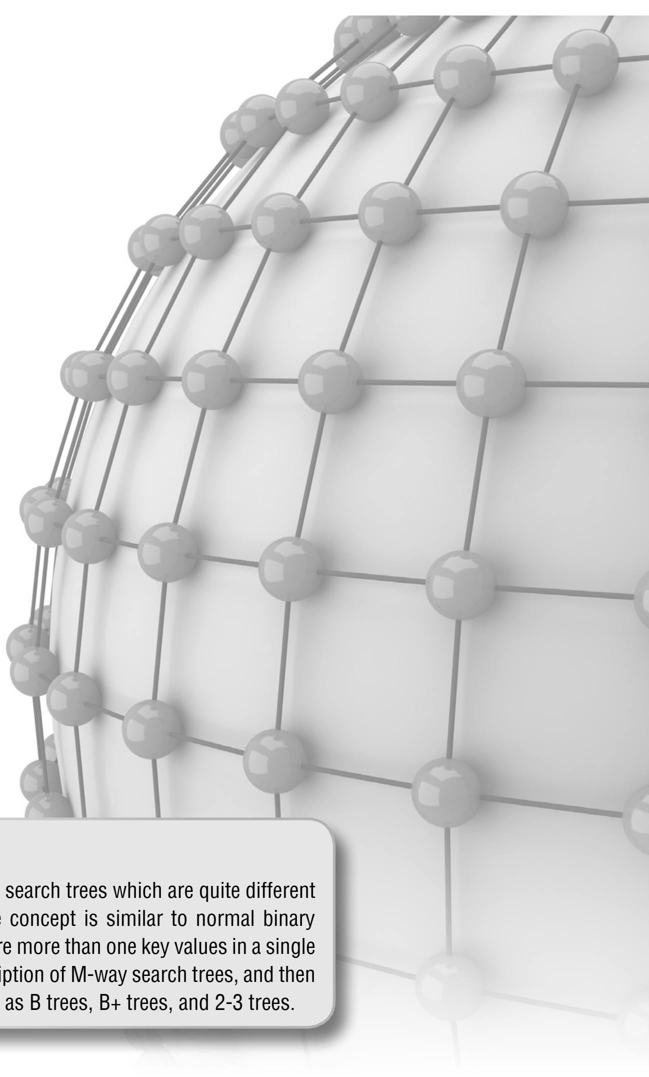
CHAPTER 11

Multi-way Search Trees



LEARNING OBJECTIVE

In this chapter we will study about multi-way search trees which are quite different from other binary search trees. Though the concept is similar to normal binary search trees, but M-way search trees can store more than one key values in a single node. The chapter starts with a general description of M-way search trees, and then discusses in detail M-way search trees such as B trees, B+ trees, and 2-3 trees.

11.1 INTRODUCTION

We have discussed that every node in a binary search tree contains one value and two pointers, *left* and *right*, which point to the node's left and right sub-trees, respectively. The structure of a binary search tree node is shown in Fig. 11.1.

The same concept is used in an *M-way* search tree which has M - 1 values per node and M sub-trees. In such a tree, M is called the degree of the tree. Note that in a binary search tree M = 2, so it has one value and two sub-trees. In other words, every internal node of an M-way search tree consists of pointers to M sub-trees and contains M – 1 keys, where M > 2.

Pointer to left sub-tree	Value or Key of the node	Pointer to right sub-tree

Figure 11.1 Structure of a binary search tree node

The structure of an M-way search tree node is shown in Fig. 11.2.

P_0	K_0	P_1	K_1	P_2	K_2	......	P_{n-1}	K_{n-1}	P_n

Figure 11.2 Structure of an M-way search tree node

In the structure shown, P_0, P_1, P_2, ..., P_n are pointers to the node's sub-trees and K_0, K_1, K_2, ..., K_{n-1} are the key values of the node. All the key values are stored in ascending order. That is, $K_i < K_{i+1}$ for $0 \leq i \leq n-2$.

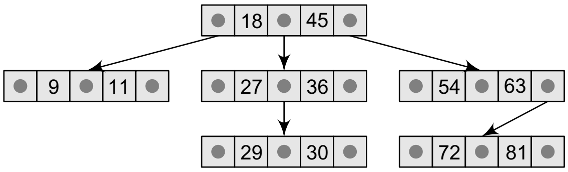


Figure 11.3 M-way search tree of order 3

In an M-way search tree, it is not compulsory that every node has exactly $M-1$ values and M sub-trees. Rather, the node can have anywhere from 1 to $M-1$ values, and the number of sub-trees can vary from 0 (for a leaf node) to $i + 1$, where i is the number of key values in the node. M is thus a fixed upper limit that defines how many key values can be stored in the node.

Consider the M-way search tree shown in Fig. 11.3. Here $M = 3$. So a node can store a maximum of two key values and can contain pointers to three sub-trees.

In our example, we have taken a very small value of M so that the concept becomes easier for the reader, but in practice, M is usually very large. Using a 3-way search tree, let us lay down some of the basic properties of an M-way search tree.

- Note that the key values in the sub-tree pointed by P_0 are less than the key value K_0. Similarly, all the key values in the sub-tree pointed by P_1 are less than K_1, so on and so forth. Thus, the generalized rule is that all the key values in the sub-tree pointed by P_i are less than K_i, where $0 \le i \le n-1$.
- Note that the key values in the sub-tree pointed by P_1 are greater than the key value K_0. Similarly, all the key values in the sub-tree pointed by P_2 are greater than K_1, so on and so forth. Thus, the generalized rule is that all the key values in the sub-tree pointed by P_i are greater than K_{i-1}, where $0 \le i \le n-1$.

In an M-way search tree, every sub-tree is also an M-way search tree and follows the same rules.

11.2 B TREES

A B tree is a specialized M-way tree developed by Rudolf Bayer and Ed McCreight in 1970 that is widely used for disk access. A B tree of order m can have a maximum of $m-1$ keys and m pointers to its sub-trees. A B tree may contain a large number of key values and pointers to sub-trees. Storing a large number of keys in a single node keeps the height of the tree relatively small.

A B tree is designed to store sorted data and allows search, insertion, and deletion operations to be performed in logarithmic amortized time. A B tree of order m (the maximum number of children that each node can have) is a tree with all the properties of an M-way search tree. In addition it has the following properties:

1. Every node in the B tree has at most (maximum) m children.
2. Every node in the B tree except the root node and leaf nodes has at least (minimum) $m/2$ children. This condition helps to keep the tree bushy so that the path from the root node to the leaf is very short, even in a tree that stores a lot of data.
3. The root node has at least two children if it is not a terminal (leaf) node.
4. All leaf nodes are at the same level.

An internal node in the B tree can have n number of children, where $0 \le n \le m$. It is not necessary that every node has the same number of children, but the only restriction is that the node should have at least $m/2$ children. As B tree of order 4 is given in Fig. 11.4.

