M 2012 **68010**

MULTIPLE CHOICE QUESTIONS

SUBJECT : MATHEMATICS

Duration : Two Hours **Maximum Marks : 100**

[Q. 1 to 60 carry one mark each]

1. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^9 + y^9 + z^9 - \frac{1}{x^9 y^9 z^9}$ is equal to

A. 0 B. 1 C. 2 D. 3

2. Let p, q, r be the sides opposite to the angles P, Q, R respectively in a triangle PQR. If $r^2 \sin P \sin Q = pq$, then the triangle is

A. equilateral
B. acute angled but not equilateral
C. obtuse angled
D. right angled

3. Let p, q, r be the sides opposite to the angles P, Q, R respectively in a triangle PQR. Then $2pr \sin\left(\frac{P - Q + R}{2}\right)$ equals

A. $p^2 + q^2 + r^2$ B. $p^2 + r^2 - q^2$ C. $q^2 + r^2 - p^2$ D. $p^2 + q^2 - r^2$

4. Let P (2, –3), Q (–2, 1) be the vertices of the triangle PQR. If the centroid of ΔPQR lies on the line 2x + 3y = 1, then the locus of R is

A. 2x + 3y = 9 B. 2x – 3y = 9 C. 3x + 2y = 5 D. 3x – 2y = 5

5. $\lim\limits_{x \to 0} \frac{\pi^x - 1}{\sqrt{1 + x} - 1}$

A. does not exist B. equals $\log_e\left(\pi^2\right)$ C. equals 1 D. lies between 10 and 11

6. If f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, then

A. f (x) must be an increasing function
B. f (x) must be a decreasing function
C. |f (x)| must be an increasing function
D. |f (x)| must be a decreasing function

7. Rolle's theorem is applicable in the interval [–2, 2] for the function

A. $f(x) = x^3$ B. $f(x) = 4x^4$ C. $f(x) = 2x^3 + 3$ D. $f(x) = \pi |x|$

8. The solution of $25\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + y = 0, \quad y(0) = 1, \quad y(1) = 2e^{\frac{1}{5}}$ is

A. $y = e^{5x} + e^{-5x}$ B. $y = (1 + x)e^{5x}$ C. $y = (1+x)e^{\frac{x}{5}}$ D. $y = (1 + x)e^{-\frac{x}{5}}$

9. Let P be the midpoint of a chord joining the vertex of the parabola $y^2 = 8x$ to another point on it. Then the locus of P is

A. $y^2 = 2x$ B. $y^2 = 4x$ C. $\frac{x^2}{4} + y^2 = 1$ D. $x^2 + \frac{y^2}{4} = 1$

10 .The line $x = 2y$ intersects the ellipse $\frac{x^2}{4} + y^2 = 1$ at the points P and Q. The equation of the circle with PQ as diameter is

A. $x^2 + y^2 = \frac{1}{2}$ B. $x^2 + y^2 = 1$ C. $x^2 + y^2 = 2$ D. $x^2 + y^2 = \frac{5}{2}$

11. The eccentric angle in the first quadrant of a point on the ellipse $\frac{x^2}{10} + \frac{y^2}{8} = 1$at a distance 3 units from the centre of the ellipse is

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$

12. The transverse axis of a hyperbola is along the x-axis and its length is 2a. The vertex of the hyperbola bisects the line segment joining the centre and the focus. The equation of the hyperbola is

A. $6x^2 - y^2 = 3a^2$ B. $x^2 - 3y^2 = 3a^2$ C. $x^2 - 6y^2 = 3a^2$ D. $3x^2 - y^2 = 3a^2$

13. A point moves in such a way that the difference of its distance from two points (8,0) and (–8,0) always remains 4. Then the locus of the point is

A. a circle B. a parabola C. an ellipse D. a hyperbola

14. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

A. 0 B. 2 C. 4 D. 1

15. If a straight line passes through the point (α,β) and the portion of the line intercepted between the axes is divided equally at that point, then $\frac{x}{\alpha} + \frac{y}{\beta}$ is

A. 0 B. 1 C. 2 D. 4

16. The maximum value of $|z|$ when the complex number z satisfies the condition $\left|z + \frac{2}{z}\right|$ is

A. $\sqrt{3}$ B. $\sqrt{3} + \sqrt{2}$ C. $\sqrt{3} + 1$ D. $\sqrt{3} - 1$

17. If $\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are real, then the ordered pair (x,y) is

A. (–3, 0) B. (0, 3) C. (0, –3) D. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

18. If $\dfrac{z-1}{z+1}$ is purely imaginary, then

A. $|z|=\dfrac{1}{2}$ B. $|z|=1$ C. $|z|=2$ D. $|z|=3$

19. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only one student passed in all the subjects. Then the number of students failing in all the three subjects.

A. is 12 B. is 4

C. is 2 D. cannot be determined from the given information

20. A vehicle registration number consists of 2 letters of English alphabet followed by 4 digits, where the first digit is not zero. Then the total number of vehicles with distinct registration numbers is

A. $26^2 \times 10^4$ B. ${}^{26}P_2 \times {}^{10}P_4$ C. ${}^{26}P_2 \times 9 \times {}^{10}P_3$ D. $26^2 \times 9 \times 10^3$

21. The number of words that can be written using all the letters of the word 'IRRATIONAL' is

A. $\dfrac{10!}{(2!)^3}$ B. $\dfrac{10!}{(2!)^2}$ C. $\dfrac{10!}{2!}$ D. 10!

22. Four speakers will address a meeting where speaker Q will always speak after speaker P. Then the number of ways in which the order of speakers can be prepared is

A. 256 B. 128 C. 24 D. 12

23. The number of diagonals in a regular polygon of 100 sides is

A. 4950 B. 4850 C. 4750 D. 4650

24. Let the coefficients of powers of x in the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^n$, where n is a positive integer, be in arithmetic progression. Then the sum of the coefficients of odd powers of x in the expansion is

A. 32 B. 64 C. 128 D. 256

25. Let $f(x) = ax^2 + bx + c, g(x) = px^2 + qx + r$ such that $f(1) = g(1)$, $f(2) = g(2)$ and $f(3) - g(3) = 2$. Then $f(4) - g(4)$ is

A. 4 B. 5 C. 6 D. 7

26. The sum $1 \times 1! + 2 \times 2! + ... + 50 \times 50!$ equals

A. 51! B. 51! – 1 C. 51! + 1 D. $2 \times 51!$

27. Six numbers are in A.P. such that their sum is 3. The first term is 4 times the third term. Then the fifth term is

A. –15 B. –3 C. 9 D. –4

28. The sum of the infinite series $1+\dfrac{1}{3}+\dfrac{1.3}{3.6}+\dfrac{1.3.5}{3.6.9}+\dfrac{1.3.5.7}{3.6.9.12}+\cdots$ is equal to

A. $\sqrt{2}$ B. $\sqrt{3}$ C. $\sqrt{\frac{3}{2}}$ D. $\sqrt{\frac{1}{3}}$

29. The equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$ have a common real root

A. for no value of a
B. for exactly one value of a
C. for exactly two values of a
D. for exactly three values of a

30. If 64, 27, 36 are the P^{th}, Q^{th} and R^{th} terms of a G.P., then $P + 2Q$ is equal to

A. R
B. 2R
C. 3R
D. 4R

31. The equation $y^2 + 4x + 4y + k = 0$ represents a parabola whose latus rectum is

A. 1
B. 2
C. 3
D. 4

32. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is equal to

A. 2 or $-\frac{3}{2}$
B. -2 or $-\frac{3}{2}$
C. 2 or $\frac{3}{2}$
D. -2 or $\frac{3}{2}$

33. If four distinct points (2k, 3k), (2, 0), (0, 3), (0, 0) lie on a circle, then

A. $k < 0$
B. $0 < k < 1$
C. $k = 1$
D. $k > 1$

34. The line joining A(b cos α, b sin α) and B(a cos β, a sin β), where $a \neq b$, is produced to the point M(x, y) so that AM : MB = b : a. Then $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2}$

A. 0
B. 1
C. -1
D. $a^2 + b^2$

35. Let the foci of the ellipse $\frac{x^2}{9} + y^2 = 1$ subtend a right angle at a point P. Then the locus of P is

A. $x^2 + y^2 = 1$
B. $x^2 + y^2 = 2$
C. $x^2 + y^2 = 4$
D. $x^2 + y^2 = 8$

36. The general solution of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$ is

A. $\log_e|3x + 3y + 2| + 3x + 6y = c$
B. $\log_e|3x + 3y + 2| - 3x + 6y = c$
C. $\log_e|3x + 3y + 2| - 3x - 6y = c$
D. $\log_e|3x + 3y + 2| + 3x - 6y = c$

37. The value of the integral $\int_{\pi/6}^{\pi/2}\left(\frac{1+\sin 2x+\cos 2x}{\sin x+\cos x}\right)dx$ is equal to

A. 16
B. 8
C. 4
D. 1

38. The value of the integral $\int_0^{\frac{\pi}{2}}\frac{1}{1+(\tan x)^{101}}dx$ is equal to

A. 1
B. $\frac{\pi}{6}$
C. $\frac{\pi}{8}$
D. $\frac{\pi}{4}$

39. The integrating factor of the differential equation $3x\log_e x\frac{dy}{dx} + y = 2\log_e x$ is given by

A. $(\log_e x)^3$ B. $\log_e(\log_e x)$ C. $\log_e x$ D. $(\log_e x)^{\frac{1}{3}}$

40. Number of solutions of the equation $\tan x + \sec x = 2\cos x,\ x \in [0, \pi]$ is

A. 0 B. 1 C. 2 `D. 3

41. The value of the integral

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x}\,dx$$ is equal to

A. $\log_e 2$ B. $\log_e 3$ C. $\frac{1}{4}\log_e 2$ D. $\frac{1}{4}\log_e 3$

42. Let $y = \left(\frac{3^x - 1}{3^x + 1}\right)\sin x + \log_e(2 + x),\ x > -1$. Then at $x = 0$, $\frac{dy}{dx}$ equals

A. 1 B. 0 C. –1 D. –2

43. Maximum value of the function $f(x) = \frac{x}{8} + \frac{2}{x}$ on the interval $[1,\ 6]$ is

A. 1 B. $\frac{9}{8}$ C. $\frac{13}{12}$ D. $\frac{17}{8}$

44. For $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, the value of $\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1 + \sin x}\right\}$ is equal to

A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. 1 D. $\frac{\sin x}{(1 + \sin x)^2}$

45. The value of the integral $\int_{-2}^{2} (1 + 2\sin x)e^{|x|}\,dx$ is equal to

A. 0 B. $e^2 - 1$ C. $2(e^2 - 1)$ D. 1

46. If $(\alpha + \sqrt{\beta})$ and $(\alpha - \sqrt{\beta})$ are the roots of the equation $x^2 + px + q = 0$ where α, β, p and q are real, then the roots of the equation $(p^2 - 4q)(p^2x^2 + 4px) - 16q = 0$ are

A. $\left(\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}\right)$ B. $\left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\beta}\right)$ and $\left(\frac{1}{\sqrt{\alpha}} - \frac{1}{\beta}\right)$

C. $\left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\sqrt{\alpha}} - \frac{1}{\sqrt{\beta}}\right)$ D. $(\sqrt{\alpha} + \sqrt{\beta})$ and $(\sqrt{\alpha} - \sqrt{\beta})$

47. The number of solutions of the equation $\log_2\left(x^2+2x-1\right)=1$ is

A. 0 B. 1 C. 2 D. 3

48. The sum of the series $1+\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2+\ldots.+\frac{1}{n+1}{}^nC_n$ is equal to

A. $\frac{2^{n+1}-1}{n+1}$ B. $\frac{3\left(2^n-1\right)}{2n}$ C. $\frac{2^n+1}{n+1}$ D. $\frac{2^n+1}{2n}$

49. The value of $\sum_{r=2}^{\infty}\frac{1+2+\ldots..+(r-1)}{r!}$ is equal to

A. e B. 2e C. $\frac{e}{2}$ D. $\frac{3e}{2}$

50. If $P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}$, $Q=PP^T$, then the value of the determinant of Q is equal to

A. 2 B. –2 C. 1 D. 0

51. The remainder obtained when $1!+2!+\ldots+95!$ is divided by 15 is

A. 14 B. 3 C. 1 D. 0

52. If P, Q, R are angles of triangle PQR, then the value of $\begin{vmatrix}-1 & \cos R & \cos Q\\ \cos R & -1 & \cos P\\ \cos Q & \cos P & -1\end{vmatrix}$ is equal to

A. –1 B. 0 C. $\frac{1}{2}$ D. 1

53. The number of real values of α for which the system of equations

$$\begin{aligned}x+3y+5z&=\alpha x\\ 5x+y+3z&=\alpha y\\ 3x+5y+z&=\alpha z\end{aligned}$$

has infinite number of solutions is

A. 1 B. 2 C. 4 D. 6

54. The total number of injections (one-one into mappings) from $\{a_1, a_2, a_3, a_4\}$ to $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ is

A. 400 B. 420 C. 800 D. 840

55. Let $(1+x)^{10} = \sum_{r=0}^{10} c_r x^r$ and $(1+x)^7 = \sum_{r=0}^{7} d_r x^r$. If $P = \sum_{r=0}^{5} c_{2r}$ and $Q = \sum_{r=0}^{3} d_{2r+1}$, then $\frac{P}{Q}$ is equal to

A. 4 B. 8 C. 16 D. 32

56. Two decks of playing cards are well shuffled and 26 cards are randomly distributed to a player. Then the probability that the player gets all distinct cards is

A. ${}^{52}C_{26} / {}^{104}C_{26}$ B. $2 \times {}^{52}C_{26} / {}^{104}C_{26}$

C. $2^{13} \times {}^{52}C_{26} / {}^{104}C_{26}$ D. $2^{26} \times {}^{52}C_{26} / {}^{104}C_{26}$

57. An urn contains 8 red and 5 white balls. Three balls are drawn at random. Then the probability that balls of both colours are drawn is

