b. $x(t) = -2 + 2u(t+10) + 2u(t) - 2u(t-5)$

c. $x(t) = -r(t+1.5) + 2r(t-0.5) - 2r(t-1.5) + r(t-2.5)$

d. $x(t) = t - r(t+1) + u(t-1.5) + u(t-2) - 2u(t-3)$

3.23 Plot the following signals:

a. $x(t) = 2r(t+2) - 2r(t+1) - 2r(t) + 2r(t-1)$

b. $x(t) = 2u(t-1.5) - r(t-1.5) + r(t-2.5) - u(t-4)$

c. $x(t) = r(t)\prod[(t-3)/2] + 2u(2-t) + 4u(t-4)$

d. $x(t) = 2 + r(t-2) - r(t-4)$

e. $x(t) = 2\prod[(t-1)/3] + (2t+3)\prod(t+1) + (7-2t)\prod(t-3)$

3.24 Write equations for the test signals shown in Figure 3.29 in terms of a sum of step and/or ramp functions.

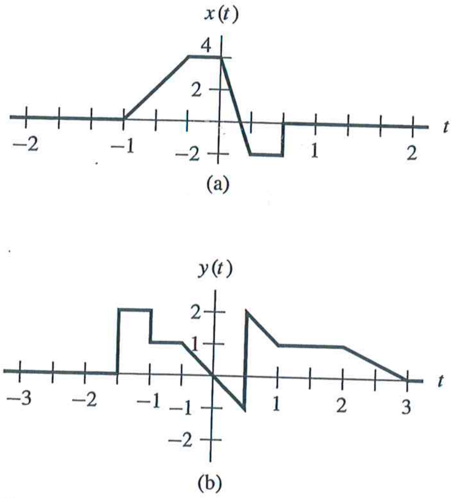


Figure 3.29

3.25 Write equations for the test signals shown in Figure 3.30 in terms of a sum of step and/or ramp functions.

3.26 Show that $x(t) = r(t)$ has infinite energy and infinite power and that $y(t) = t^{-0.1}u(t-1)$ has infinite energy and zero power.

3.27 Determine the signal energy and signal power for each signal given and indicate whether it is an energy signal or a power signal.

a. $x(t) = [1/(2+t^2)]u(t+1)$

b. $x(t) = (t^2)\prod[(t-1)/3]$

c. $x(t) = 10\cos(5t - 0.1)$

d. $x(t) = 10\sin(5t - 0.1)u(t)$

e. $x(t) = r(t)u(t+1)$

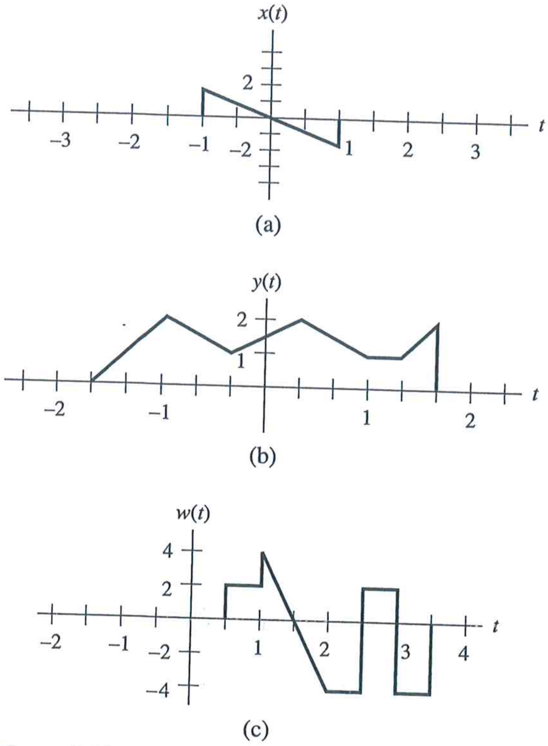


Figure 3.30

3.28 Determine the signal energy and signal power for each signal given and indicate whether it is an energy signal or power signal.

a. $x(t) = r(t) - r(t-1) - u(t-4)$

b. $x(t) = 0.5r(t+1)u(t-2)u(3-t)$

c. $x(t) = 1/\sqrt{1+t^2}$

d. $x(t) = t^{-0.1}u(t-1)$

e. $x(t) = e^{-|t|}\cos(10\pi t)$

3.29 The characteristics of the signal $y(t) = e^{bt}u(t - t_0)$, $t_0 \geq 0$, depend on the value of b. Determine the values of b, if any, that will produce (a) an energy signal, (b) a power signal, (c) a signal that is neither an energy signal nor a power signal.

3.30 Considering the signal $x(t) = \frac{A}{\tau}\prod\left(\frac{t}{\tau}\right)$ and taking the limit as τ approaches zero, show that the impulse function is neither an energy signal nor a power signal.