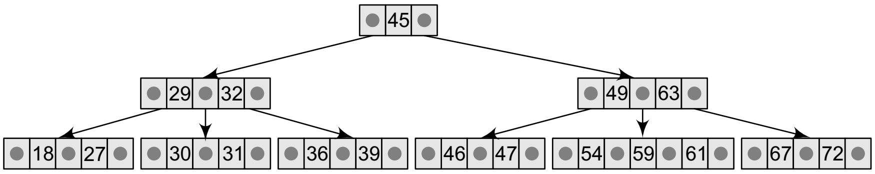


Figure 11.4 B tree of order 4

While performing insertion and deletion operations in a B tree, the number of child nodes may change. So, in order to maintain a minimum number of children, the internal nodes may be joined or split. We will discuss search, insertion, and deletion operations in this section.

11.2.1 Searching for an Element in a B Tree

Searching for an element in a B tree is similar to that in binary search trees. Consider the B tree given in Fig. 11.4. To search for 59, we begin at the root node. The root node has a value 45 which is less than 59. So, we traverse in the right sub-tree. The right sub-tree of the root node has two key values, 49 and 63. Since $49 \le 59 \le 63$, we traverse the right sub-tree of 49, that is, the left sub-tree of 63. This sub-tree has three values, 54, 59, and 61. On finding the value 59, the search is successful.

Take another example. If you want to search for 9, then we traverse the left sub-tree of the root node. The left sub-tree has two key values, 29 and 32. Again, we traverse the left sub-tree of 29. We find that it has two key values, 18 and 27. There is no left sub-tree of 18, hence the value 9 is not stored in the tree.

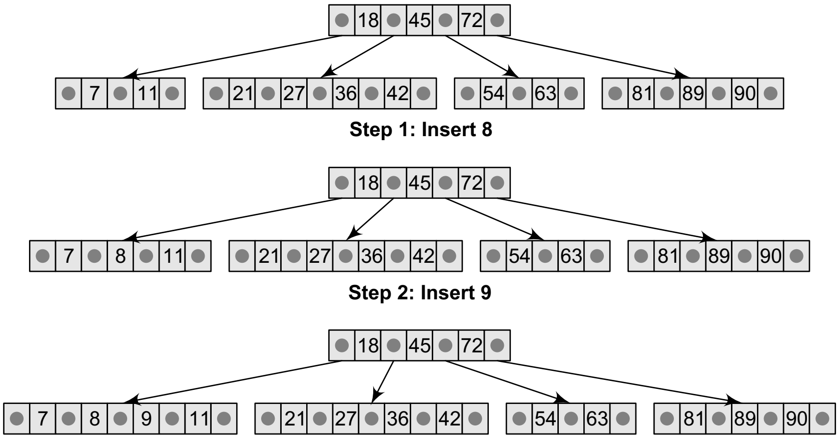
Since the running time of the search operation depends upon the height of the tree, the algorithm to search for an element in a B tree takes $O(\log_t n)$ time to execute.

11.2.2 Inserting a New Element in a B Tree

In a B tree, all insertions are done at the leaf node level. A new value is inserted in the B tree using the algorithm given below.

1. Search the B tree to find the leaf node where the new key value should be inserted.
2. If the leaf node is not full, that is, it contains less than `m-1` key values, then insert the new element in the node keeping the node's elements ordered.
3. If the leaf node is full, that is, the leaf node already contains `m-1` key values, then
   (a) insert the new value in order into the existing set of keys,
   (b) split the node at its median into two nodes (note that the split nodes are half full), and
   (c) push the median element up to its parent's node. If the parent's node is already full, then split the parent node by following the same steps.

Example 11.1 Look at the B tree of order 5 given below and insert 8, 9, 39, and 4 into it.



Step 1: Insert 8

Step 2: Insert 9

Figure 11.5(a)

Till now, we have easily inserted 8 and 9 in the tree because the leaf nodes were not full. But now, the node in which 39 should be inserted is already full as it contains four values. Here we split the nodes to form two separate nodes. But before splitting, arrange the key values in order (including the new value). The ordered set of values is given as 21, 27, 36, 39, and 42. The median value is 36, so push 36 into its parent's node and split the leaf nodes.

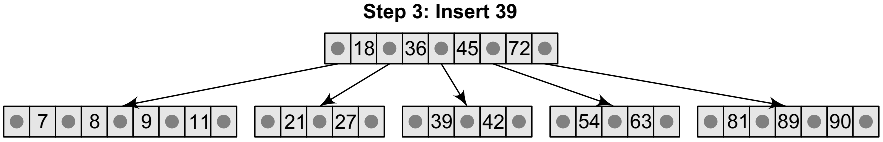


Figure 11.5(b)

Now the node in which 4 should be inserted is already full as it contains four key values. Here we split the nodes to form two separate nodes. But before splitting, we arrange the key values in order (including the new value). The ordered set of values is given as 4, 7, 8, 9, and 11. The median value is 8, so we push 8 into its parent's node and split the leaf nodes. But again, we see that the parent's node is already full, so we split the parent node using the same procedure.

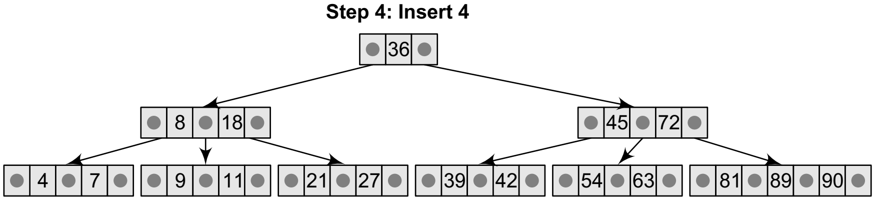


Figure 11.5(c) B tree

11.2.3 Deleting an Element from a B Tree

Like insertion, deletion is also done from the leaf nodes. There are two cases of deletion. In the first case, a leaf node has to be deleted. In the second case, an internal node has to be deleted. Let us first see the steps involved in deleting a leaf node.

1. Locate the leaf node which has to be deleted.
2. If the leaf node contains more than the minimum number of key values (more than `m/2` elements), then delete the value.
3. Else if the leaf node does not contain `m/2` elements, then fill the node by taking an element either from the left or from the right sibling.
   (a) If the left sibling has more than the minimum number of key values, push its largest key into its parent's node and pull down the intervening element from the parent node to the leaf node where the key is deleted.
   (b) Else, if the right sibling has more than the minimum number of key values, push its smallest key into its parent node and pull down the intervening element from the parent node to the leaf node where the key is deleted.
4. Else, if both left and right siblings contain only the minimum number of elements, then create a new leaf node by combining the two leaf nodes and the intervening element of the parent node (ensuring that the number of elements does not exceed the maximum number of elements a node can have, that is, `m`). If pulling the intervening element from the parent node leaves it with less than the minimum number of keys in the node, then propagate the process upwards, thereby reducing the height of the B tree.

To delete an internal node, promote the successor or predecessor of the key to be deleted to occupy the position of the deleted key. This predecessor or successor will always be in the leaf node. So the processing will be done as if a value from the leaf node has been deleted.

Example 11.2 Consider the following B tree of order 5 and delete values 93, 201, 180, and 72 from it (Fig. 11.6(a)).

