Code-δ

ANSWERS & HINTS
for
WBJEE - 2013
SUB : MATHEMATICS

CATEGORY - I

Q. 1 – Q. 60 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark

1. A point P lies on the circle $x^2 + y^2 = 169$. If Q = (5,12) and R = (–12,5), then the a gle $\angle QPR$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans : (B)

Hints : Q (5,12) R (–12,5)

0 (0,0) $m_{OQ} = \frac{12}{5}$, $m_{OR} = \frac{5}{-12}$

$m_{OQ} . m_{OR} = -1$, so $\angle QOR = \pi/2$ Hence $\angle QPR = \pi/4$

2. A circle passing through (0,0), (2,6), (6,2) cuts the x axis a the point P ≠ (0,0). Then the length of OP, where O is origin, is

(A) $\frac{5}{2}$ (B) $\frac{5}{\sqrt{2}}$ (C) 5 (D) 10

Ans : (C)

Hints : Circle passes through (0,0)

so, $x^2+y^2+2gx+2fy = 0$

(2,6) & (6,2) lies on it so, $2^2+6^2+4g+12f = 0$ — (1)

$2^2+6^2+12g+4f = 0$ — (2)

$\Rightarrow$ From (1) & (2),

$g = f = -5/2$

Eqn. of circle is $x^2+y^2-5x-5y=0$

For y = 0, $x(x-5)=0 \Rightarrow x=0, x=5$

OP = 5

3. The locus of the midpoints of the chords of an ellpse $x^2+4y^2 = 4$ that are drawn form the positive end of the minor axis, is

(A) a circle with centre $\left(\frac{1}{2},0\right)$ and radius 1

(B) a parabola with focus $\left(\frac{1}{2},0\right)$ and directrix x = –1

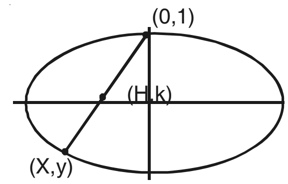
(C) an ellipse with centre $\left(0,\frac{1}{2}\right)$, major axis 1 and minor axis $\frac{1}{2}$

(D) a hyperbola with centre $\left(0,\frac{1}{2}\right)$, transverse axis 1 and conjugate axis $\frac{1}{2}$

Ans : No option is correct

Hints : Positive end of minor axis (0,1) but mid-pt be (h,k) x=2h, y=2k–1 lies on ellipse

$$4h^2 + 4(2k-1)^2 = 4 \Rightarrow \frac{h^2}{1} + \frac{\left(k-\frac{1}{2}\right)^2}{\frac{1}{4}} = 1$$



Here on ellipse of centre $(0, \frac{1}{2})$, major axis 2, minor axis 1

4. A point moves so that the sum of squares of its distances from the points (1,2) and (–2,1) is always 6. Then its locus is

(A) the straight line $y - \frac{3}{2} = -3\left(x + \frac{1}{2}\right)$

(B) a circle with centre $\left(-\frac{1}{2}, \frac{3}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$

(C) a parabola with focus (1,2) and directix pssing through (–2,1)

(D) an ellipse with foci (1,2) and (–2,1)

Ans : (B)

Hints : Let (h,k) be co-ordinates of the point

$(h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 6$

$\Rightarrow h^2+k^2+h - 3k + 2 = 0$

Circle with centre $\left(-\frac{1}{2}, \frac{3}{2}\right)$ radius $= \frac{\ }{\sqrt{2}}$

5. For the variable t, the locus of the points of intersection of lines x–2y = t and x+2y= $\frac{1}{t}$ is

(A) the straight line x=y

(B) the circle with centre at the origin and radius 1

(C) the ellipse with centre at the origin and one focus $\left(\frac{2}{\sqrt{5}}, 0\right)$

(D) the hyperbola with centre at the origin and one focus $\left(\frac{\sqrt{5}}{2}, 0\right)$

Ans : (D)

Hints : $(x-2y)(x+2y) = 1 \Rightarrow x^2 - 4y^2 = 1$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{\frac{1}{4}} = 1$$

$a = 1, b = \frac{1}{2}$ $e = \frac{\sqrt{5}}{2}$ focus $\left(\frac{\sqrt{5}}{2}, 0\right)$

6. Let $P = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $X = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$. Then P^3X is equal to

(A) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ (C) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (D) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

Ans : (C)

Hints : $P^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$P^3 = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$P^3 X = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

7. The number of solutions of the equation x+y+z = 10 in positive integers x,y,z is equal to

(A) 36 (B) 55 (C) 72 (D) 45

Ans : (A)

Hints : ${}^{10-1}C_{3-1} = {}^{9}C_{2} = 36$

8. For $0 \le P, Q \le \frac{\pi}{2}$, if sin P + cos Q=2, then the value of $\tan\left(\frac{P+Q}{2}\right)$ is equal to

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

Ans : (A)

Hints : $P = \pi/2$, $Q = 0$

9. If α and β are the roots of $x^2 - x + 1 = 0$, then the value of $\alpha^{2013} + \beta^{2013}$ is equal to

(A) 2 (B) –2 (C) –1 (D) 1

Ans : (B)

Hints : $\alpha = -\omega, -\omega^2$

$$-\omega^{2013} - \omega^{2\times2013} = -(\omega^3)^{671} - (\omega^3)^{2\times671} = -2$$

10. The value of the integral

$$\int_{-1}^{+1}\left\{\frac{x^{2013}}{e^{|x|}\left(x^2+\cos x\right)}+\frac{1}{e^{|x|}}\right\}dx$$

is equal to

(A) 0 (B) $1-e^{-1}$ (C) $2e^{-1}$ (D) $2\left(1-e^{-1}\right)$

Ans : (D)

Hints : $\frac{x^{2013}}{e^{|x|}\left(x^2+\cos x\right)}$ is odd

$$I=\int_{-1}^{1}\frac{1}{e^{|x|}}dx=2\int_{0}^{1}e^{-x}dx=2\left(1-e^{-1}\right)$$

11. Let

$f(x) = 2^{100}x+1$,

$g(x) = 3^{100}x+1$.

Then the set of real numbers x such that $f(g(x)) = x$ is

(A) empty (B) a singleton (C) a finite se with more than one element

(D) infinite

Ans : (B)

Hints : $f(x) = 2^{100}x+1$; $g(x) = 3^{100}x+1$

$$f(g(x)) = x \Rightarrow x = -\frac{\left(1+2^{100}\right)}{6^{100}-1}$$

12. The limit of $x\sin(e^{1/x})$ as $x \to 0$

(A) is equal to 0 (B) is equal to 1 (C) is equal to e/2 (D) does not exist

Ans : (A)

Hints : $-1\leq \sin e^{1/x}\leq 1$, $-x\leq x\sin(e^{1/})\leq x$

$$\lim_{x\to 0} x\sin(e^{1/x})=\lim_{x\to 0} x=0,$$

13. Let $I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ and $P=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-2\end{pmatrix}$. Then the matrix P^3+2P^2 is equal to

(A) P (B) I–P (C) 2I+P (D) 2I–P

Ans : (C)

Hints : $|P-\lambda I|=0$, characteristics equation of P is $P^3+2P^2-P-2I=0$

$$P^3+2P^2=P+2I$$

14. If α, β are the roots of the quadratic equation $x^2+ax+b=0$, $(b\neq0)$; then the quadratic equation whose roots are $\alpha-\frac{1}{\beta}, \beta-\frac{1}{\alpha}$ is

(A) $ax^2+a(b-1)x+(a-1)^2=0$ (B) $bx^2+a(b-1)x+(b-1)^2=0$ (C) $x^2+ax+b = 0$ (D) $abx^2+bx+a = 0$

Ans : (B)

Hints : $\alpha+\beta = -a$, $\alpha\beta = b$

$$\gamma = \alpha-\frac{1}{\beta},\ \delta=\beta-\frac{1}{\alpha}$$

$$\gamma + \delta = \frac{-a(b-1)}{b} \quad \gamma\delta = \frac{(b-1)^2}{b}$$

Equation is $x^2-(\gamma+\delta)\,x + \gamma\delta = 0$

$\Rightarrow bx^2 + a(b-1)x + (b-1)^2 = 0$

15. The value of $1000\left[\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+......+\frac{1}{999x1000}\right]$ is equal to

(A) 1000 (B) 999 (C) 1001 (D) 1/999

Ans : (B)

Hints : $1000\left[1-\cancel{\frac{1}{2}}+\frac{1}{2}-\cancel{\frac{1}{3}}------+\cancel{\frac{1}{999}}-\frac{1}{1000}\right]$

$= 1000\left(1-\frac{1}{1000}\right)$

$= 999$

16. The value of the determinant

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1\ \ a^2-b^2 \end{vmatrix}$$

is equal to

(A) 0 (B) $(1+a^2+b^2)$ (C) $(1+a^2+b^2)^2$ (D) $(1+a^2+b^2)^3$

Ans : (D)

Hints : $(1+a^2+b^2)^3$

$C_1 \to C_1 - bC_3$, $C_2 \to aC_3 + C_2$

$$\begin{bmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{bmatrix}$$

$$= (1+a^2+b^2)^2\begin{bmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{bmatrix}$$

$= (1+a^2+b^2)^3$

17. If the distance between the foci of an ellipse is equal to the length of the latus rectum, then its eccentricity is

(A) $\frac{1}{4}\left(\sqrt{5}-1\right)$ (B) $\frac{1}{2}\left(\sqrt{5}+1\right)$ (C) $\frac{1}{2}\left(\sqrt{5}-1\right)$ (D) $\frac{1}{4}\left(\sqrt{5}+1\right)$

Ans : (C)

Hints : $2ae = \frac{2b^2}{a}$

$\Rightarrow e = \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^2 + e - 1 = 0$

$e = \frac{\sqrt{5}-1}{2}$

18. For the curve $x^2+4xy+8y^2=64$ the tangents are parallel to the x-axis only at the points

(A) $(0,2\sqrt{2})$ and $(0,-2\sqrt{2})$

(B) (8,–4) and (–8,4)

(C) $\left(8\sqrt{2},-2\sqrt{2}\right)$ and $\left(-8\sqrt{2},2\sqrt{2}\right)$

(D) (8,0) and (–8,0)

Ans : (B)

Hints : $x^2+4xy+8y^2 = 64$

$\Rightarrow 2x+4xy'+4y+16yy'=0$

$\Rightarrow (4x+16y)y' = -(2x+4y)$

$2x + 4y = 0$

19. The value of $I = \int_0^{\frac{\pi}{4}}\left(\tan^{n+1} x\right)dx + \frac{1}{2}\int_0^{\frac{\pi}{2}}\tan^{n-1}\left(x/2\right)dx$ is equal to

(A) $\frac{1}{n}$ (B) $\frac{n+2}{2n+1}$ (C) $\frac{2n-1}{n}$ (D) $\frac{2n-3}{3n-2}$

Ans : (A)

Hints : $I = \int_0^{\pi/4}\tan^{n+1} x.dx + \frac{1}{2}\int_0^{\pi/2}\tan^{n-1}\left(\frac{x}{2}\right)dx$

For second integral substitute $\frac{x}{2} = y$

$I = \int_0^{\pi/4}\left(\tan^{n+1} x + \tan^{n-1} x\right)dx$

$\int_0^{\pi/4}\tan^{n-1} x.\sec^2 x\,dx = \frac{\tan^n x}{n}\Big|_0^{\pi/4} = \frac{1}{n}$

20. Let $f(\theta) = (1+\sin^2\theta)(2-\sin^2\theta)$. Then for all values of θ

(A) $f(\theta) > \frac{9}{4}$ (B) $f(\theta) < 2$ (C) $f(\theta) > \frac{11}{4}$ (D) $2 \le f(\theta) \le \frac{9}{4}$

Ans : (D)

Hints : $f(\theta) = (1+\sin^2\theta)\,(2-\sin^2\theta)$

$f(\theta)=(1+\sin^2\theta)(1+\cos^2\theta)$

$=2+\sin^2\theta\cos^2\theta$

$=2+\dfrac{1}{4}\sin^2\theta$

$2 \le f(\theta) \le \dfrac{9}{4}$

21. Let $f(x) = \begin{cases} x^3-3x+2, & x<2 \\ x^3-6x^2+9x+2, & x\ge 2 \end{cases}$

Then

(A) $\lim\limits_{x\to 2} f(x)$ does not exist

(B) f is not continuous at x = 2

(C) f is continuous but not differentiable at x = 2

(D) f is continuous and differentiable at x = 2

Ans : (C)

Hints : $\lim\limits_{x\to 2^+} f(x) = 4$

$\lim\limits_{x\to 2^-} f(x) = 4$

$f'(x) = \begin{cases} 3x^2-3, & x<2 \\ 3x^2-12x+9, & x\ge 2 \end{cases}$

so L.H.D at x = 2 is 9, R.H.D at x = 2 is –3

so f(x) is continuous but not differentiable at x = 2

22. The limit of $\sum\limits_{n=1}^{1000} (-1)^n x^n$ as $x\to\infty$

(A) does not exist (B) exists and equals to 0 (C) exists and approaches $+\infty$ (D) exists and approaches $-\infty$

Ans : (C)

Hints : $\lim\limits_{x\to\infty} (-x+x^2-x^3+x^4 \ldots\ \ldots\ldots+x^{1000})$

$= \lim\limits_{x\to\infty} (-x).\dfrac{\left((-x)^{1000}-1\right)}{-x-1} = \lim\limits_{x\to\infty} \dfrac{x^{1001}-x}{x+1} = +\infty$

23. If $f(x) = e^x (x-2)^2$ then

(A) f is increasing in $(-\infty,0)$ and $(2,\infty)$ and decreasing in (0, 2)

(B) f is increasing in $(-\infty,0)$ and decreasing in $(0,\infty)$

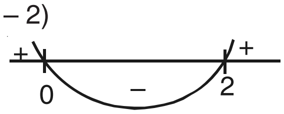
(C) f is increasing in $(2,\infty)$ and decreasing in $(-\infty,0)$

(D) f is increasing in (0, 2) and decreasing in $(-\infty,0)$ and $(2,\infty)$

Ans : (A)

Hints : $f'(x) = e^x [(x-2)^2 + 2(x-2)]$

$= e^x [x^2 - 2x] = e^x.x(x-2)$

sign scheme of f′(x) will be 

so f is increasing in $(-\infty, 0)$ and $(2, \infty)$ and decreasing in (0, 2)

24. Let f : $\mathbb{R} \to \mathbb{R}$ be such that f is injective and f(x)f(y) = f(x +y) for all x, y $\in$ $\mathbb{R}$. If f(x), f(y), f(z) are in G.P., then x, y, z are in

(A) A.P. always
(B) G.P. always
(C) A.P. depending on the values of x, y, z
(D) G.P. depending on the values of x, y, z

Ans : (A)

Hints : f (x + y) = f(x).f(y), so $f(x) = a^{kx}$

a^{kx}, a^{ky}, a^{kz} are in G.P

$a^{2ky} = a^{k(x+z)}$

$\Rightarrow 2y = x + z$, so x, y, z are in A.P

25. The number of solutions of the equation

$\frac{1}{2}\log_{\sqrt{3}}\left(\frac{x+1}{x+5}\right)+\log_9(x+5)^2=1$ is

(A) 0 (B) 1 (C) 2 (D) infinite

Ans : (B)

Hints : $\log_3\left(\frac{x+1}{x+5}\right)+\log_3(x+5)=1$

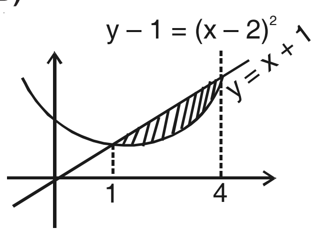
(x + 1) = 3, x = 2 so only one solution

26. The area of the region bounded by the parabola $y = x^2 - 4x + 5$ and the straight line $y = x + 1$ is

(A) 1/2 (B) 2 (C) 3 (D) 9/2

Ans : (D)

Hints :



$$\text{Required Area} = \int_1^4 \left((x+1)-(x^2-4x+5)\right)dx$$

$$= \frac{9}{2}\text{sq.unit}$$

27. The value of the integral

$\int_1^2 e^x\left(\log_e x+\frac{x+1}{x}\right)dx$ is

(A) $e^2(1 + \log_e 2)$ (B) $e^2 - e$ (C) $e^2(1 + \log_e 2) - e$ (D) $e^2 - e(1 + \log_e 2)$

Ans : (C)

Hints : $\int_1^2 e^x\left(\log_e x+1+\frac{1}{x}\right)dx = \int_1^2 e^x.dx+\int_1^2 e^x\left(\log_e x+\frac{1}{x}\right)dx$

$$= (e^2 - e^1) + \left[e^x \log_e x\right]_1^2$$

$$= e^2 - e + e^2\log_e 2$$

$$= e^2(1 + \log_e 2) - e$$

28. Let P = $1+\frac{1}{2\times 2}+\frac{1}{3\times 2^2}+........$

and Q = $\frac{1}{1\times 2}+\frac{1}{3\times 4}+\frac{1}{5\times 6}+........$

Then

(A) P = Q (B) 2P = Q (C) P = 2Q (D) P = 4Q

Ans : (C)

Hints : $\frac{P}{2}=\frac{1}{2}+\frac{\left(\frac{1}{2}\right)^2}{2}+\frac{\left(\frac{1}{2}\right)^3}{3}+.........\text{to}\infty$

$= -\log_e\left(1-\frac{1}{2}\right)$ so P = 2 $\log_e 2$

Q = $\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+........\text{to}\infty$

