

5.5

| Lmk | α_{i-1} | a_{i-1} | θ_i | d_i |
|-----|----------------|-----------|--------------|-------|
| 1 | 0 | 0 | θ_1^* | 0 |
| 2 | $-\pi/2$ | 0 | θ_2^* | 0 |
| 3 | 0 | a_2 | θ_3^* | d_3 |
| 4 | $-\pi/2$ | a_3 | θ_4^* | d_4 |
| 5 | $\pi/2$ | 0 | θ_5^* | 0 |
| 6 | $-\pi/2$ | 0 | θ_6^* | 0 |

$${}^0_1T = R_z(\theta_1) = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^1_2T = R_x(-\pi/2) R_z(\theta_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = D_x(a_2) R_z(\theta_3) D_z(d_3) = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^5_5 T \sim {}^3_4 T = R_x(-\pi/2) D_x(a_3) R_z(\theta_4) D_z(d_4)$$

$${}^3_4 T = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4_5 T = R_x(\pi/2) R_x(\theta_5) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4_5 T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^5_6 T = R_x(-\pi/2) R_z(\theta_6) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^5_6 T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0 w_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad {}^1 w_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix},$$

$${}^2 w_2 = {}^1 R^T {}^1 w_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} c_2 & 0 & -s_2 \\ -s_2 & 0 & -c_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -s_2 \dot{\theta}_1 \\ -c_2 \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix},$$

$${}^3 w_3 = {}^2 R^T {}^2 w_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s_2 \dot{\theta}_1 \\ -c_2 \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -s_2 c_3 \dot{\theta}_1 - c_2 s_3 \dot{\theta}_1 \\ s_2 s_3 \dot{\theta}_1 - c_2 c_3 \dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

$${}^3 w_3 = \begin{bmatrix} -s_{23} \dot{\theta}_1 \\ -c_{23} \dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}, \quad {}^4 w_4 = {}^3 R^T {}^3 w_3 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} c_4 & 0 & -s_4 \\ -s_4 & 0 & -c_4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -s_{23} \dot{\theta}_1 \\ -c_{23} \dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix}$$

$${}^4 w_4 = \begin{bmatrix} -s_{23} c_4 \dot{\theta}_1 + s_4 \dot{\theta}_2 - s_4 \dot{\theta}_3 \\ s_{23} s_4 \dot{\theta}_1 - c_4 \dot{\theta}_2 - c_4 \dot{\theta}_3 \\ -c_{23} \dot{\theta}_1 + \dot{\theta}_4 \end{bmatrix}, \quad {}^5 w_5 = {}^4 R^T {}^4 w_4 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_5 \end{bmatrix}$$

$${}^5 w_5 = \begin{bmatrix} c_5 & 0 & s_5 \\ -s_5 & 0 & c_5 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -s_{23} c_4 \dot{\theta}_1 + s_4 \dot{\theta}_2 - s_4 \dot{\theta}_3 \\ s_{23} s_4 \dot{\theta}_1 - c_4 \dot{\theta}_2 - c_4 \dot{\theta}_3 \\ -c_{23} \dot{\theta}_1 + \dot{\theta}_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_5 \end{bmatrix}$$

$${}^5 w_5 = \begin{bmatrix} -s_{23} c_4 c_5 \dot{\theta}_1 - c_{23} s_5 \dot{\theta}_1 - s_4 c_5 \dot{\theta}_2 - s_4 c_5 \dot{\theta}_3 + s_5 \dot{\theta}_4 \\ s_{23} c_4 s_5 \dot{\theta}_1 - c_{23} c_5 \dot{\theta}_1 + s_4 s_5 \dot{\theta}_2 + s_4 s_5 \dot{\theta}_3 + c_5 \dot{\theta}_4 \\ -s_{23} s_4 \dot{\theta}_1 + c_4 \dot{\theta}_2 + c_4 \dot{\theta}_3 + \dot{\theta}_5 \end{bmatrix},$$

$${}^6 w_6 = {}^5 R^T {}^5 w_5 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_6 \end{bmatrix} = \begin{bmatrix} c_6 & 0 & -s_6 \\ -s_6 & 0 & -c_6 \\ 0 & 1 & 0 \end{bmatrix} {}^5 w_5 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_6 \end{bmatrix}$$

