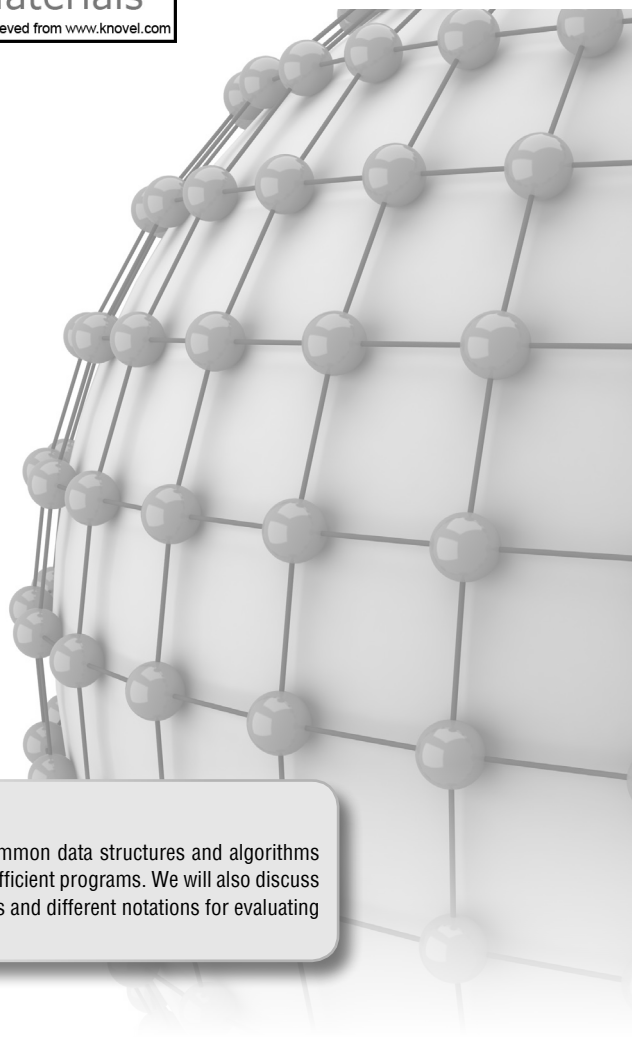
CHAPTER 2

Introduction to Data Structures and Algorithms



LEARNING OBJECTIVE

In this chapter, we are going to discuss common data structures and algorithms which serve as building blocks for creating efficient programs. We will also discuss different approaches to designing algorithms and different notations for evaluating the performance of algorithms.

2.1 BASIC TERMINOLOGY

We have already learnt the basics of programming in C in the previous chapter and know how to write, debug, and run simple programs in C language. Our aim has been to design good programs, where a good program is defined as a program that

- runs correctly
- is easy to read and understand
- is easy to debug *and*
- is easy to modify.

A program should undoubtedly give correct results, but along with that it should also run efficiently. A program is said to be efficient when it executes in minimum time and with minimum memory space. In order to write efficient programs we need to apply certain data management concepts.

The concept of data management is a complex task that includes activities like data collection, organization of data into appropriate structures, and developing and maintaining routines for quality assurance.

Data structure is a crucial part of data management and in this book it will be our prime concern. A *data structure* is basically a group of data elements that are put together under one name, and which defines a particular way of storing and organizing data in a computer so that it can be used efficiently.

Data structures are used in almost every program or software system. Some common examples of data structures are arrays, linked lists, queues, stacks, binary trees, and hash tables. Data structures are widely applied in the following areas:

- Compiler design
- Statistical analysis package
- Numerical analysis
- Artificial intelligence
- Operating system
- DBMS
- Simulation
- Graphics

When you will study DBMS as a subject, you will realize that the major data structures used in the Network data model is graphs, Hierarchical data model is trees, and RDBMS is arrays.

Specific data structures are essential ingredients of many efficient algorithms as they enable the programmers to manage huge amounts of data easily and efficiently. Some formal design methods and programming languages emphasize data structures and the algorithms as the key organizing factor in software design. This is because representing information is fundamental to computer science. The primary goal of a program or software is not to perform calculations or operations but to store and retrieve information as fast as possible.

Be it any problem at hand, the application of an appropriate data structure provides the most efficient solution. A solution is said to be efficient if it solves the problem within the required resource constraints like the total space available to store the data and the time allowed to perform each subtask. And the best solution is the one that requires fewer resources than known alternatives. Moreover, the cost of a solution is the amount of resources it consumes. The cost of a solution is basically measured in terms of one key resource such as time, with the implied assumption that the solution meets the other resource constraints.

Today computer programmers do not write programs just to solve a problem but to write an efficient program. For this, they first analyse the problem to determine the performance goals that must be achieved and then think of the most appropriate data structure for that job. However, program designers with a poor understanding of data structure concepts ignore this analysis step and apply a data structure with which they can work comfortably. The applied data structure may not be appropriate for the problem at hand and therefore may result in poor performance (like slow speed of operations).

Conversely, if a program meets its performance goals with a data structure that is simple to use, then it makes no sense to apply another complex data structure just to exhibit the programmer's skill. When selecting a data structure to solve a problem, the following steps must be performed.

1. Analysis of the problem to determine the basic operations that must be supported. For example, basic operation may include inserting/deleting/searching a data item from the data structure.
2. Quantify the resource constraints for each operation.
3. Select the data structure that best meets these requirements.

This three-step approach to select an appropriate data structure for the problem at hand supports a data-centred view of the design process. In the approach, the first concern is the data and the operations that are to be performed on them. The second concern is the representation of the data, and the final concern is the implementation of that representation.

There are different types of data structures that the C language supports. While one type of data structure may permit adding of new data items only at the beginning, the other may allow it to be added at any position. While one data structure may allow accessing data items sequentially, the other may allow random access of data. So, selection of an appropriate data structure for the problem is a crucial decision and may have a major impact on the performance of the program.

2.1.1 Elementary Data Structure Organization

Data structures are building blocks of a program. A program built using improper data structures may not work as expected. So as a programmer it is mandatory to choose most appropriate data structures for a program.

The term *data* means a value or set of values. It specifies either the value of a variable or a constant (e.g., marks of students, name of an employee, address of a customer, value of *pi*, etc.).

While a data item that does not have subordinate data items is categorized as an elementary item, the one that is composed of one or more subordinate data items is called a group item. For example, a student's name may be divided into three sub-items—first name, middle name, and last name—but his roll number would normally be treated as a single item.

A *record* is a collection of data items. For example, the name, address, course, and marks obtained are individual data items. But all these data items can be grouped together to form a record.

A *file* is a collection of related records. For example, if there are 60 students in a class, then there are 60 records of the students. All these related records are stored in a file. Similarly, we can have a file of all the employees working in an organization, a file of all the customers of a company, a file of all the suppliers, so on and so forth.

Moreover, each record in a file may consist of multiple data items but the value of a certain data item uniquely identifies the record in the file. Such a data item K is called a *primary key*, and the values K_1, K_2 ... in such field are called keys or key values. For example, in a student's record that contains roll number, name, address, course, and marks obtained, the field roll number is a primary key. Rest of the fields (name, address, course, and marks) cannot serve as primary keys, since two or more students may have the same name, or may have the same address (as they might be staying at the same place), or may be enrolled in the same course, or have obtained same marks.

This organization and hierarchy of data is taken further to form more complex types of data structures, which is discussed in Section 2.2.

2.2 CLASSIFICATION OF DATA STRUCTURES

Data structures are generally categorized into two classes: *primitive* and *non-primitive* data structures.

Primitive and Non-primitive Data Structures

Primitive data structures are the fundamental data types which are supported by a programming language. Some basic data types are integer, real, character, and boolean. The terms 'data type', 'basic data type', and 'primitive data type' are often used interchangeably.

Non-primitive data structures are those data structures which are created using primitive data structures. Examples of such data structures include linked lists, stacks, trees, and graphs.
Non-primitive data structures can further be classified into two categories: *linear* and *non-linear* data structures.