3.31 Determine which of the following pairs of signals are orthogonal over the time interval $1 < t < 3$:

a. $x(t) = t - 2$ and $y(t) = \prod[(t-3)/6]$

b. $x(t) = t$ and $y(t) = e^{-|t|}$

c. $x(t) = \cos(5\pi t)$ and $y(t) = \sin(5\pi t)$

d. $x(t) = \cos(2.5\pi t)$ and $y(t) = \sin(2.5\pi t)$

e. $x(t) = e^{j3\pi t}$ and $y(t) = e^{j2\pi t}$
f. $x(t) = e^{j2.5\pi t}$ and $y(t) = e^{-j2\pi t}$
g. $x(t) = e^{j5\pi t}$ and $y(t) = e^{-j5\pi t}$

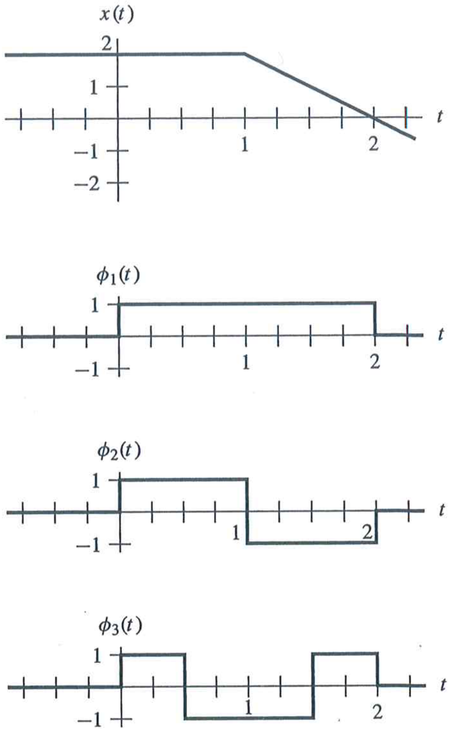


Figure 3.31

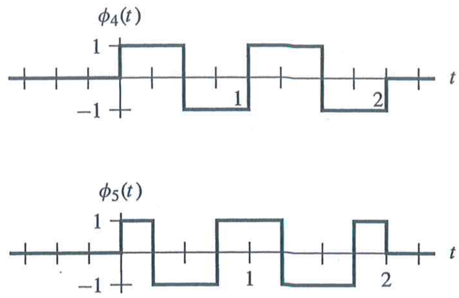


Figure 3.31 (continued)

3.32 Consider the signal $x(t)$ and the five basis signa $\phi_i(t)$, $1 \le i \le 5$, shown in Figure 3.31.

a. Show that the basis signals are mutually ortho onal over the interval $0 < t < 2$ and find t value of λ_n associated with each one.

b. Find and plot the generalized Fourier series a proximation to $x(t)$ over the interval $0 < t <$ using the five basis signals.

c. Compute the integral square error over the i terval of the approximation.

d. Compute the signal energy and the sum of t energies in each term of the generalized Fouri series approximation over the interval $0 < t <$ What is the ratio of the energy contained in t series approximation to the energy contained the signal in the interval.

Note: The basis signals used in this problem a the first five of an infinite, complete set known *Walsh functions*.

Problems for Computer Solution

3.33 **a.** On the same set of axes, having time interval $0 \le t \le 1$, plot the signal $x(t) = 2\cos(2\pi t + \theta)$ for: $\theta = -\pi/2$, $\theta = -\pi$, and $\theta = -3\pi/2$.

b. Repeat part **a** for the signal $y(t) = 2\cos(4\pi t + \theta)$

c. Explain the results observed for part **a** and part **b**.

3.34 The laser cutting tool of Example 3.6 must proceed at a constant rate to provide uniform cutting. Show that the equations given produce a constant cutting rate of 1 cm/s.

3.35 The laser cutting tool of Example 3.6 is reprogrammed to cut a different part from an 8-cm by 16-cm plate of 6-mm steel stock. The part requires three separate cuts. The first cut is produced during a time interval $0 \le t \le 2.36$ with the cutting he control signals.

$$x(t) = 3 + 0.375\cos(t/0.375)$$

and

$$y(t) = 4 + 0.375\sin(t/0.375)$$

The control signals

$$x(t) = 12 + 1.25\cos(t/1.25)$$

and

$$y(t) = 4 + 1.25\sin(t/1.25)$$

produce the second cut during a time interval 0 $t \le 7.86$. The third and final cut is produced du ing a time interval $0 \le t \le 27.65$ with the control signals

$$x(t) = a_x(t)\,[u(t) - u(t - t_1)] + x_1 u(t - t_1)$$
$$+ b_x\,[r(t - t_1) - r(t - t_2)] - x_2 u(t - t_2)$$
$$+ c_x(t)\,[u(t - t_2) - u(t - t_3)]$$
$$- b_x r(t - t_3) + x_2 u(t - t_3)$$

and

$$y(t) = a_y(t)\,[u(t) - u(t - t_1)] + y_1 u(t - t_1)$$
$$+ b_y\,[r(t - t_1) - r(t - t_2)] - y_2 u(t - t_2)$$
$$+ c_y(t)\,[u(t - t_2) - u(t - t_3)]$$
$$+ b_y r(t - t_3) + y_3 u(t - t_3)$$

where

$x_1 = 3 + 0.75\cos(2/3)$, $x_2 = 12 - 2\cos(0.5)$, $y_1 = 4 - 0.75\sin(2/3)$, $y_2 = 4 - 2\sin(0.5)$, $y_3 = 4 + 2\sin(0.5)$, $A = |(x_2 - x_1) + j(y_2 - y_1)|$, $\theta = \angle (x_2 - x_1) + j(y_2 - y_1)$, $t_1 = 1.5\pi - 1$, $t_2 = t_1 + A$, $t_3 = t_2 + 4\pi - 2$, $b_x = \cos\theta$, $b_y = \sin\theta$, $a_x(t) = 3 + 0.75\cos[(4t + 2)3]$, $c_x(t) = 12 + 2\cos\left[(t - t_2 + 2\pi + 1)/2\right]$, $a_y(t) = 4 + 0.75\sin\left[(4t + 2)/3\right]$, and $c_y(t) = 4 + 2\sin\left[(t - t_2 + 2\pi + 1)/2\right]$.

a. Plot $y(t)$ versus $x(t)$ for all cuts on the same figure to illustrate the shape of the part.

b. Show that the cutting rate is constant at 1cm/s.

