WB-JEE-2020 (Mathematics)

1. Let φ(x) = f(x) + f (1–x) and f"(x) < in [0,1], then
   a. φ is monotonic increasing in $\left[0,\frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2},1\right]$
   b. φ is monotonic increasing in $\left[\frac{1}{2},1\right]$ and monotonic decreasing in $\left[0,\frac{1}{2}\right]$
   c. φ neither increasing nor decreasing in any sub interval of [0,1]
   d. φ neither is increasing in [0,1]

2. Let $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$. Then

   $\left(\text{Here}\, yz \equiv \frac{d^2y}{dx^2}, y1 \equiv \frac{dy}{dx}\right)$
   a. $x^2 y_2 + xy_1 + n^2y = 0$
   b. $xy_2 - xy_1 + 2n^2y=0$
   c. $x^2yz + 3xy_1 - n^2y = 0$
   d. $xyz + 5xy_1 - 3y = 0$

3. $\int \frac{f(x)\phi'(x)f'(x)}{(f(x)\phi(x)+1\sqrt{f(x)\phi(x)-1})}dx =$
   a. $\sin^{-1}\sqrt{\frac{f(x)}{\phi(x)}}+c$
   b. $\cos^{-1}\sqrt{(f(x))^2-(\phi(x)^2}+c$
   c. $\sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\phi(x)+1}{2}}+c$
   d. $\sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\phi(x)+1}{2}}+c$

4. The value of $\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$ is equal to
   a. 27
   b. 54
   c. –54
   d. 0

WB-JEE-2020 (Mathematics)

5. $\int_0^2 [x^2]$ is equal

a. 1

b. $5-\sqrt{2}-\sqrt{3}$

c. $3-\sqrt{2}$

d. $\frac{8}{3}$

6. If the tangent to the curve y^2-x^3 at (m^2, m^3) is also a normal to the curve at (M^2, M^3), then the value of mM is :

a. $-\frac{1}{9}$

b. $-\frac{2}{9}$

c. $-\frac{1}{3}$

d. $-\frac{4}{9}$

7. If $x^2 + y^2 = a^2$

a. $2\pi a$

b. πa

c. $\frac{1}{2}\pi a$

d. $\frac{1}{4}\pi a$ -

8. Let f, be a continuous function in [0,1] then $\lim_{n\to\infty}\sum_{j=0}^{n}\frac{1}{n}f\left(\frac{j}{n}\right)$

a. $\frac{1}{2}\int_0^{1/2} f(x)dx$

b. $\int_{1/2}^{1} f(x)dx$

c. $\int_0^1 f(x)dx$

d. $\int_0^{1/2} f(x)dx$

WB-JEE-2020 (Mathematics)

9. Let f be a differentiable function with $\lim_{x\to\infty} f(x) = 0$. If $y' + yf'(x) - f(x) = 0$, $\lim_{x\to\infty} y(x) = 0$, then (where $y' \frac{dy}{dx}$)
   a. $y + 1 = e^{f(x)} + f(x)$
   b. $y - 1 = e^{f(x)} + f(x)$
   c. $y + 1 = e^{-f(x)} + f(x)$
   d. $y - 1 = e^{-f(x)} + f(x)$

10. Let $f(x) = 1 - \sqrt{(x^2)}$ where the square root is to be taken positive, then
    a. f has no extrema at x = 0
    b. f has minima at x = 0
    c. f has maxima at x = 0
    d. f' exist at 0

11. If $x \sin\left(\frac{y}{x}\right)dy = \left[y \sin\left(\frac{y}{x}\right) - x\right]dx$, $x > 0$ and $y(1) = \frac{\pi}{2}$ then the value of $\cos\left(\frac{y}{x}\right)$ is
    a. 1
    b. Log x
    c. e
    d. 0

12. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1 [a > 0]$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a is equal to
    a. 2
    b. $\frac{1}{2}$
    c. $\frac{1}{4}$
    d. 3