= $\log_e(1 + 1) = \log_e 2$

P = 2Q

29. Let f(x) = sin x + 2 cos² x, $\frac{\pi}{4}\leq x\leq\frac{3\pi}{4}$. Then f attains its

(A) minimum at x = $\frac{\pi}{4}$ (B) maximum at x = $\frac{\pi}{2}$

(C) minimum at x = $\frac{\pi}{2}$ (D) maximum at x = $\sin^{-1}\left(\frac{1}{4}\right)$

Ans : (C)

Hints : f(x) = sinx + 2cos²x

= – 2sin²x + sinx + 2

= – 2 (sin²x – ½ sinx) + 2

$= -2\left(\sin x-\frac{1}{4}\right)^2+2+\frac{1}{8}$

$= \frac{17}{8}-2\left(\sin x-\frac{1}{4}\right)^2$; nder the given domain

f(x) will be minimum when $\left(\sin x-\frac{1}{4}\right)^2$ is maximum which is at x = $\pi/2$

30. Each of a and b can take values 1 or 2 with equal probability. The probability that the equation $ax^2 + bx + 1 = 0$ has real roots, is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

Ans : (B)

Hints : $ax^2 + bx + 1 = 0$ has real roots for $b^2 - 4a \geq 0$

So a has to be 1 and b has to be 2

so probability is = $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

31. There are two coins, one unbiased with probability $\frac{1}{2}$ of getting heads and the other one is biased with probability $\frac{3}{4}$ of getting heads. A coin is selected at random and tossed. It shows heads up. Then the probability that the unbiased coin was selected is

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Ans : (D)

Hints : H $\rightarrow$ Event of head showing up

B $\rightarrow$ Event of biased coin chosen

UB $\rightarrow$ Event of unbiased coin chosen

$$P\left(\frac{UB}{H}\right)=\frac{P(UB).P\left(\frac{H}{UB}\right)}{P(UB).P\left(\frac{H}{UB}\right)+P(B).P\left(\frac{H}{B}\right)}$$

$$\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{3}{4}}=\frac{2}{5}$$

32. For the variable t, the locus of the point of intersection of the line $3tx - 2y + 6t = 0$ and $3x + 2ty - 6 = 0$ is

(A) the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ (B) the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

(C) the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$ (D) the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$

Ans : (A)

Hints : The point of intersection of $3tx - 2y + 6t = 0$ and $3x + 2ty - 6 = 0$ is

$$x = \frac{2(1-t^2)}{(1+t^2)},\ y = \frac{6t}{(1+t^2)}$$

Considering $t = \tan\theta$, $x = 2\cos 2\theta$, $y = 3.\sin 2\theta$

so locus of point of intersection is the ellipse

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

33. Cards are drawn one-by-one without replacement from a well shuffled pack of 52 cards. Then the probability that a face card (Jack, Queen or King) will appear for the first time on the third turn is equal to

(A) $\frac{300}{2197}$ (B) $\frac{36}{85}$ (C) $\frac{12}{85}$ (D) $\frac{4}{51}$

Ans : (C)

Hints : P (face card on third turn) = P (no face card in first turn) × P (no face card in 2nd turn) × P (face card in 3rd turn)

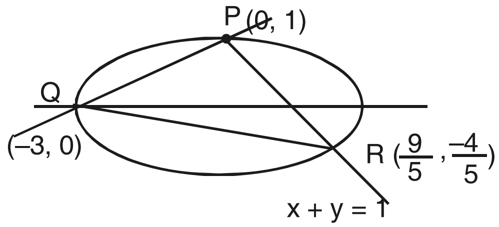
$$= \frac{40}{52}\times\frac{39}{51}\times\frac{12}{50}=\frac{12}{85}$$

34. Lines x + y = 1 and 3y = x + 3 intersect the ellipse $x^2 + 9y^2 = 9$ at the points P,Q,R. The are of the triangle PQR is

(A) $\frac{36}{5}$ (B) $\frac{18}{5}$ (C) $\frac{9}{5}$ (D) $\frac{1}{5}$

Ans : (B)

Hints : 3y = x + 3 (–3, 0)



Area of $\Delta PQR = \left\| \frac{1}{2}\begin{vmatrix} 0 & -3 & \frac{9}{5} \\ 1 & 0 & -4/5 \\ 1 & 1 & 1 \end{vmatrix} \right\|$

$= \frac{18}{5}$ sq. units

35. The number of onto functions from the set {1, 2, ..., 11} to set {1, 2, ... 10} is

(A) $5\times\underline{|11}$ (B) $\underline{|10}$ (C) $\frac{\underline{|11}}{2}$ (D) $10\times\underline{|11}$

Ans : (D)

Hints : No. of onto function $= {}^{11}C_{10}\times\underline{|10}\times10$

$= 10 \times \underline{|11}$

36. The limit of $\left[\frac{1}{x^2}+\frac{(2013)^x}{e^x-1}-\frac{1}{e^x-1}\right]$ as $x \to 0$

(A) approaches $+\infty$ (B) approaches $-\infty$ (C) is equal to $\log_e(2013)$ (D) does not exist

Ans : (A)

Hints : $\lim_{x\to0}\left(\frac{1}{x^2}+\frac{(2013)^x}{e^x-1}-\frac{1}{e^x-1}\right)$

$= \lim_{x\to0}\frac{1}{x^2}+\lim_{x\to0}\frac{(2013)^x-1}{x}\times\frac{x}{e^x-1}$

$= \infty+\log_e(2013)$

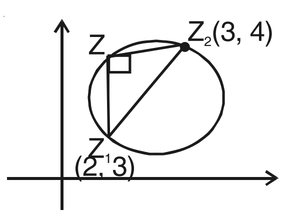
$= +\infty$

37. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$ be two points on the complex plane. Then the set of complex numbers z satisfying $|z-z_1|^2 + |z-z_2|^2 = |z_1-z_2|^2$ represents

(A) a straight line (B) a point (C) a circle (D) a pair of straight lines

Ans : (C)

Hints :



Clearly the locus of Z is a circle with Z_1 & Z_2 as end point of diameter.

38. Let p(x) be a quadratic polynomial with constant term 1. Suppose p(x) when divided by x – 1 leaves remainder 2 and when divided by x + 1 leaves remainder 4. Then the sum of the roots of p(x) = 0 is

(A) –1 (B) 1 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

Ans : (D)

Hints : $P(x) = ax^2 + bx + 1$

$P(1) = a + b + 1 = 2$

$P(-1) = a - b + 1 = 4$

so $b = -1$, $a = 2$

sum of roots of P(x) is $\frac{-b}{a} = \frac{1}{2}$

39. Eleven apples are distributed among a girl and a boy. Then which one of the following statements is true ?

(A) At least one of them will receive 7 apples

(B) The girl receives at least 4 apples or the boy receives at least 9 apples

(C) The girl receives at least 5 apples or the boy receives at least 8 apples

(D) The girl receives at least 4 apples or the boy receives at least 8 apples

Ans : ()

Hints :

40. Five numbers are in H.P. The middle term is 1 and the ratio of the second and the fourth terms is 2:1. Then the sum of the first three terms is

(A) 11/2 (B) 5 (C) 2 (D) 14/3

Ans : (A)

Hints : Let a, b, 1, c, d are H.P

so $\frac{1}{a}, \frac{1}{b}, 1, \frac{1}{c}, \frac{1}{d}$ are A.P, b = 2C.

$\frac{1}{b}+\frac{1}{c}=2$ so $b = \frac{3}{2}$, $c = \frac{3}{4}$, $a = 3$.

so, sum of the first three terms = $3+\frac{3}{2}+1=\frac{11}{2}$

41. The limit of $\left\{\frac{1}{x}\sqrt{1+x}-\sqrt{1+\frac{1}{x^2}}\right\}$ as $x \to 0$

(A) does not exist (B) is equal to 1/2 (C) is equal to 0 (D) is equal to 1

Ans : (A)

Hints : R.H.L. $\lim\limits_{x\to0}\frac{\sqrt{1+x}}{x}-\frac{\sqrt{x^2+1}}{x} = \lim\limits_{x\to0}\frac{\sqrt{1+x}-\sqrt{x^2+1}}{x}\times\frac{\sqrt{1+x}+\sqrt{x^2+1}}{\sqrt{1+x}+\sqrt{x^2+1}}$

$= \lim\limits_{x\to0}\frac{1+x-1-x^2}{x\left(\sqrt{1+x}+\sqrt{1+x^2}\right)}$

$= \lim\limits_{x\to0}\frac{x(1-x)}{x\left(\sqrt{1+x}+\sqrt{1+x^2}\right)} = \frac{1}{2}$. L.H.L. $= \lim\limits_{x\to0}\frac{\sqrt{1+x}+\sqrt{x^2+1}}{x} = \infty$

R.H.L. $\neq$ L.H.L.

42. The maximum and minimum values of $\cos^6\theta + \sin^6\theta$ are respectively

(A) 1 and 1/4 (B) 1 and 0 (C) 2 and 0 (D) 1 and 1/2

Ans : (A)

Hints : $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta.\cos^2\theta$

$= 1-\frac{3}{4}\sin^2 2\theta$

43. If a, b, c are in A.P., then the straight line $ax + 2by + c = 0$ will always pass through a fixed point whose co-ordinates are

(A) (1, –1) (B) (–1, 1) (C) (1, –2) (D) (–2, 1)

Ans : (A)

Hints : $a(x + y) + c(y + 1) = 0$

$x = 1, y = -1$

44. If one end of a diameter of the circle $3x^2 + 3y^2 - 9x + 6y + 5 = 0$ is (1, 2) then the other end is

(A) (2, 1) (B) (2, 4) (C) (2, –4) (D) (–4, 2)

Ans : (C)

Hints : Center $\left(\frac{3}{2}, -1\right)$

Let the other point be (h, k) $\frac{h+1}{2} = \frac{3}{2} \Rightarrow h = 2$

$\frac{k+2}{2} = -1 \Rightarrow k = -4$

45. The value of $\cos^2 75^0 + \cos^2 45^0 + \cos^2 15^0 - \cos^2 30^0 - \cos^2 60^0$ is

(A) 0 (B) 1 (C) 1/2 (D) 1/4

Ans : (C)

Hints : $\cos 15^\circ = \sin 75^\circ$

$\cos^2 75^\circ + \cos^2 45^\circ + \cos^2 15^\circ - \cos^2 30^\circ - \cos^2 60^\circ$

$= \cos^2 75^\circ + \sin^2 75^\circ + \cos^2 45^\circ - \cos^2 30^\circ - \cos^2 60^\circ$

$= 1 + \frac{1}{2} - \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$

46. Suppose $z = x + iy$ where x and y are real numbers and $i = \sqrt{-1}$. The points (x, y) for which $\frac{z-1}{z-i}$ is real, lie on

(A) an ellipse (B) a circle (C) a parabola (D) a straight line

Ans : (D)

Hints : $\frac{(x-1)+iy}{x+i(y-1)} = k$

$\Rightarrow \frac{(x-1)+iy}{x+i(y-1)} \times \frac{x-i(y-1)}{x-i(y-1)} = k$

$\Rightarrow$ Imaginary part $= 0 \Rightarrow x + y = 1$

47. The equation $2x^2 + 5xy - 12y^2 = 0$ represents a

(A) circle

(B) pair of non-perpendicular intersecting straight lines

(C) pair of perpendicular straight lines

(D) hyperbola

Ans : (B)

Hints : $2x^2 + 5xy - 12y^2 = 0$

$(x + 4y)(2x - 3y) = 0$

48. The line y = x intersects the hyperbola $\frac{x^2}{9}-\frac{y^2}{25}=1$ at the points P and Q. The eccentricity of ellipse with PQ as major axis and minor axis of length $\frac{5}{\sqrt{2}}$ is

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{5}{\sqrt{3}}$ (C) $\frac{5}{9}$ (D) $\frac{25}{9}$

Ans : ()

Hints : For y = x, $x^2\left(\frac{1}{9}-\frac{1}{25}\right)=1 \Rightarrow x^2=\left(\frac{5\times3}{4}\right)^2=\left(\frac{15}{4}\right)^2$

$\Rightarrow a=\frac{15\sqrt{2}}{4}=\frac{15}{2\sqrt{2}}$, $b=\frac{5}{2\sqrt{2}}$

$e^2=1-\frac{1}{9}=\frac{8}{9}$

$e=\frac{2\sqrt{2}}{3}$

49. The equation of the circle passing through the point (1, 1) and the points of intersection of $x^2+y^2-6x-8=0$ and $x^2+y^2-6=0$ is

(A) $x^2+y^2+3x-5=0$ (B) $x^2+y^2-4x+2=0$ (C) $x^2+y^2+6x-4=0$ (D) $x^2+y^2-4y-2=0$

Ans : (A)

Hints : Circle passing through point of intersection of ircles is $x^2+y^2-6x-8+\lambda(x^2+y^2-6)=0$

It passes through (1, 1) so, $\lambda=-3$

Circle is $x^2+y^2+3x-5=0$

50. Six positive numbers are in G.P., such that the r product is 1000. If the fourth term is 1, then the last term is

(A) 1000 (B) 100 (C) 1/100 (D) 1/1000

Ans : (C)

Hints : $\frac{a}{r^5},\frac{a}{r^3},\frac{a}{r}$, ar, ar^3, ar^5

$a^6=1000 \Rightarrow a^2=10$

given $ar=1, \Rightarrow a^2r^2=1$, $r^2=\frac{1}{10}$

$ar^5=\frac{1}{100}$

51. In the set of all 3×3 real matrices a relation is defined as follows. A matrix A is related to a matrix B if and only if there is a non-singular 3×3 matrix P such that $B=P^{-1}AP$. This relation is

(A) Reflexive, Symmetric but not Transitive (B) Reflexive, Transitive but not Symmetric

(C) Symmetric, Transitive but not Reflexive (D) an Equivalence relation

Ans : (D)

Hints : $R=\{(A,B) \mid B=P^{-1}AP\}$

$A=I^{-1}AI \Rightarrow (A,A)\in R \Rightarrow R$ is reflexive

Let $(A,B)\in R$, $B=P^{-1}AP$

$PB=AP \Rightarrow PBP^{-1}=A \Rightarrow A=(P^{-1})^{-1}B(P^{-1})$

$\Rightarrow$ (B, A) $\in$ R, $\Rightarrow$ R is symmetric
Let (A, B) $\in$ R, (B, C) $\in$ R
$A = P^{-1}BP$ and $B = Q^{-1}CQ$
$A = P^{-1}Q^{-1}C\,QP = (QP)^{-1}\,C(QP) \Rightarrow (A, C) \in R$

52. The number of lines which pass through the point (2, –3) and are at the distance 8 from the point (–1, 2) is
(A) infinite (B) 4 (C) 2 (D) 0
Ans : (D)
Hints : The maximum distance of the line passing through (2, –3) from (–1, 2) is $\sqrt{34}$. So there is no possible line

53. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ and $3b^2 = 16ac$ then
(A) $\alpha = 4\beta$ or $\beta = 4\alpha$ (B) $\alpha = -4\beta$ or $\beta = -4\alpha$ (C) $\alpha = 3\beta$ or $\beta = 3\alpha$ (D) $\alpha = -3\beta$ or $\beta = -3\alpha$
Ans : (C)
Hints : $3b^2 = 16ac$

$$\Rightarrow 3\left(\frac{b}{a}\right)^2 = 16\frac{c}{a}$$

$$3(\alpha+\beta)^2 = 16\alpha\beta, \quad 3\alpha^2 + 3\beta^2 = 10\alpha\beta$$

$$3\frac{\alpha}{\beta} + 3\frac{\beta}{\alpha} = 10, \text{ Let } \frac{\alpha}{\beta} = y$$

$3y^2 - 10y + 3 = 0, \Rightarrow (3y - 1)(y - 3) = 0$

$$y = \frac{1}{3} \text{ or } y = 3$$

$\Rightarrow 3\alpha = \beta$ or $\alpha = 3\beta$

54. For any two real numbers a and b, we define a R b if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is
(A) Reflexive but not Symmetric (B) Symmetric but not transitive
(C) Transitive but not Reflexive (D) an Equivalence relation
Ans : (D)
Hints : $\sin^2 a + \cos^2 b = 1$
Reflexive : $\sin^2 a + \cos^2 a = 1$
$\Rightarrow$ aRa
$\sin^2 a + \cos^2 b = 1, \; 1 - \cos^2 a + 1 - \sin^2 b = 1$
$\sin^2 b + \cos^2 a = 1$
$\Rightarrow$ bRa
Hence symmetric Let aRb bR
$\sin^2 a + \cos^2 b = 1$ .... (1)
$\sin^2 b + \cos^2 c = 1$ .... (2)
(1) + (2)
$\sin^2 a + \cos^2 c = 1$
Hence transitive therefore equivalence relation.

