

Northeastern University Khoury College of Computer Sciences
CS 5008 Algorithms Final Project
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A High-Speed Railway Across the United States

Omar Rashwan | Annie Pates (Elizabeth) | Josephine Yu

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Project Introduction

Clearly defined question and relevance to our team

Initial Inspiration

With fuel prices increasing astronomically in the past few months, travel has become more cumbersome.

Each of our group members has been affected by this issue personally lately, which prompted us the brainstorm.

This led to a discussion whether the United States had a transcontinental, high-speed railroad that facilitates an easier and relatively economical travel.

We began brainstorming the best methodology to apply algorithms we have learnt in this class and potential roadblocks that we may face through our theoretical railway building journey.

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Question to be Answered:

What is the most efficient way
to build a high-speed railway
to connect the most populous cities
in the continental United States?

Relevance to Each Team Member

Omar Rashwan

Ever since watching Thomas the Tank Engine as a young boy, I have been fascinated with rail transit as a means of transportation. This interest in trains has only grown as I learned more about the efficiency of rail versus other means of transport such as road and air travel ...

Annie Pates (Elizabeth)

This year, I have weddings in Massachusetts, Rhode Island, Ohio, Minnesota, and Texas as well as trips to Wyoming and Florida. Additionally, my friends who are getting married in the bride's hometown in Texas are committed to having their dog participate in the ceremony ...

Josephine Yu

For upcoming travels in Canada, I have been canceling flights and opting for travel via train instead. In Canada, the transcontinental train goes from the west coast in Vancouver to the east coast in Halifax, it is also heavily subsidized by the government...

Relevance to Omar:

Omar Rashwan

"Ever since watching Thomas the Tank Engine as a young boy, I have been fascinated with rail transit as a means of transportation. This interest in trains has only grown as I learned more about the efficiency of rail versus other means of transport such as road and air travel. I also grew up in a time where China's rail network grew extensively in both size and speed of transport. Today, China boasts the world's most advanced high-speed rail (HSR) network, accounting for two thirds of the world's high-speed rail. When one considers the similarities between the United States and China in terms of geographic size (3,796,742 sq mi vs. 3,705,407 sq mi) and gross domestic product (\$25 trillion vs \$20 trillion), it seems strange that China has been able to implement a new rail network so effectively while the United States struggles to maintain the infrastructure it built in the past. This is often attributed to a lack of political will to build a rail network on a national scale, ostensibly due to the car-centered perspective most Americans have.

Despite this, there are multiple incentives to build a high-speed rail network in the United States. The most talked about reason is also a looming existential threat; global warming. In terms of emissions per passenger, air travel produces the most emissions with 244gCO₂/km. Road travel is just less than half of that with emissions of 102gCO₂/km. However, rail travel is easily the cleanest mode of transportation, with only 28gCO₂ per kilometer traveled. In the United States, there are roughly 3,325 domestic flights each day, which comes out to about 1.2 million domestic flights a year. The emissions saved by switching even half of those domestic flights to rail travel would be substantial to say the least.

High-speed rail could also promote greater interstate commerce and travel. For example, a day trip to New York City from Boston would be infeasible for most people due to it being either a 4 hour drive one-way or a \$140 plane ticket. However, if a high-speed rail line were to connect NYC and Boston, the trip would only take about an hour and a half. This would be beneficial for tourism and trade for both cities, and there are many other similar corridors all over the country. Seeing how some of the algorithms we've learned about could be used to develop a high-speed rail network that may hopefully become a reality someday would be very exciting."

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Relevance to Annie:

Annie Pates (Elizabeth)

This year, I have weddings in Massachusetts, Rhode Island, Ohio, Minnesota, and Texas as well as trips to Wyoming and Florida. I love train travel for the easy boarding and uninterrupted work time it offers compared to flying, and would certainly appreciate a cheaper transportation option for some of these destinations, especially one that operates at much higher speeds than current trains. The journey from Wyoming to Minnesota especially has the potential to offer beautiful scenery in addition to the other benefits of train travel. Since the airport in Jackson Hole is quite small, it would also be a useful leg to have connected to a high speed rail system.

Additionally, my friends who are getting married in the bride's hometown in Texas are committed to having their dog participate in the ceremony. They are torn between forcing their precious puppy to fly with the luggage on a plane or beginning their marriage on a 3-day cross-country drive with a carsick dog. While we are unlikely to solve the problem before their October nuptials, it could save future dog-owners from facing a similar quandary!

As high speed rail does seem poised to take off in places like California and Florida, I am curious how the maps that wind up being created will differ from the one we create.

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Relevance to Josephine:

Josephine Yu

For upcoming travels in Canada, I have been canceling flights and opting for travel via train instead. Wait times at Canadian airports have been making headlines recently. An hour flight from New York was delayed four hours, then two hours sitting on the plane, waiting to take off with no air conditioning. In Canada, the transcontinental train goes from the west coast in Vancouver to the east coast in Halifax, it is also heavily subsidized by the government making it a viable option to travel. The Canadian government has also been working on laying tracks to operate a high speed frequency rail line train that cuts down a 6 hour car ride between Toronto and Montréal to 40 minutes. Future extensions will include connections to Québec City, Montréal, Ottawa and Toronto. If this form of transportation was available when I was living in Toronto years ago, it would have made traveling to visit family in Montréal much more convenient. Due to the travel time, I was only able to see family by taking advantage of a long weekend. Furthermore, train travel is much more baggage friendly. It requires less personnel to handle bags, minimizing instances of further delays as well as lost baggage.

I am quite interested to see how and whether it is feasible to build a transcontinental railroad that connects to all the major cities in the United States. As someone that did not grow up in the US, a nice perk is also learning more of the geography quicker than any class I could have taken. Mapping out the cities will be fun and I am looking forward to discussing the different ways of applying the algorithms with my teammates in a group setting for the first time in this program. Perhaps our theoretical railway roadmap may even come to fruition in the future.

