

Northeastern University Khoury College of Computer Sciences  
CS 5008 Algorithms Final Project - Summer 2022  
YouTube Link: [https://youtu.be/Fa-U\\_wfWcBA](https://youtu.be/Fa-U_wfWcBA)

# A High-Speed Railway Across the United States

---

Omar Rashwan | Annie Pates (Elizabeth) | Josephine Yu

# Table of Contents

00

**PROJECT INTRODUCTION**

---

01

**PROBLEM ANALYSIS**

---

02

**SOLUTION APPROACH**

---

03

**IMPLEMENTATION DEMO**

---

04

**RESULT & CONCLUSION**

---

00

# Project Introduction

---

Clearly defined question and relevance to our team

# Initial Inspiration

With fuel prices increasing astronomically in the past few months, travel has become more cumbersome.

Each of our group members has been affected by this issue personally lately, which prompted us to brainstorm.

This led to a discussion whether the United States had a transcontinental, high-speed railroad that facilitates an easier and relatively economical travel.

We began brainstorming the best methodology to apply algorithms we have learnt in this class and potential roadblocks that we may face through our theoretical railway building journey.

00

PROJECT INTRODUCTION

01

PROBLEM ANALYSIS

02

SOLUTION APPROACH

03

IMPLEMENTATION DEMO

04

RESULT & CONCLUSION

# Question to be Answered:

What is the most efficient way  
to build a high-speed railway  
to connect the most populous cities  
in the contiguous United States?

# Relevance to Each Team Member

## Omar Rashwan

*Ever since watching Thomas the Tank Engine as a young boy, I have been fascinated with rail transit as a means of transportation. This interest in trains has only grown as I learned more about the efficiency of rail versus other means of transport such as road and air travel ...*

## Annie Pates (Elizabeth)

*This year, I have weddings in Massachusetts, Rhode Island, Ohio, Minnesota, and Texas as well as trips to Wyoming and Florida. Additionally, my friends who are getting married in the bride's hometown in Texas are committed to having their dog participate in the ceremony ...*

## Josephine Yu

*For upcoming travels in Canada, I have been canceling flights and opting for travel via train instead. In Canada, the transcontinental train goes from the west coast in Vancouver to the east coast in Halifax, it is also heavily subsidized by the government...*

01

# Problem Analysis

---

How we framed the problem and how we will solve it

# Problem Analysis: Assumptions

Parameters given the short time frame

- **Only examining the most populous city in each of the contiguous US states**
  - ie. not considering Hawaii, Alaska at this time
- **Caveat: paths in MST may not be the most effective way to get from one city to another**
  - Used Prim's algorithm to create minimum weight spanning tree
  - Mitigated caveat by applying Floyd-Warshall once we'd created it
- **Ignoring terrain complexities that come with going around/over/through geographical features**
  - ie. Lakes and mountains
- **Not factoring cost in the implementation which could have skewed the paths**

02

# Solution Approach

---

Prim's Algorithm, Minimum-Spanning Tree, Floyd-Warshall Algorithm

# Methods Used to Gather Data

- Wikipedia dataset on the top 330 most populous cities in the United States including:
  - 2021 census information
  - Land area
  - Population density
  - Location longitudes and latitudes

2021 rank	City	State <sup>[c]</sup>	2021 estimate	2020 census	Change	2020 land area	2020 population density	Location
1	New York <sup>[d]</sup>	New York	8,467,513	8,804,190	-3.82%	300.5 sq mi	778.3 km <sup>2</sup>	29,298/sq mi
2	Los Angeles	California	3,849,297	3,898,747	-1.27%	469.5 sq mi	1,216.0 km <sup>2</sup>	8,304/sq mi
3	Chicago	Illinois	2,696,555	2,746,388	-1.81%	227.7 sq mi	589.7 km <sup>2</sup>	12,061/sq mi
4	Houston	Texas	2,288,250	2,304,580	-0.71%	640.4 sq mi	1,658.6 km <sup>2</sup>	3,599/sq mi
5	Phoenix	Arizona	1,624,569	1,608,139	+1.02%	518.0 sq mi	1,341.6 km <sup>2</sup>	3,105/sq mi
6	Philadelphia <sup>[e]</sup>	Pennsylvania	1,576,251	1,603,797	-1.72%	134.4 sq mi	348.1 km <sup>2</sup>	11,933/sq mi
7	San Antonio	Texas	1,451,853	1,434,625	+1.20%	498.8 sq mi	1,291.9 km <sup>2</sup>	2,876/sq mi
8	San Diego	California	1,381,611	1,386,932	-0.38%	325.9 sq mi	844.1 km <sup>2</sup>	4,256/sq mi
9	Dallas	Texas	1,288,457	1,304,379	-1.22%	339.6 sq mi	879.6 km <sup>2</sup>	3,841/sq mi
10	San Jose	California	983,489	1,013,240	-2.94%	178.3 sq mi	461.8 km <sup>2</sup>	5,683/sq mi
11	Austin	Texas	964,177	961,855	+0.24%	319.9 sq mi	828.5 km <sup>2</sup>	3,007/sq mi
12	Jacksonville <sup>[f]</sup>	Florida	954,614	949,611	+0.53%	747.3 sq mi	1,935.5 km <sup>2</sup>	1,271/sq mi
13	Fort Worth	Texas	935,508	918,915	+1.81%	342.9 sq mi	888.1 km <sup>2</sup>	2,646/sq mi

[https://en.wikipedia.org/wiki/List\\_of\\_United\\_States\\_cities\\_by\\_population](https://en.wikipedia.org/wiki/List_of_United_States_cities_by_population)