A. $\frac{40}{143}$ B. $\frac{70}{143}$ C. $\frac{3}{13}$ D. $\frac{10}{13}$

58. Two coins are available, one fair and the other two-headed. Choose a coin and toss it once ; assume that the unbiased coin is chosen with probability $\frac{3}{4}$. Given that the outcome is head, the probability that the two-headed coin was chosen is

A. $\frac{3}{5}$ B. $\frac{2}{5}$ C. $\frac{1}{5}$ D. $\frac{2}{7}$

59. Let R be the set of real numbers and the functions $f : R \to R$ and $g : R \to R$ be defined $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$. Then the value of x for which $f(g(x)) = g(f(x))$ is

A. –1 B. 0 C. 1 D. 2

60. If a, b, c are in arithmetic progression, then the roots of the equation $ax^2 - 2bx + c = 0$ are

A. 1 and $\frac{c}{a}$ B. $-\frac{1}{a}$ and $-c$ C. -1 and $-\frac{c}{a}$ D. -2 and $-\frac{c}{2a}$

[Q. 61 to 80 carry two marks each]

61. Let y be the solution of the differential equation $x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$ satisfying y(1) = 1. Then y satisfies

A. $y = x^{y-1}$ B. $y = x^y$ C. $y = x^{y+1}$ D. $y = x^{y+2}$

62. The area of the region, bounded by the curves $y = \sin^{-1} x + x(1-x)$ and $y = \sin^{-1} x - x(1-x)$ in the first quadrant, is

A. 1 B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$

63. The value of the integral $\int_1^5 \left[|x-3|+|1-x|\right]dx$ is equal to

A. 4 B. 8 C. 12 D. 16

64. If f(x) and g(x) are twice differentiable functions on (0, 3) satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$, $g(2) = 9$, then $f(1) - g(1)$ is

A. 4 B. –4 C. 0 D. –2

65. Let [x] denote the greatest integer less than or equal to x, then the value of the integral $\int_{-1}^{1} (|x| - 2[x])dx$ is equal to

A. 3 B. 2 C. –2 D. –3

66. The points representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$ lie on

A. a circle B. a straight line C. an ellipse D. a parabola

67. Let a, b, c, p, q, r be positive real numbers such that a, b, c are in G.P. and $a^p = b^q = c^r$. Then

A. p, q, r are in G.P. B. p, q, r are in A.P. C. p, q, r are in H.P. D. p^2, q^2, r^2 are in A.P.

68. Let S_k be the sum of an infinite G.P. series whose first term is k and common ratio is $\frac{k}{k+1}$ (k > 0). Then the value of $\sum_{k=1}^{\infty} \frac{(-1)^k}{S_k}$ is equal to

A. $\log_e 4$ B. $\log_e 2 - 1$ C. $1 - \log_e 2$ D. $1 - \log_e 4$

69. The quadratic equation $2x^2 - \left(a^3 + 8a - 1\right)x + a^2 - 4a = 0$ possesses roots of opposite sign. Then

A. $a \le 0$ B. $0 < a < 4$ C. $4 \le a < 8$ D. $a \ge 8$

70. If $\log_e\left(x^2 - 16\right) \le \log_e(4x - 11)$, then

A. $4 < x \le 5$ B. $x < -4$ or $x > 4$ C. $-1 \le x \le 5$ D. $x < -1$ or $x > 5$

71. The coefficient of x^{10} in the expansion of $1 + (1+x) + \ldots + (1+x)^{20}$ is

A. ${}^{19}C_9$ B. ${}^{20}C_{10}$ C. ${}^{21}C_{11}$ D. ${}^{22}C_{12}$

72. The system of linear equations

$\lambda x + y + z = 3$

$x - y - 2z = 6$

$-x + y + z = \mu$

has

A. Infinite number of solutions for $\lambda \ne -1$ and all μ

B. Infinite number of solutions for $\lambda = -1$ and $\mu = 3$

C. No solution for $\lambda \ne -1$

D. Unique solution for $\lambda = -1$ and $\mu = 3$

73. Let A and B be two events with $P(A^C) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^C) = 0.5$. Then $P(B \backslash A \cup B^C)$ is equal to

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

74. Let p, q, r be the altitudes of a triangle with area S and perimeter 2t. Then the value of $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ is

A. $\frac{s}{t}$ B. $\frac{t}{s}$ C. $\frac{s}{2t}$ D. $\frac{2s}{t}$

75. Let C_1 and C_2 denote the centres of the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 1$ respectively and let P and Q be their points of intersection. Then the areas of triangles C_1PQ and C_2PQ are in the ratio

A. 3 : 1 B. 5 : 1 C. 7 : 1 D. 9 : 1

76. A straight line through the point of intersection of the lines $x + 2y = 4$ and $2x + y = 4$ meets the coordinates axes at A and B. The locus of the midpoint of AB is

A. $3(x + y) = 2xy$ B. $2(x + y) = 3xy$ C. $2(x + y) = xy$ D. $x + y = 3xy$

77. Let P and Q be the points on the parabola $y^2 = 4x$ so that the line segment PQ subtends right angle at the vertex. If PQ intersects the axis of the parabola at R, then the distance of the vertex from R is

A. 1 B. 2 C. 4 D. 6

78. The incentre of an equilateral triangle is (1, 1) and the equation of the one side is $3x + 4y + 3 = 0$. Then the equation of the circumcircle of the triangle is

A. $x^2 + y^2 - 2x - 2y - 2 = 0$ B. $x^2 + y^2 - 2x - 2y - 14 = 0$

C. $x^2 + y^2 - 2x - 2y + 2 = 0$ D. $x^2 + y^2 - 2x - 2y + 14 = 0$

79. The value of $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n}$ is

A. 1 B. $\frac{1}{e^2}$ C. $\frac{1}{2e}$ D. $\frac{1}{e}$

80. The area of the region bounded by the curves $y = x^3$, $y = \frac{1}{x}$, $x = 2$ is

A. $4 - \log_e 2$ B. $\frac{1}{4} + \log_e 2$ C. $3 - \log_e 2$ D. $\frac{15}{4} - \log_e 2$

1. Given $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=3.\frac{\pi}{2}$

$\therefore \quad \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

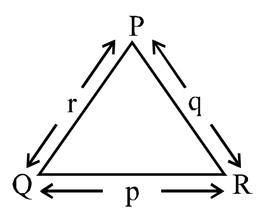
$\therefore \quad x=y=z=1$

$x^9+y^9+z^9-\frac{1}{x^9.y^9.z^9}$

$=1+1+1-\frac{1}{1}=3-1=2$

Ans. C.

2. $r^2.\sin P.\sin Q=pq$



$r^2\times\frac{p}{2R}.\frac{q}{2R}=pq$

$r^2=4R^2$

r = 2R

side = 2 × circumradius = diameter

So, triangle is right angled triangle.

Ans. D.

3. Given P + Q + R = 180°

$\therefore \quad P+R=180°-Q$

$\therefore \quad \sin\left(\frac{180°-2Q}{2}\right) = \sin(90°-Q) = \cos Q$

Now $2pr\cos Q$

$= 2pr\,\frac{p^2+r^2-q^2}{2pr}=p^2+r^2-q^2$

Ans. B.

4. Let R = (h, k)

Centroid $=\left(\frac{2-2+h}{3},\ \frac{1-3+k}{3}\right)=\left(\frac{h}{3},\ \frac{k-2}{3}\right)$

$\therefore \quad 2\frac{h}{3}+3.\frac{k-2}{3}=1$

$\frac{2h}{3}+k-2=1$

$2h+3k=9$

Required locus $2x+3y=9$

Ans. A.

5. $\lim\limits_{x\to 0} \dfrac{\pi^x - 1}{\sqrt{1+x} - 1}$

$= \lim\limits_{x\to 0} \dfrac{(\pi^x - 1)(\sqrt{1+x} + 1)}{1 + x - 1}$

$= \lim\limits_{x\to 0} \dfrac{\pi^x - 1}{x} . (\sqrt{1+x} + 1)$

$= (\log_2 \pi) \times 2 = \log_2 \pi^2$

Ans. B.

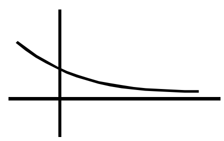
6. Given $f(x). f'(x) < 0 \quad \forall x \in R$

and f(x) is differentiable

$\therefore$ f(x) is continuous function and f(x) and $f'(x)$ are opposite of sign

$\therefore$ either f(x) > 0 or $f(x) < 0$ but

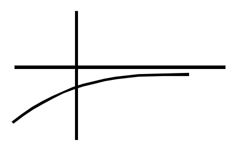
It can not cut the x-axis



$\therefore$ When f(x) > 0 then $f'(x) < 0$

$\therefore$ f(x) is decresing function

When $f(x) < 0$ then $f'(x) > 0$

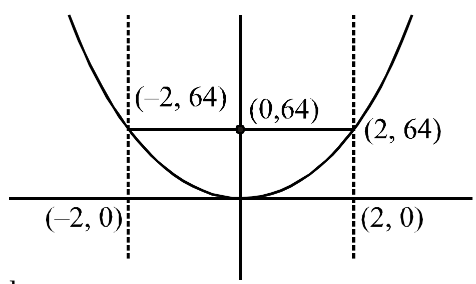


$\therefore$ f(x) is increasing function

$\therefore$ We can say that $|f(x)|$ is decreasing function.

Ans. D.

7. Check the options individually. Take option (b) $f(x) = 4x^4$



Now, Rolle's theorem is applicable.

Ans. B.

8. Let $y = e^{mx}$

$\dfrac{dy}{dx} = me^{mx}$

$\dfrac{d^2y}{dx^2} = m^2e^{mx}$

$25m^2e^{mx} - 10me^{mx} + e^{mx} = 0$

$25m^2 - 10m + 1 = 0$

$(5m - 1)^2 = 0$

$m = \frac{1}{5} \cdot \frac{1}{5}$

$\therefore$ General solution is $y = (A + Bx)e^{\frac{1}{5}x}$

$y(0) = 1$

$1 = A.1 \Rightarrow A = 1$

$y(1) = 2e^{\frac{1}{5}}$

$2e^{\frac{1}{5}} = (A + B)e^{\frac{1}{5}}$

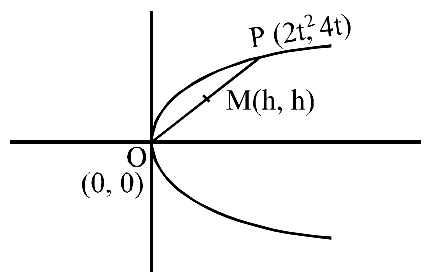
$2 = (1 + B)$

$B = 1$

$\therefore \quad y = (1 + x)e^{\frac{x}{5}}$

Ans. C.

9. $y^2 = 8x$



$y^2 = 4ax$

$4a = 8$

$a = 2$

$h = \frac{2t^2 + 0}{2} = t^2$ or $t^2 = h$

$k = \frac{4t + 0}{2} = 2t$ or $t = \frac{k}{2}$

$\therefore \quad \frac{k^2}{4} = h$

$\therefore k^2 = 4h \Rightarrow y^2 = 4x$

Ans. B.

10. $\frac{x^2}{4} + y^2 = 1$

$\frac{4y^2}{4} + y^2 = 1$

$2y^2 = 1$

$y^2 = \frac{1}{2}$

$y = \pm \frac{1}{\sqrt{2}}$

$x = \pm \sqrt{2}$

$P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$

$Q\left(-\sqrt{2},\ -\frac{1}{\sqrt{2}}\right)$

equation of circle PQ as diameter is $\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)+\left(y-\frac{1}{\sqrt{2}}\right)\left(y+\frac{1}{\sqrt{2}}\right)=0$

$x^2-2+y^2-\frac{1}{2}=0$

$x^2+y^2=\frac{5}{2}$

Ans. D.

11. Let the pt on the first quadrant $\left(\sqrt{10}\cos\theta,\ \sqrt{8}\sin\theta\right)$

distance from the centre = 3

$\therefore\ 10\cos^2\theta+8\sin^2\theta=9$

$\Rightarrow\ 8+2\cos^2\theta=9$

$\Rightarrow\ \cos^2\theta=\frac{1}{2}\Rightarrow\cos\theta=+\frac{1}{\sqrt{2}}$ [$\because$ pt lie on lst quadrant]

$\therefore\ \theta=\frac{\pi}{4}$

Ans. B.

12. Let the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore$ Length of the transverse axis = 2a

vertex = (a, 0)

focus (ae, 0) $\therefore\ \left(\frac{ae}{2},\ 0\right)=(a,\ 0)$

centre = (0, 0) $\therefore\ \frac{ae}{2}=a$

$\Rightarrow\ e=2$

$\therefore\ e^2=4$

$\Rightarrow\ 1+\frac{b^2}{a^2}=4\ \Rightarrow\ b^2=3a^2$

$\therefore$ Equation : $\frac{x^2}{a^2}-\frac{y^2}{3a^2}=1$

Ans. D.

13. The distance between (8, 0) and (–8, 0) = 16 > 4.

$\therefore$ According to the defination of hyperbola the locus is a hyperbola.

Ans. D.

14. $m \in I$

$\left.\begin{array}{l}3x+4y=9\\ y=mx+1\end{array}\right\} \Rightarrow 3x+4(mx+1)=9$

$\Rightarrow (3+4m)x=5$

$\Rightarrow x=\dfrac{5}{3+4m} \in I$

$\therefore 3+4m=-5, -1, 1, 5$ when $m \in I$

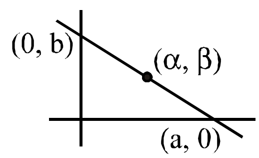
$\therefore$ m can take only two values

$m=-1, -2.$

Ans. B.