55. Let n be a positive even integer. The ratio of the largest coefficient and the 2nd largest coefficient in the expansion of $(1 + x)^n$ is 11:10. The the number of terms in the expansion of $(1 + x)^n$ is
(A) 20 (B) 21 (C) 10 (D) 11
Ans : (B)
Hints : Let n = 2m

$$\Rightarrow \frac{{}^{2m}C_m}{{}^{2m}C_{m-1}} = \frac{11}{10}$$

$\Rightarrow$ m = 10, n = 20
Total No. of term = 21

56. Let exp(x) denote exponential function e^x. If $f(x) = \exp\left(x^{\frac{1}{x}}\right)$, $x > 0$ then the minimum value of f in the interval [2, 5] is

(A) $\exp\left(e^{\frac{1}{e}}\right)$ (B) $\exp\left(2^{\frac{1}{2}}\right)$ (C) $\exp\left(5^{\frac{1}{5}}\right)$ (D) $\exp\left(3^{\frac{1}{3}}\right)$

Ans : (C)

Hints : $f(x) = e^{x^{1/x}}$

$g(x) = \log f(x) = x^{\frac{1}{x}}$

g(x) increases is (0, e) & decreases in (e, ∞) it will be minimum at either 2 or 5

$2^{\frac{1}{2}} > 5^{\frac{1}{5}} \Rightarrow$ minimum value of $f(x) = e^{5^{\frac{1}{5}}}$

57. The sum of the series $\frac{1}{1\times2}\,{}^{25}C_0 + \frac{1}{2\times3}\,{}^{25}C_1 + \frac{1}{3\times4}\,{}^{25}C_2 + \ldots\ldots\ldots + \frac{1}{26\times27}\,{}^{25}C_{25}$

(A) $\frac{2^{27}-1}{26\times27}$ (B) $\frac{2^{27}-28}{26\times27}$ (C) $\frac{1}{2}\left(\frac{2^{26}+1}{26\;\;27}\right)$ (D) $\frac{2^{26}-1}{52}$

Ans : (B)

Hints : On integrate $(1 + x)^{25}$ twice 1st under the limit 0 to x & then 0 to 1 we get sum = $\frac{2^{27}-28}{26\times27}$

58. Five numbers are in A.P. with common difference ≠ 0 If the 1st, 3rd and 4th terms are in G.P., then

(A) the 5th term is always 0 (B) the 1st term is always 0

(C) the middle term is always 0 (D) the middle term is always –2

Ans : (A)

Hints : Let a, a + d, a + 2d, a + 3d, a + 4d are five number in A.P.

Given $\frac{a+2d}{a} = \frac{a+3d}{a+2d}$

$\Rightarrow a + 4d = 0$

59. The minimum value of the function $f(x) = 2|x - 1| + |x - 2|$ is

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (B)

Hints : f(x) will be minimum at x = 1

60. If P, Q, R are angles of an isosceles triangle and $\angle P = \frac{\pi}{2}$, then the value of $\left(\cos\frac{P}{3} - i\sin\frac{P}{3}\right)^3$ + (cosQ + isinQ) (cosR – isinR) + (cosP – isinP) (cosQ – isinQ) (cosR – isinR) is equal to

(A) i (B) –i (C) 1 (D) –1

Ans : (B)

Hints : $P = \frac{\pi}{2}$, $Q = R = \frac{\pi}{4}$

$$\left(\cos\frac{P}{3} - i\sin\frac{P}{3}\right)^3 + (\cos Q + i\sin Q)(\cos R - i\sin R) + (\cos P - i\sin P)(\cos Q - i\sin Q)(\cos R - i\sin R)$$

$= e^{-ip} + e^{iQ}.e^{-iR} + e^{-iP} \times e^{-iQ} \times^{-iR}$

$= e^{-i\pi/2} + e^{i\times(Q-R)} + e^{-i(P+Q+R)}$

$= e^{-i\pi/2} + e^{0} + e^{-i\pi}$

$= \left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right) + 1 + (\cos\pi - i\sin\pi)$

$= -i + 1 - 1 - 0 = -i$

CATEGORY - II

Q. 61 – Q. 75 carry two marks each, for which only option is correct. Any wrong answer will lead to deduction of 2/3 mark.

61. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}(3/4)$ with the x-axis. It intersects the parabola $y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1-x_2|$ is equal to

(A) $\frac{16}{9}$ (B) $\frac{32}{9}$ (C) $\frac{40}{9}$ (D) $\frac{80}{9}$

Ans : (B)

Hints : A(3, 1)

$y - 1 = \frac{3}{4}(x-3) \Rightarrow y = \frac{3}{4}x + 1 - \frac{9}{4}$ or, $y = \frac{3}{4}x - \frac{5}{4}, y^2 = 4(x-3)$

$\left(\frac{3x-5}{4}\right)^2 = 4(x-3)$ or, $9x^2 - 30x + 25 = 64x - 64 \times 3$

$9x^2 - 94x + 217 = 0$

$x_1 + x_2 = \frac{94}{9}$

$x_1x_2 = \frac{217}{9}$

$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = \left(\frac{94}{9}\right)^2 \; 4\frac{217}{9}$

$= \frac{(94)^2 - 4.217.9}{9^2} = \frac{32}{9}$

62. Let [a] denote the greatest integer which is less than or equal to a. Then the value of the integral

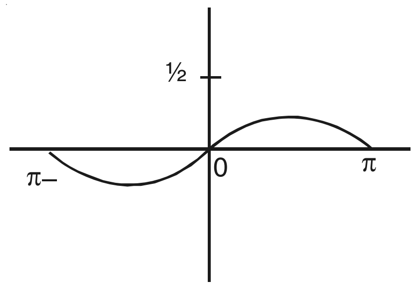
$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\sin x \cos x] dx$ is

(A) $\frac{\pi}{2}$ (B) π (C) $-\pi$ (D) $-\pi/2$

Ans : (D)

Hints : $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{1}{2}\text{Sin}\,2x\right] dx$ Put $2x = \theta$ or, $2dx = d\theta$

$$=\frac{1}{2}\int_{-\pi}^{\pi}\left[\frac{1}{2}\text{Sin}\theta\right]d\theta=\frac{1}{2}\left[\int_{-\pi}^{0}(-1)\,dx+\int_{0}^{\pi}0dx\right]$$



$$=\frac{1}{2}(-)(x)_{-\pi}^{0}+0=-\frac{1}{2}(0+\pi)=-\frac{\pi}{2}$$

63. If $P=\begin{pmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{pmatrix}$ then P^5 equals

(A) P (B) 2P (C) –P (D) –2P

Ans : (A)

Hints : $P^2=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$

$=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ $P^2 = p$; $p^4= p$; $p^5=p^2=p$

64. If $\sin^2\theta+3\cos\theta=2$, then $\cos^3\theta+\sec^3\theta$ is

(A) 1 (B) 4 (C) 9 (D) 18

Ans : (D)

Hints : $\text{Cos}^2\theta-3\text{Cos}\theta+1=0$ or, $\text{Cos}\theta+\frac{1}{\text{Cos}\theta}=3$,

$C^3+\frac{1}{C^3}+3.C.\frac{1}{C}\left(C+\frac{1}{C}\right)=27$ or, $\text{Cos}^3\theta+\sec^3\theta+3.3=27$

$\text{Cos}^3\theta+\sec^3\theta=18$

65. $x=1+\frac{1}{2\times\lfloor 1}+\frac{1}{4\times\lfloor 2}+\frac{1}{8\times\lfloor 3}+........$ and $y=1+\frac{x^2}{\lfloor 1}+\frac{x^4}{\lfloor 2}+\frac{x^6}{\lfloor 3}+......$. Then the value of $\log_e y$ is

(A) e (B) e^2 (C) 1 (D) 1/e

Ans : (A)

Hints : $y=e^{x^2}, x=e^{\frac{1}{2}}$

$\ln y = x^2 = e$

66. The value of the infinite series $\frac{1^2+2^2}{\lfloor 3}+\frac{1^2+2^2+3^2}{\lfloor 4}+\frac{1^2+2^2+3^2+4^2}{\lfloor 5}$.......... is

(A) e (B) 5e (C) $\frac{5e}{6}-\frac{1}{2}$ (D) $\frac{5e}{6}$

Ans : (C)

Hints : $\sum_{r=1}^{n}\frac{1^2+2^2+3^2+.......(r+1)^2}{\lfloor r+2}=\frac{1}{6}\sum\frac{2r+3}{\lfloor r}=\frac{1}{6}\sum_{r=1}\left(\frac{2}{\lfloor r-1}+\frac{3}{\lfloor r}\right)=\frac{1}{6}\left[2e+3(e-1)\right]=\frac{5}{6}e-\frac{1}{2}$

67. The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x-x\cos x)}{x(x+\sin x)}dx$ is equal to

(A) $\log_e\left(\frac{2(\pi+3)}{2\pi+3\sqrt{3}}\right)$ (B) $\log_e\left(\frac{\pi+3}{2(2\pi+3\sqrt{3})}\right)$ (C) $\log_e\left(\frac{2\pi+3\sqrt{3}}{2(\pi+3)}\right)$ (D) $\log_e\left(\frac{2(2\pi+3\sqrt{3})}{\pi+3}\right)$

Ans : (A)

Hints : $I=\int\frac{(x+\text{Sin}x)-x(1+\text{Cos}x)}{x(x+\text{Sin}x)}dx=\int\left[\frac{1}{x}-\frac{1+\text{Cos}x}{x+\text{Sin}x}\right]dx$

$$=\left(\ln\frac{\pi}{3}-\ln\frac{\pi}{6}\right)-\left[\ln\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)-\ln\left(\frac{\pi}{6}+\frac{1}{2}\right)\right]$$

$$=\ln\left(\frac{2\pi+6}{2\pi+3\sqrt{3}}\right)$$

68. Let $f(x)=x\left(\frac{1}{x-1}+\frac{1}{x}+\frac{1}{x+1}\right), x>1$ Then

(A) $f(x)\le 1$ (B) $1<f(x)\le 2$ (C) $2<f(x)\le 3$ (D) $f(x)>3$

Ans : (D)

Hints : $f(x)=x\left(\frac{1}{x-1}+\frac{1}{x+1}+\frac{1}{x}\right)$

$$=x\left(\frac{x+1+x-1}{x^2-1}+1\right)=\frac{2x^2}{x^2-1}+1=\frac{2}{1-\frac{1}{x^2}}+1$$

69. Let $F(x) = \int_0^x \frac{\cos t}{(1+t^2)} dt, 0 \le x \le 2\pi$. Then

(A) F is increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$

(B) F is increasing in $(0, \pi)$ and decreasing in $(\pi, 2\pi)$

(C) F is increasing in $(\pi, 2\pi)$ and decreasing in $(0, \pi)$

(D) F is increasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Ans : (D)

Hints : $F'(x) = \frac{\operatorname{Cos} x}{1+x^2}$

$$\operatorname{Cos} x > 0 \Rightarrow x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$$

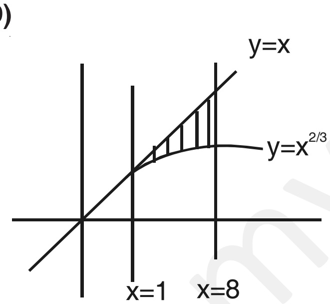
$$\operatorname{Cos} x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

70. Let $f(x) = x^{2/3}$, $x \ge 0$. Then the area of the region enclosed by the curve $y = f(x)$ and the three lines $y = x$, $x = 1$ and $x = 8$ is

(A) $\frac{63}{2}$ (B) $\frac{93}{5}$ (C) $\frac{105}{7}$ (D) $\frac{129}{10}$

Ans : (D)

Hints :



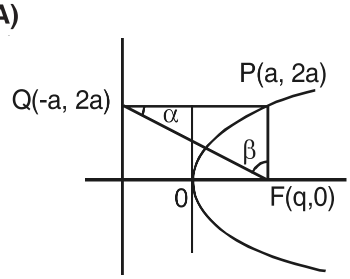
$$A = \int_1^8 \left[x - x^{\frac{2}{3}}\right] dx = \frac{1}{2}\left(x^2\right)_1^8 - \frac{3}{5}\left(x^{5/3}\right)_1^8 = \frac{1}{2}(64-1) - \frac{3}{5}(32-1) = \frac{129}{10}$$

71. Let P be a point on the parabola $y^2 = 4ax$ with focus F. Let Q denote the foot of the perpendicular from P onto the directrix. Then $\frac{\tan\angle PQF}{\tan\angle PFQ}$ is

(A) 1 (B) 1/2 (C) 2 (D) 1/4

Ans : (A)

Hints :



PQ = PF so $\alpha = \beta$

72. An objective type test paper has 5 questions. Out of these 5 questions, 3 questions have four options each (A, B, C, D) with one option being the correct answer. The other 2 questions have two options each, namely True and False. A candidate randomly ticks the options. Then the probability that he/she will tick the correct option in at least four questions, is

(A) $\frac{5}{32}$ (B) $\frac{3}{128}$ (C) $\frac{3}{256}$ (D) $\frac{3}{64}$

Ans : (D)

Hints : $n(S) = 4^3 . 2^2$, $n(e) = ({}^3C_1 .3 + {}^2C_1 . 1) + 1$

$$P = \frac{3.3+2+1}{4^3.2^2} = \frac{12}{4.4.4.4} = \frac{3}{64}$$

73. A family of curves is such that the length intercepted on the y-axis between the origin and the tangent at a point is three times the ordinate of the point of contact. The family of curves is

(A) $xy = c$, c is a constant (B) $xy^2 = c$, c is a constant

(C) $x^2y = c$, c is a constant (D) $x^2y^2 = c$, c is a constant

Ans : (C)

Hints : $y - x\frac{dy}{dx} = 3y$ or, $-x\frac{dy}{dx} = 2y$

or, $\frac{dy}{y} = -2\frac{dx}{x}$ (Integrate) or, $\ln y = -2 \ln x + \ln c$ or, $\ln y + \ln x^2 = \ln$ or, $y\ x^2 = c$

74. The solution of the differential equation $\left(y^2 + 2x\right)\frac{dy}{dx} = y$ satisfies $x = 1$, $y = 1$. Then the solution is

(A) $x = y^2(1 + \log_e y)$ (B) $y = x^2(1 + \log_e x)$ (C) $x = y^2(1 - \log_e y)$ (D) $y = x^2(1 - \log_e x)$

Ans : (A)

Hints : $\frac{dx}{dy} = \frac{y^2+2x}{y}$ or, $\frac{dx}{dy} - \frac{2}{y}.x = y$ or, $IF = e^{-\int \frac{2}{y}dy} = e^{-\ln y} = \frac{1}{y^2}$ or, $x.\frac{1}{y^2} = \int y.\frac{1}{y^2}dy + c$

or, $\frac{x}{y^2} = \ln y + c$ $\Rightarrow y(1) = 1$ or, $1 = 0 + c$

$x = y^2(\ln y + 1)$

75. The solution of the differential equatio y sin (x/y) dx= (x sin(x/y)–y) dy satisfying $y\left(\pi/4\right) = 1$ is

(A) $\cos\frac{x}{y} = -\log_e y + \frac{1}{\sqrt{2}}$ (B) $\sin\frac{x}{y} = \log_e y + \frac{1}{\sqrt{2}}$ (C) $\sin\frac{x}{y} = \log_e x - \frac{1}{\sqrt{2}}$ (D) $\cos\frac{x}{y} = -\log_e x - \frac{1}{\sqrt{2}}$

Ans : ()

Hints : $\frac{dx}{dy} = \frac{\frac{x}{y}\sin\frac{x}{y} - 1}{\sin\frac{x}{y}}$ Put $\frac{x}{y} = \theta$ or, $x = y . \theta$ then, $\frac{dx}{dy} = \theta + y\frac{d\theta}{dy}$

or, $\theta + y\frac{d\theta}{dy} = \frac{\theta \text{Sin}\theta - 1}{\text{Sin}\theta}$ or, $y\frac{d\theta}{dy} = \frac{\theta\text{Sin}\theta - 1}{\text{Sin}\theta} - \theta = \frac{-1}{\text{Sin}\theta}$ or, $\sin\theta d\theta = -\frac{dy}{y}$

or, $\ln y = \cos\theta - c$ $y\left(\frac{\pi}{4}\right) = 1$ $\Rightarrow c = \frac{1}{\sqrt{2}}$

or, $\ln y = \cos\frac{x}{y} - \frac{1}{\sqrt{2}}$

CATEGORY - 3

Q. 76 – Q. 80 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question –irrespective of the number of correct options marked

76. The area of the region enclosed between parabola $y^2 = x$ and the line $y = mx$ is $\frac{1}{48}$. Then the value of m is

(A) –2 (B) –1 (C) 1 (D) 2

Ans : (A, D)

Hints : . $A = \int_0^{\frac{1}{m}} \left(\frac{y}{m} - y^2\right) dy = \left|\frac{1}{2m}\left(y^2\right)_0^{1/m} - \frac{1}{3}\left(y^3\right)_0^{\frac{1}{m}}\right|$

$\frac{1}{48} = \left|\frac{1}{2m^3} - \frac{1}{3m^3}\right|$ or, $\frac{1}{48} = \left|\frac{1}{6m^3}\right|$

(1) $m^3 = \frac{1}{6}.48 = 1.8 = 8 \text{ or}, m = 2$

(2) $m^3 = -\frac{1}{6}.48 = -1.8 \text{ or}, m = -2$

77. Consider the system of equations:

$$x + y + z = 0$$
$$\alpha x + \beta y + \gamma z = 0$$
$$\alpha^2 x + \beta^2 y + \gamma^2 z = 0$$

Then the system of equations has

(A) A unique solution for all values α, β, γ

(B) Infinite number of solutions if any two of $\alpha, \beta\ \gamma$ are equal

(C) A unique solution if α, β, γ are distinct

(D) More than one, but finite number of solutions depending on values of α, β, γ

Ans : (B, C)

Hints : $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

78. The equations of the circles which touch both the axes and the line $4x + 3y = 12$ and have centres in the first quadrant, are

(A) $x^2 + y^2 - x - y + 1 = 0$ (B) $x^2 + y^2 - 2x - 2y + 1 = 0$

(C) $x^2 + y^2 - 12x - 12y + 36 = 0$ (D) $x^2 + y^2 - 6x - 6y + 36 = 0$

Ans : (B, C)

Hints : $\left|\frac{4h + 3h - 12}{5}\right| = h$ or, $|7h - 12| = 5h$

(i) $7h - 12 = 5h$ or, $2h = 12$ or, $h = 6$ Centre (6, 6)

$x^2 + y^2 - 12x - 12y + 36 = 0$

(ii) $7h - 12 = -5h$ or, $h = 1$ (1, 1) or, $r = 1$

79. Which of the following real valued functions is/are not even functions?

(A) $f(x) = x^3 \sin x$

(B) $f(x) = x^2 \cos x$

(C) $f(x) = e^x x^3 \sin x$

(D) $f(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to x

Ans : (C, D)

Hints : (A) $f(-x) = f(x)$ even (B) $f(-x) = f(x)$ even

(C) $f(-x) \neq f(x)$ not even (D) $f(-x) \neq f(x)$ not even

80. Let sin α, cos α be the roots of the equation $x^2 - bx + c = 0$. Then which of the following statements is/are correct?