5.5 ~ Let ${}^6w_6 = \begin{bmatrix} {}^6w_x \\ {}^6w_y \\ {}^6w_z \end{bmatrix};$

$${}^6w_x = (-s_{23}c_4c_5c_6 - c_{23}s_5c_6 + s_{23}s_4s_6)\dot{\theta}_1 + (-s_4c_5c_6 - c_4s_6)\dot{\theta}_2 + (-s_4c_5c_6 - c_4s_6)\dot{\theta}_3 + s_5c_6\dot{\theta}_4 + s_6\dot{\theta}_5$$

$${}^6w_y = (s_{23}c_4c_5s_6 + c_{23}s_5s_6 + s_{23}s_4c_6)\dot{\theta}_1 + (s_4c_5s_6 - c_4c_6)\dot{\theta}_2 + (s_4c_5s_6 - c_4c_6)\dot{\theta}_3 - s_5s_6\dot{\theta}_4 - c_6\dot{\theta}_5$$

$${}^6w_z = (s_{23}c_4s_5 - c_{23}c_5)\dot{\theta}_1 + s_4s_5\dot{\theta}_2 + s_4s_5\dot{\theta}_3 + c_5\dot{\theta}_4 + \dot{\theta}_6$$

$${}^0V_0 = {}^1V_1 = {}^2V_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{The base point doesn't move}),$$

$${}^3V_3 = {}^2R^T \left({}^2V_2 + {}^2W_2 \times \begin{bmatrix} a_2 \\ 0 \\ d_3 \end{bmatrix} \right) \quad {}^2W_2 \times \begin{bmatrix} a_2 \\ 0 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\theta}_2 & -c_2\dot{\theta}_1 \\ \dot{\theta}_2 & 0 & s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 - s_2\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \\ d_3 \end{bmatrix}$$

$${}^3V_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -d_3c_2\dot{\theta}_1 \\ a_2\dot{\theta}_2 + d_3s_2\dot{\theta}_1 \\ a_2c_2\dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} (-d_3c_2c_3 + d_3s_2s_3)\dot{\theta}_1 + a_2s_3\dot{\theta}_2 \\ (d_3c_2s_3 + d_3s_2c_3)\dot{\theta}_1 + a_2c_3\dot{\theta}_2 \\ a_2c_2\dot{\theta}_1 \end{bmatrix}$$

$${}^4V_4 = {}^3R^T \left({}^3V_3 + {}^3W_3 \times \begin{bmatrix} a_3 \\ d_4 \\ 0 \end{bmatrix} \right) \quad {}^3W_3 \times \begin{bmatrix} a_3 \\ d_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\theta}_3 & -c_3\dot{\theta}_1 \\ \dot{\theta}_3 + \dot{\theta}_1 & 0 & s_{23}\dot{\theta}_1 \\ c_{23}\dot{\theta}_1 - s_{23}\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ d_4 \\ 0 \end{bmatrix}$$

$${}^4V_4 = \begin{bmatrix} c_4 & 0 & -s_4 \\ -s_4 & 0 & -c_4 \\ 0 & 1 & 0 \end{bmatrix} \left(\begin{bmatrix} (d_3s_2s_3 - d_3c_2c_3)\dot{\theta}_1 + a_2s_3\dot{\theta}_2 \\ (d_3c_2s_3 + d_3s_2c_3)\dot{\theta}_1 + a_2c_3\dot{\theta}_2 \\ a_2c_2\dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} -d_4\dot{\theta}_1 - d_4\dot{\theta}_2 \\ a_3\dot{\theta}_2 + a_3\dot{\theta}_3 \\ a_3c_3\dot{\theta}_1 - d_4s_{23}\dot{\theta}_1 \end{bmatrix} \right)$$

$${}^5_5 \tilde{V} = \begin{bmatrix} c_4 & 0 & -s_4 \\ -s_4 & 0 & -c_4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (d_3 s_2 s_3 - d_3 c_2 c_3) \dot{\theta}_1 + (a_2 s_3 - d_4) \dot{\theta}_2 - d_4 \dot{\theta}_3 \\ (d_3 c_2 s_3 + d_3 s_2 c_3) \dot{\theta}_1 + (a_2 c_3 + a_3) \dot{\theta}_2 + a_3 \dot{\theta}_3 \\ (a_2 c_2 + a_3 c_2 - d_4 s_2 s_3) \dot{\theta}_1 \end{bmatrix}$$

$${}^4_4 \tilde{V} = \begin{bmatrix} (d_3 s_2 s_3 c_4 - d_3 c_2 c_3 c_4 - a_2 c_2 s_4 - a_3 c_2 s_4 + d_4 s_2 s_4) \dot{\theta}_1 + (a_2 s_3 c_4 - d_4 c_4) \dot{\theta}_2 - d_4 c_4 \dot{\theta}_3 \\ (-d_3 s_2 s_3 s_4 + d_3 c_2 c_3 s_4 - a_2 c_2 c_4 - a_3 c_2 c_4 + d_4 s_2 s_4) \dot{\theta}_1 + (-a_2 s_3 s_4 + d_4 s_4) \dot{\theta}_2 + d_4 s_4 \dot{\theta}_3 \\ (d_3 c_2 s_3 + d_3 s_2 c_3) \dot{\theta}_1 + (a_2 c_3 + a_3) \dot{\theta}_2 + a_3 \dot{\theta}_3 \end{bmatrix}$$

