar{z}=\frac{\hat{z}_1^c+\hat{z}_2^c}{2}$. As a result, $\frac{d^2[\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)]}{d\alpha^2}=-p(\bar{z}-\mu)^2 f_{\bar{X}}((1-\alpha)\bar{z}+\alpha\mu)<0$, which indicates that the system equilibrium profit $\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)$ is concave in α. Moreover, $\frac{d[\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)]}{d\alpha}|_{\alpha=1}=p(\beta-m)(\mu-\bar{z})>0$ if $\bar{z}<\mu$. Therefore, if $\bar{z}<\mu$, then $\hat{\pi}_1^c(\alpha)+\hat{\pi}_2^c(\alpha)$ is an increasing function because of its concavity, which implies that the overall system can benefit from overconfidence (Proposition 2).

We next check whether overconfidence can lead to the first-best equilibrium (Proposition 3). Note that the central planner solves

$$\max_{q_1,q_2} p\mathrm{E}[(X_1+(X_2-q_2)^+)\wedge q_1]-cq_1+p\mathrm{E}[(X_2+(X_1-q_1)^+)\wedge q_2]-cq_2.$$

Consequently, if $F_{\bar{X}}(\gamma\bar{z}+(1-\gamma)\mu)=\beta$, then overconfidence can yield a first-best equilibrium (Proposition 3).

EC.3.2. Asymmetric Overconfidence

We next examine whether the more biased newsvendor can earn a higher expected profit than its less biased rival. For this purpose, we assume r_1 and r_2 are identically distributed, which guarantees that newsvendors have identical expected profits when they are symmetric on overconfidence levels. Furthermore, Newsvendor i behaves as though its game with its rival were described by $\max_{q_i}\pi_i(\alpha_i)=p\mathrm{E}[(D_i(\alpha_i)+(D_{3-i}(\alpha_i)-q_{3-i})^+)\wedge q_i]-cq_i$, and $\max_{q_{3-i}}\pi_{3-i}(\alpha_i)=p\mathrm{E}[(D_{3-i}(\alpha_i)+(D_i(\alpha_i)-q_i)^+)\wedge q_{3-i}]-cq_{3-i}$. As in §4, the stocking factors are identical, i.e., $\hat{z}^c:=\hat{z}_1^c=\hat{z}_2^c$.

Proposition EC.11 shows that the less biased newsvendor can earn a lower expected profit than its rival, and this result remains true even if the less biased newsvendor is sophisticated.

Proposition EC.11. *Let $\hat{m}^c=P(X_1+(X_2-\bar{\mu})^+\le\bar{\mu})$ with $\bar{\mu}=E[r_iY]$ for $i=1,2$. If $\beta<\hat{m}^c$, then the more biased newsvendor can earn a higher expected profit than the less biased newsvendor with a random spliting. Furthermore, even a sophisticated newsvendor can earn a lower expected profit in equilibrium than its biased competitor.*

EC.4. Expanded Definition of Overconfidence

In this section, we extend our analysis to the case in which a newsvendor with demand X behaves as though the demand were $D(\alpha) = \alpha\rho + (1-\alpha)X$, where $\rho \geq 0$. Note that when $\rho = \mu$, this model reduces to the base model in §§3-5.

EC.4.1. Symmetric Overconfidence

Newsvendor i behaves as though its game with its rival satisfies

$$\max_{q_i} \pi_i(\alpha) = p\mathrm{E}[(D_i(\alpha) + (D_{3-i}(\alpha) - q_{3-i})^+) \wedge q_i] - cq_i, \tag{EC.14}$$

and

$$\max_{q_{3-i}} \pi_{3-i}(\alpha) = p\mathrm{E}[(D_{3-i}(\alpha) + (D_i(\alpha) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}, \tag{EC.15}$$

where $D(\alpha_i) = \alpha_i\rho + (1-\alpha_i)X$ for $i = 1, 2$. The equilibrium ordering quantities $(\hat{q}_1^c, \hat{q}_2^c)$ for the overconfident newsvendor system (EC.14)-(EC.15) exist and are unique. Moreover, they are identical, and $\hat{q}^c := \hat{q}_1^c = \hat{q}_2^c$ satisfies $\mathrm{P}(D_1(\alpha) + (D_2(\alpha) - \hat{q}^c)^+ \leq \hat{q}^c) = \mathrm{P}(D_2(\alpha) + (D_1(\alpha) - \hat{q}^c)^+ \leq \hat{q}^c) = \beta$. Note that the equilibrium $\hat{z}^c$ for the overconfident newsvendor system (EC.14)-(EC.15) is $\hat{z}^c = g^{-1}(\beta) = q_n$. Accordingly, $d\hat{q}^c(\alpha)/d\alpha < 0$ if and only if $\beta > \hat{m}^c$ for $i = 1, 2$, where $\hat{m}^c := g(\rho)$. Note that $\hat{m}^c$ is different from the one in §3, which is defined as $g(\mu)$.

Proposition EC.12. *Let $\hat{\pi}^c(\alpha) = \hat{\pi}_1(\alpha) = \hat{\pi}_2(\alpha)$ denote the equilibrium expected profits for the overconfident newsvendors. Then, $\hat{\pi}^c(\alpha)$ is concave in α. Moreover: (a) if $\beta \leq \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ decreases in α; (b) if $\hat{m}^c < \beta < m$, then $\hat{\pi}^c(\alpha)$ increases in α, where $m := F_{\bar{X}}(\rho)$ and $\bar{X} = \frac{X_1 + X_2}{2}$; (c) if $m \leq \beta$, then, $\hat{\pi}^c(\alpha)$ increases in α for $\alpha \in (0, \hat{\alpha}]$ and then decreases in α for $\alpha \in [\hat{\alpha}, 1]$, where $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \rho)$.*

Proposition EC.12 thus expands Proposition 2 to the more general definition of overconfidence considered here. To further illustrate its applicability, suppose $\rho = q^*$. Then, $\hat{m}^c = g(q^*) < \beta = F_X(q^*)$, and $\beta < m \iff F_X(q^*) < F_{\bar{X}}(q^*)$. Consequently, the condition for case (b) of Proposition EC.12 boils down to $F_X(q^*) < F_{\bar{X}}(q^*)$, which is equivalent to $q^* > 0.5 \iff \beta > 0.5$ when

$X \sim U[0,1]$. Thus, when $\beta > 0.5$ and $X \sim U[0,1]$, the newsvendors' expected profits are increasing with respect to α.

PROPOSITION EC.13. *If $\beta > m$, then competing newsvendors with overconfidence level $\alpha = \hat{\alpha}$ order the same quantity that an (unbiased) central planner orders, i.e., $\hat{q}^c(\hat{\alpha}) = q_c^*$.*

Again, suppose $\rho = q^*$. In the coordinated system, the order quantity of the system is $2F_{\bar{X}}^{-1}(\beta)$. The system order quantity for overconfident and competitive newsvendors is $2[\alpha F_X^{-1}(\beta) + (1-\alpha)\hat{z}^c]$, where $g(\hat{z}^c) = \beta$. Note that when $X_i \sim U[0,1]$ for $i = 1,2$ and X_1 and X_2 are independent, $\alpha F_X^{-1}(\beta) + (1-\alpha)\hat{z}^c = \alpha\beta + (1-\alpha) = F_{\bar{X}}^{-1}(\beta)$ holds if $\alpha = \frac{2\sqrt{3}-3}{\sqrt{2}(\sqrt{6}-3\sqrt{\beta})}$ and $\beta \in [0, 1/2]$. Thus, overconfidence has the potential to coordinate the system when $\rho = q^*$.

EC.4.2. Asymmetric Overconfidence

We now study the case with asymmetric overconfidence levels. Analogous to (EC.14)-(EC.15), Newsvendor i behaves as though its game with its rival were described by $\max_{q_i} \pi_i(\alpha_i) = p\mathrm{E}[(D_i(\alpha_i) + (D_{3-i}(\alpha_i) - q_{3-i})^+) \wedge q_i] - cq_i$, and $\max_{q_{3-i}} \pi_{3-i}(\alpha_i) = p\mathrm{E}[(D_{3-i}(\alpha_i) + (D_i(\alpha_i) - q_i)^+) \wedge q_{3-i}] - cq_{3-i}$.

PROPOSITION EC.14. *If $\beta < \hat{m}^c$, then the more biased newsvendor can earn a higher expected profit than the less biased newsvendor. Furthermore, the more biased newsvendor even can earn a higher expected profit than a sophisticated rival.*

Proposition EC.14 reveals that a less biased newsvendor does not necessarily earn a higher expected profit than its more biased rival when $\rho > g^{-1}(\beta)$. This essentially means that the more biased newsvendor is more prone to earn a higher profit than the less biased one when the newsvendor behaves as though the actual demand is stochastically larger than it really is as is the case if $\rho > \mu$.

Appendix: Proofs for Electronic Companion To "Overconfident Competing Newsvendors"

Proof of Lemma EC.1 Following Lippman and McCardle (1997), the equilibrium ordering quantities $(\hat{q}_1^c, \hat{q}_2^c)$ exist and are unique. Moreover, from (3), they are identical because $\mathrm{P}(D_1(\alpha) +$

$\theta(D_2(\alpha)-\hat{q}^c)^+ < \hat{q}^c) = \mathrm{P}(D_2(\alpha)+\theta(D_1(\alpha)-\hat{q}^c)^+ < \hat{q}^c) = \beta$, where $\hat{q}^c = \hat{q}_1^c = \hat{q}_2^c$. Q.E.D.

Proof of Proposition EC.1 By the definition of $\hat{q}^c(\alpha)$, $\mathrm{P}[D_1(\alpha)+\theta(D_2(\alpha)-\hat{q}^c(\alpha))^+ \le \hat{q}^c(\alpha)] = \beta$. Note that $D_i(\alpha) = \alpha\mu + (1-\alpha)X_i$. So, $g(\hat{z}^c) = \mathrm{P}[X_1 + \theta(X_2 - \hat{z}^c)^+ \le \hat{z}^c] = \beta$. From the definition of q_n and the monotonicity of $g(\cdot)$, we know that $\hat{z}^c = g^{-1}(\beta) = q_n$. A key observation here is that q_n is independent of α. Thus, $d\hat{q}^c(\alpha)/d\alpha < 0$ if and only if $q_n > \mu$, which is identical to $\beta > \hat{m}^c = g(\mu)$ for $i = 1, 2$. Moreover, $\hat{m}^c < \hat{m}$ because $g(\mu) = \mathrm{P}[X_1 + \theta(X_2 - \mu)^+ \le \mu] < F_X(\mu) = \hat{m}$. Q.E.D.