Linear and Non-linear Structures

If the elements of a data structure are stored in a linear or sequential order, then it is a linear data structure. Examples include arrays, linked lists, stacks, and queues. Linear data structures can be represented in memory in two different ways. One way is to have to a linear relationship between elements by means of sequential memory locations. The other way is to have a linear relationship between elements by means of links.

However, if the elements of a data structure are not stored in a sequential order, then it is a non-linear data structure. The relationship of adjacency is not maintained between elements of a non-linear data structure. Examples include trees and graphs.

C supports a variety of data structures. We will now introduce all these data structures and they would be discussed in detail in subsequent chapters.

Arrays

An array is a collection of similar data elements. These data elements have the same data type. The elements of the array are stored in consecutive memory locations and are referenced by an *index* (also known as the *subscript*).

In C, arrays are declared using the following syntax:

```
type name[size];
```

For example,

```
int marks[10];
```

The above statement declares an array `marks` that contains 10 elements. In C, the array index starts from zero. This means that the array marks will contain 10 elements in all. The first element will be stored in `marks[0]`, second element in `marks[1]`, so on and so forth. Therefore, the last element, that is the 10th element, will be stored in `marks[9]`. In the memory, the array will be stored as shown in Fig. 2.1.

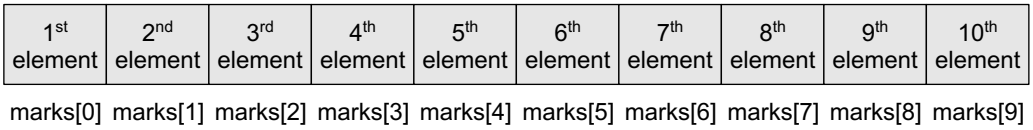


Figure 2.1 Memory representation of an array of 10 elements

Arrays are generally used when we want to store large amount of similar type of data. But they have the following limitations:

- Arrays are of fixed size.
- Data elements are stored in contiguous memory locations which may not be always available.
- Insertion and deletion of elements can be problematic because of shifting of elements from their positions.

However, these limitations can be solved by using linked lists. We will discuss more about arrays in Chapter 3.

Linked Lists

A linked list is a very flexible, dynamic data structure in which elements (called *nodes*) form a sequential list. In contrast to static arrays, a programmer need not worry about how many elements will be stored in the linked list. This feature enables the programmers to write robust programs which require less maintenance.

In a linked list, each node is allocated space as it is added to the list. Every node in the list points to the next node in the list. Therefore, in a linked list, every node contains the following two types of data:

- The value of the node or any other data that corresponds to that node
- A pointer or link to the next node in the list

The last node in the list contains a `NULL` pointer to indicate that it is the end or *tail* of the list. Since the memory for a node is dynamically allocated when it is added to the list, the total number of nodes that may be added to a list is limited only by the amount of memory available. Figure 2.2 shows a linked list of seven nodes.

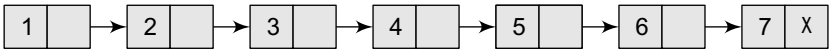


Figure 2.2 Simple linked list

Note *Advantage*: Easier to insert or delete data elements
Disadvantage: Slow search operation and requires more memory space

Stacks

A stack is a linear data structure in which insertion and deletion of elements are done at only one end, which is known as the `top` of the stack. Stack is called a last-in, first-out (LIFO) structure because the last element which is added to the stack is the first element which is deleted from the stack.

In the computer's memory, stacks can be implemented using arrays or linked lists. Figure 2.3 shows the array implementation of a stack. Every stack has a variable `top` associated with it. `top` is used to store the address of the topmost element of the stack. It is this position from where the element will be added or deleted. There is another variable `MAX`, which is used to store the maximum number of elements that the stack can store.

If `top = NULL`, then it indicates that the stack is empty and if `top = MAX-1`, then the stack is full.

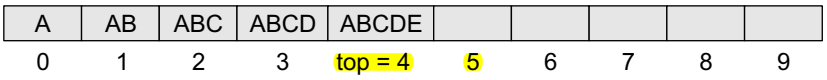


Figure 2.3 Array representation of a stack

In Fig. 2.3, `top = 4`, so insertions and deletions will be done at this position. Here, the stack can store a maximum of 10 elements where the indices range from 0–9. In the above stack, five more elements can still be stored.

A stack supports three basic operations: `push`, `pop`, and `peep`. The `push` operation adds an element to the top of the stack. The `pop` operation removes the element from the top of the stack. And the `peep` operation returns the value of the topmost element of the stack (without deleting it).

However, before inserting an element in the stack, we must check for overflow conditions. An overflow occurs when we try to insert an element into a stack that is already full.

Similarly, before deleting an element from the stack, we must check for underflow conditions. An underflow condition occurs when we try to delete an element from a stack that is already empty.

Queues

A queue is a first-in, first-out (FIFO) data structure in which the element that is inserted first is the first one to be taken out. The elements in a queue are added at one end called the `rear` and removed from the other end called the `front`. Like stacks, queues can be implemented by using either arrays or linked lists.

Every queue has `front` and `rear` variables that point to the position from where deletions and insertions can be done, respectively. Consider the queue shown in Fig. 2.4.

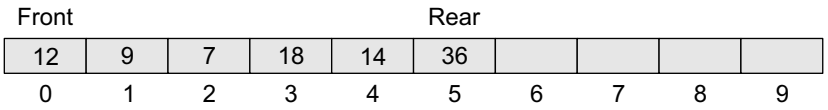


Figure 2.4 Array representation of a queue

Here, `front = 0` and `rear = 5`. If we want to add one more value to the list, say, if we want to add another element with the value 45, then the `rear` would be incremented by 1 and the value would be stored at the position pointed by the `rear`. The queue, after the addition, would be as shown in Fig. 2.5.

Here, `front = 0` and `rear = 6`. Every time a new element is to be added, we will repeat the same procedure.

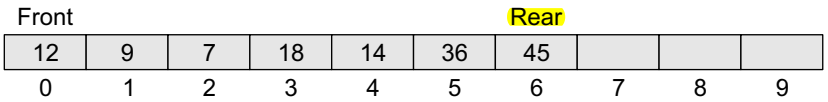


Figure 2.5 Queue after insertion of a new element

Now, if we want to delete an element from the queue, then the value of `front` will be incremented. Deletions are done only from this end of the queue. The queue after the deletion will be as shown in Fig. 2.6.

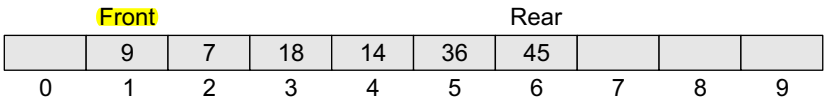


Figure 2.6 Queue after deletion of an element

However, before inserting an element in the queue, we must check for overflow conditions. An overflow occurs when we try to insert an element into a queue that is already full. A queue is full when `rear = MAX-1`, where `MAX` is the size of the queue, that is `MAX` specifies the maximum number of elements in the queue. Note that we have written `MAX-1` because the index starts from 0.

Similarly, before deleting an element from the queue, we must check for underflow conditions. An underflow condition occurs when we try to delete an element from a queue that is already empty. If `front = NULL` and `rear = NULL`, then there is no element in the queue.

Trees

A tree is a non-linear data structure which consists of a collection of nodes arranged in a hierarchical order. One of the nodes is designated as the root node, and the remaining nodes can be partitioned into disjoint sets such that each set is a sub-tree of the root.

The simplest form of a tree is a binary tree. A binary tree consists of a root node and left and right sub-trees, where both sub-trees are also binary trees. Each node contains a data element, a left pointer which points to the left sub-tree, and a right pointer which points to the right sub-tree. The root element is the topmost node which is pointed by a 'root' pointer. If `root = NULL` then the tree is empty.

