moderate size, our asymptotic approximations are accurate.

# 8 Applications

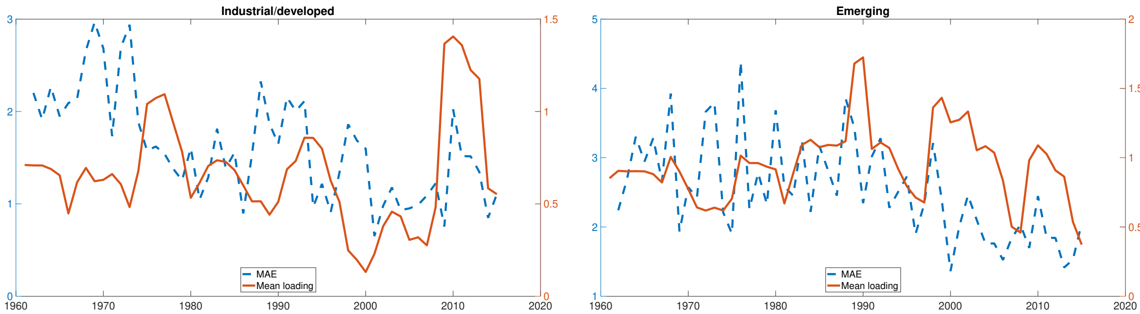
This section describes two empirical applications of IPCA to demonstrate its broad usefulness for analyzing economic data. The first is an application to international macroeconomics, where IPCA makes it easy analyze many nations' evolving relationships to global business cycles using country-level instruments. The second application builds on KPS and uses IPCA to analyze a dynamic model of asset risk and expected returns.

## 8.1 International Macroeconomics

Country-level macroeconomic fluctuations are globally connected (Backus et al., 1992). Using a static state-space model, Gregory et al. (1997) use maximum likelihood to document this for the G7 countries. Likewise, Kose et al. (2003) use a static state-space model estimated with Bayesian methods to disentangle global from regional and country-specific growth factors for a panel of countries over 30 years. Recently Kose et al. (2012) (henceforth KOP) used data from the World Development Indicators and estimated their static state-space model for a panel of countries before and after 1985 in order to analyze the convergence or decoupling of global business cycles. In essence, they ask: have countries' relationships to global growth changed as the countries themselves have evolved? This question is ideally suited for investigation with IPCA.

We use IPCA to analyze the global factor structure in GDP growth using data from the World Development Indicators database. We include as many countries as possible from the "industrial/developed" and "emerging" country groups studied by KOP. After reasonable data filters on indicators and countries, which we detail in the Appendix E, we are left with 45 countries. Within this sample there are nine variables that are available for most countries, so we use these as our instruments. The first two instruments are the import and export share of GDP—natural indicators of a country's economic connectedness with the rest of the world. Next we use the proportion of GDP relative to world GDP to measure the nation's relative size. We account for dynamics in capital intensity using gross capital formation, and we use popula-

Figure 5: MAE and Average Loading




*Notes:* The left axis shows the IPCA model mean absolute error in units of percentage annual growth and corresponds to the blue dotted line. The right axis corresponds to the equally weighted cross-sectional mean factor loading in each group, shown in red.

tion growth to account for growth in the labor force. To account for recent economic growth and risks, we include the 5-year rolling mean and volatility of the nation's GDP growth and its rate of inflation. Finally, we include a constant characteristic. Next, we double the set of instruments to 18 by interacting the nine variable above with an indicator for whether a country is in industrial/developed group. Our annual data run from 1961 to 2015 so that $T = 55$, and we demean the growth rates following KOP. About 91% of the 5,280 possible country-year observations are non-missing.

We study latent factor models with $K = 1$ and compare IPCA to the static-loading PCA estimator. We find a panel $R^2$ from the IPCA model of 32%, capturing roughly triple the variation in demeaned country growth explained by KOP. The $R^2$ from PCA is 22%, or two-thirds that achieved by IPCA. When making a head-to-head comparison of PCA and IPCA it is important to keep in mind two major differences between the estimators. The first is their stark difference in parameterization. IPCA achieves its fit using only 18 parameters to estimate its loadings, or 60% fewer parameters than the 45 used by PCA.[39] Second, IPCA accommodates dynamics in each country's loading on the global growth factor while PCA estimates static loadings. If countries converge or decouple as they evolve, IPCA's dynamic loadings are capable of detecting this. PCA's static loadings, on the other hand, cannot detect such dynamics and will instead try to fit an evolving system with a static model, and this type of misspecification is difficult to diagnose.