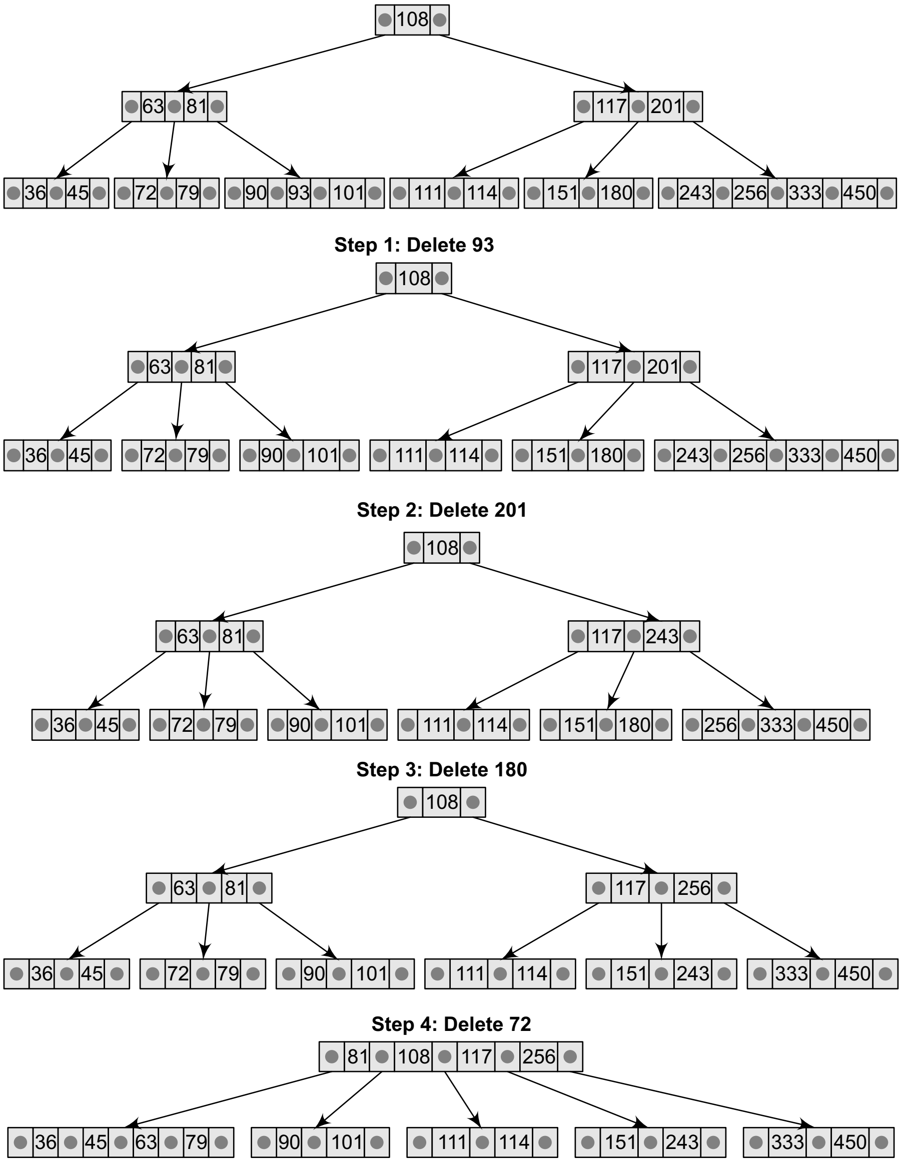


Figure 11.6 B tree

Example 11.3 Consider the B tree of order 3 given below and perform the following operations: (a) insert 121, 87 and then (b) delete 36, 109.

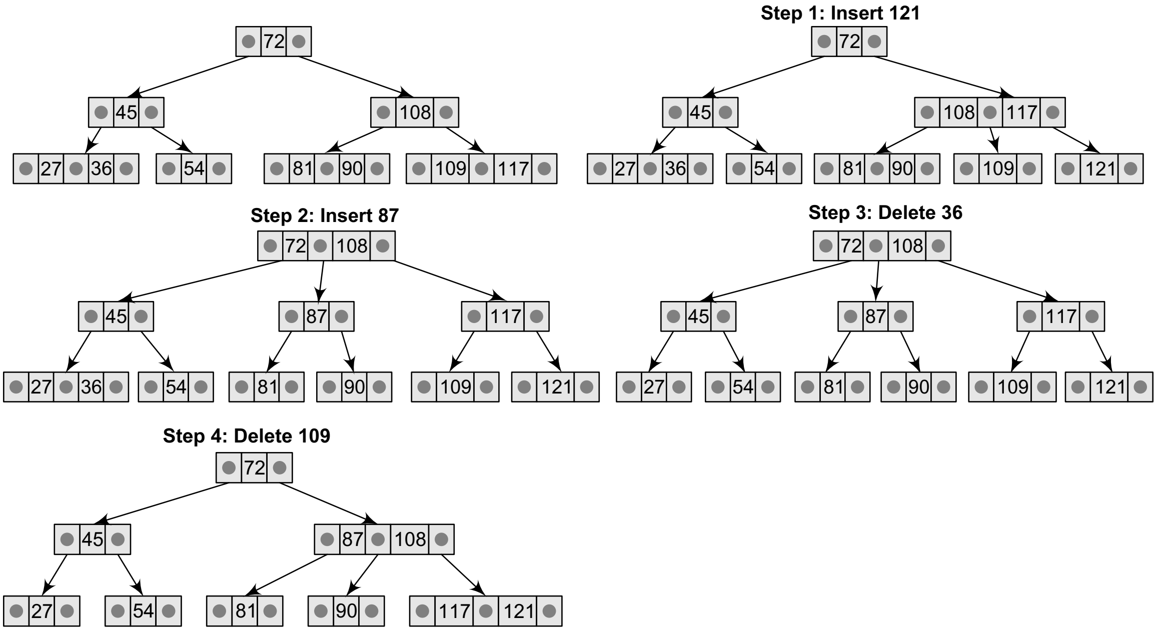
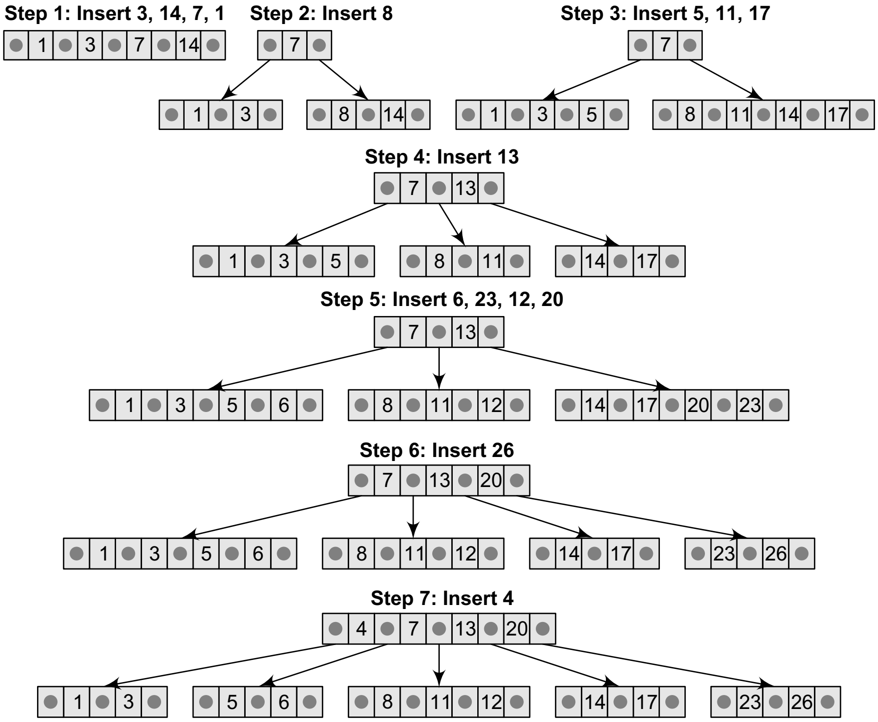
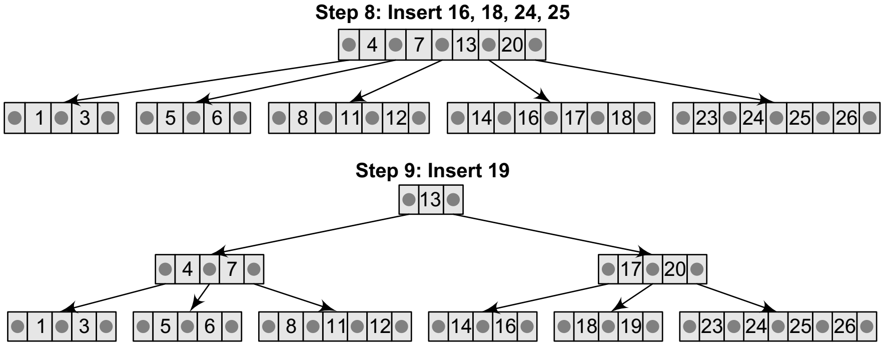