- Kaggle dataset in .csv format

	A	B	C	D	E	F	G
1	City	State	Type	Counties	Population	Latitude	Longitude
2	New York	NY	City	Bronx;Richmond;New York;Kings;Queens	8804190	40.714	-74.007
3	Los Angeles	CA	City	Los Angeles	3898747	34.052	-118.243
4	Chicago	IL	City	Cook;DuPage	2746388	41.882	-87.628
5	Houston	TX	City	Harris;Fort Bend;Montgomery	2304580	29.76	-95.363
6	Phoenix	AZ	City	Maricopa	1608139	33.448	-112.074
7	Philadelphia	PA	City	Philadelphia	1603797	39.952	-75.164
8	San Antonio	TX	City	Bexar	1434625	29.423	-98.49
9	San Diego	CA	City	San Diego	1386932	32.716	-117.165
10	Dallas	TX	City	Rockwall;Denton;Kaufman;Dallas;Collin	1304379	32.781	-96.797
11	San Jose	CA	City	Santa Clara	1013240	37.336	-121.891
12	Austin	TX	City	Williamson;Travis;Hays	961855	30.268	-97.743
13	Jacksonville	FL	City	Duval	949611	30.328	-81.657
14	Fort Worth	TX	City	Wise;Denton;Parker;Tarrant	918915	32.754	-97.331
15	Columbus	OH	City	Delaware;Franklin;Fairfield	905748	39.962	-83.001
16	Indianapolis	IN	City	Marion	887642	39.769	-86.158
17	Charlotte	NC	City	Mecklenburg	874579	35.227	-80.843
18	San Francisco	CA	City	San Francisco	873965	37.78	-122.414
19	Seattle	WA	City	King	737015	47.607	-122.338
20	Denver	CO	City	Denver	715522	39.739	-104.987
21	Washington	DC	City	District of Columbia	689545	38.895	-77.036
22	Nashville	TN	Metro Government	Davidson	689447	36.164	-86.783

<https://www.kaggle.com/datasets/axeltorbenson/us-cities-by-population-top-330>

# File Reader in Python

- Coded a file reader in Python to iterate through the data of the 330 cities
- Sorted the data by population in decreasing order (*Module 0 - Sorting*)
- Appended data into a nested array initialized as “top48”
  - sorted, most populous 48 cities used to create the graph in an adjacency matrix (*Module 5- Adjacency Matrices and List*)
- Results were outputted to a file, “top48.py”

```

1 City,State,Type,COUNTIES,Population,Latitude,Longitude
2 New York,NY,City,Bronx;Richmond;New York;Kings;Queens,8804190,40.714,-74.007
3 Los Angeles,CA,City,Los Angeles,3898747,34.052,-118.243
4 Chicago,IL,City,Cook;DuPage,2746388,41.882,-87.628
5 Houston,TX,City,Harris;Fort Bend;Montgomery,2304580,29.76,-95.363
6 Phoenix,AZ,City,Maricopa,1608139,33.448,-112.074
7 Philadelphia,PA,City,Philadelphia,1603797,39.952,-75.164
8 San Antonio,TX,City,Bexar,1434625,29.423,-98.49
9 San Diego,CA,City,San Diego,1386932,32.716,-117.165
10 Dallas,TX,City,Rockwall;Denton;Kaufman;Dallas;Collin,1304379,32.781,-96.797
11 San Jose,CA,City,Santa Clara,1013240,37.336,-121.891
12 Austin,TX,City,Williamson;Travis;Hays,961855,30.268,-97.743
13 Jacksonville,FL,City,Duval,949611,30.328,-81.657
14 Fort Worth,TX,City,Wise;Denton;Parker;Tarrant,918915,32.754,-97.331
15 Columbus,OH,City,Delaware;Franklin;Fairfield,905748,39.962,-83.001
16 Indianapolis,IN,City,Marion,887642,39.769,-86.158
17 Charlotte,NC,City,Mecklenburg,874579,35.227,-80.843
18 San Francisco,CA,City,San Francisco,873965,37.78,-122.414
19 Seattle,WA,City,King,737015,47.607,-122.338
20 Denver,CO,City,Denver,715522,39.739,-104.987
21 Washington,DC,City,District of Columbia,689545,38.895,-77.036
22 Nashville,TN,Metro Government,Davidson,689447,36.164,-86.783
23 Oklahoma City,OK,City,Canadian;Cleveland;Oklahoma,681054,35.468,-97.516
24 El Paso,TX,City,El Paso,678815,31.76,-106.488

```

&gt;&gt;&gt;

```

16 def main():
17     filename = "us2021census.csv"
18     with open(filename, mode='r') as f:
19         header = f.readline()
20         data = f.readlines()
21
22     states = {"AK", "HI", "DC"}
23     top48 = []
24     for i in range(len(data)):
25         data[i] = data[i].strip('\n')
26         data[i] = data[i].split(',')
27         data[i][4] = int(data[i][4])
28         data[i][5], data[i][6] = float(data[i][5]), float(data[i][6])
29         lst = [data[i][0], data[i][5], data[i][6]]
30         if data[i][1] not in states:
31             states.add(data[i][1])
32             top48.append(lst)
33
34     n = len(top48)
35     adj_matrix = [[0 for x in range(n)] for y in range(n)]
36
37     for i in range(n):
38         for j in range(i, n):
39             city1 = top48[i][1], top48[i][2]
40             city2 = top48[j][1], top48[j][2]
41             distance = dist(city1, city2)
42             adj_matrix[i][j] = distance
43             adj_matrix[j][i] = distance
44
45     output = "top48.py"
46
47     with open(output, mode='w') as o:
48         o.write("top48 = " + str(top48) + '\n')
49         o.write("g = " + str(adj_matrix))
50
51
52
53
54
55
56

```

.CSV file

csv\_reader.py

&gt;&gt;&gt;

```

1 top48 = [['New York', 40.714, -74.007], ['Los Angeles', 34.052, -118.243], ['Chicago', 41.882, -87.628], ['
2 g = [[0.0, 3086.703983270828, 943.298051129652, 1656.0989717924465, 2674.0426730785352, 95.59159990815074, 8

```

top48.py

# Creating the Graph

## Began with a graph structure:

- Vertices = cities
- Edges = railway path
- Weighted edge = distance between two vertices