3.36 The 15 basis functions consisting of $\phi_{2i}(t) = \cos(0.5\pi i t)$, $0 \le i \le 7$, and $\phi_{2i-1} = \sin(0.5\pi i t)$, $1 \le i \le 7$ are orthogonal over the time interval $0 \le t \le 4$.

a. Find the generalized Fourier series coefficients corresponding to these basis signals for the signal $x(t) = 0.5(t - 1)^2$ in the interval $0 \le t \le 4$. Plot the signal and the generalized Fourier series approximation for the interval $0 \le t \le 4$ and compare.

b. Repeat part **a** using the 15 periodic triangular basis functions

$$\phi_o(t) = 1$$

$$\left.\begin{aligned} \phi_{2i-1}(t) &= \phi_1(it) \\ \phi_{2i}(t) &= \phi_1(it + 1) \end{aligned}\right\} \quad 1 \le i \le 7$$

where

$$\phi_1(t) = r(t) - 2\big[r(t - 1) - r(t - 3) + r(t - 5)$$
$$- r(t - 7) + r(t - 9) - r(t - 11)$$
$$+ r(t - 13) - r(t - 15) + r(t - 17)$$
$$- r(t - 19) + r(t - 21) - r(t - 23)$$
$$+ r(t - 25) - r(t - 27)\big]$$

These basis functions are also orthogonal on the interval $0 \le t \le 4$.

3.37 Work Problem 3.36 for the signal

$$x(t) = \begin{cases} 1 + t & t < 2 \\ 7 - 2t & t \ge 2 \end{cases}$$

2.17 Draw block-diagram representations for the systems characterized by the following system equations, where $x(t)$ or $x[nT]$ is the input and $y(t)$ or $y[nT]$ is the output.

a. $5\frac{dy(t)}{dt}+2y(t)=\frac{dx(t)}{dt}+3x(t)$

b. $0.5y(t)+0.8\int_{-\infty}^{t}y(\tau)d\tau=x(t)+2\frac{dx(t)}{dt}$

c. $y[nT]-0.2y[(n-1)T]+0.5y[(n-2)T]=x[(n-1)T]-x[(n-2)T]$

d. $0.7y[(n-3)T]-y[nT]=x[(n-1)T]-2x[(n-2)T]$

2.18 Draw block-diagram representations for the systems modeled by the following sets of equations, where $x(t)$ and $x[nT]$ are the input signals and $y(t)$ and $y[nT]$ are the output signals:

a. $\frac{dy(t)}{dt}+y(t)-az(t)+\int_{-\infty}^{t}z(\tau)d\tau=x(t)$

$\int_{-\infty}^{t}y(\tau)\,d\tau+bz(t)=c\frac{dx(t)}{dt}$

b. $$y[nT]=y[(n-1)T]+Az[(n-1)T]+Bx[nT]+Cx[(n-1)T]$$

$$z[nT]=Dz[(n-2)T]-Ey[(n-1)T]+x[nT]$$

2.19 Determine whether the systems having the following system equations have memory or not. If a system has memory, what is its order? State the reasons for your answers.

a. $y(t)=3x^2(t)+\frac{dy(t)}{dt}$

b. $\frac{d^2y(t)}{dt^2}=y(t)+4tx(t)-\frac{dx(t)}{dt}$

c. $y[nT]=0.5x[nT]+3$

d. $y(t)-2x(t)\frac{dy(t)}{dt}=\frac{d^2x(t)}{dt^2}$

e. $$y[nT]=y[(n-1)T]+2x[nT]-3x[(n-1)T]+2x[(n-2)T]$$

f. $y[nT]-\frac{y[(n-2)T]}{n}=2x[nT]$

2.20 Determine whether the systems of Problem 2.19 are linear.

2.21 Determine whether the systems of Problem 2.19 are time invariant or time varying. State the reasons for your answers.

2.22 Show whether the systems with the following outputs are linear. The input is $x(t)$ or $x[nT]$, the output is $y(t)$ or $y[nT]$.

a. $y(t)=\int_{-\infty}^{t}x^2(\tau)h(t-\tau)\,d\tau$

b. $$y[nT]=0.5\Big\{x[nT]+x[(n-1)T]+[x(n-2)T]\Big\}$$

c. $y(t)=x(t+3)+2\int_{-\infty}^{0.5t}x(\tau)\,d\tau$

d. $y[nT]=\frac{2x[(n-1)T]-x[(n+1)T]}{1+x[(n-2)T]}$

2.23 Determine whether the systems of Problem 2.22 are causal. State the reasons for your answers.

2.24 Show that the system with the output given in part c of Problem 2.22 is time invariant.

Problems for Computer Solution

2.25 Write MATLAB script to generate plots of the following signals for $|t|\le 2$.

a. $x(t)=\left(2\,|t|^{1.3}-3\,|t|\right)/\left(e^{-4|t-1|}+1\right)$

b. $x(0.7t)$

c. $x(-t)$

d. $x(-2t-1)$

2.26 Write MATLAB script to generate plots of the following signals for $|t|\le 2$.

a. $x[nT]=4n^3/\left(1+0.5n+0.5n^4\right)$, $T=0.5$

b. $x[nT]$, $T=0.25$

c. $x[-nT]$, $T=-1$

d. $x[(-n+3)T]$, $T=1/3$

2.27 Use MATLAB to generate plots of the piecewise defined signals.

$$x_1(t)=\begin{cases} t^2+4t+4 & -2\le t<-1\\ 0.16t^2-0.48t+0.36 & -1\le t<1.5\\ 0 & \text{elsewhere}\end{cases}$$

