WBJEEM - 2014

PHYSICS

Q.No.	✱	⌘	❖	✪
01	B	C	A	C
02	D	C	D	A
03	B	D	A	C
04	C	A	B	C
05	C	A	A	D
06	A	D	D	C
07	B	A	C	B
08	A	A	D	A
09	C	D	B	B
10	D	B	C	A
11	C	D	A	D
12	C	B	C	A
13	B	B	D	A
14	D	B	A	D
15	B	D	C	A
16	B	B	B	A
17	B	C	A	D
18	B	B	D	A
19	A	D	D	C
20	B	D	C	D
21	C	D	C	C
22	A	C	D	D
23	D	A	C	B
24	A	C	D	B
25	A	C	A	A
26	D	A	A	B
27	B	B	B	C
28	C	B	B	D
29	A	D	B	B
30	D	A	D	B
31	D	C	D	B
32	C	C	C	C
33	D	A	D	B
34	D	D	B	A
35	A	C	A	B
36	A	A	B	D
37	C	B	C	C
38	D	A	D	C
39	D	C	C	D
40	D	B	A	D
41	A	D	A	A
42	C	A	B	D
43	A		C	C
44		C	C	C
45	C	D	A	D
6	B	A	C	D
7	C	C	C	B
4	D	A	D	C
49	D	C	D	B
50	B	D	B	B
51	D	B	A	D
52	C	D	A	A
53	B	B	B	A
54	A	B	D	D
55	A	D	B	C
56	A, D	B, C, D	A, C	C, D
57	A, C	A, C	A, C	B, C, D
58	C, D	C, D	A, D	A, D
59	A, C	A, C	B, C, D	A, C
60	B, C, D	A, D	C, D	A, C

CHEMISTRY

Q.No.	↑	←	↓	→
01	A	D	D	A
02	A	B	C	B
03	C	C	D	C
04	C	C	A	A
05	D	B	B	C
06	A	A	C	A
07	A	D	B	B*
08	C	B	C	C
09	B	A	C	D
10	A	C	A	C
11	D	B	B	B
12	D	D	A	C
13	D	A	C	A
14	C	C	B*	A
15	C	B*	A	B
16	B	B	B	D
17	B		D	A
18	A	C	D	B
19	A	C	A	A
20	A	D	D	C
21	B	C	D	D
22		C	B	C
23	B	B	C	B
24	A	A	A	D
25	C	B	A	B
26		A	A	D
27	C	A	A	A
28	B	A	D	D
29	C	C	D	C
30	D	B	A	C
31	B	B	A	B
32	A	A	B	C
33	D	D	C	A
34	C	B	A	B
35	B	B	B	A
36	A	D	C	B
37	D	D	B	B
38	C	D	B	C
39	B	C	D	B
40	D	A	C	A
41	D	B	C	A
42	C	A	B	B
43	B	A	B	D
44	B	B	B	D
45	B	B	C	B
46	B	C	C	A
47	C	A	D	B
48	C	A	A	C
49	D	D	C	C
50	B	D	B	C
51	A	C	B	C
52	C	B	C	A
53	C	C	B	D
54	A	B	C	B
55	B	C	A	B
56	A, B, D	B, D	B, C, D	B, C
57	B, C, D,	B, C	A, B, D	A, B, D
58	B, C	A, B, D	A, B, D	B, D
59	A, B, D	A, B, D	B, D	B, C, D
60	B, D	B, C, D	B, C	A, B, D

*** B and C both option are correct but as single Option B is more appropriate.**

Code - ✱

ANSWERS & HINTS
for
WBJEEM - 2014
SUB : PHYSICS

CATEGORY - I

Q.1 to Q.45 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark.

1. A whistle whose air column is open at both ends has a fundamental frequency of 5100 Hz. If the speed of sound in air is 340 ms^{-1}, the length of the whistle, in cm, is

   (A) 5/3 (B) 10/3 (C) 5 (D) 20/3

   Ans : (B)

   Hints : $f=\frac{v}{2\ell} \Rightarrow \ell=\frac{v}{2f}=\frac{340}{2\times5100}=\frac{1}{30}m=\frac{10}{3}cm$

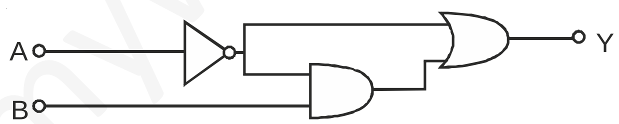
2. One mole of an ideal monoatomic gas is heated at a constant pressure from 0°C to 100°C. Then the change in the internal energy of the gas is (Given R = 8.32 Jmol^{-1}K^{-1})

   (A) 0.83×10^3 J (B) 4.6×10^3 J (C) 2.08×10^3 J (D) 1.25×10^3 J

   Ans : (D)

   Hints : $\Delta U=nC_V\Delta T=1\times\left(\frac{3}{2}R\right)\times100=1\times\frac{3}{2}\times8.32\times100=1.25\times10^3J$

3. The output Y of the logic circuit given b low is,



   (A) $\overline{A}+B$ (B) $\overline{A}$ (C) $\overline{(\overline{A}+B)}.\overline{A}$ (D) $\overline{(\overline{A}+B)}.A$

   Ans : (B)

   Hints : $\left(\overline{A}.B\right)+\overline{A}=\overline{A}.(B+1)=\overline{A}.1=\overline{A}$

4. In which of the following pairs, the two physical quantities have different dimensions?

   (A) Planck's constant and angular momentum (B) Impulse and linear momentum

   (C) Moment of inertia and moment of a force (D) Energy and torque

   Ans : (C)

5. A small metal sphere of radius *a* is falling with a velocity ν through a vertical column of a viscous liquid. If the coefficient of viscosity of the liquid is η, then the sphere encounters an opposing force of

   (A) $6\pi\eta a^2\nu$ (B) $\frac{6\eta\nu}{\pi a}$ (C) $6\pi\eta a\nu$ (D) $\frac{\pi\eta\nu}{6a^3}$

   Ans : (C)

   Hints : Stoke's Law

6. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(A) $\frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$ (B) $\frac{h_1t_1^2 + h_2t_2^2}{h_2t_1 + h_1t_2}$ (C) $\frac{h_1t_2^2 + h_2t_1^2}{h_1t_2 + h_2t_1}$ (D) $\frac{h_1t_1^2 - h_2t_2^2}{h_1t_1 - h_2t_2}$

Ans : (A)

Hints : $h_1 = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$; $h_2 = (u\sin\theta)t_2 - \frac{1}{2}gt_2^2$

$$\Rightarrow \frac{h_1 + \frac{1}{2}gt_1^2}{h_2 + \frac{1}{2}gt_2^2} = \frac{t_1}{t_2} \Rightarrow h_1t_2 - h_2t_1 = \frac{g}{2}\left(t_1t_2^2 - t_1^2t_2\right)$$

$$T = \frac{2u\sin\theta}{g} = \frac{2}{g}\left[\frac{h_1 + \frac{1}{2}gt_1^2}{t_1}\right] = \frac{2}{t_1}\left[\frac{h_1}{g} + \frac{t_1^2}{2}\right] = \frac{h_1}{t_1} \times \left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1 = \frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$$

7. A smooth massless string passes over a smooth fixed pulley. Two masses m_1 and m_2 ($m_1 > m_2$) are tied at the two ends of the string. The masses are allowed to move under gravity starting from r st. The total external force acting on the two masses is

(A) $(m_1 + m_2)\,g$ (B) $\frac{(m_1 - m_2)^2}{m_1 + m_2}g$ (C) $(m_1 - m\ )\,g$ (D) $\frac{(m_1 + m_2)^2}{m_1 - m_2}g$

Ans : (B)

Hints : $a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

so, Resultant external force = $(m_1 + m_2)\,a_{cm} = \frac{(m_1 - m_2)^2}{(m_1 + m_2)}g$

8. To determine the coefficient of friction between a rough surface and a block, the surface is kept inclined at 45° and the block is released from rest. The block takes a time *t* in moving a distance *d*.The rough surface is then replaced by a smooth surface and the same experiment is repeated. The block now takes a time *t/2* in moving down the same distance *d*. The coefficient of frict on is

(A) 3/4 (B) 5/4 (C) 1/2 (D) $1/\sqrt{2}$

Ans : (A)

Hints : $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) = 1\left[1 - \frac{1}{2^2}\right] = \frac{3}{4}$

9. A wooden block is floating on water kept in a beaker. 40% of the block is above the water surface. Now the beaker is kept inside a lift that starts going upward with acceleration equal to g/2. The block will then

(A) sink (B) float with 10% above the water surface

(C) float with 40% above the water surface (D) float with 70% above the water surface

Ans : (C)

10. An electron in a circular orbit of radius .05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in Am^2)

(A) 2.16×10^{-23} (B) 3.21×10^{-22} (C) 3.21×10^{-24} (D) 1.26×10^{-23}

Ans : (D)

Hints : $M = iA = qfA = (1.6 \times 10^{-19})(10^{16})(3.14 \times (0.05\times10^{-9})^2) = 1.26 \times 10^{-23}$

11. A very small circular loop of radius *a* is initially (at t = 0) coplanar and concentric with a much larger fixed circular loop of radius *b*. A constant current *I* flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time *t* is

(A) $\frac{\pi a^2 \mu_o I}{2b}\omega\cos(\omega t)$ (B) $\frac{\pi a^2 \mu_o I}{2b}\omega\sin(\omega^2 t^2)$

(C) $\frac{\pi a^2 \mu_o I}{2b}\omega\sin(\omega t)$ (D) $\frac{\pi a^2 \mu_o I}{2b}\omega\sin^2(\omega t)$

Ans : (C)

Hints : ε = NBAω sinωt N = 1, B = $\frac{\mu_0 I}{2b}$, A = πa^2

$= \frac{\mu_0 I}{2b}(\pi a^2)\omega \sin\omega t$

12. A drop of some liquid of volume 0.04 cm³ is placed on the surface of a glass slide. Then another glass slide is placed on it in such a way that the liquid forms a thin layer of area 20 cm^2 between the surfaces of the two slides. To separate the slides a force of 16×10^5 dyne has to be applied normal to the surfaces. The surface tension of the liquid is (in dyne-cm^{-1})

(A) 60 (B) 70 (C) 80 (D) 90

Ans : (C)

Hints : Let thickness of layer is t

$V = At,\quad t = \frac{V}{A},\quad 2r = \frac{V}{A},\ r = \frac{V}{2A},\quad \Delta P = \frac{T}{r}$

$F = \Delta P \times A = \frac{T}{r} \times A = \frac{T}{\left(\frac{V}{2A}\right)}A,\ F = \frac{2TA^2}{V} = 80$ dyne/cm

13. A proton of mass *m* and charge *q* is moving in a plane with kinetic energy *E.* If there exists a uniform magnetic field *B,* perpendicular to the plane of the motio th portion will move in a circular path of radius

(A) $\frac{2Em}{qB}$ (B) $\frac{\sqrt{2Em}}{qB}$ (C) $\frac{\sqrt{Em}}{2qB}$ (D) $\sqrt{\frac{2Eq}{mB}}$

Ans : (B)

Hints : $r = \frac{mv}{qB} = \frac{\sqrt{2Em}}{qB}$

14. In which of the following phenomena, the heat waves travel along straight lines with the speed of light ?

(A) thermal conduction (B) forced convection (C) natural convection (D) thermal radiation

Ans : (D)

15. An artificial satellite moves in a circular orbit around the earth. Total energy of the satellite is given by *E.* The potential energy of the satellite is

(A) –2E (B) 2E (C) 2E/3 (D) –2E/3

Ans : (B)

Hints : P.E. = 2(T.E.)

16. A particle moves with constant acceleration along a straight line starting from rest. The percentage increase in its displacement during the 4th second compared to that in the 3rd second is

(A) 33% (B) 40% (C) 66% (D) 77%

Ans : (B)

Hints : $S_{nth} = u + \frac{1}{2}a(2n-1)$

$S_{3rd} = \frac{5}{2}a$, $S_{4\,h} = \frac{7}{2}a$

$\frac{S_{4\,h} - S_{3rd}}{S_{3rd}} \times 100 = \frac{a}{\left(\frac{5a}{2}\right)} \times 100 = 40\%$

17. In the circuit shown assume the diode to be ideal. When V_i increases from 2 V to 6 V, the change in the current is (in mA)

(A) zero (B) 20 (C) 80/3 (D) 40

Ans : (B)

Hints : $I_{initial} = 0$, $I_{final} = 3/150 = 0.02A$

S, change in I = 0.02A = 20 mA

18. In a transistor output characteristics commonly used in common emitter con ig ration, the base current I_B, the collector current I_C and the collector-emitter voltage V_{CE} have values of the following orders of magnitude in the active region

(A) I_B and I_C both are in μA and V_{CE} in Volts
(B) I_B is in μA, and I_C is in mA and V_{CE} in Volts
(C) I_B is in mA, and I_C is in μA and V_{CE} in mV
(D) I_B is in mA, nd I_C is in mA and V_{CE} in mV

Ans : (B)

19. If *n* denotes a positive integer, *h* the Planck's constant, *q* the charge and *B* the magnetic field, then the quantity $\left(\frac{nh}{2\pi qB}\right)$ has the dimension of

(A) area (B) length (C) speed (D) acceleration

Ans : (A)

Hints : $\left[\frac{nh}{2\pi qB}\right] = \frac{[mvr]}{[qB]} = \frac{[mvr][v]}{[F]} = \frac{[mv\ r]}{\left[\frac{mv^2}{r}\right]} = [r^2]$

20. For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur ?

(A) isobar of original nucl us is produced
(B) isotope of the original nucleus is produced
(C) nuclei with higher atomic number than that of the original nucleus is produced
(D) nuclei with lower atomic number than that of the original nucleus is produced

Ans : (B)

21. A car moving with a speed of 72 km-hour^{-1} towards a roadside source that emits sound at a frequency of 850 Hz. The car driver listens to the sound while approaching the source and again while moving away from the source after crossing it. If the velocity of sound is 340 ms^{-1},the difference of the two frequencies, the driver hears is

(A) 50 Hz (B) 85 Hz
(C) 100 Hz (D) 150 Hz

Ans : (C)

Hints : νapproach = $\nu\left(\frac{V+Vo}{V}\right)$=850$\left(\frac{340+20}{340}\right)$, νseparation = 850$\left(\frac{340-20}{340}\right)$, νapproach–νseparation =

$\frac{850}{340} \times 40 = 100$ Hz

22. Same quantity of ice is filled in each of the two metal containers P and Q having the same size, shape and wall thickness but made of different materials. The containers are kept in identical surroundings. The ice in P melts completely in time t_1 whereas that in Q takes a time t_2. The ratio of thermal conductivities of the materials of P and Q is

(A) $t_2 : t_1$ (B) $t_1 : t_2$ (C) $t_1^2 : t_2^2$ (D) $t_2^2 : t_1^2$

Ans : (A)

Hints : $\left(KA\dfrac{dT}{dx}\right)t=mL$, $K \alpha \dfrac{1}{t}$ So, $\dfrac{K_1}{K_2} = \dfrac{t_2}{t_1}$

23. Three capacitors, 3μF, 6μF and 6μF are connected in series to a source of 120V. The potential difference, in volts, across the 3μF capacitor will be

(A) 24 (B) 30 (C) 40 (D) 60

Ans : (D)

Hints : $Q=CV \Rightarrow V=\dfrac{Q}{C} \Rightarrow V\alpha\dfrac{1}{C}$, so, $V = 120\left(\dfrac{1/3}{1/3+1/6+1/6}\right)$ = 60 volts

24. A galvanometer having internal resistance 10Ω requires 0.01 A for a full scale def ection. To convert this galvanometer to a voltmeter of full-scale deflection at 120V, we need to connect a resistance of

(A) 11990 Ω in series (B) 11990 Ω in parallel (C) 12010 Ω in series (D) 12010 Ω in parallel

Ans : (A)

Hints : $R = \dfrac{V}{I_g} - R_g = \dfrac{120}{0.01} - 10 = 11990\ \Omega$

25. Consider three vectors $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{C} = 2\hat{i} - 3j + 4\hat{k}$. A vector $\vec{X}$ of the form $\alpha\vec{A} + \beta\vec{B}$ (α and β are numbers) is perpendicular to $\vec{C}$. The ratio of α and β is

(A) 1:1 (B) 2:1 (C) –1:1 (D) 3:1

Ans : (A)

Hints : $(\alpha\vec{A} + \beta\vec{B}) \cdot \vec{C} = 0$, $\Rightarrow 2(\alpha+\beta)-3(\alpha-\beta)\ \ 4(\beta-2\alpha)=0$, $\Rightarrow -9\alpha+9\beta=0$, $\Rightarrow \alpha:\beta = 1:1$

26. A parallel plate capacitor is charged an then disconnected from the charging battery. If the plates are now moved farther apart by pulling at them by means of insulating handles, then

(A) the energy stored in the capacitor decreases (B) the capacitance of the capacitor increases

(C) the charge on the capacitor decreases (D) the voltage across the capacitor increases

Ans : (D)

Hints : $C = \dfrac{\varepsilon_o A}{d}$, $d\uparrow$, $c\downarrow$, Q(Const), $V\uparrow$

27. When a particle executing SHM oscillates with a frequency ν, then the kinetic energy of the particle

(A) changes periodically with a frequency of ν (B) changes periodically with a frequency of 2ν

(C) changes periodically with a frequency of ν/2 (D) remains constant

Ans : (B)

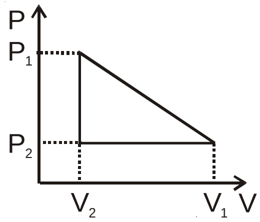
28. The ionization energy of hydrogen is 13.6eV. The energy of the photon released when an electron jumps from the first excited state (n=2) to the ground state of a hydrogen atom is