Proof of Proposition EC.2 Because the equilibrium profits of the two newsvendors are identical, we have both newsvendors' expected equilibrium profits as $\hat{\pi}^c(\alpha) = -c\hat{q}^c(\alpha) + p\mathrm{E}[\hat{q}^c(\alpha) \wedge (X_1 + \theta(X_2 - \hat{q}^c(\alpha))^+]$. We first show that $\hat{\pi}^c$ is concave in $\hat{q}^c$. Using the fact that

$$\frac{\partial \mathrm{E}[\hat{q}^c \wedge (X_1+\theta(X_2-\hat{q}^c)^+]}{\partial \hat{q}^c} = \mathrm{E}\left[\frac{\partial[\hat{q}^c \wedge (X_1+\theta(X_2-\hat{q}^c)^+]}{\partial \hat{q}^c}\right],$$

we obtain

$$\begin{aligned}
\frac{\partial \hat{\pi}^c}{\partial \hat{q}^c} &= p\int_{\hat{q}^c}\int_{\hat{q}^c-\theta(X_2-\hat{q}^c)} f(x_1,x_2)dx_1dx_2 + p\int_{\hat{q}^c}\int^{\hat{q}^c} f(x_1,x_2)dx_2dx_1 \\
&\quad - p\theta\int_{\hat{q}^c}\int^{\hat{q}^c-\theta(X_2-\hat{q}^c)} f(x_1,x_2)dx_1dx_2 - c \\
&= p - c - p\mathrm{P}[X_1+\theta(X_2-\hat{q}^c)^+ \le \hat{q}^c] - p\theta\mathrm{P}[X_2 \ge \hat{q}^c, X_1+\theta X_2 \le (1+\theta)\hat{q}^c] \\
&= p - c - ph(\hat{q}^c),
\end{aligned} \tag{EC.16}$$

where the second equality is from $\int_{\hat{q}^c}\int_{\hat{q}^c-\theta(X_2-\hat{q}^c)} f(x_1,x_2)dx_1dx_2 + \int_{\hat{q}^c}\int^{\hat{q}^c} f(x_1,x_2)dx_2dx_1 = 1 - \mathrm{P}[X_1+\theta(X_2-\hat{q}^c)^+ \le \hat{q}^c]$. Note that $h(\cdot)$ increases in $\hat{q}^c$. Consequently, $\partial\hat{\pi}^c/\partial\hat{q}^c$ decreases in $\hat{q}^c$. Therefore, $\hat{\pi}^c$ is concave in $\hat{q}^c$. It follows that $\hat{\pi}^c(\alpha)$ is concave in α because $\hat{q}^c = \alpha\mu + (1-\alpha)\hat{z}^c$ is linear in α. Accordingly, there are three cases to consider.

Case (a): $\beta \le \hat{m}^c$. In this case, we evaluate $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=0}$. For $\alpha = 0$, $\hat{q}^c = \hat{z}^c$. So,

$$\begin{aligned}
\left.\frac{\partial \hat{\pi}^c}{\partial \hat{q}^c}\right|_{\alpha=0} &= p - c - p\mathrm{P}[X_1+\theta(X_2-\hat{z}^c)^+ \le \hat{z}^c] - p\theta\int_{\hat{z}^c}\int^{\hat{z}^c-\theta(X_2-\hat{z}^c)} f(x_1,x_2)dx_1dx_2 \\
&= p(\beta - \hat{m}^c) - p\theta\int_{\hat{z}^c}\int^{\hat{z}^c-\theta(X_2-\hat{z}^c)} f(x_1,x_2)dx_1dx_2 \\
&\le 0.
\end{aligned}$$

It follows that $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=0} = \left.\left[\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c} \times \frac{\partial\hat{q}^c}{\partial\alpha}\right]\right|_{\alpha=0} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=0} \times (\mu - \hat{z}^c) \le 0$ because $\hat{z}^c \le \mu \Longleftrightarrow \beta \le \hat{m}^c$. Therefore, if $\beta \le \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ is a decreasing function because of its concavity.

Case (b): $\hat{m}^c < \beta \le m$. In this case, we evaluate $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=1}$. For $\alpha = 1$, $\hat{q}^c = \mu$. So,

$$\begin{aligned}\left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1} &= p - c - p\mathrm{P}[X_1 + \theta(X_2-\mu)^+ \le \mu] - p\theta\mathrm{P}[X_1 + \theta(X_2-\mu)^+ \le \mu, X_1 \ge \mu] \\ &= p\,(\beta - m) \le 0.\end{aligned}$$

Note that in this case, $\mu < \hat{z}^c$ because $\hat{m}^c < \beta$. Then, $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=1} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1}(\mu - \hat{z}^c) > 0$. Therefore, if $\hat{m}^c < \beta \le m$, then $\hat{\pi}^c(\alpha)$ is an increasing function because of its concavity.

Case (c): $\beta > m$. In this case, $\left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=0} = p - c - ph(\hat{z}^c) = p[\beta - h(\hat{z}^c)] < 0$ because $g(\hat{z}^c) = \beta$ and $g(\cdot) \le h(\cdot)$. As a result, $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=0}(\mu - \hat{z}^c) > 0$. Furthermore, $\left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1} = p(\beta - m) > 0$, which implies $\left.\frac{d\hat{\pi}^c(\alpha)}{d\alpha}\right|_{\alpha=1} = \left.\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c}\right|_{\alpha=1}(\mu - \hat{z}^c) < 0$. Therefore, if $\beta > m$, then $\hat{\pi}^c(\alpha)$ is an increasing-decreasing function. To ensure $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$, we must have $\frac{\partial\hat{\pi}^c}{\partial\hat{q}^c} = 0$. Using (EC.16), we have $\hat{q}^c = h^{-1}(\beta)$, that is $\alpha\mu + (1-\alpha)\hat{z}^c = h^{-1}(\beta)$. Since $g(\hat{z}^c) = \beta$, we can conclude that $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$ holds when $\alpha = \hat{\alpha} = (g^{-1}(\beta) - h^{-1}(\beta))/(g^{-1}(\beta) - \mu)$. Q.E.D.

Proof of Lemma EC.2 (a) In the competition case, the equilibrium order quantities $q_n = \hat{q}^c(\alpha = 0)$ satisfy (3). In this case, the order quantity q_n is higher than the order quantity q_c^* in the centralized system because q_c^* satisfies $h(q_c^*) = \beta$, q_n satisfies $g(q_n) = \beta$, and $h(y) \ge g(y)$ for all y.

(b) In the competition case, the infinitely overconfident newsvendors order μ, i.e., $\hat{q}(\alpha = 1) = \mu$. In the centralized system $h(q_c^*) = \beta$. Thus, $q_c^* < \hat{q}(\alpha = 1) = \mu$ if and only if $\beta < m$, where $m = h(\mu)$, as defined in (EC.4). Q.E.D.

Proof of Proposition EC.3 If $\beta > m$, then $h(\alpha\mu + (1-\alpha)\hat{z}^c) = \beta$ when $\alpha = \hat{\alpha}$ from Proposition EC.2. Thus, when $\alpha = \hat{\alpha}$, the competing newsvendors would order the same quantity that the central planner would order. Q.E.D.

Proof of Lemma EC.3 Because $D_1(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X_1$, $D_2(\alpha_i) = \alpha_i\mu + (1-\alpha_i)X_2$ and the equilibrium solution satisfies $\mathrm{P}(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \le q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) - q_i)^+ \le q_{3-i}) = \beta$, Newsvendor i solves $\mathrm{P}(D_i(\alpha_i) + \theta(D_{3-i}(\alpha_i) - q_{3-i})^+) \le q_i) = \mathrm{P}(D_{3-i}(\alpha_i) + \theta(D_i(\alpha_i) -$

$q_i)^+ \leq q_{3-i}) = \beta$ to obtain the equilibrium order quantity $\hat{q}_i^c(\alpha_i) = \alpha_i\mu + (1-\alpha_i)\hat{z}_i$. Thus, $\mathrm{P}(X_i + \theta(X_{3-i} - \hat{z}_{3-i})^+) \leq \hat{z}_i) = \mathrm{P}(X_{3-i} + \theta(X_i - \hat{z}_i)^+ \leq \hat{z}_{3-i}) = \beta$. As a result, from the proof of Proposition EC.1, $\hat{z}_1 = \hat{z}_2 = q_n$ and $g(q_n) = \beta$. Q.E.D.

Proof of Proposition EC.4 For the overconfident newsvendors, the order quantities satisfy $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\mu + (2 - \alpha_1 - \alpha_2)q_n$. In contrast, the central planner's equilibrium order quantity is $2h^{-1}(\beta)$. Thus, if $\beta > m$, then the system can be coordinated as in Proposition EC.2. In particular, $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$ when $\alpha_1 + \alpha_2 = 2\hat{\alpha}$. Q.E.D.

Proof of Proposition EC.5 To prove Proposition EC.5, we first show the following two-part lemma. <u>Lemma EC.5(a)</u> Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_i} \leq 0$, for $i = 1, 2$), and <u>Lemma EC.5(b)</u> Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_{3-i}} < 0$ for $i = 1, 2$) if and only if $\beta < \hat{m}^c$. Toward that end, first note from (EC.5) and Lemma EC.3 that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1, \alpha_2) = p\mathrm{E}[(X_1 + \theta(X_2 - \hat{q}_2^c)^+) \wedge \hat{q}_1^c] - c\hat{q}_1^c$. Thus, $\frac{\partial \hat{\pi}_1^c}{\partial \hat{q}_1^c} = p\mathrm{P}[(X_1 + \theta(X_2 - \hat{q}_2^c)^+) \geq \hat{q}_1^c] - c$ and $\frac{\partial \hat{\pi}_1^c}{\partial \hat{q}_2^c} = -p\theta\mathrm{P}[X_1 + \theta(X_2 - \hat{q}_2^c) \leq \hat{q}_1^c, X_2 \geq \hat{q}_2^c]$. And, for $i = 1, 2$, $\frac{\partial \hat{q}_i^c}{\partial \alpha_i} = \mu - q_n$ and $\frac{\partial \hat{q}_i^c}{\partial \alpha_{3-i}} = 0$. Therefore,

$$\begin{aligned}\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} &= (\mu - q_n)p[\mathrm{P}(\alpha_1\mu + (1-\alpha_1)q_n < X_1 + \theta(X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+) - (1-\beta)] \\ &= (\mu - q_n)p[\beta - \mathrm{P}(\alpha_1\mu + (1-\alpha_1)q_n > X_1 + \theta(X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+)]. \qquad \text{(EC.17)}\end{aligned}$$

Accordingly:

<u>Proof of Lemma EC.5(a)</u>: Without loss of generality, we let $i = 1$. First, if $q_n > \mu$, then $q_n > \hat{q}_i^c = \alpha_i\mu + (1-\alpha_i)q_n$, for $i = 1, 2$. So, $\mathrm{P}[X_1 + \theta(X_2 - \hat{q}_2^c)^+ \geq \hat{q}_1^c] < \mathrm{P}[X_1 + \theta(X_2 - q_n)^+ \geq q_n] = \beta$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} < 0$ if $q_n > \mu$. Second, if $q_n \leq \mu$, then $q_n \leq \hat{q}_i^c$, for $i = 1, 2$. So, $\mathrm{P}[X_1 + \theta(X_2 - \hat{q}_2^c)^+ \geq \hat{q}_1^c] \geq \mathrm{P}[X_1 + \theta(X_2 - q_n)^+ \geq q_n] = \beta$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \leq 0$ if $q_n \leq \mu$. This completes the proof of Lemma EC.5(a).