## Building our minimum spanning tree:

- Starting vertex = most populous city from our “top48” array
- Applied Prim’s algorithm (*Module 7 Greedy Algorithms 2*)
- Calculated the lowest weighted edge at each step
- Runtime complexity =  $O( (V+E) \log(V) )$

```

7  def prim(graph):
8      n = len(graph)
9      included = {0}
10     mst = [[0 for x in range(n)] for y in range(n)]
11
12     while len(included) < n:
13         minimum = 999999999
14         x, y = 0, 0
15         for u in included:
16             for v in range(n):
17                 if v not in included and graph[u][v] < minimum:
18                     minimum = graph[u][v]
19                     x, y = u, v
20         included.add(y)
21         mst[x][y] = minimum
22         mst[y][x] = minimum
23
24     return mst
25
26
27 def main():
28
29     mst = prim(g)
30
31     data = []
32
33     for i in range(len(top48)):
34         data.append((top48[i][0], top48[i][1], top48[i][2]))
35
36
37     visualize(mst, data)

```

*mst\_build.py*

# Graph Visualization

## Utilized Python library packages: NetworkX, Matplotlib and Basemap

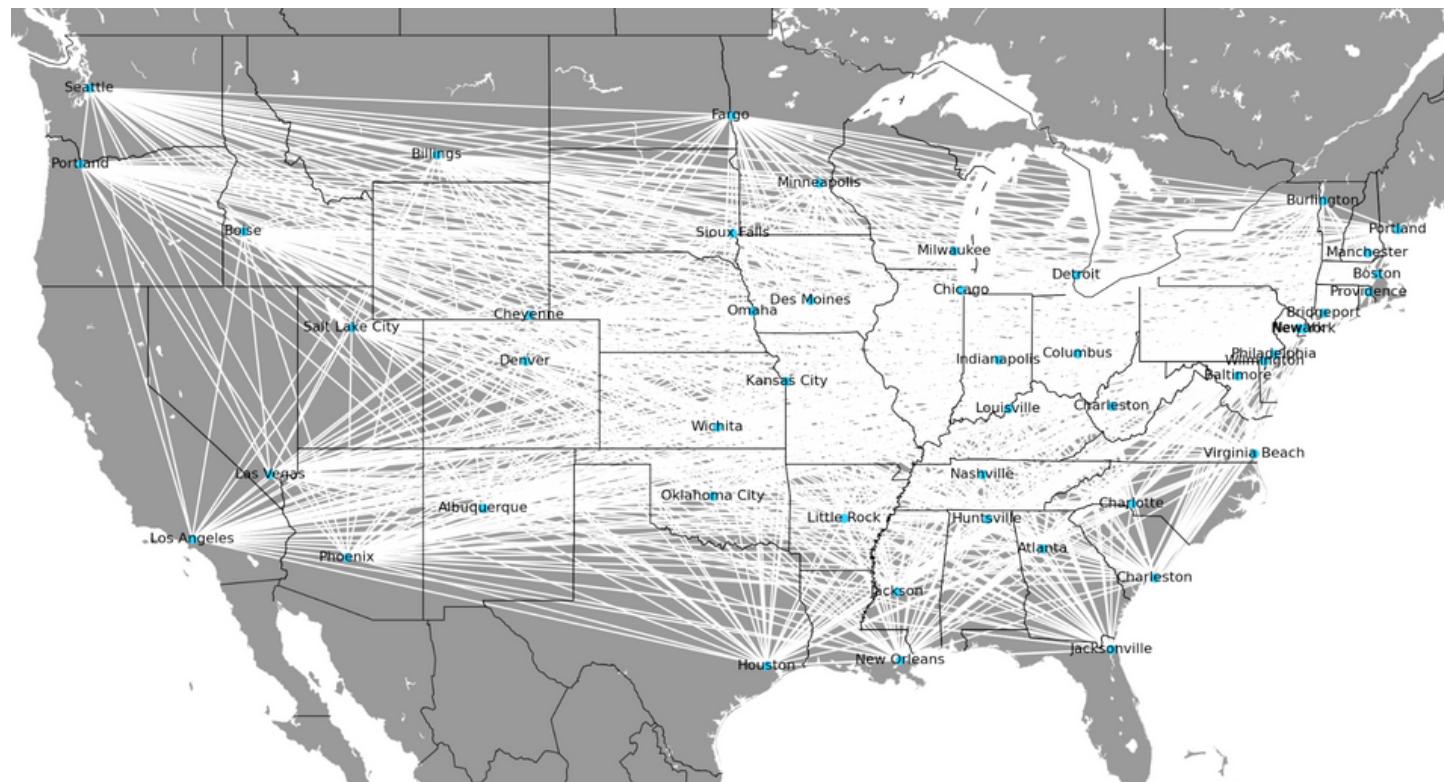
- Graphed our vertices and edges with relation to the map of the United States
- From the adjacency matrix, [n][0] = city\_name, [n][1] = latitude, and [n][2] = longitude.

```
import networkx as nx
import matplotlib.pyplot as plt
from mpl_toolkits.basemap import Basemap as Basemap

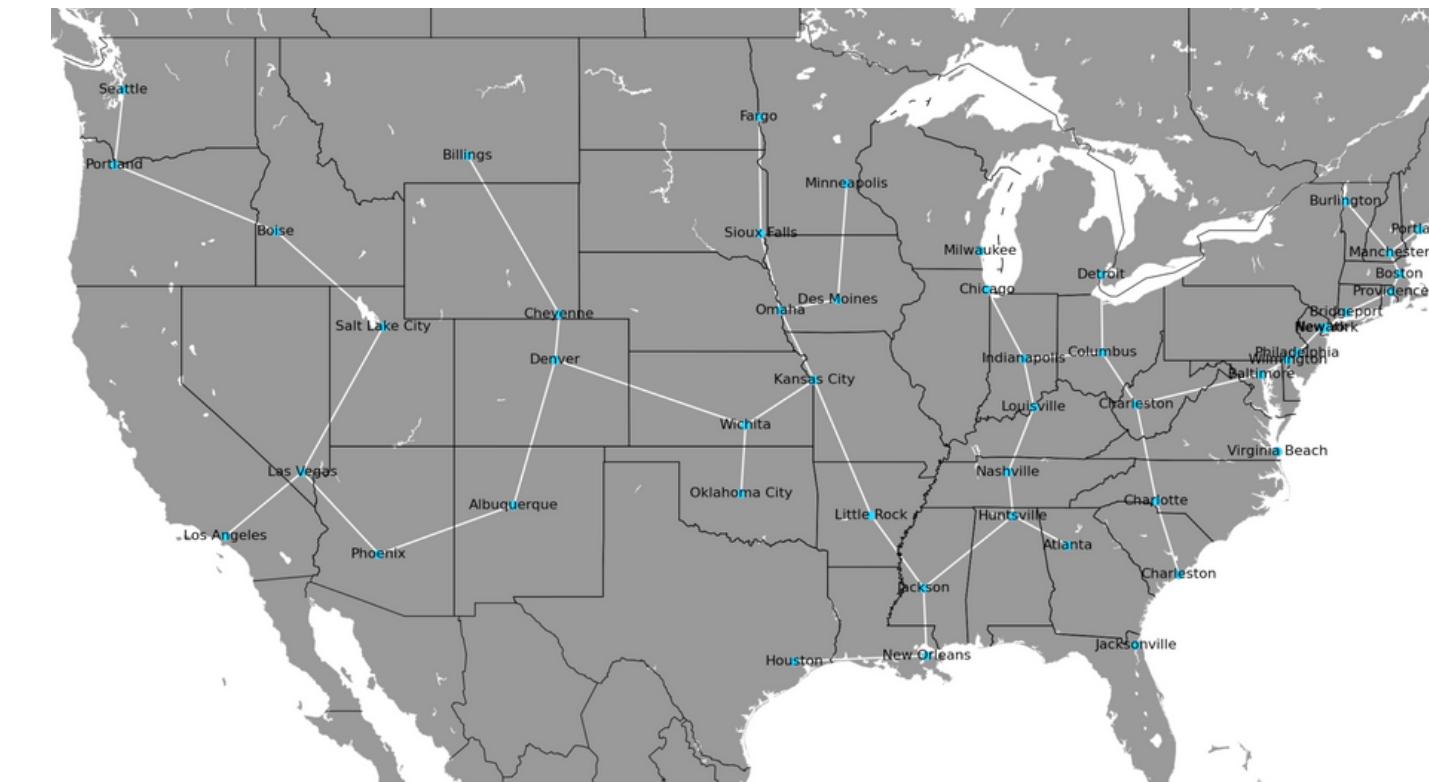
def visualize(graph, array, x_offset=0, y_offset=0, scale=1):
    ...
    x_offset: shifts all points along the x axis
    y_offset: shifts all points along the y axis
    scale: scales the distance from the origin (0,0) by input
    ...
graph_visualizer.py
```