(A) 3.4 eV (B) 4.53 eV (C) 10.2 eV (D) 13.6 eV

Ans : (C)

Hints : $13.6\left[\dfrac{1}{1^2} - \dfrac{1}{2^2}\right] = 13.6\left(1 - \dfrac{1}{4}\right) = 13.6 \times \dfrac{3}{4}$ = 10.2 ev

29. One mole of a van der Waals' gas obeying the equation $\left(P+\frac{a}{V^2}\right)$(V–b) = RT undergoes the quasi-static cyclic process which is shown in the P-V diagram. The net heat absorbed by the gas in this process is



(A) $\frac{1}{2}(P_1–P_2)(V_1–V_2)$ (B) $\frac{1}{2}(P_1+P_2)(V_1–V_2)$

(C) $\frac{1}{2}\left(P_1+\frac{a}{v_1^2}-P_2-\frac{a}{v_2^2}\right)(V_1–V_2)$ (D) $\frac{1}{2}\left(P_1+\frac{a}{v_1^2}+P_2+\frac{a}{v_2^2}\right)(V_1–V_2)$

Ans : (A)

Hints : For cyclic process, heat absorbed = work done = Area = $\frac{1}{2}(P_1–P_2)\,(V_1–V_2)$

30. A scientist proposes a new temperature scale in which the ice point is 25 X (X is the new unit of temperature) and the steam point is 305 X. The specific heat capacity of water in this new scale is (in $Jkg^{-1}\,X^{-1}$)

(A) 4.2×10^3 (B) 3.0×10^3 (C) 1.2×10^3 (D) 1.5×10^3

Ans : (D)

Hints : (305–25)X = 100°C, ⇒ 1°C = 2.8X, Sp. heat capacity of water = $4200\,\frac{J}{Kg\,°C}$, = $4200\,\frac{J}{Kg\,(2.8X)}$,

= $1.5\times10^3\ J/(Kg-X)$

31. A metal rod is fixed rigidly at two ends so as to prevent its thermal expansion. If L, α and Y respectively denote the length of the rod, coefficient of linear thermal expansion and Young's modulus of its material, then for an increase in temperature of the rod by ΔT, the longitudinal stress developed in the rod is

(A) inversely proportional to α

(B) inversely proportional to Y

(C) directly proportional to $\frac{\Delta T}{Y}$

(D) independent of L

Ans : (D)

Hints : Strain = $\alpha\Delta T$

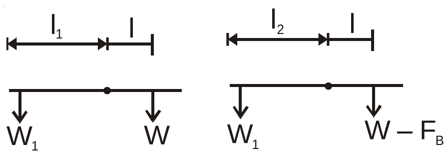
Stress = $Y\alpha\Delta T$

32. A uniform rod is suspended horizontally from its mid-point. A piece of metal whose weight is W is suspended at a distance *l* from the mid-point. Another weight W_1 is suspended on the other side at a distance l_1 from the mid-point to bring the rod to a horizontal position. When *W* is completely immersed in water, W_1 needs to be kept at a distance l_2 from the mid-point to get the rod back into horizontal position. The specific gravity of the metal piece is

(A) $\frac{W}{W_1}$ (B) $\frac{Wl_1}{Wl-W_1l_2}$ (C) $\frac{l_1}{l_1-l_2}$ (D) $\frac{l_1}{l_2}$

Ans : (C)

Hints :



ρ = specific gravity

$Wl = W_1 l_1$ $\qquad$ $W - F_B = W(1 - 1/\rho)$

$Wl\,(1 - 1/\rho) = W_1 l_2$

$1 - 1/\rho = \dfrac{l_2}{l_1}$ $\qquad \Rightarrow 1/\rho = 1 - \dfrac{l_2}{l_1} = \dfrac{l_1 - l_2}{l_1}$

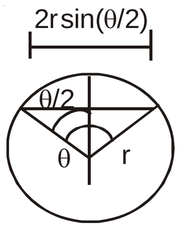
$\Rightarrow \rho = \dfrac{l_1}{l_1 - l_2}$

33. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be

(A) $2r\cos\left(\dfrac{\theta}{2}\right)$ (B) $2r\cot\left(\dfrac{\theta}{2}\right)$ (C) $2r\tan\left(\dfrac{\theta}{2}\right)$ (D) $2r\sin\left(\dfrac{\theta}{2}\right)$

Ans : (D)

Hints :



34. A luminous object is separated from a screen by distance *d*. A convex len is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is

(A) 4d (B) 2d (C) d/2 (D) d/4

Ans : (D)

Hints : From lens displacement method

35. The intensity of magnetization of a bar magnet is 5 0 × 0^4 Am^{-1}. The magnetic length and the area of cross section of the magnet are 12 cm and 1 cm^2 respectively The magnitude of magnetic moment of this bar magnet is (in SI unit)

(A) 0.6 (B) 1.3 (C) 1.24 (D) 2.4

Ans : (A)

Hints : $I = \dfrac{M}{V} \Rightarrow M = IV = 5.0 \times 10^4 \times 12 \times 10^{-6} = 60 \times 10^{-2} = 0.6$

36. An infinite sheet carrying a unif rm surface charge density σ lies on the xy-plane. The work done to carry a charge q from the point $\vec{A} = a\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ to the point $\vec{B} = a\left(\hat{i} - 2\hat{j} + 6\hat{k}\right)$ (where a is a constant with the dimension of length and ε_0 is the permittivity of free space) is

(A) $\dfrac{3\sigma aq}{2\varepsilon_0}$ (B) $\dfrac{2\sigma aq}{\varepsilon_0}$ (C) $\dfrac{5\sigma aq}{2\varepsilon_0}$ (D) $\dfrac{3\sigma aq}{\varepsilon_0}$

Ans : (A)

Hints : $\overrightarrow{AB} = a(-4\hat{j} + 3\hat{k})$

$$\text{Workdone} = q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{k}\cdot a\left(-4\hat{j} + 3\hat{k}\right) = \frac{3q\sigma a}{2\varepsilon_0}$$

37. A uniform solid spherical ball is rolling down a smooth inclined plane from a height h. The velocity attained by the ball when it reaches the bottom of the inclined plane is v. If the ball is now thrown vertically upwards with the same velocity v, the maximum height to which the ball will rise is

(A) 5h/8 (B) 3h/5 (C) 5h/7 (D) 7h/9

Ans : (C)

Hints : $mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

$\Rightarrow v = \sqrt{\frac{10gh}{7}}$

For vertical projection,

$v^2 - u^2 = 2gh'$

So, $\frac{10}{7}gh = 2gh'$ $\Rightarrow h' = 5h/7$

38. Two coherent monochromatic beams of intensities I and 4I respectively are superposed. The maximum and minimum intensities in the resulting pattern are

(A) 5I and 3I (B) 9I and 3I (C) 4I and I (D) 9I and I

Ans : (D)

Hints : $\frac{I_{max}}{I_{man}} = \left(\frac{\sqrt{4I}+\sqrt{I}}{\sqrt{4I}-\sqrt{I}}\right)^2 = \left(\frac{3\sqrt{I}}{\sqrt{I}}\right)^2 = \frac{9}{1}$

39. If the bandgap between valence band and conduction band in a material is 0 eV, then the material is

(A) semiconductor (B) good conductor (C) superconductor (D) insulator

Ans : (D)

Hints : The band gap of 5 eV corresponds to that of an insulator.

40. Consider a blackbody radiation in a cubical box at absol te temperature T. If the length of each side of the box is doubled and the temperature of the walls of the box and that of the radiation is halved, then the total energy

(A) halves (B) doubles (C) quadruples (D) remains the same

Ans : (D)

Hints : Assuming temperature of the body and cubica box is same initially i.e. T and finally it becomes T/2. Because temperature of body and surrounding remains same Hence no net loss of radiation occur through the body. Thus total energy remains constant.

41. Four cells, each of emf E and intern l resistance r, are connected in series across an external resistance R. By mistake one of the cells is connected in reverse. Then the current in the external circuit is

(A) $\frac{2E}{4r+R}$ (B) $\frac{3E}{4r+R}$ (C) $\frac{3E}{3r+R}$ (D) $\frac{2E}{3r+R}$

Ans : (A)

Hints : $i = \frac{3E - E}{4r+R} = \frac{2E}{4r+R}$

42. The energy of gamma (γ) ray photon is E_γ and that of an X-ray photon is E_x. If the visible light photon has an energy of E_v, then we can say that

(A) $E_x > E_\gamma > E_v$ (B) $E_\gamma > E_v > E_x$ (C) $E_\gamma > E_x > E_v$ (D) $E_x > E_v > E_\gamma$

Ans : (C)

43. The intermediate image formed by the objective of a compound microscope is

(A) real, inverted and magnified (B) real, erect and magnified

(C) virtual, erect and magnified (D) virtual, inverted and magnified

Ans : (A)

44. The displacement of a particle in a periodic motion is given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$. This displacement may be considered as the result of superposition of n independent harmonic oscillations, Here n is

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (C)

Hints : $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t) = 2(1+\cos t)\sin(1000t) = 2\sin 1000t + 2\cos t \cdot \sin 1000t$

$= 2\sin 1000\,t + \sin(1001\,t) + \sin(999\,t)$

45. Consider two concentric spherical metal shells of radii r_1 and r_2 ($r_2 > r_1$). If the outer shell has a charge q and the inner one is grounded, the charge on the inner shell is

(A) $\frac{-r_2}{r_1}q$ (B) zero (C) $\frac{-r_1}{r_2}q$ (D) $-q$

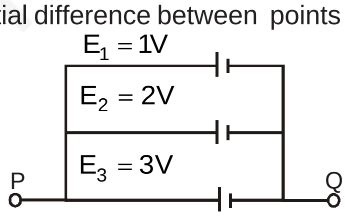
Ans : (C)

Hints : $\frac{k\,q'}{r_1} + \frac{k\,q}{r_2} = 0 \Rightarrow q' = -\left(\frac{r_1}{r_2}\right)q$

CATEGORY - II

Q.46 to Q.55 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark

46. A circuit consists of three batteries of emf E_1 = 1 V, E_2 = 2 V and E_3 = 3 V and internal resistances 1 Ω , 2 Ω and 1 Ω respectively which are connected in parallel as shown in the figure. The potential difference between points P and Q is



(A) 1.0 V (B) 2.0 V

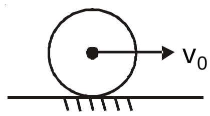
(C) 2.2 V (D) 3.0 V

Ans : (B)

Hints : $E_{eff} = \frac{\frac{1}{1}+\frac{2}{2}+\frac{3}{1}}{\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{1}\right)} = \frac{5}{5}\times 2 = 2$ volt

P.D between two point P and Q = 2 volt

47. A solid uniform sphere resting on a rough horizontal plane is given a horizontal impulse directed through its center so that it starts sliding with an initial vel city v_0. When it finally starts rolling without slipping the speed of its center is



(A) $\frac{2}{7}v_0$ (B) $\frac{3}{7}v_0$

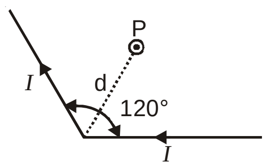
(C) $\frac{5}{7}v_0$ (D) $\frac{6}{7}v_0$

Ans : (C)

Hints : Angular momentum will remain conserved along point of contact

$mv_0R = mvR + \frac{2}{5}mR^2\left(\frac{v}{R}\right) \Rightarrow v = \frac{5v_0}{7}$

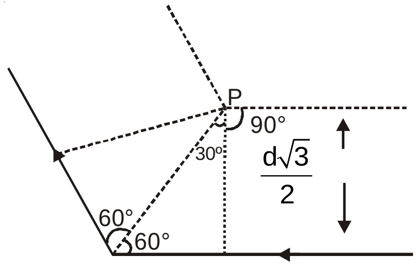
48. A long conducting wire carrying a current I is bent at 120°(see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is (μ_0 is the permeability of free space)



(A) $\frac{3\mu_0 I}{2\pi d}$ (B) $\frac{\mu_0 I}{2\pi d}$

(C) $\frac{\mu_0 I}{\sqrt{3}\pi d}$ (D) $\frac{\sqrt{3}\,\mu_0 I}{2\pi d}$

Ans : (D)



$$B_{net} = 2\left[\frac{\mu_0}{4\pi}\times\frac{i}{\left(\frac{d\sqrt{3}}{2}\right)}\times[1+\sin 30°]\right] = 2\left[\frac{\mu_0}{4\pi}\times\frac{2i}{d\sqrt{3}}\times\frac{3}{2}\right] = \frac{\sqrt{3}\mu_0 I}{2\pi d}$$

49. An object is placed 30 cm away from a convex lens of focal length 10 cm and a sharp image is formed on a screen. Now a concave lens is placed in contact with the convex lens. The screen now has to be moved by 45 cm to get a sharp image again. The magnitude of focal length of the concave len is (in cm)

(A) 72 (B) 60 (C) 36 (D) 20

Ans : (D)

Hints : $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, $\frac{1}{10}=\frac{1}{v}+\frac{1}{30}$, v = 15 cm. When concave lens is placed v′ = (45 + 15) = 60 cm

$\frac{1}{f}=\frac{1}{v'}-\frac{1}{u}$ (f = focal length of combination), $\frac{1}{f}=\frac{1}{60}+\frac{1}{30}$ = $\boxed{f = 20\text{ m}}$

$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$, $\frac{1}{20}=\frac{1}{10}+\frac{1}{f_2}$, $\frac{1}{20}-\frac{1}{10}=\frac{1}{f_2}$ $\boxed{f_2 = -20\text{ m}}$

50. A 10 watt electric heater is used to heat a container filled with 0.5 kg of water. It is found that the temperature of water and the container rises by 3° K in 15 minutes. The contain r is then emptied, dried and filled with 2 kg of oil. The same heater now raises the temperature of container-oil system by 2°K in 20 minutes. Assuming that there is no heat loss in the process and the specific heat of water as 4200 $Jkg^{-1}K^{-}$, the specific heat of oil in the same unit is equal to

(A) 1.50×10^3 (B) 2.55×10^3 (C) 3.00×10^3 (D) 5.10×10^3

Ans : (B)

Hints : $\left(\frac{1}{2}\times4200\times3\right)+(m_c\times c_c\times3)=10\times15\times60-----(1)$

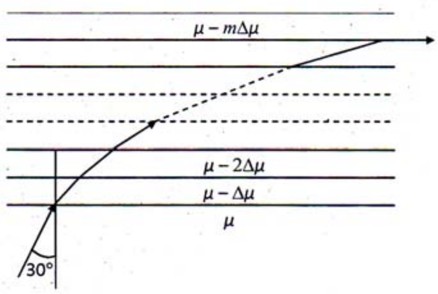
$(m_c\times c_c)=900$. In case of oil. $(2\times c_0\times2)+(m_c\times c_c\times2)=(10\times20\times60)$, $4C_0$ + (900×2)=12000

$(C_0) = 2.55\times10^3\ J\ kg^{-1}k^{-1}$

C_c = Sp. heat capacity of container

C_0 = Sp. heat capcity of o l

51. A glass slab consists of thin u iform layers of progressively decreasing refractive indices RI (see figure) such that the RI of any layer is μ-mΔμ. Here μ and Δμ denote the RI of 0th layer and the difference in RI between any two consecutive layers, respectively. The integer m = 0, 1, 2, 3..... denotes the numbers of the successive layers. A ray of light from the 0th layer enters the 1st layer at an angle of incidence of 30°. After undergoing the mth refraction, the ray emerges parallel to the interface. If μ = 1.5 and Δμ = 0.015, the value of m is



(A) 20 (B) 30 (C) 40 (D) 50

Ans : (D)

Hints : By Snell's law, $\mu \sin i$ = constant, $1.5 \sin 30° = (\mu - m\Delta\mu) \sin 90°$, $\frac{3}{2} \times \frac{1}{2} = (1.5 - m \times 0.15) \times 1$, $\therefore m = 50$

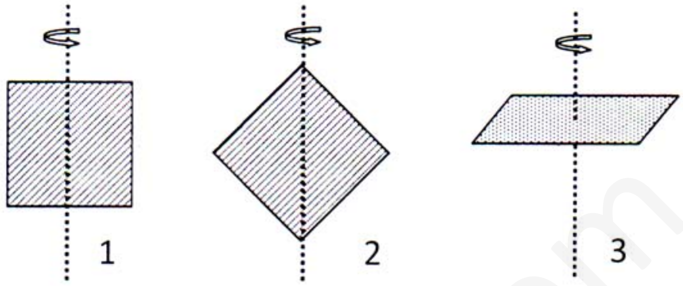
52. The de-Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass is 0.5 MeV)

(A) 1 : 50 (B) 1 : 20 (C) 20 : 1 (D) 50 : 1

Ans : (C)

Hints : $\lambda = \frac{h}{\sqrt{2mK}}$, $K_{electron} = \frac{h^2}{(\lambda^2 \times 2m)}$, $E_{photon} = \frac{hC}{\lambda}$, $\frac{E_{photon}}{K_{electron}} = \left[\frac{hc}{\lambda} \cdot \frac{\lambda^2 \times 2m}{h^2}\right] = \frac{2mC^2}{\left(hC/\lambda\right)} = \frac{2 \times 5 \times 10^5}{(50 \times 10^3)} = \frac{20}{1}$

53. Three identical square plates rotate about the axes shown in the figure in such a way that their kinetic energies are equal. Each of the rotation axes passes through the centre of the square. Then the ratio of angular speeds $\omega_1 : \omega_2 : \omega_3$ is



(A) 1 : 1 : 1 (B) $\sqrt{2} : \sqrt{2} : 1$ (C) $1 : \sqrt{2} : 1$ (D) $1 : 2 : \sqrt{2}$

Ans : (B)

Hints : $K = \frac{1}{2} I\omega^2$, $\omega \propto \frac{1}{\sqrt{I}}$, $\omega_1 : \omega_2 : \omega_3 = 1 : 1 : \frac{1}{\sqrt{2}} = \sqrt{2}\ \ \sqrt{2} : 1$

54. To determine the composition of a bimetallic alloy, a sample is first weighed in air and then in water. These weights are found to be w_1 and w_2 respectively. If the densities of t e two constituent metals are ρ_1 and ρ_2 respectively, then the weight of the first metal in the sample is (whe e ρ_w is the density of water)

(A) $\frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 - \rho_w) - w_2\rho_2]$ (B) $\frac{\rho_1}{\rho_w(\rho_2 + \rho_1)}[w_1(\rho_2 - \rho_w) + w_2\rho_2]$

(C) $\frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 + \rho_w) - w_2\rho_1]$ (D) $\frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_1 - \rho_w) - w_2\rho_1]$

Ans : (A)

Hints : $(w_1 - w_2) = v\rho_w g$, $(w_1 - w_2) = (v_1 + v_2)\rho_w g$, $(w_1 - w_2) = \left[\frac{x}{\rho_1} + \frac{(w_1 - x)}{\rho_2}\right]\rho_w$

(x - weight of the first metal) $x = \frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 - \rho_w) - w_2\rho_2]$

55. Sound waves are passing through two routes-one in straight path and the other along a semicircular path of radius r and are again combined into one pipe and superposed as shown in the figure. If the velocity of sound waves in the pipe is v, then frequencies of resultant waves of maximum amplitude will be integral multiples of

(A) $\frac{v}{r(\pi - 2)}$ (B) $\frac{v}{r(\pi - 1)}$ (C) $\frac{2v}{r(\pi - 1)}$ (D) $\frac{v}{r(\pi + 1)}$

Hints :

Path difference = $(\pi r - 2r) = (\pi - 2)r = n\lambda$

f- frequency. $v = f \times \lambda,\ \dfrac{v}{\lambda} = f \Rightarrow \left[\dfrac{v}{(\pi - 2)r}\right]n = f$

CATEGORY - III

Q.56 to Q.60 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.