<u>Proof of Lemma EC.5(b)</u> Without loss of generality, we let $i = 1$. Because $\frac{\partial \hat{\pi}_1^c}{\partial \hat{q}_2^c} < 0$, the sign of $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2}$ is determined by $q_n - \mu$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0 \Longleftrightarrow \mu > q_n$. Furthermore, from Proposition EC.1, $\mu > q_n \Longleftrightarrow \hat{m}^c > \beta$. This completes the proof of Lemma EC.5(b).

Note that if $\alpha_1 = \alpha_2$, then the newsvendors have identical expected profits. If $\alpha_1 < \alpha_2$ and $\beta > \hat{m}^c$, then Lemma EC.5(a) and Lemma EC.5(b) together imply that the less biased newsvendor has a higher expected profit. However, if $\beta < \hat{m}^c$, then we show that the more biased newsvendor can earn a higher expected profit, using an existence proof. To that end, suppose $\alpha_1 = 0$. Then,

$$\begin{aligned}\left.\frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\right|_{\alpha_2=0} &= \left.\frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \hat{q}_2^c}\right|_{\alpha_2=0}(\mu - q_n)\\ &= -p\theta \mathrm{P}[(X_1+\theta(X_2-q_n)) \le q_n, X_2 \ge q_n](\mu - q_n)\\ &< 0.\end{aligned}$$

Similarly,

$$\begin{aligned}\left.\frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\right|_{\alpha_2=0} &= \left.\frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \hat{q}_2^c}\right|_{\alpha_2=0}(\mu - q_n)\\ &= \{p\mathrm{P}[(X_1+\theta(X_2-q_n)) \ge q_n] - c\}(\mu - q_n)\\ &= 0.\end{aligned}$$

It follows that $\left[\frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2} - \frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\right]_{\alpha_2=0} < 0$. Note that $\hat{\pi}_1^c(\alpha_1=0,\alpha_2=0) = \hat{\pi}_2^c(\alpha_1=0,\alpha_2=0)$. So, there must exist some $\alpha_2 > 0$ such that $\hat{\pi}_2^c(\alpha_1=0,\alpha_2) > \hat{\pi}_1^c(\alpha_1=0,\alpha_2)$ if $\beta < \hat{m}^c$. The proof is complete. Q.E.D.

Proof of Proposition EC.6 Without loss of generality, define Newsvendor 2 as the biased newsvendor characterized by overconfidence level α_2 and define Newsvendor 1 as the sophisticated newsvendor. Accordingly, denote $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ as the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Similarly, denote $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ as the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^c(\alpha_2) = \alpha_2\mu + (1-\alpha_2)q_n$ from Lemma 4. So, $\frac{\partial \hat{q}_2^{fc}(\alpha_2)}{\partial \alpha_2} = \mu - q_n$. And $q_1^{fc}(\alpha_2)$ is determined by $\mathrm{P}(X_1 + \theta(X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \le q_1^{fc}(\alpha_2)) = \beta$. Accordingly, Newsvendor 2's equilibrium expected profit can be written as $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}[X_2 + \theta(X_1 - q_1^{fc})^+ \wedge \hat{q}_2^{fc}] - c\hat{q}_2^{fc}$, which implies that $\frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_1^{fc}} = -p\theta \mathrm{P}[X_2 + \theta(X_1 - q_1^{fc}) \le \hat{q}_2^{fc}, X_1 \ge q_1^{fc}]$ and $\frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_2^{fc}} = p\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc})^+ \ge \hat{q}_2^{fc}] - c$. It follows that

$$\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0} = \left.\frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_1^{fc}} \times \frac{\partial \hat{q}_1^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0} + \left.\frac{\partial \hat{\pi}_2^{fc}}{\partial \hat{q}_2^{fc}} \times \frac{\partial \hat{q}_2^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0}$$

$$= -p\theta \mathrm{P}[X_2 + \theta(X_1 - q_1^{fc}) \le \hat{q}_2^{fc}, X_1 \ge q_1^{fc}] \times \left.\frac{\partial \hat{q}_1^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0} + \\ \left\{p\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc})^+ \ge \hat{q}_2^{fc}] - c\right\} \times (\mu - q_n)\big|_{\alpha_2=0}.$$

However, $p\mathrm{P}[X_2 + \theta(X_1 - q_1^{fc})^+ \ge \hat{q}_2^{fc}] - c\big|_{\alpha_2=0} = p\mathrm{P}[X_2 + \theta(X_1 - q_n)^+ \ge q_n] - c = 0$. So,

$$\left.\frac{d\hat{\pi}_2^{fc}}{d\alpha_2}\right|_{\alpha_2=0} = -p\theta\, \mathrm{P}[X_2 + \theta(X_1 - q_1^{fc}) \le \hat{q}_2^{fc}, X_1 \ge q_1^{fc}] \times \left.\frac{\partial \hat{q}_1^{fc}}{\partial \alpha_2}\right|_{\alpha_2=0} > 0, \tag{EC.18}$$

where the inequality follows because $\beta < \hat{m}^c \Longrightarrow q_n < \mu$, $q_n < \mu \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^{c}(\alpha_2)}{d\alpha_2} = \mu - q_n < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$.

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[(X_1 + \theta(X_2 - \alpha_2\mu - (1-\alpha_2)q_n)^+) \wedge q_1^{fc}] - cq_1^{fc}$. Thus, by the envelope theorem,

$$\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\theta\mathrm{P}\left[X_1 + \theta\left(X_2 - \alpha_2\mu - (1-\alpha_2)q_n\right) < q_1^{fc}, X_2 > \alpha_2\mu + (1-\alpha_2)q_n\right](q_n - \mu) < 0, \tag{EC.19}$$

where the inequality holds because, again, $\beta < \hat{m}^c \Longrightarrow q_n < \mu$ from Proposition EC.1. It follows from (EC.18) and (EC.19) that $[\frac{d\hat{\pi}_2^{fc}}{d\alpha_2} - \frac{d\pi_1^{fc}}{d\alpha_2}]_{\alpha_2=0} > 0$. Hence, there must exist some $\alpha_2 > 0$ such that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ because $\hat{\pi}_1^{fc}(\alpha_2 = 0) = \hat{\pi}_2^{fc}(\alpha_2 = 0)$. Q.E.D.

Proof of Lemma EC.4. For $X \sim U[0,1]$, $D(\alpha_i) = \alpha_i/2 + (1-\alpha_i)X$ because $\mathrm{E}[X] = 1/2$. Hence,

$$F_D(x) = F\left(\frac{x - \alpha_i\mu}{1-\alpha_i}\right) = \begin{cases} 0 & x \le \frac{\alpha_i}{2}; \\ \frac{x - \alpha_i/2}{1-\alpha_i} & \frac{\alpha_i}{2} < x \le 1 - \frac{\alpha_i}{2}; \\ 1 & x > 1 - \frac{\alpha_i}{2}. \end{cases}$$

Substituting $1 - \alpha_i/2$ for $\hat{Q}^c$ in the left-hand side of (EC.8), we obtain

$$\frac{F_D(1-\alpha_i/2)}{2} + \frac{1}{2(1-\alpha_i/2)}\int_{\frac{\alpha_i}{2}}^{1-\frac{\alpha_i}{2}} F_D(x)dx = \frac{3 - 2\alpha_i}{2(2-\alpha_i)}.$$

So, if $\beta > \frac{3-2\alpha_i}{2(2-\alpha_i)}$, $\hat{Q}^c > 1 - \alpha_i/2$; and if $\beta \le \frac{3-2\alpha_i}{2(2-\alpha_i)}$, $\hat{Q}^c \le 1 - \alpha_i/2$. Suppose $\hat{Q}^c \in [\alpha_i/2, 1 - \alpha_i/2]$. Then, (EC.8) becomes $\frac{1}{2}\frac{Q - \alpha_i/2}{1-\alpha_i} + \frac{1}{2Q}\int_{\alpha_i/2}^{Q} \frac{x - \alpha_i/2}{1-\alpha_i}dx = \beta$, which is a quadratic equation in Q, and the left-hand side of the equation increases in Q. Thus, $\hat{Q}^c = \frac{1}{3}\alpha_i + \frac{2}{3}\beta - \frac{2}{3}\alpha_i\beta + \frac{1}{3}h(\alpha_i, \beta)$. Suppose $\hat{Q}^c > 1 - \alpha_i/2$. Then, (EC.8) becomes

$$\frac{1}{2} + \frac{1}{2Q}\left(\int_{\alpha_i/2}^{1-\alpha_i/2} \frac{x - \alpha_i/2}{1-\alpha_i}dx + \int_{1-\alpha_i/2}^{Q} dx\right) = \beta,$$

with the solution $\hat{Q}^c = \frac{1}{4(1-\beta)}$. The desired result then follows because $\hat{q}^c = \hat{Q}^c/2$. Q.E.D.

Proof of Proposition EC.7. Clearly, $\partial \hat{q}_i^c(\alpha)/\partial\alpha = 0$ for $\beta > \frac{3-2\alpha}{2(2-\alpha)}$. For $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$, $\hat{q}_i^c(\alpha) = \frac{1}{6}\alpha + \frac{1}{3}\beta - \frac{1}{3}\alpha\beta + \frac{1}{6}h(\alpha,\beta)$, and

$$\frac{\partial \hat{q}_i^c(\alpha)}{\partial\alpha} = \frac{1}{12\sqrt{16\alpha^2\beta^2 - 16\alpha^2\beta + \alpha^2 - 32\alpha\beta^2 + 16\alpha\beta + 16\beta^2}} \times$$
$$[\underbrace{(2-4\beta)\sqrt{16\alpha^2\beta^2 - 16\alpha^2\beta + \alpha^2 - 32\alpha\beta^2 + 16\alpha\beta + 16\beta^2}}_{A} + \underbrace{\alpha + 8\beta - 16\beta^2 - 16\alpha\beta + 16\alpha\beta^2}_{B}].$$

Note that $B \ge 0 \Longleftrightarrow \beta \le \frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right)$. So we consider two following cases.

