Air Traffic Flow Management

Team Code - 13

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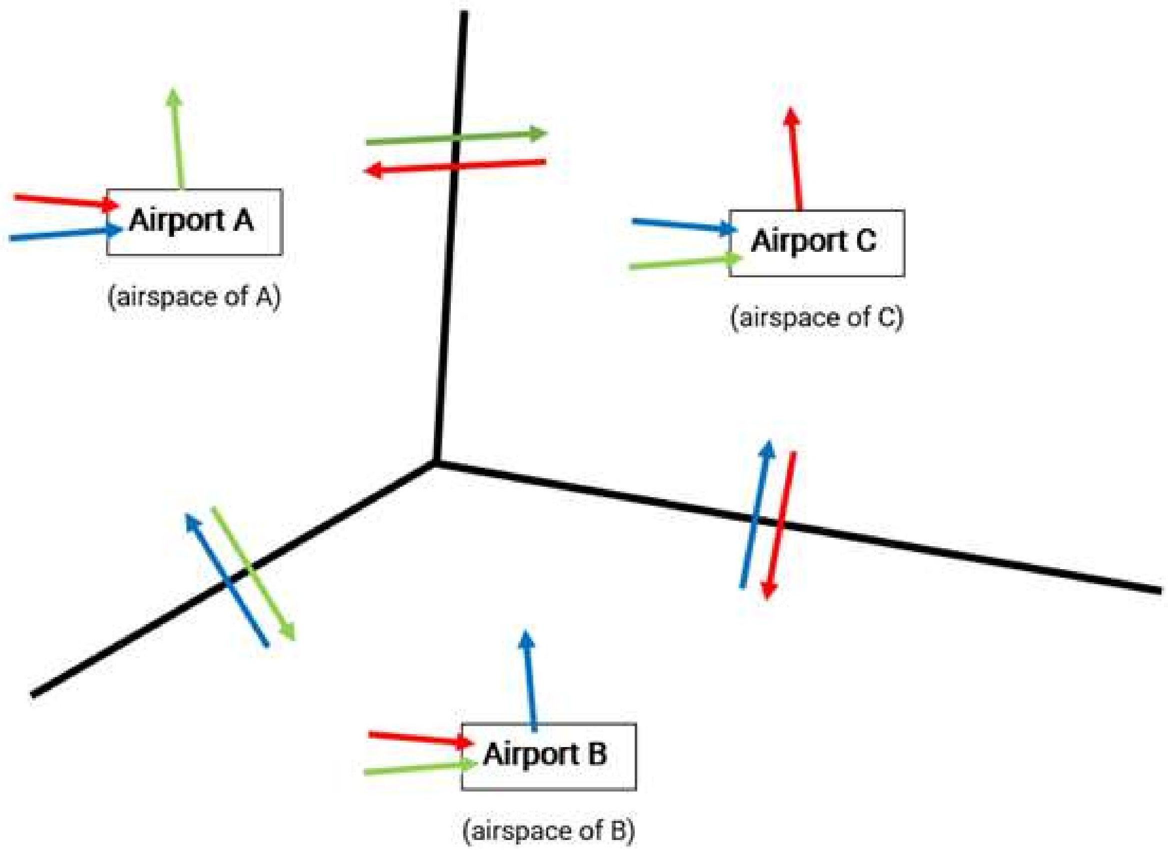
Motivation

- 22% of flights delayed or cancelled between 2004 and 2017
- Very challenging due to highly random factors involved (eg: adverse weather)
- Air transport is a major sector of the global economy
- Billions of dollars lost in rerouting aircraft & compensating passengers for cancelled flights

Problem Description

- Consider 3 centers with well defined airspace boundaries comprising multiple airports each and a day divided into ‘k’ equal time intervals
- Optimise the number of aircraft departing from & arriving at each center as well as travelling between centers in each time interval such that the cost of airborne delay and ground hold delay is minimized
- Cost is directly proportional to the number of scheduled flights that have to be delayed/cancelled and in-transit flights that undergo airborne delays

Problem Description



Assumptions

- All aircraft are identical (same cost/time delay)
- There are no connecting flights
- We consider only the flights which travel between the 3 designated centers (eg: center A may have flights travelling to a fourth center, say D - these flights are not considered)
- Arrivals at and departures from the airports of a center happen at the end boundary of the time interval
- Transition across airspace boundaries of centers happen only when there is a transition across time boundary as well
- Stochasticity in the form of adverse weather, airport crowd congestion, etc are not accounted for

