WBJEE

BONG MOTION

JEE

WBJEE (Chapter wise PYQ)
Topic - Vector

Q1. In the four points with position vectors $-2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$, $\hat{j}-\hat{k}$ and $\lambda\hat{j}+\hat{k}$ are coplanar, then λ is equal t (2015)

(A) 1 (B) 2 (C) -1 (D) 0

Solution:

$A(\vec{a}) = -2\hat{i}+\hat{j}+\hat{k}$

$B(\vec{b}) = \hat{i}+\hat{j}+\hat{k}$

$C(\vec{c}) = \hat{j}-\hat{k}$

$D(\vec{d}) = \lambda\hat{j}+\hat{k}$

$\overrightarrow{AB} = \hat{i}+0\hat{j}+0\hat{k}$

$\overrightarrow{AC} = 2\hat{i}+0\hat{j}-2\hat{k}$

$\overrightarrow{AD} = 2\hat{i}+(\lambda-1)\hat{j}+0\hat{k}$

$\therefore [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & -2 \\ 2 & (\lambda-1) & 0 \end{vmatrix} = 0$

$\Rightarrow 1\left(0+2(\lambda-1)\right) = 0$

$\Rightarrow 2\lambda-2=0 \Rightarrow \lambda=1$

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Q.2. Which of the following is not always true? (2015)

A. $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$. If $(\vec{a} \perp \vec{b})$ T

B. $|\vec{a}+\lambda\vec{b}| \geq |\vec{a}| \;\forall\, \lambda \in R$ if $(\vec{a} \perp \vec{b})$ T

C. $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$ T

D. $|\vec{a}+\lambda\vec{b}| \geq |\vec{a}| \;\forall\, \lambda \in R$ if $\vec{a} \parallel \vec{b}$. F

$\vec{a} = k\vec{b}$ — Real number

$\sqrt{\vec{a}\cdot\vec{a}} = |\vec{a}|$

$|\vec{a}+\lambda\vec{b}|$

$= \left|\vec{a} + \frac{\lambda}{k}\vec{a}\right|$

$= \left|\left(1+\frac{\lambda}{k}\right)\vec{a}\right|$

$= \left|1+\frac{\lambda}{k}\right||\vec{a}| \geq |\vec{a}|$

put, $\lambda = -1$, $k = 1$

$\left|1+\frac{\lambda}{k}\right| \geq 1 \Rightarrow |1-1| \geq 1 \Rightarrow 0 \geq 1$

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Q. 3. For non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(A) Collinear (B) Perpendicular (C) Inclined at a acute angle (D) Inclined at a obtuse angle. [2016]

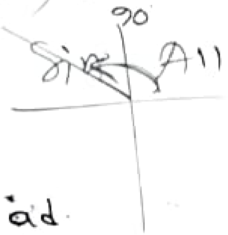
$$|\vec{a}+\vec{b}|^2 < |\vec{a}-\vec{b}|^2$$

$$\Rightarrow \cancel{|\vec{a}|^2} + 2\vec{a}\cdot\vec{b} + \cancel{|\vec{b}|^2} < \cancel{|\vec{a}|^2} - 2\vec{a}\cdot\vec{b} + \cancel{|\vec{b}|^2}$$

$$\Rightarrow 4\vec{a}\cdot\vec{b} < 0$$

$$\Rightarrow \vec{a}\cdot\vec{b} < 0$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta < 0 \Rightarrow \cos\theta < 0$$



$\therefore \theta$ belongs to 2nd Quad.

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Q. 4. If the sum of two unit vectors is a unit vector, then the magnitude of their differen e is $\boxed{2017}$

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ ✓ (D) $\sqrt{5}$

$\vec{a}, \vec{b}$

$|\vec{a}| = 1, |\vec{b}| = 1$

$|\vec{a}+\vec{b}|^2 = 1^2$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 1$

$\Rightarrow 1 + 1 + 2\vec{a}\cdot\vec{b} = 1$

$\Rightarrow 2\vec{a}\cdot\vec{b} = -1$

$\therefore |\vec{a}-\vec{b}| = \sqrt{(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})}$