13. If a and b are arbitrary positive real numbers, then the least possible value of $\frac{6a}{5b} + \frac{10b}{3a}$ is
    a. 4
    b. $\frac{6}{5}$
    c. $\frac{10}{3}$
    d. $\frac{68}{15}$

14. If $2\log(x+1)-\log(x^2-1)-\log 2$, then $x =$
    a. Only 3
    b. -1 and 3
    c. only-1
    d. 1 and 3

15. The number of complex p such that $|p| = 1$ and imaginary part of p^4 is 0, is
    a. 4
    b. 2
    c. 8
    d. Infinitely many

16. The equation $z\bar{z}\ (2-3i)z + (2 + 3i)\ \bar{z} + 4 = 0$ represents a circle of radius
    a. 2 unit
    b. 3 unit
    c. 4 unit
    d. 6 unit

17. The expression $ax^2 + bx + c$ (a, b and c are real) has the same sign as that of a for all x is
    a. $b^2 - 4ac > 0$
    b. $b^2 - 4ac \neq 0$
    c. $b^2-4ac \leq 0$
    d. b and c have the same sign as that of a

18. In a 12 storied building, 3 person enter a lift cabin. It is known that they will leave the lift at different floor. In how many ways can they do so if the lift does not stop at the second floor?
    a. 36
    b. 120
    c. 240
    d. 720

19. If the total number of m-element subsets of the set $A = \{a_1, a_2, \ldots\ldots\ldots, a_n\}$ is k times the number of m element subsets containing a_4 then n is
    a. $(m-1)k$
    b. mk
    c. $(m + 1)k$
    d. $(m+2)k$

20. Let $I(n) = n^n$, $J(n) = 1.3.5. \ldots.(2n-1)$ for all $(n > 1)$, $n\in N$, then
    a. $I(n) > J(n)$
    b. $I(n) < J(n)$
    c. $I(n) \neq J(n)$
    d. $I(n) = \frac{1}{2}J(n)$

WB-JEE-2020 (Mathematics)

21. If c_0, c_1, c_2 .....c_{15} are the Binomial co-efficients in the expansion of $(1 + x)^{15}$, then the value of $\frac{c_1}{c_0} + 2\frac{c_3}{c_1} + 3\frac{c_3}{c_2} + ... + 15\frac{c_{15}}{c_{14}}$ is
    a. 1240
    b. 120
    c. 124
    d. 140

22. Let A = $\begin{pmatrix} 3-t & 1 & 0 \\ 1 & 3-t & 1 \\ 0 & -1 & 0 \end{pmatrix}$ and det A = 5, then
    a. $t = 1$
    b. $t = 2$
    c. $t = -1$
    d. $t = -2$

23. Let A = $\begin{pmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{pmatrix}$. The value of x for which the matrix A is not in
    a. 6
    b. 12
    c. 3
    d. 2

24. Let A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2 × 2 real matrix with det A = 1. If the equation det $(A-\lambda I_2) = 0$ has imaginary roots (I_2 be the identify matrix of order 2), then
    a. $(a + d)^2 < 4$
    b. $(a + d)^2 = 4$
    c. $(a + d)^2 > 4$
    d. $(a + d)^2 = 16$

25. If $\begin{bmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{bmatrix} = ka^2b^2c^2$, then k is equal to:
    a. 2
    b. −2
    c. −4
    d. 4

26. If $f : S \to R$ where S is the set of all non-singular matrices of order 2 over R and $f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$, then
   a. f is bijective mapping
   b. f is one-one but not onto
   c. f is onto but not one-one
   d. f is neither one-one nor onto

27. Let the relation ρ be defined on R by a ρ b holds if and only if a – b is zero or irrational then
   a. ρ is equivalence relation
   b. ρ is reflexive & symmetric but is not transitive
   c. ρ is reflexive and transitive but is not symmetric
   d. ρ is reflexive only

