Operation Research II

Assignment 2

Ans1) (a) $f(x) = \begin{cases} x & 0 \le x \le c \\ 2c - x & c \le x \le 2c \end{cases}$

b) cdf $= \int_0^{\infty} f(x)\,dx$

$= \int_0^{c} x\,dx + \int_c^{x} (2c - x)\,dx$

$= \left[\frac{x^2}{2}\right]_0^c + \left[2cx - \frac{x^2}{2}\right]_c^x$

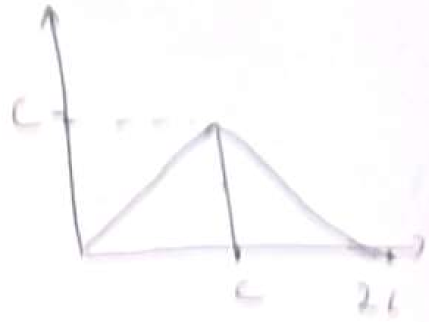
$= \frac{c^2}{2} + \frac{4cx - x^2}{2} - \frac{4c^2 + c^2}{2}$

$= \frac{c^2 + 4cx - x^2 - 4c^2 + c^2}{2}$

$= \frac{4cx - x^2 - 2c^2}{2}$

$F(x) = \begin{cases} x^2/2 & \text{for } 0 < x \le c \\ \frac{4cx - x^2 - 2c^2}{2} & \text{for } c < x \le 2c \end{cases}$

(c) $F(x=c) = \frac{1}{2}(\text{base} \times \text{Alt}) + \frac{1}{2}(\text{base} \times \text{alt})$



$= \frac{1}{2} \times c \times c + \frac{1}{2}[2c - c] \times c$

$= \frac{1}{2}c^2 + \frac{1}{2}\left[2c^2 - c^2\right]$

$= \frac{1}{2}c^2 + \frac{1}{2}c^2$

$= c^2$

Now total area $= 1$; $c^2 = 1$

$\boxed{c = 1}$

(d) $P(0.5 \leq x \leq 1.5)$

$F(1.5) = \frac{4cx - x^2 - 2c^2}{2}$ where $x = 1.5$ and $c = 1$

$= \frac{4 \times 1 \times 1.5 - (1.5)^2 - 2 \times 1^2}{2} = \frac{6 - 2.25 - 2}{2} = \frac{1.75}{2}$

$F(0.5) = \left[\frac{x^2}{2}\right]$ where $x = 0.5$

$= \frac{(0.5)^2}{2} = \frac{.25}{2}$

$F(1.5) - F(0.5) = P(0.5 \leq x \leq 1.5)$

$= \frac{1.75}{2} - \frac{0.25}{2} = \frac{1.50}{2} = \frac{15}{20} = \underline{3/4}$

e) First moment = Mean = $E(x)$

$$E(x) = \int_0^{2c} x f(x)\,dx$$

$$= \int_0^{c} x \cdot x\,dx + \int_c^{2c} x(2c-x)\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^c + \int_c^{2c} 2cx\,dx - \int_c^{2x} x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^c + 2c \cdot \left[\frac{x^2}{2}\right]_c^{2c} - \left[\frac{x^3}{3}\right]_c^{2c}$$

$$= \frac{c^3}{3} + c\left[4c^2 - c^2\right] - \frac{1}{3}\left[8c^3 - c^3\right]$$

$$= \frac{c^3}{3} + 3c^3 - \frac{1}{3} 7c^3$$

$$= \frac{c^3}{3} + \frac{9c^3 - 7c^3}{3}$$

$$= \frac{3c^3}{3}$$

$$E(x) = c^3 = 1$$

$$E(x^2) = \int_0^{2c} x^2 f(x)\,dx$$

$$= \int_0^{c} x^2 \cdot x\,dx + \int_c^{2c} x^2(2c-x)\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^c + 2c\int_c^{2c} x^2\,dx - \int_c^{2c} x^3\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^c + 2c\left[\frac{x^3}{3}\right]_c^{2c} - \left[\frac{x^4}{4}\right]_c^{2c}$$