## Total rail track:

Complete graph = 1,445,157 miles



Our railway MST using Prim's algorithm = 9622 miles



# Using Floyd-Warshall

Used Floyd-Warshall (*Module 9 Dynamic Programming*) instead of Dijkstra's algorithm (*Module 6 Greedy Algorithms 1*) to determine which additional lines (beyond those on the MST) would improve efficiency of travel

More efficient to find the shortest paths of all pairs of vertices (cities) using matrix multiplication

Our function takes in an adjacency matrix input, calculates the distances between the vertices and returns a matrix of distances

Iteration to find which additional edge reduces the sum of all shortest paths = runtime complexity  $O(n^3)$

```

16 def floyd_marshall(adj_matrix):
17     """
18         Returns a matrix of distances between vertices based on the input adjacency
19         matrix. Runs in  $O(n^3)$  time.
20     """
21     n = len(adj_matrix)
22     dist = [[INF for x in range(n)] for y in range(n)]
23
24     for u in range(n):
25         dist[u][u] = 0
26         for v in range(n):
27             if adj_matrix[u][v]:
28                 dist[u][v] = adj_matrix[u][v]
29
30     for k in range(n):
31         for i in range(n):
32             for j in range(n):
33                 dist[i][j] = min(dist[i][j], (dist[i][k] + dist[k][j]))
34
35     return dist

```

*analysis\_tools.py*

# Calculating Track Length

```
1 INF = 9999999
2
3 def track_length(adj_matrix):
4     """
5         Returns the total length of track from an input adjacency matrix
6     """
7     n = len(adj_matrix)
8     track = 0
9
10    for u in range(n):
11        for v in range(u + 1, n):
12            track += adj_matrix[u][v]
13
14    return track
```

*analysis\_tools.py*

We also utilized a `track_length` function to:

- Take in the adjacency matrix parameter
- Calculates the track length using two for-loops
- Returns the total length of the track

Runtime complexity is  $O(n^2)$

# Final Step - Railgorithm

Integrate all functions together:

- input parameters = graph and complete graph
- Stop algorithm once we laid down 20,000 miles of track to be comparable to the current Amtrak rail network of 21,400 miles (*Amtrak Corporate Profile*).

Runtime complexity =  $O(n^5)$

- Two nested loops to call track\_length function and applying floyd\_marshall to our graph

Total miles of rail track with our High-Speed Rail = 21,106 miles

```

6  def railgorithm(graph, complete_graph):
7      ...
8      O(n^5), lol
9      ...
10     if len(graph) != len(complete_graph):
11         raise ValueError("Graph lengths must be equal")
12     n = len(graph)
13     curr_sum = track_length(floyd_marshall(graph))
14
15
16     travel_saved = 0
17     x = 0
18     y = 0
19     for u in range(n):
20         for v in range(u + 1, n):
21             if graph[u][v] == 0:
22                 graph[u][v] = complete_graph[u][v]
23                 delta = curr_sum - track_length(floyd_marshall(graph))
24                 graph[u][v] = 0
25                 if delta > travel_saved:
26                     travel_saved = delta
27                     x,y = u,v
28
29     graph[x][y] = complete_graph[x][y]
30
31     return graph
32
33
34 def main():
35     ...
36     This took 10 minutes to produce an output with the current parameters
37     ...
38
39     mst = prim(g)
40
41     new = mst
42
43     while(track_length(new) < 20000):
44         new = railgorithm(mst, g)
45
46     visualize(new, top48)