Case 1: $\beta \le \frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right)$. Note that $\beta \le 1/2$. So, $A > 0$ and $B > 0$, which implies that $\partial\hat{q}_i^c/\partial\alpha > 0$.

Case 2: $\beta > \frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right)$. Note that $\frac{3-2\alpha}{2(2-\alpha)} \ge 1/2$, we need to consider two sub-cases. *Case 2A*: $\frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right) < \beta \le 0.5$. Then, $A > 0$ and $B < 0$, and we compare their squares:

$$(1-2\beta)^2(16\alpha^2\beta^2 - 16\alpha^2\beta + \alpha^2 - 32\alpha\beta^2 + 16\alpha\beta + 16\beta^2) - (\alpha + 8\beta - 16\beta^2 - 16\alpha\beta + 16\alpha\beta^2)^2$$
$$= 12\beta\Big[\underbrace{\left(1 - 16\beta^3 + 32\beta^2 - 17\beta\right)\alpha^2 - \left(48\beta^2 - 32\beta^3 - 16\beta\right)\alpha - \left(16\beta^3 - 16\beta^2 + 4\beta\right)}_{C}\Big].$$

Note that C is a cubic function of β, $\frac{\partial C}{\partial\beta}$ is a quadratic function of β, and $\frac{\partial C}{\partial\beta} < 0$ for $\frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right) < \beta \le 0.5$. Since $C|_{\beta=\frac{1}{4(1-\alpha)}\left(1-2\alpha+\sqrt{1-3\alpha+3\alpha^2}\right)} > 0$ and $C|_{\beta=0.5} < 0$, there must exist a unique solution $\beta_c \in \left[\frac{1}{4(1-\alpha)}\left(1 - 2\alpha + \sqrt{1-3\alpha+3\alpha^2}\right), 1/2\right]$ such that $C \ge 0 \Longleftrightarrow \beta \le \beta_c$. Consequently, $\frac{\partial \hat{q}_i^c}{\partial c} \ge 0 \Longleftrightarrow \beta \le \beta_c$. *Case 2B*: $1/2 < \beta \le \frac{3-2\alpha}{4-2\alpha}$. In this case, $\frac{\partial\hat{q}_i^c}{\partial\alpha} < 0$ because $A < 0$ and $B < 0$. Q.E.D.

Proof of Proposition EC.8. Because $\hat{\pi}_i^c$ is independent of α when $\beta > \frac{3-2\alpha}{2(2-\alpha)}$, it suffices to consider the case $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$, in which $\hat{q}_i^c(\alpha) = \frac{1}{6}\alpha + \frac{1}{3}\beta - \frac{1}{3}\alpha\beta + \frac{1}{6}\sqrt{4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2}$ by Lemma EC.4. Furthermore, for $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$, $\hat{q}_i^c(\alpha) - \frac{1}{2} = \frac{1}{6}\left[\sqrt{4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2} - (3 - \alpha - 2\beta + 2\alpha\beta)\right] < 0$ because $3 - \alpha - 2\beta + 2\alpha\beta \ge 0$, and $\left(4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2\right) - (3 - \alpha - 2\beta + 2\alpha\beta)^2 = 6\alpha + 12(1-\alpha)\beta - \frac{3}{4}\alpha^2 - 9 < 0$. In addition, because $\hat{\pi}_i^c = -c\hat{q}_i{}^c + p\hat{q}_i^c(1-\hat{q}_i^c)$, $\frac{\partial\hat{\pi}_i^c}{\partial\alpha} = \frac{\partial\hat{\pi}_i^c}{\partial\hat{q}_i^c} \times \frac{\partial\hat{q}_i^c}{\partial\alpha} = p[\beta - 2\hat{q}_i^c] \times \frac{\partial\hat{q}_i^c}{\partial\alpha}$, where

$\beta - 2\hat{q}_i^c = \frac{1}{3}(\beta - \alpha + 2\alpha\beta - \sqrt{4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2})$. If $\beta \le \frac{\alpha}{1+2\alpha} \Longleftrightarrow \beta - \alpha + 2\alpha\beta \ge 0$, then $\beta - 2\hat{q}_i^c < 0$. If $\beta > \frac{\alpha}{1+2\alpha}$, define $f(\beta) := (\beta - \alpha + 2\alpha\beta)^2 - (4\alpha^2\beta^2 - 4\alpha^2\beta + \frac{1}{4}\alpha^2 - 8\alpha\beta^2 + 4\alpha\beta + 4\beta^2) = (12\alpha - 3)\beta^2 - 6\alpha\beta + \frac{3}{4}\alpha^2$. Note that $f\left(\frac{\alpha}{1+2\alpha}\right) = \frac{3}{4}\frac{\alpha^2}{(2\alpha+1)^2}(4\alpha^2 + 4\alpha - 11) < 0, f'\left(\frac{\alpha}{1+2\alpha}\right) = 12\alpha\frac{\alpha-1}{2\alpha+1} < 0$, and $f\left(\frac{3-2\alpha}{2(2-\alpha)}\right) = \frac{3}{4}\frac{(\alpha-1)^2}{(\alpha-2)^2}(\alpha^2 + 6\alpha - 9) < 0$. Consequently, $f(\beta) < 0$ for $\beta \in [\frac{\alpha}{1+2\alpha}, \frac{3-2\alpha}{2(2-\alpha)}]$. In summary, $\beta - 2\hat{q}_i^c \le 0$ when $\beta \le \frac{3-2\alpha}{2(2-\alpha)}$. Appealing to Proposition EC.7, we complete the proof. Q.E.D.

Proof of Proposition EC.9 When $\alpha_1 = 0$, from Lemma EC.4,

$$\hat{q}_1^c = \begin{cases} \frac{2}{3}\beta & \beta \le \frac{3}{4}; \\ \frac{1}{8(1-\beta)} & \beta > \frac{3}{4}. \end{cases} \tag{EC.20}$$

We partition the interval $[0,1]$ into four disjoint sub-intervals to facilitate the comparison of $\hat{\pi}_1^c$ and $\hat{\pi}_2^c$: $[0, \frac{3-\alpha_2}{8}]$, $(\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$, $(\frac{3-2\alpha_2}{2(2-\alpha_2)}, \frac{3}{4}]$, and $(\frac{3}{4}, 1]$. For each sub-interval, we sign three expressions to compare $\hat{\pi}_1^c$ and $\hat{\pi}_2^c$. First, we sign $\hat{q}_1^c + \hat{q}_2^c - 1$; second, we sign $\hat{q}_1^c - \hat{q}_2^c$; and third, depending on the sign of $\hat{q}_1^c + \hat{q}_2^c - 1$, we sign either $\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}$ or $\beta(\hat{q}_1^c + \hat{q}_2^c) - \frac{1}{2}$. From (EC.10), we see that the signs of $\hat{q}_1^c + \hat{q}_2^c - 1$, $\hat{q}_1^c - \hat{q}_2^c$, and $\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}$ or $\beta(\hat{q}_1^c + \hat{q}_2^c) - \frac{1}{2}$, taken together, completely determine the sign of $\hat{\pi}_2^c - \hat{\pi}_1^c$.

<u>Case 1</u>: $\beta \in [0, \frac{3-\alpha_2}{8}]$. For this case, we present a detailed proof to illustrate the steps of analysis taken to derive the desired results. The analysis for other cases are similar, and are therefore only outlined for space. When $\beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}$, by (EC.20) and Lemma EC.4, the equilibrium order quantities are

$$\hat{q}_1^c = \frac{2}{3}\beta, \tag{EC.21}$$

$$\hat{q}_2^c = \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2, \beta). \tag{EC.22}$$

We first sign $\hat{q}_1^c + \hat{q}_2^c - 1$. By (EC.21) and (EC.22), $\hat{q}_1^c + \hat{q}_2^c - 1 = \frac{1}{6}[h(\alpha_2,\beta) - (6 - 6\beta - \alpha_2 + 2\alpha_2\beta)]$. Note that $6 - 6\beta - \alpha_2 + 2\alpha_2\beta > 0$ for $\beta \in [0, \frac{3-\alpha_2}{8}]$. So, $\hat{q}_1^c + \hat{q}_2^c - 1$ has the same sign as $h^2(\alpha_2,\beta) - (6 - 6\beta - \alpha_2 + 2\alpha_2\beta)^2$. By the definition of $h(\alpha_2,\beta)$ and collecting terms, we obtain $J(\beta) := h^2(\alpha_2,\beta) - (6 - 6\beta - \alpha_2 + 2\alpha_2\beta)^2 = (16\alpha_2 - 32)\beta^2 + (72 - 32\alpha_2)\beta + \left(12\alpha_2 - \frac{3}{4}\alpha_2^2 - 36\right)$. To sign $J(\beta)$ for $\beta \in [0, \frac{3-\alpha_2}{8}]$, we check its values and its first-order derivatives at the boundaries:

$$J\left(\frac{3-\alpha_2}{8}\right)=\frac{1}{4}\alpha_2^3+\frac{5}{4}\alpha_2^2-\frac{15}{4}\alpha_2-\frac{27}{2}\leq 0, \tag{EC.23}$$

$$J'\left(\frac{3-\alpha_2}{8}\right)=-4\alpha_2^2-12\alpha_2+48\geq 0. \tag{EC.24}$$

Note that the graph of $J(\beta)$ is a downward-opening parabola. It then follows from (EC.23) and (EC.24) that $J(\beta)\leq 0$ for $\beta\in[0,\frac{3-\alpha_2}{8}]$, which implies that $\hat{q}_1^c+\hat{q}_2^c\leq 1$. Consequently, by (EC.10),

$$\hat{\pi}_2^c-\hat{\pi}_1^c=p(\hat{q}_2^c-\hat{q}_1^c)\left(\beta-\frac{\hat{q}_1^c+\hat{q}_2^c}{2}\right). \tag{EC.25}$$

