5.10 The signal model, $x(t)$, for the output of the horizontal sweep oscillator in a monochrome monitor is shown in Figure 5.61. The sweep return time is very small; therefore, it has been modeled as negligible. The sweep time is $t_s = 1/15.75$ ms.

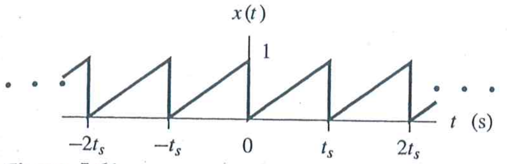


Figure 5.61

a. Plot the double-sided amplitude and phase spectra of the sweep signal up to a frequency of 65 kHz.

b. What fraction of the signal power is contained in the approximate signal corresponding to the spectrum portion plotted?

5.11 A full-wave rectifier is the first step in an AC to DC convertor. It converts a sinusoidal signal with frequency of 5 Hz into the signal $x(t)$ shown in Figure 5.62.

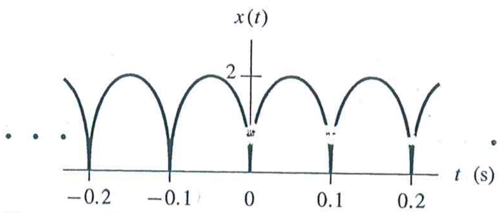


Figure 5.62

a. Plot the double-sided amplitude and phase spectra for the full-wave rectified sinusoidal signal. Plot for $|f| < 50$ Hz.

b. What fraction of the signal power is contained in the approximate signal corresponding to the spectrum portion plotted?

5.12 Repeat Problem 5.11 for the half-wave rectified signal shown in Figure 5.63.

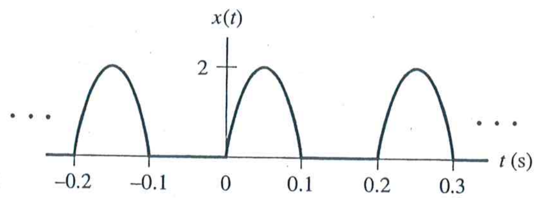


Figure 5.63

5.13 The periodic signal

$$y(t) = \sum_{i=-\infty}^{\infty} y_p(t - 0.1i)$$

where

$$y_p(t) = 3\prod\left[(t + 0.04)/0.04\right]$$

is a train of pulses used in a time-division multiplexer. It periodically turns one signal on for 40 ms and then off for 60 ms. Find and plot the double-sided amplitude and phase spectra of $y(t)$ for $|f| < 75$ Hz.

5.14 Verify the Fourier transform pair

$$\Lambda(t/\tau) \leftrightarrow \tau \sin^2(\tau f)$$

5.15 Verify the Fourier transform pair

$$e^{-a|t|} \leftrightarrow 2a/\left[a^2 + (2\pi f)^2\right] \qquad \text{where } a > 0$$

5.16 Verify the Fourier transform pair

$$te^{-at}u(t) \leftrightarrow \frac{1}{(a + j2\pi f)^2} \qquad \text{where } a > 0$$

5.17 Verify the Fourier transform pair

$$\sin 2\pi f_0 t \leftrightarrow \frac{1}{2j}\left[\delta(f - f_0) - \delta(f + f_0)\right]$$

5.18 Which of the following signals are time limited, band limited, or neither. Show your reasoning for your answers.

a. $x(t) = e^{-3t}u(t - 1)$

b. $y(t) = \prod\left[(t - 3)/4\right] - \Lambda\left[(t - 3)/2\right]$

c. $z(t) = 4\,\mathrm{sinc}^2(3t)$

d. $w(t) = \prod\left[(t - 1)/2\right] - \mathrm{sinc}(t)$

5.19 Indicate whether the following signals have complex, real and even, or imaginary and odd Fourier transforms. Which complex transforms have complex-conjugate symmetry about $f = 0$? State reasons for your answers.

