Supporting materials for “Signal processing for condition-based maintenance of rotating machines using vibrations analysis: A tutorial”

In this file, some extension of “Signal processing for condition-based maintenance of rotating machines using vibrations analysis: A tutorial” are presented. First, In Section 1, several more details about condition-based maintenance are given, and then In Section some examples of real faults are presented. In Section 3 it is proved how the synchronous average can be calculated using the order domain.

1. Extended background about condition-based maintenance

Complex mechanical systems such as helicopters, trains, and wind turbines require expensive maintenance to prevent accidents that can cost human lives or cause severe damage to the system itself [1,2]. The maintenance cost of these systems over their operational lifespan can often be much higher than the initial cost of the system. For example, as illustrated in **Fig. 1**, the purchase price of a Bell 407GX helicopter can range approximately from $3 million to $4 million. The maintenance cost per flight hour can be estimated at approximately $1,200, which becomes more expensive as the helicopter gets older. Assuming 400 flight hours per year, this results in an annual cost of approximately $0.5 million. Over an optional operational lifespan of 30 years, with several different operators, the maintenance costs can add up to $15 million or even more. This significantly exceeds the purchase price, being four to five times as much.

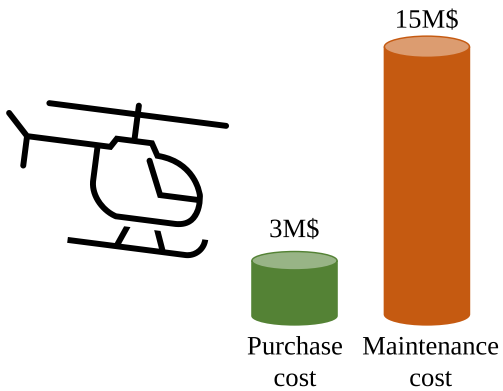

Fig. 1 – Illustration of purchase cost versus maintenance cost.

Complex mechanical systems are maintained through preventive maintenance, where inspections and replacements of various components are scheduled based on a manufacturer's predetermined schedule, which is established based on failure statistics and known maintenance principles [3]. As illustrated in **Fig. 2**, this type of maintenance presents three main issues: (1) the maintenance costs are high because many checks and component replacements are carried out under perfectly healthy conditions, (2) maintenance actions themselves can introduce failures [3] ("backward induction" in bathtub maintenance curve), and (3) misdetection of failures that can lead to catastrophic accidents or system unavailability.

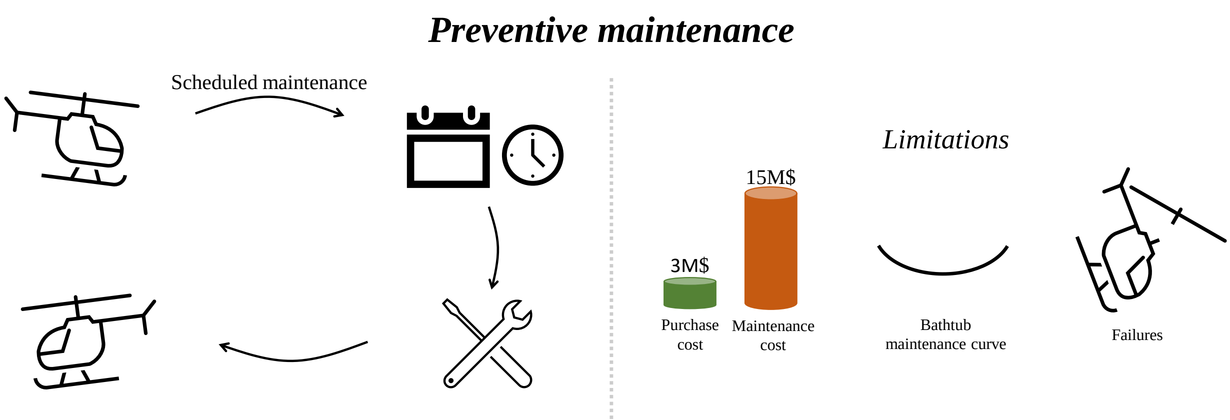

Fig. 2 – Illustration of preventive maintenance: The machine is maintained based on predetermined intervals.