28. The unit vector in ZOX plane, making angles 45° and 60° respectively with $\vec{\alpha} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{\beta} - \hat{j} - \hat{k}$ is
   a. $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$
   b. $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$
   c. $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$
   d. $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$

29. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one get an even number and win the game. What is the probability that A wins if A begins?
   a. $\frac{1}{4}$
   b. $\frac{1}{2}$
   c. $\frac{7}{12}$
   d. $\frac{8}{15}$

30. The rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds he must fire to have more than 50% chance of hitting it at least once is
   a. 5
   b. 7
   c. 9
   d. 11

31. $\cos(2x+7) = a\,(2-\sin x)$ can have a real solution for
    a. all real values of a
    b. $a \in [2, 6]$
    c. $a \in [-\infty, 2]\setminus\{0\}$
    d. $a \in (0, \infty)$

32. The differential equation of the family of curves $y = e^x\,(A \cos x + B \sin x)$ where A, B are arbitrary constant is
    a. $\frac{d^2y}{dx^2} - 9x = 13$
    b. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
    c. $\frac{d^2y}{dx^2} + 3y = 4$
    d. $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - xy = 0$

33. The equation $r \cos\left(\theta - \frac{\pi}{3}\right) = 2$ represents
    a. a circle
    b. a parabola
    c. an ellipse
    d. a straight line

34. The locus of the centre of the circles which touch both the circles $x^2 + y^2 - a^2$ and $x^2 + y^2 - 4ax$ externally is
    a. a circle
    b. a parabola
    c. an ellipse
    d. a hyperbola

35. Let each of the equations $x^2 + 2xy + ay^2 = 0$ & $ax^2 + 2xy + y^2 = 0$ represent two straight lines passing through the origin. If they have a common line, then the other two lines are given by
    a. $x - y = 0,\ x - 3y = 0$
    b. $x + 3y = 0,\ 3x + y = 0$
    c. $3x + y = 0,\ 3x - y = 0$
    d. $(3x - 2y) = 0,\ x + y = 0$

36. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at P and Q respectively. The point O divides the segment PQ in the ratio
    a. 1 : 2
    b. 3 : 4
    c. 2 : 1
    d. 4 : 3

37. Area in the first quadrant between the ellipse $x^2 + 2y^2 - a^2$ and $2x^2 + y^2 - a^2$ is
    a. $\frac{a^2}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$
    b. $\frac{3a^2}{4}\tan^{-1}\frac{1}{2}$
    c. $\frac{5a^2}{2}\sin^{-1}\frac{1}{2}$
    d. $\frac{9\pi a^2}{2}$

38. The equation of circle of radius $\sqrt{17}$ unit, with centre on the positive side of x-axis and through the point (0, 1) is
    a. $x^2 + y^2 - 8x - 1 = 0$
    b. $x^2 + y^2 + 8x - 1 = 0$
    c. $x^2 + y^2 - 9y + 1 = 0$
    d. $2x^2 + 2y^2 - 3x + 2y = 0$

39. The length of the chord of the parabola $y^2 = 4ax$ $(a > 0)$ which passes through the vertex and makes an acute angle α with the axis of the parabola is
    a. $\pm 4a \cot\alpha \operatorname{cosec}\alpha$
    b. $4a \cot\alpha \operatorname{cosec}\alpha$
    c. $-4a \cot\alpha \operatorname{cosec}\alpha$
    d. $4a \operatorname{cosec}^2\alpha$

40. A double ordinate PQ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is such that ΔOPQ is equilateral, O being the centre of the hyperbola. Then the eccentricity e satisfies the relation
    a. $1 < e < \frac{2}{\sqrt{3}}$
    b. $e = \frac{2}{\sqrt{3}}$
    c. $e = \frac{\sqrt{3}}{2}$
    d. $e > \frac{2}{\sqrt{3}}$