(A) $c \leq \frac{1}{2}$ (B) $b \leq \sqrt{2}$ (C) $c > \frac{1}{2}$ (D) $b > \sqrt{2}$

Ans : (A, B)

Hints : $\text{Sin}\alpha + \text{Cos}\alpha = b, \text{Sin}\alpha.\text{Cos}\alpha = c$

$b \leq \sqrt{2},$ $c = \frac{1}{2}\sin 2\alpha$

$c \leq \frac{1}{2}$

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Code-☼

ANSWERS & HINTS
for
WBJEE - 2013
SUB : PHYSICS

CATEGORY - I

Q. 1 – Q. 45 carry one mark each, for which only one option is correct. Any wrong answer will be lead to deduction of 1/3 mark.

1. The equation of state of a gas is given by $\left(P+\frac{a}{V^3}\right)$(V- b²) = cT, where P, V, T are pressure, volume and temperature respectively, and a, b, c are constants. The dimensions of a and b are espectively

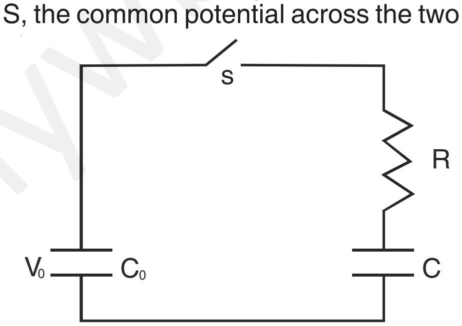
(A) ML^8T^{-2} and $L^{3/2}$ (B) ML^5T^{-2} and L^3 (C) ML^5T^{-2} a d L (D) ML^6T^{-2} and $L^{3/2}$

Ans : (A)

Hints : $[P] = \left[\frac{a}{V^3}\right] \Rightarrow ML^{-1}T^{-2} = \frac{a}{L^9} \Rightarrow a = ML^8T^{-2}$

$[V] = [b^2] \Rightarrow L^3 = b^2 \therefore b = L^{\frac{3}{2}}$

2. A capacitor of capacitance C_0 is charged to a pot ntia V_0 and is connected with another capacitor of capacitance C as shown. After closing the switch S, the common potential across the two capacitors becomes V. The capacitance C is given by



(A) $\frac{C_0(V_0 - V)}{V_0}$ (B) $\frac{C_0(V - V_0)}{V_0}$ (C) $\frac{C_0(V + V_0)}{V}$ (D) $\frac{C_0(V_0 - V)}{V}$

Ans : (D)

Hints : Charge on isolated plates remains same

$$C_0V_0 = C_0V + CV \Rightarrow \frac{C_0V_0 - C_0V}{V} = C \Rightarrow C = \frac{C_0(V_0 - V)}{V}$$

3. The r.m.s. speed of the molecules of a gas at 100°C is ν. The temperature at which the r.m.s. speed will be $\sqrt{3}\nu$ is

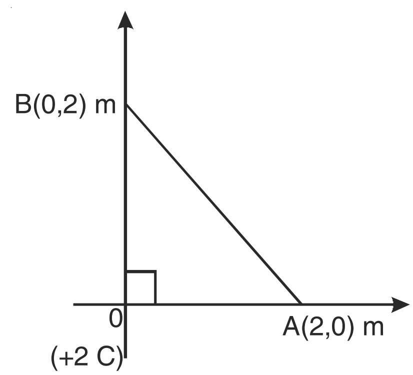
(A) 546°C (B) 646°C (C) 746°C (D) 846°C

Ans : (D)

Hints : $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow V = \sqrt{\frac{3R \times 373}{M}} \Rightarrow \sqrt{3}V = \sqrt{\frac{3R \times T}{M}} \Rightarrow \sqrt{3} = \sqrt{\frac{T}{373}} \Rightarrow 3 = \frac{T}{373} \Rightarrow T = 3 \times 373 = 1119K$

$T\,(°C) = 846°C$

4. As shown in the figure below, a charge +2C is situated at the origin O and another charge +5C is on the x-axis at the point A. The later charge from the point A is then brought to a point B on the y-axis. The work done is (given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F)



(A) 45×10^9 J (B) 90×10^9 J (C) Zero (D) -45×10^9 J

Ans : (C)

Hints : Work done = $U_{final} - U_{initial} = \frac{1}{4\pi\varepsilon_0} \times 2 \times 5 \times \left(\frac{1}{2} - \frac{1}{2}\right) = 0$

5. A frictionless piston-cylinder based enclosure contains some amount of gas at a pressure of 400 kPa. Then heat is transferred to the gas at constant pressure in a quasi-sta ic process. The piston moves up slowly through a height of 10cm. If the piston has a cross-section area of 0.3 m^2, the work done by the gas in this process is

(A) 6 kJ (B) 12 kJ (C) 7.5 kJ (D) 24 kJ

Ans : (B)

Hints : $W_{const-Press} = P \times \Delta V = 400 \times 10^3 \times 0\ 3 \times 10 \times 10^{-2} = 400 \times 10 \times 3 = 12000 = 12$ kJ

6. An electric cell of e.m.f. E is connected acro s a copper wire of diameter d and length l. The drift velocity of electrons in the wire is v_d. If the length of the wire is changed to 2l, the new drift velocity of electrons in the copper wire will be

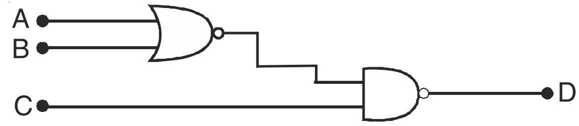
(A) ν_d (B) $2\nu_d$ (C) $\nu_d/2$ (D) $\nu_d/4$

Ans : (C)

Hints : $\nu_d = \frac{i}{neA}$ and $\nu_d^1 = \frac{E}{\rho \times 2l \times n \times e}$

$$= \frac{E}{R \times neA} \Rightarrow \frac{E \times \cancel{A}}{\rho \times l \times n \times e \times \cancel{A}} \Rightarrow \frac{E}{\rho \times l \times n \times e} \Rightarrow \frac{\nu^1_d}{\nu_d} = \frac{1}{2} \Rightarrow \nu^1_d = \frac{\nu_d}{2}$$

7. A NOR gate and a NAND gate are connected as shown in the figure. Two different sets of inputs are given to this set up. In the first case, the input to the gates are A=0, B=0, C=0. In the second case, the inputs are A=1, B=0, C=1. The output D in the first case and second case respectively are



(A) 0 and 0 (B) 0 and 1 (C) 1 and 0 (D) 1 and 1

Ans : (D)

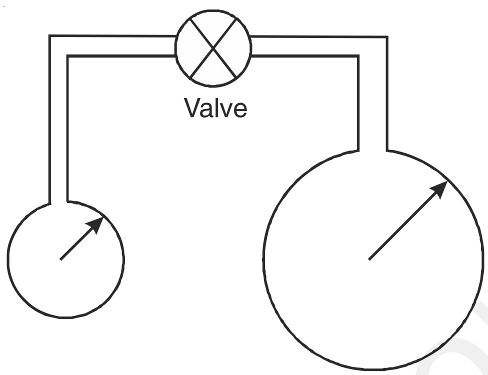
8. A bar magnet has a magnetic moment of 200 A.m^2. The magnet is suspended in a magnetic field of 0.30 NA^{-1} m^{-1}. The torque required to rotate the magnet from its equilibrium position through an angle of 30°, will be

(A) 30N m (B) $30\sqrt{3}$ N m (C) 60 N m (D) $60\sqrt{3}$ N m

Ans : (A)

Hints : $\vec{T} = \vec{M} \times \vec{B} \Rightarrow |\vec{T}| = M \times B \times \sin\theta = 200 \times 0.3 \times \frac{1}{2} = 100 \times 0.3 = 30$ Nm

9. Two soap bubbles of radii r and 2r are connected by a capillary tube-valve arrangement as shown in the diagram. The valve is now opened. Then which one of the following will result:



(A) the radii of the bubbles will remain unchanged

(B) the bubbles will have equal radii

(C) The radius of the smaller bubble will increase and tha of the bigger bubble will decrease

(D) The radius of the smaller bubble will decrease and hat of the bigger bubble will increase

Ans : (D)

Hints : Pressure – Difference = $\frac{4T}{r}$

For smaller soap $P_{atm} - P_{i1} = \frac{4T}{r}$

For bigger soap $P_{atm} \quad P_{i2} = \frac{4T}{2r}$

As pressure inside smaller bubble is greater than pressure inside bigger bubble . so air flows from smaller to bigger.

10. An ideal mono-atomic gas of given mass is heated at constant pressure. In this process, the fraction of supplied heat energy used for the increase of the internal energy of the gas is

(A) 3/8 (B) 3/5 (C) 3/4 (D) 2/5

Ans : (B)

Hints : Fraction = $\frac{\Delta U}{\Delta Q} = \frac{C_V}{C_P} = \frac{1}{\Upsilon} = \frac{3}{5}$

11. The velocity of a car travelling on a straight road is 36 kmh^{-1} at an instant of time. Now travelling with uniform acceleration for 10 s, the velocity becomes exactly double. If the wheel radius of the car is 25 cm, then which of the following numbers is the closest to the number of revolutions that the wheel makes during this 10 s?

(A) 84 (B) 95 (C) 126 (D) 135

Ans : (B)

Hints : $\theta = 2\pi n = \dfrac{\left(\dfrac{v_f^2}{r^2} - \dfrac{v_i^2}{r^2}\right)}{\left(2\dfrac{a}{r}\right)} \Rightarrow \quad n = \dfrac{v_f^2 - v_i^2}{(2ar)2\pi} \approx 95$

12. Two glass prisms P_1 and P_2 are to be combined together to produce dispersion without deviation. The angles of the prisms P_1 and P_2 are selected as 4° and 3° respectively. If the refractive index of prism P_1 is 1.54, then that of P_2 will be

(A) 1.48 (B) 1.58 (C) 1.62 (D) 1.72

Ans : (D)

Hints : $\delta_1 + \delta_2 = 0 \Rightarrow (\mu - 1)A_1 = (\mu_2 - 1)A_2 \Rightarrow \mu_2 = 1.72$

13. The ionization energy of the hydrogen atom is 13.6 eV. The potential energy of the electron in n = 2 state of hydrogen atom is

(A) + 3.4 eV (B) – 3.4 eV (C) + 6.8 eV (D) – 6.8 eV

Ans : (D)

Hints : $E_{n=2} = \dfrac{-13.6z^2}{n^2} \approx -3.4 \text{ ev}, \text{PE} = 2E_{n=2} \approx -6.8\text{ev}$

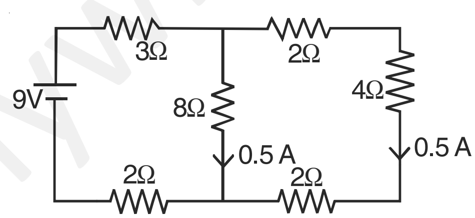
14. Water is flowing in streamline motion through a horizontal tube. The pressur at a point in the tube is p where the velocity of flow is v. At another point, where the pressure is p/2, the velocity of flow is [density of water = ρ]

(A) $\sqrt{\nu^2 + \dfrac{p}{\rho}}$ (B) $\sqrt{\nu^2 - \dfrac{p}{\rho}}$ (C) $\sqrt{\nu^2 + \dfrac{2p}{\rho}}$ (D) $\sqrt{\nu^2 - \dfrac{2p}{\rho}}$

Ans : (A)

Hints : $p + \dfrac{1}{2}\rho v^2 = \dfrac{p}{2} + \dfrac{1}{2}\rho v_1^2 \Rightarrow v_1 = \sqrt{\dfrac{p}{\rho} + v^2}$

15. In the electrical circuit shown in figure, the cu rent through the 4Ω resistor is



(A) 1A (B) 0.5 A (C) 0.25 A (D) 0.1 A

Ans : (B)

Hints : $\dfrac{1}{2}$ A

16. A wire of initial length L and radius r is stretched by a length l. Another wire of same material but with initial length 2L and radius 2r is stretched by a length 2l. The ratio of the stored elastic energy per unit volume in the first and second wire is,

(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 1 : 1

Ans : (D)

Hints : $\dfrac{U_1}{U_2} = \left\{\dfrac{(\text{strain})_1}{(\text{strain})_2}\right\}^2 = \dfrac{l^2}{L^2}\dfrac{4L^2}{4l^2} = 1 : 1$

17. A current of 1 A is flowing along positive x-axis through a straight wire of length 0.5 m placed in a region of a magnetic field given by $\vec{B} = (2\hat{i} + 4\hat{j})$ T. The magnitude and the direction of the force experienced by the wire respectively are

(A) $\sqrt{18}$N, along positive z-axis
(B) $\sqrt{20}$N, along positive x-axis
(C) 2N, along positive z-axis
(D) 4N, along positive y-axis

Ans : (C)

Hints : $i\vec{L} = \frac{1}{2}\hat{i}$; $\vec{B} = (2\hat{i} + 4\hat{j})T$

$\vec{F} = (i\vec{L}) \times \vec{B} = 2\hat{k}$ N

18. Two spheres of the same material, but of radii R and 3R are allowed to fall vertically downwards through a liquid of density σ. The ratio of their terminal velocities is

(A) 1 : 3 (B) 1 : 6 (C) 1 : 9 (D) 1 : 1

Ans : (C)

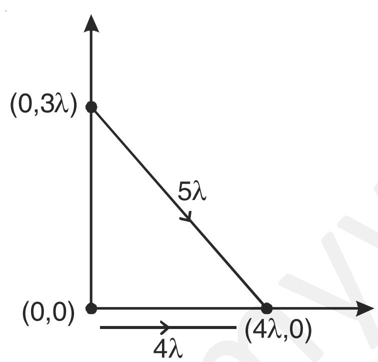
Hints : $V \propto r^2 \Rightarrow \frac{V_R}{V_{3R}} = \frac{1}{9}$

19. S_1 and S_2 are the two coherent point sources of light located in the xy-plane at points (0,0) and (0,3λ) respectively. Here λ is the wavelength of light. At which one of the following points (given as coordinates), the intensity of interference will be maximum?

(A) (3λ, 0) (B) (4λ, 0) (C) (5λ/4, 0) (D) (2λ/3, 0)

Ans : (B)

Hints :



$\Delta x = n\lambda$ (n = 1) $\Rightarrow I = I_{max}$

20. An alpha particle (^{4}He)has a mass of 4.00300 amu. A proton has mass of 1.00783 amu and a neutron has mass of 1.00867 amu respectively. The binding energy of alpha particle estimated from these data is the closest to

(A) 27.9 MeV (B) 22.3 MeV (C) 35.0 MeV (D) 20.4 MeV

Ans : (A)

Hints : $\Delta M = 2(m_p + m_n) - m_{He} = 0.0300$ amu

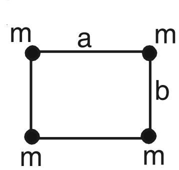
$E = \Delta M C^2 = 0.03 \times 931$ MeV ≈ 27.9 Mev

21. Four small objects each of mass *m* are fixed at the corners of a rectangular wire-frame of negligible mass and of sides a and b (a > b). If the wire frame is now rotated about an axis passing along the side of length b, then the moment of inertia of the system for this axis of rotation is

(A) $2ma^2$
(B) $4ma^2$
(C) $2m(a^2 + b^2)$
(D) $2m(a^2 - b^2)$

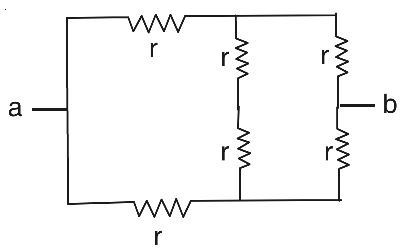
Ans : (A)

Hints :



$I = 2ma^2$

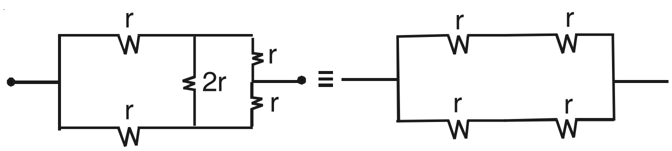
22. The equivalent resistance between the points a and b of the electrical network shown in the figure is



(A) 6 r (B) 4 r (C) 2r (D) r

Ans : (D)

Hints :



23. The de Broglie wavelength of an electron (mass = 1×10^{-30} kg, charge = 1.6×10^{-1} C) with a kinetic energy of 200 eV is (Planck's constant = 6.6×10^{-34} J s)

(A) 9.60×10^{-11} m (B) 8.25×10^{-11} m (C) 6.25×10^{-11} m (D) 5.00×10^{-11} m

Ans : (B)

Hints : $\lambda = \dfrac{h}{\sqrt{2mk}} = \dfrac{6.6\times10^{-34}}{\sqrt{2\times10^{-30}\times200\times1.6\times10^{-19}}}$

$$= \frac{6.6\times10^{-34}}{\sqrt{4\times16\times10^{-29}\times10^{-19}}} = \frac{6.6}{8}\times\frac{10^{-34}}{10^{-24}}$$