a. $x(t) = e^{-|t-1|}$

b. $y(t) = 2\prod\left[t/4\right]$

c. $z(t) = 3\sin 6\pi t$

d. $w(t) = 4\cos(2\pi t - \pi/4)$

e. $a(t) = \Lambda(t)$

f. $b(t) = 3e^{j4|t|}$

5.20 Find and plot the amplitude, phase, and energy spectra for the signal $y(t) = e^{3t}u(-t)$.

5.21 Find and plot the amplitude and phase spectra or, if possible, the total spectrum for the time-limited signals:

a. $x(t) = e^{-0.1t}\prod(t/8)$

b. $v(t) = e^{-0.1|t|}\prod(t/8)$
c. $i(t) = 2t^2\prod(t/2)$
d. $i(t) = 2t^2\prod[(t-1)/2]$

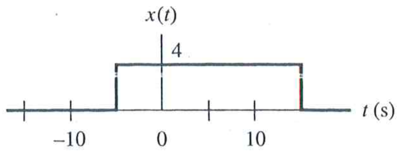
5.22 Prove the scale-change theorem (Theorem 2).

5.23 Prove the complex-conjugation theorem (Theorem 4).

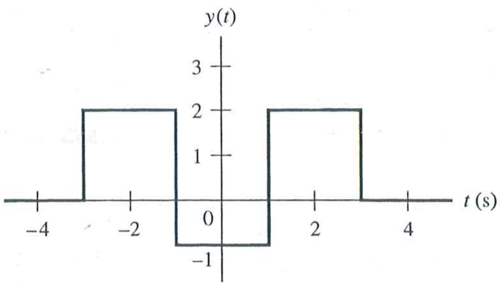
5.24 Prove the time-differentiation theorem (Theorem 9) for $n = 1$.

5.25 Prove the multiplication theorem (Theorem 12).

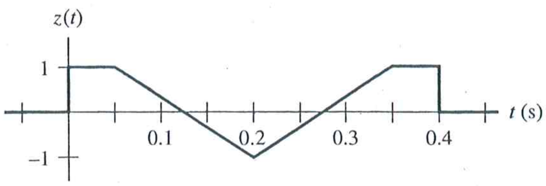
5.26 Use the table of Fourier transforms (Table C.2) and the theorems to find the spectra for the signals shown in Figure 5.64. Plot the amplitude and phase spectra or, if possible, the total spectrum for each signal.



(a)



(b)



(c)

$v(t) = 2 + te^{-0.1t}u(t-2)$

(d)

Figure 5.64

5.27 The signal $x(t)$ has a spectrum

$$X(f) = 3/(2 + j\pi f)$$

Use the theorems to find the spectrum for the lowing signals.

a. $x_a(t) = 2\frac{dx(t)}{dt}$
b. $x_b(t) = x(-2-t)$
c. $x_c(t) = x(-t/4)$
d. $x_d(t) = e^{-j3t}x(t-5)$
e. $x_e(t) = 3x(t)\cos 6\pi t$
f. $x_f(t) = x(4t-3)$

5.28 The signal $w(t)$ has a spectrum

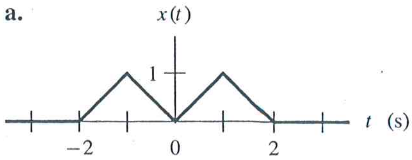
$$W(f) = (j\pi f)/(1 + j2\pi f)$$

Use the theorems to find the spectrum for the lowing signals.

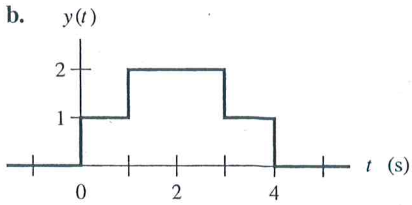
a. $w_a(t) = w(t/5)$
b. $w_b(t) = 2w(t-1)\cos 4\pi t$
c. $w_c(t) = w(t) - \frac{dw(t)}{dt}$
d. $w_d(t) = w(2t+4)$
e. $w_e(t) = e^{-j2t}w(t+3)$
f. $w_f(t) = w(-4-2t)$

5.29 Use the table of Fourier transforms (Table and the theorems to find the spectra for the sig shown in Figure 5.65. Plot the amplitude and p spectra or, if possible, the total spectrum for signal.

a.



b.



c. $v(t) = 1 - e^{-0.2|t+1|}$

Figure 5.65

5.30 A cosine carrier signal with frequency of 2 Hz is used with double-sideband amplitude mo lation to transmit the single pulse message si $m(t) = 4\prod(t/0.0025)$. Sketch the transmitted nal and plot its amplitude and phase spectra o possible, its total spectrum.