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Parameters

- $x_i(k)$: Total number of aircraft in airspace of center 'i' at the start of time interval 'k'
- $\text{Capacity}_i(k)$: Total capacity of all airports in center 'i' during time interval 'k'
- $\text{SchDept}_i(k)$: Scheduled departures from airports in center 'i' during time interval 'k'
- $\text{DeptLim}_i(k)$: Limit on departures from airports in center 'i' during time interval 'k'
- $\text{ArrivLim}_i(k)$: Limit on arrivals at airports in center 'i' during time interval 'k'
- $\text{TrafLim}_{ij}(k)$: limit on traffic flow between airspace boundaries of centers 'i' & 'j' during time interval 'k'
- C_i: cost factor for delay in center i (different for ground holding, C_g and airborne holding C_a)

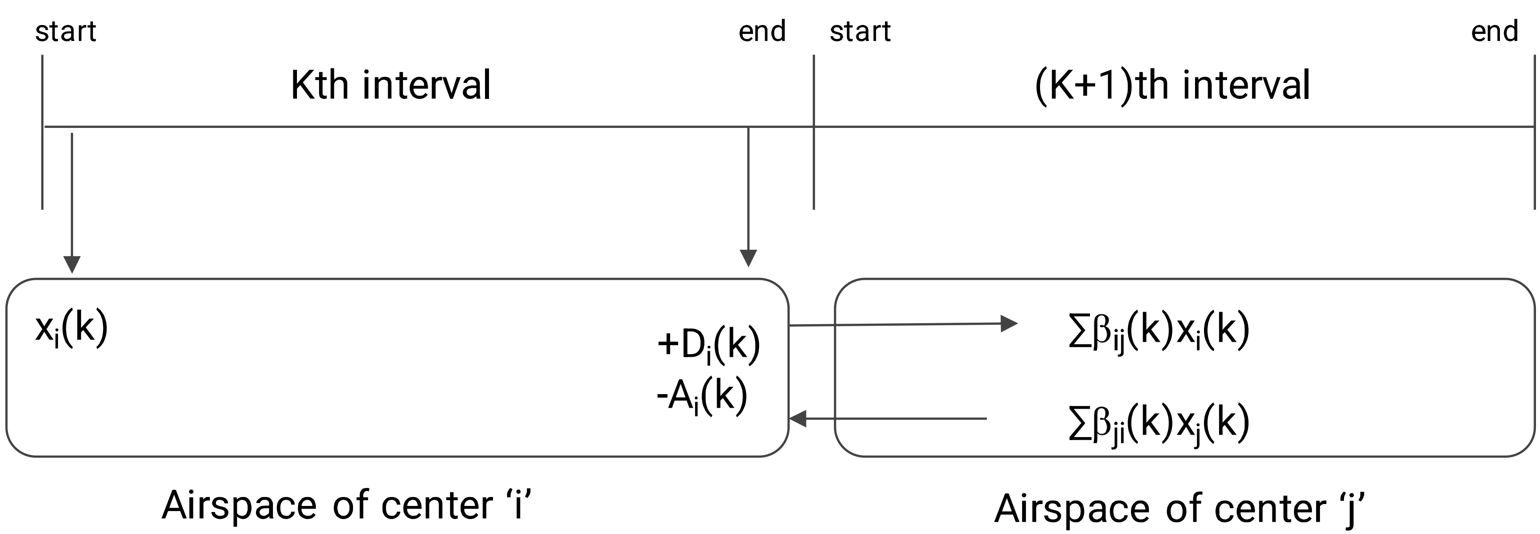
Decision Variables

- $\beta_{ij}(k)$: fraction of airborne aircraft in center ‘i’ that move to center ‘j’ during time interval ‘k’
- $D_i(k)$: Total number of departures from the airports in center ‘i’ during time interval ‘k’
- $A_i(k)$: Total number of arrivals at the airports in center ‘i’ during time interval ‘k’

Constraints

- Airborne Aircraft Balance Constraint:

$$x_i(k+1) = x_i(k) - \left(\sum_{\substack{j=1;j\neq i \\ k=1}}^{\substack{j=3 \\ k=7}} \beta_{ij}(k) x_i(k) + A_i(k) \right) + \left(\sum_{\substack{j=1;j\neq i \\ k=1}}^{\substack{j=3 \\ k=7}} \beta_{ji}(k) x_j(k) + D_i(k) \right)$$