$= 0.825 \times 10^{-10} = 8.25 \times 10^{-11}$ m

24. An object placed at a distance of 16 cm from a conv x lens produces an image of magnification m (m > 1). If the object is moved towards the lens by 8 cm then again an image of magnification m is obtained. The numerical value of the focal length of the lens is

(A) 12 cm (B) 14 cm (C) 18 cm (D) 20 cm

Ans : (A)

Hints : $m = \dfrac{f}{f+u}$

As magnification can be same for two diffeent values of u only if they are of opposite sign.

$$\frac{f}{f-16} = \frac{-f}{f-8} \Rightarrow 16 - f = f - 8$$

$\Rightarrow 2f = 24$, f = 12 cm

25. The number of atoms of a radioactive substance of half-life T is N_0 at t = 0. The time necessary to decay from $N_0/2$ atoms to $N_0/10$ atoms will be

(A) $\dfrac{5}{2}T$ (B) Tln 5 (C) $T\ln\left(\dfrac{5}{2}\right)$ (D) $T\dfrac{\ln 5}{\ln 2}$

Ans : (C)

Hints : $N(t) = N_0 \times e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 \times e^{-\lambda t_1} \text{ and } \frac{N_0}{10} = N_0 \times e^{-\lambda t_2}$$

$\ell n2 = \lambda t_1$, $t_1 = \frac{\ell n2}{\lambda} = \frac{\ell n2 \times T}{\ell n2}$, $t_2 = \frac{T \times \ell n10}{\ell n2}$

$$(t_2 - t_1) = T\left[\frac{\ell n10}{\ell n2} - 1\right] = T \times \left[\frac{\ell n10 - \ell n2}{\ell n2}\right]$$

$$= T \times \frac{\ell n5}{\ell n2}$$

26. A travelling acoustic wave of frequency 500 Hz is moving along the positive x-direction with a velocity of 300 ms^{-1}. The phase difference between two points x_1 and x_2 is 60º. Then the minimum separation between the two pints is
(A) 1 mm (B) 1 cm (C) 10 cm (D) 1 m
Ans : (C)

Hints : $\lambda = \frac{300}{500} = \frac{3}{5}$

$\phi = \frac{2\pi}{\lambda}(\Delta x)$, $\frac{\pi}{3} = \frac{2\pi}{\lambda}(\Delta x)$

$\therefore \Delta x = 10$ cm

27. A mass M at rest is broken into two pieces having masses m and (M-m). The two masses are then separated by a distance r. The gravitational force between them will be the maximum when he ratio of the masses [m:(M-m)] of the two parts is
(A) 1 : 1 (B) 1 : 2 (C) 1: 3 (D) 1 : 4
Ans : (A)

Hints : $F = \frac{Gm_1m_2}{r^2}$

$$\frac{dF}{dm} = \frac{d}{dm}[m(M-m)] = 0$$

$$\boxed{m = \frac{M}{2}}, \left(\frac{m}{M-m}\right) = \frac{1}{1}$$

28. A shell of mass 5M, acted upon by no e ternal force and initially at rest, bursts into three fragments of masses M, 2M and 2M respectively. The first tw fragments move in opposite directions with velocities of magnitudes 2V and V respectively. The third fragment will
(A) move with a velocity V in a direction perpendicular to the other two
(B) move with a velocity 2V in the direction of velocity of the first fragment
(C) be at rest
(D) move with a velocity V in the direction of velocity of the second fragment
Ans : (C)
Hints : By conservation of momentum

$$0 = M \times 2\vec{V} - 2M\vec{V} + 2M\vec{V}'$$

$$\therefore \vec{V}' = 0$$

29. A bullet of mass m travelling with a speed v hits a block of mass M initially at rest and gets embedded in it. The combined system is free to move and there is no other force acting on the system. The heat generated in the process will be

(A) Zero (B) $\frac{mv^2}{2}$ (C) $\frac{Mmv^2}{2(M-m)}$ (D) $\frac{mMv^2}{2(M+m)}$

Ans : (D)

Hints : Loss in K.E. = $\frac{m_1m_2}{2(m_1+m_2)}(u_1-u_2)^2 = \frac{Mmu^2}{2(M+m)}$

30. A particle moves along X-axis and its displacement at any time is given by $x(t) = 2t^3 - 3t^2 + 4t$ in SI units. The velocity of the particle when its acceleration is zero, is

(A) 2.5 ms^{-1} (B) 3.5 ms^{-1} (C) 4.5 ms^{-1} (D) 8.5 ms^{-1}

Ans : (A)

Hints : $x(t) = (2t^3 - 3t^2 + 4t)$

$$v = \frac{dx}{dt} = (6t^2 - 6t + 4), \quad a = \left(\frac{dv}{dt}\right) = (12t - 6) = 0$$

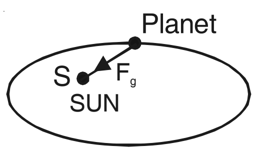
$$12t = 6, \quad t = \frac{6}{12} = \frac{1}{2}\text{sec}, \quad v = (6t^2 - 6t + 4) = 2.5 \text{ m/s}$$

31. A planet moves around the sun in an elliptical orbit with the sun at one of its foci. The physical quantity associated with the motion of the planet that remains constant with time is

(A) velocity (B) centripetal force (C) linear momentum (D) angular momentum

Ans : (D)

Hints :



Torque about the sun, S = 0

$\Rightarrow$ Angular momentum is conserved

32. The fundamental frequency of a closed pipe is equal to the frequency of the second harmonic of an open pipe. The ratio of their lengths is

(A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16

Ans : (B)

Hints : $f_{cp} = 2f_{op} \quad \Rightarrow \frac{v}{4\ell_{cp}} = 2\times\frac{v}{2\ell_{op}}$

$$\Rightarrow \frac{\ell_{cp}}{\ell_{op}} = \frac{1}{4}$$

33. A particle of mass M and charge q is released from rest in a region of uniform electric field of magnitude E. After a time t, the distance travelled by the charge is S and the kinetic energy attained by the particle is T. Then, the ratio T/S

(A) remains constant with time t
(B) varies linearly with the mass M of the particle
(C) is independent of the charge q
(D) is independent of the magnitude of the electric field E

Ans : (A)

Hints : $S = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$

$$T = \frac{1}{2}m\left(\frac{qE}{m}t\right)^2 \quad \Rightarrow \frac{T}{S} = qE$$

34. An alternating current in a circuit is given by $I = 20 \sin(100\pi t + 0.05\pi)$ A. The r.m.s. value and the frequency of current respectively are

(A) 10A & 100 Hz (B) 10A & 50 Hz (C) $10\sqrt{2}$ A & 50Hz (D) $10\sqrt{2}$ A & 100 Hz

Ans : (C)

Hints : $I = 20\sin(100\pi t + 0.05\pi)$

$$\therefore I_{rms} = \frac{20}{\sqrt{2}} = 10\sqrt{2}$$

$w = 100\pi \Rightarrow f = 50$ Hz

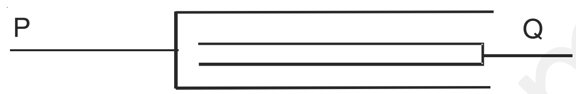
35. The specific heat c of a solid at low temperature shows temperature dependence according to the relation $c = DT^3$ where D is a constant and T is the temperature in kelvin. A piece of this solid of mass m kg is taken and its temperature is raised from 20 K to 30 K. The amount of the heat required in the process in energy units is

(A) 5×10^4 Dm (B) $(33/4) \times 10^4$ Dm (C) $(65/4) \times 10^4$ Dm (D) $(5/4) \times 10^4$ Dm

Ans : (C)

Hints : $Q = \int dQ = \int_{T_1=20}^{T_2=30} mc\,dT = \int_{20}^{30} mDT^3\,dT = \frac{65mD}{4} \times 10^4$

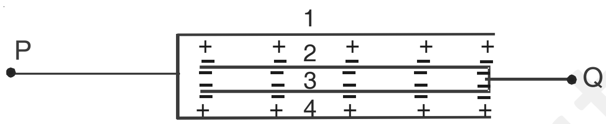
36. Four identical plates each of area a are separated by a distance d. The connection is shown below. What is the capacitance between P and Q ?

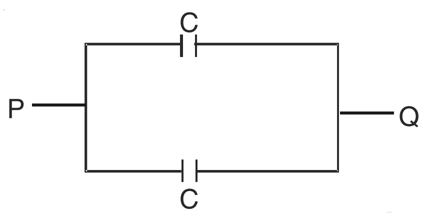


(A) $2a\varepsilon_0/d$ (B) $a\varepsilon_0/(2d)$ (C) $a\varepsilon_0/d$ (D) $4a\varepsilon_0/d$

Ans : (A)

Hints :





$$\therefore C_{eq.} = 2C = \frac{2\varepsilon_0 a}{d}$$

37. The least distance of vision of a longsighted person is 60 cm. By using a spectacle lens, this distance is reduced to 12 cm. The power of the len is

(A) +5.0 D (B) +(20/3) D (C) –(10/3) D (D) +2.0 D

Ans : (B)

Hints : Here, v = – 60 cm, u = – 12 cm

$$\therefore \frac{1}{-60} - \frac{1}{-12} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{15\,\text{cm}} = \frac{100}{15\text{m}}$$

$$\Rightarrow P = \frac{100}{15} = \frac{20}{3}\text{D}$$

38. A particle is acted upon by a constant power. Then, which of the following physical quantity remains constant ?

(A) speed (B) rate of change of acceleration

(C) kinetic energy (D) rate of change of kinetic energy

Ans : (D)

Hints : By definition, $P = \frac{dw}{dt} = \frac{dk}{dt} = \text{constant}$

39. A particle of mass M and charge q, initially at rest, is accelerated by a uniform electric field E through a distance D and is then allowed to approach a fixed static charge Q of the same sign. The distance of the closest approach of the charge q will then be

(A) $\frac{qQ}{4\pi\varepsilon_0 D}$ (B) $\frac{Q}{4\pi\varepsilon_0 ED}$ (C) $\frac{qQ}{2\pi\varepsilon_0 D^2}$ (D) $\frac{Q}{4\pi\varepsilon_0 E}$

Ans : (B)

Hints :

$\therefore (qED) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r_0} \Rightarrow r_0 = \frac{Q}{4\pi\varepsilon_0 ED}$

40. In an n-p-n transistor
(A) the emitter has higher degree of doping compared to that of the collector
(B) the collector has higher degree of doping compared to that of the emitter
(C) both the emitter and collector have same degree of doping
(D) the base region is most heavily doped

Ans : (A)

41. At two different places the angles of dip are respectively 30º and 45º. At these two places the ratio of horizontal component of earth's magnetic field is

(A) $\sqrt{3}:\sqrt{2}$ (B) $1:\sqrt{2}$ (C) 1 : 2 (D) $1:\sqrt{3}$

Ans : (A)

Hints : $\frac{H_1}{H_2} = \frac{B\cos 30^\circ}{B\cos 45^\circ}$

Note : Information is not sufficient in the given question. It can be solved only when magnetic field at these two places are equal.

42. Two vectors are given by $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 6\hat{j} + 2\hat{k}$. Another vector $\vec{C}$ has the same magnitude as $\vec{B}$ but has the same direction as $\vec{A}$. Then which of the following vectors represents $\vec{C}$?

(A) $\frac{7}{3}\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ (B) $\frac{3}{7}\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ (C) $\frac{7}{9}\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ (D) $\frac{9}{7}\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

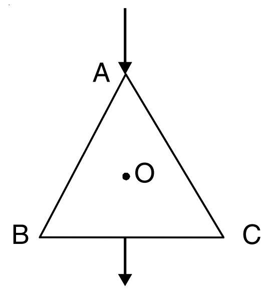
Ans : (A)

Hints : $\vec{C} = \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}} \times \sqrt{3^2 + 6^2 + 2^2}$

$= \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3} \times \sqrt{49}$

$= \frac{7}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$

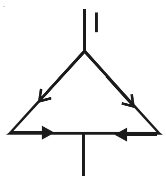
43. An equilateral triangle is made by uniform wires AB, BC, CA. A current I enters at A and leaves from the mid point of BC. If the lengths of each side of the triangle is L, the magnetic field B at the centroid O of the triangle is



(A) $\frac{\mu_0}{4\pi}\left(\frac{4I}{L}\right)$ (B) $\frac{\mu_0}{2\pi}\left(\frac{4I}{L}\right)$ (C) $\frac{\mu_0}{4\pi}\left(\frac{2I}{L}\right)$ (D) Zero

Ans : (D)

Hints :



$\vec{B}_1 + \vec{B}_2 = 0$

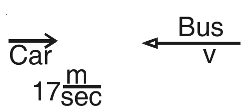
$B_1 \rightarrow$ due to left part
$B_2 \rightarrow$ due to right part

44. A car moving at a velocity of 17 ms^{-1} towards an approac ing bus that blows a horn at a frequency of 640 Hz on a straight track. The frequency of this horn appears to be 680 Hz to the car driver. If the velociy of sound in air is 340 ms^{-1}, then velocity of the approaching bus is

(A) 2 ms^{-1} (B) 4 ms^{-1} (C) 8 ms^{-1} (D) 10 ms^{-1}

Ans : (B)

Hints :



$$680 = 640\left(\frac{340+17}{340-v}\right)$$

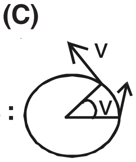
on solving, v = $4ms^{-}$

45. A particle is moving with a uniform speed *v* in a circular path of radius r with the centre at O. When the particle moves from a point P to Q on the circle such that $\angle POQ = \theta$, then the magnitude of the change in velocity is

(A) $2v\sin(2\theta)$ (B) Zero (C) $2v\sin\left(\frac{\theta}{2}\right)$ (D) $2v\cos\left(\frac{\theta}{2}\right)$

Ans : (C)

Hints :



$$|\Delta\vec{v}| = \sqrt{v^2+v^2-2v^2\cos\theta}$$

$$= 2v\sin\theta/2$$

CATEGORY - II

Q. 46 – Q. 55 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark

46. Two simple harmonic motions are given by

x_1 = a sin ωt + a cos ωt and

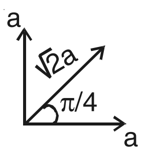
$x_2 = a \sin \omega t + \dfrac{a}{\sqrt{3}} \cos \omega t$

The ratio of the amplitudes of first and second motion and the phase difference between them are respectively

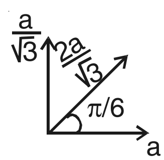
(A) $\sqrt{\dfrac{3}{2}}$ and $\dfrac{\pi}{12}$ (B) $\dfrac{\sqrt{3}}{2}$ and $\dfrac{\pi}{12}$ (C) $\dfrac{2}{\sqrt{3}}$ and $\dfrac{\pi}{12}$ (D) $\sqrt{\dfrac{3}{2}}$ and $\dfrac{\pi}{6}$

Ans : (A)

Hints :



for first S.H.M



for second S H M

Ratio of amplitude $\dfrac{a_1}{a_2} = \dfrac{\sqrt{3}}{\sqrt{2}}$

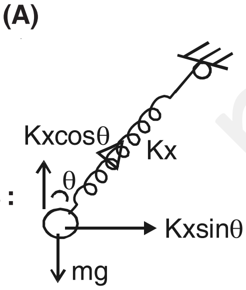
phase differ nce i $\dfrac{\pi}{4} - \dfrac{\pi}{6} = \dfrac{\pi}{12}$

47. A small mass m attached to one end of a spring with a negligi le mass and an unstretched length L, executes vertical oscillations with angular frequency ω_0. When the mass is rotated with an angular speed ω by holding the other end of the spring at a fixed point, the mass moves uniformly in a circular path in a horizontal plane. Then the increase in length of the spring during this rotation is

(A) $\dfrac{\omega^2 L}{\omega_0^2 - \omega^2}$ (B) $\dfrac{\omega_0^2 L}{\omega^2 - \omega_0^2}$ (C) $\dfrac{\omega^2 L}{\omega_0^2}$ (D) $\dfrac{\omega_0^2 L}{\omega^2}$

Ans : (A)

Hints :



$Kx\sin\theta = m\omega^2(L + x)\sin\theta$

$Kx = m\omega^2(L + x)$

and

$\sqrt{\dfrac{K}{m}} = \omega_0$

$K = m\omega_0^2$

$\cancel{m}\omega_0^2 x = \cancel{m}\omega^2 (L + x)$

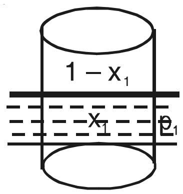
$\boxed{x = \dfrac{\omega^2 L}{\omega_0^2 - \omega^2}}$

48. A cylindrical block floats vertically in a liquid of density ρ_1 kept in a container such that the fraction of volume of the cylinder inside the liquid is x_1. then some amount of another immiscible liquid of density ρ_2 ($\rho_2 < \rho_1$) is added to the liquid in the container so that the cylinder now floats just fully immersed in the liquids with x_2 fraction of volume of the cylinder inside the liquid of density ρ_1. The ratio ρ_1/ρ_2 will be

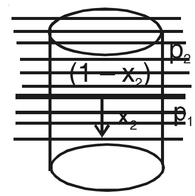
(A) $\dfrac{1-x_2}{x_1-x_2}$ (B) $\dfrac{1-x_1}{x_1+x_2}$ (C) $\dfrac{x_1-x_2}{x_1+x_2}$ (D) $\dfrac{x_2}{x_1}-1$