$= \sqrt{|\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2}$

$= \sqrt{1 - (-1) + 1}$

$= \sqrt{1+1+1} = \sqrt{3}$

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Q. 5. For any vector $\vec{x}$, where $\hat{i}, \hat{j}, \hat{k}$ have their usual meani[ng]
the value of $|\vec{x}\times\hat{i}|^2+|\vec{x}\times\hat{j}|^2+|\vec{x}\times\hat{k}|^2$ is equal to $\boxed{2017}$

(A) $|\vec{x}|^2$ (B) $2|\vec{x}|^2$ (C) $3|\vec{x}|^2$ (D) $4|\vec{x}|^2$

Let, $\vec{x} = a\hat{i} + b\hat{j} + c\hat{k} \Rightarrow |\vec{x}| = \sqrt{a^2+b^2+c^2}$

$\vec{x}\times\hat{i} = \vec{0} - b\hat{k} + c\hat{j}$

$\vec{x}\times\hat{j} = a\hat{k} + \vec{0} - c\hat{i}$

$\vec{x}\times\hat{k} = -a\hat{j} + b\hat{i} + \vec{0}$

$|\vec{x}\times\hat{i}|^2 = |-b\hat{k}+c\hat{j}|^2 = (-b\hat{k}+c\hat{j})\cdot(-b\hat{k}+c\hat{j}) = b^2+c^2$

$+\ |\vec{x}\times\hat{j}|^2 = a^2+c^2$

$|\vec{x}\times\hat{k}|^2 = a^2+b^2$

$|\vec{x}\times\hat{i}|^2+|\vec{x}\times\hat{j}|^2+|\vec{x}\times\hat{k}|^2 = 2(a^2+b^2+c^2) = 2|\vec{x}|^2$

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Q. 6. Let, $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be the three unit vectors such that $\vec{\alpha}\cdot\vec{\beta} = \vec{\alpha}\cdot\vec{\gamma} = 0$ and, the angle between $\vec{\beta}$ and $\vec{\gamma}$ is $30°$. Then $\vec{\alpha}$ is

(A) $2(\vec{\beta}\times\vec{\gamma})$, (B) $-2(\vec{\beta}\times\vec{\gamma})$ (C) $\pm 2(\vec{\beta}\times\vec{\gamma})$ (D) $(\vec{\beta}\times\vec{\gamma})$ $\boxed{2018}$

$$\vec{\alpha}\cdot\vec{\beta} = 0 \Rightarrow \vec{\alpha} \perp \vec{\beta}$$

$$\vec{\alpha}\cdot\vec{\gamma} = 0 \Rightarrow \vec{\alpha} \perp \vec{\gamma}$$

$$\boxed{\vec{\alpha} = \lambda(\vec{\beta}\times\vec{\gamma})}$$

$$|\vec{\alpha}| = |\lambda|\,|\vec{\beta}||\vec{\gamma}|\sin 30°$$

$$\Rightarrow 1 = |\lambda|\cdot 1\cdot 1\cdot \tfrac{1}{2}$$

$$\Rightarrow 2 = |\lambda|$$

$$\therefore \lambda = \pm 2$$

$$\vec{\alpha} = \pm 2(\vec{\beta}\times\vec{\gamma})$$

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Q.7. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$, $\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{S}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, whose projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by $\boxed{2018}$

(A) $-\hat{i} - 3\hat{j} - 3\hat{k}$, (B) $\hat{i} - 3\hat{j} - 3\hat{k}$ (C) $-\hat{i} + 3\hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Let, $\vec{S} = \vec{\alpha} + \mu\vec{\beta}$

$= (1+\mu)\hat{i} + (1-\mu)\hat{j} + (1-\mu)\hat{k}$

projection of $\vec{S}$ on $\vec{\gamma} = \frac{1}{\sqrt{3}}$

$\therefore \frac{\vec{S}\cdot\vec{\gamma}}{|\vec{\gamma}|} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{(1+\mu)\cdot(-1) + \cancel{(1-\mu)\cdot 1} + \cancel{(1-\mu)\cdot(-1)}}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$

$\Rightarrow -(1+\mu) = 1$

$\Rightarrow -1-\mu = 1 \Rightarrow \mu = -2$

$\vec{S} = -\hat{i} + 3\hat{j} + 3\hat{k}$