56. Find the correct statement(s) about photoelectric effect

(A) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons

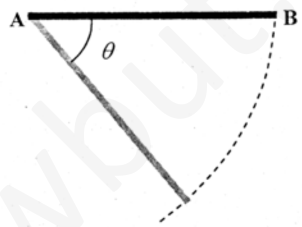
(B) Einstein analysis gives a threshold frequency above which no electron can be emitted

(C) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation

(D) The maximum kinetic energy of electrons does not depend on the intensity of radiation

Ans : (A & D)

57. A thin rod AB is held horizontally so that it can freely rotate in a vertical plane about the end A as shown in the figure. The potential energy of the rod when it hangs vertically is taken to be zero. The end B of the rod is released from rest from a horizontal position. At the instant the rod makes an angle θ with the horizontal.



(A) the speed of end B is proportiona to $\sqrt{\sin\theta}$

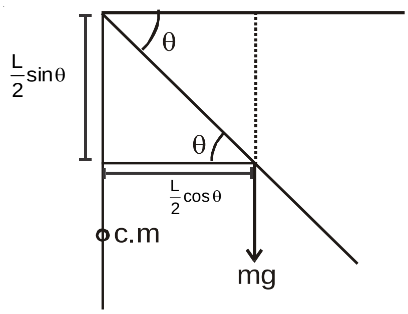
(B) the potential energy is pr portional o $(1 - \cos\theta)$

(C) the angular acceleration is proportional to $\cos\theta$

(D) the torque about A rem ins the same as its initial value

Ans : (A,C)

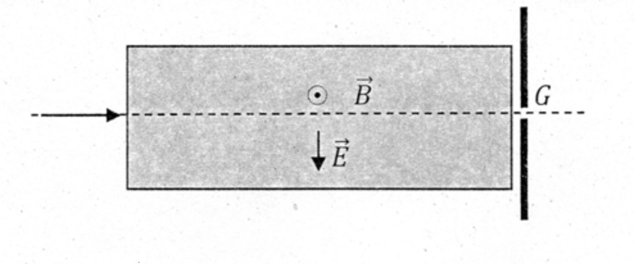
Hints :



Loss in Potential Energy = gain in Kinetic Energy, $mg\dfrac{L}{2}\sin\theta = \dfrac{1}{2}I\omega^2,\ \omega \propto \sqrt{\sin\theta},\ v \propto \sqrt{\sin\theta}$

$U = mgh = mg\ \dfrac{L}{2}(1 - \sin\theta)\ \because\ \tau = I\alpha \Rightarrow mg \times \dfrac{L}{2}\cos\theta = \dfrac{ml^2}{3} \times \alpha\ .\ \alpha \propto \cos\theta$

58. A stream of electrons and protons are directed towards a narrow slit in a screen (see figure). The intervening region has a uniform electric field $\vec{E}$ (vertically downwards) and a uniform magnetic field $\vec{B}$ (out of the plane of the figure) as shown. Then



(A) electrons and protons with speed $\dfrac{|\vec{E}|}{|\vec{B}|}$ will pass through the slit

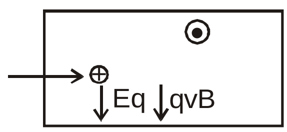
(B) protons with speed $\dfrac{|\vec{E}|}{|\vec{B}|}$ will pass through the slit, electrons of the same speed will not

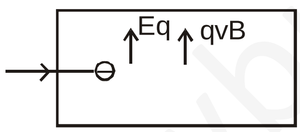
(C) neither electrons nor protons will go through the slit irrespective of their speed

(D) electrons will always be deflected upwards irrespective of their speed

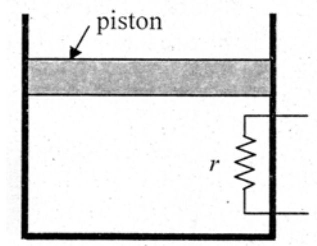
Ans : (C,D)

Hints :





59. A heating element of resistance r is fitted inside an adiabatic cylinder which carries a frictionless piston of mass m and cross-section A as shown in diagram. The cylinder contains one mole of an ideal diatomic gas. The current flows through the element such tha the temp rature rises with time t as $\Delta T = \alpha t + \frac{1}{2}\beta t^2$ (α and β are constants), while pressure remains constan The atmospheric pressure above the piston is P_0. Then



(A) the rate of increase in internal energy is $\frac{5}{2}R(\alpha + \beta t)$

(B) the current flowing in the element is $\sqrt{\frac{5}{2r}R(\alpha + \beta t)}$

(C) the piston moves upwards with constant acceleration

(D) the piston moves upwards with constant speed

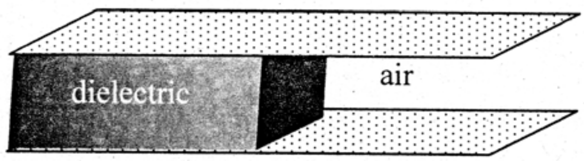
Ans : (A & C)

Hints : Internal energy $U=\frac{nfRT}{2}$, $U=\frac{5R}{2}\left[\alpha t+\frac{1}{2}\beta t^2\right]$, $\frac{dU}{dt}=\frac{5R}{2}[\alpha+\beta t]$, $dQ = nC_P dT$, $\frac{dQ}{dt}=nC_P\times\frac{dT}{dt}$,

$i^2r=\frac{7}{2}R\times[\alpha+\beta t]$, $i=\sqrt{\frac{7}{2r}R(\alpha+\beta t)}$, $PV = nRT$, $V=\frac{nRT}{P}$, $V=\frac{nR}{P}\left[\alpha t+\frac{1}{2}\beta t^2\right]$,

$x=\frac{nR}{PA}\left[\alpha t+\frac{1}{2}\beta t^2\right]$, $v=\frac{nR}{PA}[\alpha+\beta t]$, acceleration $=\frac{nR}{PA}\times\beta$

60. Half of the space between the plates of a parallel-plate capacitor is filled with a dielectric material of dielectric constant K. The remaining half contains air as shown in the figure. The capacitor is now given a charge Q. Then



(A) electric field in the dielectric-filled region is higher than that in the air-filled regi n

(B) on the two halves of the bottom plate the charge densities are unequal

(C) charge on the half of the top plate above the air-filled part is $\frac{Q}{K+}$

(D) capacitance of the capacitor shown above is $(1+K)\frac{C_0}{2}$, where C_0 is the capacitance of the same capacitor with the dielectric removed

Ans : (B, C, D)

Hints : $C_1=\frac{K\epsilon_0 A}{2d}$, $C_2=\frac{\epsilon_0 A}{2d}$, $C_{eq}=\frac{\epsilon .A}{2d}(K+1)=\frac{C_0}{2}(K+1)$, $\frac{Q_1}{Q_2}=\frac{C_1}{C_2}=\frac{K}{1}\Rightarrow\frac{\sigma_1}{\sigma_2}=\frac{K}{1}$,

$Q_1=\frac{KQ}{K+1}$ and $Q_2=\frac{Q}{K+1}$, $E=\frac{\sigma}{\epsilon_0 K}$, $\frac{E_1}{E_2}$ $\frac{\sigma}{\sigma_2}\times\frac{K_2}{K_1}=\frac{Q_1}{Q_2}\times\frac{K_2}{K_1}=\frac{K}{1}\times\frac{1}{K}=1:1$

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ANSWERS & HINTS
for
WBJEEM - 2014
SUB : CHEMISTRY

CATEGORY - I

Q.1 to Q.45 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark.

1. During the emission of a positron from a nucleus, the mass number of the daughter element remains the same but the atomic number

(A) is decreased by 1 unit (B) is decreased by 2 units
(C) is increased by 1 unit (D) remains unchanged

Ans : (A)

Hints : ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e$

Atomic number is decreased by 1

2. Four gases P, Q, R and S have almost same values of 'b but their 'a' values (a, b are van der Waals constants) are in the order Q < R < S < P. At a particular temperature among the four gases the most easily liquefiable one is

(A) P (B) Q (C) R (D) S

Ans : (A)

Hints : More the value of 'a' for the gas, more i the intermolecular forces of attraction. Thus the gas can be easily liquefied.

3. β emission is always accompanied by

(A) formation of antineutrino and α particle (B) emission of α particle and γ-ray
(C) formation of antineutrino and γ-ray (D) formation of antineutrino and positron

Ans : (C)

4. The values of ΔH and ΔS of a certain reaction are – 400 kJ mol^{-1} and –20 kJ mol^{-1}K^{-1} respectively. The temperature below which the reaction is spontaneous is

(A) 100°K (B) 20°C (C) 20°K (D) 120°C

Ans : (C)

Hints : The reaction is spontaneous when ΔG is -ve

$\Delta G < 0$

$\Delta H - T\,\Delta S < 0$

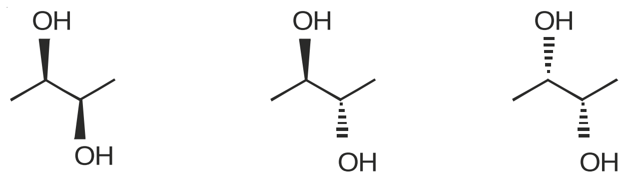
$-400 - (T)(-20) < 0$

$-400 + 20T < 0$

$20T < 400$

$T < \frac{400}{20}$; $T < 20K$

5. The correct statement regarding the following compounds is



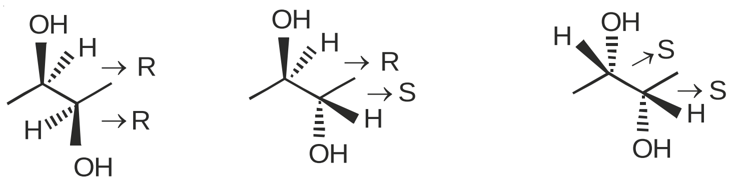
I II III

(A) all three compounds are chiral (B) only I and II are chiral

(C) I and III are diastereomers (D) only I and III are chiral

Ans : (D)

Hints :

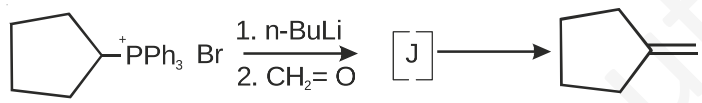


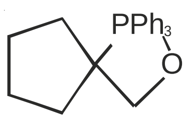
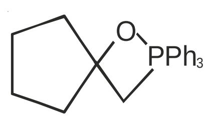
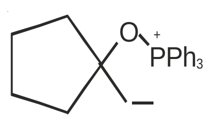
I II III

• I and III are enantiomers

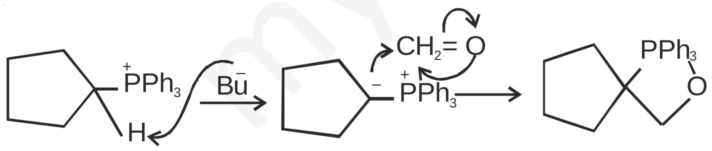
• II has plane of symmetry hence achiral

6. The intermediate J in the following Wittig reaction is



(A)  (B)  (C)  (D) 

Ans : (A)

Hints : 

7. Among the following compounds, the one(s) that gives (give)effervescence with aqueous $NaHCO_3$ solution is (are)

$(CH_3CO)_2O$	CH_3COOH	PhOH	CH_3COCHO
I	**II**	**III**	**IV**

(A) I and II (B) I and III (C) only II (D) I and IV

Ans : (A)

Hints : $CH_3COOH + NaHCO_3 \rightarrow CH_3COONa + CO_2 + H_2O$

$$CH_3-\overset{\overset{O}{\|}}{C}-O-\overset{\overset{O}{\|}}{C}-CH_3 + H_2O \rightarrow 2CH_3COOH$$

$$CH_3COOH + NaHCO_3 \rightarrow CH_3COONa + CO_2 + H_2O$$

8. The system that contains the maximum number of atoms is

(A) 4.25 g of NH_3 (B) 8 g of O_2 (C) 2 g of H_2 (D) 4 g of He

Ans : (C)

Hints : a) $4.25g\ NH_3 = \left(\frac{4.25}{17}\right)N_A \times 4 = N_A$ atoms

b) $8\ g\ O_2 = \left(\frac{8}{32}\right)N_A \times 2 = \frac{N_A}{2}$ atoms

c) $2\ g\ H_2 = \left(\frac{2}{9}\right)N_A \times 2 = 2N_A$ atoms

d) $4\ g\ He = \left(\frac{4}{4}\right)N_A = N_A$atoms

9. Metal ion responsible for the Minamata disease is

(A) Co^{2+} (B) Hg^{2+} (C) Cu^{2+} (D) Zn^{2+}

Ans : (B)

Hints : Hg^{2+} causes Minamata diseases

10. Among the following observations, the correct one that differentiates between SO_3^{2-} and SO_4^{2-} is

(A) Both form precipitate with $BaCl_2$, SO_3^{2-} dissolves in HCl but SO_4^{2-} does not

(B) SO_3^{2-} forms precipitate with $BaCl_2$, SO_4^{2-} does not

(C) SO_4^{2-} forms precipitate with $BaCl_2$, SO_3^{2-} does not

(D) Both form precipitate with $BaCl_2$, SO_4^{2-} dissolves in HCl but SO_3^{2-} does not

Ans : (A)

Hints : $BaCl_2 + SO_4^{2-} \rightarrow BaSO_4 \downarrow + 2Cl^-$

$BaCl_2 + SO_3^{2-} \rightarrow BaSO_3 \downarrow + 2Cl^-$

But $BaSO_3$ dissolves in HCl as $BaSO_3 + 2HCl \rightarrow BaCl_2 + SO_2 \uparrow + H_2O$

11. The pH of 10^{-4} M KOH solution will be

(A) 4 (B) 11 (C) 10.5 (D) 10

Ans : (D)

Hints : $[OH^-] = 10^{-4}$ M $\Rightarrow$ pOH = 4

pH + pOH = 14, $\therefore$ pH = 14 - 4 = 10

12. The reagents to carry out the following conversion are

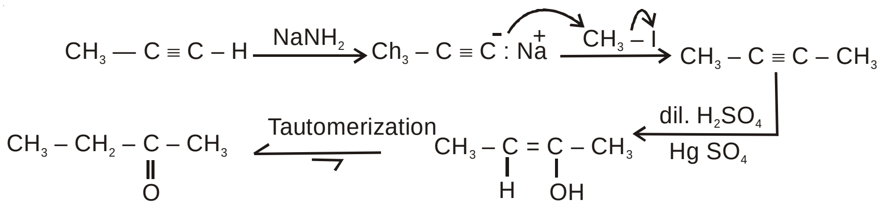
(A) $HgSO_4$/dil H_2SO_4 (B) BH_3;H_2O_2/NaOH

(C) OsO_4; HIO_4 (D) $NaNH_2$/CH_3I; $HgSO_4$/dil H_2SO_4

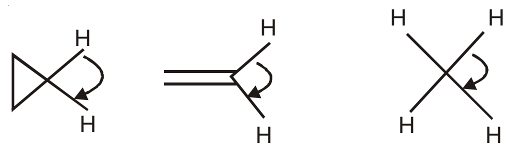
Ans : (D)

Hints : 

or $CH_3 — C \equiv C – H$



13. The correct order of decreasing H-C-H angle in the following molecules is

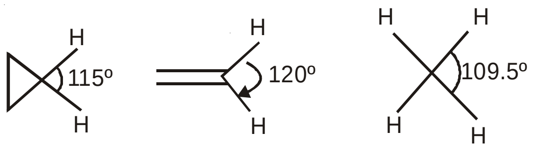


I II III

(A) I > II > III (B) II > I > III (C) III > II > I (D) I > III > II

Ans : (B)

Hints : II > I > III



I II III

14. $_{98}Cf^{246}$ was formed along with a neutron when an unknown radioactive substance was bombarded with $_{6}C^{12}$. The unknown substance was

(A) $_{91}Pa^{234}$ (B) $_{90}Th^{234}$ (C) $_{92}U^{235}$ (D) $_{92}U^{238}$

Ans : (C)

