Qn1

a) KS TEST for 10 numbers:
   Alpha = 0.05
   Critical Value = 0.410

Trail 1:

Ri
[0.05499319522517876, 0.21180958783961223, 0.2451745896082982, 0.27700784844038095, 0.3212177136803722, 0.39633507971324566, 0.40494651565115747, 0.41250661812276257, 0.6786418434443274, 0.9953977550280638]
i/N
[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+
[0.045006804774821246, 0.0, 0.0548254103917018, 0.12299215155961907, 0.17878228631962778, 0.20366492028675431, 0.2950534843488425, 0.3874933818772375, 0.2213581565556726, 0.004602244971936176]
D-
[0.05499319522517876, 0.11180958783961223, 0.04517458960829818, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.095397550280638]

max(D+,D-): 0.3874933818772375
Dalpha: 0.41
Ho is not Rejected

Trial 2

Ri[0.06582589083598667, 0.11128997467826185, 0.1454786735433894, 0.30061426591032936, 0.48133442992760167, 0.5455460652534981, 0.6376064503343324, 0.6419505635897427, 0.9485067543678792, 0.9754735201516768]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.03417410916401334, 0.08871002532173816, 0.1545213264566106, 0.09938573408967066, 0.01866557007239833, 0.05445393474650184, 0.06239354966566757, 0.15804943641025737, 0.0, 0.024526479848323235]
D-[0.0658258908359867, 0.011289974678261844, 0.0, 6.142659103293702E-4, 0.08133442992760165, 0.045546065253498136, 0.03760645033433241, 0.0,

0.1485067543678792, 0.07547352015167674]
max(D+,D-): 0.15804943641025737
Dalpha: 0.41
Ho is not Rejected

Trail 3

Ri[0.13553362284656112, 0.3454326883998672, 0.4662267270100906, 0.6237097885356061, 0.6274522525047799, 0.6276606454109102, 0.7138885297333445, 0.7353821678401599, 0.8952236441140917, 0.9168600939875273]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.06461783215984018, 0.004776355885908301, 0.08313990601247268]
D-[0.13553362284656112, 0.2454326883998672, 0.2662267270100906, 0.3237097885356061, 0.22745225250477985, 0.12766064541091016, 0.11388852973334451, 0.035382167840159906, 0.09522364411409168, 0.016860093987527303]
max(D+,D-): 0.3237097885356061
Dalpha: 0.41
Ho is not Rejected

Trail 4

Ri[0.01318027723078452, 0.021586822063231703, 0.08899871941669302, 0.10671466998041057, 0.11900435792105979, 0.14931771751264333, 0.35716643993942565, 0.49308481869078724, 0.9233175685649642, 0.9522176968760516]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.08681972276921548, 0.1784131779367683, 0.21100128058330697, 0.29328533001958945, 0.3809956420789402, 0.45068228248735664, 0.3428335600605743, 0.3069151813092128, 0.0, 0.04778230312394838]

D-[0.01318027723078452, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.12331756856496412, 0.0522176968760516]
max(D+,D-): 0.45068228248735664
Dalpha: 0.41
Ho is Rejected

B) Chi square test
Alpha is :0.05
Critical Value :16.9
Note Expected is always :100
Trial 1

Observed[0]) = 106.0
Observed[1]) = 96.0
Observed[2]) = 90.0
Observed[3]) = 93.0
Observed[4]) = 108.0
Observed[5]) = 106.0
Observed[6]) = 84.0
Observed[7]) = 115.0
Observed[8]) = 115.0
Observed[9]) = 87.0
The X0 for Chi-Square test is : 11.76
The critical value is: 16.9
Hypothesis is not Rejected

Trail 2

Observed[0]) = 99.0
Observed[1]) = 94.0
Observed[2]) = 106.0
Observed[3]) = 109.0
Observed[4]) = 97.0
Observed[5]) = 103.0
Observed[6]) = 95.0
Observed[7]) = 105.0
Observed[8]) = 97.0
Observed[9]) = 95.0
The X0 for Chi-Square test is : 2.56

```
The critical value is: 16.9
Hypothesis is not Rejected
```

Trial 3

```
Observed[0]) = 117.0
Observed[1]) = 115.0
Observed[2]) = 112.0
Observed[3]) = 85.0
Observed[4]) = 108.0
Observed[5]) = 100.0
Observed[6]) = 81.0
Observed[7]) = 104.0
Observed[8]) = 81.0
Observed[9]) = 97.0
The X0 for Chi-Square test is : 16.94
The critical value is: 16.9
Hypothesis is Rejected
```