```

*railgorithm.py*

03

# Implementation Demo

---

Python Code Demonstration

04

# Result & Conclusion

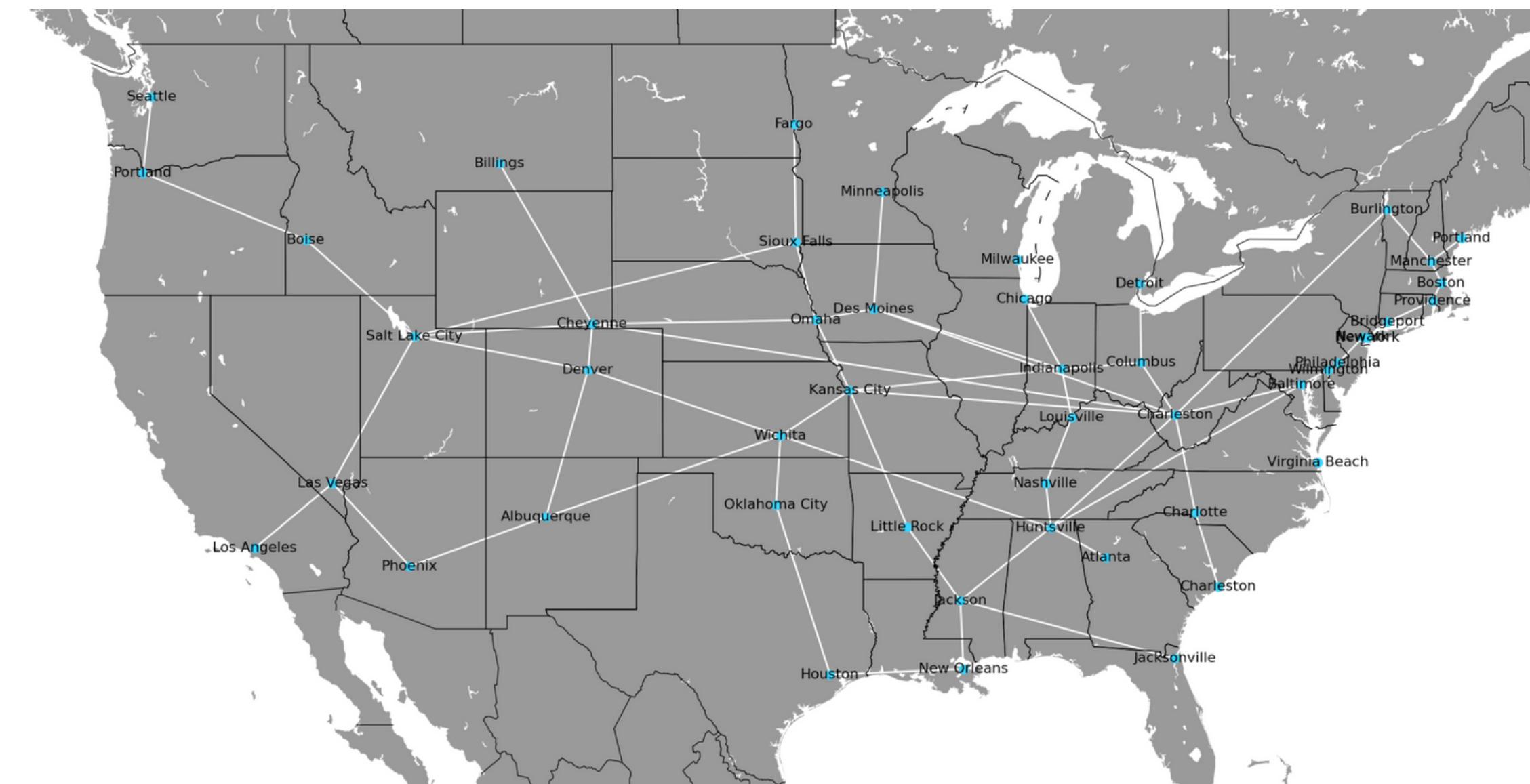
---

Summary of findings, future considerations, conclusion

# Question Being Answered:

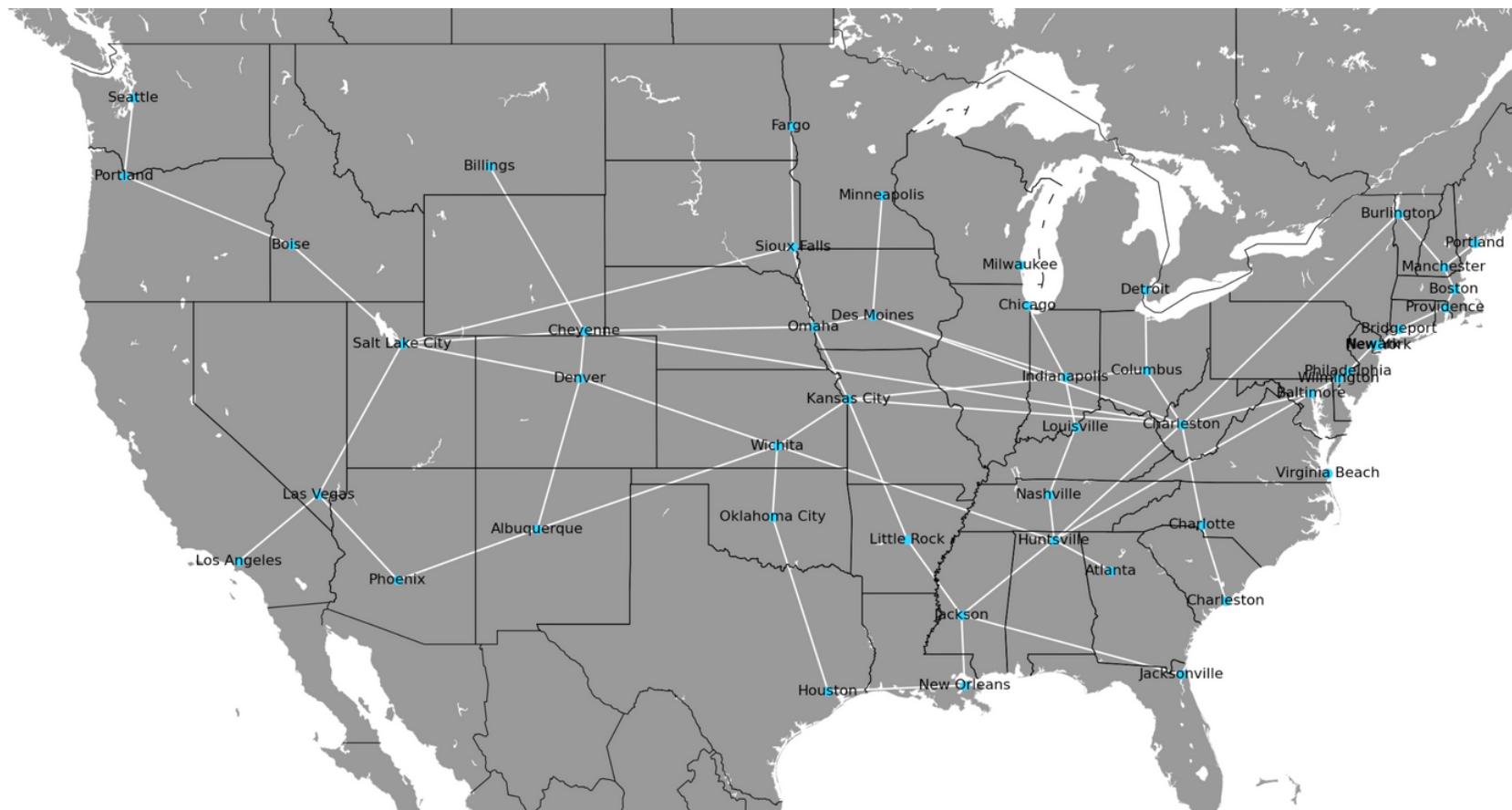
What is the most efficient way to build a high-speed railway to connect the most populous cities in the contiguous United States?

The most efficient way is shown in the graph visualization below with the cities in blue and the rail path in white :



# Comparison to Current Amtrak Map

- Laying track like this totaled 21,106 total miles
- This is a dramatic improvement from the 1,445,157 miles in the complete graph, and similar to the 21,400 miles in the current Amtrak map
- We believe that our railway tracks are more efficiently laid out than the current Amtrak