Figure 11.7 B tree

Example 11.4 Create a B tree of order 5 by inserting the following elements: 3, 14, 7, 1, 8, 5, 11, 17, 13, 6, 23, 12, 20, 26, 4, 16, 18, 24, 25, and 19.



(Contd)


Step 8: Insert 16, 18, 24, 25

Step 9: Insert 19

Figure 11.8 B tree

11.2.4 Applications of B Trees

A database is a collection of related data. The prime reason for using a database is that it stores organized data to facilitate its users to update, retrieve, and manage the data. The data stored in the database may include names, addresses, pictures, and numbers. For example, a teacher may wish to maintain a database of all the students that includes the names, roll numbers, date of birth, and marks obtained by every student.

Nowadays, databases are used in every industry to store hundreds of millions of records. In the real world, it is not uncommon for a database to store gigabytes and terabytes of data. For example, a telecommunication company maintains a customer billing database with more than 50 billion rows that contains terabytes of data.

We know that primary memory is very expensive and is capable of storing very little data as compared to secondary memory devices like magnetic disks. Also, RAM is volatile in nature and we cannot store all the data in primary memory. We have no other option but to store data on secondary storage devices. But accessing data from magnetic disks is 10,000 to 1,000,000 times slower than accessing it from the main memory. So, B trees are often used to index the data and provide fast access.

Consider a situation in which we have to search an un-indexed and unsorted database that contains `n` key values. The worst case running time to perform this operation would be `O(n)`. In contrast, if the data in the database is indexed with a B tree, the same search operation will run in `O(log n)`. For example, searching for a single key on a set of one million keys will at most require 1,000,000 comparisons. But if the same data is indexed with a B tree of order 10, then only 114 comparisons will be required in the worst case.

Hence, we see that indexing large amounts of data can provide significant boost to the performance of search operations.

When we use B trees or generalized M-way search trees, the value of `m` or the order of B trees is often very large. Typically, it varies from 128–512. This means that a single node in the tree can contain 127–511 keys and 128–512 pointers to child nodes.

We take a large value of `m` mainly because of three reasons:

1. Disk access is very slow. We should be able to fetch a large amount of data in one disk access.
2. Disk is a block-oriented device. That is, data is organized and retrieved in terms of blocks. So while using a B tree (generalized M-way search tree), a large value of `m` is used so that one single node of the tree can occupy the entire block. In other words, `m` represents the maximum number of data items that can be stored in a single block. `m` is maximized to speed up processing. More the data stored in a block, lesser the time needed to move it into the main memory.
3. A large value minimizes the height of the tree. So, search operation becomes really fast.

11.3 B+ TREES

A B+ tree is a variant of a B tree which stores sorted data in a way that allows for efficient insertion, retrieval, and removal of records, each of which is identified by a *key*. While a B tree can store both keys and records in its interior nodes, a B+ tree, in contrast, stores all the records at the leaf level of the tree; only keys are stored in the interior nodes.

The leaf nodes of a B+ tree are often linked to one another in a linked list. This has an added advantage of making the queries simpler and more efficient.

Typically, B+ trees are used to store large amounts of data that cannot be stored in the main memory. With B+ trees, the secondary storage (magnetic disk) is used to store the leaf nodes of trees and the internal nodes of trees are stored in the main memory.

B+ trees store data only in the leaf nodes. All other nodes (internal nodes) are called *index nodes* or *i-nodes* and store index values. This allows us to traverse the tree from the root down to the leaf node that stores the desired data item. Figure 11.9 shows a B+ tree of order 3.

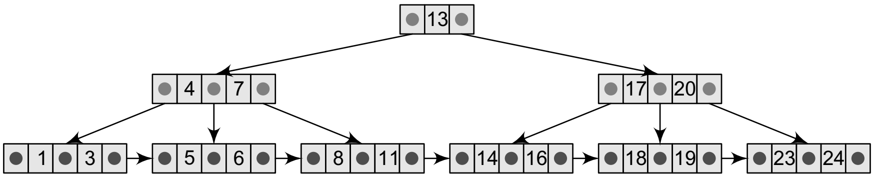


Figure 11.9 B+ tree of order 3

Many database systems are implemented using B+ tree structure because of its simplicity. Since all the data appear in the leaf nodes and are ordered, the tree is always balanced and makes searching for data efficient.

A B+ tree can be thought of as a multi-level index in which the leaves make up a dense index and the non-leaf nodes make up a sparse index. The advantages of B+ trees can be given as follows:

1. Records can be fetched in equal number of disk accesses
2. It can be used to perform a wide range of queries easily as leaves are linked to nodes at the upper level
3. Height of the tree is less and balanced
4. Supports both random and sequential access to records
5. Keys are used for indexing

Comparison Between B Trees and B+ Trees

Table 11.1 shows the comparison between B trees and B+ trees.