Ans : (A)

Hints :



first case



$p_1x_1g = p_1x_2g + p_2(1 - x_2)g$, as Bouyant force in both the cases are same

on solving, $\dfrac{p_1}{p_2} = \left(\dfrac{1-x_2}{x_1-x_2}\right)$

49. A sphere of radius R has a volume density of charge $\rho = kr$, where r is the distance from the centre of the sphere and k is constant. The magnitude of the electric field which exists at the sur ace o the sphere is given by (ε_0 = permittivity of the free space)

(A) $\dfrac{4\pi kR^4}{3\varepsilon_0}$ (B) $\dfrac{kR}{3\varepsilon_0}$ (C) $\dfrac{4\pi kR}{\varepsilon_0}$ (D) $\dfrac{kR^2}{4\varepsilon_0}$

Ans : (D)

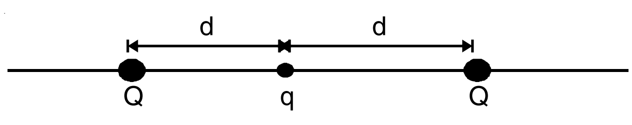
Hints : By Gauss's theorem

$$E(4\pi r^2) = \frac{\int p \times 4\pi r^2 dr}{\varepsilon_0}$$

$$= \frac{\int kr \times 4\pi r^2 dr}{\varepsilon_0}$$

$$= \boxed{E = \frac{Kr^2}{4\varepsilon_0}}$$

50. A particle of mass M and charge q is at rest at the midpoint between two other fixed similar charges each of magnitude Q placed a distance 2d apart. The system is collinear as shown in the figure. The particle is now displaced by a small amount x (x<< d) along the line joining the two charges and is left to itself. It will now oscillate about the mean position with a time period (ε_0 = permittivity of free space)



(A) $2\sqrt{\dfrac{\pi^3 M\varepsilon_0 d}{Qq}}$ (B) $2\sqrt{\dfrac{\pi^2 M\varepsilon_0 d^3}{Qq}}$ (C) $2\sqrt{\dfrac{\pi^3 M\varepsilon_0 d^3}{Qq}}$ (D) $2\sqrt{\dfrac{\pi^3 M\varepsilon_0}{Qqd^3}}$

Ans : (C)

Hints : Restoring force on displacement of x,

$$F = K\left[\frac{q}{(d-x)^2} - \frac{Qq}{(d+x)^2}\right]$$

$$= KQq\left[\frac{1}{(d-x)^2} - \frac{1}{(d+x)^2}\right]$$

$$= KQq\left[\frac{4dx}{(d^2-x^2)^2}\right]$$

$$= KQq\left[\frac{4dx}{d^4}\right] \text{If } (d \gg x)$$

$$= KQq\left[\frac{4x}{d^3}\right]$$

acceleration $a = \frac{F}{m} = \frac{4KQq}{md^3}x$

$$\omega^2 = \frac{4KQq}{md^3}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md}{4KQq}} = 2\sqrt{\frac{\pi^3 md^3 \varepsilon_0}{Qq}}$$

51. A body is projected from the ground with a velocity $\vec{v} = \left(3\hat{i} + 10\hat{j}\right)ms^{-1}$ The maximum height attained and the range of the body respectively are (given g = 10 ms^{-2})

(A) 5 m and 6 m (B) 3 m and 10 m (C) 6 m and 5 m (D) 3 m and 5 m

Ans : (A)

Hints : V = 3i + 10j

$V_x = 3$ $V_y = 10$

$$H = \frac{V_y^2}{2g} = \frac{10\cancel{0}}{2\times 1\cancel{0}} = 5\,m$$

$$R = V_x \times T = V_x \times \frac{2V_y}{g} = 6m$$

52. The stopping potential for photoelect ons from a metal surface is V_1 when monochromatic light of frequency v_1 is incident on it. The stopping potential becomes V_2 when monochromatic light of another frequency is incident on the same metal surface. If h be the Planck's onstant and e be the charge of an electron, then the frequency of light in the second case is

(A) $v_1 - \frac{e}{h}(V_2 + V_1)$ (B) $v_1 + \frac{e}{h}(V_2 + V_1)$ (C) $v_1 - \frac{e}{h}(V_2 - V_1)$ (D) $v_1 + \frac{e}{h}(V_2 - V_1)$

Ans : (D)

Hints : $h\nu_1 = \phi_0 + ev_1$ ——(1)

$h\nu_2 = \phi_0 + ev_2$ ——(2)

$h(\nu_2 - \nu_1) = e(v_2 - v_1)$

$$\nu_2 = \frac{e}{h}(v_2 - v_1) + \nu_1$$

53. A cell of e.m.f. E is connected to a resistance R_1 for time t and the amount of heat generated in it is H. If the resistance R_1 is replaced by another resistance R_2 and is connected to the cell for the same time t, the amount of heat generated in R_2 is 4H. Then the internal resistance of the cell is

(A) $\frac{2R_1 + R_2}{2}$

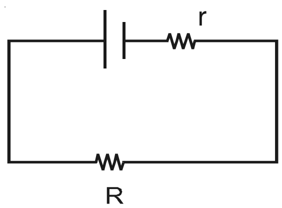
(B) $\sqrt{R_1R_2}\frac{2\sqrt{R_2}-\sqrt{R_1}}{\sqrt{R_2}-2\sqrt{R_1}}$

(C) $\sqrt{R_1R_2}\frac{\sqrt{R_2}-2\sqrt{R_1}}{2\sqrt{R_2}-\sqrt{R_1}}$

(D) $\sqrt{R_1R_2}\frac{\sqrt{R_2}-\sqrt{R_1}}{\sqrt{R_2}+\sqrt{R_1}}$

Ans : (B)

Hints :



$I_1^2R_1 = H$
$I_2^2R_2 = 4H$

$$\frac{E^2}{(R_1+r)^2}R_1 = H \quad \text{and} \quad \frac{E^2}{(R_2+r)^2}R_2 = 4H$$

$$\therefore \frac{R_2}{(R_2+r)^2} = 4\frac{R_1}{(R_1+r)^2}$$

$$\sqrt{R_2}(R_1+r) = 2\sqrt{R_1}(R_2+r)$$

$$\frac{\sqrt{R_1R_2}\left[\sqrt{R_1}-2\sqrt{R_2}\right]}{\left[2\sqrt{R_1}-\sqrt{R_2}\right]} = r$$

54. 3 moles of a mono-atomic gas ($\gamma = 5/3$) is mixed with 1 mole of a diatomic gas ($\gamma = 7/3$). The value of γ for the mixture will be

(A) 9/11 (B) 11/7 (C) 12/7 (D) 15/7

Ans : (B)

Hints : Degree of freedom

$$f_{mix} = \frac{n_1f_1 + n_2f_2}{n_1+n_2} = \frac{3\times 3 + 1\times 5}{4}$$

ie. $f = \frac{7}{2}$ $\quad\therefore \gamma = 1+\frac{2}{f}$

$$= 1+\frac{4}{7} = \frac{11}{7}$$

Note : If we take $\gamma = 7/3$ for diatomic gas as given in the question, none of the options are correct

55. The magnetic field $B = 2t^2 + 4t^2$ (where t = time) is applied perpendicular to the plane of a circular wire of radius r and resistance R. If all the units are in SI the electric charge that flows through the circular wire during t = 0 s to t = 2 s is

(A) $\frac{6\pi r^2}{R}$ (B) $\frac{20\pi r^2}{R}$ (C) $\frac{32\pi r^2}{R}$ (D) $\frac{48\pi r^2}{R}$

Ans : (B)

Hints : $\Delta Q = \dfrac{\Delta\phi}{R}$

$$= \frac{\pi r^2(B_2 - B_1)}{R} \qquad = \frac{\pi r^2[2\times 2+4\times 4]}{R} = \frac{20\pi r^2}{R}$$

CATEGORY - III

Q. 56 – Q. 60 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of twoon pro rata basis. There willbe no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of corredt options marked.

56. If E and B are the magnitudes of electric and magnetic fields respectively in some region of space, then the possibilities for which a charged particle may move in that space with a uniform velocity of magnitude v are

(A) $E = vB$ (B) $E \neq 0, B = 0$ (C) $E = 0, B \neq 0$ (D) $E \neq 0, B \neq 0$

Ans : (A, C, D)

57. An electron of charge e and mass m is moving in circular path of radius r with a uniform angular speed ω. Then which of the following statements are correct ?

(A) The equivalent current flowing in the circular path is proportional to r^2

(B) The magnetic moment due to circular current loop is independent of m

(C) The magnetic moment due to circular current loop is equal to 2e/m time th angular momentum of the electron

(D) The angular momentum of the particle is proportional to the areal veloc ty of electron.

Ans : (B , D)

Hints : Magnetic moment $\mu = IA$

$$= \frac{ev\times}{2\pi \cancel{r}}\cancel{\pi} r^{\cancel{2}} = \frac{evr}{2}$$

Angular momentum = $2\,m\,\dfrac{dA}{dt}$

58. A biconvex lens of focal length f and radii of cu vature of both the surfaces R is made of a material of refractive index n_1. This lens is placed in a liquid of refractive index n_2. Now this lens will behave like

(A) either as a convex or as a concave len depending solely on R

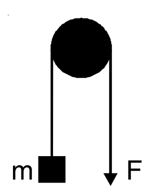
(B) a convex lens depending on n_1 and n_2

(C) a concave lens depending on n_1 and n_2

(D) a convex lens of same foc l length irrespective of R, n_1 and n_2

Ans : (B , C)

59. A block of mass m (= 0.1 kg) is hanging over a frictionless light fixed pulley by an inextensible string of negligible mass. The other end of the string is pulled by a constant force F in the verticaly downward direction. The linear momentum of the block increase by 2 kg ms^{-1} in 1 s after the block starts from rest. Then, (given g = 10 ms^{-2})



(A) The tension in the string is F

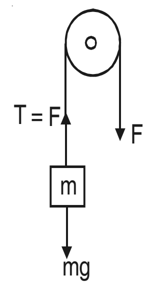
(B) The tension in the string is 3N

(C) The work done by the tension on the block is 20 J during this 1 s

(D) The work done against the force of gravity is 10 J

Ans : (A, B, D)

Hints :



F – mg = 2

F = 2 + mg = 3 N

$$a = \frac{\text{unbalanced force}}{\text{mass}} = \frac{2}{0.1} = 20\,m/s^2$$

$$\therefore S = \frac{1}{2}at^2 = \frac{1}{2}\times 20\times 1 = 10\,m$$

$\therefore$ W by tension = F × 10 = 3 × 10 = 30 J

W against gravity = mg × s = = 1 × 10 = 10 J

60. A bar of length l carrying a small mass m at one of its ends rotates with a uniform angular speed ω in a vertical plane about the mid-point of the bar. During the rotation, at some instant of time when the bar is horizontal, the mass is detached from the bar but the bar continues to rotate with same ω. The mass moves vertically up, comes back and reches the bar at the same point. At that place, the accele ation due to gravity is g.

(A) This is possible if the quantity $\frac{\omega^2\ell}{2\pi g}$ is an integer

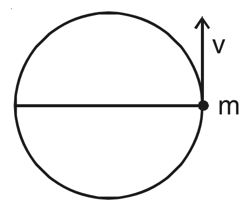
(B) The total time of flight of the mass is proportiona to ω^2

(C) The total distance travelled by the mass n air is proportional to ω^2

(D) The total distance travelled by the mass in air and its total time of flight are both independent on its mass.

Ans : (A, C, D)

Hints :



$$v = \frac{\ell}{2}\omega, \quad T = \frac{2v}{g} = \frac{\ell\omega}{g}$$

$$n\frac{2\pi}{\omega} = \frac{\ell\omega}{g} \text{ (as completes n rotations within T)} \quad \therefore n = \frac{\ell\omega^2}{2\pi g}$$

$$\text{Distance travelled} = 2h = 2\frac{v^2}{2g} = \frac{\ell^2\omega^2}{4g}.$$

❑ ❑ ❑

Code-▲

ANSWERS & HINTS
for
WBJEE - 2013
SUB : CHEMISTRY

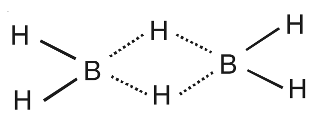
CATEGORY - I

Q. 1 – Q. 45 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark.

1. In diborane, the number of electrons that account for bonding in the bridges is

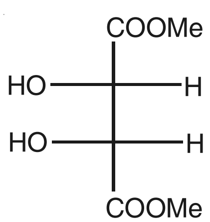
(A) Six (B) Two (C) Eight (D) Four

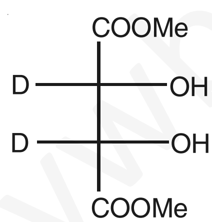
Ans : (D)

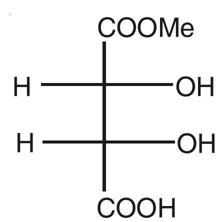
Hints : 

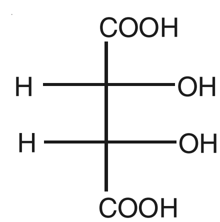
Each bridging bond is formed by two electrons. Hence four electrons account for bonding in the bridges.

2. The optically active molecule is

(A) 

(B) 

(C) 

(D) 

Ans : (C)

Hints : Others are meso ompound due to presence of plane of symmetry.

3. A van der Waals gas may behave ideally when

(A) The volume is very low

(B) The temperature is very high

(C) The pressure is very low

(D) The temperature, pressure and volume all are very high

Ans : (C)

Hints : A van der waals gas may behave ideally when pressure is very low as compressibility factor (Z) approaches 1. At high temperature Z > 1.

4. The half-life for decay of ^{14}C by β-emission is 5730 years. The fraction of ^{14}C decays, in a sample that is 22,920 years old, would be

(A) 1/8 (B) 1/16 (C) 7/8 (D) 15/16

Ans : (D)

Hints : $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}} = N_0\left(\frac{1}{2}\right)^{\frac{22920}{5730}} = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$ where N_0 = initial amount, N = amount left

So fraction reacted $N_0 - \frac{N_0}{16} = \frac{15}{16}N_0$

5. 2-Methylpropane on monochlorination under photochemical condition give

(A) 2-Chloro-2-methylpropane as major product

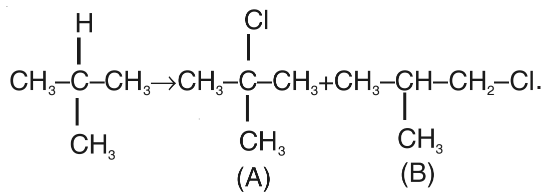
(B) (1:1) Mixture of 1-chloro-2-methylpropane and 2-chloro-2-methylpropane

(C) 1-Chloro-2-methylpropane as a major product

(D) (1:9) Mixture of 1-chloro-2-methylpropane and 2-chloro-2-methylpropane

Ans : (C)

Hints :



Ratio of (A) : (B) is 5 : 9.

6. For a chemical reaction at 27°C, the activation energy is 600 R. The ratio of the rate constants at 327°C to that of at 27°C will be

(A) 2 (B) 40 (C) e (D) e^2

Ans : (C)

Hints : $\ln\frac{K_2}{K_1} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$ or, $\ln\frac{K_2}{K_1} = \frac{600R}{R}\left(\frac{1}{300} - \frac{1}{600}\right)$ or, $\ln\frac{K_2}{K_1} = \frac{600R}{R}\left(\frac{2-1}{600}\right) = 1$

$\ln\frac{K_2}{K_1} = \ln e$ $\qquad \frac{K_2}{K_1} = e$

7. Chlorine gas reacts with red hot calcium oxide to give

(A) Bleaching powder and di hlorine monoxide (B) Bleaching powder and water

(C) Calcium chloride and chlorine dioxide (D) Calcium chloride and oxygen

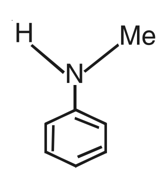
Ans : (D)

Hints : $\underset{\text{Red hot}}{2CaO} + 2Cl_2 \rightarrow CaCl_2 + O_2\uparrow$

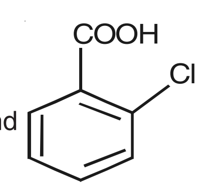
8. Correct pair of compounds which gives blue colouration/precipitate and white precipitate, respectively, when their Lassaigne's test is separately done is

(A) $NH_2NH_2.HCl$ and $ClCH_2COOH$ (B) NH_2CSNH_2 and $PhCH_2Cl$

(C) NH_2CH_2COOH and NH_2CONH_2 (D)

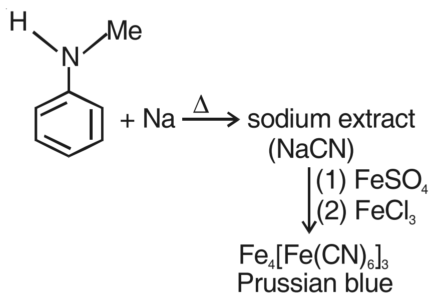


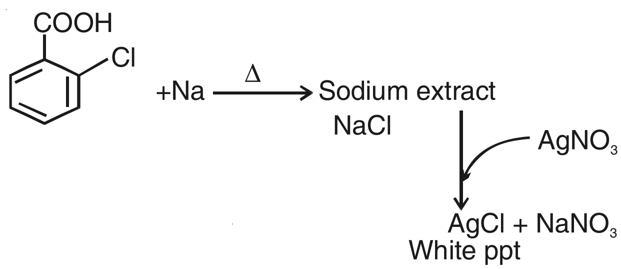
and



Ans : (D)