Trail 4

```
Observed[0]) = 87.0
Observed[1]) = 109.0
Observed[2]) = 104.0
Observed[3]) = 123.0
Observed[4]) = 87.0
Observed[5]) = 105.0
Observed[6]) = 95.0
Observed[7]) = 115.0
Observed[8]) = 84.0
Observed[9]) = 91.0
The X0 for Chi-Square test is : 15.760000000000002
The critical value is: 16.9
Hypothesis is not Rejected
```

AS we can see from the observation for chi square and KS square there was a rare instance of a reject among four trial. Its always important to run more trials on a random number generator before accepting or rejecting it.

Now changing the level of significance to 0.01

a) Ks Test

Alpha = 0.01

Critical Value : 0.490

Trail 1

Ri[0.010637938595042029, 0.04750459881255009, 0.12265786540677981, 0.26498696321689685, 0.5533212938643327, 0.5951768961326804, 0.6085632732808113, 0.6756690730079689, 0.7149522248652422, 0.8336465904204234]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.08936206140495798, 0.15249540118744992, 0.17734213459322018, 0.13501303678310317, 0.0, 0.004823103867319545, 0.09143672671918868, 0.12433092699203119, 0.18504777513475779, 0.16635340957957656]
D-[0.010637938595042029, 0.0, 0.0, 0.0, 0.1533212938643327, 0.09517689613268043, 0.008563273280811301, 0.0, 0.0, 0.0]
max(D+,D-): 0.18504777513475779
Dalpha: 0.49
Ho is not Rejected

Trail 2

Ri[0.0235134888528451, 0.17431497009149122, 0.20689455536678192, 0.2571006256482149, 0.276861475971271, 0.3100759412818265, 0.38005181352338036, 0.6121202359338487, 0.8837087136162771, 0.9377582541177282]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.0764865111471549, 0.025685029908508794, 0.09310544463321807, 0.14289937435178512, 0.22313852402872902, 0.2899240587181735, 0.3199481864766196, 0.18787976406615137, 0.016291286383722903, 0.062241745882271826]
D-[0.0235134888528451, 0.07431497009149121, 0.006894555366781907, 0.0, 0.0, 0.0, 0.0, 0.0, 0.08370871361627708, 0.03775825411772815]
max(D+,D-): 0.3199481864766196
Dalpha: 0.49
Ho is not Rejected

Trail 3

Ri[0.1746786492536907, 0.228919980204894, 0.23303286415326752, 0.24934598564131816, 0.6281593764257926, 0.6454947045782701, 0.6471082245114969, 0.7074613189958832, 0.7836871582454362, 0.8032376109125526]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.0, 0.0, 0.06696713584673247, 0.15065401435868186, 0.0, 0.0, 0.052891754885031, 0.09253868100411689, 0.1163128417545638, 0.19676238908744736]
D-[0.1746786492536907, 0.128919980204894, 0.033032864153267505, 0.0, 0.22815937642579254, 0.14549470457827007, 0.04710822451149688, 0.007461318995883204, 0.0, 0.0]
max(D+,D-): 0.22815937642579254
Dalpha: 0.49
Ho is not Rejected

Trail 4

Ri[0.13623458428090107, 0.16266758648883717, 0.2511883323326206, 0.41290393502427725, 0.4537054231935459, 0.6024917977139677, 0.6096010418970375, 0.9202748056319452, 0.9833557794802724, 0.9843079556121074]
i/N[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
D+[0.0, 0.03733241351116284, 0.04881166766737938, 0.0, 0.0462945768064541, 0.0, 0.0903989581029625, 0.0, 0.0, 0.015692044387892623]
D-[0.13623458428090107, 0.06266758648883716, 0.0511883323326206, 0.11290393502427726, 0.053705423193545876, 0.10249179771396766, 0.009601041897037477, 0.22027480563194524, 0.18335577948027237, 0.08430795561210735]
max(D+,D-): 0.22027480563194524
Dalpha: 0.49
Ho is not Rejected

With the new Critical value, This Random Generator is not rejected by the hypothesis even with many trials.

b) Chi Square Test
   Alpha = 0.01

Critical Value :