Hints : $_{z}X^{A} + {}_{6}C^{12} \rightarrow {}_{98}Cf^{246} + {}_{0}n^{1}$

$z + 6 = 98$ $\quad\quad$ $A + 12 = 246 + 1$

$\Rightarrow z = 92$ $\quad\quad$ or, $A = 247 - 12$

$= 235$

$\therefore$ The element is $_{92}U^{235}$

15. The rate of a certain reaction is given by, rate = k $[H^+]^n$. The rate increases 100 times when the pH changes from 3 to 1. The order (n) of the reaction is

(A) 2 (B) 0 (C) 1 (D) 1.5

Ans : (C)

Hints : Rate $r = k[H^+]^n$

New rate, $r' = 100\ r$

pH changes from 3 to 1

i.e. $[H^+] = 10^{-3}$M changes to $[H^+]' = 10^{-1}$M

i.e. conc. increases 100 times $\frac{[H^+]'}{[H^+]} = \frac{10^{-1}}{10^{-3}} = 100$

$\frac{r'}{r} = \left(\frac{[H^+]'}{[H^+]}\right)^n$ $\quad\quad$ or, $100 = (100)^n$

or, $n = 1$

16. $({}_{32}Ge^{76}, {}_{34}Se^{76})$ and $({}_{14}Si^{30}, {}_{16}S^{32})$ are examples of

(A) isotopes and isobars
(B) isobars and isotones
(C) isotones and isotopes
(D) isobars and isotopes

Ans : (B)

Hints : $\left({}_{32}Ge^{76}, {}_{34}Se^{76}\right)$ Same atomic mass = isobars

$\left({}_{14}Si^{30}, {}_{16}Se^{32}\right)$

$A - Z = 30 - 14 = 16$ Same no. of neutrons = isotones

and $32 - 16 = 16$

17. The enthalpy of vaporization of a certain liquid at its boiling point of 35ºC is 24.64 kJ mol^{-1}. The value of change in entropy for the process is

(A) 704 J K^{-1}mol^{-1}
(B) 80 J K^{-1}mol^{-1}
(C) 24.64 J K^{-1}mol^{-1}
(D) 7.04 J K^{-1}mol^{-1}

Ans : (B)

Hints : $\Delta S = \dfrac{q_{rev}}{T}$

At constant pressure, $q_{rev} = \Delta H_{transformation}$

$\Delta S_{vap} = \dfrac{\Delta H_{vap}}{T_b}$; T_b = boiling point, ΔH_{vap} = Enthalpy of vapourizati n

$= \dfrac{24.64 \times 10^3 \, Jmol^{-1}}{308 \quad K} = 80 \, JK^{-1}mol^{-1}$

18. Given that :

$C + O_2 \rightarrow CO_2$; $\Delta H^o = -x$ kJ

$2CO + O_2 \rightarrow 2CO_2$; $\Delta H^o = -y$ kJ

The heat of formation of carbon monoxide will be

(A) $\dfrac{y - 2x}{2}$
(B) $y + 2x$
(C) $2x - y$
(D) $\dfrac{2x - y}{2}$

Ans : (A)

Hints : i) $C + O_2 \rightarrow CO_2$; $\Delta H^o = -x$ kJ

ii) $2CO + O_2 \rightarrow 2CO_2$; $\Delta H = -y$ kJ

Eq (i) × 2

$2C + 2O_2 \rightarrow 2CO_2$ $\Delta H^o = -2x$ kJ

Writing eq. (ii) in rever e order

$2CO_2 \rightarrow 2CO + O_2$, $\Delta H^o = y$ kJ

adding, $2C + O_2 \rightarrow 2CO$, $\Delta H = (y - 2x)$ kJ

For 2 mol CO, $\Delta H = (y - 2x)$ kJ

$\therefore$ For 1 mol CO, $\Delta H_f = \left(\dfrac{y - 2x}{2}\right)$ kJ

$\therefore$ Enthalpy of formation, $\Delta H^o_f = \dfrac{y - 2x}{2}$

19. Commercial sample of H_2O_2 is labeled as 10V. Its % strength is nearly

(A) 3
(B) 6
(C) 9
(D) 12

Ans : (A)

Hints : 10 volume H_2O_2 means
1 mL H_2O_2 solution produces 10 mL O_2 at STP

$$2H_2O_2 \longrightarrow 2H_2O + O_2$$

2 mol — 1 mol
2 × 34 g — 22.4 L at STP
68 g

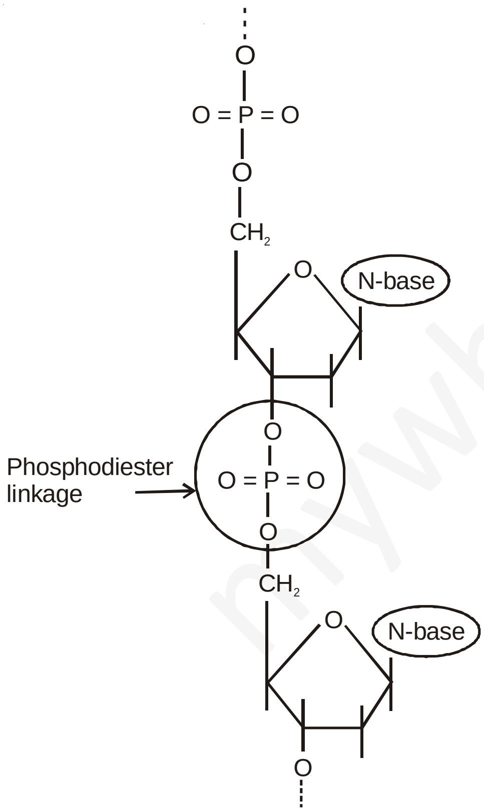
22400 mL O_2 at STP is produced from 68 g. H_2O_2

$\therefore$ 10 mL O_2 is produced from $\frac{68 \times 10}{22400}$g = 0.03036 g H_2O_2

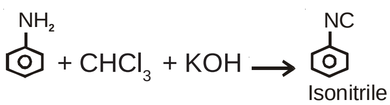
$\therefore$ 1 mL H_2O_2 solution contains 0.03 g H_2O_2 (approx.)
$\therefore$ 100 mL H_2O_2 solution contains 0.03 × 100
= 3 g H_2O_2 (approx.)

20. In DNA, the consecutive deoxynucleotides are connected via
(A) phospho diester linkage
(B) phospho monoester linkage
(C) phospho triester linkage
(D) amide linkage
Ans : (A)

Hints :

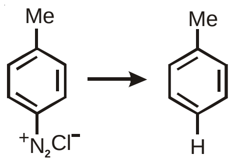


21. The reaction of aniline with chloroform under alkaline conditions leads to the formation of
(A) Phenyl cyanide (B) Phenyl isonitrile (C) Phenyl cyanate (D) Phenyl isocyanate
Ans : (B)

Hints : 

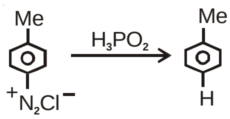
This is the carbylamine reaction

22. The reagent with which the following reaction is best accomplished is



(A) H_3PO_2 (B) H_3PO_3 (C) H_3PO_4 (D) $NaHSO_3$

Ans : (A)

Hints : 

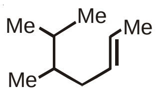
23. At a certain temperature the time required for the complete diffusion of 200 mL of H_2 gas is 30 minutes. The time required for the complete diffusion of 50 mL of O_2 gas at the same temperature will be

(A) 60 minutes (B) 30 minutes (C) 45 minutes (D) 15 minutes

Ans : (B)

Hints : $\frac{r_{H_2}}{r_{O_2}} = \frac{\sqrt{M_{O_2}}}{\sqrt{M_{H_2}}} = \frac{V_{H_2} / t_{H_2}}{V_{O_2} / t_{O_2}}$, $\sqrt{\frac{32}{2}} = \frac{200}{30} \times \frac{t_{O_2}}{50}$ = or $4 = \frac{4}{30} \times t_{O_2}$, $\therefore$ t_{O_2} = 30 min

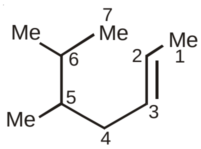
24. The IUPAC name of the following molecule is



(A) 5,6-Dimethyl hept-2-ene (B) 2,3-Dimethyl hept-5-ene

(C) 5,6-Dimethyl hept-3-ene (D) 5-I opropyl hex-2-ene

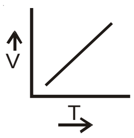
Ans : (A)

Hints :  5,6 - Dimethyl hept-2 en

25. For one mole of an ideal gas the slope of V vs T curve at constant pressure of 2 atm is X lit mol^{-1}K^{-1}. The value of the ideal universal gas constant 'R' in term of X is

(A) X lit atm mol^{-1}K^{-1} (B) X/2 lit a m mol^{-1}K^{-1} (C) 2X lit atm mol^{-1}K^{-1} (D) 2X atm lit^{-1}mol^{-1}K^{-1}

Ans : (C)

Hints : 

$PV = RT$, $V = \frac{R}{P} \times T$, $m = \frac{R}{P} = X$, or R = X.P, = 2X L.atm mol^{-1}K^{-1}

('m' is the slope)

26. An atomic nucleus having low n/p ratio tries to find stability by

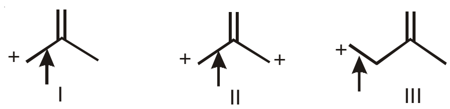
(A) the emission of an α particle (B) the emission of a positron

(C) capturing an orbital electron (K-electron capture) (D) emission of a β particle

Ans : (B)

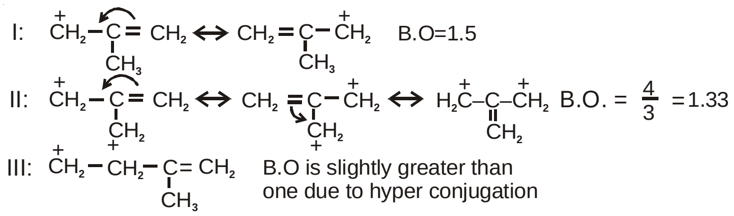
Hints : B and C both option are correct but as single option, B is more appropriate.

27. The correct order of decreasing length of the bond as indicated by the arrow in the following structure is



(A) I>II>III (B) II>I>III (C) III>II>I (D) I>III>II

Ans : (C)

Hints : I:


28. If Cl_2 is passed through hot aqueous NaOH, the products formed have Cl in different oxidation states. These are indicated as

(A) –1 and +1 (B) –1 and +5 (C) +1 and +5 (D) –1 and +3

Ans : (B)

Hints : Reaction : $3Cl_2 + 6NaOH$ (hot & conc) $\rightarrow 5\,Na\overset{-1}{Cl} + Na\overset{+5}{Cl}O_3 + 3H_2O$

29. In the following reaction, the product E is

$$\begin{matrix} CHO \\ | \\ CHO \end{matrix} \xrightarrow[2.\ H^+]{1.\ NaOH} E$$

(A) $\begin{matrix} CH_2OH \\ | \\ CHO \end{matrix}$ (B) $\begin{matrix} CHO \\ | \\ CO_2H \end{matrix}$ (C) $\begin{matrix} CH_2OH \\ | \\ CO_2H \end{matrix}$ (D) $\begin{matrix} CO_2H \\ | \\ CO_2H \end{matrix}$

Ans : (C)

Hints : Reaction : $\begin{matrix} CHO \\ | \\ CHO \end{matrix} \xrightarrow[2.\ H^+]{1.\ NaOH} \begin{matrix} CH_2OH \\ | \\ CO_2H \end{matrix}$ (Intramolecular Cannizaro's reaction)

30. The amount of electrolytes required to coagulate a given amount f Ag colloidal solution (–ve charge) will be in the order

(A) $NaNO_3 > Al_2(NO_3)_3 > Ba(NO_3)_2$ (B) $Al_2(NO_3)_3 > Ba(NO_3)_2 > NaNO_3$

(C) $Al_2(NO_3)_3 > NaNO_3 > Ba(NO_3)_2$ ( ) $NaNO_3 > Ba(NO_3)_2 > Al_2(NO_3)_3$

Ans : (D)

Hints : For [AgI] I^- Negatively charged sol, e fec ive i n for coagulation is cation and amount of electrolyte required $\propto \dfrac{1}{\text{charge content}}$. Also note that $Al(NO_3)_3$ is written as $Al_2(NO_3)_3$ in the questions paper.

31. The value of ΔH for cooling 2 mole of n deal monoatomic gas from 225°C to 125°C at constant pressure will be given $C_p = \dfrac{5}{2}R]$

(A) 250 R (B) –500 R (C) 500 R (D) –250 R

Ans : (B)

Hints : Here, n = 2

$$C_p = \frac{5}{2}R$$

$$\Delta T = 125 - 225 = -100$$

$$\Delta H = nC_p\,\Delta T = 2 \times \frac{5}{2}R \times (-100) = -500\ R$$

32. The quantity of electricity needed to separately electrolyze 1M solution of $ZnSO_4$, $AlCl_3$ and $AgNO_3$ completely is in the ratio of

(A) 2 : 3 : 1 (B) 2 : 1 : 1 (C) 2 : 1 : 3 (D) 2 : 2 : 1

Ans : (A)

Hints : $Zn^{2+} + 2e^- \rightarrow Zn$

$Al^{2+} + 3e^- \rightarrow Al$

$Ag^+ + e^- \rightarrow Ag$

$\therefore$ Quantity of electricity required = 2 : 3 : 1

33. The emission spectrum of hydrogen discovered first and the region of the electromagnetic spectrum in which it belongs, respectively are

(A) Lyman, ultraviolet (B) Lyman, visible (C) Balmer, ultraviolet (D) Balmer, visible

Ans : (D)

Hints : Fact

34. As per de Broglie's formula a macroscopic particle of mass 100 gm and moving at a velocity of 100 cm s^{-1} will have a wavelength of

(A) 6.6×10^{-29} cm (B) 6.6×10^{-30} cm (C) 6.6×10^{-31} cm (D) 6.6×10^{-32} cm

Ans : (C)

Hints : m = 100 g, v = 100 cm s^{-1} = 1 ms^{-1}

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{0.1 \times 1} = 6.626 \times 10^{-33} \text{ m} = 6.626 \times 10^{-31} \text{ cm}$$

35. The electronic configuration of Cu is

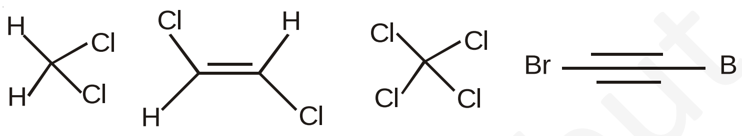
(A) $Ne3s^2 3p^6 3d^9 4s^2$ (B) $Ne3s^2 3p^6 3d^{10} 4s^1$ (C) $Ne3s^2 3p^6 3d^3 4s^2 4p^6$ (D) $Ne3s^2 3p^6 3d^5 4s^2 4p^4$

Ans : (B)

Hints : Cu : z = 29

$[Ne]\ 3s^2 3p^6 3d^{10} 4s^1$

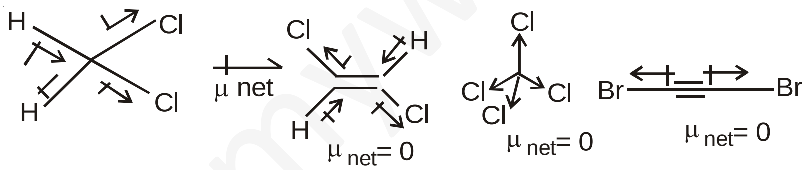
36. The compound that will have a permanent dipole moment among he following is



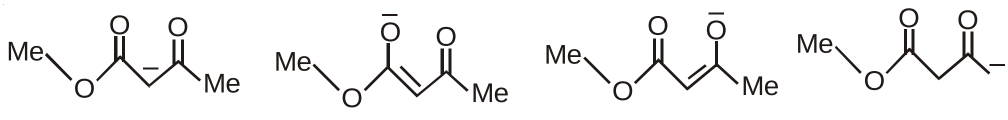
I II III IV

(A) I (B) II (C) III (D) IV

Ans : (A)

Hints : 

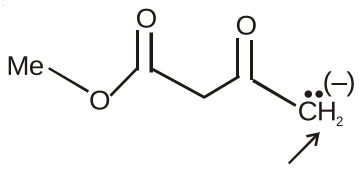
37. Among the following structures the one which is not a resonating structure of others is



I II III IV

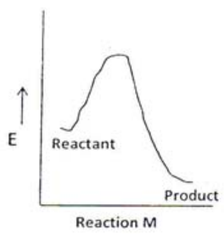
(A) I (B) II (C) III (D) IV

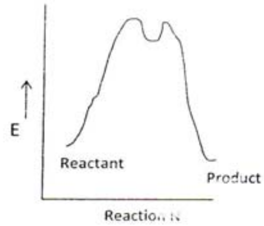
Ans : (D)

Hints : 

A hydrogen is removed from this carbon. But, in resonating structure, position of atoms do not changes.

38. The correct statement regarding the following energy diagrams is





(A) Reaction M is faster and less exothermic than Reaction N

(B) Reaction M is slower and less exothermic than Reaction N

(C) Reaction M is faster and more exothermic than Reaction N

(D) Reaction M is slower and more exothermic than Reaction N

Ans : (C)

Hints :

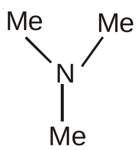
Activation energy ($\Delta E_M < \Delta E_N$)

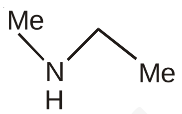
Reaction M is faster than N.