Figure 2.7 shows a binary tree, where `R` is the root node and T_1 and T_2 are the left and right sub-trees of `R`. If T_1 is non-empty, then T_1 is said to be the left successor of `R`. Likewise, if T_2 is non-empty, then it is called the right successor of `R`.

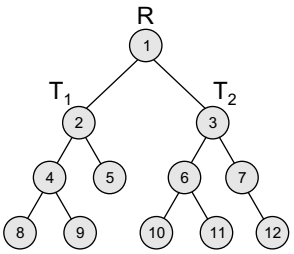


Figure 2.7 Binary tree

In Fig. 2.7, node 2 is the left child and node 3 is the right child of the root node 1. Note that the left sub-tree of the root node consists of the nodes 2, 4, 5, 8, and 9. Similarly, the right sub-tree of the root node consists of the nodes 3, 6, 7, 10, 11, and 12.

Note *Advantage*: Provides quick search, insert, and delete operations
Disadvantage: Complicated deletion algorithm

Graphs

A graph is a non-linear data structure which is a collection of *vertices* (also called *nodes*) and *edges* that connect these vertices. A graph is often viewed as a generalization of the tree structure, where instead of a purely parent-to-child relationship between tree nodes, any kind of complex relationships between the nodes can exist.

In a tree structure, nodes can have any number of children but only one parent, a graph on the other hand relaxes all such kinds of restrictions. Figure 2.8 shows a graph with five nodes.

A node in the graph may represent a city and the edges connecting the nodes can represent roads. A graph can also be used to represent a computer network where the nodes are workstations and the edges are the network connections. Graphs have so many applications in computer science and mathematics that several algorithms have been written to perform the standard graph operations, such as searching the graph and finding the shortest path between the nodes of a graph.

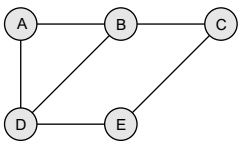


Figure 2.8 Graph

Note that unlike trees, graphs do not have any root node. Rather, every node in the graph can be connected with every another node in the graph. When two nodes are connected via an edge, the two nodes are known as *neighbours*. For example, in Fig. 2.8, node A has two neighbours: B and D.

Note *Advantage*: Best models real-world situations
Disadvantage: Some algorithms are slow and very complex

2.3 OPERATIONS ON DATA STRUCTURES

This section discusses the different operations that can be performed on the various data structures previously mentioned.

Traversing It means to access each data item exactly once so that it can be processed. For example, to print the names of all the students in a class.

Searching It is used to find the location of one or more data items that satisfy the given constraint. Such a data item may or may not be present in the given collection of data items. For example, to find the names of all the students who secured 100 marks in mathematics.

Inserting It is used to add new data items to the given list of data items. For example, to add the details of a new student who has recently joined the course.

Deleting It means to remove (delete) a particular data item from the given collection of data items. For example, to delete the name of a student who has left the course.

Sorting Data items can be arranged in some order like ascending order or descending order depending on the type of application. For example, arranging the names of students in a class in an alphabetical order, or calculating the top three winners by arranging the participants' scores in descending order and then extracting the top three.

Merging Lists of two sorted data items can be combined to form a single list of sorted data items.

Many a time, two or more operations are applied simultaneously in a given situation. For example, if we want to delete the details of a student whose name is X, then we first have to search the list of students to find whether the record of X exists or not and if it exists then at which location, so that the details can be deleted from that particular location.

2.4 ABSTRACT DATA TYPE

An *abstract data type* (ADT) is the way we look at a data structure, focusing on what it does and ignoring how it does its job. For example, stacks and queues are perfect examples of an ADT. We can implement both these ADTs using an array or a linked list. This demonstrates the 'abstract' nature of stacks and queues.

To further understand the meaning of an abstract data type, we will break the term into 'data type' and 'abstract', and then discuss their meanings.

Data type Data type of a variable is the set of values that the variable can take. We have already read the basic data types in C include `int`, `char`, `float`, and `double`.

When we talk about a primitive type (built-in data type), we actually consider two things: a data item with certain characteristics and the permissible operations on that data. For example, an `int` variable can contain any whole-number value from –32768 to 32767 and can be operated with the operators `+`, `-`, `*`, and `/`. In other words, the operations that can be performed on a data type are an inseparable part of its identity. Therefore, when we declare a variable of an abstract data type (e.g., stack or a queue), we also need to specify the operations that can be performed on it.

Abstract The word 'abstract' in the context of data structures means *considered apart from the detailed specifications or implementation*.

In C, an abstract data type can be a structure considered without regard to its implementation. It can be thought of as a 'description' of the data in the structure with a list of operations that can be performed on the data within that structure.

The end-user is not concerned about the details of how the methods carry out their tasks. They are only aware of the methods that are available to them and are only concerned about calling those methods and getting the results. They are not concerned about how they work.

For example, when we use a stack or a queue, the user is concerned only with the type of data and the operations that can be performed on it. Therefore, the fundamentals of how the data is stored should be invisible to the user. They should not be concerned with how the methods work or what structures are being used to store the data. They should just know that to work with stacks, they have `push()` and `pop()` functions available to them. Using these functions, they can manipulate the data (insertion or deletion) stored in the stack.

Advantage of using ADTs

In the real world, programs *evolve* as a result of new requirements or constraints, so a modification to a program commonly requires a change in one or more of its data structures. For example, if you want to add a new field to a student's record to keep track of more information about each student, then it will be better to replace an array with a linked structure to improve the program's efficiency. In such a scenario, rewriting every procedure that uses the changed structure is not desirable. Therefore, a better alternative is to *separate* the use of a data structure from the details of its implementation. This is the principle underlying the use of abstract data types.

2.5 ALGORITHMS

The typical definition of algorithm is 'a formally defined procedure for performing some calculation'. If a procedure is formally defined, then it can be implemented using a formal language,

and such a language is known as a *programming language*. In general terms, an algorithm provides a blueprint to write a program to solve a particular problem. It is considered to be an effective procedure for solving a problem in finite number of steps. That is, a well-defined algorithm always provides an answer and is guaranteed to terminate.

Algorithms are mainly used to achieve *software reuse*. Once we have an idea or a blueprint of a solution, we can implement it in any high-level language like C, C++, or Java.

An algorithm is basically a set of instructions that solve a problem. It is not uncommon to have multiple algorithms to tackle the same problem, but the choice of a particular algorithm must depend on the time and space complexity of the algorithm.

2.6 DIFFERENT APPROACHES TO DESIGNING AN ALGORITHM

Algorithms are used to manipulate the data contained in data structures. When working with data structures, algorithms are used to perform operations on the stored data.

A complex algorithm is often divided into smaller units called modules. This process of dividing an algorithm into modules is called modularization. The key advantages of modularization are as follows:

- It makes the complex algorithm simpler to design and implement.
- Each module can be designed independently. While designing one module, the details of other modules can be ignored, thereby enhancing clarity in design which in turn simplifies implementation, debugging, testing, documenting, and maintenance of the overall algorithm.

There are two main approaches to design an algorithm—top-down approach and bottom-up approach, as shown in Fig. 2.9.

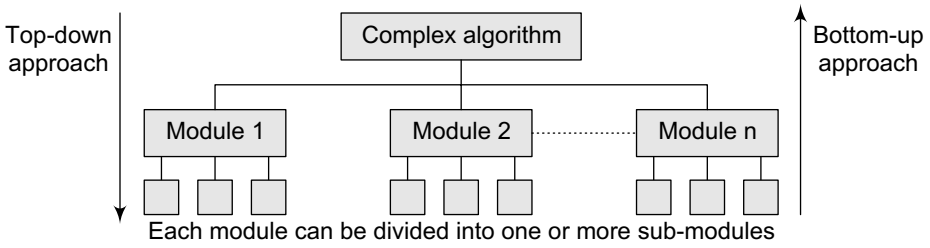


Figure 2.9 Different approaches of designing an algorithm

Top-down approach A top-down design approach starts by dividing the complex algorithm into one or more modules. These modules can further be decomposed into one or more sub-modules, and this process of decomposition is iterated until the desired level of module complexity is achieved. Top-down design method is a form of stepwise refinement where we begin with the topmost module and incrementally add modules that it calls.