$$= \left[\frac{x^4}{4}\right]_0^c + \frac{2c}{3}\left[8c^3 - c^3\right] - \frac{1}{4}\left[16c^4 - c^4\right]$$

$$= \frac{c^4}{4} + \frac{2c}{3} \cdot 7c^3 - \frac{1}{4} \cdot 15c^4$$

$$= \frac{3c^4 - 45c^4 + 15c^4}{12}$$

$$= \frac{14c^4}{12}$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2$$

$$= \frac{14c^4}{12} - [c^3]^2$$

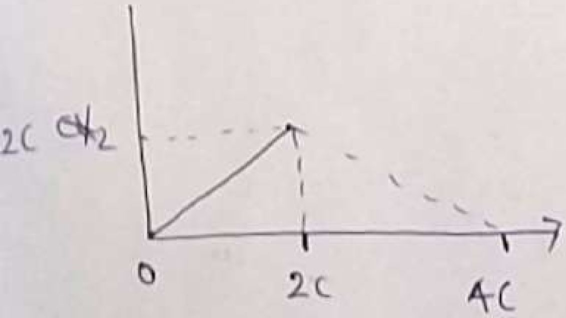
$$= \frac{7c^4 - 6c^6}{6}$$

$c = 1$;

$$= \frac{7-6}{6} = 1/6$$

$$= \frac{1}{6}$$

(f) $Y = 2X$



Area $\frac{1}{2} \times 2c \times y + \frac{1}{2} \times (4c - 2c) \times y$

$$= \frac{2cy}{2} + \frac{2cy}{2}$$

$$= 2cy = 1$$

$$= y = 1/2c$$

Area $= 1$

$$f(y) = \begin{cases} y/4 & 0 \le y \le 2c \\ 1 - y/4 & 2c < y \le 4c \end{cases}$$

Qn2) $E[Y] = E(X_1 + X_2 + \quad + X_n]$

$= E(X_1] + E[X_2] + \quad + E(X_n)$

$= h + h + \quad + h$

$\boxed{E(Y) = n \cdot h}$

$Var[Y] = E[Y^2] - (E(Y)]^2$

$= E[(X_1 + X_2 + \quad X_n)^2] - [E(X_1 + X_2 \quad + X_n)]^2$

$= E(X_1)^2 + E(X_2)^2 + E(X_3)^2 \quad + E(X_n)^2 + 2E(X_1)\,E(X_2)$

$\quad \cdot + 2E(X_n)\,E(X_{n-1})$

$- [E(X_1)]^2 - (E(X_2)]^2 \quad - [E(X_n)]^2 - 2E(X_1)E(X_2)$

$\quad \cdot -2E(X_n)\,E(X_{n-1})$

$= E(X_1)^2 - [E(X_1)]^2 + E(X_2)^2 - [E(X_2)]^2 \cdot \; E(X_n)^2 - [E(X_n)]^2$

$= \sigma^2 + \sigma^2 + \sigma^2 \cdots + \sigma^2$

$Var[Y] = n \cdot \sigma^2$

Qn3) a) Service time at Security check

This is a M|M|1 System,

$\lambda_{security} = \lambda_1 + \lambda_2 = 20 + 10 = 30$, $\rho_{security} = 0.80$

$\rho = \frac{\lambda}{\mu} \Rightarrow 0.80 = \frac{30}{\mu_{security}}$

$\Rightarrow \mu_{security} = 37.5$ Customers per hour.

b) Performance metrics at Terminal 1 check in,

$\lambda_{T_1} = 10$ Cust / hour $\quad \mu_{T_1} = 30$ Cust

$\rho = \frac{\lambda_{T_1}}{\mu_{T_1}} = \frac{10}{3} = 1/3$

$L_{T1} = \dfrac{\rho}{1-\rho} = \dfrac{1/3}{1-1/3} = \dfrac{1/3}{2/3} = 1/2$ or 30 Customer in hour

$L_Q = \dfrac{\rho^2}{1-\rho} = \dfrac{(1/3)^2}{1-1/3} = \dfrac{1/9}{2/3} = 1/6$ or 10 Customer in hour