WB-JEE-2020 (Mathematics)

41. If B and B' are the ends of the minor axis and S and S' are foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, then the area of the rhombus SBS' B' will be
   a. 12 sq. unit
   b. 48 sq. unit
   c. 24 sq. unit
   d. 36 sq. unit

42. The equation of the latus rectum of a parabola is x + y = 8 and the equation of the tangent at the vertex is x + y = 12. Then the length of the latus rectum is
   a. $4\sqrt{2}$ units
   b. $2\sqrt{2}$ units
   c. 8 units
   d. $8\sqrt{2}$ units

43. The equation of the plane through the point (2, −1, −3) and parallel to the lines $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is
   a. 8x + 14y + 13z + 37 = 0
   b. 8x – 14y – 13z – 37 = 0
   c. 8x – 14y – 13z + 37 = 0
   d. 8x – 14y + 13z + 37 = 0

44. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane 2x–2y + z = 5 is
   a. $\frac{2\sqrt{3}}{5}$
   b. $\frac{\sqrt{2}}{10}$
   c. $\frac{4}{5\sqrt{2}}$
   d. $\frac{\sqrt{5}}{6}$

WB-JEE-2020 (Mathematics)

45. Let f(x) –sin x + co sax be periodic function. Then
   a. ‘a’ is real number
   b. ‘a’ is any irrational number
   c. ‘a’ is any rational number
   d. a = 0

46. The domain of f(x) = $\sqrt{\left(\frac{1}{\sqrt{x}}-\sqrt{(x+1)}\right)}$ is
   a. x > –1
   b. (–, ∞)\{0}
   c. $\left(0, \frac{\sqrt{5}-1}{2}\right]$
   d. $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

47. Let y = f(x) = 2x^2–3x + 2. The differential of y when x changes from 2 to 1.99 is
   a. 0.01
   b. 0.18
   c. –0.05
   d. 0.07

48. If $\lim_{x\to\infty}\left[\frac{1+cx}{1-cx}\right]^{1/x}$ = 4, then $\lim_{x\to\infty}\left[\frac{1+2cx}{1-2cx}\right]^{1/x}$ is
   a. 2
   b. 4
   c. 16
   d. 64

49. Let f : R → R be twice continuously differentiable (or f” exists and is continuous) such that f(0) = f(1) = f’(0) = 0. Then
   a. f”(c) = 0 for some c ∈ R
   b. there is no point for which f” (x) = 0
   c. at all points f”(x) > 0
   d. at all points f”(x) < 0

50. Let f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12. Then
   a. f(x) has 13 non-zero real roots
   b. f(x) has exactly one real root
   c. f(x) has exactly one pair of imaginary roots
   d. f(x) has no real root

WB-JEE-2020 (Mathematics)

51. Let z_1 and z_2 be two imaginary roots of $z^2 + pz + q = 0$, where p and q are real. The points z_1, z_2 and origin form an equilateral triangle if
    a. $p^2 > 3q$
    b. $p^2 < 3q$
    c. $p^2 = 3q$
    d. $p^2 = q$

52. If the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$ and $\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc of
    a. 1
    b. 0
    c. −1
    d. 2

53. If the line y = x is a tangent to the parabola $y = ax^2 + bx + c$ at the point (1,1) and the curve passes through (−1, 0), then
    a. $a = b = -1, c = 3$
    b. $a = b = \frac{1}{2}, c = 0$
    c. $a = c = \frac{1}{4}, b = \frac{1}{2}$
    d. $a = 0, b = c = \frac{1}{2}$

54. In an open interval $\left(0, \frac{\pi}{2}\right)$,
    a. $\cos x + x \sin x < 1$
    b. $\cos x + x \sin x > 1$
    c. no specific order relation can be ascertained between $\cos x + x \sin x$ and 1
    d. $\cos x + x \sin x > 1$