Therefore, in a top-down approach, we start from an abstract design and then at each step, this design is refined into more concrete levels until a level is reached that requires no further refinement.

Bottom-up approach A bottom-up approach is just the reverse of top-down approach. In the bottom-up design, we start with designing the most basic or concrete modules and then proceed towards designing higher level modules. The higher level modules are implemented by using the operations performed by lower level modules. Thus, in this approach sub-modules are grouped together to form a higher level module. All the higher level modules are clubbed together to form even higher level modules. This process is repeated until the design of the complete algorithm is obtained.

Top-down vs bottom-up approach Whether the top-down strategy should be followed or a bottom-up is a question that can be answered depending on the application at hand.

While top-down approach follows a stepwise refinement by decomposing the algorithm into manageable modules, the bottom-up approach on the other hand defines a module and then groups together several modules to form a new higher level module.

Top-down approach is highly appreciated for ease in documenting the modules, generation of test cases, implementation of code, and debugging. However, it is also criticized because the sub-modules are analysed in isolation without concentrating on their communication with other modules or on reusability of components and little attention is paid to data, thereby ignoring the concept of information hiding.

Although the bottom-up approach allows information hiding as it first identifies what has to be encapsulated within a module and then provides an abstract interface to define the module's boundaries as seen from the clients. But all this is difficult to be done in a strict bottom-up strategy. Some top-down activities need to be performed for this.

All in all, design of complex algorithms must not be constrained to proceed according to a fixed pattern but should be a blend of top-down and bottom-up approaches.

2.7 CONTROL STRUCTURES USED IN ALGORITHMS

An algorithm has a finite number of steps. Some steps may involve decision-making and repetition. Broadly speaking, an algorithm may employ one of the following control structures: (a) sequence, (b) decision, and (c) repetition.

Sequence

By sequence, we mean that each step of an algorithm is executed in a specified order. Let us write an algorithm to add two numbers. This algorithm performs the steps in a purely sequential order, as shown in Fig. 2.10.

```
Step 1: Input first number as A
Step 2: Input second number as B
Step 3: SET SUM = A+B
Step 4: PRINT SUM
Step 5: END
```

Figure 2.10 Algorithm to add two numbers

Decision

Decision statements are used when the execution of a process depends on the outcome of some condition. For example, if `x = y`, then print `EQUAL`. So the general form of `IF` construct can be given as:

```
IF condition Then process
```

A condition in this context is any statement that may evaluate to either a true value or a false value. In the above example, a variable `x` can be either equal to `y` or not equal to `y`. However, it cannot be both true and false. If the condition is true, then the process is executed.

A decision statement can also be stated in the following manner:

```
IF condition
      Then process1
ELSE  process2
```

This form is popularly known as the `IF-ELSE` construct. Here, if the condition is true, then `process1` is executed, else `process2` is executed. Figure 2.11 shows an algorithm to check if two numbers are equal.

Repetition

Repetition, which involves executing one or more steps for a number of times, can be implemented using constructs such as `while`, `do-while`, and `for` loops. These loops execute one or more steps until some condition is true. Figure 2.12 shows an algorithm that prints the first 10 natural numbers.

```
Step 1: Input first number as A
Step 2: Input second number as B
Step 3: IF A = B
             PRINT "EQUAL"
        ELSE
             PRINT "NOT EQUAL"
        [END OF IF]
Step 4: END
```

Figure 2.11 Algorithm to test for equality of two numbers

```
Step 1: [INITIALIZE] SET I = 1, N = 10
Step 2: Repeat Steps 3 and 4 while I<=N
Step 3: PRINT I
Step 4: SET I = I+1
        [END OF LOOP]
Step 5: END
```

Figure 2.12 Algorithm to print the first 10 natural of

Programming Examples

1. Write an algorithm for swapping two values.

```
Step 1: Input first number as A
Step 2: Input second number as B
Step 3: SET TEMP = A
Step 4: SET A = B
Step 5: SET B = TEMP
Step 6: PRINT A, B
Step 7: END
```

2. Write an algorithm to find the larger of two numbers.

```
Step 1: Input first number as A
Step 2: Input second number as B
Step 3: IF A>B
           PRINT A
        ELSE
           IF A<B
              PRINT B
        ELSE
             PRINT "The numbers are equal"
             [END OF IF]
        [END OF IF]
Step 4: END
```

3. Write an algorithm to find whether a number is even or odd.

```
Step 1: Input number as A
Step 2: IF A%2 =0
               PRINT "EVEN"
        ELSE
               PRINT "ODD"
        [END OF IF]
Step 3: END
```

4. Write an algorithm to print the grade obtained by a student using the following rules.

```
Step 1: Enter the Marks obtained as M
Step 2: IF M>75
               PRINT O
Step 3: IF M>=60 AND M<75
               PRINT A
Step 4: IF M>=50 AND M<60
               PRINT B
Step 5: IF M>=40 AND M<50
               PRINT C
        ELSE
               PRINT D
```

Marks	Grade
Above 75	O
60–75	A
50–59	B
40–49	C
Less then 40	D

```
            [END OF IF]
Step 6: END
```

5. Write an algorithm to find the sum of first N natural numbers.

```
Step 1: Input N
Step 2: SET I = 1, SUM = 0
Step 3: Repeat Step 4 while I <= N
Step 4:         SET SUM = SUM + I
                SET I = I + 1
            [END OF LOOP]
Step 5: PRINT SUM
Step 6: END
```

2.8 TIME AND SPACE COMPLEXITY

Analysing an algorithm means determining the amount of resources (such as time and memory) needed to execute it. Algorithms are generally designed to work with an arbitrary number of inputs, so the efficiency or complexity of an algorithm is stated in terms of time and space complexity.

The *time complexity* of an algorithm is basically the running time of a program as a function of the input size. Similarly, the *space complexity* of an algorithm is the amount of computer memory that is required during the program execution as a function of the input size.

In other words, the number of machine instructions which a program executes is called its time complexity. This number is primarily dependent on the size of the program's input and the algorithm used.

Generally, the space needed by a program depends on the following two parts:

- *Fixed part*: It varies from problem to problem. It includes the space needed for storing instructions, constants, variables, and structured variables (like arrays and structures).
- *Variable part*: It varies from program to program. It includes the space needed for recursion stack, and for structured variables that are allocated space dynamically during the runtime of a program.

However, running time requirements are more critical than memory requirements. Therefore, in this section, we will concentrate on the running time efficiency of algorithms.

2.8.1 Worst-case, Average-case, Best-case, and Amortized Time Complexity

Worst-case running time This denotes the behaviour of an algorithm with respect to the worst-possible case of the input instance. The worst-case running time of an algorithm is an upper bound on the running time for any input. Therefore, having the knowledge of worst-case running time gives us an assurance that the algorithm will never go beyond this time limit.

Average-case running time The average-case running time of an algorithm is an estimate of the running time for an 'average' input. It specifies the expected behaviour of the algorithm when the input is randomly drawn from a given distribution. Average-case running time assumes that all inputs of a given size are equally likely.

Best-case running time The term 'best-case performance' is used to analyse an algorithm under optimal conditions. For example, the best case for a simple linear search on an array occurs when the desired element is the first in the list. However, while developing and choosing an algorithm to solve a problem, we hardly base our decision on the best-case performance. It is always recommended to improve the average performance and the worst-case performance of an algorithm.

Amortized running time Amortized running time refers to the time required to perform a sequence of (related) operations averaged over all the operations performed. Amortized analysis guarantees the average performance of each operation in the worst case.