15. Let the equation of the line $\dfrac{x}{a}+\dfrac{y}{b}=1$

Its passes through (α, β)



$\therefore \dfrac{\alpha}{a}+\dfrac{\beta}{b}=1$

(α, β) is the mid points of (a, 0) and (0, b)

$\therefore \alpha=\dfrac{a}{2}$ and $\beta=\dfrac{b}{2}$

$\Rightarrow a=2\alpha, b=2\beta$

$\therefore$ Equation of the line $\dfrac{x}{2\alpha}+\dfrac{y}{2\beta}=1 \Rightarrow \dfrac{x}{\alpha}+\dfrac{y}{\beta}=2$

Ans. C.

16. $\left|z+\dfrac{2}{z}\right|=2$

Now, $\left|z+\dfrac{2}{z}\right| \le |z|+\dfrac{2}{|z|}$

$\Rightarrow |z|+\dfrac{2}{|z|} \ge 2$

$\Rightarrow |z|^2-2|z|+2 \ge 0$

$\Rightarrow |z|^2-2|z|+1 \le 3$

$\Rightarrow (|z|-1)^2 \le 3$

$\Rightarrow -\sqrt{3} \le (|z|-1) \le \sqrt{3}$

$\Rightarrow 1-\sqrt{3} \le |z| \le 1+\sqrt{3} \Rightarrow 1-\sqrt{3}$

$\because |z| \ge 0$

Ans. C.

17. $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}\ (x+iy)$

$\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)^{50} = x+iy$

$i^{50}\left(\frac{-1+i\sqrt{3}}{2}\right)^{50} = x+iy$

$\Rightarrow\ i^{50}.w^{50} = x+iy \Rightarrow\ x+iy = i^2\times w^2 \Rightarrow\ x+iy = -w^2\ \Rightarrow\ x+iy = \frac{1}{2}+\frac{i\sqrt{3}}{2}$

Ans. D.

18. $\frac{z-1}{z+1} = ik, \quad k\in R, \quad k\neq 0$

$\frac{2z}{-2} = \frac{ik+1}{ik-1}$; by comp.-div.

$z = \frac{1+ik}{1-ik}$

$|z| = \frac{|1+ik|}{|1-ik|} = \frac{\sqrt{1+k^2}}{\sqrt{1+k^2}} = 1$

Ans. B.

19. n(M) = 50 = No. of failed in maths.

n(P) = 45

n (B) = 40

$n(M\cap P)+n(M\cap B)+n(P\cap B)-3\,n(M\cap P\cap B) = 32$

We have to find $n(M\cap P\cap B)$

Total no of student = 100

$n(M\cup P\cup B) = 99$

$\Rightarrow\ n(M)+n(P)+n(B)-\{n(M\cap P)+n(M\cap B)+n(P\cap B)\}+n(M\cap P\cap B) = 99$

$\Rightarrow\ 50+45+40-\{32+3n(M\cap P\cap B)\}+n(M\cap P\cap B) = 99$

$\Rightarrow\ 135-32-2n(M\cap P\cap B) = 99$

$\Rightarrow\ 2n(M\cap P\cap B) = 4$

$\Rightarrow\ n(M\cap P\cap B) = 2$

Ans. C.

20. $\overline{1}\ \overline{2}\quad \overline{3}\ \overline{4}\quad \overline{5}\ \overline{6}$

two alphabet we can choose 26^2 ways.

and 1st number we can choose 9 ways.

next 3 numbers we can choose 10^3 ways.

Ans. D.

21. I - 2

R - 2

A - 2

T - 1

N - 1

O - 1

L - 1

Number of words $\frac{10!}{(2!)^3}$

Ans. A.

22. Required number of ways is which the order of speakers can be prepared

$= \frac{4!}{2!}$

$= \frac{24}{2}$

= 12 [Taking speakers P & Q as identical]

Ans. D.

23. No. of diagonals in a regular polygon

$= {}^{100}C_2 - 100$

$= \frac{100 \times 99}{2} - 100$

= 50 × 99 – 100

= 4950 – 100

= 4850

Ans. B.

24. ${}^nC_1$, ${}^nC_2$, ${}^nC_3$ are in A.P.

$\therefore\ \cancel{2} \cdot \frac{n\cancel{(n-1)}}{\cancel{2}} = \frac{n\cancel{(n-1)}(n-2)}{6} + n$

or, $n - 1 = \frac{n^2 - 3n + 2}{6} + 1$

or, $6n - 6 = n^2 - 3n + 2 + 6$

or, $n^2 - 9n + 14 = 0$

$n = 7$, $n = 2$ not acceptable.

$$\text{sum} = \frac{2^n}{2} = \frac{2^7}{2} = 2^6 = 64$$

Ans. B.

25. $f(x) = ax^2 + bx + c$, $g(x) = px^2 + qx + r$

Now $f(1) = g(1) \Rightarrow a + b + c = p + q + r \Rightarrow \boxed{(a-b)+(b-q)+(c-r)=0}$ ...... (1)

$f(2) = g(2) \Rightarrow 4a + 2b + c = 4p + 2q + r \Rightarrow \boxed{4(a-b)+2(b-q)+(c-r)=0}$

$\Rightarrow \boxed{3(a-b)+(b-q)=0}$ .... (2)

[using (1)]

$f(3) - g(3) = 2$

$\Rightarrow 9(a - p) + 3(b - q) + (c - r) = 2$

$\Rightarrow 8(a - p) + 2(b - q) = 2$ [using (1)]

$\Rightarrow 4(a - p) + (b - q) = 1$

$\Rightarrow (a - p) = 1$ (using (2))

Now, $f(4) - g(4) = 16(a - p) + 4(b - q) + (c - r)$

$= 15\ (a - p) + 3(b - q)$ (using (1))

$= 15.(1) + 3(-3)$ $\left\{\because \begin{array}{l} a - p = 1 \\ b - q = -3 \end{array}\right\}$

$= 15 - 9$

$= 6$

Ans. C.

26. $1 \times 1! + 2 \times 2! + 3 \times 3! + ... + n \times n!$

$= (n + 1)! - 1$

$1 \times 1! + 2 \times 2! + ... + 50 \times 50!$

$= 51! - 1$

Ans. B.

27. Six numbers are in A.P.

$a - 5d$, $a - 3d$, $a - d$ $a + d$, $a + 3d$, $a + 5d$

$6a = 3$

$$\therefore\ a = \frac{1}{2}$$

$$\frac{1}{2} - 5d = 4\left(\frac{1}{2} - d\right)$$

$$\frac{1}{2} - 5d = 2 - 4d$$

$$d = -\frac{3}{2}$$

Fifth term = a + 3d

$$= \frac{1}{2} + 3\left(-\frac{3}{2}\right)$$

$$= \frac{1}{2} - \frac{9}{2}$$

$$= \frac{-8}{2}$$

$= -4$

Ans. D.

28. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots.$

comparing, $nx = \frac{1}{3}$

$$\frac{nx(nx - x)}{2} = \frac{1}{3} \cdot \frac{3}{6}$$

$$\frac{\cancel{\frac{1}{3}}\left(\frac{1}{3} - x\right)}{\cancel{2}} = \frac{\cancel{1}}{3} \cdot \frac{1}{\cancel{2}}$$

$$\frac{1}{3} - x = 1$$

$$x = -\frac{2}{3}$$

$$n\left(-\frac{2}{3}\right) = \frac{1}{3}$$

$$n = -\frac{1}{2}$$

$$\therefore \left(1 - \frac{2}{3}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{3}\right)^{-\frac{1}{2}} = \sqrt{3}$$

Ans. B.

29. Let α be the common root

$$\alpha^2 + \alpha + a = 0$$
$$\alpha^2 + a\alpha + 1 = 0$$
$$(-)$$
$$\therefore\ \alpha(1-a) + a - 1 = 0$$

$$(1-a)(\alpha - 1) = 0$$

Either $a = 1$

or, $\alpha = 1$

Put $\alpha = 1$ in the 1st equation

$1 + 1 + a = 0$

$a = -2$

Put $a = 1$

$^2 + x + 1 = 0$

$x^2 + x + 1 = 0$

They have no real common root.

Put $a = -2$

$x^2 + x - 2 = 0$ & $x^2 - 2x + 1 = 0$ or, $x = 1$

they have a one real common root.

Ans. B.

30. Let A be the 1st term & r be the c.r.

$A \,.\, r^{p-1} = 64 = 2^6$

$A \,.\, r^{q-1} = 27 = 3^3$

$A \,.\, r^{R-1} = 36 = 2^2 \cdot 3^2$

Now, $2 = A^{\frac{1}{6}} \cdot r^{\frac{p-1}{6}}$

$$3 = A^{\frac{1}{3}} \cdot r^{\frac{q-1}{3}}$$

$$2 \cdot 3 = A^{\frac{1}{2}} \cdot r^{\frac{R-1}{2}}$$

$$A^{\frac{1}{6}} \cdot r^{\frac{p-1}{6}} \cdot A^{\frac{1}{3}} \cdot r^{\frac{q-1}{3}} = A^{\frac{1}{2}} \cdot r^{\frac{R-1}{2}}$$

$$r^{\frac{p-1}{6}+\frac{2q-2}{6}} = r^{\frac{3R-3}{6}}$$

$$\frac{p-1+2q-2}{6} = \frac{3R-3}{6}$$

$\therefore\ p + 2q = 3R$

Ans. C.

31. $y^2 + 4y = -4x - k$

$y^2 + 4y + 4 = -4x + 4 - k$

$(y+2)^2 = -4x - (k-4) = -4\left[x + \frac{k-4}{4}\right]$

$Y^2 = -4AX$

L.R. = 4A = 4 unit.

Ans. D.

32. Apply, $2(g_1g_2 + f_1f_2) = C_1 + C_2$

$2(1 \times 0 + k \cdot k) = 6 + k$

$\therefore \quad 2k^2 = 6 + k$

or, $\quad 2k^2 - k - 6 = 0$

or, $\quad 2k^2 - 4k + 3k - 6 = 0$

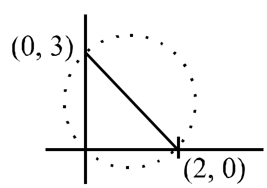
or, $\quad 2k(k-2) + 3(k-2) = 0$

or, $\quad (k-2)(2k+3) = 0$

$k = 2, \ -\frac{3}{2}$

Ans. A.

33. Equation of circle is $(x-2)(x-0) + (y-0)(y-3) = 0$



$x^2 - 2x + y^2 - 3y = 0$

$x^2 + y^2 - 2x - 3y = 0$

$4k^2 + 9k^2 - 4k - 9k = 0$

$13k^2 = 13k$

$\therefore \quad k = 0, \ 1$

$k = 1$

Ans. C.

34. $x = \dfrac{ab\cos\beta - ab\cos\alpha}{b-a}$

$$= \frac{ab}{b-a} = (\cos\beta - \cos\alpha)$$

$$= \frac{ab}{b-a}\left[2\cdot\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}\right]$$

$$y = \frac{ab\sin\beta - ab\sin\alpha}{b-a}$$

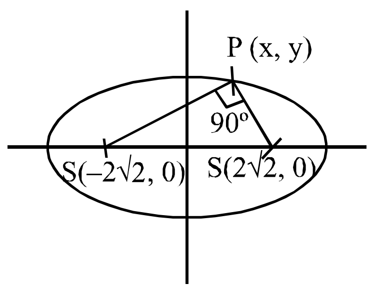
$$= \frac{ab}{b-a}\left[2\cdot\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\beta-\alpha}{2}\right]$$

$$\frac{x}{y} = -\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$$

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0$$

Ans. A.

35. $\dfrac{x^2}{9} + \dfrac{y^2}{1} = 1$



$a = 3,\ b = 1$

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$e = \frac{2\sqrt{2}}{3}$$

$$ae = 2\sqrt{2}$$

$$m_{ps} \times m_{ps'} = -1$$

$$\frac{y-0}{x+2\sqrt{2}} \times \frac{y-0}{x-2\sqrt{2}} = -1$$

$$\frac{y^2}{x^2-8} = -1$$

$$y^2 = -x^2 + 8$$

$$x^2 + y^2 = 8$$

Ans. D.

36. $\frac{dy}{dx} = \frac{x+y+1}{2(x+y)+1}$

Let $x + y = z$

$1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dy}{dx} = \frac{dz}{dx} - 1$

$\frac{dz}{dx} - 1 = \frac{z+1}{2z+1}$

$\frac{dz}{dx} = \frac{z+1}{2z+1} + 1$

$= \frac{z+1+2z+1}{2z+1}$

$= \frac{3z+2}{2z+1}$

or, $\frac{2z+1}{3z+1} dz = dx$

or, $\frac{2}{3}\int \frac{3z+\frac{3}{2}}{3z+1} dz = \int dx + c$

or, $\frac{2}{3}\int \frac{3z+2-\frac{1}{2}}{3z+2} dz = x + c$

or, $\frac{2}{3}z - \frac{1}{3}\cdot\frac{1}{3}\int \frac{d(3z+2)}{3z+2} = x + c$

or, $\frac{2}{3}z - \frac{1}{9}\log|3z+2| = x + c$

or, $\frac{2}{3}(x+y) - \frac{1}{9}\log|3x+3y+2| = x + c$

or, $6x + 6y - \log|3x+3y+2| = 9x + c'$

or, $3x - 6y + \log|3x+3y+2| = c$

Ans. D.

37. $$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2 + (\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[(\cos x + \sin x) + (\cos x - \sin x)\right]dx$$

$$= 2\sin x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 2\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$$

$$= 2\left[1 - \frac{1}{2}\right]$$

$$= 2 \cdot \frac{1}{2} = 1$$

Ans. D.

38. $$I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \left(\frac{\sin x}{\cos x}\right)^{101}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{101}}{(\cos x)^{101} + (\sin x)^{101}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{101}}{(\sin x)^{101} + (\cos x)^{101}} dx \qquad \left[\text{Apply} \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$\therefore \quad 2I = \int_0^{\frac{\pi}{2}} dx = x\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

Ans. D.