Results from IPCA show that beta dynamics are indeed critical to understanding

[39]These parameter counts are net of the 45 country-specific means used to demean the data in both IPCA and PCA.

Table 1: Global Growth Model Estimates

| | | | |
|---|---|---|---|
| GDP | -0.27* | Ind×GDP | 0.44* |
| Capital Formation | -0.09 | Ind×Capital Formation | 0.21 |
| Exports | 0.45 | Ind×Exports | -0.48 |
| Imports | -0.34 | Ind×Imports | 0.32 |
| Inflation | 0.23* | Ind×Inflation | -0.05 |
| Pop. Growth | -0.31 | Ind×Pop. Growth | 0.17 |
| Growth Vol. | 0.79* | Ind×Growth Vol. | 0.06 |
| Mean Growth | 0.00 | Ind×Mean Growth | -0.19* |
| Constant | -0.60* | Ind×Constant | 0.69* |

*Notes:* Estimated Γ coefficients scaled by the panel standard deviation of each instrument (the exceptions are Constant and Ind×Constant which are unscaled). The instruments are the log of GDP as a fraction of world GDP, gross capital formation, export and import share of GDP, inflation, population growth, 5-year rolling mean and volatility of GDP growth, and a constant. Each instrument is also interacted with an indicator for inclusion in the "industrial/developed" country group. An asterisk denotes statistical significance at the 10% level or better (using a bootstrap test following KPS).

the global business cycle. Figure 5 shows the time series of loadings on the global growth factor broken out by industrial/developed economies and emerging economies. For readability, we report the equally weighted cross-sectional average loading within each group of countries.

We see substantial variation in global growth sensitivity in each group. This is underpinned by an interesting state dependence in loadings—they rise sharply in economic downturns. While this is visually evident in the plot for industrial/developed countries, the precise nature of state dependence in loadings can be read from the estimated Γ matrix, which is shown in Table 1. To make estimates more interpretable, we scale each element of Γ to describe the effect on factor loadings from a one standard deviation increase in the associated instrument.

First, the constant and its interaction with the industrial dummy show that loadings in the industrial/developed group are significantly higher than those in emerging economies. But the largest and perhaps most interesting finding is the role of growth volatility in describing state dependence in global growth sensitivity. A well documented pattern in the business cycle literature is the spike in growth volatility associated with recessions (Bloom, 2014).Table 1 shows that such rises in volatility are accompanied by concomitant rises in sensitivity to the global growth factor. It also shows that the dependence of growth sensitivity is one of the few instruments that plays a similar role in both developed and emerging countries. For the emerging group, a one standard deviation increase in growth volatility associates with an increased loading of 0.79 on the global growth factor, versus 0.85 for the industrial group (the difference is insignificant). The only other instrument that the two groups agree on is inflation. We see that higher inflation associates with high global sensitivity, that this effect is slightly stronger in emerging economies (estimate of 0.23), but that the difference versus developed economies is insignificant. For the remaining instruments, our estimates show significant differences in the drivers of global exposure. Emerging economies are especially sensitive to global fluctuations if they have high exports, low imports, are relatively small, have high inflation, and have low population growth. In industrial/developed group, the effects of most instruments other than volatility net to nearly zero. One other significant effect in industrial/developed countries is that global sensitivity rises when recent growth has been low (based on the significant coefficient of $-0.19$ on recent mean growth). This compounds the jump in global sensitivity associated with a rise in volatility, because recent growth volatility and recent mean growth are negatively correlated.

Finally, we see some broad evidence of global convergence from the dotted lines shown in Figure 5. These describe the mean absolute error (MAE) from the IPCA model each year. They show that over time the global growth factor has become increasingly successful at describing the full panel of growth rates. This is evident from the downward trend in MAE among both industrial and emerging economies. Overall, IPCA results illustrate an important role for dynamic loadings in global business cycle models and show that IPCA's ability to accommodate such dynamics ultimately delivers a more accurate description of the data than leading alternatives.

## 8.2 Asset Pricing

KPS apply IPCA to describe systematic risk and associated risk premium (cost of capital) of US stocks, where systematic risk is defined as dynamic loadings on latent factors that are instrumented by stock characteristics. Here, we expand upon their asset pricing context and use IPCA to calculate systematic risk and cost of capital for *newly listed* firms that the model has not seen before. This is motivated by the challenging question of how to value private firms and if it is possible to use information in publicly traded equity prices for this purpose. Like in the macroeconomic example, IPCA is ideally suited to address this question because the model parameterizes risk and cost of capital as a function of firm characteristics. IPCA finds the mapping between the return behavior of traded firms and their characteristics, which can then be extrapolated to non-traded firms to approximate their as-if traded value. Note that this is not possible with standard empirical asset pricing methodologies, which require the history of publicly traded prices for individual assets to infer their future risk premia (and thus cost of capital).