5.31 Use the Fourier transform to find the amplitude and phase spectra for the following signals.

a. $x(t) = 4\cos(100\pi t + \pi/2.5) + 2$

b. $y(t) = 12\sin(10\pi t - \pi/2) + 5\sin(25\pi t + \pi/4)$

c. $z(t) = 3\cos(3\pi t) + 4\cos(3\pi t)\cos(6\pi t - \pi/3)$

5.32 An infrared sensor mounted on the belly of an aircraft sweeps linearly back and forth over the angle range from vertical to 1 radian from vertical. In this way, it produces an infrared image of the ground on one side of the aircraft flight path. The angular sweep signal $\theta(t)$ used to drive the sensor is shown in Figure 5.66. Use the Fourier transform table and theorems to find the spectrum of the sweep signal. Plot the amplitude and phase spectra or, if possible, the total spectrum for frequencies less than or equal to 10 Hz.

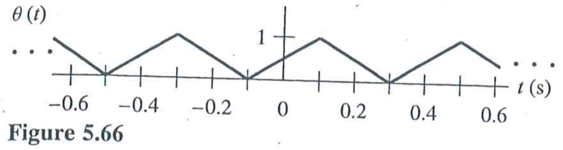


Figure 5.66

5.33 A video camera used to monitor a scene is mounted on the corner of a building and scans by continual rotation. Since the camera points at the building walls for 1/4 of its scan, we want to blank its video signal for this portion of time. Figure 5.67 shows the signal that turns the video on and off. Use the Fourier transform table and theorems to find the spectrum of $x(t)$. Plot the amplitude and phase spectra or, if possible, the total spectrum for frequencies less than or equal to 0.1 Hz.

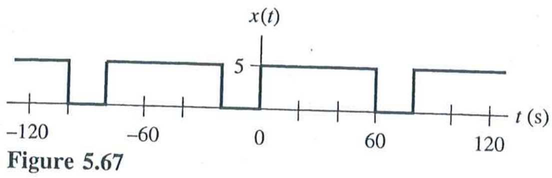


Figure 5.67

5.34 Use the Fourier transform table and theorems to find the spectrum of the periodic signal shown in Figure 5.68. Plot the amplitude and phase spectra or, if possible, the total spectrum for frequencies less than or equal to 1 Hz.

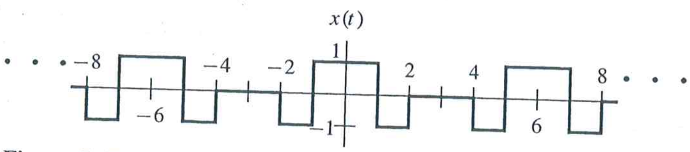


Figure 5.68

5.35 Use the Fourier transform table and theorems to find the spectrum of the signal

$$x(t) = A\prod(f_0 t/N)\cos 2\pi f_0 t$$

for (a) $N = 1$, (b) $N = 2$, (c) $N = 4$. Plot the three signals. Also plot the spectra of the signals for frequencies less than $3f_0$.