39. $\frac{dy}{dx}+\frac{y}{3x\log_e x}=\frac{2}{3x}$

$I = e^{\int \frac{dx}{3x\log_e x}}$

$= e^{\frac{1}{3}\int \frac{d(\log_e x)}{\log_e x}}$

$= e^{\frac{1}{3}\log(\log_e x)}$

$= e^{\log_e (\log_e x)^{\frac{1}{3}}}$

$= (\log_e x)^{\frac{1}{3}}$

Ans. D.

40. $\frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$

$\sin x+1=2\cos^2 x=2(1-\sin^2 x)$

$\sin x+1=2(1+\sin x)(1-\sin x)$

$(1+\sin x)[1-2(1-\sin x)]=0$

$(1+\sin x)(1-2+2\sin x)=0$

$(1+\sin x)(2\sin x-1)=0$

$\sin x=-1$

$\sin x=\frac{1}{2}$

$\therefore \quad x=\frac{\pi}{6},\frac{5\pi}{6}$

Ans. C.

41. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

$= \int_0^{\pi/4} \frac{(\sin x + \cos x)dx}{4 - (\sin x - \cos x)^2}$

$= \int_0^1 \frac{dz}{2^2 - z^2}$ Putting $\sin x - \cos x = z \Rightarrow (\cos x + \sin x)dx = dz$, $\begin{array}{c|c|c} x & 0 & \frac{\pi}{4} \\ \hline z & 1 & 0 \end{array}$

$= \frac{1}{2 \times 2}\left[\log\left(\frac{2+z}{2-z}\right)\right]_0^1$

$= \frac{1}{4}[\log(3) - \log(1)]$

$= \frac{1}{4}\log(3)$

Ans. D.

42. $y = \left(\frac{3^x - 1}{3^x + 1}\right)\sin x + \log_e(1+x)$

$= \frac{3^x + 1 - 2}{3^x + 1}(\sin x) + \log_e(1+x)$

$= \left(1 - \frac{2}{3^x + 1}\right)\sin x + \log_e(1+x)$

$= \sin x - \frac{2\sin x}{3^x + 1} + \log(1+x)$

$\frac{dy}{dx} = \cos x - 2\frac{(3^x + 1)\cdot \cos x - (\sin x)3^x \cdot \log_e 3}{(3^x + 1)^2} + \frac{1}{1+x}$

$\therefore \left.\frac{dy}{dx}\right|_{x=0} = 1 - 2\frac{2-0}{2^2} + \frac{1}{1+0}$

$= 1 - 1 + 1 = 1$

Ans. A.

43. $f(x)=\frac{x}{8}+\frac{2}{x}$

$f'(x)=\frac{1}{8}-\frac{2}{x^2}$

For max & min.

$f'(x)=0$

$\frac{1}{8}-\frac{2}{x^2}=0$

$\frac{1}{8}=\frac{2}{x^2}$

$x^2 = 16$

$x = +4 \ [x \in [1, 6]]$

$f'(4^-)>0$

$f'(4^+)<0$

at $x = 4$ $f(x)$ is max.

$f(4)=\frac{4}{8}+\frac{2}{4}=\frac{1}{2}+\frac{1}{2}=1$

Ans. A.

44. Exp. $=\frac{d}{dx}\left\{\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)\right\}$

$=\frac{d}{dx}\left\{\tan^{-1}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right\}$

$=\frac{d}{dx}\left(\tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

$=\frac{d}{dx}\left\{\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right\}$

$\Rightarrow -\frac{\pi}{2}<x<\frac{3\pi}{2}$

$\Rightarrow \ -\frac{\pi}{4} < \frac{x}{2} < \frac{3\pi}{4}$

$\Rightarrow \ \frac{\pi}{4} > -\frac{x}{2} > -\frac{3\pi}{4}$

$\Rightarrow \ \frac{\pi}{2} > \frac{\pi}{4} - \frac{\pi}{2} > -\frac{\pi}{2}$

$\therefore$ Exp. $= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = 0 - \frac{1}{2}$

Ans. B.

45. $\int_{-2}^{2}(1+2\sin x)e^{|x|}dx$

$= \int_{-2}^{2} e^{|x|}dx + 2\int_{-2}^{2}\sin x e^{|x|}dx$

$= 2 \times \int_{0}^{2} e^{|x|}dx + 0$

$= 2e^{x}\Big|_{0}^{2}$

$= 2(e^2 - 1)$

Ans. C.

46. $x^2 + px + q = 0 \ \rightarrow$ roots are $\alpha + \sqrt{\beta}$ and $\alpha - \sqrt{\beta}$

$\therefore 2\alpha = -p \ \Rightarrow \boxed{p = -2\alpha} \Rightarrow \alpha = -\frac{p}{2}$

$\alpha^2 - \beta = q \Rightarrow \beta = \alpha^2 - q = \frac{p^2}{4} - q = \left(\frac{p^2 - 4q}{4}\right) \ \Rightarrow \boxed{\left(p^2 - 4q\right) = 4\beta}$

Now, $4\beta\left(4\alpha^2x^2 - 8\alpha x\right) - 16\left(\alpha^2 - \beta\right) = 0$

$\Rightarrow 16\alpha^2\beta x^2 - 32\alpha\beta x - 16\alpha^2 + 16\beta = 0$

$\Rightarrow x^2 - \frac{2}{\alpha}x - \frac{1}{\beta} + \frac{1}{\alpha^2} = 0$

$\therefore$ Sum of the roots $= \frac{2}{\alpha} = \left(\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}\right) + \left(\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}\right)$

Product of the roots $= \frac{1}{\alpha^2} - \frac{1}{\beta} = \left(\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}\right) \cdot \left(\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}\right)$

Ans. A.

47. $x^2 + 2x - 1 = 2$

$x^2 + 2x = 3$

$x^2 + 2x + 1 = 4$

$(x+1)^2 = 2^2$

$x + 1 = \pm 2$

$x = 1, -3$

Ans. C.

48. $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_n}{n+1}$

$$\int_0^1 (1+x)^n dx = \int_0^1 C_0 dx + \int_0^1 C_1 x dx + \int_0^1 C_2 x^2 dx + \cdots + \int_0^1 C_n x^n dx$$

$$\left.\frac{(1+x)^{n+1}}{n+1}\right|_0^1 = C_0 \cdot x\Big|_0^1 + C_1 \left.\frac{x^2}{2}\right|_0^1 + C_2 \left.\frac{x^3}{3}\right|_0^1 + \cdots + C_n \left.\frac{x^{n+1}}{n+1}\right|_0^1$$

$$\frac{2^{n+1}-1}{n+1} = \frac{C_0}{1} + \frac{C_1}{2} + \cdots + \frac{C_n}{n+1}$$

Ans. A.

49. $\sum_{r=2}^{\infty} \frac{(r-1)r}{2 \cdot r!}$

$$= \sum_{r=2}^{\infty} \frac{1}{2} \cdot \frac{1}{(r-2)!}$$

$$= \frac{1}{2} \cdot \sum_{r=2}^{\infty} \frac{1}{(r-2)!}$$

$$= \frac{1}{2} \cdot e$$

Ans. C.

50. $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$

$$P^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$$

$Q = PP^T$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+1 & 1+6+1 \\ 1+6+1 & 1+9+1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ 8 & 11 \end{bmatrix}$$

$|Q| = 66 - 64 = 2$

Ans. A.

51. $1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33$

$$15 \,\big)\, 33 \,\big(\, 2$$
$$\underline{30}$$
$$3$$

Required remainder = 3

Ans. B.

52. Putting $P = Q = R = \dfrac{\pi}{3}$

$$\begin{vmatrix} -1 & \cos R & \cos Q \\ \cos R & -1 & \cos P \\ \cos Q & \cos P & -1 \end{vmatrix} = \begin{vmatrix} -1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{vmatrix}$$

$$= -\left(1 - \frac{1}{4}\right) - \frac{1}{2}\left(-\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{4} + \frac{1}{2}\right)$$

$$= -\frac{3}{4} + \frac{3}{4} = 0$$

Ans. B.

53. $(1-\alpha)x + 3y + 5z = 0$

$5x + (1-\alpha)y + 3z = 0$

$3x + 5y + (1-\alpha)z = 0$

$$\therefore \begin{vmatrix} 1-\alpha & 3 & 5 \\ 5 & 1-\alpha & 3 \\ 3 & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow\ (1-\alpha)^3 + 27 + 125 - 45(1-\alpha) \qquad \left[\because \quad \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = \left(a^3 + b^3 + c^3 - 3abc\right)\right]$$

$$\Rightarrow\ 1 - \alpha^3 - 3\alpha(1-\alpha) + 152 - 45 + 45\alpha = 0$$

$$\Rightarrow\ 3\alpha^2 - 3\alpha - \alpha^3 + 108 + 45\alpha = 0$$

$$\Rightarrow\ -\alpha^3 + 3\alpha^2 + 42\alpha + 108 = 0$$

$$\Rightarrow\ \alpha^3 - 3\alpha^2 - 42\alpha - 108 = 0$$

It has one real solution and two imaginary solution.

Ans. A.

54. $n(A) = 4$

$n(B) = 7$

no. of mappings $= {}^7p_4 = \dfrac{7!}{3!}$

$$= \frac{\cancel{3!} \times 4 \times 5 \times 6 \times 7}{\cancel{3!}} = 20 \times 6 \times 7 = 840$$

Ans. D.

55. $2^{10} = C_0 + C_1 + \ldots + C_{10}$

$$= C_0 + C_2 + C_4 + C_6 + C_8 + C_{10} = 2^9$$

$2^7 = d_0 + d_1 + \ldots + d_7$

$= d_1 + d_3 + d_5 + d_7 = 2^6$

$\therefore\ \dfrac{P}{Q} = \dfrac{2^9}{2^6} = 2^3 = 8$

Ans. B.

56. Total no. of possible outcomes $= {}^{104}C_{26}$, which are equally likely.

Number of casses that the player gets all distinct cards $= \left({}^2C_1\right)^{26} \times {}^{52}C_{26}$

$$= 2^{26} \times {}^{52}C_{26}$$

$\therefore$ Required probability $= \dfrac{2^{26} \times {}^{52}C_{26}}{{}^{104}C_{26}}$

Ans. D.

57. $\text{caseI} \to 2R, 1W$

$\text{caseII} \to 1R, 2W$

$\begin{array}{|c|} 8R \\ 5W \\ \hline \end{array}$

$$\frac{{}^8C_2 \times {}^5C_1 + {}^5C_2 \times {}^8C_1}{{}^{13}C_3}$$

$$= \frac{\frac{8.7}{2} \times 5 + \frac{5.4}{2} \times 8}{\frac{13 \times 12 \times 11}{6}}$$

$$= \frac{(140+80) \times \cancel{6}}{13 \times \cancel{12}_2 \times 11} = \frac{220}{13 \times 22} = \frac{10}{13}$$

Ans. D.

58. Let X = the event that outcame is head

Given that

A	B
H/T	H/H

$P(A) = \frac{3}{4}$ $\qquad P = (B) = \frac{1}{4}$

$$\therefore \quad P\left(\frac{B}{X}\right) = \frac{1/4 \times 1}{\frac{3}{4} \times \frac{1}{2} + \frac{1}{4} \times 1} = \frac{1/4}{\frac{3}{8} + \frac{1}{4}}$$

$$= \frac{1/4}{\frac{3+2}{8}} = \frac{\frac{1}{\cancel{4}}}{\frac{5}{\cancel{8}_2}} = \frac{2}{5}$$

Ans. B.

59. $f : R \to R$

$g : R \to R$

$f(x) = x^2 + 2x - 3 = x^2 + 2x + 1 - 4 = (x+1)^2 - 4$

$g(x) = x + 1$

$f[g(x)] = f(x+1)$

$= (x+2)^2 - 4$

$= x^2 + 4x + 4 - 4 = x^2 + 4x$

$g[f(x)] = g\left[(x+1)^2 - 4\right]$

$= (x+1)^2 - 4 + 1 = (x+1)^2 - 3$

$= x^2 + 2x + 1 - 3 = x^2 + 2x - 2$

$\therefore \quad x^2 + 4x = x^2 + 2x - 2$

$\Rightarrow 2x = -2 \Rightarrow x = -1$

Ans. A.

60. $2b = a + c$

$ax^2 - 2bx + c = 0$

$ax^2 - (a + c)x + c = 0$

$ax^2 - ax - cx + c = 0$

$ax(x - 1) - c(x - 1) = 0$

$(ax - c)\,(x - 1) = 0$

$x = \frac{c}{a},\;\; x = 1$

Ans. A.

61. $x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$

$\frac{1}{x}\cdot\frac{dx}{dy} = \frac{1}{y^2} - \frac{1}{y}\log x$ $\qquad \log x = t$

$\frac{1}{x}\cdot\frac{dx}{dy} = \frac{dt}{dy}$

or, $\frac{dt}{dy} = \frac{1}{y^2} - \frac{t}{y}$

or, $\frac{dt}{dy} + \frac{t}{y} = \frac{1}{y^2}$

$I.F. = e^{\int\frac{1}{y}dy} = e^{\log y} = y$

$\therefore \int d(t.y) = \int \frac{1}{y}dy + k$

$t.y = \log y + k$

$y.\log x = \log y + k$

$1 \times 0 = \log 1 + k$

$\therefore \quad k = 0$

$\therefore \quad y.\log_e x = \log_e y \quad \therefore \; y = x^y$

Ans. B.

62. Solving, $\sin^{-1}x + x(1 - x) = \sin^{-1}x - x(1 - x)$

or, $2x(1 - x) = 0$

$\therefore \; x = 0,\; 1$

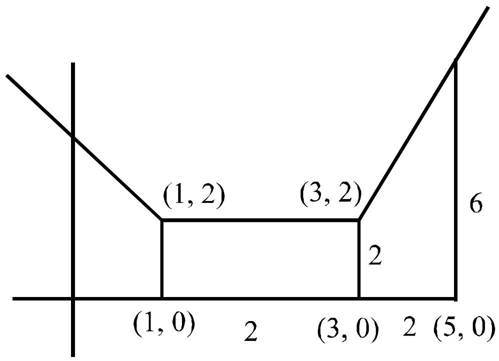
$\therefore \quad$ Required area $= \int_0^1 2x(1 - x)dx$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2 \times \frac{1}{6} = \frac{1}{3} \text{ sq. unit.}$$

Ans. C.