$W = \dfrac{1}{\mu(1-\rho)} = \dfrac{1}{30(1-1/3)} = \dfrac{1}{30\times\frac{2}{3}} = \dfrac{1}{20}$ or 3 minutes

$W_Q = \dfrac{\rho}{\mu(1-\rho)} = \dfrac{1/3}{30(1-\frac{1}{3})} = \dfrac{1/3}{30\times\frac{2}{3}} = \dfrac{1}{60}$ or 1 minute

(c) Performance metrics for Security check

$\rho_S = 0.8;\ \lambda_S = 30;\ \mu_S = 37.5$. $\left[\rho = \dfrac{\lambda_1+\lambda_2}{\mu_S} = \dfrac{30}{37.5} = 0.8\right]$

$L = \dfrac{\rho}{1-\rho} = \dfrac{0.8}{1-0.8} = \dfrac{0.8}{0.2} \Rightarrow 4$ Customers per hour.

$L_Q = \dfrac{\rho^2}{1-\rho} = \dfrac{(0.8)^2}{1-0.8} = \dfrac{0.8\times \overset{4}{\cancel{0.8}}}{\cancel{(0.2)}} \Rightarrow 3.2$ Customers per hour

$W = \dfrac{1}{\mu_S(1-\rho)} = \dfrac{1}{37.5(1-0.8)} = \dfrac{1}{37.5\times0.2} = 0.133$ hours

$= 8$ minutes.

$W_Q = \dfrac{\rho}{\mu_S(1-\rho)} = \dfrac{0.8}{37.5(1-0.8)} = \dfrac{\cancel{0.8}\,4}{37.5\times\cancel{0.2}} = 6.4$ minutes.

e
(d) Total avg time spent by Terminal 1 passengers

= Time spent at Terminal 1 + time spent at Security check

= 3+8

= 11 minutes

f
(e) The total average time spent by terminal 2 passengers = time spent at Security check = 8 minutes

(e) The total average number of Terminal 1 passengers in System (8)

= No of passengers at Terminal 1 + passengers at Security check

$$= L_{T1} + \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right) L_{security}$$

$$= 1/2 + \frac{10/60}{(20/60 + 10/60)} \cdot 4$$

$$= \frac{1}{2} + \frac{4}{3}$$

$$= \frac{3+8}{6}$$

$= 11/6$ Customer ~~per hour~~.

(5) $\lambda = 30$ per hour

Mean = 90 sec

Variance $= \sigma^2 = (90s)^2$

$$\mu = \frac{1}{90} \times 3600$$

$\mu = 40$ per hour.

$$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = 3/4$$

The above is a M/G/1 System

$$L_Q = \frac{\rho^2(1+\sigma^2\mu^2}{2(1-\rho)} = \frac{\left(\frac{3}{4}\right)^2\left[1+(90)^2 \times (1/90)^2\right]}{2(1-3/4]}$$

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{2}{2 \times \frac{1}{4}}$$

$= 9/4$ planes per Sec

Scanned by CamScanner

$$W_Q = \frac{\lambda\left(\frac{1}{\mu^2} + \sigma^2\right)}{2(1-\rho)} = \frac{\frac{40}{3600}\left(\frac{1}{(1/90)^2} + (90)^2\right)}{2(1-3/4)}$$

$$= \frac{1/120}{2/4}(90^2 + 90^2)$$

$$= 1/60 \cdot 2 \cdot 90^2$$

$$= 270 \text{ Sec}$$

$$L = \rho + L_Q = \frac{3}{4} + \frac{9}{4} = \frac{12}{4} = 3 \text{ planes per Sec}$$

$$W = \frac{1}{\mu} + W_Q = \frac{1}{\frac{1}{90}} + 270 = 360 \text{ Sec.}$$

Now Delayed plane cost 5000\$ fuel per hour,

Then per Second $\frac{5000}{3600} = \frac{25}{18}$

Total Cost per aircraft in delay $= W_Q \times \frac{25}{18}$

$$= 270 \times \frac{25}{18}$$

$$= 375 \$$$

6) For a M/M/1 System; the below are performance measures,

$$\rho = \lambda/\mu, \quad L = \frac{\rho}{1-\rho}, \quad L_Q = \frac{\rho^2}{1-\rho}, \quad W = \frac{1}{\mu(1-\rho)}, \quad W_Q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu(1-\rho)}$$