ΔH_M is more negative than ΔH_N

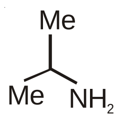
Reaction M is more extothermic than N

39. An amine C_3H_9N reacts with benzene sulfonyl chloride to form a white pr cipitate which is insoluble in aq. NaOH. The amine is

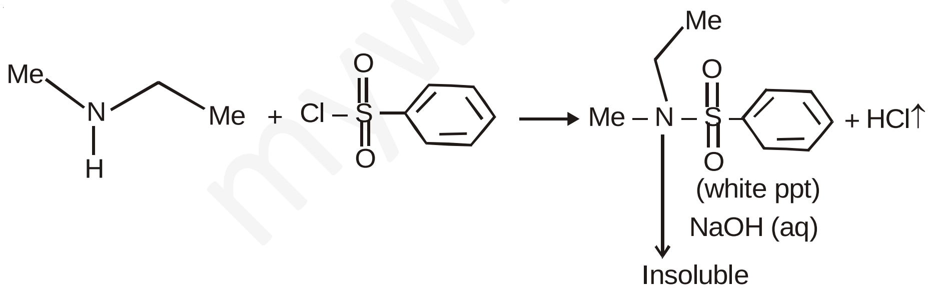
(A) 

(B) 

(C) 

(D) 

Ans : (B)

Hints : 

40. Among the followings, the one which is not a "greenhouse gas", is

(A) N_2O (B) CO_2 (C) CH_4 (D) O_2

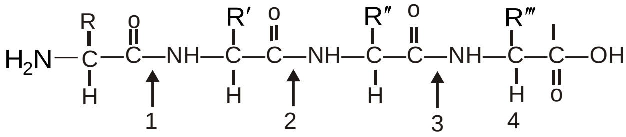
Ans : (D)

Hints : O_2 is not a green house gas

41. The number of amino acids and number of peptide bonds in a linear tetrapeptide (made of different amino acids) are respectively

(A) 4 and 4 (B) 5 and 5 (C) 5 and 4 (D) 4 and 3

Ans : (D)

Hints : 

No. of amino acids = 4

No. of Peptide bonds = 3

42. The 4th higher homologue of ethane is

(A) Butane (B) Pentane (C) Hexane (D) Heptane

Ans : (C)

Hints : homologus differ by CH_2 unit

$\therefore$ 4th homologue of ethene is C_6H_{14} $\{C_2H_6 + (CH_2)_4\}$

43. The hydrides of the first elements in groups 15 - 17, namely NH_3, H_2O and HF respectively show abnormally high values for melting and boiling points. This is due to

(A) small size of N, O and F

(B) the ability to form extensive intramolecular H-bonding

(C) the ability to form extensive intramolecular H-bonding

(D) effective van der Walls interaction

Ans : (B)

Hints : NH_3, H_2O and HF form extensive intermolecular Hydrogen bonding due to high ionic potential of N, O and F.

44. The two half cell reactions of an electrochemical cell is given as

$Ag^{+} + e^{-} \rightarrow Ag$; $E^{0}_{Ag+/Ag} = -0.3995$ V

$Fe^{++} \rightarrow Fe^{+++} + e^{-}$; $E^{0}_{Fe^{+++}/Fe^{++}} = -0.7120$ V

(A) –0.3125 V (B) 0.3125 V (C) 1.114 V (D) –1.114 V

Ans : (B)

Hints :

$$Ag^{+} + e \rightarrow Ag \quad -0.3995\text{ V (cathode)}$$
$$Fe^{+2} - e \rightarrow Fe^{+3} \quad -(-0.7120)\text{V (An de)}$$
$$Ag^{+} + Fe^{+2} \rightarrow Ag + Fe^{+3} \quad \Delta E = 0.3125\text{ V}$$

E^{o} cell $= E^{o}_{C} - E^{o}_{A}$ E^{o} cell= 0.3125 V

45. In case of heteronuclear diatomics of the type AB, where A is more electronegative than B, bonding molecular orbital resembles the character of A more than that of B. The statement

(A) is false

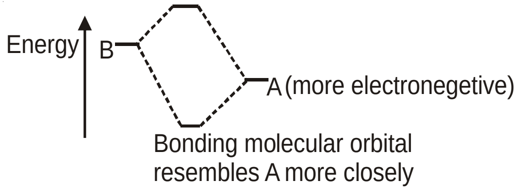
(B) is true

(C) can not be evaluated since data is not sufficient

(D) is true only for certain systems

Ans : (B)

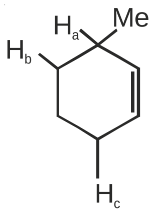
Hints :



CATEGORY - II

Q.46 to Q.55 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark

46. The order of decreasing ease of abstraction of Hydrogen atoms in the following molecule is

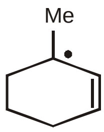


(A) $H_a > H_b > H_c$ (B) $H_a > H_c > H_b$

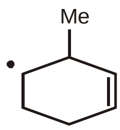
(C) $H_b > H_a > H_c$ (D) $H_c > H_b > H_a$

Ans : (B)

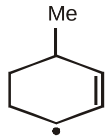
Hints : The more stable is the radical formed after H atom abstraction, easier is the abstraction



radical after H_a abstraction (tertiary allyl radical)



radical after H_b abstraction (secondary radical)



radical after H_c abstraction (secondary allyl adica

stability order of free adical is 3 allyl > 2 allyl > 2 a kyl

$\therefore\ H_a > H_c > H_b$

47. The bond angle in NF_3(102.3°) is smaller than NH_3(107.2°). This is because of

(A) large size of F compared to H (B) large size of N compared to F

(C) opposite polarity of N in the two molecules (D) small size of H compared to N

Ans : (C)

Hints : In NF_3, dipole moment vector point in the direction of F. Thus electron cloud shifts towards F in N–F bond. This reduces bond pair-bond pai r pulsi n in N–F and hence a decrease in bond angle FNF.

48. The compressibility factor (Z) of one mole of a van der Waals gas of negligible 'a' value is

(A) 1 (B) $\frac{bp}{RT}$ (C) $1+\frac{bp}{RT}$ (D) $1-\frac{bp}{RT}$

Ans : (C)

Hints: Vander Waal's Equation

$\left(P+\frac{a}{V^2}\right)(V-b)=RT$ (for 1 mole of gas) $\Rightarrow$ P(V–b) = RT$\Rightarrow$PV – Pb = RT $\Rightarrow$ PV = RT + Pb$\Rightarrow$ Z = $\frac{PV}{RT}=1+\frac{Pb}{RT}$

Z= Compressibility on neglecting "a".

49. At 25°C, the molar conductance of 0.007 M hydrofluoric acid is 150 mho cm^2mol^{-1} and $\Lambda^\circ_m = 500$ mho cm^2mol^{-1}. The value of the dissociation constant of the acid at the gas concentration at 25°C is

(A) 7×10^{-4} M (B) 7×10^{-5} M (C) 9×10^{-3} M (D) 9×10^{-4} M

Ans : (D)

Hints : α(degree of dissociation) = $\frac{150}{500}=0.3 \therefore K_a = \frac{C\alpha^2}{1-\alpha}=\frac{0.007\times(0.3)^2}{1-0.3}=9\times10^{-4}M$.

Here, α can't be neglected w.r.t 1 due to large value

50. A piece of wood from an archaeological sample has 5.0 counts min^{-1} per gram of C-14, while a fresh sample of wood has a count of 15.0 min^{-1} $gram^{-1}$. If half life of C-14 is 5770 years, the age of the archaeological sample is

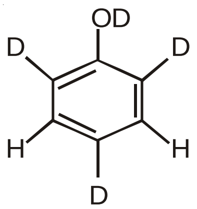
(A) 8,500 years (B) 9,200 years (C) 10,000 years (D) 11,000 years

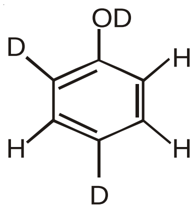
Ans : (B)

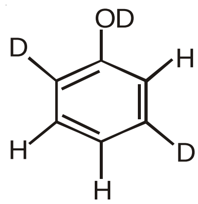
Hints : $\frac{0.693}{t_{1/2}} t = 2.303 \log \frac{[\text{Activity of fresh sample}]}{[\text{Activity of fossil}]}$, $\frac{0.693}{5770} t = 2.303 \log \frac{15}{5} \Rightarrow t = \frac{2.303(\log 3)(5770)}{0.693}$ yrs

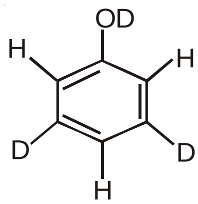
= 9,200 Yrs (approx)

51. When phenol is treated with D_2SO_4/D_2O, some of the hydrogens get exchanged. The final product in this exchange reaction is

(A) 

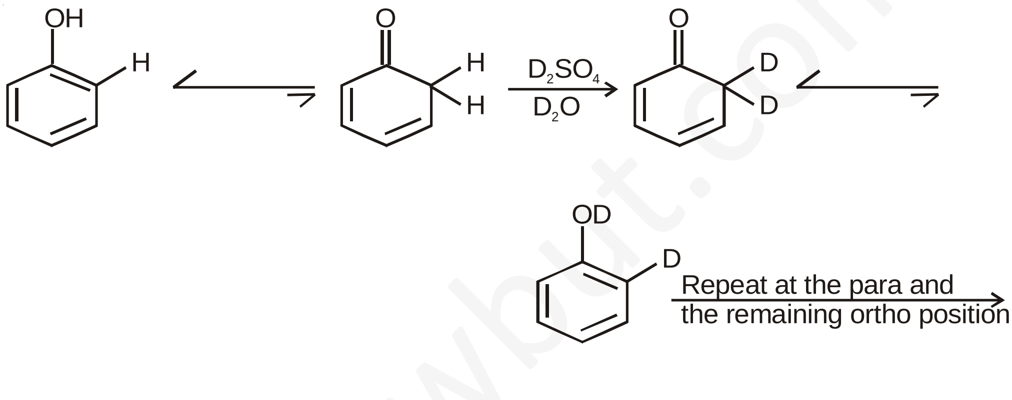
(B) 

(C) 

(D) 

Ans : (A)

Hints :



52. To observe an elevation of boiling point of 0 05°C, the amount of solute (Mol. Wt. = 100) to be added to 100 g of water (K_b=0.5) is

(A) 2 g (B) 0.5 g (C) 1 g (D) 0.75 g

Ans : (C)

Hints : $\Delta T_b = K_b\, m$, 0.05 = .5 × X 0.05= $\frac{0.5x}{100} \times 10$; X = 1 g.

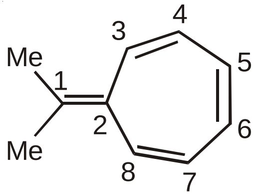
53. The structure of XeF_6 is experimentally determined to be distorted octahedron. Its structure according to VSEPR theory is

(A) Octahedron (B) Trigonal bipyramid (C) Pentagonal bipyramid (D) Tetragonal bipyramid

Ans : (C)

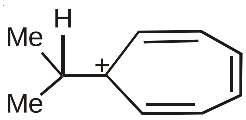
Hints : Xe is surrounded by 6 bond pairs and one lone pair. The geometry (geometry of electron pairs) is pentagonal bipyramid.

54. The most likely protonation site in the following molecule is



(A) C-1 (B) C-2 (C) C-3 (D) C-6

Ans : (A)

Hints :  Aromatic as well as tartiary carbocation

55. The volume of ethyl alcohol (density 1.15 g/cc) that has to be added to prepare 100 cc of 0.5 M ethyl alcohol solution in water is

(A) 1.15 cc (B) 2 cc (C) 2.15 cc (D) 2.30 cc

Ans : (B)

Hints : Mass of ethyl alcohol before and after the preparation must be equal.

x(volume in cc) × $\frac{1.15g}{mL} = \frac{100 \times 0.5}{1000} \times 46$, x = 2 cc

CATEGORY - III

Q.56 to Q.60 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.

56. Cupric compounds are more stable than their cuprous counterparts in solid state. This is because

(A) the endothermic character of the 2^{nd} I P of Cu is not so high

(B) size of Cu^{2+} is less than Cu^{+}

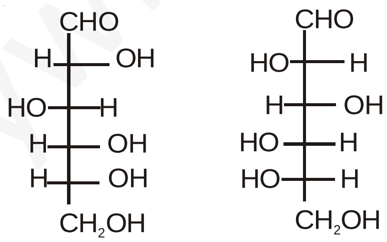
(C) Cu^{2+} has stabler electronic configuration as compared to Cu

(D) the lattice energy released for cupric compounds is much higher than Cu^{+}

Ans : (A, B, D)

Hints : Actually 2^{nd} IP of Cu (1958 kJ/mol) is not very high as compared to 1st IP (745 kJ/mol). In addition the gain in lattice energy due to +2 state and small size of Cu favour the divalent state in the solid.

57. Among the following statements about the molecules X and Y, the one (s) which is (are) correct is (are)



X Y

(A) X and Y are diastereomers (B) X and Y are enantiomers

(C) X and Y are both aldohexoses (D) X is a D-sugar and Y is an L-sugar

Ans : (B, C, D)

Hints : 'X' and 'Y' are mirror images of each other. They are aldohexoses too. In 'X', –OH of the asymmetric 'C' farthest from –CHO is on the right, so it is 'D'-Sugar. 'Y' , on the other hand, has –OH on the left. Thus it is a L-sugar.

58. For a spontaneous process, the correct statement(s) is (are)

(A) $(\Delta G_{system})_{T,P} > 0$ (B) $(\Delta S_{system}) + (\Delta S_{surroundings}) > 0$

(C) $(\Delta G_{system})_{T,P} < 0$ (D) $(\Delta U_{system})_{T,V} > 0$

Ans : (B, C)

Hints : Spontaneity of of the process can be expressed either by taking entropy changes of system and surrounding together or by considering free energy change of the system alone at constant temperature and pressure. The known criteria are : $(\Delta G_{sys})_{T,P} < 0$ and $(\Delta S_{sys}) + (\Delta S_{sur}) > 0$

59. The formal potential of Fe^{3+}/Fe^{2+} in a sulphuric acid and phosphoric acid mixture (E°=+0.61V) is much lower than the standard potential (E°=+0.77V). This is due to

(A) formation of the species $[FeHPO_4]^+$

(B) lowering of potential upon complexation

(C) formation of the species $[FeSO_4]^+$

(D) high acidity of the medium

Ans : (A, B, D)

Hints : Formation of complex by Fe^{3+} reduces its concentration. Thereby lowers the formal reduction potential.

60. Two gases X (Mol. Wt. M_x) and Y(Mol. Wt. M_Y; $M_Y > M_x$) are at the same temperature T in two different containers. Their root mean square velocities are C_X and C_Y respectively. If the average kinetic energies per molecule of two gases X and Y are E_x and E_Y respectively, then which of the following relation (s) is (are) true?

(A) $E_x > E_Y$

(B) $C_x > C_Y$

(C) $E_x = E_Y = \frac{3}{2}RT$

(D) $E_X = E_Y = \frac{3}{2}k_BT$

Ans : (B, D)

Hints : For same temperature, higher the molar mass, lower is the rms velocity.KE of individual molecules is expressed in terms of K_B not R

❑ ❑ ❑

WBJEEM - 2014

MATHEMATICS

Q.No.	μ	β	γ	δ
01	C	A	C	B
02	B	A	C	C
03	A	B	C	A
04	B	B	D	B
05	A	C	A	C
06	A	A	C	C
07	B	A	B	D
08	C	B	B	C
09	A	C	A	A
10	C	C	A	B
11	B	A	C	A
12	B	D	A	C
13	D	A	A	B
14	C	B	A	A
15	C	A	C	B
16	A	C	D	C
17	B	A	C	A
18	A	A	A	A
19	A	C	B	B
20	A	D	A	A
21	D	A	A	C
22	D	A	C	A
23	C	B	D	D
24	A	A	B	A
25	B	C	C	A
26	C	B	B	A
27	A	A	A	A
28	A	C	A	C
29	B	A	D	A
30	C	C	C	
31	D	B	C	C
32	C	C	B	A
33	C	B	B	D
34	A	B	A	A
35	C	A	B	C
36	A	C	C	D
37	D		A	A
38	A	B	A	C
39	C	C	A	B
40	D	C	A	D
41	A	D	B	C
42	C		A	A
4	A	A	D	C
44	A	A	A	A
5	A	B	C	D
4	B	C	A	C
47	A	C	A	B
48	C	D	A	B
49	B	C	D	B
50	B	C	C	C
51	A	C	D	C
52	A	D	C	C
53	A	A	C	B
54	A	A	B	A
55	C	A	C	A
56	C	A	C	B
57	D	B	B	A
58	B	A	B	A
59	C	C	A	C
60	C	D	A	D
61	D	B	C	A
62	A	A	A	C
63	D	B	A	A
64	B	C	A	C
65	B	C	D	A
66	A	D	D	C
67	A	D	C	C
68	C	D	A	D
69	A	A	A	D
70	D	A	B	A
71	C	C	B	D
72	C	A	C	A
73	A	A	D	B
74	C	A	A	B
75	A	C	C	A
76	A,D	A,B,D	C,D	A,B
77	A,B	A,B	A,B	A,B
78	A,B	C,D	A,B,D	A,D
79	C,D	A,B	A,D	A,B,D
80	A,B,D	A,D	A,B	C,D

Code-μ

ANSWERS & HINTS
for
WBJEEM - 2014
SUB : MATHEMATICS

CATEGORY - I

Q.1 to Q.60 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark.