63.



Area $= 2 \times 2 + \frac{1}{2}(2+6).2$

$= 4 + \frac{1}{2} \times 8 \times 2$

$= 4 + 8 = 12$ sq. unit

Ans. C.

64. $h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x)$

$h''(x) = f''(x) - g''(x)$

$h''(x) = 0$

$\therefore h'(x) = c$

$f'(x) - g'(x) = c$

$f'(1) - g'(1) = c$

$\therefore\ 4 - 6 = c$

$\therefore\ c = -2$

$f'(x) - g'(x) = -2$

$f(x) - g(x) = -2x + c'$

$f(2)-g(2)=-2.2+c'$

$3-9=-4+c'$

$-6=-4+c \Rightarrow c'=-2$

$f(x)-g(x)=-2x-2$

$f(1)-g(1)=-2.1-2$

$=-4$

Ans. B.

65. $\int_{-1}^{1}(|x|-2[x])dx$

$$=\int_{-1}^{1}|x|dx-2\int_{-1}^{1}[x]dx$$

$$=\int_{-1}^{0}(-x)dx+\int_{0}^{1}xdx-2\left[\int_{-1}^{0}(-1)dx+0\right]$$

$$=-\left[\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{x^2}{2}\right]_{0}^{1}+2[x]_{-1}^{0}$$

$$=-\left(-\frac{1}{2}\right)+\frac{1}{2}+2$$

$=3$

Ans. A.

66. $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$

$\therefore$ z lies on a circle

Ans. A.

67. a, b, c are in G.P.

$\therefore\ b^2 = ac \ \therefore 2\log b=\log a+\log c$ .... (1)

Now, $a^p=b^q=c^r$

$$\therefore \quad p\log a = q\log b = r\log c$$

$$\therefore \frac{\log a}{\log b} = \frac{q}{p}, \quad \frac{\log c}{\log b} = \frac{q}{r}$$

From (1)

$$\therefore \frac{q}{p} + \frac{q}{r} = 2$$

$$\Rightarrow \frac{1}{p} + \frac{1}{r} = \frac{2}{q}$$

$\therefore$ p, q, r in H.P.

Ans. C.

68. $$\sum_{k=1}^{\infty} \frac{(-1)^k}{s_k}$$

$$= \sum_{k=1}^{\infty} \frac{\frac{(-1)^k}{k}}{1 - \frac{k}{k+1}}$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)}$$

$$= \sum_{k=1}^{\infty} (-1)^k \left[\frac{1}{k} - \frac{1}{k+1}\right]$$

$$= -1 + \frac{1}{2} + \frac{1}{2} - \frac{1}{3} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{5} - \frac{1}{5} + \frac{1}{6} + \frac{1}{6} \ldots\ldots.$$

$$= -\frac{2}{3} + \frac{2}{4} - \frac{2}{5} + \frac{2}{6} \ldots\ldots\ldots$$

$$= 2\left[-\frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} \ldots\ldots.\right]$$

$$= 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} \ldots\ldots\right] - 1$$

$$= -2\left[-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}.....\right]-1$$

$$= -2\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}.....\right]+1$$

$= -2\ ln2 + 1$

$= 1 - ln4$

Ans. D.

69. $\therefore$ O lies between the roots

$\therefore$ $f(0) < 0$

$\Rightarrow a^2 - 4a < 0$

$\Rightarrow a(a-4) < 0$

$\Rightarrow 0 < a < 4$

Ans. B.

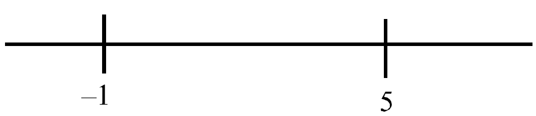
70. $\log_e\left(x^2-16\right) \le \log_e(4x-11)$

$x^2 - 16 \le 4x - 11$

$x^2 - 4x - 5 \le 0$

$x^2 - 5x + x - 5 \le 0$

$(x-5)(x+1) \le 0$



$-1 \le x \le 5$

$4x - 11 > 0 \;\Rightarrow x > \frac{11}{4}$

$x^2 - 16 > 0 \;\Rightarrow x > 16$

$\Rightarrow x > 4,\ x < -4$

Req. Answer $4 < x \le 5$

Ans. A.

71. $1+(1+x)+(1+x)^2+....+(1+x)^{20}$

$= 1.\dfrac{(1+x)^{21}-1}{(1+x)-1}$

$= \dfrac{(1+x)^{21}-1}{x}$

$\therefore$ Required coefficient = ${}^{21}C_{11}$

[From N^r found coefficien of x^{11}]

Ans. C.

72. To get infinite no of solution.

u = 3

Now $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix} = 0$

$\lambda\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0$

or, $\lambda(-1+2)+1(+2-1)+1(1-1)=0$

or, $\lambda+1=0$

$\therefore$ $\lambda = -1$

Ans. B.

73. $P(A^C) = 0.3$ | $P(B) = 0.4$

$P(A) = 0.7$ | $P(B^C) = 0.6$

$P(A \cap B^C) = 0.5$

$P(B / A \bigcup B^C)$

$= \dfrac{P(B \bigcap (A \bigcup B^C))}{P(A \cup B^C)}$

$= \dfrac{P((B \cap A) \cup \phi)}{P(A \cup B^C)}$

$$= \frac{P(B \cap A)}{P(A \cup B^C)}$$

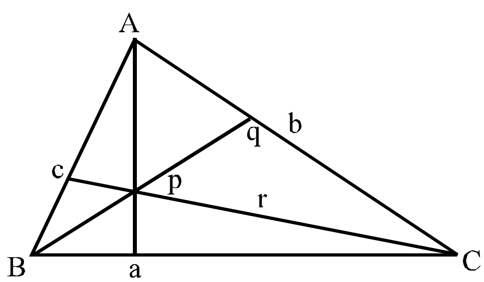
$$= \frac{P(A) - P(A \cap B^C)}{P(A \cup B^C)}$$

$$= \frac{0.7 - 0.5}{P(A) + P(B^C) - P(A \cap B^C)}$$

$$= \frac{0.2}{0.7 + 0.6 - 0.5} = \frac{1}{4}$$

Ans. A.

74.



$$S = \frac{1}{2}ap = \frac{1}{2}bq = \frac{1}{2}cr,$$

where a, b, c are the sides of the triangle

$\therefore$ a + b + c = 2t

Now $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{a}{2s} + \frac{b}{2s} + \frac{c}{2s} = \frac{2t}{2s} = \frac{t}{s}$

Ans. B.

75. $x^2 + y^2 = 4$

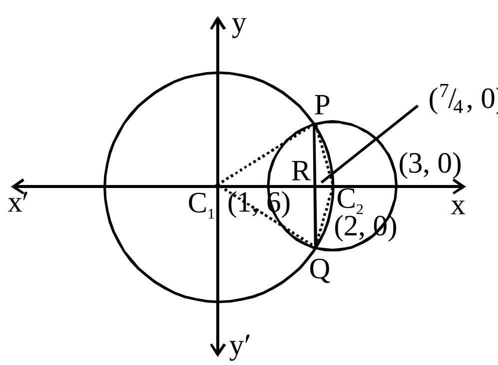
$(x-2)^2 + y^2 = 1$

equation of common chord.

$x^2 + y^2 - 4 - \left[(x-2)^2 + y^2 - 1\right] = 0$

$x^2 + y^2 - 4 - \left[x^2 - 4x + 4 + y^2 - 1\right] = 0$

$x^2 + y^2 - 4 - x^2 - y^2 + 4x - 3 = 0$



$4x = 7$

$x = \frac{7}{4}$

Here $R = \left(\frac{7}{4},\ 0\right)$

$C_1Q = 2$

$RQ = \sqrt{4 - \frac{49}{16}} = \frac{\sqrt{15}}{4}$

$\therefore\ PQ = \frac{\sqrt{15}}{2}$

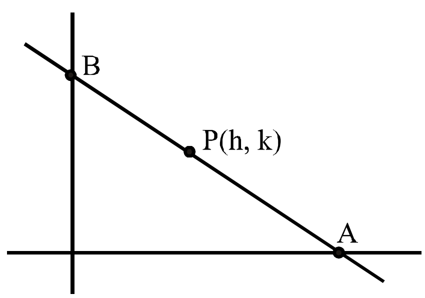
$\Delta C_1PQ = \frac{1}{2}.\frac{7}{4}.\frac{\sqrt{15}}{2} = \frac{7.\sqrt{15}}{16}$

$\Delta C_2PQ = \frac{1}{2}.\frac{1}{4}.\frac{\sqrt{15}}{2} = \frac{\sqrt{15}}{16}$

$\therefore\ \frac{\Delta C_1PQ}{\Delta C_2PQ} = \frac{7}{1}$

Ans. C.

76.



The point of intersectgion of the given lines is $\left(\frac{4}{3},\frac{4}{3}\right)$.

Any line through this point is $y - \frac{4}{3} = m\left(x - \frac{4}{3}\right)$

Then coordinates of A & B are $A\left(\frac{4(m-1)}{3m},0\right)$ and $B\left(0,\ \frac{4(1-m)}{3}\right)$

Let P (h, k) be the mid-point of AB. Then $h = \frac{2(m-1)}{3m}$ and $k = \frac{2(1-m)}{3}$.

Eliminating 'm' from the above two repations, we get $2(h+k) = 3hk$.

$\therefore$ The required locus is $2(x+y) = 3xy$.

Ans. B.

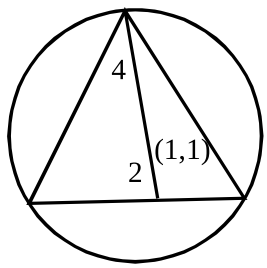
77. Any chord of a parabola $y^2 = 4ax$, which subtends right angle at the vertex, always pass through a fixed point (4a, 0) on the axis of the parabola.

$\therefore$ In this case R is (4, 0)

$\therefore$ The distance of R from the vertex is 4.

Ans. C.

78.



The incentre, circumcentre, centroid of an equlateral triangle are same.

$\therefore$ Inradius $= \dfrac{|3(1)+4(1)+3|}{\sqrt{3^2+4^2}} = 2$

$\therefore$ Circumradius $= 4$

$\therefore$ Circumcircle is $(x-1)^2 + (y-1)^2 = 4^2$

i.e., $x^2 + y^2 - 2x - 2y - 14 = 0$

Ans. B.

79. Let $L = \lim\limits_{n\to\infty} \left(\dfrac{n!}{n^n}\right)^{\frac{1}{n}}$

$\therefore \log L = \lim\limits_{n\to\infty} \dfrac{1}{n} \log\left(\dfrac{1}{n}\cdot\dfrac{2}{n}\ldots.\dfrac{n}{n}\right)$

$= \lim\limits_{n\to\infty} \dfrac{1}{n}\left[\log\dfrac{1}{n} + \log\dfrac{2}{n} + \ldots. + \log\dfrac{n}{a}\right]$

$= \lim\limits_{n\to\infty} \dfrac{1}{n}\sum\limits_{r=1}^{n} \log\left(\dfrac{r}{n}\right)$

$= \int_0^1 \log x \, dx$

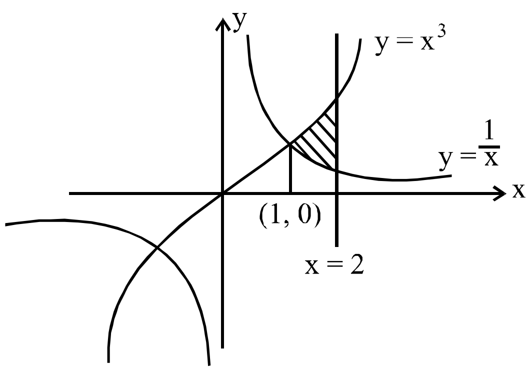
$= \left[(x\log x - x)\right]_0^1$

$= -1$

$\therefore L = e^{-1} = \dfrac{1}{e}$

Ans. D.

80.



$\therefore$ Area $= \int_1^2 \left(x^3 - \frac{1}{x}\right)dx$

$= \left[\frac{x^4}{4} - \log x\right]_1^2$

$= \left(\frac{16}{4} - \frac{1}{4}\right) - (\log 2 - \log 1)$

$= \left(\frac{15}{4} - \log 2\right)$ sq. unit.

Ans. D.

PC 2012 49528

MULTIPLE CHOICE QUESTIONS

Duration : Two Hours **Maximum Marks : 100**

SUBJECT : PHYSICS

[Q. 1 to 30 carry one mark each]

1. A train approaching a railway platform with a speed of 20 ms^{-1} starts blowing the whistle. Speed of sound in air is 340 ms^{-1}. If the frequency of the emitted sound from the whistle is 640 Hz, the frequency of sound to a person standing on the platform will appear to be

   A. 600 Hz B. 640 Hz C. 680 Hz D. 720 Hz

2. A straight wire of length 2 m carries a current of 10A If this wire is placed in a uniform magnetic field of 0.15 T making an angle of 45° with the magnetic field, the applied force on the wire will be

   A. 1.5 N B. 3 N C. $3\sqrt{2}$ N D. $\frac{3}{\sqrt{2}}$N

3. What is the phase difference between two simple harmonic motions represented by $x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = A\cos(\omega t)$?