For the Combined $2 \times M/M/1$, System

$$\rho_1 = \rho_2 = \frac{\lambda_1}{\mu_1} = \frac{\lambda_2}{\mu_2}$$

$$L = L_1 + L_2 = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} = \frac{2\rho}{1-\rho}$$

$$L_Q = L_{Q1} + L_{Q2} = \frac{\rho_1^2}{1-\rho_1} + \frac{\rho_2^2}{1-\rho_2} = \frac{2\rho^2}{1-\rho}$$

$W_{combined} = W_1 = W_2$

$$= \frac{1}{\mu_1(1-\rho_1)} = \frac{1}{\mu_2(1-\rho_2)} = \frac{1}{\mu(1-\rho)}$$

W_Q Combined $= W_{Q1} = W_{Q2}$

$$= \frac{\rho_1}{\mu_1(1-\rho_1)} = \frac{\rho_2}{\mu_2(1-\rho_2)} = \frac{\rho}{\mu(1-\rho)}$$

For a M|M|2 Queue,

P_0 = Probablity that there are no Customers in the Queue

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + (c\rho)^c \frac{1}{c!}\left(\frac{1}{1-\rho}\right)} = \frac{1}{1+2\rho+(2\rho)^2 \cdot \frac{1}{2!} \cdot \frac{1}{(1-\rho)}}$$

$$= \frac{1}{1+2\rho+\frac{4\rho^2}{2}(1-\rho)}$$

$$= \frac{1-\rho}{1-\rho+2\rho(1-\rho)+2\rho^2}$$

$$= \frac{1-\rho}{1-\rho+2\rho-2\rho^2+2\rho^2}$$

$$P_0 = \frac{1-\rho}{1+\rho}$$

$$L_{(M|M|2)} = \frac{2\lambda}{\mu} + P_0 \frac{\rho(c\rho)^c}{(1-\rho)^2 c!}$$

$$= 2\rho + \left(\frac{1-\rho}{1+\rho}\right)\frac{\rho(2\rho)^2}{(1-\rho)^2 2!}$$

$$= 2\rho + \frac{(\cancel{1-\rho})}{1+\rho}\frac{\rho \cdot \cancel{4}^{2}\rho^2}{(1-\rho)^{\cancel{2}}\cancel{2}!} = 2\rho + \frac{2\rho^3}{1-\rho^2} = \frac{2\rho-2\rho^3+2\rho^3}{1-\rho^2}$$

$L(M|M|2) = \frac{2\rho}{1-\rho^2}$; $L_Q(M|M|2) = \frac{2\rho^3}{1-\rho^2}$

$W = \frac{1}{\mu} + P_0 \frac{\rho(c\rho)^c}{\lambda(1-\rho^2)(c!}$

$W = \frac{L_{M|M|2}}{\lambda_{M|M|2}}$

$= \frac{\frac{2\rho}{1-\rho^2}}{2\lambda} = \frac{2\rho}{(1-\rho^2)2\lambda}$

$\frac{\lambda}{\mu} = \rho \qquad \lambda = \mu\rho$

$= \frac{\rho}{\mu\rho(1-\rho^2)} = \frac{1}{\mu(1-\rho^2)}$

$W_Q = W_{M|M|2} - \frac{1}{\mu}$

$= \frac{1}{\mu(1-\rho^2)} - \frac{1}{\mu}.$

$W_Q = \frac{\rho^2}{\mu(1-\rho^2)}$

Substitute $\lambda = 5$, $\mu = 10$.

2 x M|M|1
Performance metrics

$P_{0\,2M|M|1} = \overline{(1-\rho)}(1-\rho)$

$= (1-0.5)^2$

$= 0.25$

M|M|2
Performance metric

$P_{0\,M|M|2} = \left(\frac{1-\rho}{1+\rho}\right) \quad \frac{1-0.5}{1-0.5}$

$= 0.33.$

The probablity of Zero customers greater for M|M|2 System.