1. Let the equation of an ellipse be $\frac{x^2}{144}+\frac{y^2}{25}=1$. Then the radius of the circle with entre $\left(0,\sqrt{2}\right)$ and passing through the foci of the ellipse is

(A) 9 (B) 7 (C) 11 (D) 5

Ans : (C)

Hints : $a^2 = 144$, $b^2 = 25$ $P\left(0, \sqrt{2}\right)$, S(ae, 0)

Radius = PS, $S\left(\sqrt{119}, 0\right)$ PS = 11

2. If y = 4x + 3 is parallel to a tangent to the parabola $y^2 = 12x$, then its distance from the normal parallel to the given line is

(A) $\frac{213}{\sqrt{17}}$ (B) $\frac{219}{\sqrt{17}}$ (C) $\frac{211}{\sqrt{17}}$ (D) $\frac{210}{\sqrt{17}}$

Ans : (B)

Hints : m = slope of line = 4 ; a = 3

$y = mx - 2am - am^3$ (Norm l)

$y = 4x - 216$.. Distance = $\frac{219}{\sqrt{17}}$

3. In a ΔABC, tan*A* and tan*B* are the roots of $pq(x^2 + 1) = r^2x$. Then ΔABC is

(A) a right angled triangle (B) an acute angled triangle

(C) an obtuse angled triangle (D) an equilateral triangle

Ans : (A)

Hints : $pqx^2 - r^2x + pq = 0$

tanA tanB = 1 so tan(A+B) is undefined $\therefore \angle C = \pi / 2$

4. Let the number of elements of the sets *A* and *B* be p and q respectively. Then the number of relations from the set *A* to the set *B* is

(A) 2^{p+q} (B) 2^{pq} (C) p + q (D) pq

Ans : (B)

Hints : 0(A) = p 0(B) = q ; 0(A × B) = pq

5. The function $f(x)=\dfrac{\tan\left\{\pi[x-\frac{\pi}{2}]\right\}}{2+[x]^2}$, where [x] denotes the greatest integer $\le x$, is

(A) continuous for all values of x
(B) discontinuous at $x=\dfrac{\pi}{2}$
(C) not differentiable for some values of x
(D) discontinuous at x = –2

Ans : (A)

Hints : $f(x) = 0\ \forall x \in R$

6. Let z_1, z_2 be two fixed complex numbers in the Argand plane and z be an arbitrary point satisfying $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$. Then the locus of z will be

(A) an ellipse
(B) a straight line joining z_1 and z_2
(C) a parabola
(D) a bisector of the line segment joining z_1 and z_2

Ans : (A)

Hints : Possibility of ellipse P(z), $S_1(z_1)$, $S_2(z_2)$

$PS_1 + PS_2 = 2a = 2S_1S_2 = 4ae$

$\therefore$ So $e = \dfrac{1}{2}$ it is an ellipse

7. Let $S=\dfrac{2}{1}{}^nC_0+\dfrac{2^2}{2}{}^nC_1+\dfrac{2^3}{3}{}^nC_2+.......+\dfrac{2^{n+1}}{n+1}{}^nC_n$. Then *S* equals

(A) $\dfrac{2^{n+1}-1}{n+1}$
(B) $\dfrac{3^{n+1}-1}{n+1}$
(C) $\dfrac{3^n-1}{n}$
(D) $\dfrac{2^n-1}{n}$

Ans : (B)

Hints : $P=\sum\dfrac{2^{r+1}}{r+1}{}^nC_r$; $S_0=\sum {}^nC_r.x^r$, $\int_0^2 S_0=\int_0^2\sum {}^nC_r.x^r$

$\therefore \dfrac{3^{n+1}-1}{n+1}$

8. Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

(A) 24800
(B) 25100
(C) 25200
(D) 25400

Ans : (C)

Hints : ${}^7C_3 \times {}^4C_2 \times 5!$

9. The remainder obtained when 1! + 2! + 3! + ... + 11! is divided by 12 is

(A) 9
(B) 8
(C) 7
(D) 6

Ans : (A)

Hints : 12 divides 4!, 5! etc .

Remainder = 1 + 2 + 6 = 9

10. Let S denote the sum of the infinite series $1+\frac{8}{2!}+\frac{21}{3!}+\frac{40}{4!}+\frac{65}{5!}+\ldots\ldots$. Then

(A) $S < 8$ (B) $S > 12$ (C) $8 < S < 12$ (D) $S = 8$

Ans : (C)

Hints : n^{th} term of 1, 8, 21, 40, 65, ...= $n(3n - 2)$

$$\sum_{r=1}^{\infty}\frac{r.(3r-2)}{r!} = 3e + 3e - 2e = 4e$$

11. For every real number x, let $f(x) = \frac{x}{1!}+\frac{3}{2!}x^2+\frac{7}{3!}x^3+\frac{15}{4!}x^4+\ldots$ .Then the equation $f(x) = 0$ has

(A) no real solution (B) exactly one real solution

(C) exactly two real solutions (D) inifinite number of real solutions

Ans : (B)

Hints : $x = 0$ is a solution

$$\sum_{r=1}^{\infty}\frac{x^r}{r!}(2^r-1) = e^{2x} - e^x$$

12. The coefficient of x^3 in the infinite series expansion of $\frac{2}{(1-x)(2-x)}$, for $|x| < 1$, is

(A) $-1/16$ (B) $15/8$ (C) $-1/8$ (D) $15/16$

Ans : (B)

Hints : Exp = $2(1-x)^{-1}\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1} = (1 + x + x^2 \ldots.)\left(1+\frac{x}{2}+\frac{x^2}{2^2}+\ldots.\right)$

Coefficient = 15/8

13. If α, β are the roots of the quadratic quation $x^2 + px + q = 0$, then the values of $\alpha^3 + \beta^3$ and $\alpha^4 + \alpha^2\beta^2 + \beta^4$ are respectively

(A) $3pq - p^3$ and $p^4 - 3p^2q + 3q^2$ (B) $-p(3q - p^2)$ and $(p^2 - q)(p^2 + 3q)$

(C) $pq - 4$ and $p^4 - q^4$ (D) $3pq - p^3$ and $(p^2 - q)(p^2 - 3q)$

Ans : (D)

Hints : $\alpha^4 + \beta^4 + \alpha^2\beta^2$ $\quad$ $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (\alpha^2 + \beta^2)^2 - \alpha^2\beta^2$ $\quad$ $= -p^3 + 3pq$

$= (p^2 - 2q)^2 - q^2$

14. A fair six-faced die is rolled 12 times. The probability that each face turns up twice is equal to

(A) $\frac{12!}{6!6!6^{12}}$ (B) $\frac{2^{12}}{2^6 6^{12}}$ (C) $\frac{12!}{2^6 6^{12}}$ (D) $\frac{12!}{6^2 6^{12}}$

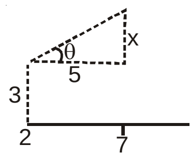
Ans : (C)

Hints : ${}^{12}C_2\,{}^{10}C_2 \ldots\ldots\ldots\ldots\ldots\ldots {}^{2}C_2 . \frac{1}{6^{12}}$

15. Let $f(x)$ be a differentiable function in [2, 7]. If $f(2) = 3$ and $f'(x) \leq 5$ for all x in (2, 7), then the maximum possible value of $f(x)$ at x = 7 is

(A) 7 (B) 15 (C) 28 (D) 14

Ans : (C)

Hints : 

$\frac{x}{5} = \tan\theta = 5 \Rightarrow x = 25$

So $f(7) = 3 + 25 = 28$

16. The value of $\tan\frac{\pi}{5} + 2\tan\frac{2\pi}{5} + 4\cot\frac{4\pi}{5}$ is

(A) $\cot\frac{\pi}{5}$ (B) $\cot\frac{2\pi}{5}$ (C) $\cot\frac{4\pi}{5}$ (D) $\cot\frac{3\pi}{5}$

Ans : (A)

Hints : Add, subtract $\cot\frac{\pi}{5}$

17. Let $\mathbb{R}$ be the set of all real numbers and $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 3x^2 + 1$. Then the set $f^{-1}([1, 6])$ is

(A) $\left\{-\sqrt{\frac{5}{3}}, 0, \sqrt{\frac{5}{3}}\right\}$ (B) $\left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right]$ (C) $\left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$ (D) $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

Ans : (B)

Hints : $f'(x) = 6x > 0$ if $x > 0$, < 0 if $x < 0$

$f(0) = 1$ $f(\alpha) = 6$. So $\alpha = \pm\sqrt{\frac{5}{3}}$

[Note : $f(x)$ is is invertible either f r x > 0 or for x < 0 so the right answer should be either $\left[-\sqrt{\frac{5}{3}}, 0\right]$ or $\left[0, \sqrt{\frac{5}{3}}\right]$

18. The area of the region bounded by the curves $y = x^2$ and $x = y^2$ is

(A) 1/3 (B) 1/2 (C) 1/4 (D) 3

Ans : (A)

Hints : $\int_0^1\left(\sqrt{x} - x^2\right)dx = 1/3$

19. The point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 35 = 0$ has coordinates

(A) (9, –24) (B) (1, 81) (C) (4, –16) (D) (–9, –24)

Ans : (A)

Hints : Normal at $P(am^2, -2am)$ has slope m. $a = 16$, $m = \frac{3}{4}$

20. The equation of the common tangent with positive slope to the parabola $y^2 = 8\sqrt{3}x$ and the hyperbola $4x^2 - y^2 = 4$ is

(A) $y = \sqrt{6}x + \sqrt{2}$ (B) $y = \sqrt{6}x - \sqrt{2}$ (C) $y = \sqrt{3}x + \sqrt{2}$ (D) $y = \sqrt{3}x - \sqrt{2}$

Ans : (A)

Hints : $y^2 = 4ax$, $a = 2\sqrt{3}$

$y = mx + \frac{a}{m}$; $m > 0$, $\left(\frac{a}{m}\right)^2 = 1.m^2 - 4$, $m^2 = 6$

21. Let p,q be real numbers. If α is the root of $x^2+3p^2x+5q^2 = 0$, β is a root of $x^2+9p^2x+15q^2 = 0$ and $0<\alpha<\beta$, then the equation $x^2+6p^2x+10q^2 = 0$ has a root γ that always satisfies

(A) $\gamma = \alpha/4+\beta$ (B) $\beta < \gamma$
(C) $\gamma = \alpha/2+\beta$ (D) $\alpha < \gamma < \beta$

Ans : (D)

Hints : Let, $f(x) = x^2+6p^2x+10q^2$

$f(\alpha) = \alpha^2+6p^2\alpha+10q^2 = (\alpha^2+3p^2\alpha+5q^2) + (3p^2\alpha+5q^2) = 0 + 3p^2\alpha+5q^2 > 0$

Again, $f(\beta) = \beta^2+6p^2\beta+10q^2 = (\beta^2+9p^2\beta+15q^2) - (3p^2\beta+5q^2) = 0 - (3p^2\beta + 5q^2) < 0$

So, there is one root γ such that, $\alpha < \gamma < \beta$

22. The value of the sum $({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + ... + ({}^nC_n)^2$ is

(A) $({}^{2n}C_n)^2$ (B) ${}^{2n}C_n$ (C) ${}^{2n}C_n +1$ (D) ${}^{2n}C_n-1$

Ans : (D)

Hints : $(1+x)^n = C_0 + C_1x + C_2x^2 + \text{---------} + C_nx^n$, $(x+1)^n = C_0x^n + C_1x^{n-1}+C_2x^{n-2} + \text{------} + C_n$

So, coefficient of x^n in $[(1+x)^n \times (x+1)^n]$ i.e. $(1+x)^{2n}$ is $(C_0^2 +C_1^2+ \text{-------} +C_n^2)$, which is ${}^{2n}C_n$

So, ${}^{2n}C_n = C_0^2 +C_1^2+C_2^2 + \text{-------} + C_n^2$, $\Rightarrow ({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + \text{----} + ({}^nC_n)^2 = {}^{2n}C_n - C_0^2 = {}^{2n}C_n - 1$

23. Ram is visiting a friend. Ram knows that his friend has 2 children and 1 of them is a boy. Assuming that a child is equally likely to be a boy or a girl, then the probability that the other child is a girl, is

(A) 1/2 (B) 1/3 (C) 2/3 (D) 7/10

Ans : (C)

Hints : Event that at least one of them is a boy $\rightarrow$ A, Event that other is girl $\rightarrow$ B, So, probability required P(B/A)

$= \frac{P(B\cap A)}{P(A)}$, Now, total cases are 3 (BG, BB GG) $\therefore$ $\frac{P(B\cap A)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = 1/2$

(As, $B\cap A = \{BG\}$ and $A = \{BG,BB\}$)

24. Let $n\geq 2$ be an integer, $A = \begin{pmatrix} \cos(2\pi/n) & \sin(2\pi/n) & 0 \\ \sin(2\pi/n) & \cos(2\pi/n) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and I is the identity matrix of order 3. Then

(A) $A^n = I$ and $A^{n-1}\neq I$ (B) $A^m \neq I$ for any positive integer m
(C) A is not invertible (D) $A^m = 0$ for a positive integer m

Ans : (A)

Hints : $A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\Rightarrow A^n = \begin{pmatrix} \cos n\theta & \sin n\theta & 0 \\ -\sin n\theta & \cos n\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$, So, here, $A^n = \begin{pmatrix} \cos 2\pi & \sin 2\pi & 0 \\ -\sin 2\pi & \cos 2\pi & 0 \\ 0 & 0 & 1 \end{pmatrix}$, =

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and $A^{n-1} \neq I$

25. Let I denote the 3 × 3 identity matrix and P be a matrix obtained by rearranging the columns of I. Then
(A) There are six distinct choices for P and det(P) = 1
(B) There are six distinct choices for P and det(P) = ±1
(C) There are more than one choices for P and some of them are not invertible
(D) There are more than one choices for P and $P^{-1} = I$ in each choice
Ans : (B)

Hints : $I_{3\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, 3 different columns can be arranged in, 3! i.e. 6 ways, In each case, if there are even number of interchanges of columns, determinant remains 1 and for odd number of interchanges, determinant takes the negative value i.e. –1

26. The sum of the series $\sum_{n=1}^{\infty} \sin\left(\frac{n!\pi}{720}\right)$ is
(A) $\sin\left(\frac{\pi}{180}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{540}\right)$
(B) $\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)$
(C) $\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{720}\right)$
(D) $\sin\left(\frac{\pi}{180}\right)+\text{s n}\left(\frac{\pi}{360}\right)$
Ans : (C)

Hints : $\sum_{n=1}^{\infty} \sin\left(\frac{n!\pi}{720}\right), = \left(\sin\frac{1!\pi}{720}+\sin\frac{2!\pi}{720}+-----+\frac{\sin 5!\pi}{720}\right)+\sum_{n=6}^{\infty} \sin\frac{n!\pi}{720}$

$= \sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)\ \text{s n}\left(\frac{\pi}{720}\right)+\sum_{n=6}^{\infty} \sin k_n\pi$, where $k_n \in \mathbb{N}$, so $\sin k_n \pi = 0\ \forall\ k_n$

27. Let α,β be the roots of $x^2-x-1 = 0$ and $S_n = \alpha^n+\beta^n$, for all integers $n \ge 1$. Then for every integer $n\ge2$
(A) $S_n+S_{n-1} = S_{n+1}$ (B) $S_n-S_{n-1} = S_n$ (C) $S_{n-1} = S_{n+1}$ (D) $S_n+S_{n-1} = 2S_{n+1}$
Ans : (A)
Hints : $\alpha+\beta=1$, $S_n + S_{n-1}$, $= (\alpha^n+\alpha^{n-1})+ (\beta^n+\beta^{n-1})$, $= \alpha^{n-1}(\alpha+1) + \beta^{n-1}(\beta+1)$, now since $\alpha^2-\alpha-1 = 0$ & $\beta^2-\beta-1=0 = \alpha^{n-1}.\alpha^2+ \beta^{n-1}.\beta^2 = \alpha^{n+1}+\beta^{n+1} = S_{n+}$

28. In a ΔABC, a,b,c are the side of the triangle opposie to the angles A,B,C respectively. Then the value of $a^3\sin(B-C) + b^3 \sin(C-A) + c^3\sin(A-B)$ is equqal to
(A) 0 (B) 1 (C) 3 (D) 2
Ans : (A)

29. In the Argand plane, the distinct roots of $1+z+z^3+z^4 = 0$ (z is a complex number) represent vertices of
(A) a square (B) an equilateral triangle (C) a rhombus (D) a rectangle
Ans : (B)
Hints : $1+z+z^3+z^4 = 0, \Rightarrow (1+z)(1+z^3)= 0$, $z = -1, -1, -\omega, -\omega^2$, where ω is a cube root of unity, so, distinct roots are : $(-1,0), \left(\frac{1}{2},\frac{\sqrt{3}}{2}\right), \left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$. Distance between each of them is $\sqrt{3}$. So, they form an equilateral triangle

30. The number of digits in 20^{301} (given $\log_{10}2 = 0.3010$) is
(A) 602 (B) 301 (C) 392 (D) 391
Ans : (C)
Hints : $\log 20^{301} = 301 \times \log 20 = 301 \times 1.3010 = 391.6010$, so, 392 digits

31. If $\sqrt{y} = \cos^{-1}x$, then it satisfies the differential equation $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$ = c, where c is equal to

(A) 0 (B) 3 (C) 1 (D) 2

Ans : (D)

Hints : $\sqrt{y} = \cos^{-1}x \Rightarrow y = (\cos^{-1}x)^2$, $\therefore \dfrac{dy}{dx} = -\dfrac{2\cos^{-1}x}{\sqrt{1-x^2}}$, $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{2 - \dfrac{2x\cos^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$, $= \dfrac{2 + x\dfrac{dy}{dx}}{1-x^2}$, $\Rightarrow \dfrac{d^2y}{dx^2}(1-x^2) - x\dfrac{dy}{dx} = 2 \therefore 2$

32. The integrating factor of the differential equation

$(1+x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}$ is

(A) $\tan^{-1}x$ (B) $1+x^2$ (C) $e^{\tan^{-1}x}$ (D) $\log_e(1+x^2)$

Ans : (C)

Hints : $\dfrac{dy}{dx} + \dfrac{y}{1+x^2} = \dfrac{e^{\tan^{-1}}}{1+x}$, I.F = $e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

33. The solution of the equation

$\log_{101}\log_7\left(\sqrt{x+7} + \sqrt{x}\right) = 0$ is

(A) 3 (B) 7 (C) 9 (D) 49

Ans : (C)