Trail 1

```
Observed[0]) = 98.0
Observed[1]) = 102.0
Observed[2]) = 113.0
Observed[3]) = 93.0
Observed[4]) = 109.0
Observed[5]) = 112.0
Observed[6]) = 93.0
Observed[7]) = 78.0
Observed[8]) = 92.0
Observed[9]) = 110.0
The X0 for Chi-Square test is : 11.48
The critical value is: 21.7
Hypothesis is not Rejected
```

Trail 2

```
Observed[0]) = 94.0
Observed[1]) = 86.0
Observed[2]) = 126.0
Observed[3]) = 105.0
Observed[4]) = 90.0
Observed[5]) = 117.0
Observed[6]) = 99.0
Observed[7]) = 104.0
Observed[8]) = 101.0
Observed[9]) = 78.0
The X0 for Chi-Square test is : 18.240000000000002
The critical value is: 21.7
Hypothesis is not Rejected
```

Trail 3

```
Ho is not Rejected
Observed[0]) = 87.0
Observed[1]) = 106.0
```

```
Observed[2]) = 76.0
Observed[3]) = 97.0
Observed[4]) = 106.0
Observed[5]) = 101.0
Observed[6]) = 96.0
Observed[7]) = 105.0
Observed[8]) = 120.0
Observed[9]) = 106.0
The X0 for Chi-Square test is : 13.04
The critical value is: 21.7
Hypothesis is not Rejected
```

Trail 4

```
Observed[0]) = 101.0
Observed[1]) = 114.0
Observed[2]) = 103.0
Observed[3]) = 90.0
Observed[4]) = 89.0
Observed[5]) = 110.0
Observed[6]) = 96.0
Observed[7]) = 99.0
Observed[8]) = 95.0
Observed[9]) = 103.0
The X0 for Chi-Square test is : 5.779999999999999
The critical value is: 21.7
Hypothesis is not Rejected
```

with an increased Critical value, the Random Generator seems to be not rejected by the Chi square test but we might need to run a large number of simulations to come up with any conclusions.

Qn2 :
Number of Values = 1000
Intervals = 10
Level of Significance =0.05
Critical Value : 16.9

Performing Chi Square test.

Trial 1

X[0] = 0.134567788;

X[1] = 0.645372899;

```
Observed[0]) = 85.0
Observed[1]) = 94.0
Observed[2]) = 104.0
Observed[3]) = 96.0
Observed[4]) = 102.0
Observed[5]) = 104.0
Observed[6]) = 110.0
Observed[7]) = 113.0
Observed[8]) = 97.0
Observed[9]) = 95.0
The X0 for Chi-Square test is : 6.16
The critical value is: 16.9
Hypothesis is not Rejected
```

Trial 2

```
X0:0.6102254660695621
X1:0.6062135593541157
Observed[0]) = 88.0
Observed[1]) = 96.0
Observed[2]) = 97.0
Observed[3]) = 121.0
Observed[4]) = 87.0
Observed[5]) = 97.0
Observed[6]) = 100.0
Observed[7]) = 95.0
Observed[8]) = 112.0
Observed[9]) = 107.0
The X0 for Chi-Square test is : 10.059999999999999
The critical value is: 16.9
Hypothesis is not Rejected
```

Trial 3

```
X0:0.7670518126538496
X1:0.49081135124435227
Observed[0]) = 106.0
Observed[1]) = 85.0
Observed[2]) = 96.0
Observed[3]) = 104.0
Observed[4]) = 100.0
Observed[5]) = 101.0
Observed[6]) = 92.0
Observed[7]) = 106.0
Observed[8]) = 96.0
Observed[9]) = 114.0
The X0 for Chi-Square test is : 6.06
The critical value is: 16.9
Hypothesis is not Rejected
```

Even though the Hypothesis is not rejected, this random number generator highly predictable and forms an ascending order of sequence till the value reaches above 1.0. Moreover, the initial values X0 and X1 were chosen at random and affects the whole random number generator.

Qn3)

CODE ATTACHED

Given x0= 7,a= 11,m= 16
The Sequence is:
7
13
15
5
Given x0= 8,a= 11,m= 16
The Sequence is:

8
Given x0= 7,a= 7,m= 16
The Sequence is:
7
1
Given x0= 8,a= 7,m= 16
The Sequence is:
8

We haven't achieved the maximum period for the sequences. Max period is m/4 since X0 and a is odd. This method is not good for generating random numbers without using the increment.