2.8.2 Time–Space Trade-off

The best algorithm to solve a particular problem at hand is no doubt the one that requires less memory space and takes less time to complete its execution. But practically, designing such an ideal algorithm is not a trivial task. There can be more than one algorithm to solve a particular problem. One may require less memory space, while the other may require less CPU time to execute. Thus, it is not uncommon to sacrifice one thing for the other. Hence, there exists a time–space trade-off among algorithms.

So, if space is a big constraint, then one might choose a program that takes less space at the cost of more CPU time. On the contrary, if time is a major constraint, then one might choose a program that takes minimum time to execute at the cost of more space.

2.8.3 Expressing Time and Space Complexity

The time and space complexity can be expressed using a function `f(n)` where `n` is the input size for a given instance of the problem being solved. Expressing the complexity is required when

- We want to predict the rate of growth of complexity as the input size of the problem increases.
- There are multiple algorithms that find a solution to a given problem and we need to find the algorithm that is most efficient.

The most widely used notation to express this function `f(n)` is the Big O notation. It provides the upper bound for the complexity.

2.8.4 Algorithm Efficiency

If a function is linear (without any loops or recursions), the efficiency of that algorithm or the running time of that algorithm can be given as the number of instructions it contains. However, if an algorithm contains loops, then the efficiency of that algorithm may vary depending on the number of loops and the running time of each loop in the algorithm.

Let us consider different cases in which loops determine the efficiency of an algorithm.

Linear Loops

To calculate the efficiency of an algorithm that has a single loop, we need to first determine the number of times the statements in the loop will be executed. This is because the number of iterations is directly proportional to the loop factor. Greater the loop factor, more is the number of iterations. For example, consider the loop given below:

```
for(i=0;i<100;i++)
      statement block;
```

Here, 100 is the loop factor. We have already said that efficiency is directly proportional to the number of iterations. Hence, the general formula in the case of linear loops may be given as

```
f(n) = n
```

However calculating efficiency is not as simple as is shown in the above example. Consider the loop given below:

```
for(i=0;i<100;i+=2)
       statement block;
```

Here, the number of iterations is half the number of the loop factor. So, here the efficiency can be given as

```
f(n) = n/2
```

Logarithmic Loops

We have seen that in linear loops, the loop updation statement either adds or subtracts the loop-controlling variable. However, in logarithmic loops, the loop-controlling variable is either multiplied or divided during each iteration of the loop. For example, look at the loops given below:

```
for(i=1;i<1000;i*=2)
      statement block;
```

```
for(i=1000;i>=1;i/=2)
      statement block;
```

Consider the first `for` loop in which the loop-controlling variable `i` is multiplied by 2. The loop will be executed only 10 times and not 1000 times because in each iteration the value of `i` doubles. Now, consider the second loop in which the loop-controlling variable `i` is divided by 2. In this case also, the loop will be executed 10 times. Thus, the number of iterations is a function of the number by which the loop-controlling variable is divided or multiplied. In the examples discussed, it is 2. That is, when `n = 1000`, the number of iterations can be given by `log 1000` which is approximately equal to 10.

Therefore, putting this analysis in general terms, we can conclude that the efficiency of loops in which iterations divide or multiply the loop-controlling variables can be given as

```
f(n) = log n
```

Nested Loops

Loops that contain loops are known as *nested loops*. In order to analyse nested loops, we need to determine the number of iterations each loop completes. The total is then obtained as the product of the number of iterations in the inner loop and the number of iterations in the outer loop.

In this case, we analyse the efficiency of the algorithm based on whether it is a linear logarithmic, quadratic, or dependent quadratic nested loop.

Linear logarithmic loop Consider the following code in which the loop-controlling variable of the inner loop is multiplied after each iteration. The number of iterations in the inner loop is `log 10`. This inner loop is controlled by an outer loop which iterates 10 times. Therefore, according to the formula, the number of iterations for this code can be given as `10 log 10`.

```
for(i=0;i<10;i++)
      for(j=1; j<10;j*=2)
            statement block;
```

In more general terms, the efficiency of such loops can be given as `f(n) = n log n`.

Quadratic loop In a quadratic loop, the number of iterations in the inner loop is equal to the number of iterations in the outer loop. Consider the following code in which the outer loop executes 10 times and for each iteration of the outer loop, the inner loop also executes 10 times. Therefore, the efficiency here is 100.

```
for(i=0;i<10;i++)
      for(j=0; j<10;j++)
            statement block;
```

The generalized formula for quadratic loop can be given as $f(n) = n^2$.

Dependent quadratic loop In a dependent quadratic loop, the number of iterations in the inner loop is dependent on the outer loop. Consider the code given below:

```
for(i=0;i<10;i++)
      for(j=0; j<=i;j++)
            statement block;
```

In this code, the inner loop will execute just once in the first iteration, twice in the second iteration, thrice in the third iteration, so on and so forth. In this way, the number of iterations can be calculated as

```
1 + 2 + 3 + ··· + 9 + 10 = 55
```

If we calculate the average of this loop (55/10 = 5.5), we will observe that it is equal to the number of iterations in the outer loop (10) plus 1 divided by 2. In general terms, the inner loop iterates `(n + 1)/2` times. Therefore, the efficiency of such a code can be given as

```
f(n) = n (n + 1)/2
```

2.9 BIG O NOTATION

In today's era of massive advancement in computer technology, we are hardly concerned about the efficiency of algorithms. Rather, we are more interested in knowing the generic order of the magnitude of the algorithm. If we have two different algorithms to solve the same problem where one algorithm executes in 10 iterations and the other in 20 iterations, the difference between the two algorithms is not much. However, if the first algorithm executes in 10 iterations and the other in 1000 iterations, then it is a matter of concern.

We have seen that the number of statements executed in the program for `n` elements of the data is a function of the number of elements, expressed as `f(n)`. Even if the expression derived for a function is complex, a dominant factor in the expression is sufficient to determine the order of the magnitude of the result and, hence, the efficiency of the algorithm. This factor is the `Big O`, and is expressed as `O(n)`.

The Big O notation, where O stands for 'order of', is concerned with what happens for very large values of `n`. For example, if a sorting algorithm performs n^2 operations to sort just `n` elements, then that algorithm would be described as an $O(n^2)$ algorithm.

When expressing complexity using the Big O notation, constant multipliers are ignored. So, an `O(4n)` algorithm is equivalent to `O(n)`, which is how it should be written.

If `f(n)` and `g(n)` are the functions defined on a positive integer number `n`, then

```
f(n) = O(g(n))
```

That is, `f of n is Big-O of g of n` if and only if positive constants `c` and `n` exist, such that $f(n) \leq cg(n)$. It means that for large amounts of data, `f(n)` will grow no more than a constant factor than `g(n)`. Hence, `g` provides an upper bound. Note that here `c` is a constant which depends on the following factors:

- the programming language used,
- the quality of the compiler or interpreter,
- the CPU speed,
- the size of the main memory and the access time to it,
- the knowledge of the programmer, and
- the algorithm itself, which may require simple but also time-consuming machine instructions.

We have seen that the Big O notation provides a strict upper bound for `f(n)`. This means that the function `f(n)` can do better but not worse than the specified value. Big O notation is simply written as $f(n) \in O(g(n))$ or as $f(n) = O(g(n))$.

Here, `n` is the problem size and $O(g(n)) = \{h(n): \exists \text{ positive constants } c, n_0 \text{ such that } 0 \leq h(n) \leq cg(n), \forall n \geq n_0\}$. Hence, we can say that $O(g(n))$ comprises a set of all the functions `h(n)` that are less than or equal to $cg(n)$ for all values of $n \geq n_0$.

> If $f(n) \leq cg(n)$, $c > 0$, $\forall n \geq n_0$, then $f(n) = O(g(n))$ and `g(n)` is an asymptotically tight upper bound for `f(n)`.