Hints : $\log_{101}\log_7\left(\sqrt{x+7} + \sqrt{x}\right) = 0$, $\Rightarrow \log_7\left(\sqrt{x+7} + \sqrt{x}\right) = 1$, $\sqrt{x+7} + \sqrt{x} = 7$, $\Rightarrow \sqrt{x+7} = 7 - \sqrt{x} \Rightarrow x+7 = 49 - 14\sqrt{x} + x$, $\Rightarrow \sqrt{x} = 3$, $\Rightarrow x = 9$

34. If α, β are the roots of $ax^2+bx+c = 0$ $(a \neq 0)$ a d $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$ $(p \neq 0)$ then the ratio of the squares of their discriminants is

(A) $a^2:p^2$ (B) $a:p^2$ (C) $a^2:p$ (D) $a:2p$

Ans : (A)

Hints : $D_1 = a^2(\alpha-\beta)^2$, $D_2 = P^2(\alpha-\beta)^2$; $\dfrac{D_1}{D_2} = \dfrac{a^2}{p^2}$ [Note : Correct answer is $\dfrac{a^4}{p^4}$ for ${D_1}^2/{D_2}^2$]

35. Let $f(x) = 2x^2+5x+1$. If we write f(x) as

$f(x) = a(x+1)(x-2) + b(x-2)(x-1) + c(x-1)(x+1)$ for real numbers a,b,c then

(A) there are infinite number of choices for a,b,c

(B) only one choice for a but infinite number of choices for b and c

(C) exactly one choice for each of a,b,c

(D) more than one but finite number of choices for a,b,c

Ans : (C)

Hints : $f(x) = (a+b+c)x^2 + (-a-3b)x - 2a + 2b - c$, $a+b+c = 2$, $-a-3b = 5$, $-2a+2b-c = 1$, $a = -4$, $b = -1/3$, $c = 19/3$

36. Let $f(x) = x + 1/2$. then the number of real values of x for which the three unequal terms f(x), f(2x), f(4x) are in H.P. is

(A) 1 (B) 0 (C) 3 (D) 2

Ans : (A)

Hints : $f(x) = x+\frac{1}{2} = \frac{2x+1}{2}$, $f(2x) = \frac{4x+1}{2}$, $f(4x) = \frac{8x+1}{2}$, f(x), f(2x), f(4x) are in H.P., So, $f(2x) = \frac{2f(x)f(4x)}{f(x)+f(4x)}$, $\Rightarrow$

$x=0, \frac{1}{4}$, at x= 0, terms are equal so only solution is $x=\frac{1}{4}$

37. The function $f(x)= x^2+bx+c$, where b and c real constants, describes

(A) one-to-one mapping (B) onto mapping
(C) not one-to-one but onto mapping (D) neither one-to-one nor onto mapping

Ans : (D)

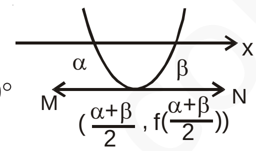
Hints : Upward parabola f(x) has a minimum value. So, it is not onto, also symmetric about its axis which is a straight line parallel to Y-axis, so it is not one-to-one

38. Suppose that the equation $f(x) = x^2+bx+c = 0$ has two distinct real roots α and β. The angle between the tangent to the curve $y = f(x)$ at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ and the positive direction of the x-axis is

(A) 0° (B) 30° (C) 60° (D) 90°

Ans : (A)

Hints : $f(x) = x^2+bx+c$ represents upward parabola which cuts x-axis at α and β. As the graph is symmetric, so, tangent at $\left(\frac{\alpha+\beta}{2}, f\frac{\alpha+\beta}{2}\right)$ parallel to x-axis. Hence, 0°



39. The solution of the differential equation $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\varphi\left(\frac{y^2}{x^2}\right)}{\varphi'\left(\frac{y^2}{x^2}\right)}\right]$ is (where c is a constant)

(A) $\varphi\left(\frac{y^2}{x^2}\right) = cx$ (B) $x\varphi\left(\frac{y^2}{x^2}\right) = c$ (C) $\varphi\left(\frac{y^2}{x^2}\right) = cx^2$ (D) $x^2\varphi\left(\frac{y^2}{x^2}\right) = c$

Ans : (C)

Hints : Let, $\frac{y}{x} = \nu$, $\Rightarrow y = \nu x \Rightarrow \frac{dy}{dx} = \nu + x\frac{d\nu}{dx}$ substituting, $\nu x\left(\nu + x\frac{d\nu}{dx}\right) = x\left(\nu^2 + \frac{\phi(\nu^2)}{\phi'(\nu^2)}\right)$, $\Rightarrow \int\frac{dx}{x} = \int\frac{\nu\phi'(\nu^2)}{\phi(\nu^2)}d\nu$,

[Let, $\phi(\nu^2)= z$, $\therefore 2\phi'(\nu^2)\nu\, d\nu = dz$], $\Rightarrow \frac{1}{2}\int\frac{dz}{z} = \ell nx$, $\Rightarrow \ell n\, z^{1/2} = \ell nx + k$, $\Rightarrow z = cx^2$, $\phi\left(\frac{y^2}{x^2}\right) = cx^2$

40. Let f(x) be a differentiable function and $f'(4) = 5$. Then $\lim_{x\to 2}\frac{f(4)-f(x^2)}{x-2}$ equals

(A) 0 (B) 5
(C) 20 (D) –20

Ans : (D)

Hints : $\lim_{x\to 2}\dfrac{f(4)-f(x^2)}{x-2}$, ($\dfrac{0}{0}$ form, so using L′Hospital's rule), = $\lim_{x\to 2}\dfrac{0-f'(x^2)\times 2x}{1}$, = –f′(4)×4 = –5×4 = –20

41. The value of $\lim_{x\to 0}\dfrac{\int_0^{x^2}\cos(t^2)dt}{x\sin x}$ is

(A) 1 (B) –1 (C) 2 (D) $\log_e 2$

Ans : (A)

Hints : $\lim_{x\to 0}\dfrac{\cos x^4\cdot 2x}{1\cdot\sin x+x\cos x}$

$$\lim_{x\to 0}\frac{2\cos x^4}{\frac{\sin x}{x}+\cos x}=\frac{2}{1+1}=1$$

42. The range of the function y = 3 sin $\left(\sqrt{\dfrac{\pi^2}{16}-x^2}\right)$ is

(A) $\left[0,\sqrt{3/2}\right]$ (B) [0, 1] (C) $\left[0, 3/\sqrt{2}\right]$ (D) $[0, \infty)$

Ans : (C)

Hints : y = 3 sin $\sqrt{\dfrac{\pi^2}{16}-x^2}$

y_{max} = 3sin π/4 = $3/\sqrt{2}$, y_{min} = 0

43. There is a group of 265 persons who like either s nging or dancing or painting. In this group 200 like singing, 110 like dancing and 55 like painting. If 60 persons like bot singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of pers ns who like *only* dancing and painting is

(A) 10 (B) 20 (C) 30 (D) 40

Ans : (A)

Hints : $n(S \cup P \cup D) = 265$

$n(S) = 200$

$n(D) = 110$

$n(P) = 55$

$n(S \cap D) = 60$

$n(S \cap P) = 30$

$n(S \cap D \cap P) = 10$

$n(S \cup P \cup D) = n(S) + n(D) + n(P) - n(S \cap D) - n(D \cap P) - n(P \cap S) + n(S \cap D \cap P)$

$265 = 200 + 110 + 55 - 60 - 30 - n(P \cap D) + 10$

$n(P \cap D) = 285 - 265 = 20$

$n(P \cap D) - n(P \cap D \cap S) = 20 - 10 = 10$

44. The curve y = $(\cos x + y)^{1/2}$ satisfies the differential equation

(A) $(2y-1)\dfrac{d^2y}{dx^2}+2\left(\dfrac{dy}{dx}\right)^2+\cos x=0$ (B) $\dfrac{d^2y}{dx^2}-2y\left(\dfrac{dy}{dx}\right)^2+\cos x=0$

(C) $(2y-1)\dfrac{d^2y}{dx^2}-2\left(\dfrac{dy}{dx}\right)^2+\cos x=0$ (D) $(2y-1)\dfrac{d^2y}{dx^2}-\left(\dfrac{dy}{dx}\right)^2+\cos x=0$

Ans : (A)

Hints : $y = (\cos x + y)^{1/2}$

$y^2 = \cos x + y$

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$2\left(\frac{dy}{dx}\right)^2 + 2.y.\frac{d^2y}{dx^2} = -\cos x + \frac{d^2y}{dx^2},\ (2y-1)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + \cos x = 0$$

45. Suppose that z_1, z_2, z_3 are three vertices of an equilateral triangle in the Argand plane. Let $\alpha = \frac{1}{2}\left(\sqrt{3}+i\right)$ and β be a non-zero complex number. The points $\alpha z_1 + \beta$, $\alpha z_2 + \beta$, $\alpha z_3 + \beta$ will be

(A) The vertices of an equilateral triangle
(B) The vertices of an isosceles triangle
(C) Collinear
(D) The vertices of an scalene triangle

Ans : (A)

Hints : $$\frac{1}{(\alpha z_1+\beta)-(\alpha z_2+\beta)}+\frac{1}{(\alpha z_2+\beta)-(\alpha z_3+\beta)}+\frac{1}{(\alpha z_3+\beta)-(\alpha z_1+\beta)}$$

$$=\frac{1}{\alpha(z_1-z_2)}+\frac{1}{\alpha(z_2-z_3)}+\frac{1}{\alpha(z_3-z_1)}$$

$$=\frac{1}{\alpha}\left[\frac{1}{(z_1-z_2)}+\frac{1}{(z_2-z_3)}+\frac{1}{(z_3-z_1)}\right]=0$$

Hence, $\alpha z_1 + \beta$, $\alpha z_2 + \beta$, $\alpha z_3 + \beta$ are vertices of equilat ral triangle.

46. If $\lim\limits_{x\to 0}\frac{2a\sin x-\sin 2x}{\tan^3 x}$ exists and is equal to 1, then th value of a is

(A) 2 (B) 1 (C) 0 (D) –1

Ans : (B)

Hints : $$\lim_{x\to 0}\frac{2a(x-\frac{x^3}{\underline{|3}})-(2x-\frac{8x^3}{\underline{|3}})+.\ ....}{x^3+......}$$

$$\lim_{x\to 0}\frac{2(a-1)x+\left(\frac{4}{3}\ \ \frac{a}{3}\right)x^3+.......}{x^3+.......}$$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1$

47. If $f(x) = \begin{cases}2x^2+1, & x\le 1\\ 4x^3-1, & x>1\end{cases}$, then $\int_0^2 f(x)dx$ is

(A) 47/3 (B) 50/3 (C) 1/3 (D) 47/2

Ans : (A)

Hints : $\int_0^1 f(x)dx+\int_1^2 f(x)dx$

$$=\int_0^1(2x^2+1)dx+\int_1^2(4x^3-1)dx$$

$$=\left(2\cdot\frac{x^3}{3}+x\right)_0^1+\left(\cancel{4}\cdot\frac{x^4}{\cancel{4}}-4\right)_1^2 = 5/3 + 14 = 47/3$$

48. The value of $|z|^2 + |z-3|^2 + |z-i|^2$ is minimum when z equals

(A) $2-\frac{2}{3}i$ (B) $45 + 3i$ (C) $1+\frac{i}{3}$ (D) $1-\frac{i}{3}$

Ans : (C)

Hints : $|z|^2 + |z-3|^2 + |z-i|^2 = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-1)^2$

$= 3x^2 + 3y^2 - 6x - 2y + 10$

$= 3\left[x^2 + y^2 - 2x - 2.y.\frac{1}{3}\right] + 10$

$= 3\left|z-\left(1+\frac{i}{3}\right)\right|^2 + \frac{20}{3}$

49. The number of solution(s) of the equation $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ is/are

(A) 2 (B) 0 (C) 3 (D) 1

Ans : (B)

Hints : $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$

Squaring,

$x + 1 + x - 1 - 2\sqrt{x^2-1} = 4x - 1$

$1 - 2x = 2\sqrt{x^2-1}$

$1 + 4x^2 - 4x = 4x^2 - 4$

$4x = 5$

$x = 5/4$

Which does not satisfies the equat on.

Hence, no solution

50. The values of λ for which the curve $(7x + 5)^2 + (7y + 3)^2 = \lambda^2(4x + 3y - 24)^2$ represents a parabola is

(A) $\pm\frac{6}{5}$ (B) $\pm\frac{7}{5}$ (C) $\pm\frac{1}{5}$ (D) $\pm\frac{2}{5}$

Ans : (B)

Hints : $49\left[(x + 5/7)^2 + (y + 3/7)^2\right] = 25\lambda^2\left(\frac{4x+3y-24}{5}\right)^2$

$\Rightarrow \frac{25\lambda^2}{49}=1$

$\lambda^2 = \frac{49}{25}$

$\lambda = \pm 7/5$

51. If $\sin^{-1}\left(\frac{x}{13}\right) + \operatorname{cosec}^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2}$, then the value of x is

(A) 5 (B) 4 (C) 12 (D) 11

Ans : (A)

Hints : $\sin^{-1}\left(\frac{x}{13}\right) = \frac{\pi}{2} - \operatorname{cosec}^{-1}\left(\frac{13}{12}\right)$

$= \sec^{-1}\left(\frac{13}{12}\right)$

$= \cos^{-1}\frac{12}{13}$

$\sin^{-1}\left(\frac{x}{13}\right) = \sin^{-1}\frac{5}{13}$

$x = 5$

52. The straight lines x + y = 0, 5x + y = 4 and x + 5y = 4 form

(A) an isosceles triangle (B) an equilateral triangle (C) a scalene tri ngle (D) a right angled triangle

Ans : (A)

Hints : Their point of intersection are (–1, 1) (1, –1) and (2/3, 2/3) which is the vertices of isoceles triangle.

53. If $I = \int_0^2 e^{x^4}(x-\alpha)dx = 0$, then α lies in the interval

(A) (0, 2) (B) (–1, 0) (C) (2, 3) (D) (–2, –1)

Ans : (A)

Hints : $I = \int_0^2 e^{x^4}(x-\alpha)dx = 0$

$e^{x^4} > 0$

$(x-\alpha)$ should be somewhere positive and somewhere negative so $\alpha \in (0, 2)$

Hence, $a \in (0, 2)$

54. If the coefficient of x^8 in $\left(ax^2 + \frac{1}{bx}\right)^{13}$ is equal to the coefficient of x^{-8} in $\left(ax - \frac{1}{bx^2}\right)^{13}$, then a and b will satisfy the relation

(A) ab + 1 = 0 (B) ab = 1 (C) a = 1 – b (D) a + b = –1

Ans : (A)

Hints : $\left(ax^2 + \frac{1}{bx}\right)^{13}$

Co-efficient of x^8 in $\left(ax^2 + \frac{1}{bx}\right)^{13}$

$^{13}C_6\, a^7 . \frac{1}{b^6}$ — (1)

Co-efficient x^{-8} in $\left(ax - \frac{1}{bx^2}\right)^{13} = {}^{13}C_7\, a^6 \times \left(-\frac{1}{b}\right)^7$

$= -\,{}^{13}C_7\, a^6 . \frac{1}{b^7}$ — (2)

Since, $^{13}C_6\, a^7/b^6 = -\,{}^{13}C_7\, a^6/b^7$

$a = -\frac{1}{b}$

$ab + 1 = 0$

55. The function $f(x) = a \sin|x| + be^{|x|}$ is differentiable at $x = 0$ when

(A) $3a + b = 0$ (B) $3a - b = 0$ (C) $a + b = 0$ (D) $a - b = 0$

Ans : (C)

Hints : $f(x) = a \sin|x| + be^{|x|}$

$f(x) = a \sin x + be^{x}$ $x \geq 0$

$= -a \sin x + be^{-x}$ $x < 0$

$f'(x) = a\cos x + be^{x}$ $x \geq 0$

$= -a\cos x - be^{-x}$ $x < 0$

at $x = 0$

$a + b = -a - b$

$a + b = 0$

56. If a, b and c are positive numbers in a G.P., then the roots of the quadratic equation $(\log_e a)x^2 - (2\log_e b)x + (\log_e c) = 0$ are

(A) -1 and $\frac{\log_e c}{\log_e a}$ (B) 1 and $-\frac{\log_e c}{\log_e a}$ (C) 1 and $\log_a c$ (D) -1 and $\log_c a$

Ans : (C)

Hints : $b^2 = ac \Rightarrow \log_e a - 2\log_e b + \log_e c = 0$

$(\log_e a)x^2 - (2\log_e b)x + \log_e c = 0$

Since, 1 satisfies the equation

Therefore 1 is one root and other root say β

$1.\beta = \frac{\log_e c}{\log_e a} = \log_c a$

$\beta = \frac{\log_e c}{\log_e a} = \log_a c$

57. Let $\mathbb{R}$ be the set of all real numbers and f: [–1, 1] $\to$ $\mathbb{R}$ be defined by f(x) = $\begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, Then

(A) f satisfies the conditions of Rolle's theorem on [–1, 1]

(B) f satisfies the conditions of Lagrange's Mean Value Theorem on [–1, 1]

(C) f satisfies the conditions of Rolle's theorem on [0, 1]

(D) f satisfies the conditions of Lagrange's Mean Value Theorem on [0, 1]

Ans : (D)

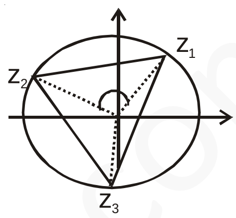
Hints : *f*(x) is nondifferentiable at x = 0

58. Let z_1 be a fixed point on the circle of radius 1 centered at the origin in the Argand plane and $z_1 \neq \pm 1$. Consider an equilateral triangle inscribed in the circle with z_1, z_2, z_3 as the vertices taken in the counter clockwise direction. Then $z_1z_2z_3$ is equal to