$${}^5_5 \tilde{V} = {}^4_5 R^T {}^4_4 \tilde{V} \quad \text{and} \quad {}^6_6 \tilde{V} = {}^5_6 R^T {}^5_5 \tilde{V} \Rightarrow {}^6_6 \tilde{V} = {}^5_6 R^T {}^4_5 R^T {}^4_4 \tilde{V}$$

$${}^6_6 \tilde{V} = \begin{bmatrix} c_6 & 0 & -s_6 \\ -s_6 & 0 & -c_6 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_5 & 0 & s_5 \\ -s_5 & 0 & c_5 \\ 0 & -1 & 0 \end{bmatrix} {}^4_4 \tilde{V} = \begin{bmatrix} c_5 c_6 & s_6 & s_5 c_6 \\ -c_5 s_6 & c_6 & -s_5 s_6 \\ -s_5 & 0 & c_5 \end{bmatrix} {}^4_4 \tilde{V} = \begin{bmatrix} {}^6_6 \tilde{V}_x \\ {}^6_6 \tilde{V}_y \\ {}^6_6 \tilde{V}_z \end{bmatrix}$$

$${}^6_6 \tilde{V}_x = (d_3 s_2 s_3 c_4 c_5 c_6 - d_3 c_2 c_3 c_4 c_5 c_6 - a_2 c_2 s_4 c_5 c_6 - a_3 c_2 s_4 c_5 c_6 + d_4 s_2 s_4 c_5 c_6 \\ - d_3 s_2 s_3 s_4 s_6 + d_3 c_2 c_3 s_4 s_6 - a_2 c_2 c_4 s_6 - a_3 c_2 c_4 s_6 + d_4 s_2 s_4 c_4 s_6 \\ + d_3 c_2 s_3 s_5 c_6 + d_3 s_2 c_3 s_5 c_6) \dot{\theta}_1 + (a_2 s_3 c_4 c_5 c_6 - d_4 c_4 c_5 c_6 - a_2 s_3 s_4 s_6 \\ + d_4 s_4 s_6 + a_2 c_3 s_5 c_6 + a_3 s_5 c_6) \dot{\theta}_2 + (-d_4 c_4 c_5 c_6 + d_4 s_4 s_6 + a_3 s_5 c_6) \dot{\theta}_3$$

$${}^6_6 \tilde{V}_y = (-d_3 s_2 s_3 c_4 c_5 s_6 + d_3 c_2 c_3 c_4 c_5 s_6 + a_2 c_2 s_4 c_5 s_6 + a_3 c_2 s_4 c_5 s_6 - d_4 s_2 s_4 c_4 s_6 \\ - d_3 s_2 s_3 s_4 c_6 + d_3 c_2 c_3 s_4 c_6 - a_2 c_2 c_4 c_6 - a_3 c_2 c_4 c_6 + d_4 s_2 s_4 c_4 c_6 \\ - d_3 c_2 s_3 s_5 s_6 - d_3 s_2 c_3 s_5 s_6) \dot{\theta}_1 + (-a_2 s_3 c_4 c_5 s_6 + d_4 c_4 c_5 s_6 + a_2 s_3 s_4 c_6 \\ + d_4 s_4 c_6 - a_2 c_3 s_5 s_6 - a_3 s_5 s_6) \dot{\theta}_2 + (d_4 c_4 c_5 s_6 + d_4 s_4 c_6 - a_3 s_5 s_6) \dot{\theta}_3$$

$${}^6_6 \tilde{V}_z = (-d_3 s_2 s_3 s_5 + d_3 c_2 c_3 c_4 s_5 + a_2 c_2 s_4 s_5 + a_3 c_2 s_4 s_5 - d_4 s_2 s_4 s_5 \\ + d_3 c_2 s_3 c_5 + d_3 s_2 c_3 c_5) \dot{\theta}_1 + (-a_2 s_3 c_4 s_5 + d_4 c_4 s_5 + a_2 c_3 c_5 + a_3 c_5) \dot{\theta}_2 \\ + (d_4 c_4 s_5 + a_3 c_5) \dot{\theta}_3$$

$$\text{in which } \begin{bmatrix} {}^6_6 \tilde{V} \\ {}^6_6 \tilde{W} \end{bmatrix} = {}^6_6 J(\theta) \dot{\theta}$$