Qn6) Code attached

Snippets below:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

int main(void){
        double randomnumber,arr[10];
        int seed = 12325;
        srand(seed);
        int lamda = 1;
        int r=0;
        printf("SEED is %d \n",seed);
        printf("RAND_MAX is %d \n",RAND_MAX);
        for(r=0;r<10;r++)
                {
                   randomnumber = (rand()/(double)(RAND_MAX));
         arr[r]= (-1)*(1/lamda)*log(1-randomnumber);
         randomnumber = 0;
                        printf("   %f \n",arr[r]);
                }
        return 0;
}
```

Output:

```
SEED is 12325
RAND_MAX is 2147483647
  0.101435
  0.227200
  1.805429
  1.986869
  0.440369
  1.114655
  1.742369
  0.007665
  0.397797
  0.094247

SEED is 62325
RAND_MAX is 2147483647
  0.668998
  0.097223
  0.107258
  0.461198
  1.266103
  0.997739
  0.057317
  0.279025
  0.160748

SEED is 82332
RAND_MAX is 2147483647
  1.033838
  1.466089
  0.687935
  0.873858
  1.571134
  0.427295
  0.324819
  0.342306
```

1.781920
0.145614

Qn7. Code Submitted on connex

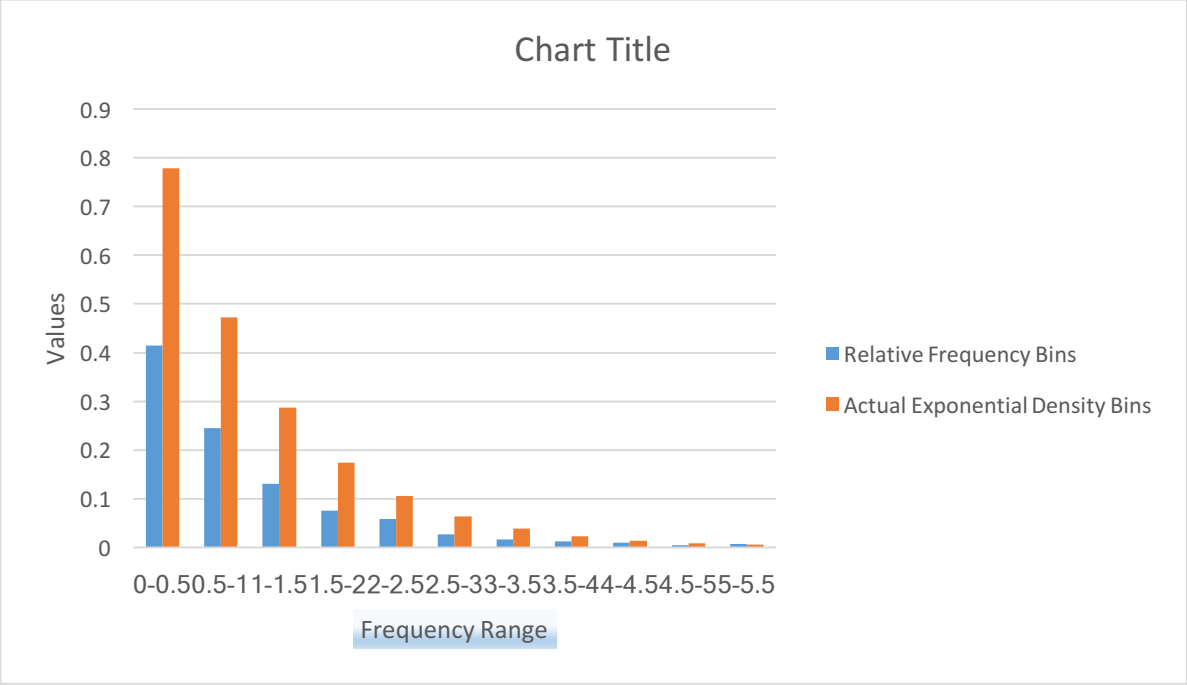
The calculated values from Program are:

SEED is 82332
The relative freq Bins are:
0.414000
0.245000
0.131000
0.075000
0.059000
0.027000
0.016000
0.012000
0.010000
0.004000
0.007000
exponential_theoretical_density Bins:
0.778801
0.472367
0.286505
0.173774
0.105399
0.063928
0.038774
0.023518
0.014264
0.008652
0.005248

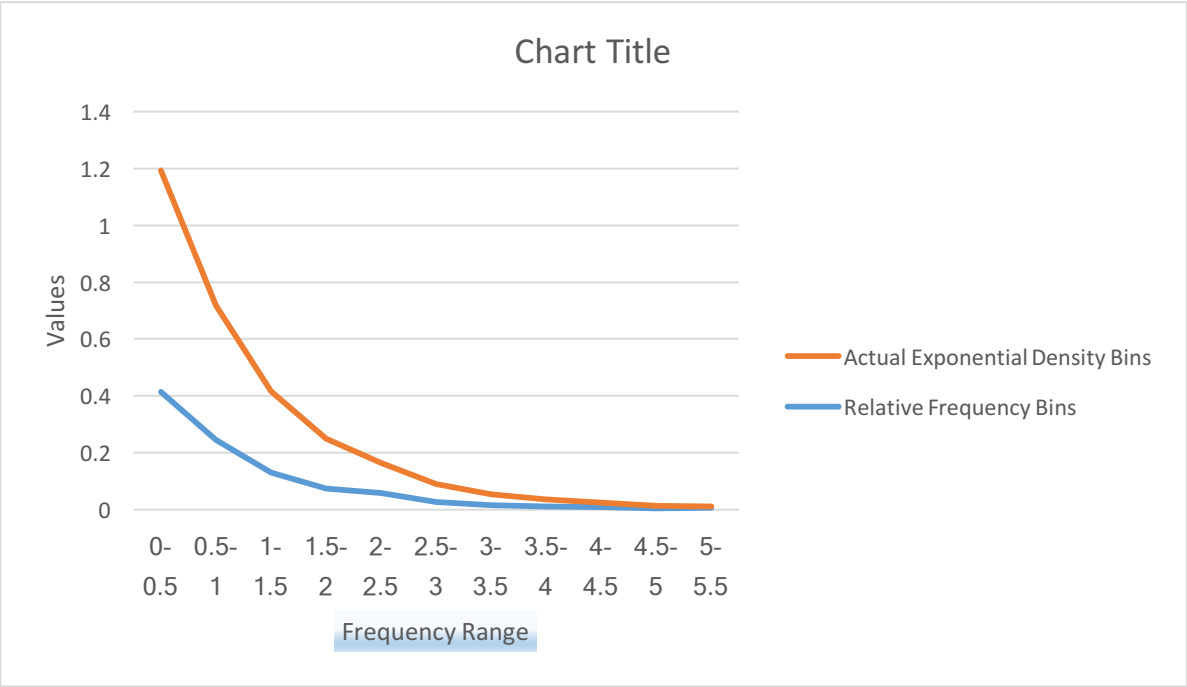
The cumulative freq are :
0.414000
0.659000
0.790000