(A) z_1^2 (B) z_1^3 (C) z_1^4 (D) z_1

Ans : (B)

Hints : Let $z_1 = re^{i\alpha}$, $z_2 = r\,e^{i(\alpha+\frac{2\pi}{3})}$, $z_3 = r\,e^{i(\alpha+\frac{4\pi}{3})}$



$z_1z_2z_3 = r^3\,e^{i(\alpha+\alpha+\frac{2\pi}{3}+\alpha+\frac{4\pi}{3})}$

$= r^3e^{i(3\alpha + 2\pi)}$

$= r^3e^{i3\alpha}$

$= (re^{i\alpha})^3$

$= z_1^3$

59. Suppose that f(x) is a differentiable function s ch tha f′(x) is continuous, f′(0) = 1 and f″(0) does not exist. Let g(x) = xf′(x). Then

(A) g′(0) does not exist (B) g′(0) = 0 (C) g′(0) = 1 (D) g′(0) = 2

Ans : (C)

Hints : g(x) = x.f′(x)

$$g'(x) = \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$g'(x) = \lim_{h\to 0}\frac{(x+h)\cdot f'(x+h)-xf'(x)}{h}$$

$$g'(0) = 0 + \lim_{h\to 0} f'(0+h)$$

$= f'(0)$

$= 1$

60. Let [x] denote the greatest integer less than or equal to x for any real number x. Then $\lim_{n\to\infty}\frac{[n\sqrt{2}]}{n}$ is equal to

(A) 0 (B) 2 (C) $\sqrt{2}$ (D) 1

Ans : (C)

Hints : $\lim_{n\to\infty}\frac{[\sqrt{2}n]}{n}$

$$\sqrt{2}n \le [\sqrt{2}n] < \sqrt{2}n + 1$$

$$\sqrt{2} \le \frac{[\sqrt{2}n]}{n} < \sqrt{2} + \frac{1}{n}$$

$$\sqrt{2} \le \lim_{n\to\infty}\frac{[\sqrt{2}n]}{n} < \sqrt{2}$$

$$\Rightarrow \lim_{n\to\infty}\frac{[\sqrt{2}n]}{n} = \sqrt{2}$$

CATEGORY - II

Q.61 to Q.75 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark

61. We define a binary relation ~ on the set of all 3 × 3 real mat ices as A ~ B if and only if there exist invertible matrices P and Q such that $B = PAQ^{-1}$. The binary relation ~ is

(A) Neither reflexive nor symmetric
(B) Reflexive and symmetric but not transitive
(C) Symmetric and transitive but not refle ive
(D) An equivalence relation

Ans : (D)

Hints : For Reflexive, A.I = IA, $A = IAI^{-}$ so reflexive.

For Symmetric, $B = PAQ^{-1}$, BQ = PA, $P^{1}BQ = A$ or $A = (P^{-1})B.(Q^{-1})^{-1}$,so symmetric.

For Transitive, $B = PAQ^{-1}$, $C = PBQ^{1} = P.PAQ^{-1}.Q^{-1} = (PP)A(QQ)^{-1}$, so transitive

62. The minimum value of $2^{\sin} + 2^{\text{co } x}$ is

(A) $2^{1-1/\sqrt{2}}$ (B) $2^{1+1/\sqrt{2}}$ (C) $2^{\sqrt{2}}$ (D) 2

Ans : (A)

Hints : $\frac{2^{\sin x}+2^{\cos x}}{2} \ge \sqrt{2^{\sin x+\cos x}} \ge \sqrt{2^{-\sqrt{2}}}$, $2^{\sin x}+2^{\cos x} \ge 2.2^{-1/\sqrt{2}}$, $2^{\sin x}+2^{\cos x} \ge 2^{1-1/\sqrt{2}}$

63. For any two real numbers θ and φ, we define θRφ if and only if $\sec^2\theta - \tan^2\varphi = 1$. The relation R is

(A) Reflexive but not transitive
(B) Symmetric but not reflexive
(C) Both reflexive and symmetric but not transitive
(D) An equivalence relation

Ans : (D)

Hints: For reflexive, θ = ϕ so $\sec^2\theta - \tan^2\theta = 1$, Hence Reflexive

For symmetric, $\sec^2\theta - \tan^2\phi = 1$ so, $(1 + \tan^2\theta) - (\sec^2\phi - 1) = 1$ so, $\sec^2\phi - \tan^2\theta = 1$. Hence symmetric

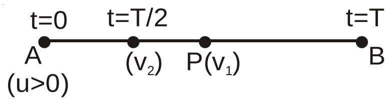
For Transitive, let $\sec^2\theta - \tan^2\phi = 1$ and $\sec^2\phi - \tan^2\gamma = 1$ so, $1 + \tan^2\phi - \tan^2\gamma = 1$ or, $\sec^2\theta - \tan^2\gamma = 1$. Hence Transitive

64. A particle starting from a point A and moving with a positive constant acceleration along a straight line reaches another point B in time T. Suppose that the initial velocity of the particle is u > 0 and P is the midpoint of the line AB. If the velocity of the particle at point P is v_1 and if the velocity at time $\frac{T}{2}$ is v_2, then

(A) $v_1 = v_2$ (B) $v_1 > v_2$ (C) $v_1 < v_2$ (D) $v_1 = \frac{1}{2} v_2$

Ans : (B)

Hints :



Since the particle is moving with a positive constant acceleration hence it's velocity should increase. So the time taken to travel AP is more that the timetaken for PB. So the instant $T/2$ is before P. Hence $v_1 > v_2$ since velocity increases from A to B.

65. Let t_n denote the nth term of the infinite series $\frac{1}{1!}+\frac{10}{2!}+\frac{21}{3!}+\frac{34}{4!}+\frac{49}{5!}+..........$. Then $\lim_{n\to\infty} t_n$ is

(A) e (B) 0 (C) e^2 (D) 1

Ans : (B)

Hints : $t_n = \frac{n^2+6n-6}{\lfloor n}$, $\lim_{n\to\infty} \frac{n^2+6n-6}{\lfloor n} = 0$ since denominator is very large compared to numerator

66. Let α, β denote the cube roots of unity other than 1 and α β. Let $s = \sum_{n=0}^{02} (-1)^n \left(\frac{\alpha}{\beta}\right)^n$. Then the value of s is

(A) Either -2ω or $-2\omega^2$ (B) Either -2ω or $2\omega^2$ (C) Either 2ω or $-2\omega^2$ (D) Either 2ω or $2\omega^2$

Ans : (A)

Hints : $a = \omega^2$, $\beta = \omega \Rightarrow \frac{\alpha}{\beta} = \omega$, $S = \sum_{n=0}^{302} (-1)^n.(\omega)^n = \omega^0 - \omega^1 + \omega^2 - \omega^3 + \omega^4 ....+ \omega^{302} = \frac{1-(-\omega)^{303}}{1-(-\omega)} = \frac{2}{-\omega^2} = -2\omega$

$\alpha = \omega$, $\beta = \omega^2 \Rightarrow \frac{\alpha}{\beta} = \frac{1}{\omega} = \omega^2$, $S = (\omega^2)^0 - (\omega)^1 + (\omega^2)^2 ....+(\omega^2)^{302} = \frac{1-(-\omega^2)^{303}}{1-(-\omega^2)} = \frac{2}{-\omega} = -2\omega^2$

67. The equation of hyperbola whose coordinates of the foci are (±8, 0) and the length of latus rectum is 24 units, is

(A) $3x^2 - y^2 = 48$ (B) $4x^2 - y^2 = 48$ (C) $x^2 - 3y^2 = 48$ (D) $x^2 - 4y^2 = 48$

Ans : (A)

Hints : $ae = 8$, $\frac{2b^2}{a} = 24$, $a^2e^2 = a^2 + b^2$ or, $64 = a^2 + 12a$ so $a = 4$, $b^2 = 48$, $\frac{x^2}{16} - \frac{y^2}{48} = 1$ so $3x^2 - y^2 = 48$

68. Applying Lagrange's Mean Value Theorem for a suitable function f(x) in [0, h], we have $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$. Then for $f(x) = \cos x$, the value of $\lim_{h\to 0^+} \theta$ is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Ans : (C)

Hints : For f(x) = cos x, cos h = 1 + h (–sin(θh)), $\sin\theta h = \dfrac{1-\cos h}{h}$, $\theta = \dfrac{\sin^{-1}\left(\dfrac{1-\cos h}{h}\right)}{h}$

$$\lim_{h\to 0^+}\theta = \lim_{h\to 0^+}\frac{\sin^{-1}\left(\frac{1-\cos h}{h}\right)}{h} = \lim_{h\to 0^+}\frac{\sin^{-1}\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\frac{1}{2} = \frac{1}{2}$$

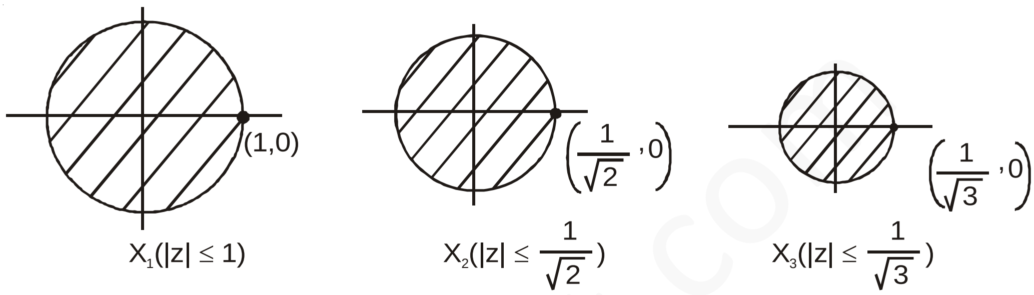
69. Let $X_n = \{z = x + iy : |z|^2 \le \frac{1}{n}\}$ for all integers n ≥ 1. Then $\bigcap_{n=1}^{\infty} X_n$ is

(A) A singleton set (B) Not a finite set

(C) An empty set (D) A finite set with more than one elements

Ans : (A)

Hints :



The required regions are shaded for n = 1, 2, 3 so clearly $\bigcap_{n=1} X_n$ will be only the point circle origin. So a singleton set

70. Suppose $M = \int_0^{\pi/2}\frac{\cos x}{x+2}dx$, $N = \int_0^{\pi/4}\frac{\sin x\cos x}{(x+1)^2}dx$. Then the value of (M – N) equals

(A) $\frac{3}{\pi+2}$ (B) $\frac{2}{\pi-4}$ (C) $\frac{4}{\pi-2}$ (D) $\frac{2}{\pi+4}$

Ans : (D)

Hints : $N = \frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{\sin 2x}{(x+1)^2} = \frac{1}{2}\left[\sin 2x\times\left(-\frac{1}{x+1}\right)\Bigg|_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}}\frac{2\cos 2x}{(x+1)}dx\right] = \frac{-2}{\pi+4} + \int_0^{\frac{\pi}{4}}\frac{\cos 2x}{(x+1)}dx$

Replacing 2x = t, $\int_0^{\frac{\pi}{4}}\frac{\cos 2x}{(x+1)}dx = \int_0^{\frac{\pi}{2}}\frac{\cos t}{(t+2)}dt = M$. So M–N = $\frac{2}{\pi+4}$

71. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

(A) is equal to zero (B) lies between 0 and 3 (C) is a negative number (D) lies between 3 and 6

Ans : (c)

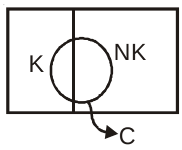
Hints : $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=\frac{\sin\frac{3\pi}{7}}{\sin\pi/7}\cdot\cos\frac{4\pi}{7}$. Clearly it is a negative no.

72. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, 0 < p < 1. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

(A) $\frac{3p}{4p+3}$ (B) $\frac{5p}{3p+2}$ (C) $\frac{5p}{4p+1}$ (D) $\frac{4p}{3p+1}$

Ans : (c)

Hints : K = He knows the answers, NK = He randomly ticks the answers, C = He is correct



$$P\left(\frac{K}{C}\right)=\frac{P(K)\cdot P\left(\frac{C}{K}\right)}{P(K)\cdot P\left(\frac{C}{K}\right)+P(NK)\cdot P\left(\frac{C}{NK}\right)}=\frac{P\times1}{P\times1+(1-P)\times\frac{1}{5}}=\frac{5P}{4P+1}$$

73. A poker hand consists of 5 cards drawn at random from a well-shuffled pack of 52 cards. Then the probability that a poker hand consists of a pair and a triple of equal face values (for example 2 sevens and 3 kings or 2 aces and 3 queens, etc.) is

(A) $\frac{6}{4165}$ (B) $\frac{23}{4165}$ (C) $\frac{1797}{4165}$ (D) $\frac{1}{4165}$

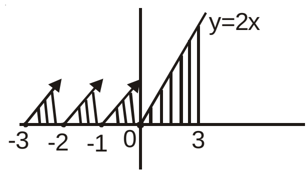
Ans : (A)

Hints : $\frac{{}^{13}C_1\times{}^{4}C_2\times{}^{12}C_1\times{}^{4}C_3}{{}^{52}C_5}=\frac{6}{4165}$.

74. Let $f(x)=\max\{x+|x|,x-[x]\}$, where [x] den tes the greatest integer $\le$ x. Then the value of $\int_{-3}^{3}f(x)dx$ is

(A) 0 (B) 51/2 (C) 21/2 (D) 1

Ans : (c)



Hints : Required area $=3\times\frac{1}{2}+\frac{1}{2}\times3\times\quad=\frac{21}{2}$

75. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x\log_e x}=\frac{1}{x}$ under the condition y = 1 when x = e is

(A) $2y=\log_e x+\frac{1}{\log_e x}$ (B) $y=\log_e x+\frac{2}{\log_e x}$ (C) $y\log_e x=\log_e x+1$ (D) $y=\log_e x+e$

Ans : (A)

Hints : Integrating factor $=e^{\int\frac{dx}{x\log_e x}}=\log_e x$ $y\cdot\log_e x=\int\frac{\log_e x}{x}\cdot dx+c=\frac{(\log_e x)^2}{2}+c$, $c=\frac{1}{2}$

$$2y=(\log_e x)+\frac{1}{\log_e x}$$

CATEGORY - III

Q. 76 – Q. 80 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.

76. Let $f(x)=\begin{cases}\int_0^x |1-t|dt, & x>1\\ x-\frac{1}{2}, & x\le 1\end{cases}$ Then,

(A) f(x) is continuous at x = 1 (B) f(x) is not continuous at x = 1

(C) f(x) is differentiable at x = 1 (D) f(x) is not differentiable at x = 1

Ans : (A,D)

Hints : $\int\limits_0^x |1-t|\,dt=\int\limits_0^1(1-t)dt+\int\limits_1^x (t-1)\,dt=\frac{x^2}{2}-x+1,$ $f(x)=\begin{cases}\frac{x^2}{2}-x+1, & x>1\\ x-\frac{1}{2}, & x\le 1\end{cases}$ $f'(x)=\begin{cases}x-1, & x>1\\ 1, & x\le 1\end{cases}$

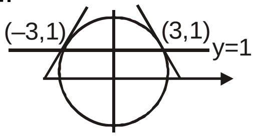
Clearly f(x) is continuous but not differentiable at x = 1.

77. The angle of intersection between the curves $y=\big[|\sin x|+|\cos x|\big]$ and $x^2 + y^2 = 10$, where [x] denotes the greatest integer $\le$ x, is

(A) $\tan^{-1}3$ (B) $\tan^{-1}(-3)$ (C) $\tan^{-1}\sqrt{3}$ (D) $\tan^{-1}\left(1/\sqrt{3}\right)$

Ans : (A,B)

Hints : $|\sin x|+|\cos x|=\sqrt{1+|\sin 2x|}$ So, $1\le|s\,n\,x|+|\cos x|\le\sqrt{2}.$ $y=\big[|\sin x|+|\cos x|\big]=1.$



$2x+2y\frac{dy}{dx}=0$, $\frac{dy}{dx}=\frac{-x}{y}$, So, angle is either $\tan^{-1}(-3)$ or $\tan^{-1}(3)$.

78. If u(x) and v(x) are two indepe dent solutions of the differential equation $\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0,$ then additional solution(s) of the given differential equa ion is (are)

(A) y = 5 u(x) + 8 v(x)

(B) $y = c_1\{u(x) - v(x)\} + c_2 v(x)$, c_1 and c_2 are arbitrary constants

(C) $y = c_1 u(x) v(x) + c_2 u(x)/v(x)$, c_1 and c_2 are arbitrary constants

(D) y = u(x) v(x)

Ans : (A,B)

Hints : Any linear combination of u(x) and v(x) will also be a solution.

79. For two events A and B, let P(A) = 0.7 and P(B) = 0.6. The necessarily false statements(s) is/are

(A) $P(A\cap B)=0.35$ (B) $P(A\cap B)=0.45$ (C) $P(A\cap B)=0.65$ (D) $P(A\cap B)=0.28$

Ans : (C,D)

Hints : $P(A\cup B)=1\cdot 3-P(A\cap B)$ now $P(A)\le P(A\cup B)\le 1,$ $0.7\le 1.3-P(A\cap B)\le 1,$ $0.3\le P(A\cap B)\le 0.6$

80. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the three circles $x^2 + y^2 - 5 = 0$, $x^2 + y^2 - 8x - 6y + 10 = 0$ and $x^2 + y^2 - 4x + 2y - 2 = 0$ at the extremities of their diameters, then

(A) $C = -5$ (B) $fg = 147/25$ (C) $g + 2f = c + 2$ (D) $4f = 3g$

Ans : (A,B,D)

Hints : Common chords of the circle will pass through the centres.

$c = -5$, $8g + 6f = -35$, $4g - 2f = -7$ so, $g = \frac{-14}{5}$, $f = \frac{-21}{10}$

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