02

Problem Analysis

How we framed the problem and how we will solve it

Problem Analysis: Assumptions

Parameters given the short time frame

- **Only examining the most populous city in each of the contiguous US states**
 - ie. not considering Hawaii, Alaska at this time
- **Caveat: paths in MST may not be the most effective way to get from one city to another**
 - Hope to mitigate this by using a minimum weight spanning tree of all 48 cities with Prim's algorithm
 - Applying Floyd-Warshall
- **Ignoring terrain complexities that come with going around/over/through geographical features**
 - ie. Lakes and mountains
- **Not factoring cost budget in the implementation which could have skewed the paths.**

03

Solution Approach

Prim's Algorithm, Minimum-Spanning Tree, Floyd-Warshall Algorithm

Methods Used to Gather data

- Wikipedia dataset on the top 330 most populous cities in the United States including:
 - 2020 census information
 - Land area
 - Population density
 - Location longitudes and latitudes

2021 rank	City	State ^[c]	2021 estimate	2020 census	Change	2020 land area	2020 population density	Location
1	New York ^[d]	New York	8,467,513	8,804,190	-3.82%	300.5 sq mi	778.3 km ²	29,298/sq mi
2	Los Angeles	California	3,849,297	3,898,747	-1.27%	469.5 sq mi	1,216.0 km ²	8,304/sq mi
3	Chicago	Illinois	2,696,555	2,746,388	-1.81%	227.7 sq mi	589.7 km ²	12,061/sq mi
4	Houston	Texas	2,288,250	2,304,580	-0.71%	640.4 sq mi	1,658.6 km ²	3,599/sq mi
5	Phoenix	Arizona	1,624,569	1,608,139	+1.02%	518.0 sq mi	1,341.6 km ²	3,105/sq mi
6	Philadelphia ^[e]	Pennsylvania	1,576,251	1,603,797	-1.72%	134.4 sq mi	348.1 km ²	11,933/sq mi
7	San Antonio	Texas	1,451,853	1,434,625	+1.20%	498.8 sq mi	1,291.9 km ²	2,876/sq mi
8	San Diego	California	1,381,611	1,386,932	-0.38%	325.9 sq mi	844.1 km ²	4,256/sq mi
9	Dallas	Texas	1,288,457	1,304,379	-1.22%	339.6 sq mi	879.6 km ²	3,841/sq mi
10	San Jose	California	983,489	1,013,240	-2.94%	178.3 sq mi	461.8 km ²	5,683/sq mi
11	Austin	Texas	964,177	961,855	+0.24%	319.9 sq mi	828.5 km ²	3,007/sq mi
12	Jacksonville ^[f]	Florida	954,614	949,611	+0.53%	747.3 sq mi	1,935.5 km ²	1,271/sq mi
13	Fort Worth	Texas	935,508	918,915	+1.81%	342.9 sq mi	888.1 km ²	2,646/sq mi

https://en.wikipedia.org/wiki/List_of_United_States_cities_by_population

- Kaggle dataset in .csv format

	A	B	C	D	E	F	G
1	City	State	Type	Counties	Population	Latitude	Longitude
2	New York	NY	City	Bronx;Richmond;New York;Kings;Queens	8804190	40.714	-74.007
3	Los Angeles	CA	City	Los Angeles	3898747	34.052	-118.243
4	Chicago	IL	City	Cook;DuPage	2746388	41.882	-87.628
5	Houston	TX	City	Harris;Fort Bend;Montgomery	2304580	29.76	-95.363
6	Phoenix	AZ	City	Maricopa	1608139	33.448	-112.074
7	Philadelphia	PA	City	Philadelphia	1603797	39.952	-75.164
8	San Antonio	TX	City	Bexar	1434625	29.423	-98.49
9	San Diego	CA	City	San Diego	1386932	32.716	-117.165
10	Dallas	TX	City	Rockwall;Denton;Kaufman;Dallas;Collin	1304379	32.781	-96.797
11	San Jose	CA	City	Santa Clara	1013240	37.336	-121.891
12	Austin	TX	City	Williamson;Travis;Hays	961855	30.268	-97.743
13	Jacksonville	FL	City	Duval	949611	30.328	-81.657
14	Fort Worth	TX	City	Wise;Denton;Parker;Tarrant	918915	32.754	-97.331
15	Columbus	OH	City	Delaware;Franklin;Fairfield	905748	39.962	-83.001
16	Indianapolis	IN	City	Marion	887642	39.769	-86.158
17	Charlotte	NC	City	Mecklenburg	874579	35.227	-80.843
18	San Francisco	CA	City	San Francisco	873965	37.78	-122.414
19	Seattle	WA	City	King	737015	47.607	-122.338
20	Denver	CO	City	Denver	715522	39.739	-104.987
21	Washington	DC	City	District of Columbia	689545	38.895	-77.036
22	Nashville	TN	Metro Government	Davidson	689447	36.164	-86.783