55. The area of the region $\{(x,y) : x^2 + y^2 \leq 1 \geq x + y\}$ is
    a. $\frac{\pi^2}{2}$
    b. $\frac{\pi}{4}$
    c. $\frac{\pi}{4} - \frac{1}{2}$
    d. $\frac{\pi^2}{3}$

56. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$ [a, b, c, d are all real], then $P(x).Q(x) = 0$ has
   a. at least two real roots
   b. two real roots
   c. four real roots
   d. no real root

57. Let $A = \{x \in R : -1 \leq x \leq 1\}$ & $f : A \to A$ be a mapping defined by $f(x) - x|x|$. Then f is
   a. Injective but not surjective
   b. Surjective but not injective
   c. Neither injective nor surjective
   d. bijective

58. Let $f(x) = \sqrt{x^2 - 3x + 2}$ and $g(x) = \sqrt{x}$ be two given functions. If S be the domain of f o g and T be the domain of g o f, then
   a. $S = T$
   b. $S \cap T = \varphi$
   c. $S \cap T$ is a singleton
   d. $S \cap T$ is an interval

59. Let ρ_1 and ρ_2 be two equivalence relations defined on a non-void set S. Then
   a. both $\rho_1 \cap \rho_2$ and $\rho_1 \cup \rho_2$ are equivalence relations
   b. $\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so
   c. $\rho_1 \cup \rho_2$ is equivalence relation but $\rho_1 \cap \rho_2$ is not so
   d. Neither $\rho_1 \cap \rho_2$ nor $\rho_1 \cup \rho_2$ is equivalence relation

60. Consider the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The portion of the tangent at any point of the curve intercepted between the point of contact and the directrix subtends at the corresponding focus an angle of:
   a. $\frac{\pi}{4}$
   b. $\frac{\pi}{3}$
   c. $\frac{\pi}{2}$
   d. $\frac{\pi}{6}$

WB-JEE-2020 (Mathematics)

61. A line cuts x-axis at A(7,0) and the y-axis at B(0, –5). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P(a, 0) and the y-axis at Q(0,b). If AQ and BP intersect at R, the locus of R is
   a. $x^2 + y^2 + 7x + 5y = 0$
   b. $x^2 + y^2 + 7x - 5y = 0$
   c. $x^2 + y^2 - 7x + 5y = 0$
   d. $x^2 + y^2 - 7x - 5y = 0$

62. Let $0 < \alpha < \beta < 1$. Then $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\int_{1/(k+\beta)}^{1/(k+\alpha)}\frac{dx}{1+x}$ is
   a. $\log_e \frac{\beta}{\alpha}$
   b. $\log_e \frac{1+\beta}{1+\alpha}$
   c. $\log_e \frac{1+\alpha}{1+\beta}$
   d. ∞

63. $\lim\limits_{x\to 1}\left[\frac{1}{\ln x} - \frac{1}{(x-1)}\right]$
   a. Does not exist
   b. 1
   c. $\frac{1}{2}$
   d. 0

64. Let $y = \frac{1}{1+x+\ln x}$, Then
   a. $x\frac{dy}{dx} + y = x$
   b. $x\frac{dy}{dx} = y\,(y \ln x - 1)$
   c. $x^2\frac{dy}{dx} = y^2 + 1 - x^2$
   d. $x\left[\frac{dy}{dx}\right]^2 = y - x$

WB-JEE-2020 (Mathematics)

65. Consider the curve $y = be^{-x/a}$ where a and b are non-zero real numbers. Then
    a. $\frac{x}{a}+\frac{y}{b} = 1$ is tangent to the curve at (0,0)
    b. $\frac{x}{a}+\frac{y}{b} = 1$ is tangent to the curve where the curve crosses the axis of y
    c. $\frac{x}{a}+\frac{y}{b} = 1$ is tangent to the curve at (a, 0)
    d. $\frac{x}{a}+\frac{y}{b} = 1$ is tangent to the curve at (2a, 0)