$$x_2(t)=\begin{cases} e^{(0.5t+1)} & t<-2\\ -0.5t & -2\le t<-0.5\\ 1.21 & -0.5\le t<1.5\\ -t^2+3t-1.04 & 1.5\le t<2.6\\ \sin(\pi t-1.6\pi) & 2.6\le t\end{cases}$$

for $|t|\le 5$.

Concept	Equation	Page
• A memoryless system's output signals at time $t = t_1$ depend on the input signal value at only time $t = t_1$.		33
• A system with memory has output signals at time $t = t_1$ that depend on input signal values corresponding to times $t \neq t_1$ in addition to time $t = t_1$.		33
• A causal system's output signal depends only on past and present input-signal values.		33
• The system order is equal to the order of the differential, or difference, system equation.		35
• A system is linear if and only if superposition holds for the system.		36
• A system is time invariant if the output shifts in time by Δt, without changing shape, when the input shifts in time by Δt.	(2.24) through (2.27)	40
• System equations for linear, time-invariant systems are linear equations with constant coefficients.		38, 41
• A system possesses bounded-input, bounded-output (BIBO) stability if every bounded-input signal produces a bounded-output signal.		42

Problems

2.1 Plot the continuous-time signal $x(t) = t/\left(1+t^4\right)$. Find the equation for and plot the signals.

a. $x(1.5t)$
b. $x(0.8t)$
c. $x(-t)$
d. $x(t+3.6)$
e. $x(-2t)$
f. $x(2t-1)$
g. $x(-0.7t-0.35)$

2.2 Plot the discrete-time signal $x[nT] = 4n/\left(2+n^2\right)$, $T = 2$. Find the equations for and plot the signals.

a. $x[nT], \quad T = 3$
b. $x[nT], \quad T = 0.5$
c. $x[-nT], \quad T = 2$
d. $x\left[(n+4)T\right], \quad T = 2$
e. $x[-nT], \quad T = 2.5$
f. $x\left[(n-2)T\right], \quad T = 0.75$
g. $x\left[(-n-2)T\right], \quad T = 0.1$

2.3 Repeat Problem 2.1 for the signal $x(t) = 3e^{-|t|}$.

2.4 Repeat Problem 2.2 for the signal $x[nT] = n^2 - n$, $T = 1.5$.

2.5 Plot the discrete-time signals $x[nT] = 4nT/\left(2+(nT)^2\right)$ and $y[nT] = \left(0.5n^2\right) - nT$ over the time interval $|t| \leq 9.5$s for $T = 2$, 1, and 3 s.

2.6 Sketch the continuous-time signals defined by the following equations; also indicate which of these signals are piecewise defined.

a. $x(t) = 0.08\left|t^3\right| \qquad -\infty \leq t \leq \infty$

b. $y(t) = \begin{cases} 2+t & -2 \leq t < 0 \\ 2-0.5t & 0 \leq t < 2 \\ 0 & \text{elsewhere} \end{cases}$

c. $w(t) = \left(2-t^2\right)/\left(1+t^2\right) \qquad -\infty \leq t \leq \infty$

d. $z(t) = \begin{cases} 4 & 1 \leq t \leq 2 \\ 0 & \text{elsewhere} \end{cases}$

2.7 Sketch the continuous-time signals defined by th following equations:

a. $x(t) = \begin{cases} t-1 & 1 \leq t < 2 \\ 2 & 2 \leq t < 5 \\ 0 & \text{elsewhere} \end{cases}$

b. $x(t) = e^{-2|t|} \qquad -\infty \leq t \leq \infty$

c. $x(t) = (t-1)/\left(0.25t^2+0.5\right)$
$-\infty \leq t \leq \infty$

2.8 Sketch the discrete-time signals defined by t following equations; also indicate which of the signals are piecewise defined.

a. $z[nT] = \{1 - \exp[-0.1n^2]$ $\qquad -\infty \leq n \leq$ o
$T = 2s$

b. $x[nT] \quad = \quad \left\{\begin{matrix} 2e^{-0.5n} & 0 \leq n \leq 5 \\ 0 & \text{elsewhere} \end{matrix}\right\},$
$T = 0.1s$

3.8 A sinusoidal signal is a sine with a phase angle of 2.1 rad. Find the time delay of this signal with respect to the reference cosine signal if its frequency is (a) 0.1 Hz, (b) 1 Hz, (c) 100 Hz.

3.9 Write the household voltage signal $v(t) = 120\sqrt{2}\cos\left[120\pi t + \pi/8\right]$ as a sum of counterrotating phasors and plot these phasors and their sum for (a) $t = 0$, (b) $t = 0.8$ ms, (c) $t = 5.0$ ms, (d) $t = 9.4$ms.