   A. $\frac{\pi}{6}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{2\pi}{3}$

4. Heat is produced at a rate given by H in a resistor when it is connected across a supply of voltage V. If now the resistance of the resistor is doubled and the supply voltage is made $\frac{V}{3}$ then the rate of production of heat in the resistor will be

   A. $\frac{H}{18}$ B. $\frac{H}{9}$ C. 6H D. 18H

5. Two elements A and B with atomic numbers Z_A and Z_B are used to produce characteristic x-rays with frequencies v_A and v_B respectively. If $Z_A : Z_B$ = 1:2, then $v_A : v_B$ will be

   A. $1:\sqrt{2}$ B. 1 : 8 C. 4 : 1 D. 1 : 4

6. The de Broglie wavelength of an electron moving with a velocity $\frac{C}{2}$ (C = velocity of light in vacuum) is equal to the wavelength of a photon. The ratio of the kinetic energies of electron and photon is

   A. 1 : 4 B. 1 : 2 C. 1 : 1 D. 2 : 1

7. Two infinite parallel metal planes, contain electric charges with charge densities $+\sigma$ and $-\sigma$ respectively and they are separated by a small distance in air. If the permittivity of air is ε_0 then the magnitude of the field between the two planes with its direction will be

   A. $\frac{\sigma}{\varepsilon_0}$ towards the positively charged plane B. $\frac{\sigma}{\varepsilon_0}$ towards the negatively charged plane

   C. $\frac{\sigma}{(2\varepsilon_0)}$ towards the positively charged plane D. 0 and towards any direction.

8. A box of mass 2 kg is placed on the roof of a car. The box would remain stationary until the car attains a maximum acceleration. Coefficient of static friction between the box and the roof of the car is 0.2 and $g = 10\ ms^{-2}$.

   This maximum acceleration of the car, for the box to remain stationary, is

   A. $8\ ms^{-2}$ B. $6\ ms^{-2}$ C. $4\ ms^{-2}$ D. $2\ ms^{-2}$

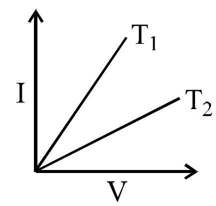
9. The decimal number equivalent to a binary number 1011001 is

   A. 13 B. 17 C. 89 D. 178

10. The frequency of the first overtone of a closed pipe of length l_1 is equal to that of the first overtone of an open pipe of length l_2. The ratio of their lengths $(l_1 : l_2)$ is

    A. 2 : 3 B. 4 : 5 C. 3 : 5 D. 3 : 4

11. The I-V characteristics of a metal wire at two different temperatures (T_1 and T_2) are given in the adjoining figure. Here, we can conclude that



    A. $T_1 > T_2$ B. $T_1 < T_2$ C. $T_1 = T_2$ D. $T_1 = 2T_2$

12. In a slide calipers, (m + 1) number of vernier divisions is equal to m number of smallest main scale divisions. If d unit is the magnitude of the smallest main scale division, then the magnitude of the vernier constant is

    A. $\frac{d}{(m+1)}$ unit B. $\frac{d}{m}$ unit C. $\frac{md}{(m+1)}$ unit D. $\frac{(m+1)d}{m}$ unit

13. From the top of a tower, 80 m high from the ground, a stone is thrown in the horizontal direction with a velocity of $8\ ms^{-1}$. The stone reaches the ground after a time 't' and falls at a distance of 'd' from the foot of the tower.

    Assuming $g = 10ms^{-2}$, the time t and distance d are given respectively by

    A. 6 s, 64 m B. 6 s, 48 m C. 4 s, 32 m D. 4s, 16 m

14. A wheatstone bridge has the resistances 10Ω, 10Ω, 10Ω and 30Ω in its four arms. What resistance joined in parallel to the 30Ω resistance will bring it to the balanced condition?

    A. 2Ω B. 5Ω C. 10Ω D. 15Ω

15. An electric bulb marked as 50 W-200 V is connected across a 100 V supply. The present power of the bulb is

    A. 37.5 W B. 25 W C. 12.5 W D. 10 W

16. In a mercury thermometer the ice point (lower fixed point) is marked as 10° and the steam point (upper fixed point) is marked as 130°. At 40°C temperature, what will this thermometer read?

    A. 78° B. 66° C. 62° D. 58°

17. The magnetic flux linked with a coil satisfies the relation $\phi = 4t^2 + 6t + 9$ Wb, where t is the time in second. The e.m.f. induced in the coil at t = 2 second is

    A. 22 V B. 18 V C. 16 V D. 40 V

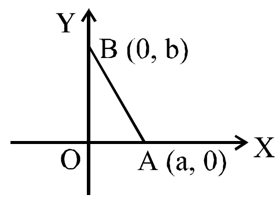
18. Water is flowing through a very narrow tube. The velocity of water below which the flow remains a streamline flow is known as

A. Relative velocity B. Terminal velocity C. Critical velocity D. Particle velocity

19. If the velocity of light in vacuum is 3×10^8 ms^{-1}, the time taken (in nanosecond) to travel through a glass plate of thickness 10 cm and refractive index 1.5 is

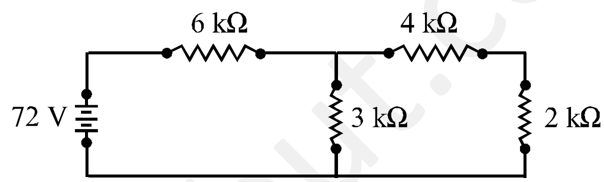
A. 0.5 B. 1.0 C. 2.0 D. 3.0

20. A charge +q is placed at the origin O of X-Y axes as shown in the figure. The work done in taking a charge Q from A to B along the straight line AB is



A. $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{a-b}{ab}\right)$ B. $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$ C. $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{b}{a^2}-\frac{1}{b}\right)$ D. $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{a}{b^2}-\frac{1}{b}\right)$

21. What current will flow through the $2k\Omega$ resistor in the circuit shown in the figure?



A. 3 mA B. 6 mA C. 12 mA D. 36 mA

22. In a region, the intensity of an electric field is given by $\vec{E} = 2\hat{i} + 3\hat{j} + \hat{k}$ in NC^{-1}. The electric flux through a surface $\vec{S} = 10\hat{i}$ m^2 in the region is

A. 5 Nm^2C^{-1} B. 10 Nm^2C^{-1} C. 15 Nm^2C^{-1} D. 20 Nm^2C^{-1}

23. The dimension of angular momentum is

A. $M^0L^1T^{-1}$ B. $M^1L^2T^{-2}$ C. $M^1L^2T^{-1}$ D. $M^2L^1T^{-2}$

24. If $\vec{A} = \vec{B} + \vec{C}$ and $\vec{A}$, $\vec{B}$, $\vec{C}$ have scalar magnitudes of 5, 4, 3 units respectively then the angle between $\vec{A}$ and $\vec{C}$ is

A. $\cos^{-1}\left(\frac{3}{5}\right)$ B. $\cos^{-1}\left(\frac{4}{5}\right)$ C. $\frac{\pi}{2}$ D. $\sin^{-1}\left(\frac{3}{4}\right)$

25. A particle is travelling along a straight line OX. The distance x (in metres) of the particle from O at a time t is given by $x = 37 + 27t - t^3$ where t is time in seconds. The distance of the particle from O when it comes to rest is

A. 81 m B. 91 m C. 101 m D. 111 m

26. A particle is projected from the ground with a kinetic energy E at an angle of 60° with the horizontal. Its kinetic energy at the highest point of its motion will be

A. $\frac{E}{\sqrt{2}}$ B. $\frac{E}{2}$ C. $\frac{E}{4}$ D. $\frac{E}{8}$

27. A bullet on penetrating 30 cm into its target loses it's velocity by 50%. What additional distance will it penetrate into the target before it comes to rest?

A. 30 cm B. 20 cm C. 10 cm D. 5 cm.

28. When a spring is stretched by 10 cm, the potential energy stored is E. When the spring is stretched by 10 cm more, the potential energy stored in the spring becomes

A. 2E B. 4E C. 6E D. 10E

29. Average distance of the Earth from the Sun is L_1. If one year of the Earth = D days, one year of another planet whose average distance from the Sun is L_2 will be

A. $D\left(\frac{L_2}{L_1}\right)^{\frac{1}{2}}$ days B. $D\left(\frac{L_2}{L_1}\right)^{\frac{3}{2}}$ days C. $D\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$ days D. $D\left(\frac{L_2}{L_1}\right)$ days

30. A spherical ball A of mass 4 kg, moving along a straight line strikes another spherical ball B of mass 1 kg at rest. After the collision, A and B move with velocities v_1 ms^{-1} and v_2 ms^{-1} respectively making angles of 30° and 60° with respect to the original direction of motion of A. The ratio $\frac{v_1}{v_2}$ will be

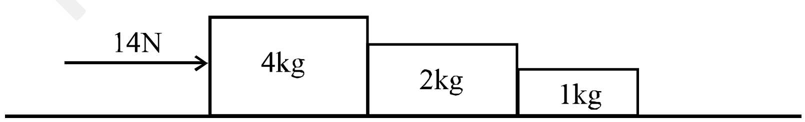
A. $\frac{\sqrt{3}}{4}$ B. $\frac{4}{\sqrt{3}}$ C. $\frac{1}{\sqrt{3}}$ D. $\sqrt{3}$

Q. 31 to 40 carry two marks each

31. When a certain metal surface is illuminated with light of frequency v, the stopping potential for photoelectric current is V_0. When the same surface is illuminated by light of frequency $\frac{v}{2}$, the stopping potential is $\frac{V_0}{4}$. The threshold frequency for photoelectric emission is

A. $\frac{v}{6}$ B. $\frac{v}{3}$ C. $\frac{2v}{3}$ D. $\frac{4v}{3}$

32. Three blocks of mass 4kg, 2kg, 1kg respectively are in contact on a frictionless table as shown in the figure. If a force of 14N is applied on the 4kg block, the contact force between the 4kg and the 2kg block will be



A. 2N B. 6N C. 8N D. 14N

33. Let L be the length and d be the diameter of cross section of a wire. Wires of the same material with different L and d are subjected to the same tension along the length of the wire. In which of the following cases, the extension of wire will be the maximum?

A. L = 200cm, d = 0.5mm B. L = 300cm, d = 1.0mm

C. L = 50cm, d = 0.05mm D. L = 100cm, d = 0.2mm

34. An object placed in front of a concave mirror at a distance of x cm from the pole gives a 3 times magnified real image. If it is moved to a distance of (x+5) cm, the magnification of the image becomes 2. The focal length of the mirror is

A. 15cm B. 20cm C. 25cm D. 30cm

35. 22320 cal heat is supplied to 100g of ice at 0°C. If the latent heat of fusion of ice is 80cal g^{-1} and latent heat of vaporization of water is 540 cal g^{-1}, the final amount of water thus obtained and its temperature respectively are

A. 8g, 100°C B. 100g, 90°C C. 92g, 100°C D. 82g, 100°C

36. A progressive wave moving along x-axis is represented by $y = A\sin\left[\frac{2\pi}{\lambda}(\nu t - x)\right]$. The wavelength ($\lambda$) at which the maximum particle velocity is 3 times the wave velocity is

A. A/3 B. 2A/(3π) C. (3/4)πA D. (2/3)πA

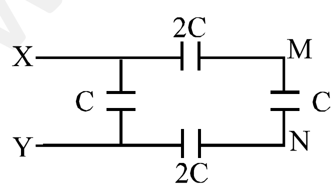
37. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, they have the same number of nuclei. The ratio of number of nuclei of A to that of B will be $(1/e)^2$ after a time interval of

A. $\frac{1}{\lambda}$ B. $\frac{1}{2\lambda}$ C. $\frac{1}{3\lambda}$ D. $\frac{1}{4\lambda}$

38. A magnetic needle is placed in a uniform magnetic field and is aligned with the field. The needle is now rotated by an angle of 60° and the work done is W. The torque on the magnetic needle at this position is

A. $2\sqrt{3}W$ B. $\sqrt{3}W$ C. $\frac{\sqrt{3}}{2}W$ D. $\frac{\sqrt{3}}{4}W$

39. In the adjoining figure the potential difference between X and Y is 60V. The potential difference the points M and N will be



A. 10V B. 15V C. 20V D. 30V

40. A body when fully immersed in a liquid of specific gravity 1.2 weighs 44gwt. The same body when fully immersed in water weighs 50 gwt. The mass of the body is

A. 36g B. 48g C. 64g D. 80g

SUBJECT : CHEMISTRY

Q. 41 to 70 carry one mark each

41. Which one of the following characteristics belongs to an electrophile?
    A. It is any species having electron deficiency which reacts at an electron rich C-centre
    B. It is any species having electron enrichment, that reacts at an electron deficient C-centre
    C. It is cationic in nature
    D. It is anionic in nature

42. Which one of the following methods is used to prepare Me_3COEt with a good yield?
    A. Mixing EtONa with Me_3CCl
    B. Mixing Me_3CONa with EtCl
    C. Heating a mixture of (1:1) EtOH and Me_3COH in presence of conc. H_2SO_4
    D. Treatment of Me_3COH with EtMgI

43. 58.5 gm of NaCl and 180gm of glucose were separately dissolved in 1000ml of water. Identify the correct statement regarding the elevation of boiling point (b.p.) of the resulting solutions.
    A. NaCl solution will show higher elevation of b.p.
    B. Glucose solution will show higher elevation of b.p.
    C. Both the solution will show equal elevation of b.p.
    D. The b.p. elevation will be shown by neither of the solutions

44. Equal weights of CH_4 and H_2 are mixed in an empty container at 25°C. The fraction of the total pressure exerted by H_2 is
    A. $\frac{1}{9}$ B. $\frac{1}{2}$ C. $\frac{8}{9}$ D. $\frac{16}{17}$

45. Which of the following will show a negative deviation from Raoult's law?
    A. Acetone-benzene B. Acetone-ethanol
    C. Benzene-methanol D. Acetone-chloroform

46. In a reversible chemical reaction at equilibrium, if the concentration of any one of the reactants is doubled, then the equilibrium constant will
    A. also be doubled B. be halved
    C. remains the same D. becomes one-fourth

47. Identify the correct statement from the following in a chemical reaction.
    A. The entropy always increases
    B. The change in entropy along with suitable change in enthalpy decides the fate of a reaction
    C. The enthalpy always decreases
    D. Both the enthalpy and the entropy remain constant