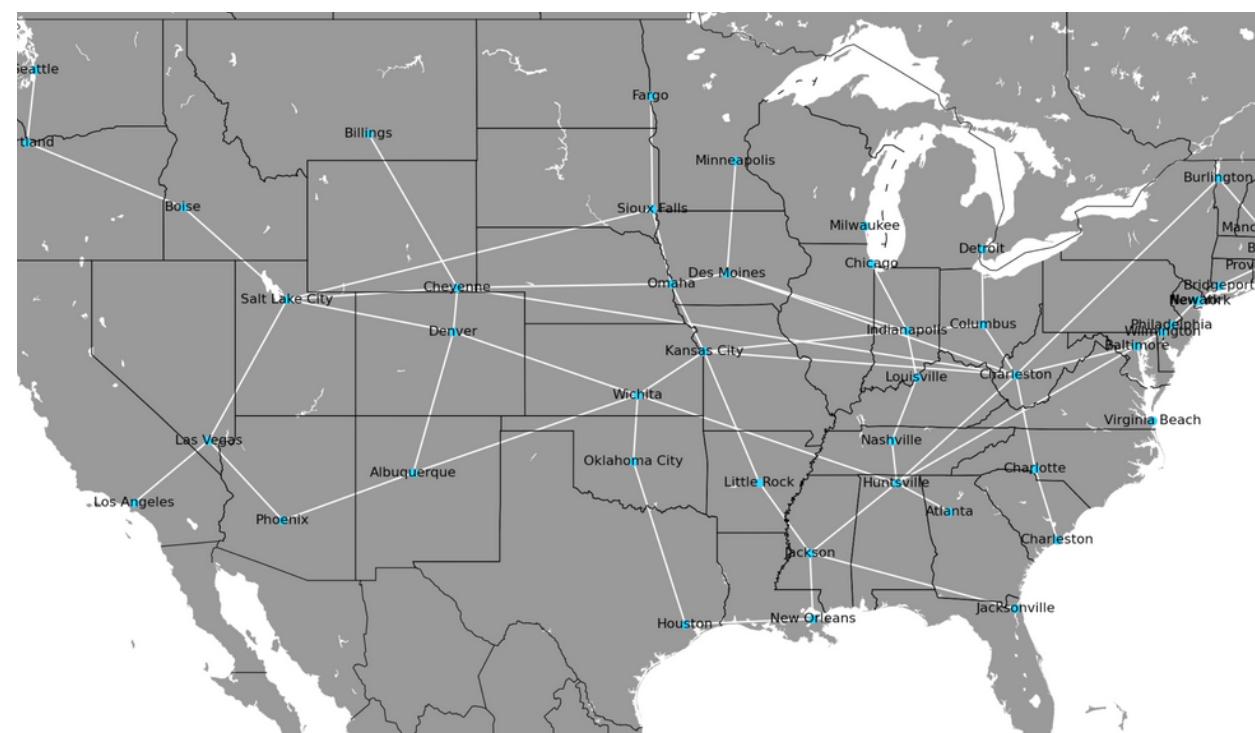


*Our High-Speed railgorithm Map*



*Amtrak Network Railway Map*

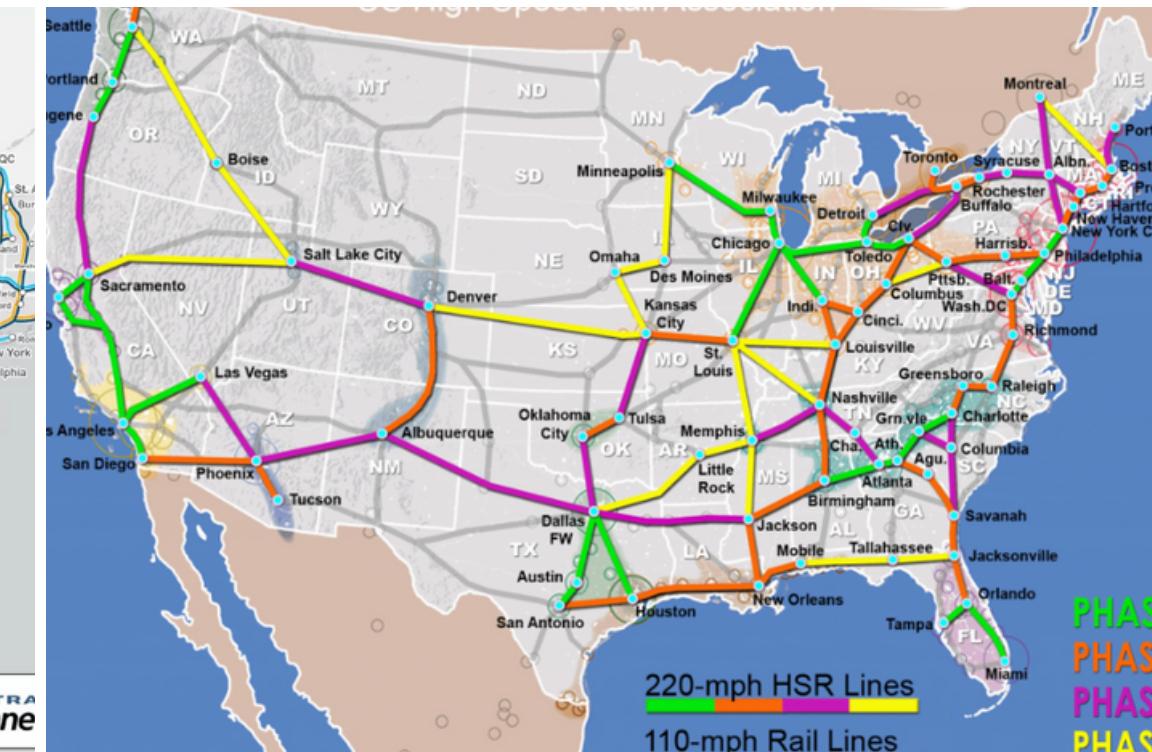
# Comparison to Amtrak Proposal



Our High-Speed Railgorithm Map



Amtrak Proposed High Speed Rail Map



US High Speed Rail Association Proposed Map

Our rail lines are similar to proposed high speed rail maps from Amtrak and the US High Speed Rail Association