<https://www.kaggle.com/datasets/axeltorbenson/us-cities-by-population-top-330>

File Reader in Python

- Coded a file reader in Python to iterate through the data of the 330 cities
- Sorted the data by population in decreasing order (*Module 0 - Sorting*)
- Appended data into a nested array initialized as “top48”
 - sorted, most populous 48 cities used to create the graph in an adjacency matrix (*Module 5- Adjacency Matrices and List*)
- Results were outputted to a file, “top48.py”

```

1 City,State,Type,COUNTIES,Population,Latitude,Longitude
2 New York,NY,City,Bronx;Richmond;New York;Kings;Queens,8804190,40.714,-74.007
3 Los Angeles,CA,City,Los Angeles,3898747,34.052,-118.243
4 Chicago,IL,City,Cook;DuPage,2746388,41.882,-87.628
5 Houston,TX,City,Harris;Fort Bend;Montgomery,2304580,29.76,-95.363
6 Phoenix,AZ,City,Maricopa,1608139,33.448,-112.074
7 Philadelphia,PA,City,Philadelphia,1603797,39.952,-75.164
8 San Antonio,TX,City,Bexar,1434625,29.423,-98.49
9 San Diego,CA,City,San Diego,1386932,32.716,-117.165
10 Dallas,TX,City,Rockwall;Denton;Kaufman;Dallas;Collin,1304379,32.781,-96.797
11 San Jose,CA,City,Santa Clara,1013240,37.336,-121.891
12 Austin,TX,City,Williamson;Travis;Hays,961855,30.268,-97.743
13 Jacksonville,FL,City,Duval,949611,30.328,-81.657
14 Fort Worth,TX,City,Wise;Denton;Parker;Tarrant,918915,32.754,-97.331
15 Columbus,OH,City,Delaware;Franklin;Fairfield,905748,39.962,-83.001
16 Indianapolis,IN,City,Marion,887642,39.769,-86.158
17 Charlotte,NC,City,Mecklenburg,874579,35.227,-80.843
18 San Francisco,CA,City,San Francisco,873965,37.78,-122.414
19 Seattle,WA,City,King,737015,47.607,-122.338
20 Denver,CO,City,Denver,715522,39.739,-104.987
21 Washington,DC,City,District of Columbia,689545,38.895,-77.036
22 Nashville,TN,Metro Government,Davidson,689447,36.164,-86.783
23 Oklahoma City,OK,City,Canadian;Cleveland;Oklahoma,681054,35.468,-97.516
24 El Paso,TX,City,El Paso,678815,31.76,-106.488

```

>>>

```

16 def main():
17     filename = "us2021census.csv"
18     with open(filename, mode='r') as f:
19         header = f.readline()
20         data = f.readlines()
21
22     states = {"AK", "HI", "DC"}
23     top48 = []
24     for i in range(len(data)):
25         data[i] = data[i].strip('\n')
26         data[i] = data[i].split(',')
27         data[i][4] = int(data[i][4])
28         data[i][5], data[i][6] = float(data[i][5]), float(data[i][6])
29         lst = [data[i][0], data[i][5], data[i][6]]
30         if data[i][1] not in states:
31             states.add(data[i][1])
32             top48.append(lst)
33
34     n = len(top48)
35     adj_matrix = [[0 for x in range(n)] for y in range(n)]
36
37     for i in range(n):
38         for j in range(i, n):
39             city1 = top48[i][1], top48[i][2]
40             city2 = top48[j][1], top48[j][2]
41             distance = dist(city1, city2)
42             adj_matrix[i][j] = distance
43             adj_matrix[j][i] = distance
44
45     output = "top48.py"
46
47     with open(output, mode='w') as o:
48         o.write("top48 = " + str(top48) + '\n')
49         o.write("g = " + str(adj_matrix))
50
51
52
53
54
55
56

```

.CSV file

csv_reader.py

>>>

```

1 top48 = [['New York', 40.714, -74.007], ['Los Angeles', 34.052, -118.243], ['Chicago', 41.882, -87.628], ['
2 g = [[0.0, 3086.703983270828, 943.298051129652, 1656.0989717924465, 2674.0426730785352, 95.59159990815074, 8

```

top48.py

Creating the Graph

Began with a graph structure:

- Vertices = cities
- Edges = railway path
- Weighted edge = distance between two vertices

Building our minimum spanning tree:

- Starting vertex = most populous city from our “top48” array
- Applied Prim’s algorithm (*Module 7 Greedy Algorithms 2*)
- Calculated the lowest weighted edge at each step
- Runtime complexity = $O((V+E) \log(V))$

```

7  def prim(graph):
8      n = len(graph)
9      included = {0}
10     mst = [[0 for x in range(n)] for y in range(n)]
11
12     while len(included) < n:
13         minimum = 999999999
14         x, y = 0, 0
15         for u in included:
16             for v in range(n):
17                 if v not in included and graph[u][v] < minimum:
18                     minimum = graph[u][v]
19                     x, y = u, v
20         included.add(y)
21         mst[x][y] = minimum
22         mst[y][x] = minimum
23
24     return mst
25
26
27 def main():
28
29     mst = prim(g)
30
31     data = []
32
33     for i in range(len(top48)):
34         data.append((top48[i][0], top48[i][1], top48[i][2]))
35
36
37     visualize(mst, data)