48. Which one of the following is wrong about molecularity of a reaction?

A. It may be whole number or fractional

B. It is calculated from reaction mechanism

C. It is the number of molecules of the reactants taking part in a single step chemical reaction

D. It is always equal to the order of elementary reaction.

49. Upon treatment with I_2 and aqueous NaOH, which of the following compounds will from iodoform?

A. $CH_3CH_2CH_2CH_2CHO$

B. $CH_3CH_2COCH_2CH_3$

C. $CH_3CH_2CH_2CH_2CH_2OH$

D. $CH_3CH_2CH_2CH(OH)CH_3$

50. Upon treatment with $Al(OEt)_3$ followed by usual reaction (work up), CH_3CHO will produce

A. only $CH_3COOCH_2CH_3$

B. a mixture of CH_3COOH and EtOH

C. only CH_3COOH

D. only EtOH

51. Friedel-Craft's reaction using MeCl and anhydrous $AlCl_3$ will take place most efficiently with

A. Benzene B. Nitrobenzene C. Acetophenone D. Toluene

52. Which one of the following properties is exhibited by phenol?

A. It is soluble in aq. NaOH and evolves CO_2 with aq. $NaHCO_3$

B. It is soluble in aq. NaOH and does not evolve CO_2 with aq. $NaHCO_3$

C. It is not soluble in aq. NaOH but evolves CO_2 with aq. $NaHCO_3$

D. It is insoluble in aq. NaOH and does not evolve CO_2 with aq. $NaHCO_3$

53. The basicity of aniline is weaker in comparison to that of methyl amine due to

A. hyperconjugative effect of Me-group in $MeNH_2$

B. resonance effect of phenyl group in aniline

C. lower molecular weight of methyl amine as compared to that of aniline

D. resonance effect of $-NH_2$ group in $MeNH_2$

54. Under identical conditions, the S_N1 reaction will occur most efficiently with

A. tert-butyl chloride

B. 1-chlorobutane

C. 2-methyl-1-chloropropane

D. 2-chlorobutane

55. Identify the method by which Me_3CCO_2H can be prepared.

A. Treating 1 mol of MeCOMe with 2 mole of MeMgI

B. Treating 1 mol of $MeCO_2Me$ with 3 moles of MeMgI

C. Treating 1 mol of MeCHO with 3 moles of MeMgI

D. Treating 1 mol of dry ice with 1 mol of Me_3CMgI

56. Li occupies higher position in the electrochemical series of metals as compared to Cu since

A. the standard reduction potential of Li^+ / Li is lower than that of Cu^{2+} / Cu

B. the standard reduction potential of Cu^{2+} / Cu is lower than that of Li^+ / Li

C. the standard oxidation potential of Li^+ / Li is lower than that of Cu / Cu^{2+}

D. Li is smaller in size as compared to Cu

57. ${}_{11}Na^{24}$ is radioactive and it decays to

A. ${}_{9}F^{20}$ and α-particles B. ${}_{13}Al^{24}$ and positron C. ${}_{11}Na^{23}$ and neutron D. ${}_{12}Mg^{24}$ and β-particles

58. The paramagnetic behaviour of B_2 is due to the presence of

A. 2 unpaired electrons in π_b MO B. 2 unpaired electrons in π^* MO

C. 2 unpaired electrons in σ^* MO D. 2 unpaired electrons in σ_b MO

59. A 100 ml 01. (M) solution of ammonium acetate is diluted by adding 100 ml of water. The pH of the resulting solution will be (pK_a of acetic acid is nearly equal to pK_b of NH_4OH)

A. 4.9 B. 5.0 C. 7.0 D. 10.0

60. In 2-butene, which one of the following statements is true ?

A. $C_1 - C_2$ bond a sp^3-sp^3 σ-bond B. $C_2 - C_3$ bond a sp^3-sp^2 σ-bond

C. $C_1 - C_2$ bond a sp^3-sp^2 σ-bond D. $C_1 - C_2$ bond a sp^2-sp^2 σ-bond

61. The well known compounds, (+) – lactic acid and (–) – lactic acid, have the same molecular formula, $C_3H_6O_3$. The correct relationship between them is

A. constitutional isomerism B. geometrical isomerism

C. identicalness D. optical isomerism

62. The stability of $Me_2C = CH_2$ is more than that of $MeCH_2CH = CH_2$ due to

A. inductive effect of the Me group

B. resonance effect of the Me group

C. hyperconjugative effect of the Me group

D. resonance as well as inductive effect of the Me group

63. Which of the following does not represent the mathematical expression for the Heisenberg uncertainty principle?

A. $\Delta x \cdot \Delta p \geq \frac{h}{(4\pi)}$ B. $\Delta x \cdot \Delta v \geq \frac{h}{(4\pi m)}$ C. $\Delta E \cdot \Delta t \geq \frac{h}{(4\pi)}$ D. $\Delta E \cdot \Delta x \geq \frac{h}{(4\pi)}$

64. The stable bivalency of Pb and trivalency of Bi is

A. due to d contraction in Pb and Bi

B. due to relativistic contraction of the 6s orbitals of Pb and Bi, leading to inert pair effect

C. due to screening effect

D. due to attainment of noble liquid configuration

65. The equivalent weight of $K_2Cr_2O_7$ in acidic medium is expressed in terms of its molecular weight (M) as

A. $\frac{M}{3}$ B. $\frac{M}{4}$ C. $\frac{M}{6}$ D. $\frac{M}{7}$

66. Which of the following is correct?

A. radius of $Ca^{2+} < Cl^- < S^{2-}$ B. radius of $Cl^- < S^{2-} < Ca^{2+}$

C. radius of $S^{2-} = Cl^- = Ca^{2+}$ D. radius of $S^{2-} < Cl^- < Ca^{2+}$

67. CO is practically non-polar since

A. the σ-electron drift from C to O is almost nullified by the π-electron drift from O to C

B. the σ-electron drift from O to C is almost nullified by the π-electron drift from C to O

C. the bond moment is low

D. there is a triple bond between C and O

68. The number of acidic protons in H_3PO_3 are

A. 0 B. 1 C. 2 D. 3

69. When H_2O_2 is shaken with an acidified solution of $K_2Cr_2O_7$ in presence of ether, the ethereal layer turns blue due to the formation of

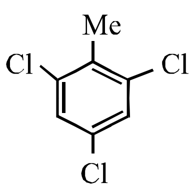
A. Cr_2O_3 B. CrO_4^{2-} C. $Cr_2(SO_4)_3$ D. CrO_5

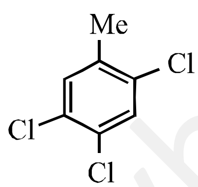
70. The state of hybridization of the central atom and the number of lone pairs over the central atom in $POCl_3$ are

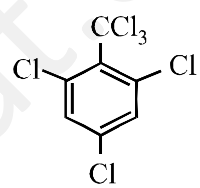
A. sp, 0 B. sp^2, 0 C. sp^3, 0 D. dsp^2, 1

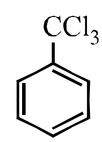
Q. 71 to 80 carry two marks each

71. By passing excess Cl_2(g) in boiling toluene, which one of the following compounds is exclusively formed?

A. 

B. 

C. 

D. 

72. An equimolar mixture of toluene and chlorobenzene is treated with a mixture of conc. H_2SO_2 and conc. HNO_3. Indicate the correct statement from the following.

A. p-nitrotoluene is formed in excess

B. equimolar amounts of p-nitrotoluene and p-nitrochlorobenzene are formed

C. p-nitrochlorobenzene is formed in excess

D. m-nitrochlorobenzene is formed in excess

73. Among the following carbocations : $Ph_2C^+CH_2Me(I)$, $PhCH_2CH_2CH^+Ph(II)$, $Ph_2CHCH^+M(III)$ and $Ph_2C(Me)CH_2^+(IV)$, the order of stability is

A. IV > II > I > III B. I > II > III > IV C. II > I > IV > III D. I > IV > III > II

74. Which of the followings is correct?

A. Evaporation of water causes an increase in disorder of the system

B. Melting of ice causes a decrease in randomness of the system

C. Condensation of steam causes an increase in disorder of the system

D. There is practically no change in the randomness of the system when water is evaporated

75. On passing ‘C’ Ampere of current for time ‘t’ sec through 1 litre of 2(M)$CuSO_4$ solution (atomic weight of Cu = 63.5), the amount ‘m’ of Cu (in gm) deposited on cathode will be

A. $m = \dfrac{Ct}{(63.5 \times 96500)}$

B. $m = \dfrac{Ct}{(31.25 \times 96500)}$

C. $m = \dfrac{(C \times 96500)}{(31.25 \times t)}$

D. $m = \dfrac{(31.25 \times C \times t)}{96500}$

76. If the 1st ionization energy of H atom is 13.6 eV, then the 2nd ionization energy of He atom is

A. 27.2 eV B. 40.8 eV C. 54.4 eV D. 108.8 eV

77. The weight of oxalic acid that will be required to prepare a 1000 ml $\left(\dfrac{N}{20}\right)$ solution is

A. 126/100 gm B. 63/40 gm C. 63/20 gm D. 126/20 gm

78. 20 ml 0.1 (N) acetic acid is mixed with 10 ml 0.1 (N) solution of NaOH. The pH of the resulting solution is (pK_a of acetic acid is 4.74)

A. 3.74 B. 4.74 C. 5.74 D. 6.74

79. In the brown ring complex $\left[Fe(H_2O)_5(NO)\right]SO_4$, nitric oxide behaves as

A. NO^+

B. neutral NO molecule

C. NO^-

D. NO^{2-}

80. The most contributing tautomeric enol form of $MeCOCH_2CO_2Et$ is

A. $CH_2 = C(OH)CH_2CO_2Et$

B. $MeC(OH) = CHCO_2Et$

C. $MeCOCH = C(OH)OEt$

D. $CH_2 = C(OH)CH = C(OH)OEt$

PC 2012 49528

SUBJECT : PHYSICS & CHEMISTRY **KEY ANSWERS WITH EXPLANATIONS**

PHYSICS

1. $\upsilon = \left(\frac{v}{v - u_S}\right)\upsilon_0$

$\upsilon = \left(\frac{340}{340 - 20}\right)640 = 680$

Ans. C.

2. $F = ilB\sin\theta$

$= 10 \times 2 \times 0.15 \sin 45^o$

$= \frac{3}{\sqrt{2}} N$

Ans. D.

3. $x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$

$x_2 = A\cos\omega t = A\sin\left(\omega t + \frac{\pi}{2}\right)$

$\Delta\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

Ans. B.

4. $H = \frac{V^2}{R}$

$H' = \frac{\left(\frac{V}{3}\right)^2}{2R}$

$H' = \frac{H}{18}$

Ans. A.

5. $\upsilon \;\alpha\; z^2$

$\Rightarrow \upsilon_A : \upsilon_B :: 1 : 4$

Ans. D.

6. $\lambda_c = \frac{h}{m_c \frac{C}{2}}$, $\lambda_p = \frac{h}{m_p c}$

$\Rightarrow \frac{m_e}{m_p} = 2$

$\frac{k_e}{k_p} = \frac{\frac{1}{2} m_c v_c^2}{\frac{1}{2} m_p v_p^2} = \frac{1}{2}$

Ans. B.

7. $E = \frac{\sigma}{2 \in_0} + \frac{\sigma}{2 \in_0} = \frac{\sigma}{\in_0}$ towards negative charged plate

Ans. B.

8. μ mg = ma

a = μg = 0.2 × 10 = 2 m/sec^2.

Ans. D.

9. 1011001 (binary number)

Its decimal equivalent is equal to

$2^0 + 0 + 0 + 2^3 + 2^4 + 2^6$

$= 1 + 8 + 16 + 64 = 89$

Ans. C.

10. $\upsilon_1 = \frac{(2n-1)}{4l_1} \nu$

$\upsilon_2 = \frac{n}{2l_2} \nu$

$\upsilon_1 = \upsilon_2 \Rightarrow \frac{l_1}{l_2} = \frac{3}{4}$

Ans. D.

11. R increases with temperature and slope of V – i graph gives resistance

Ans. B.

12. (m + 1) vernier division
= m no. of main scale division.
1 division on vernier scale

$$=\left(\frac{m}{m+1}\right) \text{ division on main scale.}$$

$$\text{Vernier constant } =\left(1-\frac{m}{m+1}\right)d$$

$$=\frac{d}{m+1}$$

Ans. A.

13. $y=\frac{1}{2}gt^2$

$$80=\frac{1}{2}\times 10\times t^2$$

t = 4 sec.
x = v × t
= 8 × 4
= 32 meter.

Ans. C.

14. Since Wheat stone bridge is balanced.

$$\frac{1}{10}=\frac{1}{x}+\frac{1}{30}$$

x = 15Ω

Ans. D.

15. $R=\frac{V^2}{P}=\frac{200\times 200}{50}$

$$P'=\frac{V'^2}{R}=\frac{100\times 100\times 50}{200\times 200}=12.5 \text{ W}$$

Ans. C.

16. $\frac{x-10}{130-10}=\frac{40}{100}$

x = 58°

Ans. D.

17. $\varepsilon = \dfrac{d\phi}{dt} = 8t + 6$

at t = 2 sec.

ε = 22 volt.

Ans. A.

18. **Ans. C.**

19. $t = \dfrac{d}{\frac{c}{\mu}} = \dfrac{10 \times 10^{-2}}{2 \times 10^{8}} = 0.5 \times 10^{-9}$ sec.

Ans. A.

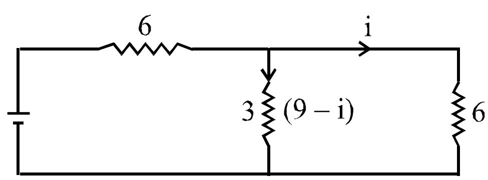
20. $W = q\Delta V$

$$= q\left[\frac{Q}{4\pi \in_0 b} - \frac{Q}{4\pi \in_0 a}\right]$$

$$= \frac{Qq}{4\pi \in_0}\left(\frac{a-b}{ab}\right)$$

Ans. A.