Our data consists of over 2.9 million stock-month observations of excess stock returns ($x$) and 93 associated firm characteristics ($z$) from 1965-2018.[40] We use lagged firm characteristics to instrument for the conditional systematic risk loadings ($c_{i,t} = z_{i,t-1}$).[41] The out-of-sample evaluation is performed for newly listed firms, defined as the first 12 months following a firm's initial public offering (IPO). In our data, 21,275 firms have an IPO, comprising 249,414 out-of-sample stock-month observations. In-sample estimation is based on the complementary 2.6 million observations of incumbent stocks that excludes those in the test sample of new listings.

We calculate the total $R^2$ and predictive $R^2$, as defined in KPS, to evaluate the estimates of stocks' systematic risk and expect returns, respectively. The procedure starts by first estimating $\widehat{\Gamma}$ and $\{\widehat{f}_t\}$ within the in-sample data set of incumbent stocks. Then, the estimates are brought to the data of new listings to calculate fitted values as

$$\widehat{x}_{i,t+1}^{\text{Tot}} := z_{i,t}\widehat{\Gamma}\widehat{f}_{t+1}, \qquad \widehat{x}_{i,t+1}^{\text{Pred}} := z_{i,t}\widehat{\Gamma}\widehat{\lambda}$$

for all $i, t$ in the out-of-sample data set. The first term $\widehat{x}_{i,t+1}^{\text{Tot}}$ reconstructs the realized return as the factor model's fitted value (that is, using the factor realization, $\widehat{f}_{t+1}$). The second term is a prediction of the new listing return and replaces the factor realization with the factor's estimated mean $\widehat{\lambda}$, directly following KPS.[42] Based on these fits, we calculate the out-of-sample total and predictive $R^2$ as the explained variation in $x_{i,t+1}$ due to $\widehat{x}_{i,t+1}^{\text{Tot}}$ and $\widehat{x}_{i,t+1}^{\text{Pred}}$, respectively.

Table 2 reports the results for the IPCA model (with $K = 4$, as advocated by KPS). The close similarity of the in-sample and out-of-sample total $R^2$ indicates the

[40] The dataset is from Gu et al. (2020). Firm characteristics are transformed into ranks on the interval $[-0.5, 0.5]$ as in KPS. Any missing characteristic is assigned the value 0, which is replacement with the cross-sectional mean/median.

[41] The information content of unexpected idiosyncratic return shocks is formalized by Assumption A.

[42] $\widehat{\lambda}$ is calculated as the in-sample time series mean of $\widehat{f}_{t+1}$.

Table 2: Explained Variation of Stock Returns

| | Total $R^2$ | Predictive $R^2$ |
|---|---|---|
| Incumbent stocks (in-sample) | 15.66 | 0.25 |
| New listings (out-of-sample) | 13.44 | 0.22 |

*Notes:* $R^2$ in percentage. Based on IPCA with $K = 4$ for the 1965-2018 US stock-month panel.

same characteristics that determine the systematic riskiness of incumbent stocks also determine the riskiness of new listings. In other words, once we condition on firm characteristics, the model finds a highly similar description for the common variation among returns on newly listed stocks compared to the common variation in returns on incumbents.

While the total $R^2$ is indicative of the model's ability to describe systematic risks of new listings, the predictive $R^2$ summarizes the model's description of their expected returns (or, in equivalent terms, cost of capital or discount rates). That is, the predictive $R^2$ is especially informative about the usefulness of the model for asset valuation. The close similarity of the in-sample and out-of-sample predictive $R^2$ indicates that the IPCA model is as effective at "pricing" new listings as it is for pricing incumbent stocks. The most important takeaway is that the model does this without using the individual return history of the new listings (which is of course unavailable and the crux of the research question), but manages to price them nonetheless by extrapolating what it learns from data on incumbent stocks.[43]

# 9 Conclusion

This paper has introduced a new approach of modeling and estimating the latent factor structure of panel data, called Instrumented Principal Component Analysis (IPCA). The key innovation is using additional panel data to instrument for the dynamic factor loadings. Mainly, each individual's time-varying factor loading is related to instrumental data according to a common and constant mapping.

Estimating this mapping, rather than the factor loadings directly, has many econometric advantages compared to other latent variable estimators like PCA. On one

[43]This performance is even more remarkable when we recognize that new firms' stock returns are more variable than incumbents.