66. The area of the figure bounded by the parabola $x = -2y^2$, $x = 1-3y^2$ is
    a. $\frac{1}{3}$ square unit
    b. $\frac{4}{3}$ square unit
    c. 1 square unit
    d. 2 square unit

67. A particle is projected vertically upwards. If it has to stay above the ground for 12 seconds, then
    a. velocity of projection is 192 ft/sec
    b. greatest height attained is 600 ft
    c. velocity of projection is 196 ft/sec
    d. greatest height attained is 576 ft

68. The equation $x^{(\log_3 x)^2-\frac{9}{2}\log_3 x+5} - 3\sqrt{3}$
    a. at least one real root
    b. exactly one real root
    c. exactly one irrational root
    d. complex roots

69. In a certain test, there are n questions. In this test 2^{n-I} students gave wrong answers to at least i questions, where i = 1,2…… , n. If the total number of wrong answer given is 2047, then n is equal to
    a. 10
    b. 11
    c. 12
    d. 13

70. A and B are independent events. The probability that both A and B occur is $\frac{1}{20}$ and the probability that neither of them occurs is $\frac{3}{5}$. The probability of occurrence of A is

a. $\frac{1}{2}$

b. $\frac{1}{10}$

c. $\frac{1}{4}$

d. $\frac{1}{5}$

71. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is

a. $\frac{x}{2}-\frac{y}{3}=1$

b. $\frac{x}{-2}+\frac{y}{1}=1$

c. $-\frac{x}{3}+\frac{y}{1}=1$

d. $\frac{x}{1}-\frac{y}{2}=1$

72. Let f(x) = $\frac{1}{3}x\sin x-(1-\cos x)$. The smallest positive integer k such that $\lim_{x\to 0}\frac{f(x)}{x^k}\neq 0$ is

a. 4

b. 3

c. 2

d. 1

73. Consider a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ at any point. The locus of the midpoint of the portion intercepted between the axes is

a. $\frac{x^2}{2}+\frac{y^2}{4}=1$

b. $\frac{x^2}{4}+\frac{y^2}{2}=1$

c. $\frac{1}{3x^2}+\frac{1}{4y^2}=1$

d. $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

74. Tangent is drawn at any point P(x, y) on a curve, which passes through (1, 1). The tangent cuts X-axis and Y-axis at A and B respectively. If AP : BP = 3 : 1, then

a. the differential equation of the curve is $3x\frac{dy}{dx}+y=0$

b. the differential equation of the curve is $3x\frac{dy}{dx}-y=0$

c. the curve passes through $\left(\frac{1}{8},2\right)$

d. the normal at (1, 1) is x +3y = 4

75. Let $y=\frac{x^2}{(x+1)^2(x+2)}$. Then $\frac{d^2y}{dx^2}$ is

a. $2\left[\frac{3}{(x+1)^4}-\frac{3}{(x+1)^3}+\frac{4}{(x+2)^3}\right]$

b. $3\left[\frac{2}{(x+1)^3}+\frac{4}{(x+1)^2}+\frac{5}{(x+2)^3}\right]$

c. $\left[\frac{6}{(x+1)^3}-\frac{4}{(x+1)^2}+\frac{3}{(x+1)^3}\right]$

d. $\left[\frac{7}{(x+1)^3}-\frac{3}{(x+1)^2}+\frac{2}{(x+1)^3}\right]$

WB-JEE-2020 (Mathematics)