Table 11.1 Comparison between B trees and to B+ trees

B Tree	B+ Tree
1. Search keys are not repeated	1. Stores redundant search key
2. Data is stored in internal or leaf nodes	2. Data is stored only in leaf nodes
3. Searching takes more time as data may be found in a leaf or non-leaf node	3. Searching data is very easy as the data can be found in leaf nodes only
4. Deletion of non-leaf nodes is very complicated	4. Deletion is very simple because data will be in the leaf node
5. Leaf nodes cannot be stored using linked lists	5. Leaf node data are ordered using sequential linked lists
6. The structure and operations are complicated	6. The structure and operations are simple

11.3.1 Inserting a New Element in a B+ Tree

A new element is simply added in the leaf node if there is space for it. But if the data node in the tree where insertion has to be done is full, then that node is split into two nodes. This calls for adding a new index value in the parent index node so that future queries can arbitrate between the two new nodes.

However, adding the new index value in the parent node may cause it, in turn, to split. In fact, all the nodes on the path from a leaf to the root may split when a new value is added to a leaf node. If the root node splits, a new leaf node is created and the tree grows by one level.
The steps to insert a new node in a B+ Tree are summarized in Fig. 11.10.

Step 1: Insert the new node as the leaf node.
Step 2: If the leaf node overflows, split the node and copy the middle element to next index node.
Step 3: If the index node overflows, split that node and move the middle element to next index page.

Figure 11.10 Algorithm for inserting a new node in a B+ tree

Example 11.5 Consider the B+ tree of order 4 given and insert 33 in it.

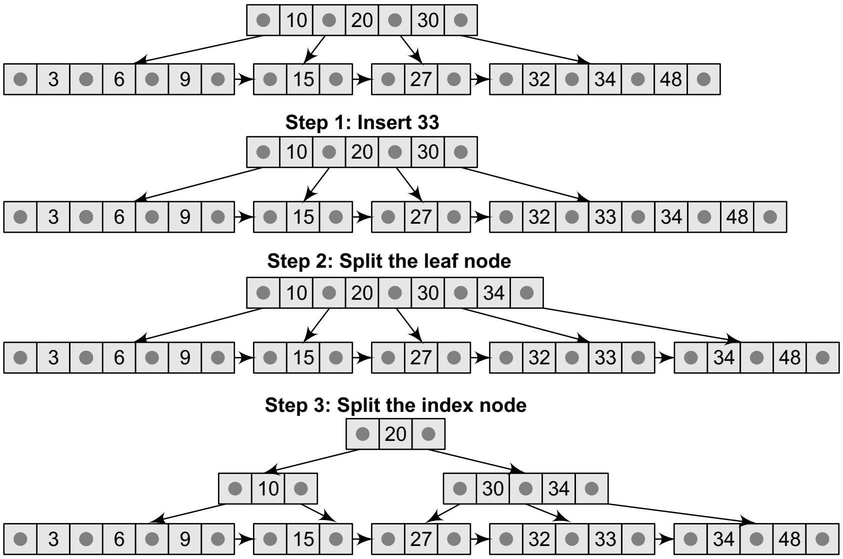


Figure 11.11 Inserting node 33 in the given B+ Tree

11.3.2 Deleting an Element from a B+ Tree

As in B trees, deletion is always done from a leaf node. If deleting a data element leaves that node empty, then the neighbouring nodes are examined and merged with the *underfull* node.

This process calls for the deletion of an index value from the parent index node which, in turn, may cause it to become empty. Similar to the insertion process, deletion may cause a merge-delete wave to run from a leaf node all the way up to the root. This leads to shrinking of the tree by one level. The steps to delete a node from a B+ tree are summarized in Fig. 11.12.

Step 1: Delete the key and data from the leaves.
Step 2: If the leaf node underflows, merge that node with the sibling and delete the key in between them.
Step 3: If the index node underflows, merge that node with the sibling and move down the key in between them.

Figure 11.12 Algorithm for deleting a node from a B+ Tree

Example 11.6 Consider the B+ tree of order 4 given below and delete node 15 from it.

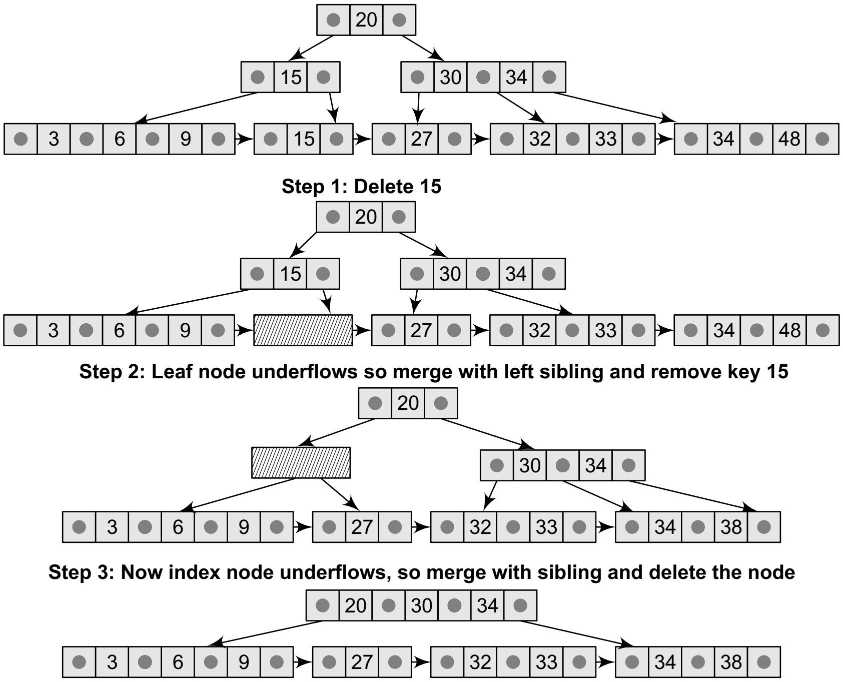

20
15
30 34
3 6 9
15
27
32 33
34 48
Step 1: Delete 15
20
15
30 34
3 6 9
27
32 33
34 48
Step 2: Leaf node underflows so merge with left sibling and remove key 15
20
30 34
3 6 9
27
32 33
34 38
Step 3: Now index node underflows, so merge with sibling and delete the node
20 30 34
3 6 9
27
32 33
34 38

Figure 11.13 Deleting node 15 from the given B+ Tree

Note Insertion and deletion operations are recursive in nature and can cascade up or down the B+tree, thereby affecting its shape dramatically.

11.4 2-3 TREES

In the last chapter, we have seen that for binary search trees the average-case time for operations like search/insert/delete is `O(log N)` and the worst-case time is `O(N)` where `N` is the number of nodes in the tree. However, a balanced tree that has height `O(log N)` always guarantees `O(log N)` time for all three methods. Typical examples of height balanced trees include AVL trees, red-black trees, B trees, and 2-3 trees. We have already discussed these data structures in the earlier chapter and section; now we will discuss 2-3 trees.

In a 2-3 tree, each interior node has either two or three children.

- Nodes with two children are called 2-nodes. The 2-nodes have one data value and two children
- Nodes with three children are called 3-nodes. The 3-nodes have two data values and three children (left child, middle child, and a right child)

This means that a 2-3 tree is not a binary tree. In this tree, all the leaf nodes are at the same level (bottom level). Look at Fig. 11.14 which shows a 2-3 tree.