```

mst_build.py

Graph Visualization

Utilized Python library packages: NetworkX, Matplotlib and Basemap

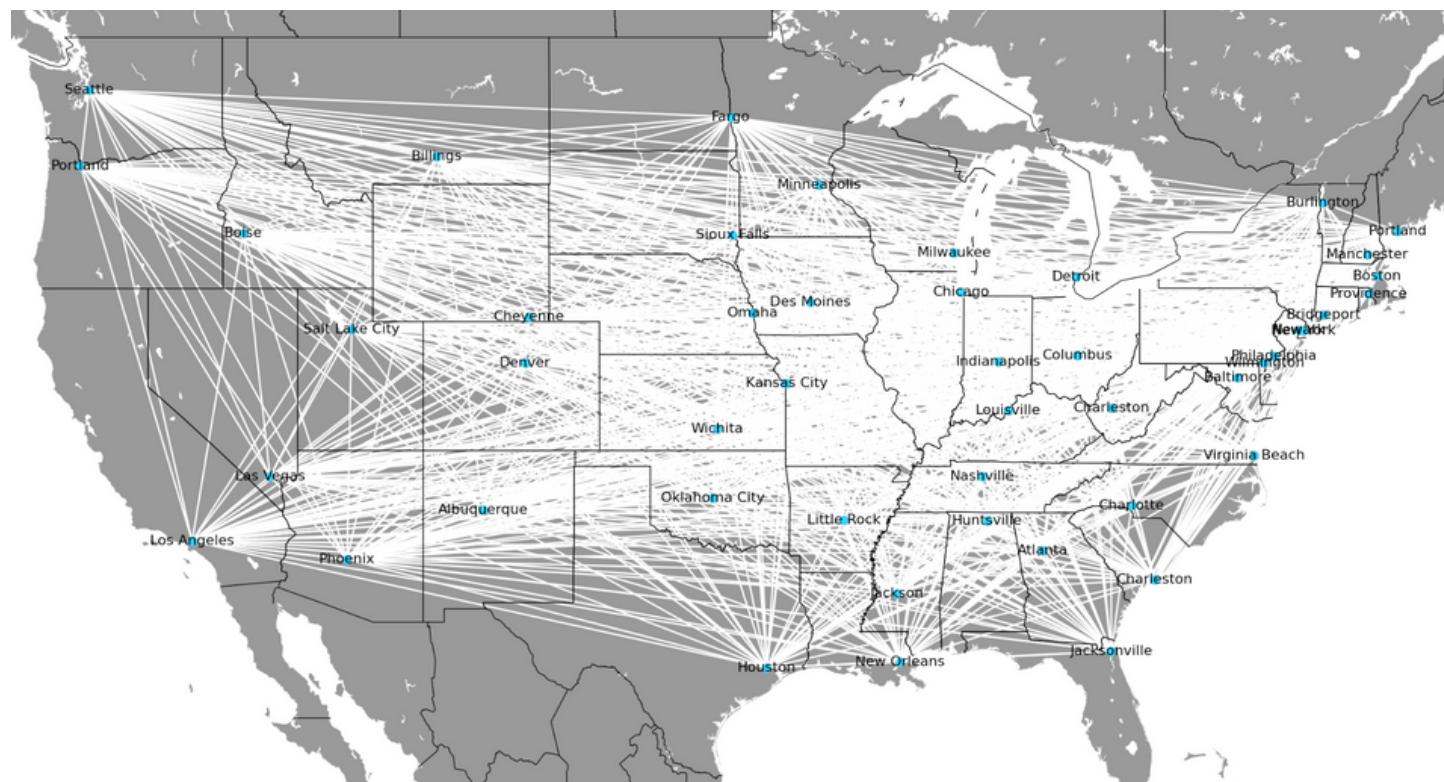
- Graphed our vertices and edges with relation to the map of the United States
- From the adjacency matrix, [n][0] = city_name, [n][1] = latitude, and [n][2] = longitude.

```
import networkx as nx
import matplotlib.pyplot as plt
from mpl_toolkits.basemap import Basemap as Basemap

def visualize(graph, array, x_offset=0, y_offset=0, scale=1):
    ...
    x_offset: shifts all points along the x axis
    y_offset: shifts all points along the y axis
    scale: scales the distance from the origin (0,0) by input
    ...
graph_visualizer.py
```

Total rail track:

Complete graph = 1,445,157 miles



Our railway MST using Prim's algorithm = 9622 miles



Using Floyd-Warshall

From Module 9: Dynamic Programming 2

Used Floyd-Warshall (*Module 6 Greedy Algorithms 1*)

instead of Dijkstra's algorithm (*Module 6 Greedy Algorithms 1*)

More efficient to find the shortest paths of all pairs of vertices (cities) using matrix multiplication

Our function takes in an adjacency matrix input, calculates the distances between the vertices and returns a matrix of distances

Iteration to find which additional edge reduces the sum of all shortest paths = runtime complexity $O(n^3)$

```

16 def floyd_marshall(adj_matrix):
17     """
18         Returns a matrix of distances between vertices based on the input adjacency
19         matrix. Runs in  $O(n^3)$  time.
20     """
21     n = len(adj_matrix)
22     dist = [[INF for x in range(n)] for y in range(n)]
23
24     for u in range(n):
25         dist[u][u] = 0
26         for v in range(n):
27             if adj_matrix[u][v]:
28                 dist[u][v] = adj_matrix[u][v]
29
30     for k in range(n):
31         for i in range(n):
32             for j in range(n):
33                 dist[i][j] = min(dist[i][j], (dist[i][k] + dist[k][j]))
34
35     return dist

```

analysis_tools.py

Calculating Track Length

```
1 INF = 9999999
2
3 def track_length(adj_matrix):
4     """
5         Returns the total length of track from an input adjacency matrix
6     """
7     n = len(adj_matrix)
8     track = 0
9
10    for u in range(n):
11        for v in range(u + 1, n):
12            track += adj_matrix[u][v]
13
14    return track
```

analysis_tools.py

We also utilized a `track_length` function to:

- Take in the adjacency matrix parameter
- Calculates the track length using two for-loops
- Returns the total length of the track

Runtime complexity is $O(n^2)$

Final Step - Railgorithm

Integrate all functions together:

- input parameters = graph and complete graph
- Stop algorithm once we laid down 20,000 miles of track to be comparable to the current Amtrak rail network of 21,400 miles (*Amtrak Corporate Profile*).

Runtime complexity = $O(n^5)$

- Two nested loops to call track_length function and applying floyd_marshall to our graph

Total miles of rail track with our High-Speed Rail = 21,106 miles

```

6  def railgorithm(graph, complete_graph):
7      ...
8      O(n^5), lol
9      ...
10     if len(graph) != len(complete_graph):
11         raise ValueError("Graph lengths must be equal")
12     n = len(graph)
13     curr_sum = track_length(floyd_marshall(graph))
14
15
16     travel_saved = 0
17     x = 0
18     y = 0
19     for u in range(n):
20         for v in range(u + 1, n):
21             if graph[u][v] == 0:
22                 graph[u][v] = complete_graph[u][v]
23                 delta = curr_sum - track_length(floyd_marshall(graph))
24                 graph[u][v] = 0
25                 if delta > travel_saved:
26                     travel_saved = delta
27                     x,y = u,v
28
29     graph[x][y] = complete_graph[x][y]
30
31     return graph
32
33
34 def main():
35     ...
36     This took 10 minutes to produce an output with the current parameters
37     ...
38
39     mst = prim(g)
40
41     new = mst
42
43     while(track_length(new) < 20000):
44         new = railgorithm(mst, g)
45
46     visualize(new, top48)