Another maintenance option is corrective maintenance. As illustrated **Fig. 3**, after a failure occurs, maintenance action is applied. It has three limitations: (1) it is not relevant for critical machines, (2) small faults can escalate into larger failures that may cost much more, and (3) the machine is unscheduled not available after the failure.

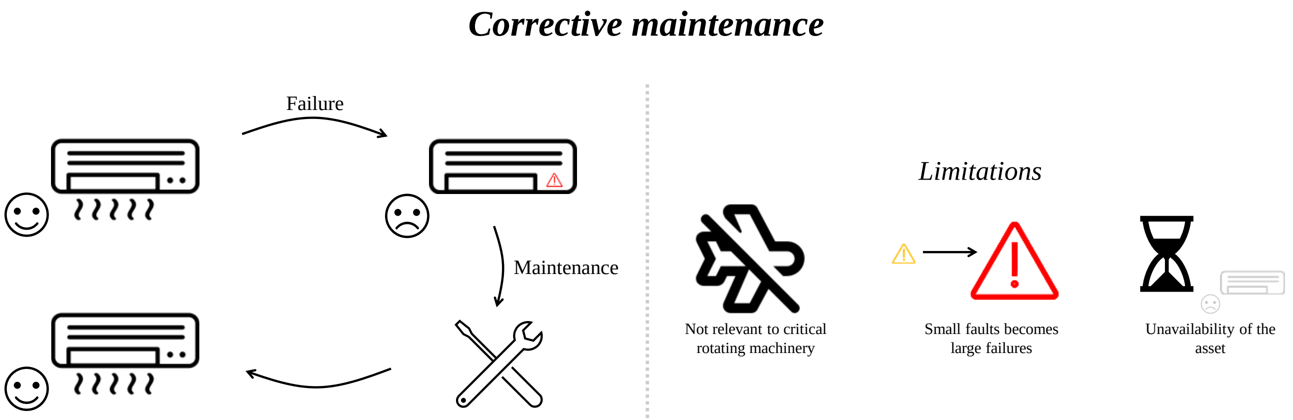

Fig. 3 – Illustration of corrective maintenance: After a failure occurs, maintenance action is applied.

In recent decades, condition-based maintenance methods have been developed to enhance maintenance practices [3]. One common approach to condition-based maintenance in complex rotating machinery is the use of vibrations analysis [4,5]. In this approach, vibration and shaft speed sensors are installed on the rotating parts of the system (e.g., a helicopter's rotor, gear casing, etc.), and as illustrated in **Fig. 4**, signal processing algorithms are employed to detect faults, identify their sources, estimating their severity and finally predicting the remaining useful life of the component. Estimating the remaining useful life and assessing the severity of the fault usually requires a historical case of faults (which are less relevant for systems such as helicopters and airplanes) and are currently active areas of research.

A profound understanding of the component's physical behavior guides signal processing algorithms tailored uniquely for each rotating component type. Most of the algorithms are based on two principles: (1) the vibrations of the rotating components are periodic due to the periodic nature of the rotation, hence most of the information about the health status of the rotating component is concentrated in a finite number of frequencies. (2) These frequencies can be calculated based on the specification of the component, such as dimensions, component type, and so on.