5.5 ~

$$\text{Let } \begin{bmatrix} {}^6V_x \\ {}^6V_y \\ {}^6V_z \\ {}^6W_x \\ {}^6W_y \\ {}^6W_z \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} & r_{56} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & r_{66} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix}$$

$$\underline{r_{11}} = d_3 s_2 s_3 c_4 c_5 c_6 - d_3 c_2 c_3 c_4 c_5 c_6 - a_2 c_2 s_4 c_5 c_6 - a_3 c_2 s_4 c_5 c_6 \\ + d_4 s_2 s_3 s_4 c_5 c_6 - d_3 s_2 s_3 s_4 s_6 + d_3 c_2 c_3 s_4 s_6 - a_2 c_2 c_4 s_6 - a_3 c_2 s_4 s_6 \\ + d_4 s_2 s_3 c_4 s_6 + d_3 c_2 s_3 s_5 c_6 + d_3 s_2 c_3 s_5 c_6,$$

$$\underline{r_{12}} = a_2 s_3 c_4 c_5 c_6 - d_4 c_4 c_5 c_6 - a_2 s_3 s_4 s_6 + d_4 s_4 s_6 + a_2 c_3 s_5 s_6 + a_3 s_5 c_6,$$

$$\underline{r_{13}} = -d_4 c_4 c_5 c_6 + d_4 s_4 s_6 + a_3 s_5 c_6, \quad \underline{r_{14}} = \underline{r_{15}} = \underline{r_{16}} = 0,$$

$$\underline{r_{21}} = -d_3 s_2 s_3 c_5 s_6 + d_3 c_2 c_3 c_4 c_5 s_6 + a_2 c_2 s_4 c_5 s_6 + a_3 c_2 s_4 c_5 s_6 \\ - d_4 s_2 s_3 s_4 c_5 s_6 - d_3 s_2 s_3 s_4 c_6 + d_3 c_2 c_3 s_4 c_6 - a_2 c_2 c_4 c_6 - a_3 c_2 s_4 c_6 \\ + d_4 s_2 s_3 c_4 c_6 - d_3 c_2 s_3 s_5 s_6 - d_3 s_2 c_3 s_5 s_6,$$

$$\underline{r_{22}} = -a_2 s_3 c_4 c_5 s_6 + d_4 c_4 c_5 s_6 - a_2 s_3 s_4 c_6 + d_4 s_4 c_6 - a_2 c_3 s_5 s_6 - a_3 s_5 s_6,$$

$$\underline{r_{23}} = d_4 c_4 c_5 s_6 + d_4 s_4 c_6 - a_3 s_5 s_6, \quad \underline{r_{24}} = \underline{r_{25}} = \underline{r_{26}} = 0,$$

$$\underline{r_{31}} = -d_3 s_2 s_3 s_5 + d_3 c_2 c_3 c_4 s_5 + a_2 c_2 s_4 s_5 + a_3 c_2 s_4 s_5 - d_4 s_2 s_3 s_4 s_5 \\ + d_3 c_2 s_3 c_5 + d_3 s_2 c_3 c_5,$$

$$\underline{r_{32}} = -a_2 s_3 c_4 s_5 + d_4 c_4 s_5 + a_2 c_3 c_5 + a_3 c_5,$$

$$\underline{r_{33}} = d_4 c_4 s_5 + a_3 c_5, \quad \underline{r_{34}} = \underline{r_{35}} = \underline{r_{36}} = 0,$$

$$\underline{r_{41}} = -s_2 s_3 c_4 c_5 c_6 - c_2 s_3 s_5 c_6 + s_2 s_3 s_4 s_6, \quad \underline{r_{42}} = -s_4 c_5 c_6 - c_4 s_6,$$

$$\underline{r_{43}} = -s_4 c_5 c_6 - c_4 s_6, \quad \underline{r_{44}} = s_5 c_6, \quad \underline{r_{45}} = -s_6, \quad \underline{r_{46}} = 0,$$

$$\underline{r_{51}} = s_2 s_3 c_4 c_5 s_6 + c_2 s_3 c_5 s_6 + s_2 s_3 s_4 c_6, \quad \underline{r_{52}} = s_4 c_5 s_6 - c_4 c_6,$$

$$\underline{r_{53}} = s_4 c_5 s_6 - c_4 c_6, \quad \underline{r_{54}} = -s_5 s_6, \quad \underline{r_{55}} = -c_6, \quad \underline{r_{56}} = 0,$$

$$\underline{r_{61}} = s_2 s_3 c_4 s_5 - c_2 s_3 c_5, \quad \underline{r_{62}} = s_4 s_5, \quad \underline{r_{63}} = s_4 s_5, \quad \underline{r_{64}} = c_5,$$

$$\underline{r_{65}} = 0, \quad \underline{r_{66}} = 1$$