3.10 The shock absorber on the front wheel of an automobile is worn out, causing the wheel to oscillate up and down as the automobile is driven. The vertical displacement $y(t)$ of the wheel with respect to its average vertical location is given by the signal $y(t) = 0.75\cos\left[1.25\pi t - \pi/4\right]$. Write $y(t)$ as a sum of counterrotating phasors and plot these phasors and their sum for (a) $t = 0$, (b) $t = 1/10$s, (c) $t = 1/5$s, (d) $t = 2/5$s.

3.11 Evaluate $\int_{-\infty}^{\infty} x(\tau)\delta(\tau - t_0)\, d\tau$ for:

a. $x(\tau) = 3e^{-\tau}$ and $t_0 = 2$

b. $x(\tau) = 1/(1+\tau^2)$ and $t_0 = -0.5$

c. $x(\tau) = \cos(2\pi\tau)$ and $t_0 = 0.25$

d. $x(\tau) = (\tau - 1)^3$ and $t_0 = 3$

3.12 Evaluate $\int_{t_1}^{t_2} (10 - t^2)\delta(t - t_0)\, dt$ for:

a. $t_1 = 0.1,\ t_2 = 2,\ t_0 = 1.5$

b. $t_1 = -2,\ t_2 = 2,\ t_0 = -3$

c. $t_1 = 0.5,\ t_2 = 3.5,\ t_0 = 2.5$

d. $t_1 = 0,\ t_2 = \infty,\ t_0 = 10$

3.13 Show that Property IV for impulse functions is valid.

3.14 Plot the following signals:

a. $y(t) = 3\delta(t+1) - 4\delta(t) + 2\delta(t-2)$

b. $w(t) = -\delta(t-0.5) + 2\delta(t-0.75) + 3\delta(t+0.25)$

c. $x(t) = 4\delta(t-2) + 5\delta(t+3) - 8\delta(t-2)$

d. $z(t) = 3\delta(t+1.5) + 2\delta(t-2.5) - 4\delta(t+1.5) - 3\delta(t-1.5) + \delta(t+1.5)$

3.15 Write the expressions for and plot $y(t) = x(t)\delta(t - t_0)$ when:

a. $x(t) = 2e^{-|t|}$ and $t_0 = 0.25$

b. $x(t) = 3\sqrt{t^3}$ and $t_0 = 14$

c. $x(t) = 3e^{-3t}\cos(2\pi t - \pi/4)$ and $t_0 = 0.125$

d. $x(t) = t(t-0.1)^2$ and $t_0 = 0.2$

3.16 Write the expressions for and plot $w(t) = y(t)\,[A\delta(t - t_0)]$ when:

a. $y(t) = t^2,\ A = 2,\ t_0 = 2$

b. $y(t) = 3 + \sin(10\pi t),\ A = 1.5,\ t_0 = -0.1$

c. $y(t) = 1/(1+t^2),\ A = -5,\ t_0 = 2$

d. $y(t) = |t-1|,\ A = 3,\ t_0 = 0.75.$

3.17 Write single-equation expressions for the following signals using the unit step function and then plot the signals:

a. $x(t) = \begin{cases} 1/(1+t^2) & t > 3 \\ 0 & t < 3 \end{cases}$

b. $x(t) = \begin{cases} e^{-|t|} & t < -2 \\ 0 & t > -2 \end{cases}$

c. $x(t) = \begin{cases} 1 - \cos(3t) & 1 < t < 3 \\ 0 & \text{elsewhere} \end{cases}$

d. $x(t) = \begin{cases} e^{-(t-1)} & t > 1 \\ \cos\,(3(t-1)) & t < 1 \end{cases}$

e. $x(t) = \begin{cases} 2 & |t| < 1 \\ 3 - t & 1 < t < 2 \\ 0 & \text{elsewhere} \end{cases}$

3.18 Repeat Problem 3.17 for the signals

a. $x(t) = \begin{cases} te^{-t} & t > -0.2 \\ 0 & t < -0.2 \end{cases}$

b. $x(t) = \begin{cases} \sin(2\pi t) & -2 < t < 0 \\ \sin(4\pi t) & 0 < t < 2 \\ 0 & \text{elsewhere} \end{cases}$

c. $x(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 < t < 2 \\ 4 & 2 < t \end{cases}$

d. $x(t) = \begin{cases} 0 & t < 0.2 \\ \tan^{-1} t) & 0.2 < t < 1 \\ \cos(2\pi t) & 1 < t \end{cases}$

3.19 Plot the signals:

a. $x(t) = r(t - 1.5)$

b. $x(t) = r(-t - 1.5)$

c. $x(t) = r(-t + 1.5)$

d. $x(t) = r(t + 1.5)$

e. $x(t) = -2r(t - 1.5)$

Comment on the relationships between the signals.

3.20 Plot the signals:

a. $x(t) = r(t+2)$

b. $x(t) = 3r(t - 1.5)$

c. $x(t) = 2.5r(0.5t + 1)$

d. $x(t) = 4r(6 - t)$

e. $x(t) = r(3 - 0.5t)$

3.21 Plot the following signals:

a. $x(t) = 2u(t+2) - u(t-0.5) + r(t-0.5) - r(t-2)$

b. $x(t) = 2r(t+1) - 4r(t-1) + u(t-2) + 2r(t-3) - u(t-4)u(5-t)$

c. $x(t) = 2u(t) - u(t-1) - u(t+1)$

d. $x(t) = r(1-t)u(t+1) - 2u(t+1) + 2u(t) + 2r(t+1)u(-t)$.