We next sign $\hat{q}_1^c-\hat{q}_2^c$. By (EC.21) and (EC.22), $\hat{q}_2^c-\hat{q}_1^c=\frac{1}{6}h(\alpha_2,\beta)-(-\frac{1}{6}\alpha_2+\frac{1}{3}\beta+\frac{1}{3}\alpha_2\beta)$. For $\beta\in[0,\frac{3-\alpha_2}{8}]$, $-\frac{1}{6}\alpha_2+\frac{1}{3}\beta+\frac{1}{3}\alpha_2\beta\geq 0\iff\beta\geq\frac{\alpha_2}{2\alpha_2+2}$. Consequently, $\hat{q}_2^c-\hat{q}_1^c\geq 0$ for $\beta\leq\frac{\alpha_2}{2\alpha_2+2}$, and $\hat{q}_2^c-\hat{q}_1^c$ has the same sign as $h^2(\alpha_2,\beta)-(-\alpha_2+2\beta+2\alpha_2\beta)^2$ for $\beta\geq\frac{\alpha_2}{2\alpha_2+2}$. However, $h^2(\alpha_2,\beta)-(-\alpha_2+2\beta+2\alpha_2\beta)^2=-\frac{1}{4}\alpha_2\left(64\beta^2-32\beta+3\alpha_2\right)$. Using properties of quadratic functions, we can verify that for $\beta\in[\frac{\alpha_2}{2\alpha_2+2},\frac{3-\alpha_2}{8}]$, $64\beta^2-32\beta+3\alpha_2<0$, which implies that $\hat{q}_2^c>\hat{q}_1^c$. In summary, for $\beta\in[0,\frac{3-\alpha_2}{8}]$,

$$\hat{q}_2^c>\hat{q}_1^c. \tag{EC.26}$$

We last sign $\beta-\frac{\hat{q}_1^c+\hat{q}_2^c}{2}$. First, for $\beta\leq\frac{\alpha_2}{6+2\alpha_2}$, $\hat{q}_1^c+\hat{q}_2^c-2\beta=\frac{1}{6}[h(\alpha_2,\beta)-(-\alpha_2+6\beta+2\alpha_2\beta)]\geq 0$ because $-\alpha_2+6\beta+2\alpha_2\beta\geq 0\iff\beta\geq\frac{\alpha_2}{6+2\alpha_2}$. For $\beta>\frac{\alpha_2}{6+2\alpha_2}$, $h^2(\alpha_2,\beta)-(-\alpha_2+6\beta+2\alpha_2\beta)^2=(-32\alpha_2-32)\,\beta^2+16\alpha_2\beta-\frac{3}{4}\alpha_2^2$. Again, using properties of quadratic function, we can verify that

$$\hat{q}_1^c+\hat{q}_2^c-2\beta=\begin{cases}>0 & \frac{\alpha_2}{6+2\alpha_2}<\beta\leq\frac{8\alpha_2+2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32};\\ \leq 0 & \beta>\frac{8\alpha_2+2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}.\end{cases}$$

In summary, for $\beta\in[0,\frac{3-\alpha_2}{8}]$,

$$\hat{q}_1^c+\hat{q}_2^c-2\beta=\begin{cases}>0 & \beta\leq\frac{8\alpha_2+2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32};\\ \leq 0 & \beta>\frac{8\alpha_2+2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}.\end{cases} \tag{EC.27}$$

Using (EC.25)-(EC.27), we obtain

$$\hat{\pi}_2^c-\hat{\pi}_1^c=\begin{cases}\leq 0 & \beta\leq\frac{8\alpha_2+2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32};\\ \geq 0 & \beta>\frac{8\alpha_2+2\sqrt{2}\alpha_2\sqrt{-3\alpha_2+5}}{32\alpha_2+32}.\end{cases}$$

<u>Case 2</u>: $\beta\in(\frac{3-\alpha_2}{8},\frac{3-2\alpha_2}{2(2-\alpha_2)}]$. In this case, $\hat{q}_1^c=\frac{2}{3}\beta$ and $\hat{q}_2^c=\frac{1}{6}\alpha_2+\frac{1}{3}\beta-\frac{1}{3}\alpha_2\beta+\frac{1}{6}h(\alpha_2,\beta)$.

First, $\hat{q}_1^c + \hat{q}_2^c - 1 = \frac{1}{6}\left[h(\alpha_2,\beta) - (6-\alpha_2-6\beta+2\alpha_2\beta)\right]$. Note that $6-\alpha_2-6\beta+2\alpha_2\beta \geq 0$ for $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$. So, we show $h^2(\alpha_2,\beta) - (6-\alpha_2-6\beta+2\alpha_2\beta)^2 = (16\alpha_2-32)\beta^2 + (72-32\alpha_2)\beta + \left(12\alpha_2 - \frac{3}{4}\alpha_2^2 - 36\right) < 0$ for $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)})$ using the properties of quadratic functions. Therefore, $\hat{q}_1^c + \hat{q}_2^c < 1$. It follows from (EC.10) that for $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$,

$$\hat{\pi}_2^c - \hat{\pi}_1^c = p(\hat{q}_2^c - \hat{q}_1^c)\big(\beta - \frac{\hat{q}_1^c + \hat{q}_2^c}{2}\big). \tag{EC.28}$$

Second, $2\beta - (\hat{q}_1^c + \hat{q}_2^c) = 6\beta - \alpha_2 + 2\alpha_2\beta - h(\alpha_2,\beta)$ where $6\beta - \alpha_2 + 2\alpha_2\beta > 0$ for $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$. From properties of quadratic functions, $(6\beta-\alpha_2+2\alpha_2\beta)^2 - h^2(\alpha_2,\beta) = 32\beta^2\alpha_2 + 32\beta^2 - 16\beta\alpha_2 + \frac{3}{4}\alpha_2^2 > 0$ for $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$. Thus,

$$2\beta > (\hat{q}_1^c + \hat{q}_2^c). \tag{EC.29}$$

Third, $\hat{q}_2^c - \hat{q}_1^c = h(\alpha_2,\beta) - (2\beta - \alpha_2 + 2\alpha_2\beta)$. Note that for $\beta \in (\frac{3-\alpha_2}{8}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$, $2\beta - \alpha_2 + 2\alpha_2\beta \geq 0$ and $h^2(\alpha_2,\beta) - (2\beta-\alpha_2+2\alpha_2\beta)^2 = \frac{1}{4}\alpha_2\left(-64\beta^2 + 32\beta - 3\alpha_2\right) \geq 0 \Longleftrightarrow \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}$. Consequently,

$$\hat{q}_2^c - \hat{q}_1^c = \begin{cases} \geq 0 & \frac{3-\alpha_2}{8} < \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ < 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases} \tag{EC.30}$$

From (EC.28)-(EC.30), we see that

$$\hat{\pi}_2^c - \hat{\pi}_1^c = \begin{cases} \geq 0 & \frac{3-\alpha_2}{8} < \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \leq 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

Case 3: $\beta \in (\frac{3-2\alpha_2}{2(2-\alpha_2)}, 3/4]$. In this case, by (EC.20) and Lemma EC.4, $\hat{q}_1^c = \frac{2}{3}\beta$ and $\hat{q}_2^c = \frac{1}{8(1-\beta)}$. First, $\frac{2}{3}\beta + \frac{1}{4(1-\beta)} \geq 1 \Longleftrightarrow \beta \geq \frac{5}{4} - \frac{1}{4}\sqrt{7}$. Consequently, we consider two sub-cases. For $\beta \in (\frac{3-2\alpha_2}{2(2-\alpha_2)}, \frac{5}{4} - \frac{1}{4}\sqrt{7}]$, $\hat{\pi}_2^c \geq \hat{\pi}_1^c$ because $2\beta \geq \hat{q}_1^c + \hat{q}_2^c$ and $\hat{q}_1^c \leq \hat{q}_2^c$. For $\beta \in (\frac{5}{4} - \frac{1}{4}\sqrt{7}, \frac{3}{4}]$, $\hat{\pi}_2^c \geq \hat{\pi}_1^c$ because $\beta(\hat{q}_1^c + \hat{q}_2^c) - \frac{1}{2} \geq 0$ and $\hat{q}_1^c \leq \hat{q}_2^c$.

Case 4: $\beta > \frac{3}{4}$. In this case, $\hat{q}_1^c = \hat{q}_2^c = \frac{1}{8(1-\beta)}$. Consequently, $\hat{\pi}_1^c = \hat{\pi}_2^c$.

Combining all four cases, we obtain

$$\hat{\pi}_2^c - \hat{\pi}_1^c = \begin{cases} \leq 0 & 0 < \beta \leq \frac{4\alpha_2 + \sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16\alpha_2+16}; \\ \geq 0 & \frac{4\alpha_2 + \sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16\alpha_2+16} < \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \leq 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}; \\ \geq 0 & \frac{3-2\alpha_2}{2(2-\alpha_2)} < \beta \leq \frac{3}{4}; \\ = 0 & \frac{3}{4} < \beta \leq 1. \end{cases}$$

Notice 1) for $\alpha_2 \in [0,1]$, $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} \leq \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4} \leq \frac{3-2\alpha_2}{2(2-\alpha_2)} \leq \frac{3}{4}$, and 2) as a_2 increases from 0 to 1, $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2}$ increases from 0 to 3/16, $\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$ decreases from 1/2 to 3/8, and $\frac{3-2\alpha_2}{2(2-\alpha_2)}$ decreases from 3/4 to 1/2.

For $\beta \in [0,3/16]$, if $\alpha_2 \leq \frac{8}{3}\beta\left(4-8\beta-\sqrt{2}\sqrt{32\beta^2-32\beta+5}\right)$, then $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} \leq \beta \leq \frac{3-\alpha_2}{8}$, which implies $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$, and if $\alpha_2 \geq \frac{8}{3}\beta\left(4-8\beta-\sqrt{2}\sqrt{32\beta^2-32\beta+5}\right)$, then $\beta \leq \frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \leq 0$.

For $\beta \in (3/16, 3/8]$, $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} < \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$ holds for all $\alpha_2 \in [0,1]$. Therefore, for $\beta \in (3/16, 3/8]$, $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$ for all α_2.

For $\beta \in [3/8, 1/2]$, if $\alpha_2 \leq 4/3-(8\beta-2)^2/3$ then $\frac{4\alpha_2+\sqrt{2}\alpha_2\sqrt{5-3\alpha_2}}{16+16\alpha_2} \leq \beta \leq \frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$; and if $\alpha_2 \geq 4/3-(8\beta-2)^2/3$, then $\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4} \leq \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \leq 0$.

For $\beta \in [1/2, 3/4]$, if $\alpha_2 \leq \frac{1}{2\beta-2}(4\beta-3)$, $\frac{1}{8}\sqrt{4-3\alpha_2}+\frac{1}{4} < \beta \leq \frac{3-2\alpha_2}{2(2-\alpha_2)}$, which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \leq 0$. If $\alpha_2 \geq \frac{1}{2\beta-2}(4\beta-3)$, $\frac{3-2\alpha_2}{2(2-\alpha_2)} < \beta \leq \frac{3}{4}$ which implies that $\hat{\pi}_2^c - \hat{\pi}_1^c \geq 0$.

For $\beta \geq 3/4$. $\hat{\pi}_2^c - \hat{\pi}_1^c = 0$ for all α_2.