```

railgorithm.py

04

Implementation Demo

Python Code Demonstration

04

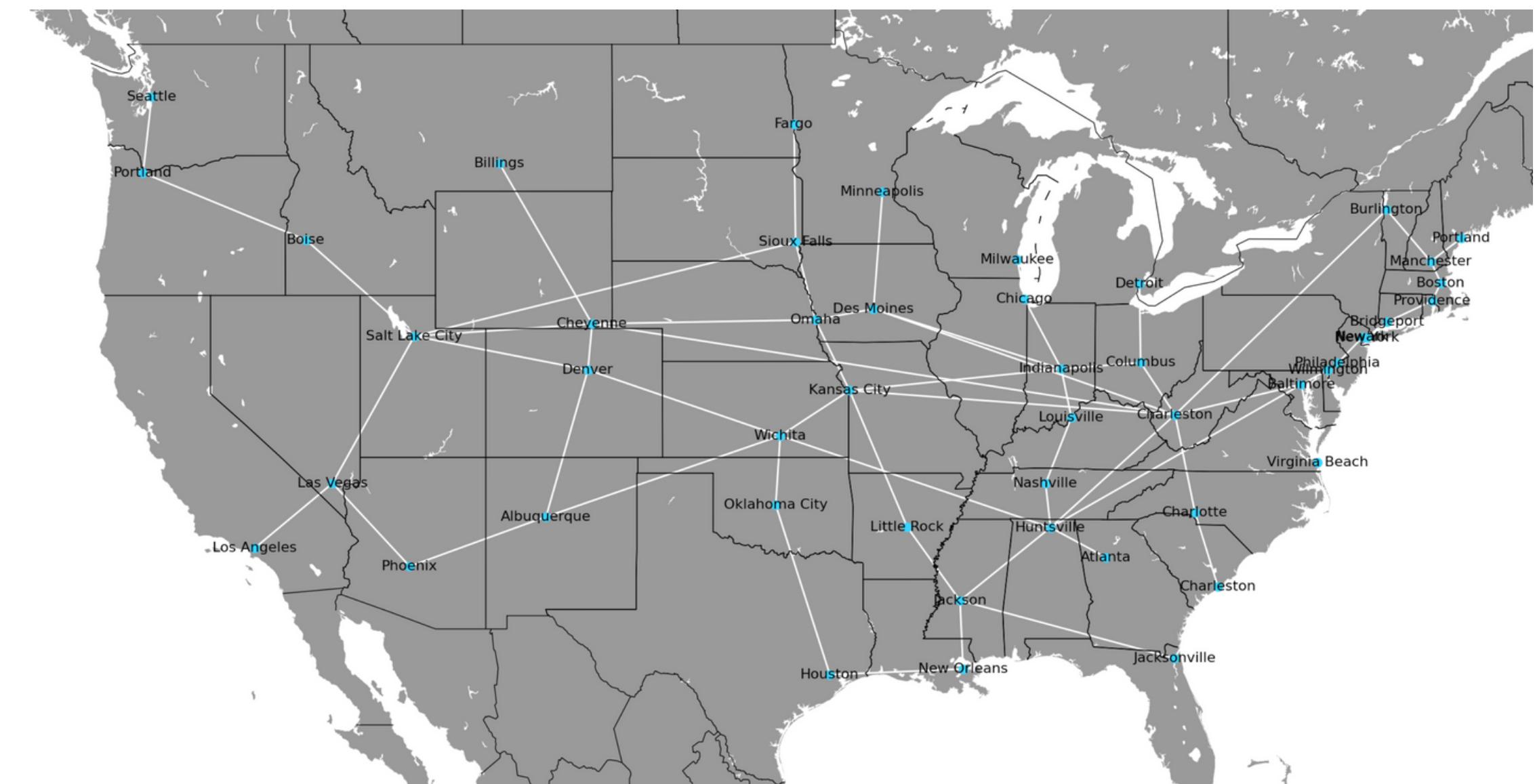
Result & Conclusion

Summary of findings, future considerations, conclusion

Question Being Answered:

What is the most efficient way to build a high-speed railway to connect the most populous cities in the continental United States?