Constraints

- Airport Departures Contribution To Airspace Constraint:

$$\sum_{j\in[1,3]} \beta_{ij}(k+1)x_i(k+1) = D_i(k)$$

- Airport Arrivals Contribution To Airspace Constraint:

$$\sum_{j\in[1,3]} \beta_{ji}(k-1)x_j(k-1) \geqslant A_i(k)$$

- Beta Constraint:

$$\sum_{j} \beta_{ij}(k) \leqslant 1$$

Constraints

- Capacity Constraint:

$$A_i(k) + D_i(k) \leqslant \text{Capacity}_i(k)$$

- Departures Limit Constraint:

$$D_i(k) \leqslant \text{DeptLim}_i(k)$$

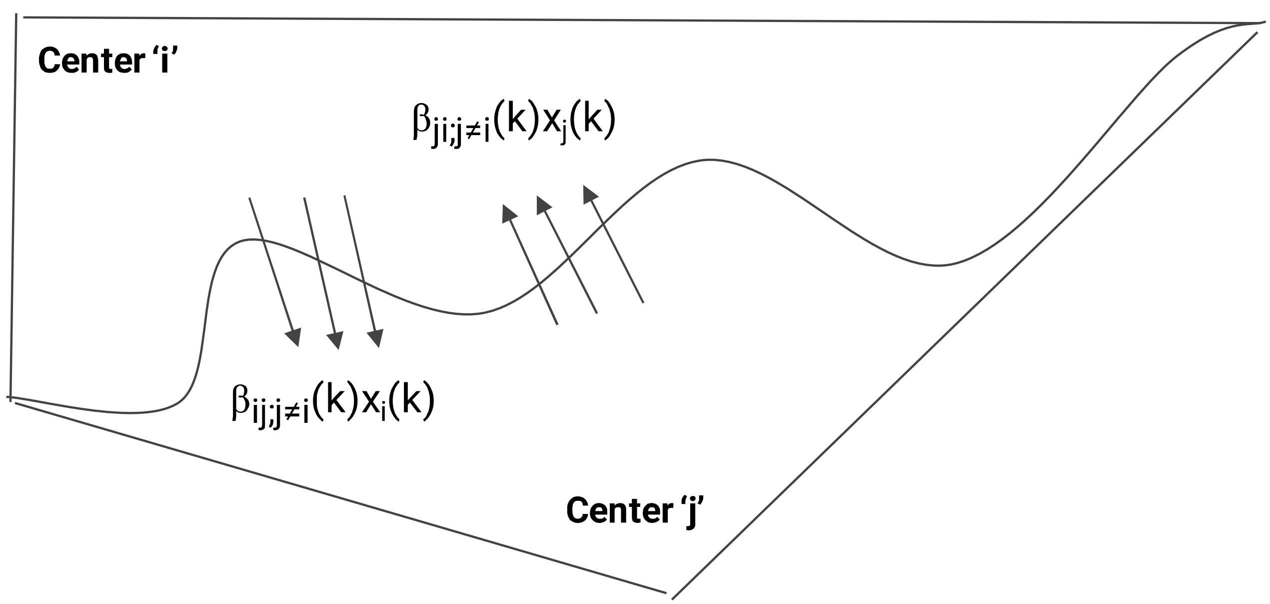
- Arrivals Limit Constraint:

$$A_i(k) \leqslant \text{ArrivLim}_i(k)$$

Constraints

- Boundary Traffic Limit Constraint:

$$\beta_{ij;j\neq i}(k)x_i(k) + \beta_{ji;j\neq i}(k)x_j(k) \leqslant \mathrm{Traflim}_{i,j}(k)$$



Objective

- Minimize the cost function:

$$\mathbf{z}: \quad \sum_{\substack{i=1\\k=1}}^{\substack{i=3\\k=8}} C_{g,i}\left[\text{Schdept}_i(k) - D_i(k)\right] + \sum_{\substack{i=1\\k=1}}^{\substack{i=3\\k=8}} C_{a,i}\left[\sum_{j=1}^{j=3} \beta_{ji}(k-1)x_j(k-1) - A_i(k)\right]$$

- First summation corresponds to ground holding costs (i.e. cost of holding flights that were scheduled for departure but did not actually depart)
- Second summation corresponds to airborne delay costs (i.e. delay costs of flights that were scheduled to arrive in a specific time interval but did not actually arrive)
- $C_{a,i} > C_{g,i}$: cost of holding aircraft in flight is costlier than holding on ground

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Optimizing Tools

- Data for scheduled departures has been taken from real-life average data for Mumbai, Delhi and Hyderabad airports respectively
- The optimization software used for modelling & solving this problem is AMPL and the solver used is Couenne
- Model statistics:

Parameter	**Value**
Optimal Objective Value	118.41
Total Constraints	104
Total Variables	129 (48 integer)
Execution Time	0.169 sec