Examples of functions in $O(n^3)$ include: $n^{2.9}$, n^3, $n^3 + n$, $540n^3 + 10$.

Examples of functions not in $O(n^3)$ include: $n^{3.2}$, n^2, $n^2 + n$, $540n + 10$, $2n$
To summarize,

- Best case O describes an upper bound for all combinations of input. It is possibly lower than the worst case. For example, when sorting an array the best case is when the array is already correctly sorted.
- Worst case O describes a lower bound for worst case input combinations. It is possibly greater than the best case. For example, when sorting an array the worst case is when the array is sorted in reverse order.
- If we simply write O, it means same as worst case O.

Now let us look at some examples of `g(n)` and `f(n)`. Table 2.1 shows the relationship between `g(n)` and `f(n)`. Note that the constant values will be ignored because the main purpose of the Big O notation is to analyse the algorithm in a general fashion, so the anomalies that appear for small input sizes are simply ignored.

Table 2.1 Examples of `f(n)` and `g(n)`

g(n)	f(n) = O(g(n))
10	O(1)
$2n^3 + 1$	$O(n^3)$
$3n^2 + 5$	$O(n^2)$
$2n^3 + 3n^2 + 5n - 10$	$O(n^3)$

Categories of Algorithms

According to the Big O notation, we have five different categories of algorithms:

- Constant time algorithm: running time complexity given as `O(1)`
- Linear time algorithm: running time complexity given as `O(n)`
- Logarithmic time algorithm: running time complexity given as `O(log n)`
- Polynomial time algorithm: running time complexity given as $O(n^k)$ where `k > 1`
- Exponential time algorithm: running time complexity given as $O(2^n)$

Table 2.2 shows the number of operations that would be performd for various values of `n`.

Table 2.2 Number of operations for different functions of `n`

n	O(1)	O(log n)	O(n)	O(n log n)	$O(n^2)$	$O(n^3)$
1	1	1	1	1	1	1
2	1	1	2	2	4	8
4	1	2	4	8	16	64
8	1	3	8	24	64	512
16	1	4	16	64	256	4096

Example 2.1 Show that $4n^2 = O(n^3)$.

Solution By definition, we have

$$0 \le h(n) \le cg(n)$$

Substituting $4n^2$ as `h(n)` and n^3 as `g(n)`, we get

$$0 \le 4n^2 \le cn^3$$

Dividing by n^3

$$0/n^3 \le 4n^2/n^3 \le cn^3/n^3$$
$$0 \le 4/n \le c$$

Now to determine the value of `c`, we see that `4/n` is maximum when `n=1`. Therefore, `c=4`.

To determine the value of n_0,

$$0 \le 4/n_0 \le 4$$
$$0 \le 4/4 \le n_0$$

$0 \le 1 \le n_0$

This means $n_0=1$. Therefore, $0 \le 4n^2 \le 4n^3 \; \forall \; n \ge n_0=1$.

Example 2.2 Show that $400n^3 + 20n^2 = O(n^3)$.

Solution By definition, we have

$0 \le h(n) \le cg(n)$

Substituting $400n^3 + 20n^2$ as $h(n)$ and n^3 as $g(n)$, we get

$0 \le 400n^3 + 20n^2 \le cn^3$

Dividing by n^3

$0/n^3 \le 400n^3/n^3 + 20n^2/n^3 \le cn^3/n^3$

$0 \le 400 + 20/n \le c$

Note that $20/n \rightarrow 0$ as $n \rightarrow \infty$, and $20/n$ is maximum when $n = 1$. Therefore,

$0 \le 400 + 20/1 \le c$

This means, $c = 420$

To determine the value of n_0,

$0 \le 400 + 20/n_0 \le 420$

$-400 \le 400 + 20/n_0 - 400 \le 420 - 400$

$-400 \le 20/n_0 \le 20$

$-20 \le 1/n_0 \le 1$

$-20\ n_0 \le 1 \le n_0$. This implies $n_0 = 1$.

Hence, $0 \le 400n^3 + 20n^2 \le 420n^3 \; \forall \; n \ge n_0=1$.

Example 2.3 Show that $n = O(n \log n)$.

Solution By definition, we have

$0 \le h(n) \le cg(n)$

Substituting n as $h(n)$ and $n \log n$ as $g(n)$, we get

$0 \le n \le c\ n \log n$

Dividing by $n \log n$, we get

$0/n \log n \le n/n \log n \le c\ n \log n/\ n \log n$

$0 \le 1/\log n \le c$

We know that $1/\log n \rightarrow 0$ as $n \rightarrow \infty$

To determine the value of c, it is clearly evident that $1/\log n$ is greatest when $n=2$. Therefore,

$0 \le 1/\log 2 \le c = 1$. Hence $c = 1$.

To determine the value of n_0, we can write

$0 \le 1/\log n_0 \le 1$

$0 \le 1 \le \log n_0$

Now, $\log n_0 = 1$, when $n_0 = 2$.

Hence, $0 \le n \le cn \log n$ when $c= 1$ and $\forall \; n \ge n_0=2$.

Example 2.4 Show that $10n^3 + 20n \ne O(n^2)$.

Solution By definition, we have

$0 \le h(n) \le cg(n)$

Substituting $10n^3 + 20n$ as $h(n)$ and n^2 as $g(n)$, we get

$0 \le 10n^3 + 20n \le cn^2$

Dividing by n^2

$0/n^2 \le 10n^3/n^2 + 20n/n^2 \le cn^2/n^2$

$0 \le 10n + 20/n \le c$

$0 \le (10n^2 + 20)/n \le c$

Hence, $10n^3 + 20n \ne O^2(n^2)$

Limitations of Big O Notation

There are certain limitations with the Big O notation of expressing the complexity of algorithms. These limitations are as follows:

- Many algorithms are simply too hard to analyse mathematically.
- There may not be sufficient information to calculate the behaviour of the algorithm in the average case.
- Big O analysis only tells us how the algorithm grows with the size of the problem, not how efficient it is, as it does not consider the programming effort.
- It ignores important constants. For example, if one algorithm takes $O(n^2)$ time to execute and the other takes $O(100000n^2)$ time to execute, then as per Big O, both algorithm have equal time complexity. In real-time systems, this may be a serious consideration.

2.10 OMEGA NOTATION (Ω)

The Omega notation provides a tight lower bound for $f(n)$. This means that the function can never do better than the specified value but it may do worse.

Ω notation is simply written as, $f(n) \in \Omega(g(n))$, where n is the problem size and $\Omega(g(n)) = \{h(n): \exists$ positive constants $c > 0$, n_0 such that $0 \leq cg(n) \leq h(n)$, $\forall\ n \geq n_0\}$.

Hence, we can say that $\Omega(g(n))$ comprises a set of all the functions $h(n)$ that are greater than or equal to $cg(n)$ for all values of $n \geq n_0$.

If $cg(n) \leq f(n)$, $c > 0$, $\forall\ n \geq n_0$, then $f(n) \in \Omega(g(n))$ and $g(n)$ is an asymptotically tight lower bound for $f(n)$.

Examples of functions in $\Omega(n^2)$ include: n^2, $n^{2.9}$, $n^3 + n^2$, n^3

Examples of functions not in $\Omega(n^3)$ include: n, $n^{2.9}$, n^2

To summarize,

- Best case Ω describes a lower bound for all combinations of input. This implies that the function can never get any better than the specified value. For example, when sorting an array the best case is when the array is already correctly sorted.
- Worst case Ω describes a lower bound for worst case input combinations. It is possibly greater than best case. For example, when sorting an array the worst case is when the array is sorted in reverse order.
- If we simply write Ω, it means same as best case Ω.

Example 2.5 Show that $5n^2 + 10n = \Omega(n^2)$.