Hints : Organic compound





9. The change of entropy (dS) is defined as

(A) $dS = \delta q / T$ (B) $dS = dH / T$ (C) $dS = \delta q_{eqv} / T$ (D) $dS = (dH - dG) / T$

Ans : (C)

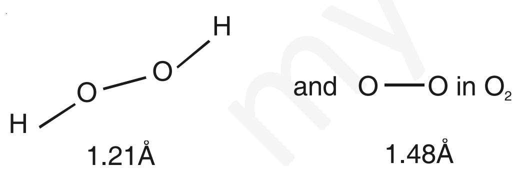
Hints : It's a fact

10. In O_2 and H_2O_2, the O–O bond lengths are 1 21 and 1.48 Å respectively. In ozone, the average O–O bond length is

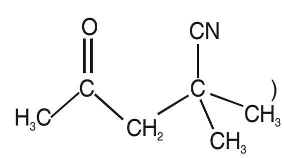
(A) 1.28 Å (B) 1.18 Å (C) 1.44 Å (D) 1.52 Å

Ans : (A)

Hints : Bond length is nearly average of bond length of O – O in



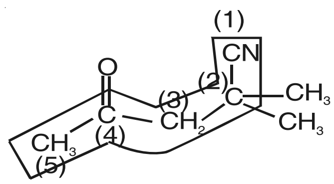
Hence it is 1.28 Å

11. The IUPAC name of the compound X is (X= )

(A) 4-cyano-4-methyl-2-oxopentane (B) 2-cyano-2-methyl-4-oxopentane

(C) 2,2-dimethyl-4-oxopentanenitrile (D) 4-cyano-4-methyl-2-pentanone

Ans : (C)

Hints : 

2, 2-Dimethyl-4-oxopentanenitrile

12. At 25°C, the solubility product of a salt of MX_2 type is 3.2×10^{-8} in water. The solubility (in moles/lit) of MX_2 in water at the same temperature will be

(A) 1.2×10^{-3} (B) 2×10^{-3} (C) 3.2×10^{-3} (D) 1.75×10^{-3}

Ans : (B)

Hints : $K_{sp}(MX_2) = 4s^3 = 3.2 \times 10^{-8} \Rightarrow s = \sqrt{\dfrac{3.2 \times 10^{-8}}{4}}$

$= 2 \times 10^{-3}$

13. In $SOCl_2$, the Cl–S–Cl and Cl–S–O bond angles are

(A) 130° and 115° (B) 106° and 96° (C) 107° and 108° (D) 96° and 106°

Ans : (D)

Hints : Fact

14. (+)-2-chloro-2-phenylethane in toluene racemises slowly in the pre ence of mall amount of $SbCl_5$, due to the formation of

(A) Carbanion (B) Carbene (C) Free-radical (D) Carbocation

Ans : (D)

Hints : $SbCl_5$ removes Cl^- from the substrate to generate a planar carbocation, which is then subsequently attacked by Cl^- from both top and bottom to result in a racemic mixture.

15. Acid catalysed hydrolysis of ethyl acetate follows a *pseudo*-first order kinetics with respect to ester. If the reaction is carried out with large excess of ester, the order wi h respect to ester will be

(A) 1.5 (B) 0 (C) 2 (D) 1

Ans : (B)

Hints : With large excess of ester th rate of reaction is independent of ester concentration.

16. The different colours of litmus in acidic, neutral and basic solutions are, respectively

(A) Red, orange and blue (B) Blue, violet and red

(C) Red, colourless and blue (D) Red, violet and blue

Ans : (D)

Hints :

17. Baeyer's reagent is

(A) Alkaline potassium permanganate (B) Acidified potassium permanganate

(C) Neutral potassium permanganate (D) Alkaline potassium manganate

Ans : (A)

Hints :

18. The correct order of equivalent conductances at infinite dilution in water at room temperature for H^+, K^+, CH_3COO^- and HO^- ions is

(A) $HO^- > H^+ > K^+ > CH_3COO^-$ (B) $H^+ > HO^- > K^+ > CH_3COO^-$

(C) $H^+ > K^+ > HO^- > CH_3COO^-$ (D) $H^+ > K^+ > CH_3COO^- > HO^-$

Ans : (B)

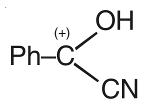
19. Nitric acid can be obtained from ammonia via the formations of the intermediate compounds
    - (A) Nitric oxides and nitrogen dioxides
    - (B) Nitrogen and nitric oxides
    - (C) Nitric oxide and dinitrogen pentoxide
    - (D) Nitrogen and nitrous oxide

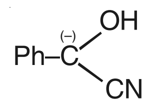
    Ans : (A)

    Hints :

20. In the following species, the one which is likely to be the intermediate during benzoin condensation of benzaldehyde, is

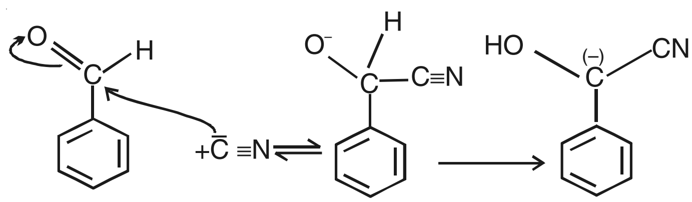
    (A) $Ph-C\equiv\overset{(+)}{O}$

    (B) 

    (C) 

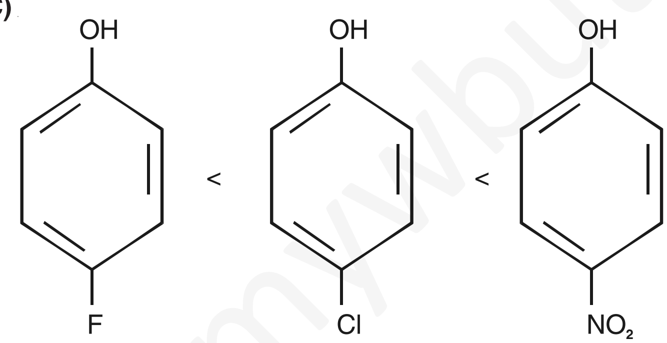
    (D) $Ph-\overset{(-)}{C}=O$

    Ans : (C)

    Hints : 

21. The correct order of acid strength of the following substituted phenols in wate at 28°C is
    - (A) p-nitrophnenol<p-fluorophenol<p-chlorophenol
    - (B) p-chlorophenol<p-fluorophenol<p-nitrophnenol
    - (C) p- fluorophenol<p-chlorophenol<p-nitrophnenol
    - (D) p-fluorophenol<p-nitrophnenol<p-chlorophenol

    Ans : (C)

    Hints : 

    (Acidic strength)

    As order of electron withdrawing nature from benzene ring : $-NO_2 > -Cl > -F$

22. For isothermal expansion of an ideal gas, the correct combination of the thermodynamic parameters will be
    - (A) $\Delta U = 0$, $Q=0$, $w \neq 0$ and $\Delta H \neq 0$
    - (B) $\Delta U \neq 0$, $Q \neq 0$, $w \neq 0$ and $\Delta H \neq 0$
    - (C) $\Delta U = 0$, $Q \neq 0$, $w = 0$ and $\Delta H \neq 0$
    - (D) $\Delta U = 0$, $Q \neq 0$, $w \neq 0$ and $\Delta H \neq 0$

    Ans : (D)

    Hints : For isothermal process, $\Delta T=0$

    $\therefore \Delta U = nC_v\Delta T = 0$

    $\Delta H = nC_p\Delta T = 0$

    From first law of thermodynamics

    $\Delta U = Q + W$

    As $\Delta U = 0$

    $\therefore Q = W \neq 0$

23. Addition of excess potassium iodide solution to a solution of mercuric chloride gives the halide complex

(A) tetrahedral $K_2[HgI_4]$ (B) trigonal $K[HgI_3]$

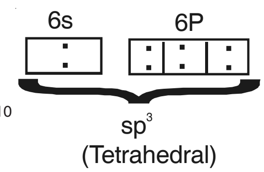
(C) linear Hg_2I_2 (D) square planar $K_2[HgCl_2I_2]$

Ans : (A)

Hints : $HgCl_2 + 4KI \longrightarrow K_2[HgI_4] + 2KCl$

Hg : [xe] $4f^{14}5d^{10}6s^2$

Hg^{2+} : [xe] $4f^{14}5d^{10}$



24. Amongst the following, the one which can exist in free state as a stable compound is

(A) C_7H_9O (B) $C_8H_{12}O$ (C) $C_6H_{11}O$ (D) $C_{10}H_{17}O_2$

Ans : (B)

Hints : Degree of unsaturation = $\dfrac{\sum n(v-2)}{2}+1$

; n = no. of atoms of a particular type

v = valency of the atom

C_7H_9O ; DU = $\dfrac{7(4-2)+9(1-2)+1(2-2)}{2}+1=3.5$

$C_8H_{12}O$; DU = $\dfrac{8(4-2)+12(1-2)+1(2-2)}{2}+1=3$

$C_6H_{11}O$: DU = $\dfrac{6(4-2)+11(1-2)+1(2-2)}{2}+1=1.5$

$C_{10}H_{17}O_2$: DU = $\dfrac{10(4-2)+17(1-2)+2(2-2)}{2}+1=2.5$

Molecules with fractional degree of unsaturation cannot exist with stability

25. A conducitivity cell has been calibrated with a 0.01 M 1:1 electrolyte solution (specific conductance, k=1.25 x 10^{-3} S cm^{-1}) in the cell and the measured resistance was 800 ohms at 25°C. The constant will be

(A) 1.02cm (B) 0.102cm^{-1} (C) 1.00cm^{-1} (D) 0.5cm^{-1}

Ans : (C)

Hints : K = 1.25x10^{-3} S cm^{-1} : $\rho = \dfrac{1}{K} = \dfrac{1}{1.25\text{x}10^{-3}}$

$R = \rho\,\dfrac{l}{A}$

$\therefore\ 800 = \dfrac{1}{1.25\times10^{-3}}\times\left(\dfrac{l}{A}\right)$, where $\dfrac{l}{A}$ = cell constant

$\dfrac{l}{A} = 800\times1.25\times10^{-3} = 1$

26. The orange solid on heating gives a colourless gas and a greensolid which can be reduced to metal by aluminium powder. The orange and the green solids are, respectively

(A) $(NH_4)_2Cr_2O_7$ and Cr_2O_3 (B) $Na_2Cr_2O_7$ and Cr_2O_3 (C) $K_2Cr_2O_7$ and CrO_3 (D) $(NH_4)_2Cr_2O_4$ and CrO_3

Ans : (A)

Hints :

$$\underset{\text{Orange solid}}{(NH_4)_2Cr_2O_7} \longrightarrow \underset{\text{Colourless gas}}{N_2} + \underset{\text{Green solid}}{Cr_2O_3} + 4H_2O$$

27. The best method for the preparationof 2,2 -dimethylbutane is via the reaction of

(A) Me_3CBr and $MeCH_2Br$ in Na/ether

(B) $(Me_3C)_2CuLi$ and $MeCH_2Br$

(C) $(MeCH_2)_2CuLi$ and Me_3CBr

(D) Me_3CMgI and $MeCH_2I$

Ans : (B)

Hints : Corey-House alkane synthesis gives the alkane in best yield

$$(Me_3C)_2CuLi + \underset{(1^\circ)}{MeCH_2Br} \xrightarrow{S_N2} Me_3C\text{–}CH_2CH_3$$

28. The condition of spontaneity of process is

(A) lowering of entropy at constant temperature and pressure

(B) lowering of Gibbs free energy of system at constant temperature and pressure

(C) increase of entropy of system at constant temp rature and pressure

(D) increase of Gibbs free energy of the universe at constant temperature and pressure

Ans : (B)

Hints : $dG_{P,T} = -ve$ is the criterion for spontaneity

29. The increasing order of O-N-O bond angle in the species NO_2, NO_2^+ and NO_2^- is

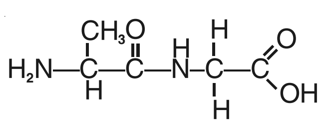
(A) $NO_2+ < NO_2 < NO_2^-$ (B) $NO_2 < NO_2^- < NO_2^+$ (C) $NO_2^+ < NO_2^- < NO_2$ (D) $NO_2 < NO_2^+ < NO_2^-$

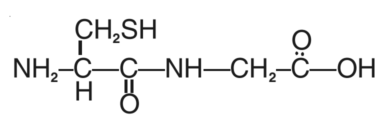
Ans : ()

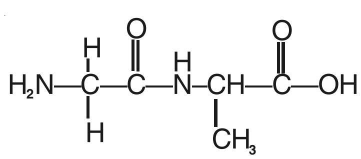
Hints : No option is correct

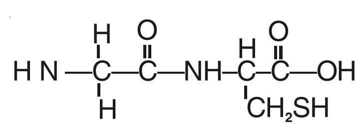
correct ans : $NO_2^+ > NO_2 > NO_2^-$

30. The correct structure of the dipeptide gly-ala is

(A) 

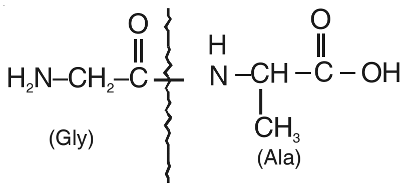
(B) 

(C) 

(D) 

Ans : (C)

Hints :



31. Equivalent conductivity at infinite dilution for sodium-potassium oxalate ($(COO^-)_2Na^+K^+$) will be [given, molar conductivities of oxalate, K^+ and Na^+ ions at infinite dilution are 148.2, 50.1, 73.5 S cm^2mol^{-1}, respectively]

(A) 271.8 S cm^2 eq^{-1} (B) 67.95 S cm^2 eq^{-1} (C) 543.6 S cm^2 eq^{-1} (D) 135.9 S cm^2 eq^{-1}

Ans : (D)

Hints : $\overset{\infty}{\lambda}_M = \overset{\infty}{\lambda}_M \text{(Oxa late)} + \overset{\infty}{\lambda}_M (Na^+) + \overset{\infty}{\lambda}_M (k^+)$

$\overset{\infty}{\lambda}_M = (148.+50.1+73.5)\text{S cm}^2 \text{ mol}^{-1}$

$\overset{\infty}{\lambda}_M = 271.8\text{S cm2 mol}^{-1}$

$\therefore \overset{\infty}{\lambda}_{Eq} = \dfrac{271.8}{2} = 135.9 \text{ S cm}^2 \text{eq}^{-1} \left(\lambda^{\infty}_{eq} = \dfrac{\lambda_M^{\infty}}{\text{n.factor}} \right)$

32. For BCl_3, $AlCl_3$ and $GaCl_3$ te increasing order of ionic character is

(A) $BCl_3<AlCl_3<GaCl_3$ (B) $GaCl_3<AlCl_3<BCl_3$ (C) $BCl_3<GaCl_3<AlCl_3$ (D) $AlCl_3<BCl_3<GaCl_3$

Ans : (C)

Hints : Ionic character is inversely proportional to polarising powe of cation.

$AlCl_3 > GaCl_3 > BCl_3$

33. At 25°C, pH of a 10^{-8} M aqueous KOH solution will be

(A) 6.0 (B) 7.02 (C) 8.02 (D) 9.02

Ans : (B)

Hints : $[O\bar{H}]_{Total} = (10^{-8} + 10^{-7})M$

$\therefore P^{OH} = -\log[10^{-8}+10^{-7}]$

~ 6.98

$\therefore pH = 14 - 6.98 = 7.02$

34. The reaction of nitropruss de anion with sulphide ion gives purple colouration due to the formation of

(A) the tetranionic complex of iron(II) coordinating to one NOS^- ion

(B) the dianionic complex of iron (II) coordinating to one NCS^- ion

(C) the trianionic complex of (III) coordinating to one NOS^- ion

(D) the tetranionic complex of iorn (III) coordinating to one NCS^- ion

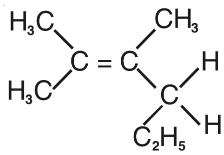
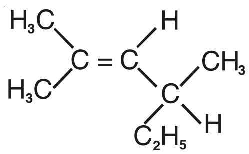
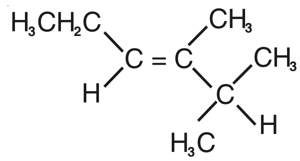
Ans : (A)

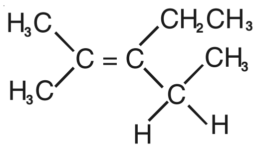
Hints : $Na_2S + Na_2[Fe(CN)_5NO] \longrightarrow Na_4[Fe(CN)_5NOS]$

Sod. Nitroprusside Violet color

$\left[Fe^{+2}(CN)_5^{-5}NoS\right]^{-4} \Rightarrow$ Tetra anionic complex of iron (II) co-ordinating to one NOS^- ion

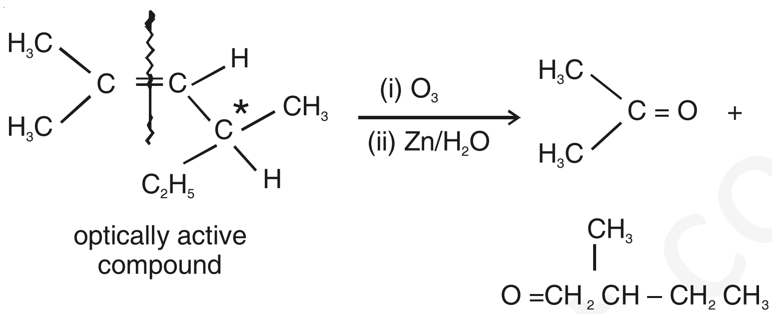
35. An optically active compound having molecular formula C_8H_{16} on ozonolysis gives acetone as one of the products. The structure of the compound is