Obtained Solution

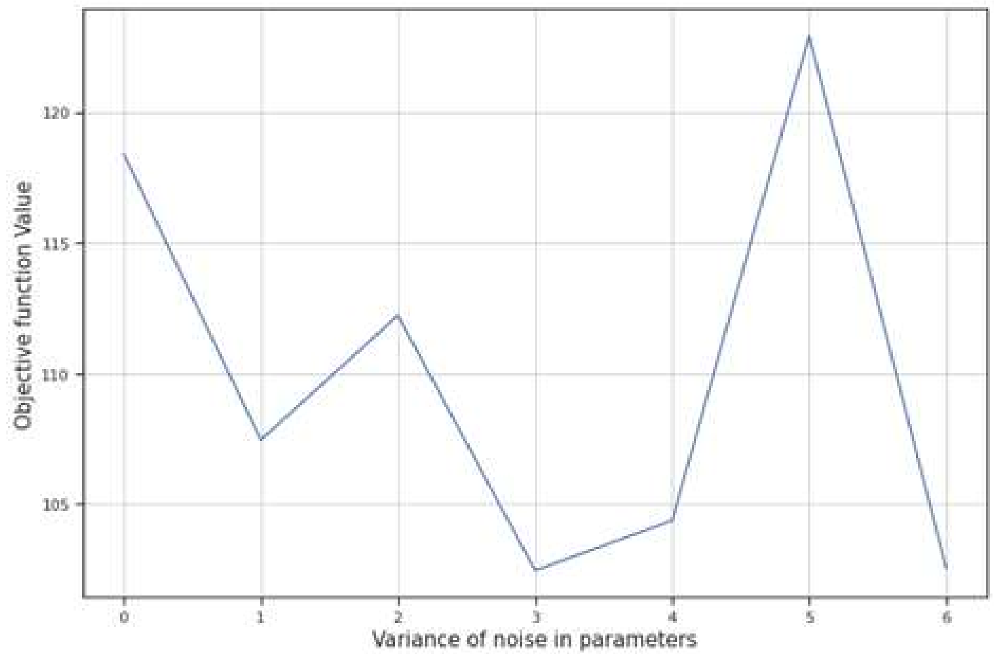
```
:        A      D      :=
1 0     20     15
1 1     60     15
1 2    100     25
1 3     50     59
1 4     52     67
1 5     49     14
1 6     78      8
1 7     43     31
1 8     13     37
2 0     20     15
2 1     52    148
2 2     43     86
2 3     81    116
2 4     36    106
2 5    110     58
2 6     84     68
2 7     14     35
2 8     35     58
3 0     20     15
3 1     19     18
3 2     50     21
3 3     37     30
3 4     31     37
3 5     46     10
3 6     48     27
3 7     25     11
3 8     17     23
;
```

```
Beta [1,*,*] (tr)
:       1          2          3         :=
0    0.964655   0.026724   0.015848
1    0.228241   0.314352   0.125484
2    0.0515152  0.095452   0.051416
3    0.051534   0.247222   0.623481
4    0.352315   0.041637   0.143981
5    0.139823   0.403097   0.015324
6    0.089846   0.078695   0.425760
7    0.0666667  0.812546   0.087462
8    0.656098   0.047213   0.141635

 [2,*,*] (tr)
:       1          2          3         :=
0    0.025503   0.210084   0.080147
1    0.569492   0.032740   0.011452
2    0.305056   0.407022   0.119382
3    0.298377   0.102273   0.157792
4    0.0553957  0.729137   0.157463
5    0.284975   0.096031   0.187192
6    0.274806   0.334104   0.124806
7    0.0666667  0.734783   0.184058
8    0.354545   0.532111   0.009778

 [3,*,*] (tr)
:       1          2          3         :=
0    0.084161   0.048631   0.715456
1    0.129365   0.001364   0.614286
2    0.055318   0.074182   0.634471
3    0.741255   0.079681   0.063265
4    0.021212   0.100412   0.361538
5    0.415520   0.325287   0.095024
6    0.742314   0.014935   0.061290
7    0.136842   0.074763   0.524694
8    0.243752   0.642477   0.032748
;
```

