Measuring Seismic Movement Using Accelerometers

ME 226 Course Project
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Overview and Working Principle

The seismic accelerometer is a form of transducer measures the motion of the ground. The main problem faced with measurement of seismic movement is its randomness, due to which there is usually no fixed reference available. Seismic accelerometers measure ground motion by using the change in the length of a spring.

Two of the most common types of accelerometers are cantilever system and piezoelectric system. The cantilever system is fairly simple and uses a strain gauge to measure relative motion. It can measure down to very low frequencies, but it is limited in high frequency ranges. The piezoelectric system converts pressure caused by seismic oscillations into electronic signals. The signals are then read and processed to output the final reading of seismic movement. The piezoelectric system has a lower limit due to the resolution of electronic devices, but it works well for high frequency motion.

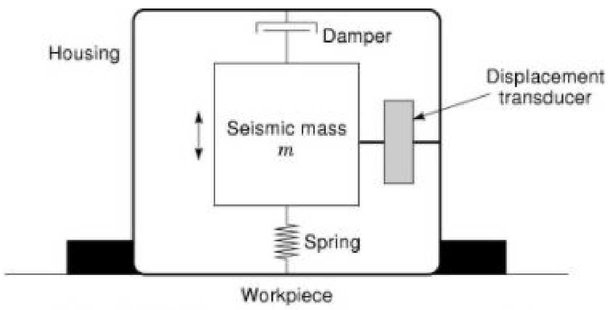
Both work on the same dynamic principles and use the time-varying relative motion of a seismic mass with respect to the moving ground, y_s, to determine the acceleration of the ground, $\ddot{s}$. The equation governing this system is very similar to that of a spring-mass-dashpot system used in force gauges. Hence this measuring instrument is of 2^{nd} order, with characteristic equation as follows:

$$m\ddot{y}_s + c\dot{y}_s + ky_s = m\ddot{s}$$

y_s is only the raw output of the transducer. We convert the displacement output to an acceleration quantity as follows:

$$Y_A = \omega_n^2 Y_s$$

A very rough and basic schematic of the seismic accelerometer (or any type of spring-based transducer in general) is shown in the figure below:



Input Response and Error Analysis

We will first consider a step input, which is given by:

$$a = \begin{cases} 0 \text{ for } t < 0 \\ A \text{ for } t > 0 \end{cases}$$

The output response of the accelerometer depends on the viscous damping ratio ζ as follows:

$$\zeta > 1 \implies \frac{y_A}{A} = 1 - e^{-\zeta\omega_n t}\left\{\cosh\left(\omega_n t\sqrt{\zeta^2 - 1}\right) + \frac{\zeta \sinh\left(\omega_n t\sqrt{\zeta^2 - 1}\right)}{\omega_n t\sqrt{\zeta^2 - 1}}\right\}$$

$$\zeta < 1 \implies \frac{y_A}{A} = 1 - e^{-\zeta\omega_n t}\left\{\frac{\sin\left(\omega_n t\sqrt{1 - \zeta^2} - \arcsin\left(\sqrt{1 - \zeta^2}\right)\right)}{\sqrt{1 - \zeta^2}}\right\}$$

Step inputs are particularly difficult to measure because after the ground has moved through the step input, the seismic mass vibrates until the transient effect eventually decays. To get a steady output, the mass must be in static equilibrium, meaning the spring force should be equal to the weight of the mass. Since this condition is already satisfied before the input is applied, uniqueness theorem tells us that the final output of the transducer will always be $y_s = (s - x) = 0$. We can get decently accurate results at low frequencies, meaning that piezoelectric systems cannot be used to measure step inputs.

Of much more interest to us is a sinusoidal input, which is the most common type of input in seismic applications. For sinusoidal input, we have $a = Asin(\omega t)$. For an easier analysis, we only consider the steady state of the accelerometer $y_s = Y_s \sin(\omega t - \phi)$, which is converted to acceleration output by the equation $Y_A = \omega_n^2 Y_s$. The graphs for amplitude errors vs. $\omega_n t$ for both step input and sinusoidal input are shown below:

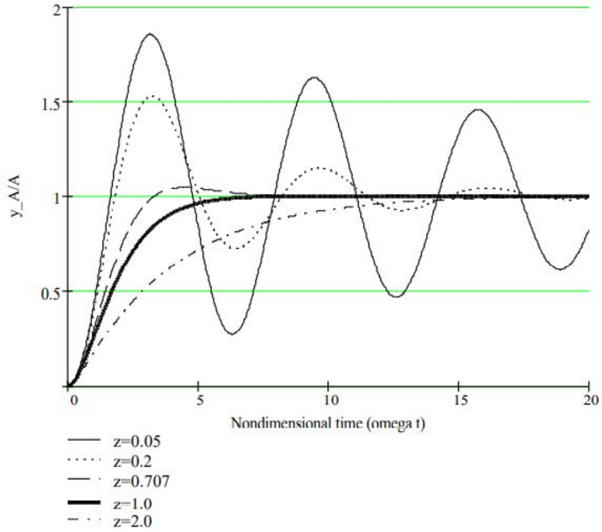


Figure 1: Step input amplitude error

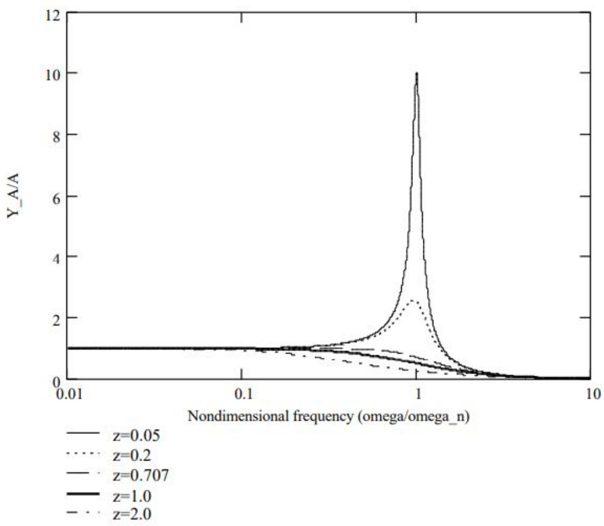


Figure 2: Sinusoidal input amplitude error

When measuring sinusoidal acceleration signals with an accelerometer, for accurate measurements we need to ensure that the natural frequency of the accelerometer is well above the frequency of the acceleration signal we are measuring.

Specifications

Suppose we want to achieve an amplitude error less than 3% at an operating frequency of 10 kHz and damping ratio 0.05. This level of precision would be expected in a high-end system like earthquake detection mechanisms, and can be calculated based on the following formulae (which can be derived from the dynamic response equations discussed above in the report):

$$\left|\frac{Y_A}{A}\right| = 1.03 = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} = \frac{1}{\sqrt{\left(1-\left(\frac{20000\pi}{2\pi f_n}\right)^2\right)^2 + 4(0.05)^2\left(\frac{20000\pi}{2\pi f_n}\right)^2}}$$

This results in $f_n = 58.59$ kHz or 7.2 kHz. Since the natural frequency must be greater than the excitation frequency, we select $f_n = 58.59$ kHz. Based on this value of f_n, the phase error is:

$$\phi = \arctan\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right) = \arctan\left(\frac{2(0.05)\frac{10000}{58590}}{1-\left(\frac{10000}{58590}\right)^2}\right) = 1.007^{\text{o}}$$

As for general medium-grade seismic accelerometers available in the market, high quality piezoelectric material (around 10 V/g or 9000 pC/g) is required. They are also limited on their high frequency capabilities (usually 50 to 100 kHz).

References

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