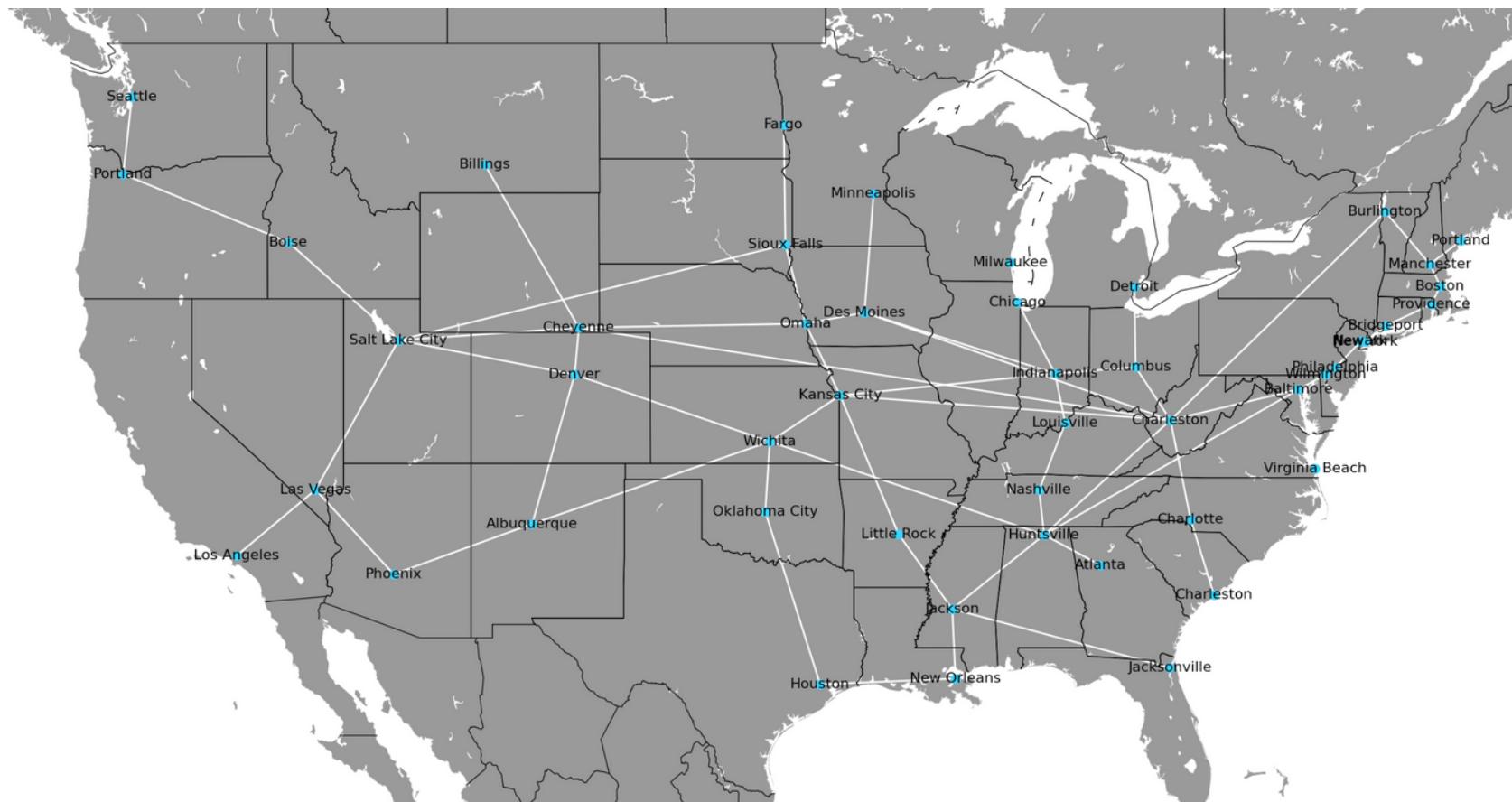
The most efficient way is shown in the graph visualization below with the cities in blue and the rail path in white :



Comparison to Current Amtrak Map

Laying track like this uses 21,106 total miles. This is a dramatic improvement from the 1,445,157 miles in the complete graph, and similar to the 21,400 miles in the current Amtrak map.

We believe that our railway tracks are more efficiently laid out than the current Amtrak.