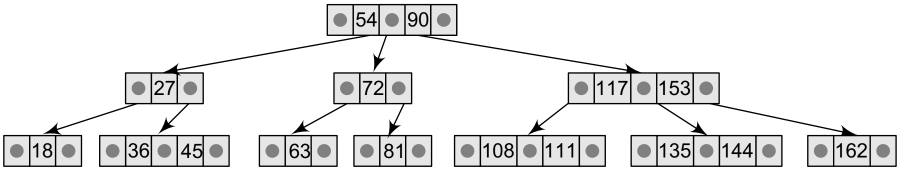


Figure 11.14 2-3 Tree

11.4.1 Searching for an Element in a 2-3 Tree

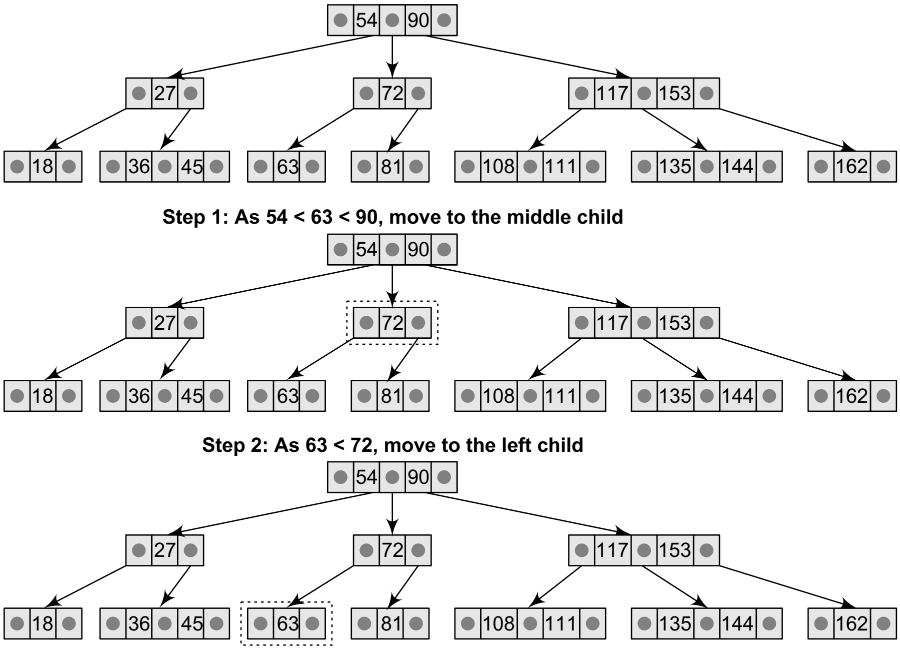
The search operation is used to determine whether a data value `x` is present in a 2-3 tree `T`. The process of searching a value in a 2-3 tree is very similar to searching a value in a binary search tree.

The search for a data value `x` starts at the root. If k_1 and k_2 are the two values stored in the root node, then

- if $x < k_1$, move to the left child.
- if $x \geq k_1$ and the node has only two children, move to the right child.
- if $x \geq k_1$ and the node has three children, then move to the middle child if $x < k_2$ else to the right child if $x \geq k_2$.

At the end of the process, the node with data value `x` is reached if and only if `x` is at this leaf.

Example 11.7 Consider the 2-3 tree in Fig. 11.14 and search 63 in the tree.



Step 1: As 54 < 63 < 90, move to the middle child

Step 2: As 63 < 72, move to the left child

Figure 11.15 Searching for element 63 in the 2-3 tree of Fig. 11.14

11.4.2 Inserting a New Element in a 2-3 Tree

To insert a new value in the 2-3 tree, an appropriate position of the value is located in one of the leaf nodes. If after insertion of the new value, the properties of the 2-3 tree do not get violated then insertion is over. Otherwise, if any property is violated then the violating node must be split (Fig. 11.16).

Splitting a node A node is split when it has three data values and four children. Here, `P` is the parent and `L`, `M`, `R` denote the left, middle, and right children.

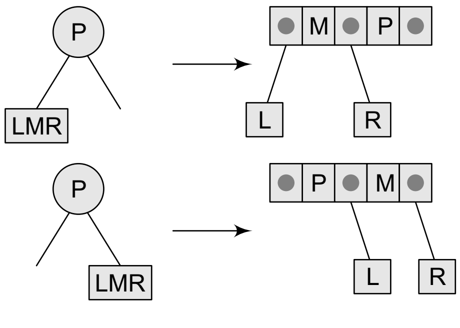


Figure 11.16(a)

Example 11.8 Consider the 2-3 tree given below and insert the following data values into it: 39, 37, 42, 47.

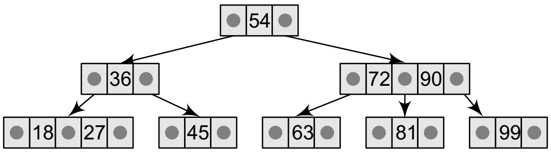


Figure 11.16(b)

Step 1: Insert 39 in the leaf node The tree after insertion can be given as

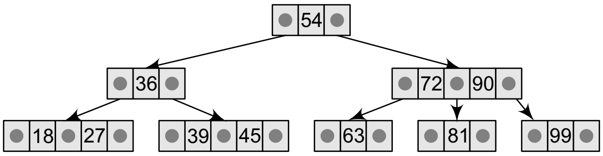


Figure 11.16(c)

Step 2: Insert 37 in the leaf node The tree after insertion can be given as below. Note that inserting 37 violates the property of 2-3 trees. Therefore, the node with values 37 and 39 must be split.

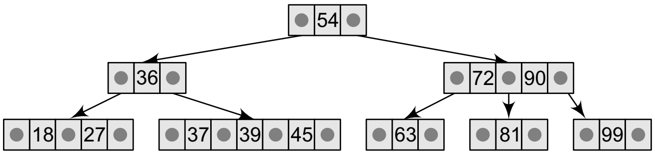


Figure 11.16(d)

After splitting the leaf node, the tree can be given as below.

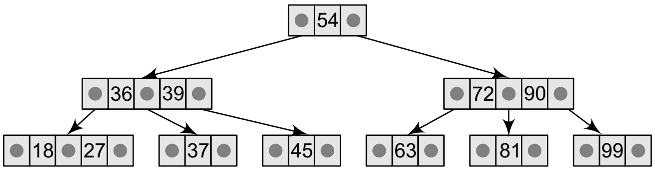


Figure 11.16(e)

Step 3: Insert 42 in the leaf node The tree after insertion can be given as follows.

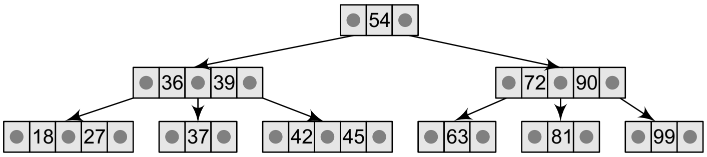


Figure 11.16(f)

Step 4: Insert 47 in the leaf node The tree after insertion can be given as follows.