The desired result then follows. Q.E.D.

Proof of Lemma EC.5 When choosing its order quantity, Newsvendor 1 knows that Newsvendor 2 orders $\hat{q}_2^c$. Therefore, Newsvendor 1 chooses q_1 to maximize $\pi_1(q_1, \hat{q}_2^c)$ defined in (EC.11). Note that $\pi_1(q_1, \hat{q}_2^c)$ is a piece-wise function with two segments joined at $q_1 = 1-\hat{q}_2^c$. For $q_1 \geq 1-\hat{q}_2^c$, $\frac{\partial \pi_1}{\partial q_1}\Big|_{q_1=1-\hat{q}_2^c} = \frac{1}{2}p\hat{q}_2^c - c = p(\beta+\frac{1}{2}\hat{q}_2^c-1)$ because $\frac{\partial \pi_1}{\partial q_1} = \frac{p\hat{q}_2^c}{2(q_1+\hat{q}_2^c)^2} - c$. For $q_1 \leq 1-\hat{q}_2^c$, $\frac{\partial \pi_1}{\partial q_1}|_{q_1=1-\hat{q}_2^c} = p(\beta-1+\frac{1}{2}\hat{q}_2^c)$ because $\frac{\partial \pi_1}{\partial q_1} = p(\beta-q_1-\frac{1}{2}\hat{q}_2^c)$. So, the two segments of $\pi_1(q_1, \hat{q}_2^c)$ have equal first-order derivatives at the point they join. Consequently, Newsvendor 1's equilibrium order quantity $\hat{q}_1^{fc}$ satisfies

$$\hat{q}_1^{fc} = \begin{cases} \leq 1-\hat{q}_2^c & \beta-1+\frac{1}{2}\hat{q}_2^c \leq 0; \\ > 1-\hat{q}_2^c & \beta-1+\frac{1}{2}\hat{q}_2^c > 0. \end{cases} \tag{EC.31}$$

Suppose $\beta-1+\frac{1}{2}\hat{q}_2^c > 0$. Solving the corresponding first-order condition $\frac{\partial \pi_1}{\partial q_1} = -c+\frac{1}{2}p\frac{\hat{q}_2^c}{(q_1+\hat{q}_2^c)^2} = 0$ yields

$$q_1 = \frac{1}{2-2\beta}\left(\sqrt{2\left(1-\beta\right)\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right). \tag{EC.32}$$

Suppose $\beta - 1 + \frac{1}{2}\hat{q}_2^c \le 0$. Then solving the corresponding first-order condition yields:

$$q_1 = \beta - \frac{1}{2}\hat{q}_2^c. \tag{EC.33}$$

Hence, by (EC.31)-(EC.33), Newsvendor 1's order quantity $\hat{q}_1^{fc}$ satisfies

$$\hat{q}_1^{fc} = \begin{cases} \max(\beta - \frac{1}{2}\hat{q}_2^c, 0) & \hat{q}_2^c \le 2(1-\beta); \\ \frac{1}{2-2\beta}\left(\sqrt{2\,(1-\beta)\,\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right) & \hat{q}_2^c > 2(1-\beta). \end{cases} \tag{EC.34}$$

We next compare $\hat{q}_2^c$ and $2(1-\beta)$. Recall from Lemma EC.4,

$$\hat{q}_2^c = \begin{cases} \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2,\beta) & \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}; \\ \frac{1}{8(1-\beta)} & \beta > \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

Suppose $\beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}$. Then, $\hat{q}_2^c - 2(1-\beta) = \frac{1}{6}\left[h(\alpha_2,\beta) - ((-14+2\alpha_2)\beta + 12 - \alpha_2)\right]$. It can be verified that $(-14+2\alpha_2)\beta + 12 - \alpha_2 \ge 0$ for $\beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}$. So, $\hat{q}_2^c - 2(1-\beta)$ and $h^2(\alpha_2,\beta) - ((-14+2\alpha_2)\beta + 12 - \alpha_2)^2$ have identical signs. Using some algebraic operations and properties of quadratic functions, we find that $h^2(\alpha_2,\beta) - ((-14+2\alpha_2)\beta + 12 - \alpha_2)^2 < 0$, which implies that $\hat{q}_2^c \le 2(1-\beta)$. Suppose $\beta > \frac{3-2\alpha_2}{2(2-\alpha_2)}$. Then, $\hat{q}_2^c - 2(1-\beta) = \frac{1}{8(1-\beta)} - 2(1-\beta) \ge 0 \Longleftrightarrow \beta \ge 3/4$. In summary

$$\hat{q}_2^c = \begin{cases} \le 2(1-\beta) & \beta \le \frac{3}{4}; \\ \ge 2(1-\beta) & \beta > \frac{3}{4}. \end{cases} \tag{EC.35}$$

Combining (EC.34) and (EC.35), we obtain

$$\hat{q}_1^{fc} = \begin{cases} \max(\beta - \frac{1}{2}\hat{q}_2^c, 0) & \beta \le \frac{3}{4}; \\ \frac{1}{2-2\beta}\left(\sqrt{2\,(1-\beta)\,\hat{q}_2^c} - 2\hat{q}_2^c(1-\beta)\right) & \beta > \frac{3}{4}. \end{cases} \tag{EC.36}$$

To completely characterize $\hat{q}_1^{fc}$, we need to compare $\beta - \frac{1}{2}\hat{q}_2^c$ and 0. Suppose $\beta \in (\frac{3-2\alpha_2}{2(2-\alpha_2)}, \frac{3}{4}]$. Then, $\hat{q}_2^c = \frac{1}{8(1-\beta)}$ by Lemma EC.4, which implies that $\beta - \frac{1}{2}\hat{q}_2^c \ge 0$. Suppose $\beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}$. Then, by Lemma EC.4, $\beta - \frac{1}{2}\hat{q}_2^c$ and $(10\beta - \alpha_2 + 2\alpha_2\beta) - h(\alpha_2,\beta)$ have the same sign. If $\beta \le \frac{\alpha_2}{10+2\alpha_2}$, then $10\beta - \alpha_2 + 2\alpha_2\beta \le 0$, which implies that $\beta - \frac{1}{2}\hat{q}_2^c \le 0$. If $\beta > \frac{\alpha_2}{10+2\alpha_2} \Longleftrightarrow 10\beta - \alpha_2 + 2\alpha_2\beta \ge 0$, we can use properties of quadratic functions to show that $(10\beta - \alpha_2 + 2\alpha_2\beta)^2 - h^2(\alpha_2,\beta) \ge 0 \Longleftrightarrow \beta \ge \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$. So, $\beta - \frac{1}{2}\hat{q}_2^c \ge 0 \Longleftrightarrow \beta \ge \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$. In summary,

$$\beta - \frac{1}{2}\hat{q}_2^c = \begin{cases} \le 0 & \beta \le \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}; \\ \ge 0 & \beta > \alpha_2\frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}. \end{cases} \tag{EC.37}$$

Applying (EC.37) to (EC.36), we complete the proof. Q.E.D.

Proof of Proposition EC.10 We have three cases. Case 1: $\beta \le \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$. In this case, $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = p\hat{q}_2^c(\beta - \hat{q}_2^c/2)$ by (EC.13). So, $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} \le 0 \Longleftrightarrow \beta \le \hat{q}_2^c/2$, which holds because $\beta \le \hat{q}_2^c/2$ when $\beta \le \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$ (EC.37).

Case 2: $\alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{3}{4}$. By (EC.13), $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \frac{p}{2}\left(\frac{3}{2}\hat{q}_2^c - \beta\right)\left(\beta - \frac{\hat{q}_2^c}{2}\right) > 0 \Longleftrightarrow \frac{1}{2}\hat{q}_2^c < \beta < \frac{3}{2}\hat{q}_2^c$. Note that $\frac{1}{2}\hat{q}_2^c < \beta$ holds for all $\beta > \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}$; see (EC.37). We only need to rank β and $\frac{3}{2}\hat{q}_2^c$. Recall that by Lemma EC.4,

$$\hat{q}_2^c = \begin{cases} \frac{1}{6}\alpha_2 + \frac{1}{3}\beta - \frac{1}{3}\alpha_2\beta + \frac{1}{6}h(\alpha_2,\beta) & \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}; \\ \frac{1}{8(1-\beta)} & \beta > \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

So, we consider two sub-cases. Case 2.1: $\alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}$. In this subcase, $\frac{3\hat{q}_2^c}{2} - \beta = \frac{1}{4}h(\alpha_2,\beta) - (-\frac{1}{4}\alpha_2 + \frac{1}{2}\beta + \frac{1}{2}\alpha_2\beta) \ge 0$ because $-\frac{1}{4}\alpha_2 + \frac{1}{2}\beta + \frac{1}{2}\alpha_2\beta \le 0$ for $\beta \in (\alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16}, \frac{\alpha_2}{2(1+\alpha_2)}]$. For $\beta \in (\frac{\alpha_2}{2(1+\alpha_2)}, \frac{3-2\alpha_2}{2(2-\alpha_2)}]$, $\frac{3\hat{q}_2^c}{2} - \beta \ge 0 \Longleftrightarrow \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}$ because of the properties of quadratic functions. Hence,

$$\frac{3\hat{q}_2^c}{2} - \beta = \begin{cases} \ge 0 & \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \le 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \le \frac{3-2\alpha_2}{2(2-\alpha_2)}. \end{cases}$$

Case 2.2: $\frac{3-2\alpha_2}{2(2-\alpha_2)} < \beta \le \frac{3}{4}$. In this subcase $\frac{3\hat{q}_2^c}{2} - \beta = \frac{3}{16(1-\beta)} - \beta \le 0$. In summary, for Case 2,

$$\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = \begin{cases} \le 0 & \alpha_2 \frac{\sqrt{2-\alpha_2}+2}{8\alpha_2+16} < \beta \le \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4}; \\ \ge 0 & \frac{1}{8}\sqrt{4-3\alpha_2} + \frac{1}{4} < \beta \le \frac{3}{4}. \end{cases}$$

Case 3: $\beta > 3/4$. In this case, $\hat{\pi}_2^{fc} - \hat{\pi}_1^{fc} = p(\hat{q}_2^c - \hat{q}_1^{fc})\left(1 - \beta + \frac{1}{2(\hat{q}_1^{fc} + \hat{q}_2^c)}\right) = 0$ because $\hat{q}_2^c - \hat{q}_1^{fc} = 0$ from Lemma EC.4 and (EC.12). Q.E.D.

Proof of Proposition EC.11 To prove Proposition EC.11, we first show the following three-part lemma. Lemma EC.11(a) $\mu > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$, Lemma EC.11(b) Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_i} \le 0$, for $i = 1,2$), and Lemma EC.11(c) Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial \hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial \alpha_{3-i}} < 0$ for $i = 1,2$) if and only if $\beta < \hat{m}^c$.