0.865000
0.924000
0.951000
0.967000
0.979000
0.989000
0.993000
1.000000

exponential_theoretical_cumulative:
0.221199
0.527633
0.713495
0.826226
0.894601
0.936072
0.961226
0.976482
0.985736
0.991348
0.994752

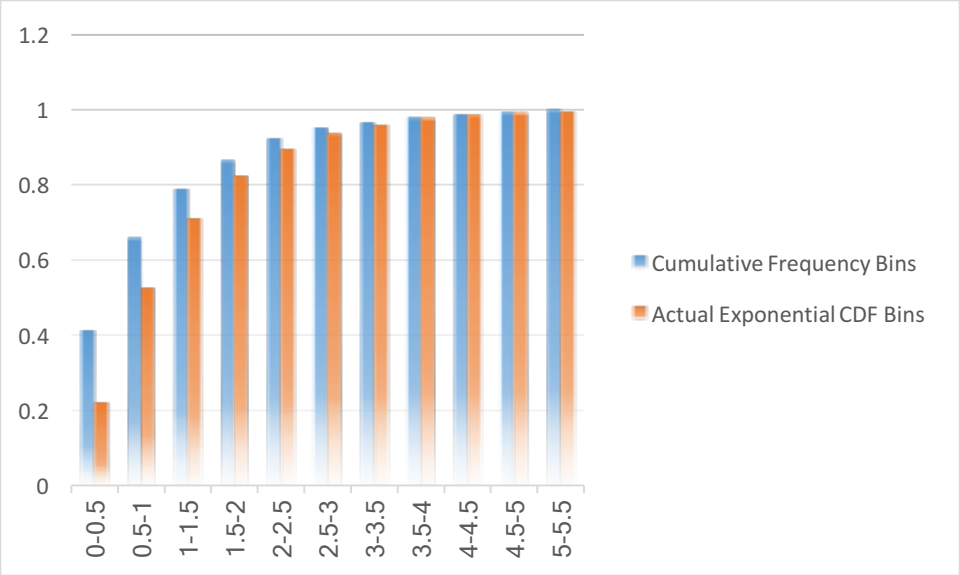
Intervals	Relative Frequency Bins	Actual Exponential Density Bins
0-0.5	0.414	0.778801
0.5-1	0.245	0.472367
1-1.5	0.131	0.286505
1.5-2	0.075	0.173774
2-2.5	0.059	0.105399
2.5-3	0.027	0.063928
3-3.5	0.016	0.038774
3.5-4	0.012	0.023518
4-4.5	0.01	0.014264
4.5-5	0.004	0.008652
5-5.5	0.007	0.005248

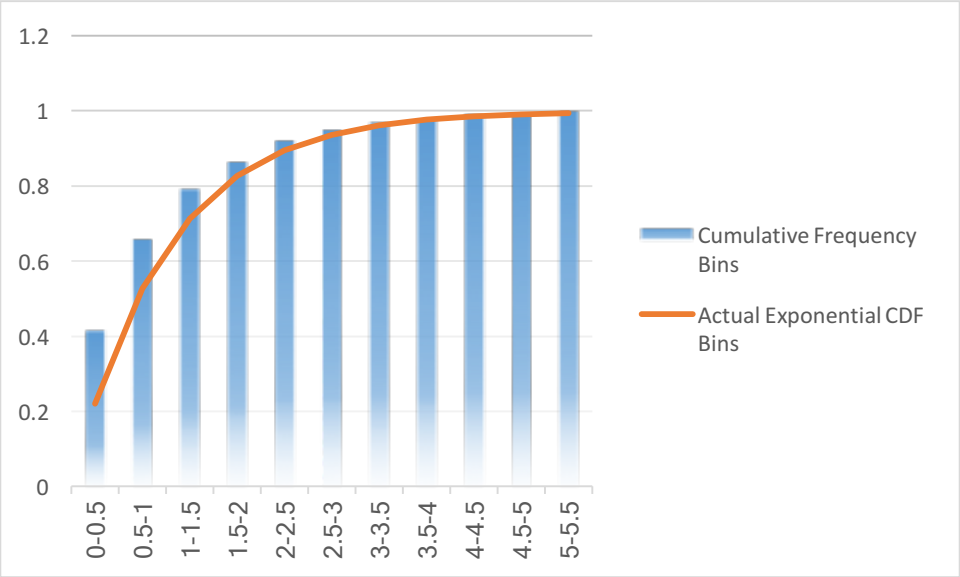


When we compare the Generated with Theoretical exponential density, we can observe that there is a huge difference in the generated and theoretical values. However, the generated data follows an exponential pattern.



Intervals	Cumulative Frequency Bins	Actual Exponential CDF Bins
0-0.5	0.414	0.221199
0.5-1	0.659	0.527633
1-1.5	0.79	0.713495
1.5-2	0.865	0.826226
2-2.5	0.924	0.894601
2.5-3	0.951	0.936072
3-3.5	0.967	0.961226
3.5-4	0.979	0.976482
4-4.5	0.989	0.985736
4.5-5	0.993	0.991348
5-5.5	1	0.994752





The cumulative frequency of generated data seems to be more concentrated to the lower intervals. But at values closer to one, the generated cdf becomes nearly identical to theoretical one.

Qn4) Given $f(x) = \begin{cases} e^{2x} & -\infty < x \leq 0 \\ e^{-2x} & 0 < x < \infty \end{cases}$

Case $-\infty < x \leq 0$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_{-\infty}^{x} = \frac{1}{2}(e^{2x})$$