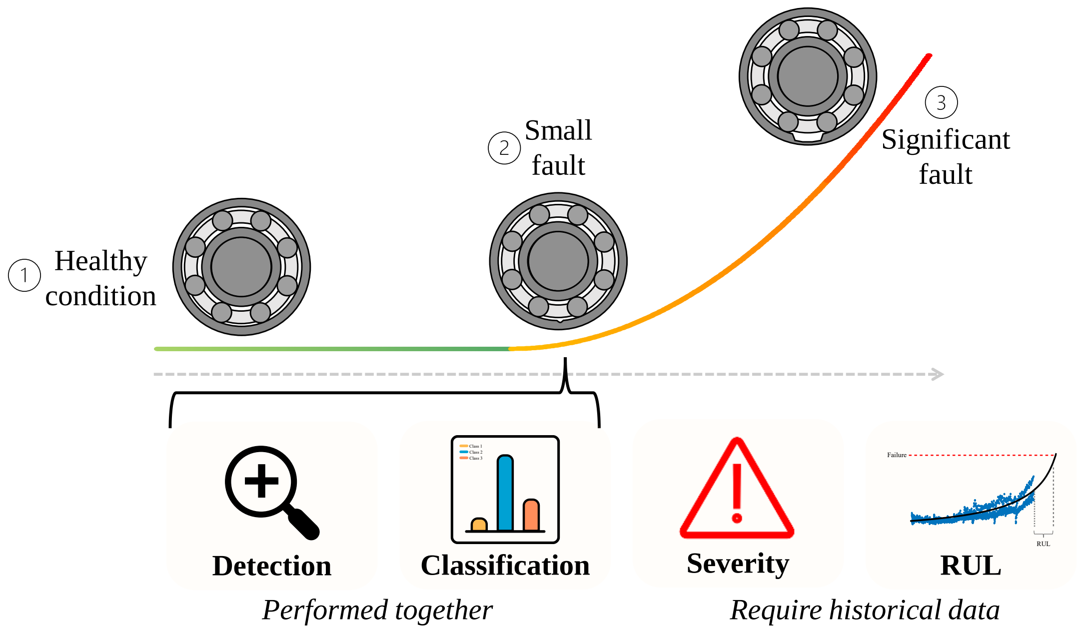

Fig. 4 – An illustration of condition-based maintenance algorithms goals: detection, classification, severity estimation and remaining useful life (RUL) estimation.

2. Fault examples

As explained in the tutorial, in bearings, there are four common faults that literature usually deals with, as shown in **Fig. 5(a)**: outer race fault, cage fault, inner race fault, and rolling element fault [6,7]. Gears also have a variety of tooth faults that literature addresses, such as illustrated in **Fig. 5(b)**: tooth breakage, pitting, missing tooth and root crack (there are also more types of faults in these rotating components).

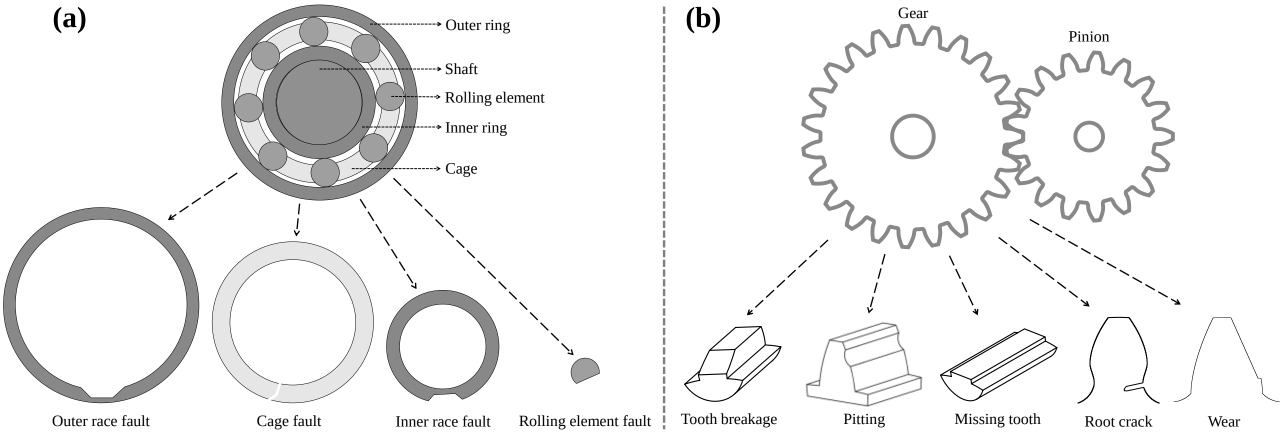

Fig. 5 – Examples of faults in rotating components: (a) examples of bearing faults, (b) examples of gear faults.

In this supporting materials file, we have also included several examples of faults observed in real rotating components. Figure 1 presents a type of pitting fault, Figure 2 depicts chipping and missing tooth faults, and Figure 3 shows an outer race spall.

