Question 2

(a) MLE for gaussian $= \frac{\sum_i x_i}{n}$

$$\therefore \hat{\mu}_1 = \frac{\sum_i x_i \; (x_i \in \text{Class}_1)}{\cancel{\text{to}}\; |\text{class}_1|}$$

$$= 0.26$$

$$\therefore \hat{\mu}_2 = \frac{\sum_i x_i \,(x_i \in \text{Class}_2)}{|\text{Class}_2|}$$

$$= 0.8625$$

Now the class probabilities:

$$p1 = \frac{10}{14} = 0.7143$$

$$p2 = \frac{4}{14} = 0.2857$$

Now for $x = 0.6$ using Bays theorm:

$$P(\text{class}=1 \mid x=0.6) = \frac{P(x=0.6 \mid \text{class}=1) \cdot p1}{p(x=0.6)}$$

$$= \frac{P(x=0.6 \mid \text{class}=1) \cdot p1}{P_1 P(x \mid \text{class}=1) + P_2 P(x \mid \text{class}=2)}$$

$\longrightarrow$ 1

using gaussian liklehood,

$$P(x|C_k) = \frac{1}{\sqrt{2\pi \sigma_k^2}} e\left[-\frac{1}{2}\frac{(x-\mu_x)^2}{\sigma_k^2}\right]$$

given $\sigma_1^2 = 0.0149$ $\quad \sigma_2^2 = 0.0092$

$$P(x/C_1) = \frac{1}{\sqrt{2\pi\, 0.0149}} \exp\left[\frac{-1}{2}\frac{(0.6-0.26)^2}{0.0149}\right]$$

$$= 0.06756$$

similarly

$$P(x|C_2) = 0.09834$$

~~Plug~~

using these in equation 1:

$$P(c=\text{class}1 \mid x=0.66) = 0.6305$$

(b) From the data given we can observe that 7th attribute in sport matrix is always 0 and in given instance the 7th attribute has value 1

$\therefore P(x \mid c = x_{sport}) = 0.$

$\therefore$ By total probability theorm

$$P(c = x_{politics} \mid x) = 1$$

$\Rightarrow$ this problem is also called as zero frequency problem.

$\rightarrow$ To overcome this we add rows (0,0,0,0,0,0,0,0) and (1,1,1,1,1,1,1,1) to both tables.

Now $P(c = x_{politics} \mid x) = \frac{\left(P(x / c = x_{pol}) . P(c = x_{pol}) \right.}{P(x)}$

$$= \frac{\left(\frac{3}{8} \times \frac{6}{8} \times \frac{6}{8} \times \frac{6}{8} \times \frac{6}{8} \times \frac{2}{8} \times \frac{5}{8} \times \frac{2}{8}\right) \times \frac{1}{2}}{P(x)}$$

$= 0.878$