Proof of Lemma EC.11(a): In the equilibrium, $\mathrm{P}(X_1 + (X_2 - \hat{z}_2^c)^+ \le \hat{z}_1^c) = \mathrm{P}(X_2 + (X_1 - \hat{z}_1^c)^+ \le \hat{z}_2^c) = \beta$, where $X_1 = r_1 Y$ and $X_2 = r_2 Y$. When r_1 and r_2 are identically distributed, X_1 and X_2 are identically distributed. As a result, $\hat{z}^c = \hat{z}_1^c = \hat{z}_2^c$, and $\bar{\mu} = \mu_1 = \mu_2$. Thus, $dq_i^c(\alpha)/d\alpha < 0$ if and only if $\hat{z}^c > \bar{\mu}$, which implies that $\hat{z}^c > \bar{\mu}$ if and only if $\beta > \hat{m}^c$.

Proof of Lemma EC.11(b): Without loss of generality, we let $i = 1$. First note that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+ \wedge (\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c)] - c[\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c]$. Thus,

$$\begin{aligned}\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} &= (\bar{\mu} - \hat{z}^c)p[\mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c < X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) - (1-\beta)] \\ &= (\bar{\mu} - \hat{z}^c)p[\beta - \mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+)]. \qquad \text{(EC.38)}\end{aligned}$$

First, if $\hat{z}^c > \bar{\mu}$, then $\mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c < \hat{z}^c$, the second inequality is from $\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c < \hat{z}^c$, and the equality is from Lemma 4. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} < 0$ if $\hat{z}^c > \bar{\mu}$. Second, if $\hat{z}^c \le \bar{\mu}$, then $\mathrm{P}(\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c \ge X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) \ge \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) \ge \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c \ge \hat{z}^c$, the second inequality is from $\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c \ge \hat{z}^c$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \le 0$ if $\hat{z}^c \le \bar{\mu}$. In conclusion, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \le 0$.

Proof of Lemma EC.11(c) Without loss of generality, we let $i = 1$. Note that

$$\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} = p(\hat{z}^c - \bar{\mu})\mathrm{P}[\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c, X_2 \ge \alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c].$$

Because $\mathrm{P}[\alpha_1\bar{\mu} + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - (1-\alpha_2)\hat{z}^c - \alpha_2\bar{\mu}, X_2 \ge \alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c] \ge 0$, the sign of $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2}$ is determined by $\hat{z}^c - \bar{\mu}$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0 \Longleftrightarrow \bar{\mu} > \hat{z}^c$. Furthermore, from Lemma EC.11(a), $\bar{\mu} > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$.

Note that if $\alpha_1 = \alpha_2$, then the newsvendors have identical expected profits. Next, we show that the less biased newsvendor can have a lower expected profit. To that end, suppose $\alpha_1 = 0$. Then, from (EC.38) and Lemma EC.11(c), $\frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2} - \frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\Big|_{\alpha_2=0} = (\bar{\mu} - \hat{z}^c)p[\beta - \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+)] \quad + p(\bar{\mu} - \hat{z}^c)\mathrm{P}(\hat{z}^c > X_1 + X_2 - \hat{z}^c, X_2 \ge \hat{z}^c) > 0$ because $\mathrm{P}(X_1 + (X_2 - \hat{z}^c)^+ < \hat{z}^c) = \beta$ and $\beta < \hat{m}^c \Longrightarrow \bar{\mu} > \hat{z}^c$ from Lemma (a). As a result, the more biased newsvendor (Newsvendor 2) can have a higher expected profit than the less biased newsvendor (Newsvendor 1).

Next, we show that a sophisticated newsvendor can earn a lower expected profit than its overconfident competitor. Define Newsvendor 2 as the biased newsvendor characterized by overconfidence level α_2 and define Newsvendor 1 as the unbiased newsvendor cognizant of its competitor's α_2. Accordingly, denote $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ as the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Similarly, denote $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ as the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^c(\alpha_2) = \alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c$, and $q_1^{fc}(\alpha_2)$ is defined implicitly by $\mathrm{P}[X_1 + (X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \leq q_1^{fc}(\alpha_2)] = \beta$.

Accordingly, Newsvendor 2's equilibrium expected profit can be written as $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}[(X_2 + (X_1 - q_1^{fc}(\alpha_2))^+) \wedge (\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c)] - c[\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c]$, which implies that $\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2} = (\mu - \hat{z}^c)p\{\mathrm{P}[\alpha_2\mu + (1-\alpha_2)\hat{z}^c < X_2 + (X_1 - q_1^{fc}(\alpha_2))^+] - (1-\beta)\} - \mathrm{E}\left[\mathbb{1}_{X_2+(X_1-q_1^{fc}(\alpha_2))^+<\alpha_2\bar{\mu}+(1-\alpha_2)\hat{z}^c, X_1>q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right] = (\bar{\mu} - \hat{z}^c)p\{\beta - \mathrm{P}[\alpha_2\bar{\mu} + (1-\alpha_2)\hat{z}^c > X_2 + (X_1 - q_1^{fc}(\alpha_2))^+]\} - \mathrm{E}\left[\mathbb{1}_{X_2+X_1-q_1^{fc}(\alpha_2)<\alpha_2\bar{\mu}+(1-\alpha_2)\hat{z}^c, X_1>q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right]$. This, in turns, implies that $\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0} = (\bar{\mu} - \hat{z}^c)p\{\beta - \mathrm{P}[\hat{z}^c > X_2 + (X_1 - \hat{z}^c)^+]\} - \mathrm{E}[\mathbb{1}_{X_2+X_1-\hat{z}^c<\hat{z}^c, X_1>\hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] = -\mathrm{E}[\mathbb{1}_{X_2+X_1-\hat{z}^c<\hat{z}^c, X_1>\hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] > 0$, where the second equality follows because $\mathrm{P}[X_2 + (X_1 - \hat{z}^c)^+ < \hat{z}^c] = \beta$, and the inequality follows because $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \bar{\mu}$ from Lemma EC.11(a), $\hat{z}^c < \bar{\mu} \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^c(\alpha_2)}{d\alpha_2} = \bar{\mu} - \hat{z}^c < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$.

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[(X_1 + (X_2 - \alpha_2\bar{\mu} - (1-\alpha_2)\hat{z}^c)^+) \wedge q_1^{fc}(\alpha_2)] - cq_1^{fc}(\alpha_2)$. Thus, from the envelope theorem, $\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\mathrm{E}\left[\mathbb{1}_{X_1+X_2-\alpha_2\bar{\mu}-(1-\alpha_2)\hat{z}^c<q_1^{fc}(\alpha_2), X_2>\alpha_2\bar{\mu}+(1-\alpha_2)\hat{z}^c}(\hat{z}^c - \bar{\mu})\right] < 0$, where the inequality is because, again, $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \bar{\mu}$ from Lemma EC.11(a).

Taken together, Newsvendor 2's equilibrium expected profit is strictly increasing in α_2 around $\alpha_2 = 0$, whereas Newsvendor 1's equilibrium expected profit is strictly decreasing in α_2 around $\alpha_2 = 0$. However, at $\alpha_2 = 0$, $\hat{\pi}_2^{fc}(\alpha_2) = \pi_1^{fc}(\alpha_2)$. Thus, all told, this means that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ around $\alpha_2 = 0$, which suffices to complete the proof. Q.E.D.

Proof of Proposition EC.12 Because the equilibrium profits of the two newsvendors are identical, we have both newsvendors' expected equilibrium profits as $\hat{\pi}^c(\alpha) = \mu(p-c) - \frac{p-c}{2}\mathrm{E}[X_1 + X_2 -$

$2\hat{q}^c(\alpha)]^+ - \frac{c}{2}\mathrm{E}[2\hat{q}^c(\alpha) - X_1 - X_2]^+ = \mu(p-c) - (p-c)\mathrm{E}[\bar{X} - \alpha\rho - (1-\alpha)\hat{z}^c]^+ - c\mathrm{E}[\alpha\rho + (1-\alpha)\hat{z}^c - \bar{X}]^+$. Thus,

$$\begin{aligned}\frac{d\hat{\pi}^c(\alpha)}{d\alpha} &= (p-c)\bar{F}_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)(\rho - \hat{z}^c) + cF_{\bar{X}}(\alpha\mu + (1-\alpha)\hat{z}^c)(\hat{z}^c - \rho)\\ &= (\rho - \hat{z}^c)[(p-c)\bar{F}_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c) - cF_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)]\\ &= (\rho - \hat{z}^c)[(p-c) - pF_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)]\\ &= p[\beta - F_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c)](\rho - \hat{z}^c),\end{aligned}$$

and $\frac{d^2\hat{\pi}^c(\alpha)}{d\alpha^2} = -p(\hat{z}^c - \rho)^2 f_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c) < 0$. As a result, $\hat{\pi}^c(\alpha)$ is concave in α. Accordingly, there are three cases to consider.

Case (a): If $\beta \le \hat{m}^c$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = p(\beta - F_{\bar{X}}(\hat{z}^c))(\rho - \hat{z}^c) \le 0$ because $\hat{z}^c \le \rho$ and $\beta = g(\hat{z}^c) \le F_{\bar{X}}(\hat{z}^c)$. Therefore, if $\beta \le \hat{m}^c$, then $\hat{\pi}^c(\alpha)$ is a decreasing function because of its concavity.

Case (b): If $\hat{m}^c < \beta \le m$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=1} = p(\beta - m)(\rho - \hat{z}^c) \ge 0$ because $\hat{z}^c > \rho \Longleftrightarrow \beta > \hat{m}^c$. Therefore, if $\hat{m}^c < \beta \le m$, then $\hat{\pi}^c(\alpha)$ is an increasing function because of its concavity.

Case (c): If $\beta > m$, then $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=0} = p(\beta - F_{\bar{X}}(\hat{z}^c))(\rho - \hat{z}^c) > 0$ because $g(\hat{z}^c) = \beta$ and $g(\cdot) \le F_{\bar{X}}(\cdot)$. Furthermore, $\frac{d\hat{\pi}^c(\alpha)}{d\alpha}|_{\alpha=1} = p(\beta - m)(\rho - \hat{z}^c) < 0$. Therefore, if $\beta > m$, then $\hat{\pi}^c(\alpha)$ is an increasing-decreasing function. In order to let $\frac{d\hat{\pi}^c(\alpha)}{d\alpha} = 0$, we have $\beta - F_{\bar{X}}(\alpha\rho + (1-\alpha)\hat{z}^c) = 0$. Since $g(\hat{z}^c) = \beta$, we can conclude that $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \rho)$. Q.E.D.