5.36 A pulse radar transmits a cosine carrier signal of frequency 4 MHz modulated by the pulse train signal

$$x(t) = \sum_{n=-\infty}^{\infty}\prod\left(\frac{t - iT}{\tau}\right)$$

where $\tau = 1\mu s$ and $T = 20\mu s$. Use the Fourier transform table and theorems to find the spectrum

Concept	Equation	
• The system frequency response can be found by using phasor concepts to solve for the output/input phasor ratio for a sinusoidal signal having frequency f.	(6.28)	
• System phase delay is the time delay experienced by a single-frequency signal in passing through the system.	(6.33)	
• System group delay is the time delay experienced by the amplitude modulation on a cosine carrier in passing through the system.	(6.43)	
• The amplitude-response Bode plot is amplitude in dB plotted on a logarithmic frequency scale.	(6.45)	
• The phase-response Bode plot is phase in degrees plotted on a logarithmic frequency scale.		
• Straight-line approximations to Bode plots for frequency-response factors and their sum are easily constructed. They show basic frequency-response characteristics and their dependence on frequency-response parameters.		
• The straight-line approximation and amplitude-response values computed at break and peak frequencies can be used to produce a quick, reasonably accurate, amplitude-response Bode plot sketch.		

Problems

6.1 A system has the frequency response $H(f) = 500/[(500 - f^2) + j45f]$ and an input signal $x(t) = \cos(10\pi t) + \cos(20\pi t + \pi/3) + \cos(80\pi t - \pi/4)$.
a. Plot the system amplitude and phase responses.
b. Find and plot the amplitude and phase spectra for the input and output signals.

6.2 A system has the frequency response $H(f) = 2000/[(40{,}000 - f^2) + j10f]$ and an input signal $x(t) = 2 - 4\sin(360\pi t) + 4\cos(600\pi t - \pi)$.
a. Plot the system amplitude and phase responses.
b. Find and plot the amplitude and phase spectra for the input and output signals.

6.3 A system has the frequency response $H(f) = 900f^2 \big/ \big[(-f^4 + 2500f^2 - 640{,}000) + j\left(60f^3 - 48{,}000f\right)\big]$ and an input signal $x(t) = 105\cos(30\pi t - 1.75) + 42\cos(50\pi t - 0.5) + 57\cos(80\pi t + 1.2) + 90\cos(100\pi t + 1.7)$.
a. Plot the system amplitude and phase responses.
b. Find and plot the amplitude and phase spectra for the input and output signals.

6.4 The system of Problem 6.3 has the following input signal: $x(t) = 40\,\mathrm{sinc}(20t)\cos(60\pi t)$.
a. Plot the system amplitude and phase responses.
b. Find and plot the amplitude and phase spectra for the input and output signals.

6.5 Plot the amplitude and phase responses, proximate the system half-power bandwi half-power cutoff frequency or frequencies systems with the frequency responses liste cate whether the systems are low-pass sys bandpass systems.
a. $H(f) = 100/(50 + jf)$
b. $H(f) = 100/[(2 + jf)(5 + jf)]$
c. $H(f) = 100f/[100f + j(f^2 - 60{,}000)]$
d. The frequency response of Problem 6.1
e. The frequency response of Problem 6.2

6.6 Find the approximate half-power bandwi a low-pass electric filter having the followi quency response.

$$H(f) = 1500\big/\big[(3000 - 45f^2) + jf(350 -$$

6.7 The input signal to the system of Problem a rectangular pulse centered at $t = -2.25$ m a width of 4.5 ms.
a. Plot the system amplitude and phase resp
b. Find and plot the amplitude and phase for the input and output signals.

6.8 A signal-repeater system in a communicatio lite provides signal filtering and delays the before retransmission. The repeater system frequency response

$$H(f) = \exp(-0.1\pi f)/[1 + j(f/2)]$$

a. Plot the system amplitude and phase responses.
b. Find and sketch the system impulse response.

6.9 An electric filter has the frequency response

$$H(f) = j0.02f/(1 + j0.02f)$$

a. Plot the filter amplitude and phase responses.
b. Find and sketch the filter impulse response. (*Hint*: Use the time differentiation theorem.)
c. Approximate the filter half-power cutoff frequency or frequencies.
d. From its amplitude response, what type of filter is this?