We offer more connections between states in the Mountain West, whereas the existing proposals skew more toward connecting a few cities in the West with more populous cities like Seattle and Chicago.

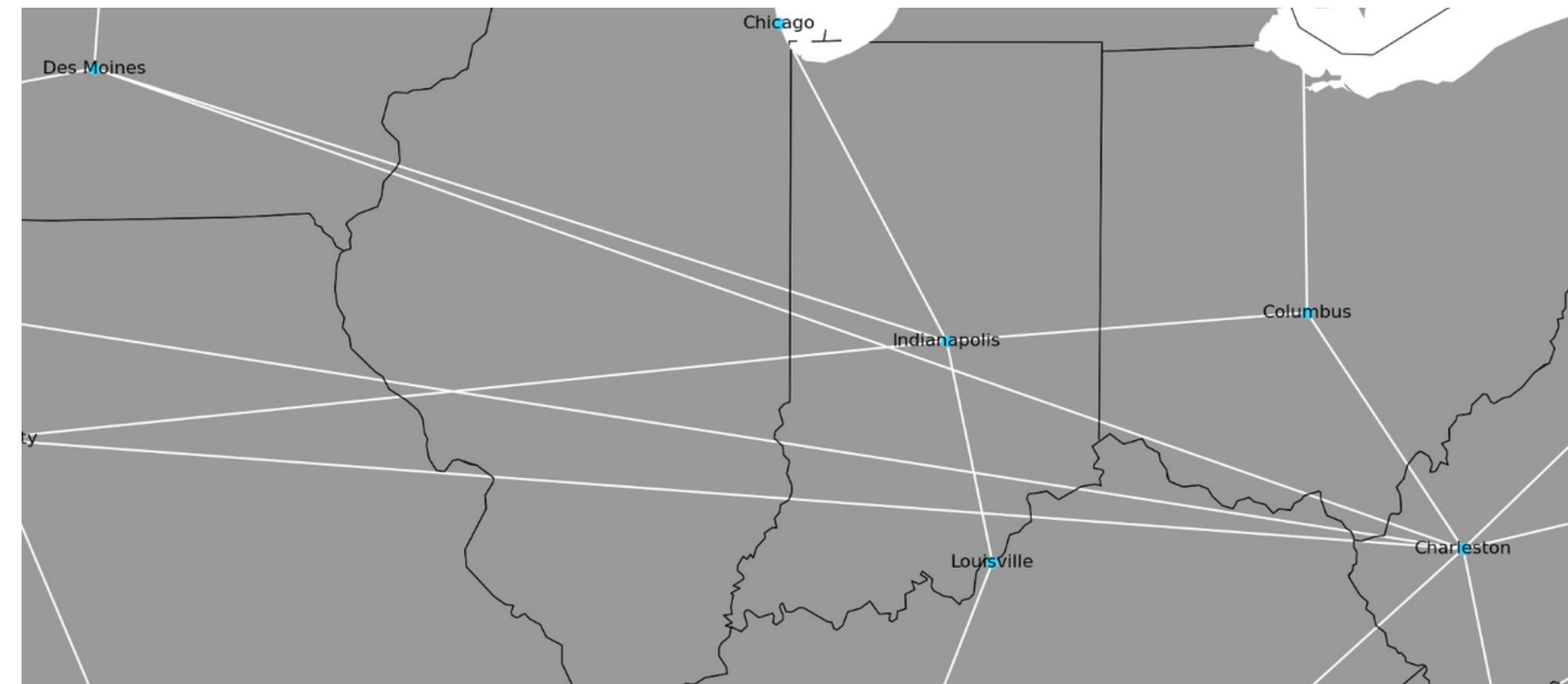
# Limitations & Improvements

Floyd-Warshall takes a very long time to run,  $O(n^3)$ , leading to a total runtime of  $O(n^5)$  for our algorithm