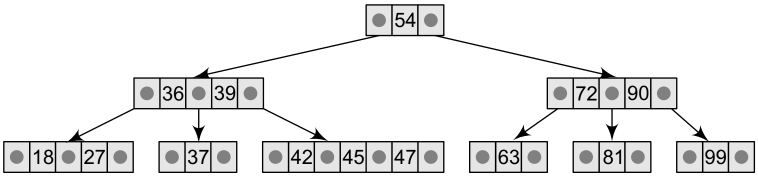


Figure 11.16(g)

The leaf node has three data values. Therefore, the node is violating the properties of the tree and must be split.

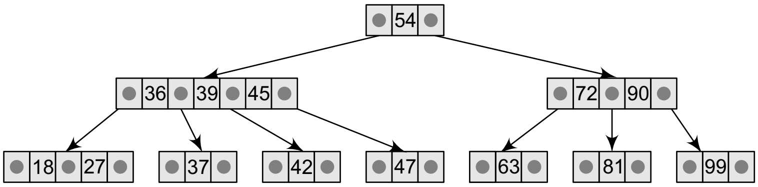


Figure 11.16(h)

The parent node has three data values. Therefore, the node is violating the properties of the tree and must be split.

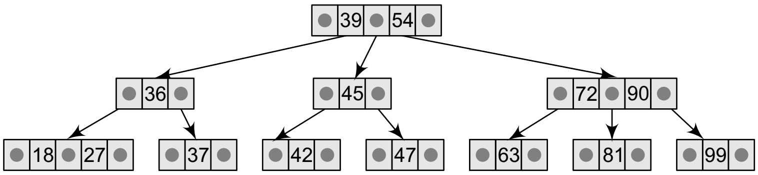


Figure 11.16(i) Inserting values in the given 2-3 Tree

11.4.3 Deleting an Element from a 2-3 Tree

In the deletion process, a specified data value is deleted from the 2-3 tree. If deleting a value from a node violates the property of a tree, that is, if a node is left with less than one data value then two nodes must be merged together to preserve the general properties of a 2-3 tree.

In insertion, the new value had to be added in any of the leaf nodes but in deletion it is not necessary that the value has to be deleted from a leaf node. The value can be deleted from any of the nodes. To delete a value `x`, it is replaced by its in-order successor and then removed. If a node becomes empty after deleting a value, it is then merged with another node to restore the property of the tree.

Example 11.9 Consider the 2-3 tree given below and delete the following values from it: 69, 72, 99, 81.

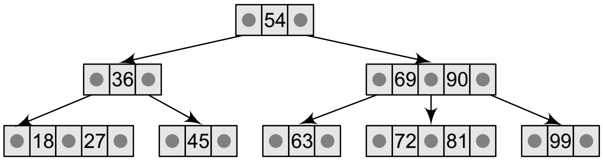


Figure 11.17(a)

To delete 69, swap it with its in-order successor, that is, 72. 69 now comes in the leaf node. Remove the value 69 from the leaf node.

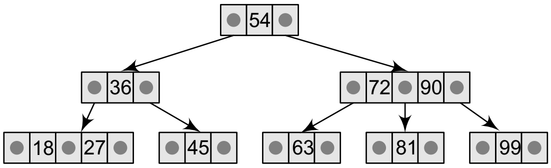


Figure 11.17(b)

72 is an internal node. To delete this value swap 72 with its in-order successor 81 so that 72 now becomes a leaf node. Remove the value 72 from the leaf node.

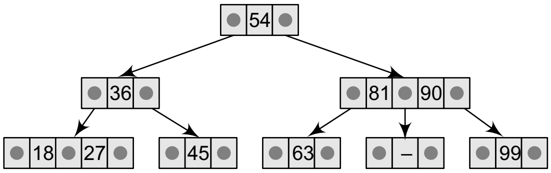


Figure 11.17(c)

Now there is a leaf node that has less than 1 data value thereby violating the property of a 2-3 tree. So the node must be merged. To merge the node, pull down the lowest data value in the parent's node and merge it with its left sibling.

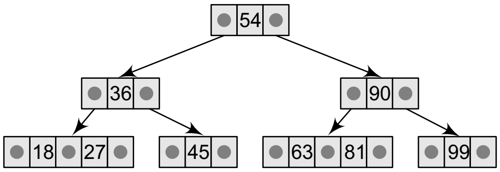


Figure 11.17(d)

99 is present in a leaf node, so the data value can be easily removed.

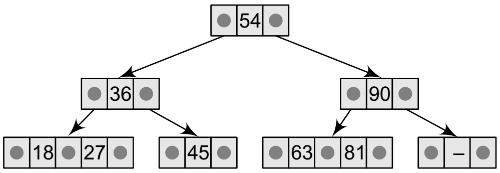


Figure 11.17(e)

Now there is a leaf node that has less than 1 data value, thereby violating the property of a 2-3 tree. So the node must be merged. To merge the node, pull down the lowest data value in the parent's node and merge it with its left sibling.

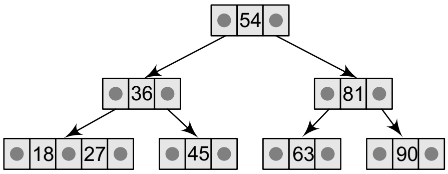


Figure 11.17(f)

81 is an internal node. To delete this value swap 81 with its in-order successor 90 so that 81 now becomes a leaf node. Remove the value 81 from the leaf node.

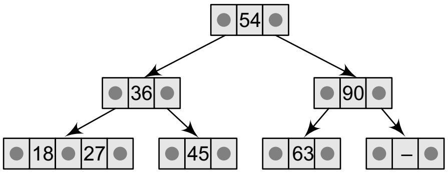


Figure 11.17(g)

Now there is a leaf node that has less than 1 data value, thereby violating the property of a 2-3 tree. So the node must be merged. To merge the node, pull down the lowest data value in the parent's node and merge it with its left sibling.

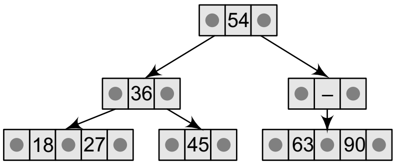


Figure 11.17(h)

An internal node cannot be empty, so now pull down the lowest data value from the parent's node and merge the empty node with its left sibling.

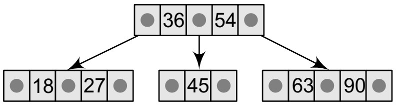


Figure 11.17(i) Deleting values from the given 2-3 tree

11.5 TRIE

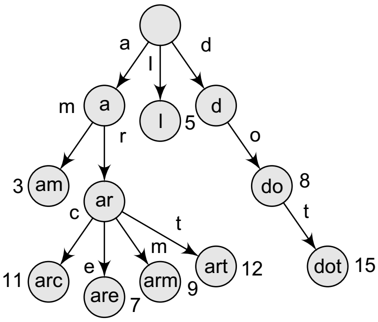


Figure 11.18 Trie data structure

The term *trie* has been taken from the word 'retrieval'. A trie is an ordered tree data structure, which was introduced in the 1960s by Edward Fredkin. Trie stores keys that are usually strings. It is basically a *k-ary* position tree.