21. $i = \dfrac{72}{8 \times 10^{3}} = 9 \times 10^{-3}$ Amp



i × 6 = (9 – i) × 3

i = 3 milli ampere.

Ans. A.

22. $\phi = \vec{E} \cdot \vec{S} = 20\,Nm^2C^{-1}$

Ans. D.

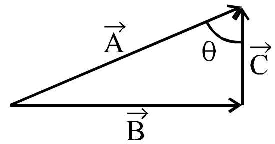
23. $\vec{L} = \vec{r} \times \vec{p}$

$$= [L]\left[MLT^{-1}\right]$$

$$= \left[ML^2T^{-1}\right]$$

Ans. C.

24. $\theta = \cos^{-1}\left(\frac{|\vec{C}|}{|\vec{A}|}\right) = \cos^{-1}\left(\frac{3}{5}\right)$



Ans. A.

25. $v = \frac{dx}{dt} = 27 - 3t^2$

$v = 0. \quad \Rightarrow 27 - 3t^2 = 0$

$t = 3$ sec.

$x = 37 + 27 \times 3 - (3)^3$

$= 91$ meter.

Ans. B.

26. At ground. $E = \frac{1}{3}mu^2$

At highest point

$k' = \frac{1}{2}m(u\cos 60°)^2$

$k' = \frac{E}{4}$

Ans. C.

27. $v^2 = u^2 + 2as$

$\frac{u^2}{4} = u^2 + 2a \times 30 \times 10^{-2}$ .... (1)

$0 = \frac{u^2}{4} + 2a \times x$ .... (2)

Solving (1) & (2)

$x = 10$ cm.

Ans. C.

28. $E = \frac{1}{2}k\left(10 \times 10^{-2}\right)^2$

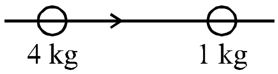
$E' = \frac{1}{2}k\left(20 \times 10^{-2}\right)^2 = 4E$

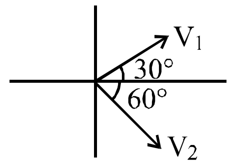
Ans. B.

29. $T^2 \;\alpha\; R^3$.

$$\Rightarrow \; T = D\left(\frac{L_2}{L_1}\right)^{\frac{3}{2}}$$

Ans. B.

30. 



Along y-axis momentum remains zero

$$4\,\nu_1 \sin 30 = \nu_2 \sin 60$$

$$\frac{\nu_1}{\nu_2} = \frac{\sqrt{3}}{4}$$

Ans. A.

31. $h\nu = h\nu_o + eV_o$ .... (1)

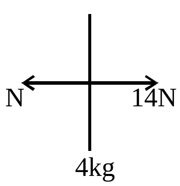
$$\frac{h\nu}{2} = h\nu_0 + \frac{eV_0}{4} \quad .... \; (2)$$

Solving (1) & (2)

$$\nu_0 = \frac{\nu}{3}.$$

Ans. B.

32. $a = \frac{F}{m} = \frac{14}{7} = 2\,m/\sec^2$



$14 - N = 4a$

$N = 6N$

Ans. B.

33. $\Delta l = \frac{F \times l}{A \times Y} = \frac{4L}{\pi d^2}$

$$\Rightarrow \Delta l \;\alpha\; \frac{L}{d^2}$$

Ans. C.

34. $\upsilon_1 = 3x$

$\upsilon_2 = 2(x+5)$

$$\frac{1}{-x}+\frac{1}{-3x}=\frac{1}{f} \quad ..... \quad (1)$$

$$\frac{1}{-(x+5)}+\frac{1}{-2(x+5)}=\frac{1}{f} \quad .... \quad (2)$$

solving (1) & (2)

f = 30 cm

Ans. D.

35. Heat required to convert ice to water at 100ºC.

$Q = m \times L + ms\Delta T = 18000$ cal.

Amount of heat left = 4320 cal.

$\Rightarrow m \times L = 4320$

m = 8g steam.

Ans. C.

36. $$\left(\upsilon_p\right)_{max} = \frac{2\pi}{\lambda}\upsilon A$$

$$\left(\upsilon_p\right)_{max} = 3\upsilon$$

$$\lambda = \frac{2\pi}{3}A$$

Ans. D.

37. $N_A = N_0 e^{-\lambda t}$

$N_B = N_0 e^{-\lambda t}$

$$\frac{N_A}{N_B} = \frac{1}{e^2} = \frac{1}{e^{4\lambda t}}$$

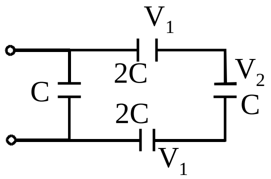
$$t = \frac{1}{2\lambda}$$

Ans. B.

38. $W = MB\ (1 - \cos 60) = \frac{MB}{2}$

$\left|T\right| = MB \sin 60 = \sqrt{3}\ W$

Ans. B.

39. 

$2C \times V_1 = C \times V_2$ (since capacitors are in series)

$2V_1 = V_2$ ..... (1)

$V_1 + V_2 + V_1 = 60$ .... (2)

solving (1) & (2)

$V_2 = 30$ V

Ans. D.

40. $W = mg - V\rho g$

$44 = m - 1.2\,V$ ..... (1)

$50 = m - V$ .... (2)

solving (1) & (2)

$m = 80$ g

Ans. D.

CHEMISTRY

41. Electrophiles are electron defficient species (neutral or cationic).

Ans. A.

42. $Me_3CONa \xrightarrow[(1^\circ \text{ alkyl halide})]{EtCl} Me_3COEt$

Ans. B.

43. Because molality for both is same and 'i' value for NaCl is 2 while for glucose it is 1.

Ans. A.

44. Ratio of no. of moles of $CH_4 : H_2$

$= \frac{x}{16} : \frac{x}{2}$

$= 1 : 8$

Hence partial pressure of hydrogen $= \frac{8}{9} \times P_{total}$

Ans. C.

45. Acetone and chloroform will show a negative deviation due to $CH_3 - \overset{\overset{\displaystyle O \text{-----}}{\|}}{C} - CH_3 \quad \overset{\overset{\displaystyle H}{|}}{C}Cl_3$ association after mixing.

Ans. D.

46. Because equilibrium constant is independent of conc. of any species.

    Ans. C.

47. Because $\Delta G_{(T,\ P)} = \Delta H - T\Delta S$ determines the course of a chemical reaction.

    Ans. B.

48. Because molecularity of a reaction can never be fractional.

    Ans. A.

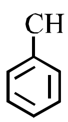
49. Because $CH_3CH_2CH_2 - \underset{OH}{\overset{H}{C}} - CH_3$ when oxidised form $CH_3CH_2CH_2 - \overset{O}{\overset{\|}{C}} - CH_3$ which contains a keto-methyl group.

    Ans. D.

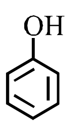
50. It is an example of Tischenko reaction

$$CH_3CHO \xrightarrow{Al(OC_2H_5)_3} CH_3COOH + C_2H_5OH \longrightarrow CH_3COOC_2H_5$$

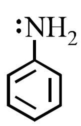
    Ans. A.

51. Because in  . The alkyl group is electron donating.

    Ans. D.

52. Because  reacts with NaOH, but is not sufficiently acidic to evolve CO_2 from $NaHCO_3$.

    Ans. B.

53. Because  the N-lone pair in aniline is involved in the ring resonance.

    Ans. B.

54. tert-butyl chloride because the S_N1 reaction is most effective in 3º carbon.

    Ans. A.

55. $$CH_3 - \underset{CH_3}{\overset{CH_3}{\overset{\delta-}{C}}} - \overset{\delta+}{MgI} \xrightarrow{\overset{\delta-}{O} = \overset{\delta+}{C} = \overset{\delta-}{O}} Me_3CCO_2H$$

    Ans. D.

56. Because Li^+ has least tendency to get converted to Li

Ans. A.

57. $^{24}_{11}Na \rightarrow \, ^{24}_{12}Mg + \, ^{0}_{-1}e$

because for stable $\frac{n}{p}$ ratio.

Ans. D.

58. $\sigma_{1s^2}\,\sigma^*_{1s^2}\,\sigma_{2s^2}\,\sigma^*_{2s^2}\,\pi^1_{2P_x}\,\pi^1_{2P_y}$.

So there are 2e′s in π-bonding molecular orbital

Ans. A.

59. Because for ammonium acetate which is a soln of weak acid-weak base, ${}_pK_a$ of $CH_3COOH = {}_pK_b$ of NH_4OH.

Ans. C.

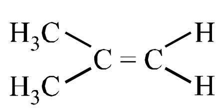
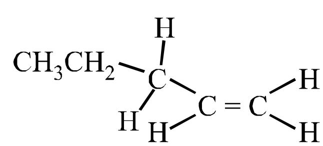
60. $-\overset{|}{\underset{|}{C_2}}-\overset{|}{C_2}=\overset{|}{C}-\overset{|}{\underset{|}{C}}-$

because C_1 is a sp^3-carbon and C_2 is a sp^2-carbon.

Ans. C.

61. They are optical isomers which rotate the plane of polarised light in opposite direction.

Ans. D.

62.  has 6-hyperconjugative forms while.  has 2 hyper-conjugative forms.

Ans. C.

63. $\Delta E.\Delta x \geq \frac{h}{4\pi}$

Ans. D.

64. Because inert-pair effect is prominent in group 14 and 15 element.

Ans. B.

65. Equivalent mass of $K_2Cr_2O_7 \xrightarrow{H^+} Cr^{3+} = \frac{M}{6}$ because no. of e^- s transfer is 3 for each Cr and 6 for two Cr-atoms.

Ans. C.

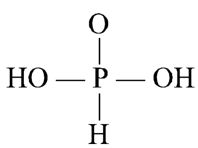
66. $Ca^{2+} < Cl^- < S^{2-}$ because for isoelectronic species, more the atomic number, lesser the size.

Ans. A.

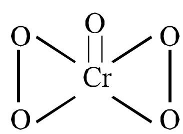
67. $:C \overset{\pi}{\underset{\sigma}{\overset{\longleftarrow}{\underset{\pi}{\longrightarrow}}}} O:$ because the C — O σ moment and O — C π-moment cancels out each other.

Ans. A.

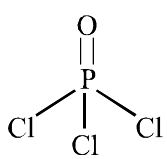
68. The no. of acidic protons in H_3PO_2 is 2.

 (the H′s linked via oxygen are replacable or acidic)

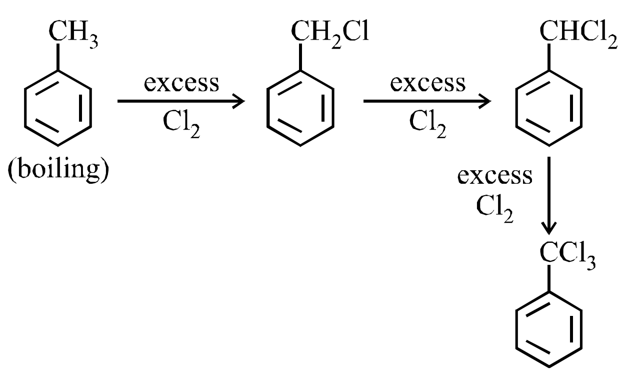
Ans. C.

69. Because of the formation of CrO_5. 

Ans. D.

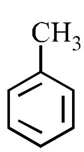
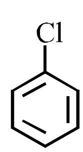
70.  $\rightarrow$ sp^3 hybridised with no lone pair.

Ans. C.

71. 

Higher temp favours side-chain substitution.

Ans. D.

72. Because  is more reactive than  and will add $\overset{+}{N}O_2$ at para position.

Ans. A.

73. $\mathrm{Ph}-\overset{\mathrm{Ph}}{\underset{(+)}{\mathrm{C}}}-\mathrm{CH_2CH_3} > \mathrm{PhCH_2CH_2}-\overset{\mathrm{H}}{\underset{(+)}{\mathrm{C}}}-\mathrm{Ph} >$

$\mathrm{Ph_2CH}-\overset{\mathrm{H}}{\underset{\mathrm{Me}}{\mathrm{C}}}\,(+) > \mathrm{Ph}-\overset{\mathrm{Me}}{\underset{\mathrm{Ph}}{\mathrm{C}}}-\underset{(+)}{\mathrm{CH_2}}$

because 3° benzylic > 2° benzylic > 2° > 1°.

Ans. B.

74. Because when water is converted into steam, its disorder or randomness increases.

Ans. A.

75. $m = \dfrac{E\,C\,t}{F} = \dfrac{\frac{63.5}{2} \times C \times t}{96500}$

$= \dfrac{31.75 \times C \times t}{96500}$

Wrongly given as 31.25 in place of 31.75 in J.E.E. paper.

Ans. D.

76. 2nd ionization energy $= 13.6 \times \dfrac{2^2}{1^2}$

$= 54.4$ eV.

Ans. C.

77. 1000 ml $\dfrac{N}{20} \equiv 50$ ml (N)

$= 0.5$ gm eq. $= 0.05 \times \dfrac{126}{2} = \dfrac{63}{20}$ gm

Ans. C.

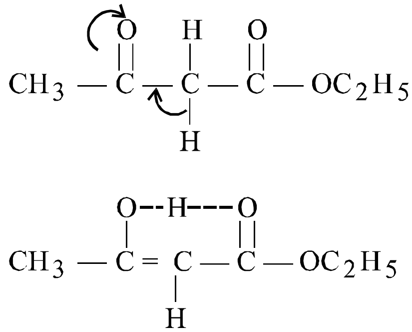
78. $pH = pK_a + \log\dfrac{\text{salt}}{\text{base}} = 4.74 + \log\dfrac{1}{1}$

$= 4.74.$

Ans. B.

79. Because in Fe-complexes. NO behaves as NO^+.

Ans. A.

80. 

Ans. B.