Proof of Proposition EC.13 For the overconfident newsvendors, the order quantities satisfy $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = (\alpha_1 + \alpha_2)\rho + (2 - \alpha_1 - \alpha_2)q_n$. In contrast, the central planner's equilibrium order quantity is $2F_{\bar{X}}^{-1}(\beta)$. Thus, if $\beta > m$, then the system can be coordinated. In particular, $\hat{q}_1^c(\alpha_1) + \hat{q}_2^c(\alpha_2) = 2q_c^*$ when $\hat{\alpha} := (g^{-1}(\beta) - F_{\bar{X}}^{-1}(\beta))/(g^{-1}(\beta) - \mu)$. Q.E.D.

Proof of Proposition EC.14 To prove Proposition EC.14, we first show the following three-part lemma. Lemma EC.14(a) $\rho > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$, Lemma EC.14(b) Newsvendor i's equilibrium expected profit is decreasing in α_i (i.e., $\frac{\partial\hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial\alpha_i} \le 0$, for $i = 1, 2$), and Lemma EC.14(c) Newsvendor i's equilibrium expected profit is decreasing in α_{3-i} (i.e., $\frac{\partial\hat{\pi}_i^c(\alpha_1,\alpha_2)}{\partial\alpha_{3-i}} < 0$ for $i = 1, 2$) if and only if $\beta < \hat{m}^c$.

Proof of Lemma EC.14(a): In the equilibrium, $g(z_1^c) = g(z_2^c) = \beta$. As a result, $\hat{z}^c = \hat{z}_1^c = \hat{z}_2^c$. Thus, $dq_i^c(\alpha)/d\alpha < 0 \Longleftrightarrow \hat{z}^c > \rho \Longleftrightarrow \beta > \hat{m}^c$.

Proof of Lemma EC.14(b): Without loss of generality, we let $i = 1$. First note that Newsvendor 1's profit is $\hat{\pi}_1^c(\alpha_1, \alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+ \wedge (\alpha_1\rho + (1-\alpha_1)\hat{z}^c)] - c[\alpha_1\rho + (1-\alpha_1)\hat{z}^c]$. Thus,

$$\begin{aligned}\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} &= (\rho - \hat{z}^c)p[\mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c < X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) - (1-\beta)] \\ &= (\rho - \hat{z}^c)p[\beta - \mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+)]. \qquad \text{(EC.39)}\end{aligned}$$

First, if $\hat{z}^c > \rho$, then $\mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) < \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\rho + (1-\alpha_1)\hat{z}^c < \hat{z}^c$, the second inequality is from $\alpha_2\rho + (1-\alpha_2)\hat{z}^c < \hat{z}^c$, and the equality is from Lemma 4. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} < 0$ if $\hat{z}^c > \rho$. Second, if $\hat{z}^c \le \rho$, then $\mathrm{P}(\alpha_1\rho + (1-\alpha_1)\hat{z}^c \ge X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) \ge \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+) \ge \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+) = \beta$, where the first inequality is from $\alpha_1\rho + (1-\alpha_1)\hat{z}^c \ge \hat{z}^c$, the second inequality is from $\alpha_2\rho + (1-\alpha_2)\hat{z}^c \ge \hat{z}^c$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_1} \le 0$ if $\hat{z}^c \le \rho$.

Proof of Lemma EC.14(c) Without loss of generality, we let $i = 1$. Note that $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} = p(\hat{z}^c - \rho)\mathrm{P}[\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c, X_2 \ge \alpha_2\rho + (1-\alpha_2)\hat{z}^c]$. Because $\mathrm{P}[\alpha_1\rho + (1-\alpha_1)\hat{z}^c > X_1 + X_2 - (1-\alpha_2)\hat{z}^c - \alpha_2\rho, X_2 \ge \alpha_2\rho + (1-\alpha_2)\hat{z}^c] \ge 0$, the sign of $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2}$ is determined by $\hat{z}^c - \rho$. As a result, $\frac{\partial \hat{\pi}_1^c(\alpha_1,\alpha_2)}{\partial \alpha_2} < 0 \Longleftrightarrow \rho > \hat{z}^c$. Furthermore, from Lemma EC.14(a), $\rho > \hat{z}^c \Longleftrightarrow \hat{m}^c > \beta$.

If $\alpha_1 = \alpha_2$, then the newsvendors have identical expected profits. We next show that if $\beta < \hat{m}^c$, then the less biased newsvendor can earn a lower expected profit. To that end, suppose $\alpha_1 = 0$. Then, from (EC.39) and Lemma EC.14(c), $\frac{\partial \hat{\pi}_2^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2} - \frac{\partial \hat{\pi}_1^c(\alpha_1=0,\alpha_2)}{\partial \alpha_2}\Big|_{\alpha_2=0} = (\rho - \hat{z}^c)p[\beta - \mathrm{P}(\hat{z}^c > X_1 + (X_2 - \hat{z}^c)^+)] \quad + p(\rho - \hat{z}^c)\mathrm{P}(\hat{z}^c > X_1 + X_2 - \hat{z}^c, X_2 \ge \hat{z}^c) > 0$ because $\mathrm{P}(X_1 + (X_2 - \hat{z}^c)^+ < \hat{z}^c) = \beta$ and $\beta < \hat{m}^c \Longrightarrow \rho > \hat{z}^c$ from Lemma EC.14(a). As a result, the more biased newsvendor (Newsvendor 2) can earn a higher expected profit than the less biased newsvendor (Newsvendor 1).

Now, define Newsvendor 2 as the biased newsvendor with overconfidence level α_2 and define Newsvendor 1 as a sophisticated newsvendor. Accordingly, denote $\pi_1^{fc}(\alpha_2)$ and $\hat{\pi}_2^{fc}(\alpha_2)$ as the equilibrium expected profits of Newsvendor 1 and Newsvendor 2, respectively, for this case. Similarly, denote $q_1^{fc}(\alpha_2)$ and $\hat{q}_2^{fc}(\alpha_2)$ as the equilibrium order quantities of Newsvendor 1 and Newsvendor 2, respectively, for this case. Then, $\hat{q}_2^{fc}(\alpha_2) = \hat{q}_2^{c}(\alpha_2) = \alpha_2\rho + (1-\alpha_2)\hat{z}^c$, and $q_1^{fc}(\alpha_2)$ is defined implicitly by

$$\mathrm{P}(X_1 + (X_2 - \hat{q}_2^{fc}(\alpha_2))^+ \leq q_1^{fc}(\alpha_2)) = \beta. \tag{EC.40}$$

Accordingly, Newsvendor 2's equilibrium expected profit $\hat{\pi}_2^{fc}(\alpha_2) = p\mathrm{E}\left[\left(X_2 + (X_1 - q_1^{fc}(\alpha_2))^+\right) \wedge (\alpha_2\rho + (1-\alpha_2)\hat{z}^c)\right] - c[\alpha_2\rho + (1-\alpha_2)\hat{z}^c]$, which implies that $\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2} = (\rho - \hat{z}^c)[\{\mathrm{P}\left[\alpha_2\rho + (1-\alpha_2)\hat{z}^c < X_2 + (X_1 - q_1^{fc}(\alpha_2))^+\right] - (1-\beta)\} - \mathrm{E}\left[\mathbb{1}_{X_2 + (X_1 - q_1^{fc}(\alpha_2))^+ < \alpha_2\rho + (1-\alpha_2)\hat{z}^c, X_1 > q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right] = (\rho - \hat{z}^c)p\{\beta - \mathrm{P}[\alpha_2\rho + (1-\alpha_2)\hat{z}^c > X_2 + (X_1 - q_1^{fc}(\alpha_2))^+]\} - \mathrm{E}\left[\mathbb{1}_{X_2 + X_1 - q_1^{fc}(\alpha_2) < \alpha_2\rho + (1-\alpha_2)\hat{z}^c, X_1 > q_1^{fc}(\alpha_2)} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}\right]$. This, in turns, implies that $\left.\frac{d\hat{\pi}_2^{fc}(\alpha_2)}{d\alpha_2}\right|_{\alpha_2=0} = (\rho - \hat{z}^c)p\{\beta - \mathrm{P}[\hat{z}^c > X_2 + (X_1 - \hat{z}^c)^+]\} - \mathrm{E}[\mathbb{1}_{X_2 + X_1 - \hat{z}^c < \hat{z}^c, X_1 > \hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] = -\mathrm{E}[\mathbb{1}_{X_2 + X_1 - \hat{z}^c < \hat{z}^c, X_1 > \hat{z}^c} \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2}|_{\alpha_2=0}] > 0$, where the second equality follows because $\mathrm{P}\left[X_2 + (X_1 - \hat{z}^c)^+ < \hat{z}^c\right] = \beta$, and the inequality follows because $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \rho$, $\hat{z}^c < \rho \Longrightarrow \frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} = \frac{d\hat{q}_2^{c}(\alpha_2)}{d\alpha_2} = \rho - \hat{z}^c < 0$, and $\frac{d\hat{q}_2^{fc}(\alpha_2)}{d\alpha_2} > 0 \Longrightarrow \frac{dq_1^{fc}(\alpha_2)}{d\alpha_2} < 0$ from (EC.40).

Next, note that Newsvendor 1's equilibrium expected profit can be written as $\pi_1^{fc}(\alpha_2) = p\mathrm{E}[X_1 + (X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c)^+ \wedge q_1^{fc}(\alpha_2)] - cq_1^{fc}(\alpha_2)$. Thus, from the envelope theorem, $\frac{d\pi_1^{fc}(\alpha_2)}{d\alpha_2} = p\mathrm{E}\left[\mathbb{1}_{X_1 + X_2 - \alpha_2\rho - (1-\alpha_2)\hat{z}^c < q_1^{fc}(\alpha_2), X_2 > \alpha_2\rho + (1-\alpha_2)\hat{z}^c}(\hat{z}^c - \rho)\right] < 0$, where the inequality is because, again, $\beta < \hat{m}^c \Longrightarrow \hat{z}^c < \rho$ from Lemma EC.14(a).

Taken together, Newsvendor 2's equilibrium expected profit is strictly increasing in α_2 around $\alpha_2 = 0$, whereas Newsvendor 1's equilibrium expected profit is strictly decreasing in α_2 around $\alpha_2 = 0$. However, at $\alpha_2 = 0$, $\hat{\pi}_2^{fc}(\alpha_2) = \pi_1^{fc}(\alpha_2)$. Thus, all told, this means that $\hat{\pi}_2^{fc}(\alpha_2) > \pi_1^{fc}(\alpha_2)$ around $\alpha_2 = 0$, which suffices to establish Proposition EC.14. Q.E.D.