18.

2 × M/M/1

Utilization rate (ρ)

$$\rho = \frac{\lambda}{\mu}$$

The utilization rates are same

M/M/2

Utilisation rate ρ

$$\rho = \frac{2\lambda}{c\mu} \quad \text{where } c = 2$$

$$\rho = \lambda / \mu$$

The utilization rates are same for both systems.

L for 2 M/M/1

$$L_{2 \times M/M/1} = \frac{2\rho}{1-\rho} = \frac{2 \times 0.5}{1-0.5} = 2.$$

L for M/M/2

$$L_{M/M/2} = \frac{2\rho}{1-\rho^2} = \frac{2 \times 0.5}{1-(0.5)^2} = 1.33.$$

The number of Customers in System with 2 M/M/1 is greater than number of Customers in M/M/2

Average waiting time / delay w

$$W_{2 \times M/M/1} = \frac{1}{\mu(1-\rho)} = \frac{1}{10(1-0.5)} = \frac{1}{5}$$

Avg waiting time / delay (w)

$$W(M/M/2) = \frac{1}{\mu(1-\rho^2)} = \frac{1}{10(1-0.5)^2} = \frac{1}{10(1-0.25)} = \frac{1}{10 * 0.75} = \frac{1}{7.5}.$$

The Average waiting time in the System with 2 M/M/1 is slightly higher than the System with M/M/2

(7) Current System M|M|4|7

Arrival rate(λ) = 34 Cust | hour = 34/60

$$\mu = \frac{1}{0.2\times3+0.7\times7+0.1\times12}$$

$$= \frac{1}{6.7}$$

$$\rho = \frac{34/60}{1/6.7} = 3.8$$

Given: A Customer is turned away once there are 7 customers in the System, we need to calculate probablity of having 7 Customers in System for M|M|4|7 model.

$$P_0 = \frac{1}{\sum_{k=0}^{c} \frac{\lambda^k}{\mu^k k!} + \frac{\lambda^c}{\mu^c c!} \sum_{k=c+1}^{k} \frac{\lambda^{k-c}}{\mu^{k-c} \cdot c^{k-c}}}$$

$c = 4$
$k = 7$

$$= \frac{1}{\sum_{k=0}^{c} \frac{\rho^k}{k!} + \frac{\rho^c}{c!} \sum_{k=c+1}^{k} \frac{\rho^{(k-c)}}{c^{k-c}}}$$

$$= \frac{1}{1+\frac{3.8}{1!}+\frac{(3.8)^2}{2!}+\frac{(3.8)^3}{3!}+\frac{(3.8)^4}{4!}+\frac{\rho^4}{4!}\left[\frac{\rho^1}{c^1}+\frac{\rho^2}{c^2}+\frac{\rho^3}{c^3}\right]}$$

$$= \frac{1}{1+3.8+\frac{(3.8)^2}{2!}+\frac{(3.8)^3}{3!}+\frac{(3.8)^4}{4!}+\frac{(3.8)^4}{4!}\left[\frac{3.8}{4}+\frac{(3.8)^2}{4^2}+\frac{(3.8)^3}{4^3}\right]}$$

$P_0 = 0.019$

$$P_7 = \frac{\left(\frac{\lambda}{\mu}\right)^k \times 0.019}{c^{k-c} \cdot c!} \rightarrow P_0$$

$$= \frac{\rho^k * 0.019}{4! * 4^3} = \frac{(3.8)^7 \times 0.019}{24 \times 64} = 0.1415$$

Probablity of Customers Lost = 14.1 %

Now Trying for M|M|5|7, adding 1 Queue as Server.