Can you find a simpler expression for the signal of part **d**?

3.22 Plot the following signals:

a. $x(t) = 2u(t+0.5) + 2r(t+0.5) - r(t) - 3u(t-1.5) - r(t-2)$

c. $x(t) = 3\delta(t+4) - 2\delta(t+5)$, $y(t) = \delta(t-2) + \delta(t-3)$

4.33 Plot $x(t)$, $\delta(t-t_1)$, and $y(t) = x(t) * \delta(t-t_1)$ for

a. $x(t) = r(t+1) - 2r(t) + r(t-1)$, $t_1 = 1$
b. $x(t) = r(t) - 2r(t-1) + r(t-2)$, $t_1 = 0$
c. $x(t) = \sin(0.2\pi t)\prod(t/10)$, $t_1 = -2.5$
d. $x(t) = \prod[(t-1)/2]$, $t_1 = 1$

4.34 Show that $y(t) = B\delta(t-t_1) * A\delta(t-t_0)$ is the impulse $w(t) = AB\delta[t-(t_1+t_0)]$. *Hint*: evaluate $\int_{-\infty}^{\infty} x(t)y(t)\,dt$ and $\int_{-\infty}^{\infty} x(t)w(t)\,dt$ where $x(t)$ is continuous at $t = t_1 + t_0$.

Problems for Computer Solution

4.35 The single-tone signal $w(t) = \sin(400\pi t)$ is transmitted to an audio amplifier and speaker to produce a high-temperature warning for a silicon crystal growing facility. A filter having the impulse response $h(t) = 400e^{-200t}\cos(400\pi t)u(t)$ has been designed to reduce additive interference in the received signal. Use numerical integration to find the filter output signal, $y(t)$, when the received input signal is $x(t) = [\cos(100\pi t) + \sin(400\pi t) - \cos(800\pi t)]u(t)$. Plot the output signal, the input signal, and each component of the input signal. Comment on the effect of the filter on the signal.

4.36 Use numerical integration to evaluate the convolution integral for the following pairs of signals.

a. $x(v) = \sqrt{v}e^{-0.5v^{1.5}}u(v-2)$

$$y(v) = (1.5)^v u(-v+3)$$

b.
$$\begin{aligned} a(t) &= 2u(t+2.5) + r(t+1) - 2r(t-1) \\ &\quad + r(t-3.25) - 2.5u(t-4.5) \\ &\quad + 2r(t-5.75) - 2r(t-7) \\ &\quad - 1.75u(t-8.25) \end{aligned}$$

$$\begin{aligned} b(t) &= r(t-1) - r(t-2.5) - 1.5u(t-3) \\ &\quad + 0.75r(t-3.5) - r(t-5.25) \\ &\quad + 0.375r(t-8) \end{aligned}$$

Plot the signals and their convolution.

4.37 Work Problem 4.35 using the MATLAB command conv.

4.38 Work Problem 4.36 using the MATLAB command conv.

4.39 Use the five signals $x(t) = e^{-t}[\cos(3t) - \sin(3t)]u(t)$, $y(t) = 2\Pi\left(\frac{t-1}{4}\right)$, $z(t) = e^{-|1.5t|}$, $w(t) = r(t) - 2r(t-1) + r(t-2)$, and $v(t) = 0.75u(t+0.5) + 0.75u(t-0.5) - 1.5u(t-3.5)$ to illustrate that $y(t) * [x(t) * z(t)] + v(t) * [w(t) * x(t)]$ equals $x(t) * [y(t) * z(t) + w(t) * v(t)]$.

5.31 Use the Fourier transform to find the amplitude and phase spectra for the following signals.

a. $x(t) = 4\cos(100\pi t + \pi/2.5) + 2$

b. $y(t) = 12\sin(10\pi t - \pi/2) + 5\sin(25\pi t + \pi/4)$

c. $z(t) = 3\cos(3\pi t) + 4\cos(3\pi t)\cos(6\pi t - \pi/3)$

5.32 An infrared sensor mounted on the belly of an aircraft sweeps linearly back and forth over the angle range from vertical to 1 radian from vertical. In this way, it produces an infrared image of the ground on one side of the aircraft flight path. The angular sweep signal $\theta(t)$ used to drive the sensor is shown in Figure 5.66. Use the Fourier transform table and theorems to find the spectrum of the sweep signal. Plot the amplitude and phase spectra or, if possible, the total spectrum for frequencies less than or equal to 10 Hz.

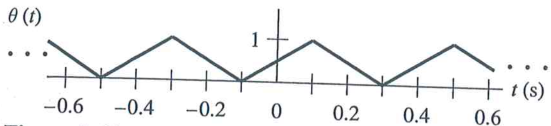


Figure 5.66

5.33 A video camera used to monitor a scene is mounted on the corner of a building and scans by continual rotation. Since the camera points at the building walls for 1/4 of its scan, we want to blank its video signal for this portion of time. Figure 5.67 shows the signal that turns the video on and off. Use the Fourier transform table and theorems to find the spectrum of $x(t)$. Plot the amplitude and phase spectra or, if possible, the total spectrum for frequencies less than or equal to 0.1 Hz.