In contrast to binary search trees, nodes in a trie do not store the keys associated with them. Rather, a node's position in the tree represents the key associated with that node. All the descendants of a node have a common prefix of the string associated with that node, and the root is associated with the empty string. Figure 11.18 shows a trie.

In the given tree, keys are listed in the nodes and the values below them. Note that each complete English word is assigned an arbitrary integer value. We can find each of these words by traversing the various branches in the tree until a leaf node is encountered. Any path from the root to a leaf represents a word.

Advantages Relative to Binary Search Tree

When compared with a binary search tree, the trie data structure has the following advantages.

Faster search Searching for keys is faster, as searching a key of length `m` takes `O(m)` time in the worst case. On the other hand, a binary search tree performs `O(logn)` comparisons of keys, where `n` is the number of nodes in the tree. Since search time depends on the height of the tree which is logarithmic in the number of keys (if the tree is balanced), the worst case may take `O(m log n)` time. In addition to this, `m` approaches `log(n)` in the worst case. Hence, a trie data structure provides a faster search mechanism.

Less space Trie occupies less space, especially when it contains a large number of short strings. Since keys are not stored explicitly and nodes are shared between the keys with common initial subsequences, a trie calls for less space as compared to a binary search tree.

Longest prefix-matching Trie facilitates the longest-prefix matching which enables us to find the key sharing the longest possible prefix of all unique characters. Since trie provides more advantages, it can be thought of as a good replacement for binary search trees.

Advantages Relative to Hash Table

Trie can also be used to replace a *hash table* as it provides the following advantages:

- Searching for data in a trie is faster in the worst case, `O(m)` time, compared to an imperfect hash table, discussed in Chapter 15, which may have numerous key collisions. Trie is free from collision of keys problem.
- Unlike a hash table, there is no need to choose a hash function or to change it when more keys are added to a trie.
- A trie can sort the keys using a predetermined alphabetical ordering.

Disadvantages

The disadvantages of having a trie are listed below:

- In some cases, tries can be slower than hash tables while searching data. This is true in cases when the data is directly accessed on a hard disk drive or some other secondary storage device that has high random access time as compared to the main memory.

- All the key values cannot be easily represented as strings. For example, the same floating point number can be represented as a string in multiple ways (1 is equivalent to 1.0, 1.00, +1.0, etc.).

Applications

Tries are commonly used to store a dictionary (for example, on a mobile telephone). These applications take advantage of a trie's ability to quickly search, insert, and delete the entries. Tries are also used to implement approximate matching algorithms, including those used in spell-checking software.

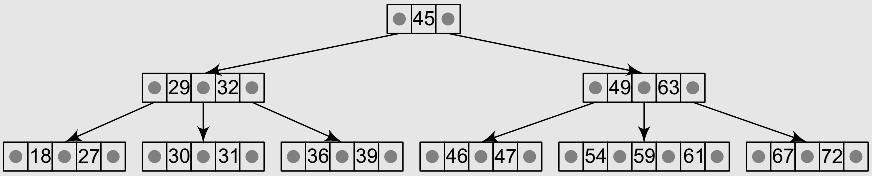
Points to Remember

- An M-way search tree has `M – 1` values per node and `M` sub-trees. In such a tree, `M` is called the degree of the tree. M-way search tree consists of pointers to `M` sub-trees and contains `M – 1` keys, where `M > 2`.
- A B tree of order `m` can have a maximum of `m–1` keys and `m` pointers to its sub-trees. A B tree may contain a large number of key values and pointers to its sub-trees.
- A B+ tree is a variant of B tree which stores sorted data in a way that allows for efficient insertion, retrieval, and removal of records, each of which is identified by a key. B+ tree record data at the leaf level of the tree; only keys are stored in interior nodes.
- A trie is an ordered tree data structure which stores keys that are usually strings. It is basically a k-ary position tree.
- In contrast to binary search trees, nodes in a trie do not store the keys associated with them. Rather, a node's position in the tree represents the key associated with that node.
- In a 2-3 tree, each interior node has either two or three children. This means that a 2-3 tree is not a binary tree.

Exercises

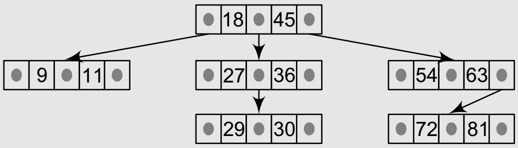
Review Questions

1. Why is a large value of `m` needed in a B tree?
2. Compare B trees with B+ trees.
3. In what conditions will you prefer a B+ tree over a B tree?
4. Write a short note on trie data structure.
5. Compare binary search trees with trie. Also, list the merits and demerits of using the trie data structure.
6. Compare hash tables with trie.
7. Give a brief summary of M-way search trees.
8. Consider the B tree given below
   (a) Insert 1, 5, 7, 11, 13, 15, 17, and 19 in the tree.
   (b) Delete 30, 59, and 67 from the tree.
9. Write an essay on B+ trees.
10. Create a B+ tree of order 5 for the following data arriving in sequence:
    90, 27, 7, 9, 18, 21, 3, 4, 16, 11, 21, 72



(B tree for Q.8)

11. List down the applications of B trees.
12. B trees of order 2 are full binary trees. Justify this statement.
13. Consider the 3-way search tree given below. Insert 23, 45, 67, 87, 54, 32, and 11 in the tree. Then, delete 9, 36, and 54 from it.



(3-way search tree for Q.13)

Multiple-choice Questions

1. Every internal node of an M-way search tree consists of pointers to M sub-trees and contains how many keys?
   (a) M (b) M–1
   (c) 2 (d) M+1
2. Every node in a B tree has at most ______ children.
   (a) M (b) M–1
   (c) 2 (d) M+1
3. Which data structure is commonly used to store a dictionary?
   (a) Binary Tree (b) Splay tree
   (c) Trie (d) Red black tree
4. In M-way search tree, M stands for
   (a) Internal nodes (b) External nodes
   (c) Degree of node (d) Leaf nodes
5. In best case, searching a value in a binary search tree may take
   (a) O(n) (b) O(n log n)
   (c) O(log n) (d) O(n^2)

True or False

1. All leaf nodes in the B tree are at the same level.
2. A B+ tree stores data only in the i-nodes.
3. B tree stores unsorted data.
4. Every node in the B-tree has at most (maximum) m–1 children.
5. The leaf nodes of a B tree are often linked to one another.
6. B+ tree stores redundant search key.
7. A trie is an ordered tree data structure.
8. A trie uses more space as compared to a binary search tree.
9. External nodes are called index nodes.

Fill in the Blanks

1. An M-way search tree consists of pointers to ______ sub-trees and contains ______ keys.
2. A B-tree of order _______ can have a maximum of _______ keys and m pointers to its sub-trees.
3. Every node in the B-tree except the root node and leaf nodes have at least _______ children.
4. In _______ data is stored in internal or leaf nodes.
5. A balanced tree that has height O(log N) always guarantees _______ time for all three methods.