6.10 A system has the frequency response

$$H(f) = [1/(1 + j0.5f)] + [j0.05/(1 + j0.05f)]$$

a. Plot the system amplitude and phase responses.
b. Find and sketch the system impulse response. (*Hint*: Use the linearity and time-differentiation theorems.)
c. Approximate the system half-power cutoff frequency or frequencies.
d. From its amplitude response, what type of filter is this?

6.11 A control system is characterized by the following system differential equation.

$$\frac{d^3y(t)}{dt^3} + 125\frac{d^2y(t)}{dt^2} + 5000\frac{dy(t)}{dt} + 10^5y(t) = 10^5x(t)$$

a. Find the frequency response for this system.
b. Plot the system amplitude and phase responses.

6.12 A system is characterized by the system differential equation

$$\frac{dy(t)}{dt} + 0.5y(t) = \frac{dx(t)}{dt} + x(t)$$

a. Find the system frequency response.
b. Plot the system amplitude and phase responses.
c. Find the system impulse response.

6.13 Consider the pipeline flow rate–control system of Problem 4.6. It is represented by the block diagram in Figure 6.38.

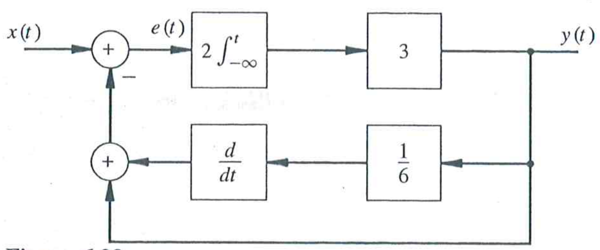


Figure 6.38

a. Find the system differential equation.
b. Find the system frequency response.
c. Plot the system amplitude and phase responses.
d. Find the system impulse response.

6.14 Work Problem 6.13 by using the phasor concept instead of finding and using the system differential equation.

6.15 Consider the electric signal transmission network shown in Figure 6.39 where $v_i(t)$ and $v_o(t)$ are the input and output signals, respectively.

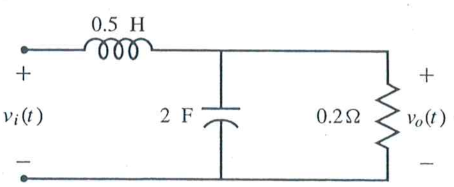


Figure 6.39

a. Find the system differential equation and use it to find the system frequency response.
b. Find the system frequency response by using phasor techniques.

6.16 Repeat Problem 6.15 for the network shown in Figure 6.40, where $i_i(t)$ and $i_o(t)$ are the input and output signa respectively.

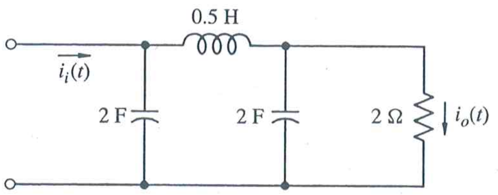


Figure 6.40

6.17 A simple model of an operational amplifier used as a low-frequency, low-pass filter in a temperature-contr system is shown in Figure 6.41.

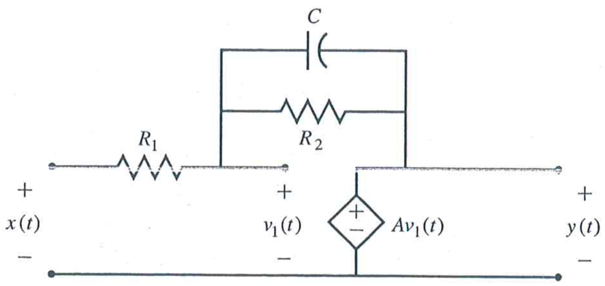


Figure 6.41

a. Find the frequency response of the circuit.
b. Plot the amplitude and phase responses, when $R_1 = 1\text{M}\Omega$, $R_2 = 2\text{M}\Omega$, $C = 0.1\mu\text{F}$, and $A = 1000$.

6.18 Consider the electric circuit shown in Figure 6.42, where the input signal is $v(t)$ and the output signal is $i(t)$.