Contents

Uncertainty Analysis

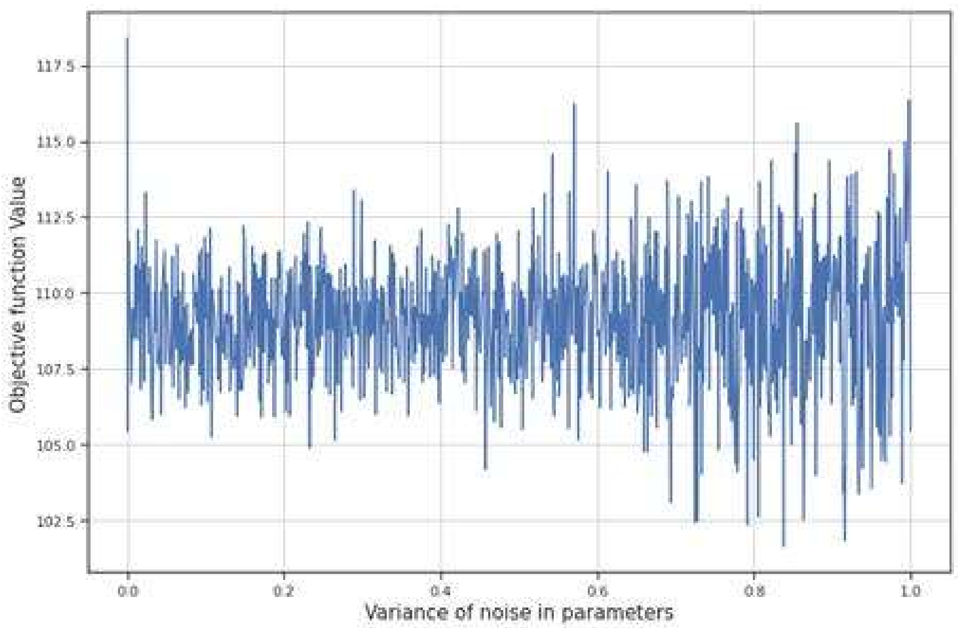
- On adding Gaussian noise to the parameter $x_i(k)$ and $SchDept_i(k)$, we observe the variation in the value of the objective function when the variables $\beta_{ij}(k)$, $A_i(k)$ and $D_i(k)$ take the optimal values for original value of parameters
- % maximum change in cost function value = *100* (118.41-102.45)/118.41* = ***13.48%***



Maximum variance = 6

Uncertainty Analysis

- On adding a small gaussian noise (maximum variance = 1) mean value of cost function is **109.13**
- % Difference w.r.t the optimal value = *100 * (118.41 - 109.13)/118.41* = ***7.84%***
- For small perturbations in environment parameters, the solution is reasonably robust




```
def costfn(beta,A,D,Cg,Ca,SchDept,x):
  a=0
  for i in range(3):
    for k in range(8):
      a+= Cg[i]*(SchDept[i][k]-D[i][k+1])
  b=0
  for i in range(3):
    for k in range(8):
      c=0
      for j in range(3):
        c+=beta[j][i][k]*x[j][k]
      b+= Ca[i]*(c-A[i][k+1])

  return a+b
```

```
print(costfn(B,a,d,Cg_mean,Ca_mean,SchDept_mean,x_mean))
```

```
118.40987369100003
```

```
costs=[]
l = np.linspace(0,1,1000)
for i in l:
  x=np.random.normal(x_mean,i).astype(int)
  SchDept = np.random.normal(SchDept_mean,i).astype(int)
  costs.append(costfn(B,a,d,Cg_mean, Ca_mean, SchDept,x))
print(costs)
```

```
l = np.linspace(0,1,1000)
plt.figure(figsize=(12,8))
sb.set(style="ticks")
plt.grid()
plt.xlabel("Variance of noise in parameters",size=15)
plt.ylabel("Objective function Value",size=15)
sb.lineplot(x=l,y=costs)
plt.show()
```

Contents

Future Work

- Develop a suitable disaggregation algorithm to extract finer instructions for every airport from the obtained results
- Extend the problem to even more airports/centers (can theoretically be extended to all airports in the world)
- Extend the model to account for different types of aircraft (size and capacity)
- Extend the model to account for connecting flights
- Increase complexity of problem by shrinking time intervals to ensure more exact predictions
- Incorporate stochasticity in the form of uncertain weather conditions, airport congestion, wind turbulence, etc
- Employ open source air traffic simulators such as bluesky and openscope for better visualization of obtained results

References & Links

1. An Integer Optimization Approach to Large Scale Air Traffic Flow Management - Bertsimas, Lulli, Odoni (2011)
2. The Traffic Flow Management Rerouting Problem in Air Traffic Control: A Dynamic Network Flow Approach - Bertsimas, Patterson (2000)
3. Disaggregation Method for an Aggregate Traffic Flow Management Model - Sun, Shridhar, Grabbe (2010)
4. Control and Optimization Algorithms for Air Transportation Systems - Balakrishnan (2016)

5. Link to Github repository for our project: https://github.com/omprabhu31/ME308-Project

Thank You