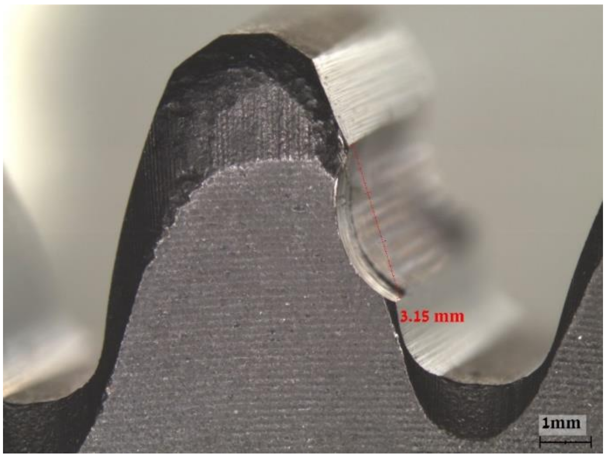

Fig. 6 – An example of a type of pitting fault.

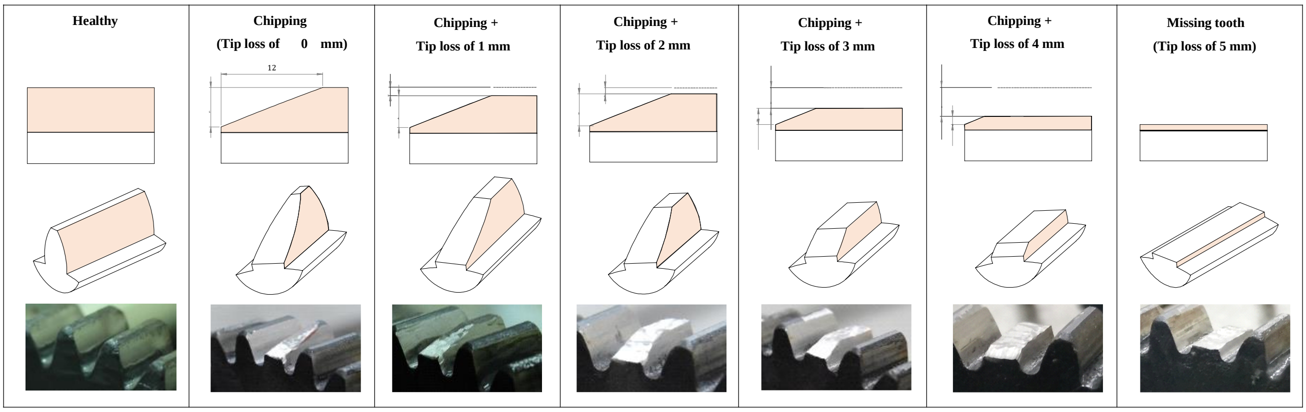

Fig. 7 – Examples of chipping and missing tooth faults.

