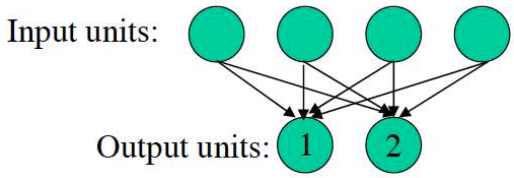
MECA 533 Neural Networks Spring 2018 Final Exam

1. (28 pts) A self organizing map (SOM) contains 4 input and 2 output units as depicted in the figure below. The neighborhood function of this network includes only the winning neuron.



There are five training samples to be clustered using this SOM:

[1, 1, 0, 0] & [0 ,0, 1, 1] & [1, 0, 1, 0] & [1, 0, 0, 0] & [0, 0, 0, 1]

The initial learning rate (η) is 0.6 for the first five weight updates. After that, learning rate is halved every five weight updates.

$\eta(t) = 0.6, \ 1 <= t <= 5$
$\eta(t) = 0.5\eta(t-5), \ t > 5$

Initial weight matrices, which contain random starting weights of each output unit, are given in Table 1. Only use the weight matrix that is assigned to you.

a) Using these five training patterns, find the weight matrices after 10 updates (two epochs).

b) How are the five training patterns clustered after the 10^{th} update?

c) Which output unit does the new pattern [1, 1, 1, 0] cluster with?

d) What are the Euclidean distances between the 6 patterns? Present as a matrix. Are these distances consistent with the results in b and c? Explain.

2. (24 pts) A 5-neuron Hopfield network with feedback loops is depicted in the figure below.

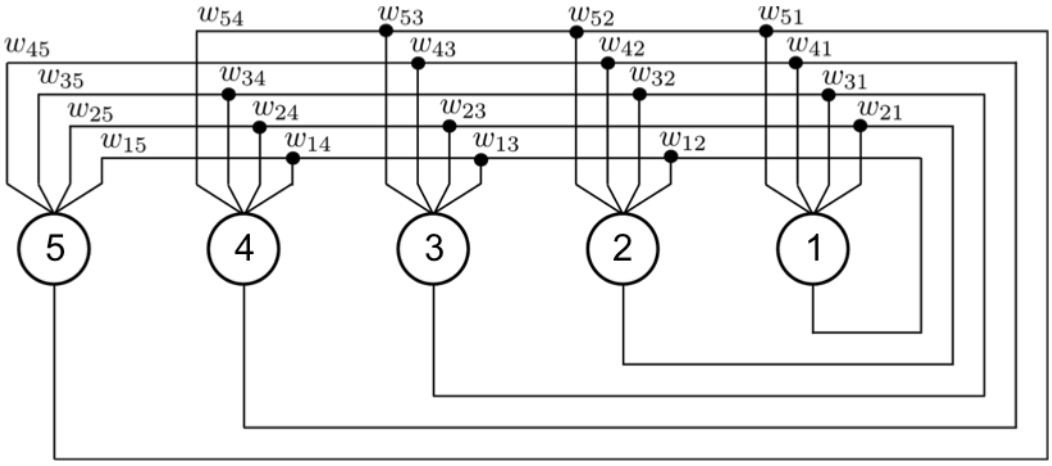


Figure 2

a) Find the weight matrix for storing these two patterns: [1 0 1 0 0] & [0 1 1 0 1]

b) If a noisy input [1 0 1 0 1] is applied, which pattern does it attract to? How many updates does it take to converge? Use the order assigned to you in Table 2 for updating neurons.

Note: For binary inputs use {0, 1} and for bipolar inputs use {-1, 1}. Remember that binary inputs use $\Sigma(2s_1-1)(2s_2-1)$ and bipolar inputs use $\Sigma s_1 s_2$ for weight training.