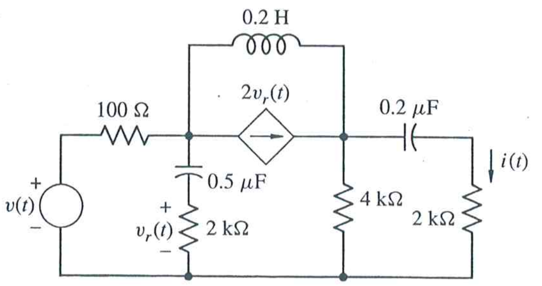


Figure 6.42

a. Find the frequency response of this circuit.
b. Plot the amplitude and phase response.

6.19 Find the phase and group delay for the system of Problem 6.2 at the frequencies (a) 100 Hz, (b) 190 Hz, (c) 210 Hz, and (d) 300 Hz.

6.20 Find the phase delay and group delay for th flow-rate control system of Problem 6.13 at the fre quencies (a) 0.1 Hz, (b) 0.4 Hz, (c) 0.6 Hz, (d) 1 Hz and (e) 2 Hz.

6.21 A system has the frequency response

$$H(f) = j0.01f/(1 + j0.01f)$$

Find the phase delay and group delay for this sys tem at the frequencies: (a) 10 Hz, (b) 50 Hz, (c) 100 Hz, (d) 200 Hz, and (e) 1000 Hz.

6.22 Given the system frequency response

$$H_\omega(\omega) = \frac{20(j\omega + 25)^2}{j\omega(j\omega + 500)}$$

a. Find and plot the straight-line approximations to the amplitude- and phase-response Bode plots for the system.
b. Sketch the approximate amplitude-response Bod plot for the system using the straight-line ap proximation and approximate values at break and/or peak frequencies.

c. Sketch the approximate phase-response Bode plot for the system using the straight-line approximation and a few calculated points.

6.23 Work Problem 6.22 with the frequency responses

a. $H_\omega(\omega) = \frac{500(j\omega+15)}{j\omega(j\omega+350)}$

b. $H_\omega(\omega) = \frac{600j\omega+600}{0.25(j\omega)^2+51(j\omega)+200}$

6.24 Work Problem 6.22 with the frequency responses

a. $H_\omega(\omega) = \frac{180(j\omega)+360}{0.006(j\omega)^2+3.06(j\omega)+30}$

b. $H_\omega(\omega) = \frac{(j\omega)[(2\times10^6)(j\omega)+10^7]}{(j\omega+35)^2[(j\omega)^2+200(j\omega)+10^6]}$

6.25 Work Problem 6.22 with the frequency responses

a. $H_\omega(\omega) = \frac{225,000}{(j\omega+1000)[(j0.01\omega)^2+j0.1\omega+225]}$

b. $H_\omega(\omega) = \frac{1800(j\omega)^2(j\omega+400)}{(j0.1\omega+2)^2[(j0.01\omega)^2+j2.4\omega+160000](j\omega+800)}$

6.26 Work Problem 6.22 with the frequency responses

a. $H_{a\omega}(\omega) = 100\big/(j\omega+100)$ and

b. $H_{b\omega}(\omega) = 6,850,000(j\omega+110)^2\,/$

$$\left[(j\omega+100)(j\omega+170)^4\right]$$

Compare the two frequency responses.

Problems for Computer Solution

6.27 The input signal to the system of Problem 6.3 is

$$x(t) = 10\cos(50\pi t)\Pi[(t-0.5)/0.2]$$
$$+\,2\sin(100\pi t)\Pi(t-0.5)$$

a. Plot the input signal.
b. Plot the system amplitude and phase responses.
c. Plot the amplitude and phase spectra for the input and output signals.
d. Plot the output signal.

6.28 Plot the positive frequency portion of the amplitude response for the system defined in Problem 6.2. Compute, to a precision of 0.5 Hz, the system cutoff frequency, or frequencies, and bandwidth.