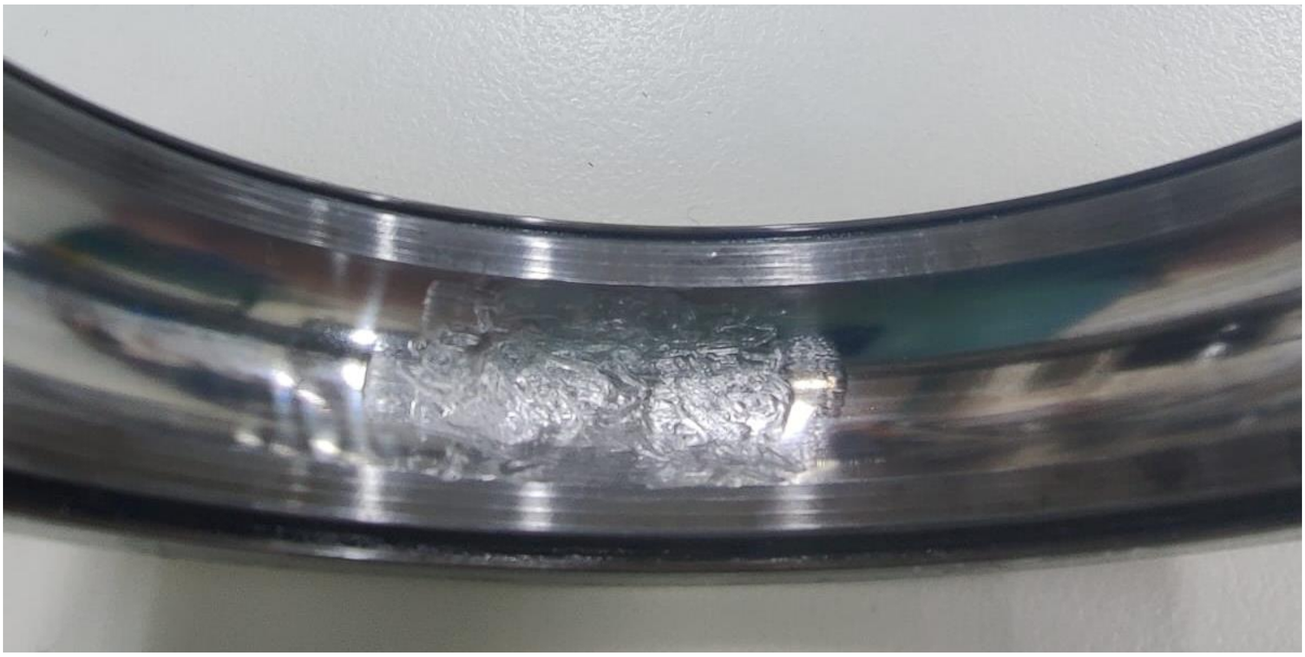

Fig. 8 – An example of an outer race spall.

3. Synchronous averaging using order analysis

Synchronous averaging [8–10] is implemented in the cycle domain after angular resampling by averaging the segments, as explained in the tutorial. This section will explain how synchronous averaging can be calculated using the order. It is less computationally efficient, but it sheds light on why synchronous averaging is a process designed to extract specific orders of interest.

The synchronous average is calculated using Eqn. 1, where sa is the calculated synchronous average with N samples, M is the number of averaged segments, sig is the signal in the cycle domain with a length of $M \cdot N$.

$$sa[n] = \frac{1}{M}\sum_{m=0}^{M-1} sig[m \cdot N + n] \tag{1}$$

As known, any signal s with the length L can be written as a sum of exponent vectors where the coefficients c_k are the coordinate values after discrete Fourier transform as presented in Eqn. 2.

$$s[n] = \frac{1}{L}\sum_{k=0}^{L-1} c_k \cdot \exp\left(2\pi i \frac{k \cdot n}{L}\right) \tag{2}$$

Henceforth, the signal sig from Eqn. 1 can be written using Eqn. 3, where a_k are its coordinate values after discrete Fourier transform.

$$sig[n] = \frac{1}{M \cdot N}\sum_{k=0}^{M \cdot N-1} a_k \cdot \exp\left(2\pi i \frac{k \cdot n}{M \cdot N}\right) \tag{3}$$

The effect of the synchronous averaging on the exponent vectors is analyzed in Eqns. 4-7, where $l \cdot M + d = k$ from Eq.2, and $d < M$.

$$\sum_{m=0}^{M-1} \exp\left(2\pi i \frac{(l \cdot M + d)(m \cdot N + n)}{M \cdot N}\right) = \sum_{m=0}^{M-1} \exp\left(2\pi i \frac{(l \cdot M + d) \cdot m \cdot N}{M \cdot N}\right) \cdot \exp\left(2\pi i \frac{(l \cdot M + d) \cdot n}{M \cdot N}\right) = \tag{4}$$

$$= \sum_{m=0}^{M-1} \exp\left(2\pi i \frac{l \cdot M \cdot m \cdot N}{M \cdot N}\right) \cdot \exp\left(2\pi i \frac{d \cdot m \cdot N}{M \cdot N}\right) \cdot \exp\left(2\pi i \frac{(l \cdot M + d) \cdot n}{M \cdot N}\right) = \tag{5}$$

$$= \sum_{m=0}^{M-1} \exp(2\pi i \cdot l \cdot m) \cdot \exp\left(2\pi i \frac{d \cdot m}{M}\right) \cdot \exp\left(2\pi i \frac{(l \cdot M + d) \cdot n}{M \cdot N}\right) \tag{6}$$

$$= \exp\left(2\pi i \frac{(l \cdot M + d) \cdot n}{M \cdot N}\right) \cdot \sum_{m=0}^{M-1} \exp\left(2\pi i \frac{d \cdot m}{M}\right) \tag{7}$$

Now, if $d = 0$ we get Eqn 8, and else we get Eqn 9.