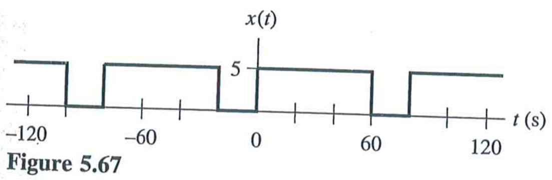


Figure 5.67

5.34 Use the Fourier transform table and theorems to find the spectrum of the periodic signal shown in Figure 5.68. Plot the amplitude and phase spectra or, if possible, the total spectrum for frequencies less than or equal to 1 Hz.

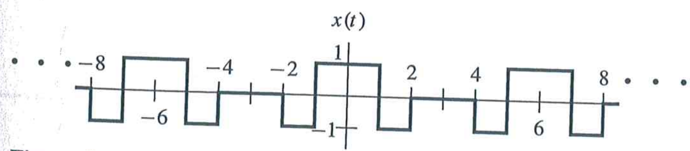


Figure 5.68

5.35 Use the Fourier transform table and theorems to find the spectrum of the signal

$$x(t) = A\prod(f_0 t/N)\cos 2\pi f_0 t$$

for (a) $N = 1$, (b) $N = 2$, (c) $N = 4$. Plot the three signals. Also plot the spectra of the signals for frequencies less than $3f_0$.

5.36 A pulse radar transmits a cosine carrier signal of frequency 4 MHz modulated by the pulse train signal

$$x(t) = \sum_{n=-\infty}^{\infty} \prod\left(\frac{t - iT}{\tau}\right)$$

where $\tau = 1\mu s$ and $T = 20\mu s$. Use the Fourier transform table and theorems to find the spectrum of the transmitted radar signal. Plot the transmitted signal and its amplitude and phase spectra or, if possible, its total spectrum. (*Note:* These radar parameters are not practical; however, the result does show the nature of a pulsed radar signal and its spectrum.

5.37 Find the power-density spectrum of the video blanking signal of Problem 5.33.

5.38 Find the power-density spectrum for the pe[…] signal of Problem 5.34. What bandwidth o[…] quencies contains at least 95% of the signal p[…]

5.39 Find the power density spectrum for the […] mitted radar signal of Problem 5.36. What […] of frequencies contains at least 95% of the s[…] power.

Problems for Computer Solution

5.40 To illustrate periodic and nonperiodic sums of sinusoids, plot the signal $x(t) = 2\cos(\pi t) + \cos(3\pi t - 0.4)$, $y(t) = 2\cos(\pi t) + \cos(2.8\pi t - 0.4)$, and $z(t) = 2\cos(\pi t) + \cos(\sqrt{8}\pi t - 0.4)$ for the time interval $0 \le t \le 10$. Which of the signals are periodic and which are not periodic? What are the periods of the periodic signals (see Appendix A)? Also plot the double-sided amplitude and phase spectra for each signal.

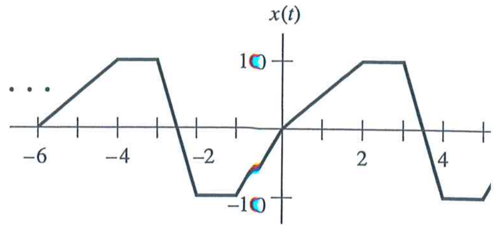


Figure 5.69

5.41 Find the 11 complex-exponential Fourier-series coefficients, X_n for $|n| \le 5$, corresponding to the signal $x(t) = (1+t^2)^{-1}$ for the expansion intervals (a) $|t| \le 1$, (b) $0 \le t \le 2$, and (c) $1 \le t \le 3$. Also, plot $x(t)$ and $\hat{x}_5(t)$ over the interval $|t| \le 4$ for each expansion interval.

5.42 Plot the truncated Fourier series approximation, $\hat{x}_N(t)$, to the signals of Problem 5.4 over the time interval $-2 \le t \le 4$ when $N = 10$. Also plot the signals and indicate the expansion interval on the same set of axes used for each approximation (refer to Figure 5.9).

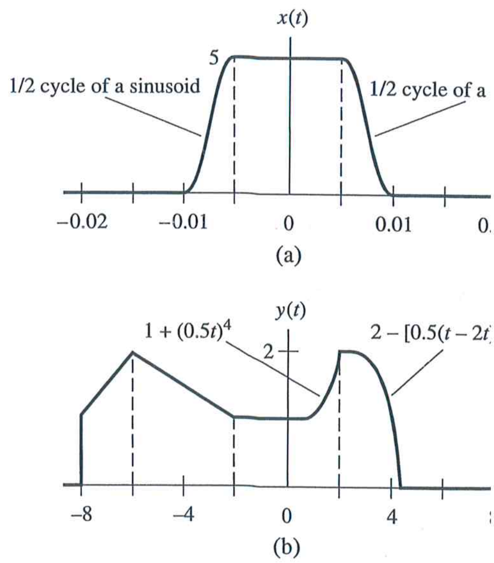
5.43 As an illustration of Gibbs phenomenon, find the complex-exponential Fourier series representation for the signal $x(t) = 4\prod(t/4)$ over the interval $|t| < 4$. Plot $\hat{x}_5(t)$, $\hat{x}_{10}(t)$ and $\hat{x}_{20}(t)$ over this same interval.

5.44 The location of a light source in an advertising sign is controlled by the periodic signal shown in Figure 5.69.

a. Plot the amplitude and phase spectra for this signal.

b. What is the signal bandwidth if the significance factor is defined to be $\alpha = 0.2$.