ANSWER KEYS

1. (a)	2. (a)	3. (c)	4. (d)	5. (b)	6. (d)	7. (c)	8. (c)	9. (c)	10. (c)
11. (b)	12. (a)	13. (a)	14. (a)	15. (c)	16. (b)	17. (c)	18. (d)	19. (b)	20. (a)
21. (b)	22. (d)	23. (c)	24. (a)	25. (d)	26. (d)	27. (b)	28. (b)	29. (d)	30. (b)
31. (G)	32. (b)	33. (d)	34. (d)	35. (b)	36. (b)	37. (a)	38. (a)	39. (b)	40. (d)
41. (c)	42. (d)	43. (G)	44. (b)	45. (c)	46. (c)	47. (c)	48. (c)	49. (a)	50. (b)
51. (c)	52. (c)	53. (c)	54. (b)	55. (c)	56. (a)	57. (d)	58. (d)	59. (b)	60. (c)
61. (c)	62. (b)	63. (c)	64. (b)	65. (b)	66. (b)	67. (a,d)	68. (a,c)	69. (b)	70. (c,d)
71. (a,b)	72. (c)	73. (d)	74. (a,c)	75. (a)					

* G – Indicates GRACE MARK awarded for the question number

WB-JEE-2020 (Mathematics)

SOLUTIONS

1. (a)

$\varphi(x) = f(x) + f(1 - x)$

Differentiate w.r.t. x

$\varphi'(x) = f'(x) - f'(1 - x)$ .....(i)

$f'(x) - f'(1 - x) \geq 0$ (for monotonic increasing)

$f'(x) \geq f'(1 - x)$, $x \leq 1 - x$ ($\because f'(x)$ is decreasing)

$\therefore x \leq 1 - x$

$2x \leq 1$

$x \leq \frac{1}{2}$ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2}, 1\right]$

2. (a)

Given that

$\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$

Differentiate w.r.t. x

$$\Rightarrow \frac{-1}{\sqrt{1-\left(\frac{y}{b}\right)^2}}.\frac{1}{b}.\frac{dy}{dx} = n.\ \frac{n}{x}.\frac{1}{n}$$

$$\Rightarrow \left(\frac{-b}{\sqrt{b^2 - y^2}}\right).\frac{1}{b}.\frac{dy}{dx} = \frac{n}{x}$$

$$\Rightarrow \left(\frac{-1}{\sqrt{b^2 - y^2}}\right).\frac{dy}{dx} = \frac{n}{x}$$

$\Rightarrow xy_1 = -n\ \sqrt{b^2 - y^2}$

Squaring both sides

$\Rightarrow x^2y_1^2 = n^2\ (b^2 - y^2)$

$\Rightarrow x^2y_1^2 = n^2b^2 - n^2y^2$

Differentiate both sides

$\Rightarrow 2xy_1^2 + x^2.\ 2y_1.\ y_2 = -n^2.\ 2y.\ y_1$

$\Rightarrow xy_1 + x^2y_2 = -\ n^2y$

$\Rightarrow x^2y_2 + xy_1 + n^2y = 0$

3. (c)

Let $I = \int \frac{f(x)\phi'(x)+\phi(x)f'(x)}{(f(x)\phi(x)+1)\sqrt{f(x)\phi(x)-1}}dx$

Let $f(x)\ \phi(x)-1 = t^2 \Rightarrow f'(x)\phi(x) + f(x)\phi'(x) = 2t\ \frac{dt}{dx}$

$\Rightarrow [f'(x)\phi(x) + f(x)\ \phi'(x)]\ dx = 2tdt$

$\therefore I = \int \frac{2t}{(t^2+2)(t)}dt$

$\Rightarrow I = 2\ \int \frac{1}{(t^2)+(\sqrt{2})^2}dt$

$\Rightarrow I = \frac{2}{\sqrt{2}}\tan^{-1}\ \frac{t}{\sqrt{2}} + c$

$\Rightarrow I = 2\ \tan^{-1}\left(\sqrt{\frac{f(x)\phi(x)-1}{2}}\right) + c$

4. (d)

$$\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x dx$$

Let $x = -t - \left[\begin{smallmatrix}2n\\2n+1\end{smallmatrix}\right.$

$\Rightarrow dx = -\ dt$

$$\Rightarrow \sum_{n=1}^{10}\int_{2n+1}^{2n}\sin^{27}(-t)(-dt)+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$$

$$\Rightarrow \sum_{n=1}^{10}\int_{2n+1}^{2n}\sin^{27}t.dt+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$$

$$\left\{\because \int_a^b f(x)dx = -\int_b^a f(x)dx\right\}$$

$$\Rightarrow \sum_{n=1}^{10}-\int_{2n}^{2n+1}\sin^{27}t\,dt+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dt$$