- Even though we do not use all the edges in the complete graph, we need to calculate them all in order to decide which ones are the best to add
- In the future, explore other ways of adding edges, perhaps find a clever heuristic we could use with A\*

Final graph has busyness around the rust belt

- Several intersecting lines recommended, however, worth doing a cost/benefit analysis before creating all of them.
- Because our algorithm is greedy, there are some inefficiencies such as this edge from Charleston to Des Moines that bypasses Indianapolis, when a more efficient use of track would have been to connect through Indianapolis.



*Busyness around the rust belt in the final map*

# More Considerations

## Factor in Terrain

- Potential critical impediment to laying track in certain areas, ie. the Rocky Mountains
- Possible reason why our map differs from the formal proposals

## Factor in cost

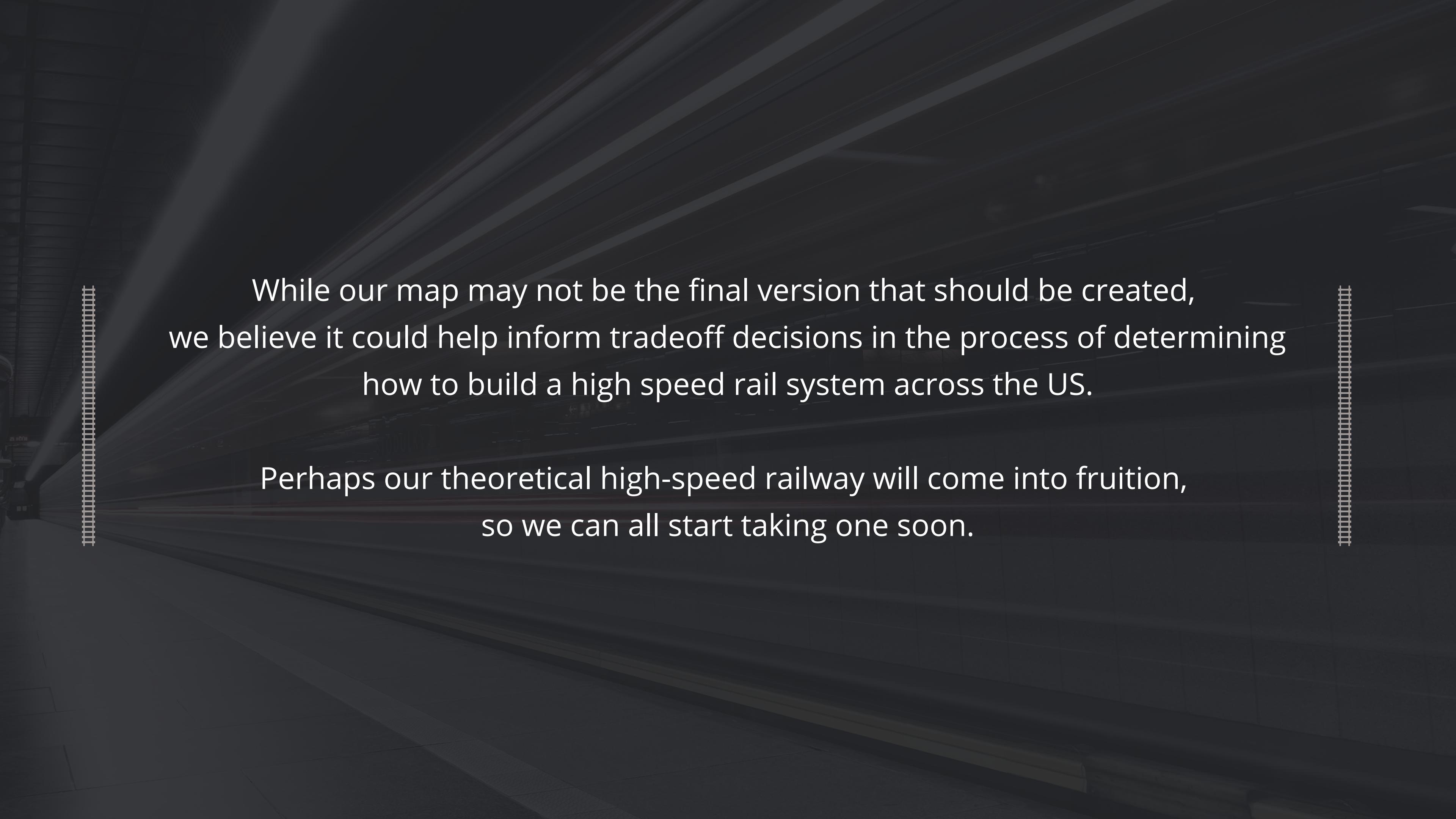
- Consider budget parameters
- Explore cost to connect to Alaska

## Populous cities not being served by our map

- Create railway connection between Houston - Dallas, Los Angeles - San Jose rather than as many lines in the rust belt
- Advantages to laying track alongside existing railways

## Ease of construction and ridership demands

- Balancing ridership (connecting most populous cities) with serving those in remote areas (cities in each state)



While our map may not be the final version that should be created, we believe it could help inform tradeoff decisions in the process of determining how to build a high speed rail system across the US.

Perhaps our theoretical high-speed railway will come into fruition, so we can all start taking one soon.

Northeastern University Khoury College of Computer Sciences  
CS 5008 Algorithms Final Project  
Summer 2022

# Enjoy the rest of your summer

---

Omar Rashwan | Annie Pates (Elizabeth) | Josephine Yu