Case $0 < x < \infty$.

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx \Rightarrow \int_{-\infty}^{0} e^{2x}\,dx + \int_{0}^{x} e^{-2x}\,dx$$

$$\Rightarrow \left[\frac{e^{2x}}{2}\right]_{-\infty}^{0} + -\left[\frac{e^{-2x}}{2}\right]_{0}^{x}$$

$$\Rightarrow \frac{1}{2} - \frac{e^{-2x}}{2} + 1/2$$

$$= 1 - \frac{e^{-2x}}{2}$$

So, $F(x) = \begin{cases} \frac{1}{2}e^{2x} & ; -\infty < x \leq 0 \\ 1 - \frac{e^{-2x}}{2} & ; 0 < x < \infty. \end{cases}$

Lets Assume $F(x) = R$ on $-\infty < x < \infty$;

$R = \frac{1}{2}e^{2x}$; $-\infty < x \leq 0$

$2R = e^{2x}$

$\frac{1}{2}\ln(2R) = x$

When $x = 0$, $R = 1/2$

When $x = -\infty$ $R = 0$.

So $0 < R < 1/2$; $x = \frac{1}{2}\ln(2R)$

$R = 1 - \frac{e^{-2x}}{2}$; $0 < x < \infty$.

$-2R + 2 = e^{-2x}$

$\ln(2 - 2R) = -2x$

$x = -\frac{1}{2}\ln(2 - 2R)$

When $x = 0$, $R = 1/2$

When $x = \infty$, $R = 0$

So $1/2 < R < 0$; $x = -1/2\ln(2 - 2R)$

$$X = \begin{cases} \frac{1}{2}\ln(2R) & 0 < R \leq \frac{1}{2} \\ -\frac{1}{2}\ln(2-2R) & \frac{1}{2} < R < 1 \end{cases}$$

Qn8) The methods to follow are below:

a) Set the value of $n = 0$

b) Generate Random Variable R

c) check if $R \leq p$, if true set $X = n$

else

if $R > p$, increment n by 1 and Generate another random Variate R.

Then Repeat step C.

In case you need to Generate more Geometric Variate, do step a again.

$P(x) = p(1-p)^x$, $X = q + \left[\frac{\ln(1-R)}{\ln(1-p)} - 1\right]$.

Let us take mean = 2, $q = 1$ $\frac{1}{p}$ = mean, so $p = \frac{1}{2}$

$R_1 = 0.0932$, $X_1 = 1 + [-1.443\ln(1-0.932) - 1] = 4$