$$\exp\left(2\pi i \frac{(l \cdot M) \cdot n}{M \cdot N}\right) \cdot \sum_{m=0}^{M-1} \exp\left(2\pi i \frac{0 \cdot m}{M}\right) = \exp\left(2\pi i \frac{(l \cdot M) \cdot n}{M \cdot N}\right) \cdot \sum_{m=0}^{M-1} 1 = M \cdot \exp\left(2\pi i \frac{k \cdot n}{M \cdot N}\right) \tag{8}$$

$$\exp\left(2\pi i \frac{(l \cdot M + d) \cdot n}{M \cdot N}\right) \cdot \sum_{m=0}^{M-1} \exp\left(2\pi i \frac{d \cdot m}{M}\right) = \exp\left(2\pi i \frac{(l \cdot M + d) \cdot n}{M \cdot N}\right) \cdot 0 = 0 \tag{9}$$

So, over all we get Eqns. 10-12:

$$sa[n] = \frac{1}{M}\sum_{m=0}^{M-1} sig[m\cdot N + n] = \frac{1}{M}\cdot\frac{1}{M\cdot N}\sum_{m=0}^{M-1}\sum_{k=0}^{M\cdot N-1} a_k \exp\left(2\pi i \frac{k\cdot(m\cdot N+n)}{M\cdot N}\right) = \tag{10}$$

$$= \frac{1}{M}\cdot\frac{1}{M\cdot N}\sum_{k=0}^{M\cdot N-1} a_k \exp\left(2\pi i\frac{k\cdot n}{M\cdot N}\right)\sum_{m=0}^{M-1}\exp\left(2\pi i\frac{k\cdot m\cdot N}{M\cdot N}\right) = \frac{1}{M}\cdot\frac{1}{M\cdot N}\sum_{k=0}^{N-1} a_{k\cdot M}\exp\left(2\pi i\frac{k\cdot n}{N}\right)\cdot M = \tag{11}$$

$$= \frac{1}{M\cdot N}\sum_{k=0}^{N-1} a_{k\cdot M}\exp\left(2\pi i\frac{k\cdot n}{N}\right) = \frac{1}{M}\cdot\frac{1}{N}\sum_{k=0}^{N-1} a_{k\cdot M}\exp\left(2\pi i\frac{k\cdot n}{N}\right) \tag{12}$$

I.E., we can present the synchronous average using Eq. 13.

$$sa[n] = \frac{1}{M}\cdot\frac{1}{N}\sum_{k=0}^{N-1} a_{k\cdot M}\exp\left(2\pi i\frac{k\cdot n}{N}\right) = \frac{1}{N}\sum_{k=0}^{N-1}\frac{a_{k\cdot M}}{M}\exp\left(2\pi i\frac{k\cdot n}{N}\right) \tag{13}$$

Thus, as depicted in **Fig. 9**, for calculating the synchronous average using the order domain, the signal is converted to the order domain, and then the values of the complete orders are extracted. After division by the number of averaged segments M, the signal is converted back to the cycle domain.