3. (30 pts) Below is a diagram of a single artificial neuron (unit):

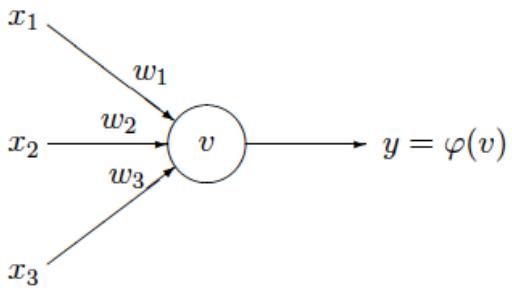


Figure 3: Single unit with three inputs.

a) The node has three inputs x = (x_1, x_2, x_3) that receive only binary signals (either 0 or 1). How many different input patterns can this node receive? What if the node had four inputs? Five? Can you give a formula that computes the number of binary input patterns for a given number of inputs?

b) Suppose that the weights corresponding to the three inputs have the following values:
$w_1 = 2$, $w_2 = -4$, $w_3 = 1$, bias = 0

and the activation of the unit is given by the step-function:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Calculate what will be the output value y of the unit for each of the following input patterns:

Pattern	P_1	P_2	P_3	P_4
x_1	1	0	1	1
x_2	0	1	0	1
x_3	0	1	1	1

c) If the weights for a 2-input AND function are w1 = 1, w2 = 1, bias = 2 and the activation function is:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

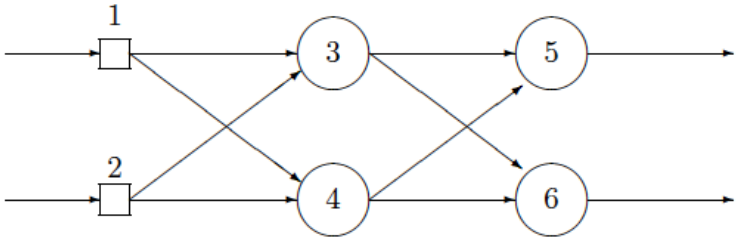
test how the neural AND function works.

d) Suggest how to change either the weights or the threshold level of this single-unit in order to implement the logical OR function

e) Do you think it is possible to implement XOR function using a single unit? A network of several units in the same layer? XOR function truth table is given below:

x_1 :	0	1	0	1
x_2 :	0	0	1	1
x_1 XOR x_2 :	0	1	1	0

4. (18 pts) The following figure represents a multi-layer feed-forward neural network with one hidden layer:



A weight on connection between nodes i and j is denoted by w_{ij} , such as w_{13} is the weight on the connection between nodes 1 and 3. The following table lists all the weights in the network:

$w_{13} = -2$ $w_{23} = 3$	$w_{35} = 1$ $w_{45} = -1$
$w_{14} = 4$ $w_{24} = -1$	$w_{36} = -1$ $w_{46} = 1$

Each of the nodes 3, 4, 5 and 6 uses the following activation function:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where v denotes the weighted sum of a node. Each of the input nodes (1 and 2) can only receive binary values (either 0 or 1).

a) Calculate the output of the network (y5 and y6) for each of the input patterns:

Pattern:	P_1	P_2	P_3	P_4
Node 1:	0	1	0	1
Node 2:	0	0	1	1

b) What is a training set and how is it used to train neural networks? What is an epoch?

Table 1

Student ID	unit1 weights				unit2 weights			
2017721051	0,60	0,87	0,60	0,52	0,45	0,33	0,69	0,67
2017721054	0,96	0,82	0,26	0,06	0,40	0,19	0,25	0,85
2017721027	0,72	0,59	0,76	0,63	0,06	0,96	0,97	0,53
2016721042	0,93	0,03	0,84	0,62	0,02	0,70	0,35	0,95
2017721069	0,45	0,73	0,71	0,42	0,69	0,49	0,58	0,21
2016721048	0,61	0,37	0,75	0,23	0,04	0,91	0,61	0,13
2017721072	0,81	0,16	0,45	0,90	0,75	0,02	0,56	0,63
2016721054	0,16	0,96	0,80	0,42	0,79	0,66	0,85	0,68
2017721048	0,97	0,49	0,14	0,92	0,96	0,04	0,93	0,76

Table 2

Student ID	neuron update order
2017721051	1 2 4 5 3
2017721054	1 3 2 5 4
2017721027	5 3 2 4 1
2016721042	3 1 4 2 5
2017721069	4 3 2 5 1
2016721048	1 4 3 2 5
2017721072	4 1 3 5 2
2016721054	4 2 3 5 1
2017721048	1 3 4 2 5