(A)  (B)  (C) 

(D) 

Ans : (B)

Hints :



optically active compound

36. Mixing of two different ideal gases under istohermal reversible condition will lead to

(A) inccrease of Gibbs free energy of the system

(B) no change of entropy of the system

(C) increase of entropy of the system

(D) increase of enthalpy of the system

Ans : (C)

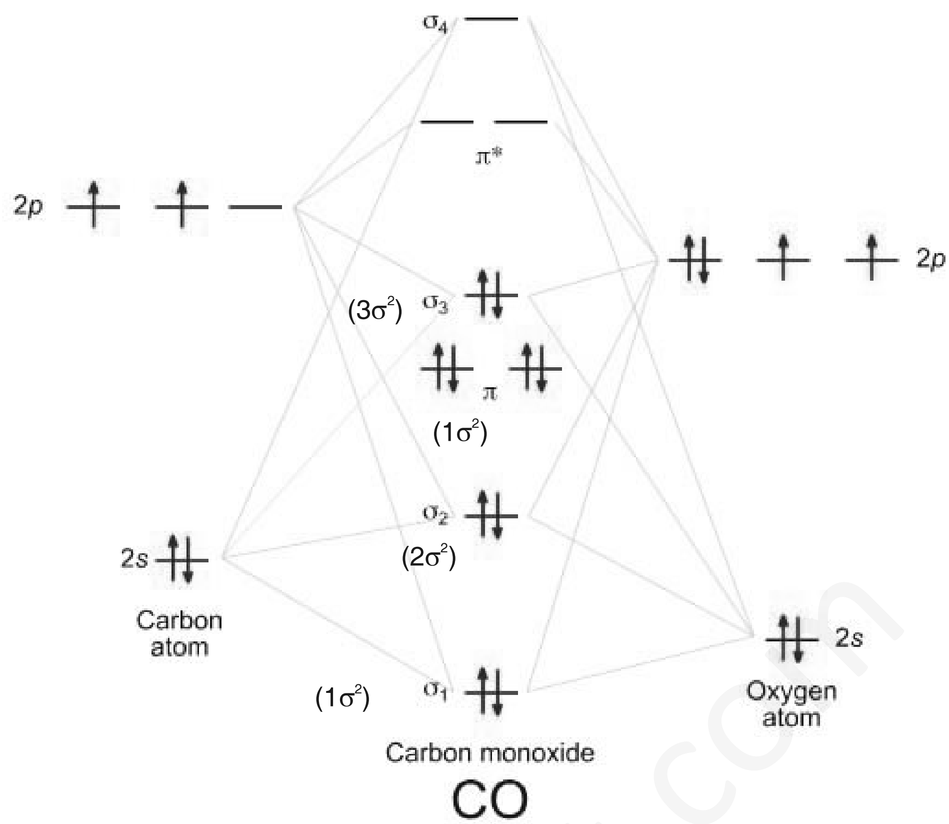
Hints : During mixing, Δs_{mix} is always positve

37. The ground state electronic configuration of CO molecule is

(A) $1\sigma^2 2\sigma^2 1\pi^4 3\sigma^2$ (B) $1\sigma^2 2\sigma^2 3\sigma^2 1\pi^2 2\pi^2$ (C) $1\sigma^2 2\sigma^2 1\pi^2 3\sigma^2 2\pi^2$ (D) $1\sigma^2 1\pi^4 2\sigma^2 3\sigma^2$

Ans : (A)

Hints :



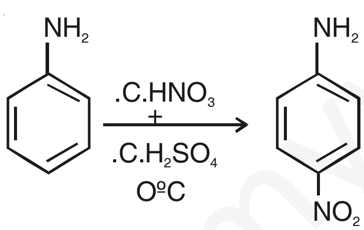
E.C for CO : $1\sigma^2\ 2\sigma^2\ 1\sigma^4\ 3\sigma^2$

38. When aniline is nitrated with nitrating mixturte in ice cold condition, the major product obtained is

(A) p-nitroaniline (B) 2,4-dinitroaniline (C) o-nitroaniline (D) m-nitroaniline

Ans : (A)

Hints :



p-nitro amiline
(Major)

39. The measured freezing point depression for a 0.1 m aqueous CH3COOH solution is 0.19°C. The acid dissociation constant K_a at this concentration will be (Given K_f, the molal cryoscopic constant = 1.86 K kg mol^{-1})

(A) $4.76\text{x}10^{-5}$ (B) $4\text{x}10^{-5}$ (C) $8\text{x}10^{-5}$ (D) $2\text{x}10^{-5}$

Ans : (B)

Hints : $\Delta T_f = i \times k_f \times m$

$$i = \frac{0.9}{1.86 \times 0.1} = 1.02$$

$$\alpha = \frac{i-1}{n-1} = \frac{0.02}{1} = 2 \times 10^{-2}$$

$$k_a = c\alpha^2 = 1 \times 10^{01} \times (2 \times 10^{-2})^2 = 4 \times 10^{-5}$$

40. The ore chromite is

(A) $FeCr_2O_4$ (B) $CoCr_2O_3$ (C) $CrFe_2O_4$ (D) $FeCr_2O_3$

Ans : (A)

Chromite ore is $FeCr_2O_4$

41. 'Sulphan' is

(A) a mixture of SO_3 and H_2SO_5

(B) 100% conc. H_2SO_4

(C) a mixture of gypsum and conc. H_2SO_4

(D) 100% oleum (a mixture of 100% SO_3 in 100% H_2SO_4)

Ans : (D)

Hints : Sulphan is pure liquid SO_3

42. Pressure-volume (PV) work done by an ideal gaseous system at constant volume is (where E is internal energy of the system)

(A) $-\Delta P/P$ (B) Zero (C) $-V\Delta P$ (D) $-\Delta E$

Ans : (B)

Hints : From 1st law of thermodynamic

$\Delta E = q+w$. Now $w = P\Delta V$. for $\Delta v = 0$

$w = 0$

43. Amongst $[NiCl_4]^{2-}$, $[Ni(H_2O)_6]^{2+}$, $[Ni(PPh_3)_2Cl_2]$, $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$, the paramagnetic species are

(A) $[NiCl_4]^{2-}$, $[Ni(H_2O)_6]^{2+}$, $[Ni(PPh_3)_2Cl_2]$

(B) $[Ni(CO)_4]$, $[Ni(PPh_3)_2Cl_2]$, $[NiCl_4]^{2-}$

(C) $[Ni(CN)_4]^{2-}$, $[Ni(H_2O)_6]^{2+}$, $[NiCl_4]^{2-}$

(D) $[Ni(PPh_3)_2Cl_2]$, $[Ni(CO)_4]$, $[Ni(CN)_4]^{2-}$

Ans : (A)

Hints : $Ni^{+2} = 3d^8\,4s^0$

(i) $[NiCl_4]^{2-}$ Cl^- weak l gand (spectrochemical series), so no pairing possible CFSE < Pairing energy)

(ii) $[Ni(H_2O)_6]^{2+}$ H O weak field ligand. So no pairing possible. CFSE < pairing energy)

(iii) $[Ni(PPh_3)_2Cl_2]$ alough. PPh_3 has d-acceptance but presence of Cl makes complex t ahedral.

44. Number of hydrogen ions present in 10 millionth part of 1.33 cm^3 of pure water at 25°C is

(A) 6.023 million (B) 60 milli n (C) 8.01 million (D) 80.23 million

Ans : (C)

Hints :

Now $[H^+] = 10^{-7}$ mole / litre

Now 1000 ml contains 10^{-7} mole. H^+

1ml " " $\dfrac{10^{-7}}{1000}$ mole H^+

1.33×10^{-7} ml — " 1.33×10^{-17}

10 million $= 10^{-7}$

so, 10 millionth part of 1.33 cm^3

$= 1.33\times10^{-7}$ ml

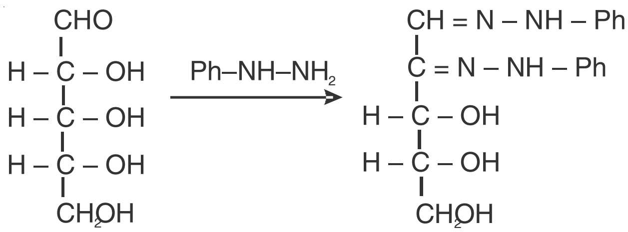
so, no of H^+ ions $= 1.33\times10^{-17}\times N_A$

45. Ribose and 2-deoxyribose can be differentiated by

(A) Fehling's reagent (B) Tollens's reagent (C) Barfoed's reagent (D) Osazone formation

Ans : (D)

Hints :



Ribose (osazone)

In deoxyribose, one –OH group is missing, which will prevent the formation of osazone.

CATEGORY - II

Q. 46 – Q. 55 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark

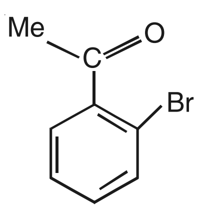
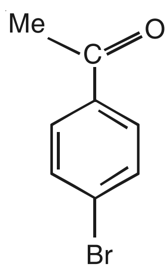
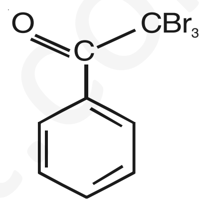
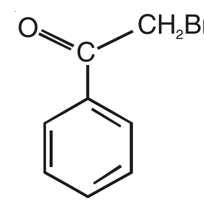
46. The standard Gibbs free energy change (ΔG^0) at 25°C for the dissociation of N_2O_4(g) to NO_2(g) is (given, equilibrium constant = 0.15, R=8.314 JK/mol)

(A) 1.1 kj (B) 4.7 kj (C) 8.1 kj (D) 38.2 kj

Ans : (B)

Hints : $\Delta G^0 = -RT\ln k$

47. Bromination of PhCOMe in acetic acid medium produces mainly

(A)  (B)  (C)  (D) 

Ans : (D)

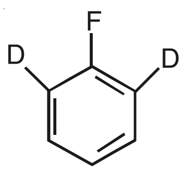
Hints : Reaction in acid media proceeds upto m nob omination stage.

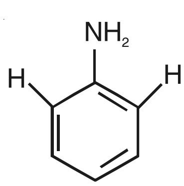
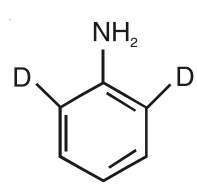
48. Silicone oil is obtained from the hydrolysis and polymerisation of

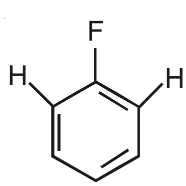
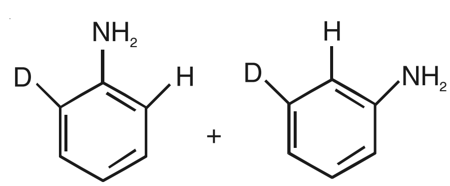
(A) trimethylchlorosilane and dimethydich orosilane
(B) trimethylchlorosilane and methyl richlorosilane
(C) methyltrichlorosilane and dime hy dichlorosilane
(D) triethylchlorosiland and diethyldichl rosilane

Ans : (A)

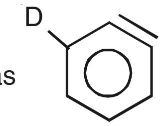
Hints : Silicone oils are formed on low degree of polymerisation

49. Treatment of  with $NaNH_2$/liq. NH_3 gives

(A)  (B) 

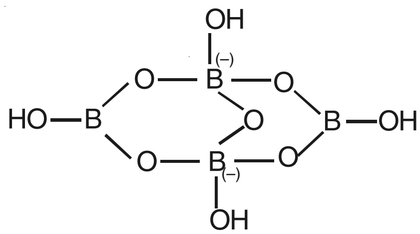
(C)  (D) 

Ans : (D)

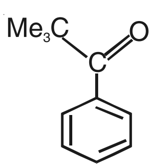
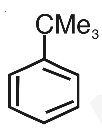
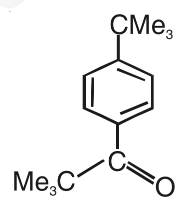
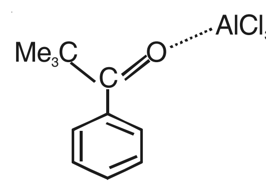
Hints : Reaction proceeds via benzyne mechanism with intermediate as 

50. Identify the CORRECT statement
(A) Quantum numbers (n,l,m,s) are obtained arbitrarily
(B) All the Quantum numbers (n,l,m,s) for any pair of electrons in an atom can be idential under special circum stance
(C) all the quantum numbers (n,l,m,s) may not be required to described an electron of an atom completely
(D) All the quantum numbers (n,l,m,s) are required to describe an electron of an atom completely
Ans : (D)
Hints : Fact

51. In borax the number of B–O–B links and B–OH bonds present are, respectively,
(A) Five and four (B) Four and five (C) Three and four (D) Five and five
Ans : (A)

Hints : 

52. Reaction of benzene with Me_3COCl in the presence of anhydrous $AlCl_3$ gives

(A) 
(B) 
(C) 
(D) 

Ans : (B)
Hints : It is because of rearrangemen during which initially formed acyl cation loses CO to form stable tertiary butyl cation

53. 1×10^{-3} mole of HCl is added t a buffer solution made up of 0.01 M acetic and 0.01 M sodium acetate. The final pH of the buffer will be (given, pK_a of acetic acid is 4.75 at 25°C)
(A) 4.60 (B) 4.66 (C) 4.75 (D) 4.8
Ans : (B)

Hints :

CH_3COO^-	+	H^+	$\rightarrow$	CH_3COOH
0.01		0.001		0.01
0.01–0.001				0.01+0.001
=0.009				=0.011

$$pH = pKa + \log\frac{[salt]}{[acid]} = 4.75 + \log\frac{0.009}{0.011} = 4.66$$

54. The best method for preparation of Me_3CCN is
(A) To react Me_3COH with HCN (B) To react Me_3CBr with NaCN
(C) To react Me_3CMgBr with ClCN (D) To react Me_3CLi with NH_2CN
Ans : (C)
Hints : It's a S_N2 reaction where $Me_3C–MgBr + Cl – CN \rightarrow Me_3C – CN + Mg (Cl)Br$

55. On heating, chloric acid decompose to

(A) $HClO_4$, Cl_2, O_2 and H_2O (B) $HClO_2$, Cl_2, O_2 and H_2O

(C) $HClO$, Cl_2O and H_2O_2 (D) HCl, $HClO$, Cl_2O and H_2O

Ans : (A)

Hints : Fact

CATEGORY - III

Q. 56 – Q. 60 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question-irrespective of the number of correct options marked.

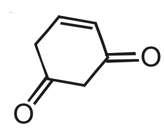
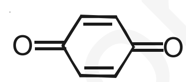
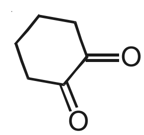
56. Consider the following reaction for $2NO_2(g) + F_2(g) \rightarrow 2NO_2F(g)$. The expression for the rate of reaction interms of the rate of change of partial pressures of reactant and product is/are

(A) rate = $-½[dp(NO_2)/dt]$ (B) rate = $½[dp(NO_2)/dt]$ (C) rate = $-½[dp(NO_2F)/dt]$ (D) rate = $½[dp(NO_2F)/dt]$

Ans : (A, D)

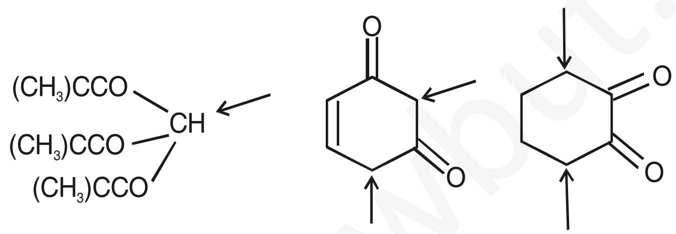
Hints : Fact

57. Tautomerism is exhibited by

(A) $(Me_3CCO)_3CH$ (B)  (C)  (D) 

Ans : (A, B, D)

Hints :



Availability of acidic α H-atoms at these positions(shown by arrow marks) enable the compounds to show keto-enol tautomerism

58. The important advantage(s) of Lintz and Donawitz (L.D.) process for the manufacture of steel is (are)

(A) The process is very quick (B) Operating costs are low

(C) Better quality steel is obtained (D) Scrap iron can be used

Ans : (A, C, D)

Hints : Fact

59. In basic medium the amount of Ni^{2+} in a solution can be estimated with the dimethylglyoxime reagent. The correct statement(s) about the reaction and the product is(are)

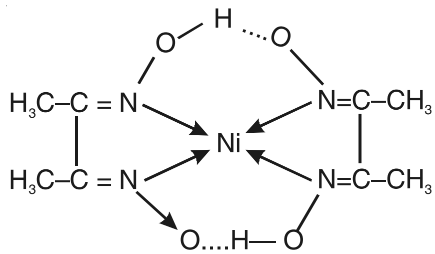
(A) In ammoniacal solution Ni^{2+} salts give cherry-red precipitate of nickel (II) dimethylglyoximate

(B) Two dimethylglyoximate units are bound to one Ni^{2+}

(C) In the complex two dimethylglyoximate units are hydrogen bonded to each other

(D) Each dimethylglyoximate unit forms a six-membered chelate ring with Ni^{2+}

Ans : (A, B, C)

Hints : 

60. Correct statement(s) in cases of n-butanol and t-butanol is (are)

(A) Both are having equal solubility in water
(B) t-butanol is more soluble in water than n-butanol
(C) Boiling point of t-butanol is lower than n-butanol
(D) Boiling point of n-butanol is lower than t-butanol

Ans : (B, C)

Hints : More branching means less boiling point and high solubility

❑ ❑ ❑