6.29 An acoustic sensor used in a remote underwater noise detecting system is characterized by the differential equation

$$y^{(6)}(t) + 3160y^{(5)}(t) + 6.14\times10^6y^{(4)}(t)$$
$$+\,4.09\times10^9y^{(3)}(t) + 2.91\times10^{12}y^{(2)}(t)$$
$$+\,2.40\times10^{14}y^{(1)}(t) + 2.56\times10^{16}y(t)$$
$$= 10^9x^{(3)}(t) + 1.022\times10^{12}x^{(2)}(t)$$
$$+\,2.204\times10^{13}x^{(1)}(t) + 4\times10^{13}x(t)$$

where $x^{(n)}(t) = d^nx(t)/dt^n$. Plot the system amplitude and phase response for $10 \le \omega \le 10{,}000$. Use a logarithmic frequency scale.

6.30 Find the phase delay and approximate group delay for the acoustic sensor in Problem 6.29 for the frequencies: (a) 20 Hz, (b) 30 Hz, (c) 40 Hz, and (d) 50 Hz.

6.31 Find the phase delay and approximate group delay for the acoustic sensor in Problem 6.29 for the frequencies (a) 20 Hz, (b) 60 Hz, (c) 1 kHz, and (d) 2 kHz.

6.32 Use MATLAB to find the amplitude- and phase-response Bode plots and their straight-line approximations for the system frequency response given in part **b** of Problem 6.25.

6.33 Use MATLAB to find the amplitude and phase-response Bode plots and their straight-line approximations for the acoustic sensor defined in Problem 6.29.

Concept	Equation	Page
• Ideal filters may be low-pass (LPF), high-pass (HPF), bandpass (BPF), or band-reject (BRF).		315
• Ideal filters are noncausal.	(8.7)	317
• Physical filters can be constructed to approximate either the ideal filter amplitude response or the ideal filter phase response.		321
• It is not possible to optimize arbitrarily both the amplitude and phase response approximations.		321
• Better approximations always result in increased filter time delay.		328
• More than one transfer function can produce the same amplitude response.		321
• The unique minimum-phase, stable system that produces a given amplitude response has all poles and zeros in the left-half plane.		323
• The Butterworth approximation to the ideal low-pass filter is maximally flat at DC and has gain that is down 3 dB from the maximum gain at the cutoff frequency.	(8.19)	326
• Poles of the Butterworth filter lie on a circle with radius equal to the cutoff frequency.		326
• As filter order increases, the approximation to the ideal low-pass filter improves, more components are required, larger components are required, and time delay increases.		328
• The Chebyshev approximation to the ideal low-pass filter is generated with Chebyshev polynomials. It permits gain ripple in the passband so the cutoff can be sharper.	(8.26)	329
• Chebyshev filter stopband gain can be decreased by increasing the filter order or the permitted gain ripple.		330
• High-pass, bandpass, and band-reject filters can be generated from a low-pass filter by using frequency transformations.	(8.33) (8.34) (8.35)	334
• Normalized low-pass filters are used in the Butterworth and Chebyshev filter design procedure so data are required for only one filter configuration, that is, for a low-pass filter with cutoff frequency $\omega_c = 1$ rad/s.		335
• The normalized low-pass filter design is converted to the desired filter design by frequency scaling or transformation.	(8.43)	339
• The bandpass Butterworth and Chebyshev filter design procedure is similar to that for a low-pass filter. Differences are: 1. Stopband-gain specifications both above and below the passband must be converted to equivalent normalized low-pass filter specifications before normalized low-pass filter design. 2. A frequency transformation is used rather than a frequency scaling to produce the bandpass filter from the normalized low-pass filter design.		342

Problems

8.1 Define the ideal high-pass filter in the manner in which the ideal low-pass and bandpass filters are defined in Section 8.2. Sketch the amplitude and phase responses.

8.2 Write the mathematical expression for the frequency response of the ideal high-pass filter defined in Problem 8.1. Find the filter impulse response.

8.3 Define the ideal band-rejection filter in the manner in which the ideal low-pass and bandpass filters are defined. Sketch the amplitude and phase responses.

8.4 Write the mathematical expression for the frequency response of the ideal band-rejection filter