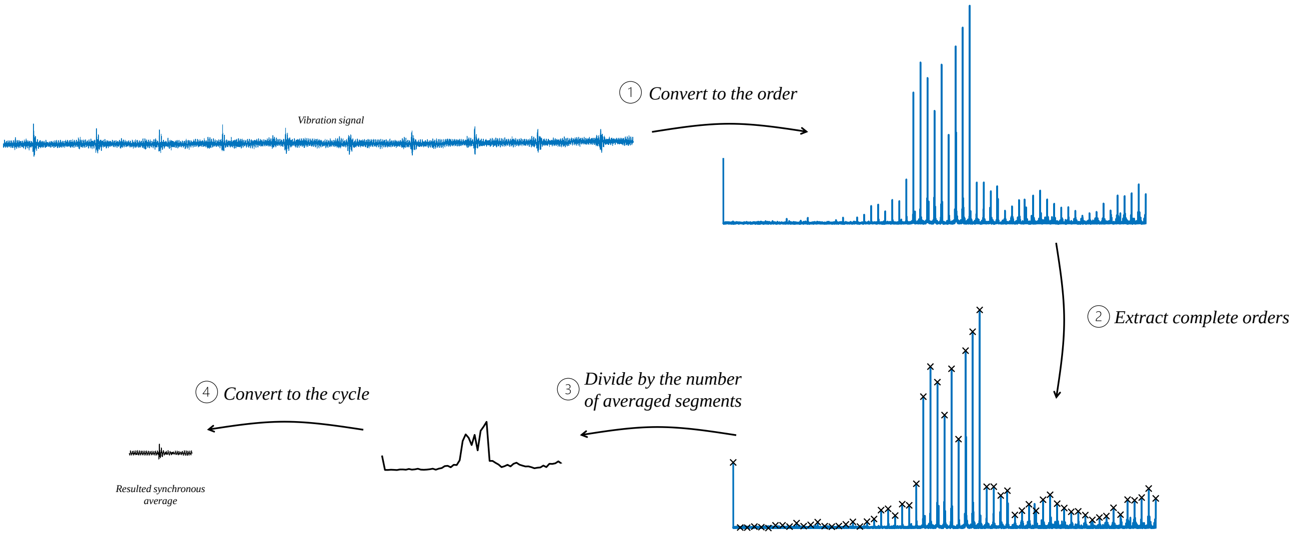

Fig. 9 – Block diagram of synchronous average calculated using the order domain by extracting orders of interest.

References:

[1] A.K.S. Jardine, D. Lin, D. Banjevic, A review on machinery diagnostics and prognostics implementing condition-based maintenance, Mech Syst Signal Process. 20 (2006) 1483–1510. https://doi.org/10.1016/J.YMSSP.2005.09.012.

[2] S. Kumar, D. Goyal, R.K. Dang, S.S. Dhami, B.S. Pabla, Condition based maintenance of bearings and gears for fault detection – A review, Mater Today Proc. 5 (2018) 6128–6137. https://doi.org/10.1016/J.MATPR.2017.12.219.

[3] R.B. Randall, Section 1.2: Maintenance Strategies, in: Vibration-Based Condition Monitoring: Industrial, Automotive and Aerospace Applications, 2nd ed., WILEY, 2021: pp. 2–3. https://www.wiley.com/en-us/Vibration+based+Condition+Monitoring%3A+Industrial%2C+Automotive+and+Aerospace+Applications%2C+2nd+Edition-p-9781119477556 (accessed October 23, 2023).

[4] R.B. Randall, State of the art in monitoring rotating machinery - Part 1, Sound and Vibration. 38 (2004) 14–21. https://www.researchgate.net/publication/293551810_State_of_the_Art_in_Monitoring_Rotating_Machinery_-_Part_1.

[5] R.B. Randall, State of the art in monitoring rotating machinery - Part 2, Sound and Vibration. 38 (2004) 10–17. https://www.researchgate.net/publication/283868453_State_of_the_art_in_monitoring_rotating_machinery_-_Part_2 (accessed September 8, 2021).

[6] R.B. Randall, J. Antoni, Rolling element bearing diagnostics—A tutorial, Mech Syst Signal Process. 25 (2011) 485–520. https://doi.org/10.1016/J.YMSSP.2010.07.017.

[7] W.A. Smith, R.B. Randall, Rolling element bearing diagnostics using the Case Western Reserve University data: A benchmark study, Mech Syst Signal Process. 64–65 (2015) 100–131. https://doi.org/10.1016/J.YMSSP.2015.04.021.

[8] S. Braun, The synchronous (time domain) average revisited, Mech Syst Signal Process. 25 (2011) 1087–1102. https://doi.org/10.1016/J.YMSSP.2010.07.016.

[9] R.B. Randall, Section 5.3.1: Time Synchronous Averaging, in: Vibration-Based Condition Monitoring: Industrial, Automotive and Aerospace Applications, 2nd ed., WILEY, 2021: pp. 178–180. https://www.wiley.com/en-us/Vibration+based+Condition+Monitoring%3A+Industrial%2C+Automotive+and+Aerospace+Applications%2C+2nd+Edition-p-9781119477556 (accessed October 21, 2023).

[10] E. Bechhoefer, M. Kingsley, A review of time synchronous average algorithms, in: Annual Conference of the Prognostics and Health Management Society, PHM 2009, 2009: pp. 1–10. https://www.researchgate.net/publication/265948472_A_Review_of_Time_Synchronous_Average_Algorithms.