Step a) Lets set $n = 0$, $F(x) = \sum_{j=0}^{n} p(1-p)^j = 1-(1-p)^{x+1}$.

When $n = 0$, $p = \frac{1}{2}$ [mean = $\frac{1}{p}$]

Step b) Let Generated Random Variable $R = 0.105$

Now $X_1 = 1 + \left[\frac{\ln(1-0.105)}{\ln(1-0.5)}\right] - 1 = \underline{1}$

Step c) $R \leq p$ [$0.105 \leq 0.5$], so we set $X = 0$

5) Given: Beginning with first number, test for Auto Correlation for every third number

Sequence = [0.594, 0.055, 0.262, 0.442, 0.227, 0.825, 0.929].

Here; $i = 5$, $m = 3$, $N = 20$

Let us calculate M:

$$I + (M+1)m \leq N$$

$$\Rightarrow 1 + (M+1)3 \leq 20$$

$$3M \leq 16$$

$$M \leq 16/3$$

$$M = 16/3 \quad [\text{Approx}] = 5.3$$

$$\hat{\rho}_{13} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} \cdot R_{i(k+i)m}\right] - 0.25$$

$$= \frac{1}{6.3}[(0.594 * 0.055) + (0.055 * 0.262) + (0.262 * 0.442) + (0.442 * 0.227) + (0.227 * 0.825) + (0.825 * 0.929)] - 0.25$$

$$= \frac{1}{6.3}[0.03267 + 0.01441 + 0.115804 + 0.100334 + 0.187275 + 0.766425] - 0.25$$

$$= \frac{1}{6.3}[1.216918] - 0.25$$

$$= -0.056838$$

$$\hat{\sigma}_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)} = \frac{\sqrt{13\times 53+7}}{12(6.8)} = \frac{\sqrt{75.9}}{75.6} = \frac{8.7120}{75.6}$$

$$= 0.115238896$$

$$Z_0 = \frac{\hat{\rho}_{im}}{\hat{\sigma}_{\hat{\rho}_{im}}} = \frac{-0.056838}{0.115238896}$$

$$= -0.49321 \in (-Z_{0.025}, Z_{0.025})$$

Do not reject the null hypothesis of independence.

There seems to a effective negative correlation between every third numbers.