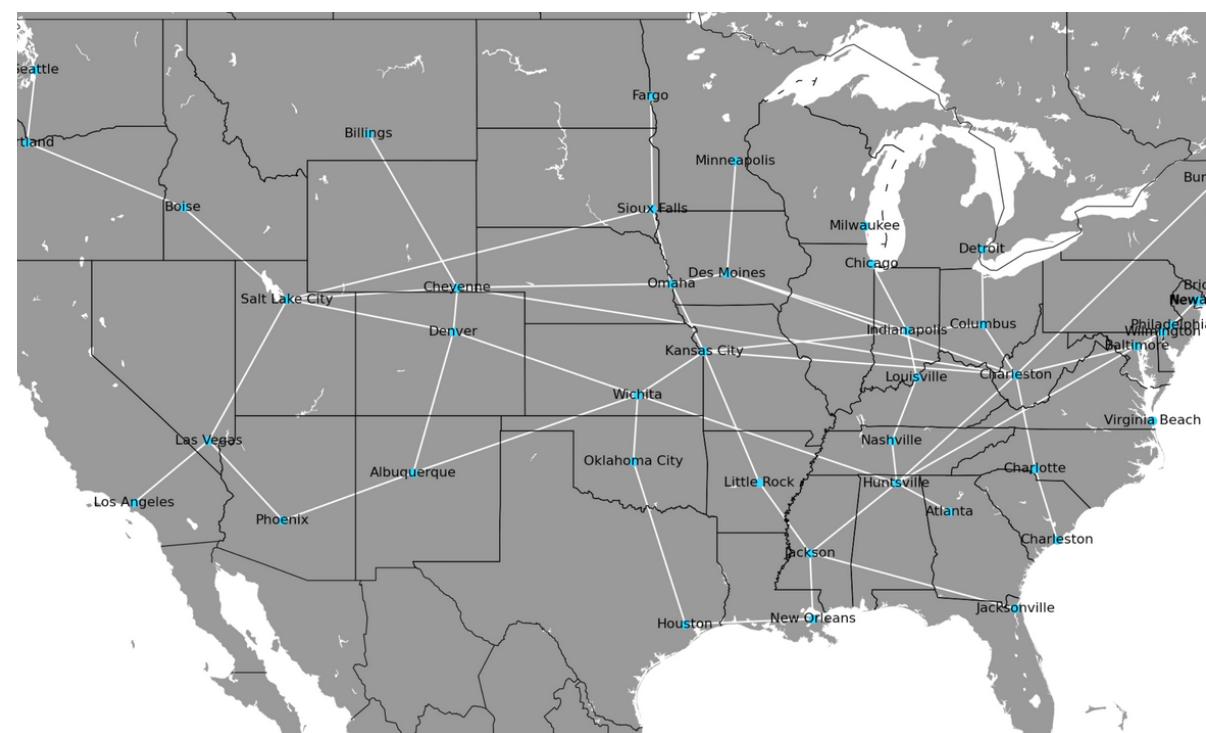


Our High-Speed railgorithm Map



Amtrak Network Railway Map

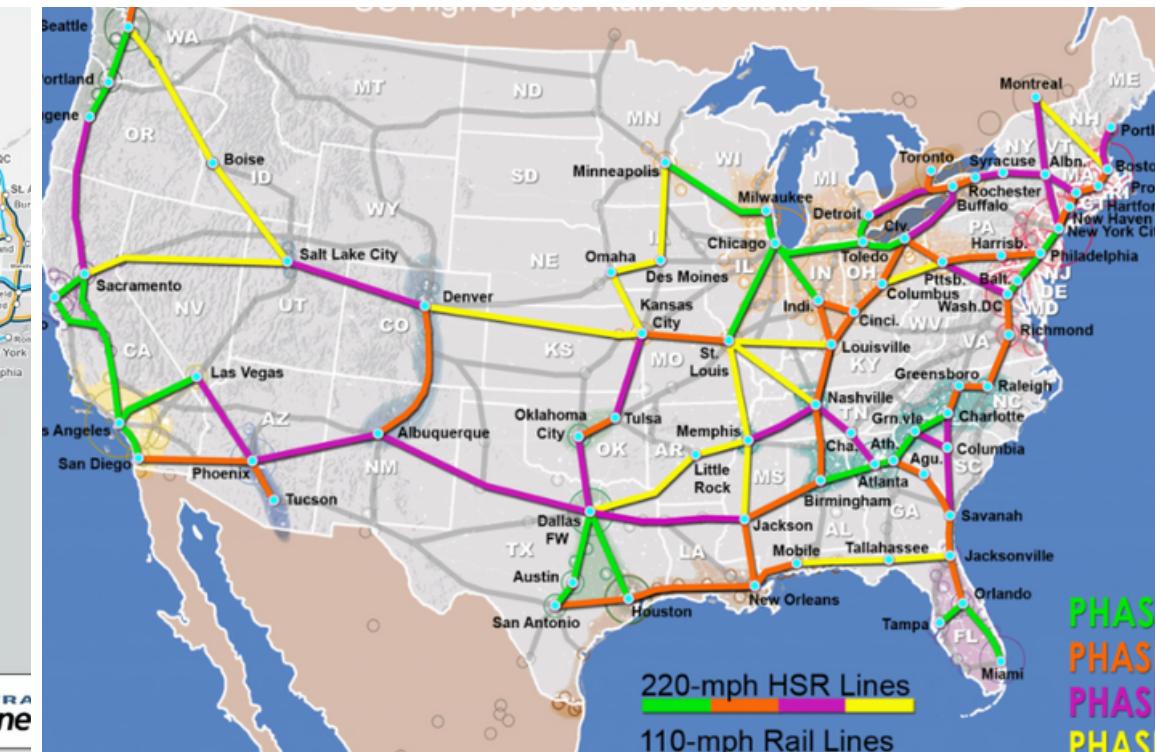
Comparison to Amtrak Proposal



Our High-Speed Railgorithm Map



Amtrak Proposed High Speed Rail Map



US High Speed Rail Association Proposed Map

Our rail lines are similar to proposed high speed rail maps from Amtrak and the US High Speed Rail Association

We offer more connections between states in the Mountain West, whereas the existing proposals skew more toward connecting a few cities in the West with more populous cities like Seattle and Chicago.

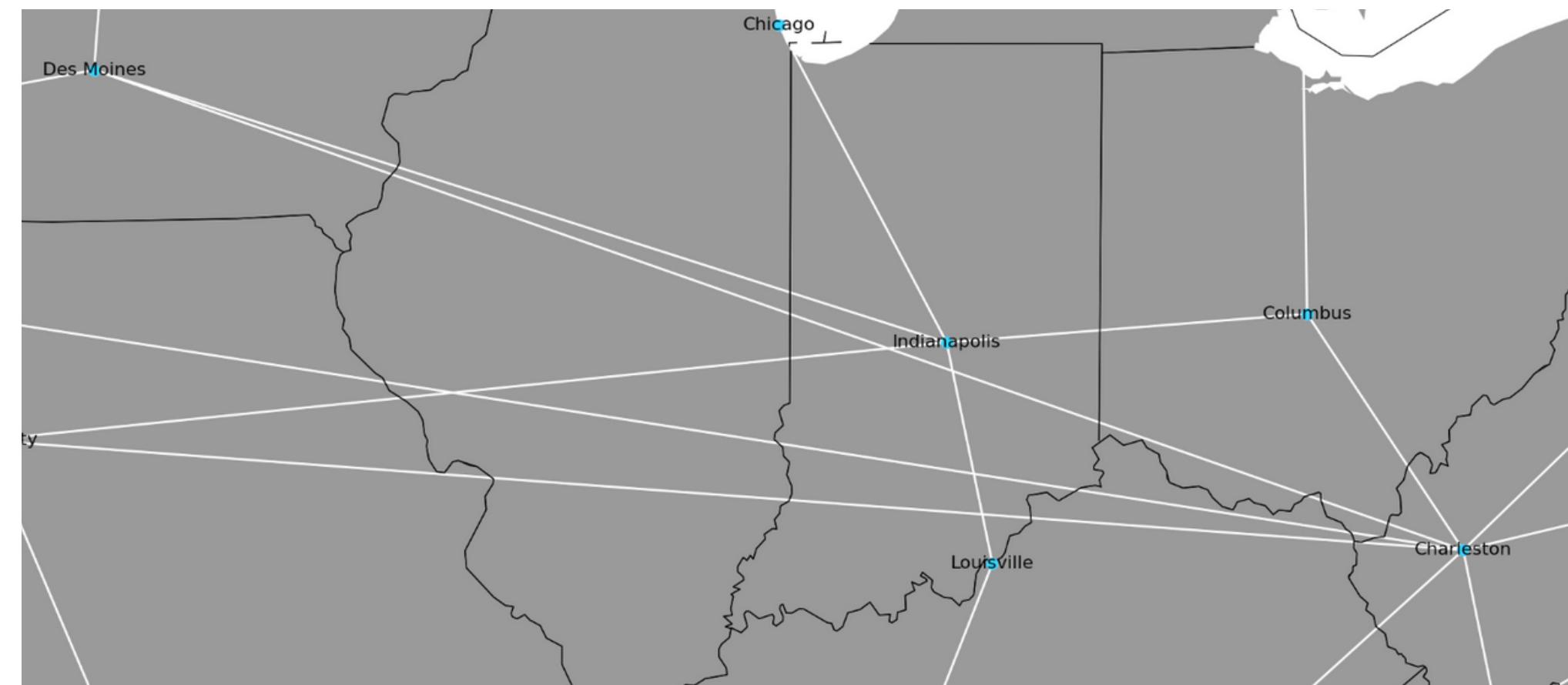
Limitations & Improvements

Floyd-Warshall takes a very long time to run, $O(n^3)$, leading to a total runtime of $O(n^5)$ for our algorithm