$$P_0 = \left[\sum_{k=0}^{c} \frac{\rho^k}{k!} + \frac{\rho^c}{c!} \sum_{k=c+1}^{k} \frac{\rho^{k-c}}{c^{k-c}}\right]^{-1}$$

$$= 1 + \frac{3.8}{1!} + \frac{3.8^2}{2!} + \frac{3.8^3}{3!} + \frac{3.8^4}{4!} + \frac{3.8^5}{5!} + \frac{3.8^5}{5!}\left[\frac{3.8}{5} + \frac{3.8^2}{5^2}\right]$$

$$= 0.022$$

$$P_7 = \frac{\rho^k \cdot P^0}{c^{k-c} \cdot c!} = \frac{(3.8)^7 * 0.022}{5! \times 5^2}$$

$$P_7 = 0.083$$

Rate of Customers Lost : $0.083 \times 100 = 8.3\,\%$

The new System M|M|5|7 is better Since Customers Lost is Comparatively Less to M|M|4|7 System

8(a) $\mu = \mu_1 + \mu_2 + \mu_3$

$= 15 + 15 + 15$

$= 45$

Exponential distribution

Mean $= \mu$ and Var $= \mu^2$

$E(x) = 45$.

$Var(x) = (\mu_1)^2 + (\mu_2)^2 + \mu_3)^3$

$= (15)^2 + 15^2 + 15^2$

$= 675$

(b) No, Combination of three Exponential distribution is not an Exponential distribution.

(c) This is a M/G/1 System.

$\mu = 1/45$, $\lambda = 1/60$

$$\rho = \frac{\lambda}{\mu} = \frac{1/60}{1/45} = 3/4.$$

$$L_Q = \frac{\rho^2 (1 + \sigma^2 \mu^2)}{2(1-\rho)}$$

$$= \frac{\left(\frac{3}{4}\right)^2 \left(1 + (675) * \left(\frac{1}{45}\right)^2\right)}{2(1 - 0.75)} = \frac{\left(\frac{3}{4}\right)^2 \times \left(1 + \frac{1}{3}\right)}{0.50} = \frac{\frac{\cancel{9}^{3}}{\cancel{16}_{4}} * \left(\frac{\cancel{4}}{\cancel{3}}\right)}{0.50}$$

$$= \frac{(0.75)^2 [}{0.50} \qquad = \frac{3}{2}$$

(d) $$W = \frac{1}{\mu} + \frac{\lambda\left(\frac{1}{\mu^2} + \sigma^2\right)}{2(1-\rho)} = 45 + \frac{1}{60} \frac{(45^2 + 675)}{0.5}$$

$= 45 + \frac{1}{30}(2700)$

$= 45 + 90$

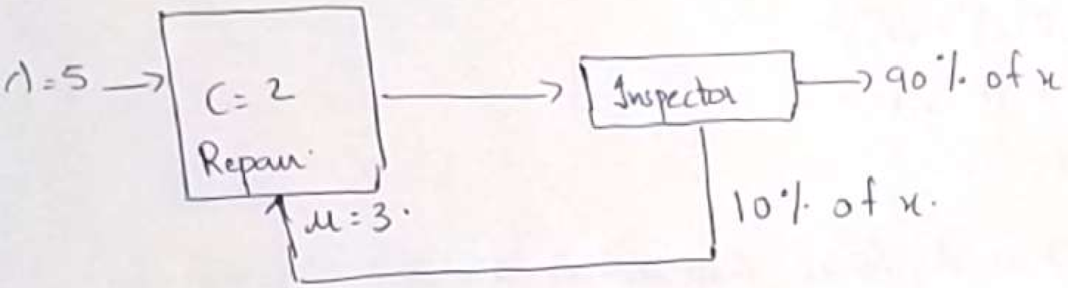
$= 135$ min

$W = 135$ min or $\frac{9}{4}$ hours. — Ans

$W_Q = 90 = 90/60 = 3/2$ hours

$W_S = 45 = 45/60 = 2/3$ hours

(9)



By Conservation, $\lambda + 0.1x = x$

$5 = 0.9x$

$x = 5.556$/hr.

$\left[W = \frac{1}{\mu(1-\rho^2)} \text{ for } M|M|2, \text{ derived in Qn } 6 \right]$

(a) Long term delay W

Repair station

$\lambda = 5 + 0.1 \times 5.55$

$= 5.56$

$\mu = 3$ but $c = 2$.