Solution By the definition, we can write

$$0 \leq cg(n) \leq h(n)$$
$$0 \leq cn^2 \leq 5n^2 + 10n$$

Dividing by n^2

$$0/n^2 \leq cn^2/n^2 \leq 5n^2/n^2 + 10n/n^2$$
$$0 \leq c \leq 5 + 10/n$$

Now, $\lim_{n \to \infty} 5 + 10/n = 5$.

Therefore, $0 \leq c \leq 5$.

Hence, $c = 5$

Now to determine the value of n_0

$$0 \leq 5 \leq 5 + 10/n_0$$
$$-5 \leq 5 - 5 \leq 5 + 10/n_0 - 5$$

$-5 \leq 0 \leq 10/n_0$

So $n_0 = 1$ as $\lim_{n\to\infty} 1/n = 0$

Hence, $5n^2 + 10n = \Omega(n^2)$ for $c=5$ and $\forall\ n \geq n_0=1$.

Example 2.6 Show that $7n \neq \Omega(n^2)$.

Solution By the definition, we can write

$0 \leq cg(n) \leq h(n)$

$0 \leq cn^2 \leq 7n$

Dividing by n^2, we get

$0/n^2 \leq cn^2/n^2 \leq 7n/n^2$

$0 \leq c \leq 7/n$

Thus, from the above statement, we see that the value of c depends on the value of n. There does not exist a value of n_0 that satisfies the condition as n increases. This could fairly be possible if $c = 0$ but it is not allowed as the definition by itself says that $\lim_{n\to\infty} 1/n = 0$.

2.11 THETA NOTATION (Θ)

Theta notation provides an asymptotically tight bound for $f(n)$. Θ notation is simply written as, $f(n) \in \Theta(g(n))$, where n is the problem size and $\Theta(g(n)) = \{h(n)$: ∃ positive constants c_1, c_2, and n_0 such that $0 \leq c_1 g(n) \leq h(n) \leq c_2 g(n)$, $\forall\ n \geq n_0\}$.

Hence, we can say that $\Theta(g(n))$ comprises a set of all the functions $h(n)$ that are between $c_1 g(n)$ and $c_2 g(n)$ for all values of $n \geq n_0$.

If $f(n)$ is between $c_1 g(n)$ and $c_2 g(n)$, $\forall\ n \geq n_0$, then $f(n) \in \Theta(g(n))$ and $g(n)$ is an asymptotically tight bound for $f(n)$ and $f(n)$ is amongst $h(n)$ in the set.

To summarize,

- The best case in Θ notation is not used.
- Worst case Θ describes asymptotic bounds for worst case combination of input values.
- If we simply write Θ, it means same as worst case Θ.

Example 2.7 Show that $n^2/2 - 2n = \Theta(n^2)$.

Solution By the definition, we can write

$c_1 g(n) \leq h(n) \leq c_2 g(n)$

$c_1 n^2 \leq n^2/2 - 2n \leq c_2 n^2$

Dividing by n^2, we get

$c_1 n^2/n^2 \leq n^2/2n^2 - 2n/n^2 \leq c_2 n^2/n^2$

$c_1 \leq 1/2 - 2/n \leq c_2$

This means $c_2 = 1/2$ because $\lim_{n\to\infty} 1/2 - 2/n = 1/2$ (Big O notation)

To determine c_1 using Ω notation, we can write

$0 < c_1 \leq 1/2 - 2/n$

We see that $0 < c_1$ is minimum when $n = 5$. Therefore,

$0 < c_1 \leq 1/2 - 2/5$

Hence, $c_1 = 1/10$

Now let us determine the value of n_0

$1/10 \leq 1/2 - 2/n_0 \leq 1/2$

$2/n_0 \leq 1/2 - 1/10 \leq 1/2$

$2/n_0 \leq 2/5 \leq 1/2$

$n_0 \geq 5$

You may verify this by substituting the values as shown below.

$c_1n^2 \leq n^2/2 - 2n \leq c_2n^2$

$c_1 = 1/10$, $c_2 = 1/2$ and $n_0 = 5$

$1/10(25) \leq 25/2 - 20/2 \leq 25/2$

$5/2 \leq 5/2 \leq 25/2$

Thus, in general, we can write, $1/10n^2 \leq n^2/2 - 2n \leq 1/2n^2$ for $n \geq 5$.

2.12 OTHER USEFUL NOTATIONS

There are other notations like little o notation and little ω notation which have been discussed below.

Little o Notation

This notation provides a non-asymptotically tight upper bound for $f(n)$. To express a function using this notation, we write

$f(n) \in o(g(n))$ where

$o(g(n)) = \{h(n) : \exists$ positive constants c, n_0 such that for any $c > 0$, $n_0 > 0$, and $0 \leq h(n) \leq cg(n)$, $\forall\ n \geq n_0\}$.

This is unlike the Big O notation where we say for some $c > 0$ (not any). For example, $5n^3 = o(n^3)$ is asymptotically tight upper bound but $5n^2 = o(n^3)$ is non-asymptotically tight bound for $f(n)$.

Examples of functions in $o(n^3)$ include: $n^{2.9}$, $n^3 / \log n$, $2n^2$

Examples of functions not in $o(n^3)$ include: $3n^3$, n^3, $n^3 / 1000$

Example 2.8 Show that $n^3 / 1000 \neq o(n^3)$.

Solution By definition, we have

$0 \leq h(n) < cg(n)$, for any constant $c > 0$

$0 \leq n^3 / 1000 \leq cn^3$

This is in contradiction with selecting any $c < 1/1000$.

An imprecise analogy between the asymptotic comparison of functions $f(n)$ and $g(n)$ and the relation between their values can be given as:

$f(n) = O(g(n)) \approx f(n) \leq g(n)$ $\quad$ $f(n) = o(g(n)) \approx f(n) < g(n)$ $\quad$ $f(n) = \Theta(g(n)) \approx f(n) = g(n)$

Little Omega Notation (ω)

This notation provides a non-asymptotically tight lower bound for $f(n)$. It can be simply written as, $f(n) \in \omega(g(n))$, where

$\omega(g(n)) = \{h(n) : \exists$ positive constants c, n_0 such that for any $c > 0$, $n_0 > 0$, and $0 \leq cg(n) < h(n), \forall\ n \geq n_0\}$.

This is unlike the Ω notation where we say for some $c > 0$ (not any). For example, $5n^3 = \Omega(n^3)$ is asymptotically tight upper bound but $5n^2 = \omega(n^3)$ is non-asymptotically tight bound for $f(n)$.

Example of functions in $\omega(g(n))$ include: $n^3 = \omega(n^2)$, $n^{3.001} = \omega(n^3)$, $n^2 \log n = \omega(n^2)$

Example of a function not in $\omega(g(n))$ is $5n^2 \neq \omega(n^2)$ (just as $5 \neq 5$)

Example 2.9 Show that $50n^3/100 \neq \omega(n^3)$.

Solution By definition, we have

$0 \leq cg(n) < h(n)$, for any constant $c > 0$

$0 \leq cn^3 < 50n^3/100$

Dividing by n^3, we get

$0 \leq c < 50/100$

This is a contradictory value as for any value of c as it cannot be assured to be less than 50/100 or 1/2.