- Even though we do not use all the edges in the complete graph, we need to calculate them all in order to decide which ones are the best to add
- In the future, explore other ways of adding edges, perhaps find a clever heuristic we could use with A*

Final graph has busyness around the rust belt

- Several intersecting lines recommended, however, worth doing a cost/benefit analysis before creating all of them.
- Perhaps not enough ridership to justify ie. line from Des Moines (top left) to Charleston (bottom right)



Busyness around the rust belt in the final map

More Considerations

Factor in Terrain

- Potential critical impediment to laying track in certain areas, ie. the Rocky Mountains
- Possible reason why our map differs from the formal proposals

Populous cities not being served by our map

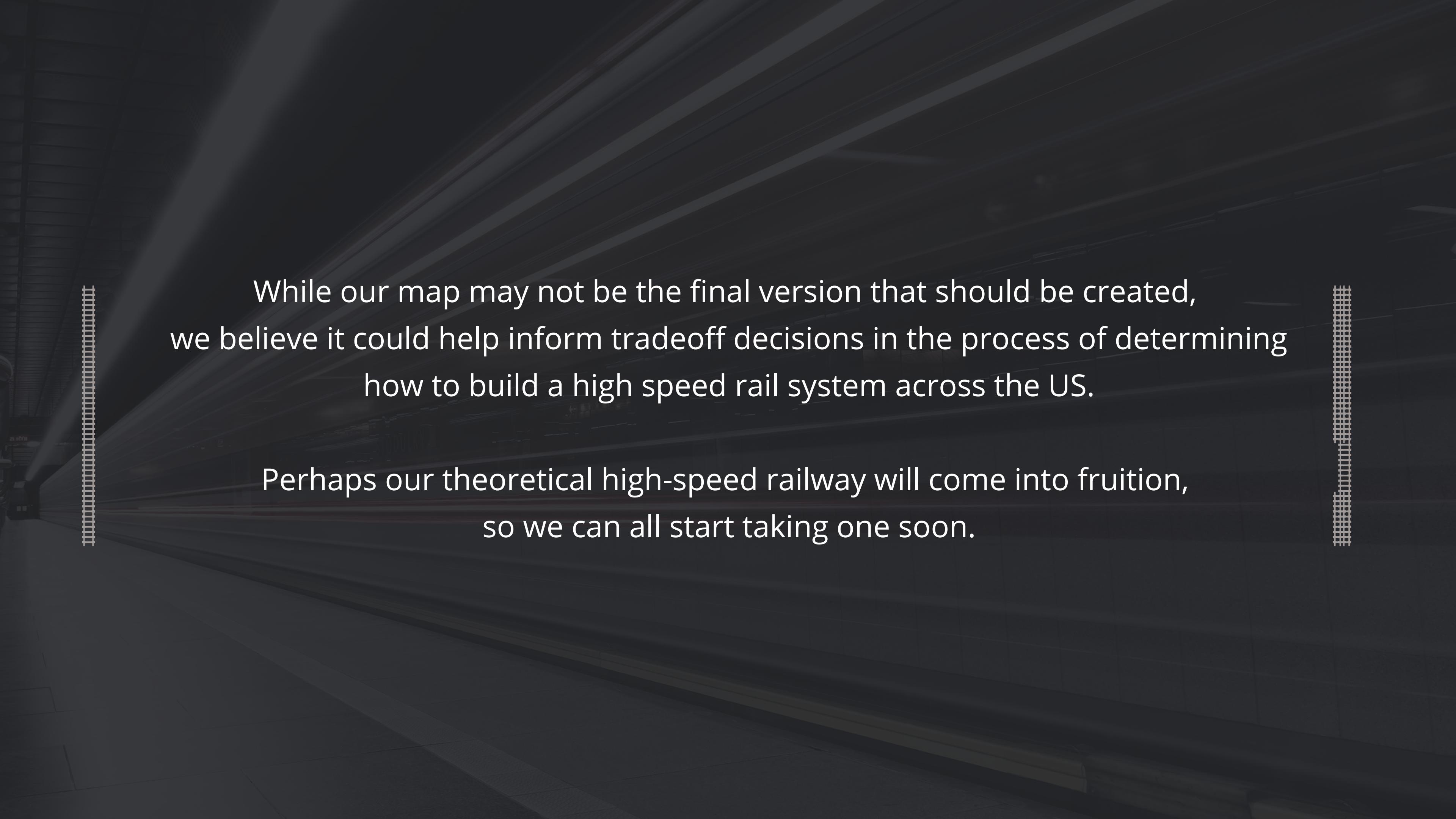
- Create railway connection between Houston - Dallas, Los Angeles - San Jose than areas in the rust belt
- Advantages to laying track alongside existing railways

Factor in cost

- Consider budget parameters
- Explore cost to connect to Alaska

Ease of construction and ridership demands

- Balancing ridership (connecting most populous cities) with serving those in remote areas (cities in each state)



While our map may not be the final version that should be created,
we believe it could help inform tradeoff decisions in the process of determining
how to build a high speed rail system across the US.

Perhaps our theoretical high-speed railway will come into fruition,
so we can all start taking one soon.

Northeastern University Khoury College of Computer Sciences
CS 5008 Algorithms Final Project
Summer 2022

Enjoy the rest of your summer

Omar Rashwan | Elizabeth Pates | Josephine Yu