$\rho = \frac{5.56}{2 \times 3} = \frac{5.56}{6}$

$W = \frac{1}{\mu(1-\rho)} = \frac{1}{3\left(1-\left(\frac{5.56}{6}\right)^2\right)}$

M|M|2

$= 2.37$ hour.

Inspection facility.

$\lambda = 5 + 0.1 \times 5.55$

$= 5.56$

$\mu = 8$

$\rho = \frac{5.56}{8}$

M|M|1.

$W = \frac{1}{8\left(1-\left(\frac{5.56}{8}\right)\right)}$

$= \frac{1}{8 - 5.56} = \frac{1}{2.44}$

$= 0.41$ hour

(b) $\rho_{repair} = \frac{5.56}{2 \times 3} = \frac{5.56}{6} = 0.92$

$\rho_{inspector} = \frac{5.56}{8} = 0.695.$

Utilization rate is greater at repair station.

For Conservation Law,

$\lambda + 0.1x = x$

$\lambda - 0.9x = 0 \quad -(1)$

$\rho_{repair} = \frac{\lambda + 0.1x}{6}.$

Max value of $\rho_{repair} = 1$

$1 = \frac{\lambda + 0.1x}{6} \Rightarrow 6 = \lambda + 0.1x$

$54 = 9\lambda + 0.9x \quad -(2)$

Using (1) and (2)

$$9\lambda + 0.9x = 54$$
$$\lambda - 0.9x = 0$$

$$10\lambda = 54$$

$$\lambda = 5.4$$

max value of $\lambda = 5.4$

(10) Given $\sum_{k=0}^{\infty} k \cdot \rho^k = \rho \frac{d}{d\rho} \sum_{k=0}^{\infty} \rho^k$ —(1)

$$L = \sum_{k=0}^{\infty} k \cdot p^k \qquad p^k = (1-\rho)\rho^k$$

$$L = \sum_{k=0}^{\infty} k \cdot (1-\rho)\rho^k$$

$$L = \sum_{k=0}^{\infty} k \cdot \rho^k - \sum_{k=0}^{\infty} k \cdot \rho^{k+1}$$

$$= \sum_{k=0}^{\infty} k \cdot \rho^k - \rho \sum_{k=0}^{\infty} k \cdot \rho^k$$

$$= \sum_{k=0}^{\infty} k \cdot \rho^k (1-\rho) \quad -(2)$$

Using (1) in (2)

$$= \left(\rho \left[\frac{d}{d\rho} \sum_{k=0}^{\infty} \rho^k\right]\right)(1-\rho)$$

$$= \rho\left(\frac{d}{d\rho}\left(\frac{1}{1-\rho}\right)\right) 1-\rho$$

$\sum_{k=0}^{\infty} \rho^k = \frac{1}{1-\rho}$ Geometric Series Sum.

$$\rho \left[\frac{1}{(1-\rho)^2}\right](1-\rho)$$

$$P(L=k) = \frac{\rho}{1-\rho}$$

Qn4) Simulation table for Security Queue of Airport.

Seed	RHO	L Customer	L(Q) Customer	L(S) Customer	W (Mins)	W(Q) Mins	W(S) mins
12345	0.789	3.748981	2.9595533	0.7894285	7.54343	5.955003	1.588432
98765	0.80911671	4.2388033	3.429686594	0.80911671	8.394288953	6.79195947	1.60232947
54567	0.801105	4.02779848	3.22669269	0.801105791	8.05580062	6.453548533	1.602252088
67234	0.7934097	3.840499323	3.04708958	0.793409743	7.688450765	6.100091744	1.588359021
78123	0.799687	3.992189289	3.192502207	0.799687083	7.963511411	6.368317209	1.595194202
Average	**0.798463682**	**3.969654278**	**3.171104874**	**0.798549565**	**7.92909635**	**6.333783991**	**1.595313356**
Theoretical	**0.8**	**4**	**3.2**	**0.8**	**8**	**6.4**	**1.6**

The results of Simulation are Consistent with the Theoretical calculations in Qn 3.