An imprecise analogy between the asymptotic comparison of functions $f(n)$ and $g(n)$ and the relation between their values can be given as:

$f(n) = \Omega(g(n)) \approx f(n) \geq g(n)$ $\quad\quad$ $f(n) = \omega(g(n)) \approx f(n) > g(n)$

Points to Remember

- A data structure is a particular way of storing and organizing data either in computer's memory or on the disk storage so that it can be used efficiently.
- There are two types of data structures: primitive and non-primitive data structures. Primitive data structures are the fundamental data types which are supported by a programming language. Non-primitive data structures are those data structures which are created using primitive data structures.
- Non-primitive data structures can further be classified into two categories: linear and non-linear data structures.
- If the elements of a data structure are stored in a linear or sequential order, then it is a linear data structure. However, if the elements of a data structure are not stored in sequential order, then it is a non-linear data structure.
- An array is a collection of similar data elements which are stored in consecutive memory locations.
- A linked list is a linear data structure consisting of a group of elements (called nodes) which together represent a sequence.
- A stack is a last-in, first-out (LIFO) data structure in which insertion and deletion of elements are done at only one end, which is known as the top of the stack.
- A queue is a first-in, first-out (FIFO) data structure in which the element that is inserted first is the first to be taken out. The elements in a queue are added at one end called the rear and removed from the other end called the front.
- A tree is a non-linear data structure which consists of a collection of nodes arranged in a hierarchical tree structure.
- The simplest form of a tree is a binary tree. A binary tree consists of a root node and left and right sub-trees, where both sub-trees are also binary trees.
- A graph is often viewed as a generalization of the tree structure, where instead of a purely parent-to-child relationship between tree nodes, any kind of complex relationships can exist between the nodes.
- An abstract data type (ADT) is the way we look at a data structure, focusing on what it does and ignoring how it does its job.
- An algorithm is basically a set of instructions that solve a problem.
- The time complexity of an algorithm is basically the running time of the program as a function of the input size.
- The space complexity of an algorithm is the amount of computer memory required during the program execution as a function of the input size.
- The worst-case running time of an algorithm is an upper bound on the running time for any input.
- The average-case running time specifies the expected behaviour of the algorithm when the input is randomly drawn from a given distribution.
- Amortized analysis guarantees the average performance of each operation in the worst case.
- The efficiency of an algorithm is expressed in terms of the number of elements that has to be processed and the type of the loop that is being used.

Exercises

Review Questions

1. Explain the features of a good program.
2. Define the terms: data, file, record, and primary key.

3. Define data structures. Give some examples.
4. In how many ways can you categorize data structures? Explain each of them.
5. Discuss the applications of data structures.
6. Write a short note on different operations that can be performed on data structures.
7. Compare a linked list with an array.
8. Write a short note on abstract data type.
9. Explain the different types of data structures. Also discuss their merits and demerits.
10. Define an algorithm. Explain its features with the help of suitable examples.
11. Explain and compare the approaches for designing an algorithm.
12. What is modularization? Give its advantages.
13. Write a brief note on trees as a data structure.
14. What do you understand by a graph?
15. Explain the criteria that you will keep in mind while choosing an appropriate algorithm to solve a particular problem.
16. What do you understand by time–space trade-off?
17. What do you understand by the efficiency of an algorithm?
18. How will you express the time complexity of a given algorithm?
19. Discuss the significance and limitations of the Big O notation.
20. Discuss the best case, worst case, average case, and amortized time complexity of an algorithm.
21. Categorize algorithms based on their running time complexity.
22. Give examples of functions that are in Big O notation as well as functions that are not in Big O notation.
23. Explain the little o notation.
24. Give examples of functions that are in little o notation as well as functions that are not in little o notation.
25. Differentiate between Big O and little o notations.
26. Explain the Ω notation.
27. Give examples of functions that are in Ω notation as well as functions that are not in Ω notation.
28. Explain the Θ notation.
29. Give examples of functions that are in Θ notation as well as functions that are not in Θ notation.
30. Explain the ω notation.
31. Give examples of functions that are in ω notation as well as functions that are in ω notation.
32. Differentiate between Big omega and little omega notations.
33. Show that $n^2 + 50n = O(n^2)$.
34. Show that $n^2+n^2+n^2 = 3n^2 = O(n^3)$.
35. Prove that $n^3 \neq O(n^2)$.
36. Show that $\sqrt{n} = \Omega(\lg n)$.
37. Prove that $3n + 5 \neq \Omega(n^2)$.
38. Show that $\frac{1}{2}n^2 - 3n \in \Theta(n^2)$.

Multiple-choice Questions

1. Which data structure is defined as a collection of similar data elements?
   (a) Arrays (b) Linked lists
   (c) Trees (d) Graphs
2. The data structure used in hierarchical data model is
   (a) Array (b) Linked list
   (c) Tree (d) Graph
3. In a stack, insertion is done at
   (a) Top (b) Front
   (c) Rear (d) Mid
4. The position in a queue from which an element is deleted is called as
   (a) Top (b) Front
   (c) Rear (d) Mid
5. Which data structure has fixed size?
   (a) Arrays (b) Linked lists
   (c) Trees (d) Graphs
6. If `TOP = MAX-1`, then that the stack is
   (a) Empty (b) Full
   (c) Contains some data (d) None of these
7. Which among the following is a LIFO data structure?
   (a) Stacks (b) Linked lists
   (c) Queues (d) Graphs
8. Which data structure is used to represent complex relationships between the nodes?
   (a) Arrays (b) Linked lists
   (c) Trees (d) Graphs
9. Examples of linear data structures include
   (a) Arrays (b) Stacks
   (c) Queues (d) All of these
10. The running time complexity of a linear time algorithm is given as
    (a) $O(1)$ (b) $O(n)$
    (c) $O(n \log n)$ (d) $O(n^2)$

11. Which notation provides a strict upper bound for f(n)?
(a) Omega notation (b) Big O notation
(c) Small o notation (d) Theta Notation

12. Which notation comprises a set of all functions h(n) that are greater than or equal to cg(n) for all values of $n \geq n_0$?
(a) Omega notation (b) Big O notation
(c) Small o notation (d) Theta Notation

13. Function in $o(n^2)$ notation is
(a) $10n^2$ (b) $n^{1.9}$
(c) $n^2/100$ (d) n^2

True or False

1. Trees and graphs are the examples of linear data structures.
2. Queue is a FIFO data structure.
3. Trees can represent any kind of complex relationship between the nodes.
4. The average-case running time of an algorithm is an upper bound on the running time for any input.
5. Array is an abstract data type.
6. Array elements are stored in continuous memory locations.
7. The pop operation adds an element to the top of a stack.
8. Graphs have a purely parent-to-child relationship between their nodes.
9. The worst-case running time of an algorithm is a lower bound on the running time for any input.
10. In top-down approach, we start with designing the most basic or concrete modules and then proceed towards designing higher-level modules.
11. o(g(n)) comprises a set of all functions h(n) that are less than or equal to cg(n) for all values of $n \geq n_0$.
12. Simply Ω means same as best case Ω.
13. Small omega notation provides an asymptotically tight bound for f(n).
14. Theta notation provides a non-asymptotically tight lower bound for f(n).
15. $n^{3.001} \neq \omega(n^3)$.

Fill in the Blanks

1. _____ is an arrangement of data either in the computer's memory or on the disk storage.
2. _____ are used to manipulate the data contained in various data structures.
3. In _____, the elements of a data structure are stored sequentially.
4. _____ of a variable specifies the set of values that the variable can take.
5. A tree is empty if _____.
6. Abstract means _____.
7. The time complexity of an algorithm is the running time given as a function of _____.
8. _____ analysis guarantees the average performance of each operation in the worst case.
9. The elements of an array are referenced by an _____.
10. _____ is used to store the address of the topmost element of a stack.
11. The _____ operation returns the value of the topmost element of a stack.
12. An overflow occurs when _____.
13. _____ is a FIFO data structure.
14. The elements in a queue are added at _____ and removed from _____.
15. If the elements of a data structure are stored sequentially, then it is a _____.
16. _____ is basically a set of instructions that solve a problem.
17. The number of machine instructions that a program executes during its execution is called its _____.
18. _____ specifies the expected behaviour of an algorithm when an input is randomly drawn from a given distribution.
19. The running time complexity of a constant time algorithm is given as _____.
20. A complex algorithm is often divided into smaller units called _____.
21. _____ design approach starts by dividing the complex algorithm into one or more modules.
22. _____ case is when the array is sorted in reverse order.
23. _____ notation provides a tight lower bound for f(n).
24. The small o notation provides a _____ tight upper bound for f(n).
25. $540n^2 + 10$ ____ $\Omega(n^2)$.