5.45 Use MATLAB to plot the amplitude and phase spectra or, if possible, the total spectrum for the signals shown in Figure 5.70. Also, plot $w(t)$.



$w(t) = 4e^{-15|t|}[\sin^2(10\pi t) - \cos^2(15\pi t)]$

(c)

Figure 5.70

5.46 The signal $x(t)$ has the spectrum

$$X(f) = \frac{137 + j40\pi f}{(87 - 28\pi^2 f^2) + j(62\pi f - 8\pi^3 f^3}$$

a. Plot the amplitude and phase spectra.
b. Compute values for the inverse Fourier transform of $X(f)$ and plot them to see $x(t)$.

5.47 A random noise signal can be constructed with the MATLAB function:

```
function[N,n]=rn(t)
  N=size(t,2);    rand('seed',0);
 n(1)=rand-0.5;
 for  i=2:N;
    n(i)=0.3*n(i-1)+1.5*(rand-0.5);
 end;
```

Add this noise to the triangular pulse signal

$$x(t) = 0.5\,[r(t-2) - 2r(t-4) + r(t-6)]$$

a. Plot the signal for $0 \le t \le 10$ and the amplitude and phase spectra for $|f| \le 5$. Use the time vector `t=0:0.02:10`.
b. Create the signal $w(t)$ by computing the following average, $w(t) = \frac{1}{9}\sum_{i=1}^{9} x(t - 0.02i + 0.1)$. Plot $w(t)$ and its amplitude and phase spectra.
c. Comment on the effect of the shifted signal average both in terms of signal waveform and spectrum.

c. Sketch the approximate phase-response Bode plot for the system using the straight-line approximation and a few calculated points.

6.23 Work Problem 6.22 with the frequency responses

a. $H_\omega(\omega) = \frac{500(j\omega+15)}{j\omega(j\omega+350)}$

b. $H_\omega(\omega) = \frac{600j\omega+600}{0.25(j\omega)^2+51(j\omega)+200}$

6.24 Work Problem 6.22 with the frequency responses

a. $H_\omega(\omega) = \frac{180(j\omega)+360}{0.006(j\omega)^2+3.06(j\omega)+30}$

b. $H_\omega(\omega) = \frac{(j\omega)[(2\times10^6)(j\omega)+10^7]}{(j\omega+35)^2[(j\omega)^2+200(j\omega)+10^6]}$

6.25 Work Problem 6.22 with the frequency responses

a. $H_\omega(\omega) = \frac{225,000}{(j\omega+1000)[(j0.01\omega)^2+j0.1\omega+225]}$

b. $H_\omega(\omega) = \frac{1800(j\omega)^2(j\omega+400)}{(j0.1\omega+2)^2[(j0.01\omega)^2+j2.4\omega+160000](j\omega+800)}$

6.26 Work Problem 6.22 with the frequency responses

a. $H_{a\omega}(\omega) = 100\big/(j\omega+100)$ and

b. $H_{b\omega}(\omega) = 6,850,000(j\omega+110)^2\,/$

$$\left[(j\omega+100)(j\omega+170)^4\right]$$

Compare the two frequency responses.

Problems for Computer Solution

6.27 The input signal to the system of Problem 6.3 is

$$x(t) = 10\cos(50\pi t)\Pi[(t-0.5)/0.2]$$
$$+\,2\sin(100\pi t)\Pi(t-0.5)$$

a. Plot the input signal.
b. Plot the system amplitude and phase responses.
c. Plot the amplitude and phase spectra for the input and output signals.
d. Plot the output signal.

6.28 Plot the positive frequency portion of the amplitude response for the system defined in Problem 6.2. Compute, to a precision of 0.5 Hz, the system cutoff frequency, or frequencies, and bandwidth.

6.29 An acoustic sensor used in a remote underwater noise detecting system is characterized by the differential equation

$$y^{(6)}(t) + 3160y^{(5)}(t) + 6.14\times10^6y^{(4)}(t)$$
$$+\,4.09\times10^9y^{(3)}(t) + 2.91\times10^{12}y^{(2)}(t)$$
$$+\,2.40\times10^{14}y^{(1)}(t) + 2.56\times10^{16}y(t)$$
$$= 10^9x^{(3)}(t) + 1.022\times10^{12}x^{(2)}(t)$$
$$+\,2.204\times10^{13}x^{(1)}(t) + 4\times10^{13}x(t)$$

where $x^{(n)}(t) = d^nx(t)/dt^n$. Plot the system amplitude and phase response for $10 \le \omega \le 10,000$. Use a logarithmic frequency scale.

6.30 Find the phase delay and approximate group delay for the acoustic sensor in Problem 6.29 for the frequencies: (a) 20 Hz, (b) 30 Hz, (c) 40 Hz, and (d) 50 Hz.

6.31 Find the phase delay and approximate group delay for the acoustic sensor in Problem 6.29 for the frequencies (a) 20 Hz, (b) 60 Hz, (c) 1 kHz, and (d) 2 kHz.

6.32 Use MATLAB to find the amplitude- and phase-response Bode plots and their straight-line approximations for the system frequency response given in part **b** of Problem 6.25.

6.33 Use MATLAB to find the amplitude and phase-response Bode plots and their straight-line approximations for the acoustic sensor defined in Problem 6.29.