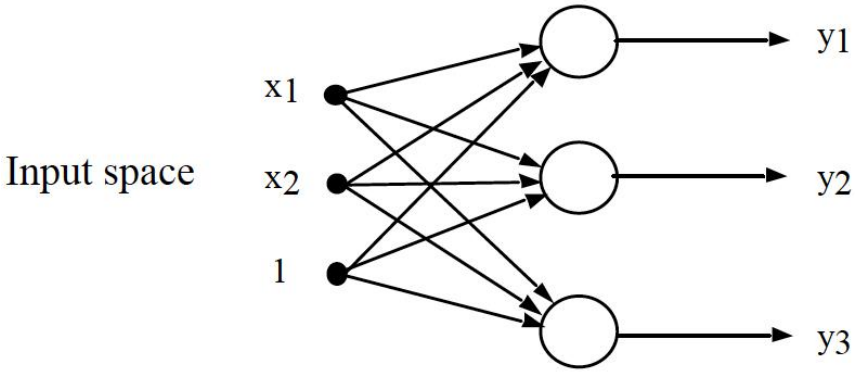
26.4.2018

MECA 533 Homework 1

Q1. Give weights and bias for a McCulloch-Pitts (M-P) neuron with inputs x, y, and z, and whose output is z if $x = -1$ and $y = 1$, and is -1 otherwise.

Q2. Give the following 3-class classification problem:
C_1: {(4, 1), (2, 3), (3, 5), (5, 4), (1, 6)}
C_2: {(0, 2), (-2, 2), (-3, 2), (-2, 4)}
C_3: {(1, -2), (3, -2)}
and the following single layer perceptron:



(a) Can the net learn to separate the samples, given that you want: if $\mathbf{x} \in C_i$ then $y_i = 1$ and $y_j = -1$ for $j \neq i$. No need to solve for the weights, but justify your answer.
(b) Add the sample (-1, 6) to C_1. Repeat part (a).

Q3. (a) For the following training samples:

$\mathbf{x}_1 = (0, 0)^T \in C_1$
$\mathbf{x}_2 = (0, 1)^T \in C_1$
$\mathbf{x}_3 = (1, 0)^T \in C_2$
$\mathbf{x}_4 = (1, 1)^T \in C_2$

Plot them in input space. Apply the perceptron learning rule to the above samples one-at-a-time to obtain weights that separate the training samples. Set η to 0.5. Work in the space with the bias as another input element. Use $\mathbf{w}(0) = (0, 0, 0)^T$. Write the expression for the resulting decision boundary.

(b) Repeat (a) for the XOR function where $\mathbf{x}_2, \mathbf{x}_3 \in C_1$ and $\mathbf{x}_1, \mathbf{x}_4 \in C_2$. Describe your observation when you apply the perceptron learning rule.

Q4. Given the following input points and corresponding desired outputs:

X = {-0.5, -0.2, -0.1, 0.3, 0.4, 0.5, 0.7}
D = {-1, 1, 2, 3.2, 3.5, 5, 6}

write down the cost function with respect to w (setting the bias to zero). Compute the gradient at the point $w = 2$ using both direct differentiation and LMS approximation (average for all data samples in both cases), and see if they agree.