Let $t = x$

$dt = dx$

$$\Rightarrow \sum_{n=1}^{10}-\int_{2n}^{2n+1}\sin^{27}x\,dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$$

$\Rightarrow 0$

5. (b)

$\Rightarrow \int_0^2 [x^2]dx$

$\Rightarrow \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$

$\Rightarrow 0 + [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$

$\Rightarrow \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$\Rightarrow \sqrt{5} - \sqrt{3} - \sqrt{2}$

6. (d)

C: $y^2 = x^3$

Differentiate w.r.t. x

$2y \frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2y}$ = m (slope)

Slope of tangent at (m^2, m^3)

$\Rightarrow m_1 = \frac{3(m^2)^2}{2(m^3)}$

$\Rightarrow m_1 = \frac{3m^4}{2m^3} = \frac{3m}{2}$

Slope at (M^2, M^3)

$\Rightarrow m_1 = \frac{3(M^2)^2}{2M^3}$

$m_2 = \frac{3M^4}{2M^3} = \frac{3M}{2}$

Now tangent and normal are perpendicular to each other.

$\therefore m_1 m_2 = -1$

$\frac{3m}{2} . \frac{3M}{2} = -1$

$m M = \frac{-4}{9}$

7. (c)

Given that

$x^2 + y^2 = a^2$

Differentiate w.r.t. x

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y} \quad ....(1)$$

Now,

$$\Rightarrow \int_0^a \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_0^a \sqrt{1+\frac{x^2}{y^2}}\,dx \text{ \{from equation ...(1)\}}$$

$$= a\int_0^a \sqrt{\frac{1}{a^2 - x^2}}\,dx$$

$$= a\left[\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= a\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi a}{2}$$

8. (c)

$$\lim_{n\to\infty}\sum_{i=0}^{n}\frac{1}{n}f\left(\frac{i}{n}\right)$$

Let $\frac{1}{n} \to dx$

$$\frac{i}{n} \to x$$

upper limit

$$\lim_{n\to\infty}\frac{n}{n} = 1$$

lower limit

$$\lim_{n\to\infty}\frac{0}{n} = 0$$

$\therefore \int_0^1 f(x)dx$

9. (c)

$y' + y f'(x) - f(x)f'(x) = 0$

$\frac{dy}{dx} + yf'(x) = f(x)f'(x)$

Compare to linear differential equation $\frac{dy}{dx} + py = q$

So $p = f'(x)$

$q = f(x) f'(x)$

Integrating factor $= e^{\int pdx} = e^{\int f'(x)dx} = e^{f(x)}$

Now, $y.e^{f(x)} = \int e^{f(x)}.f(x).f'(x)dx$

Let $f(x) = t \Rightarrow f'(x)dx = dt$

$\therefore y.e^{f(x)} = \int t.e^t dt + c$

{Using by part}

$\Rightarrow ye^{f(x)} = te^t - \int e^t dt + c$

$\Rightarrow ye^{f(x)} = e^t(t-1) + c$

$\Rightarrow ye^{f(x)} = e^{f(x)}(f(x)-1) + c$

{Given $\lim_{x\to\infty} f(x)=0$, $\lim_{x\to\infty} y(x) = 0$}

$\Rightarrow (0)(e^{\circ}) = e^{\circ}(0-1) + c \Rightarrow c = 1$

$\therefore ye^{f(x)} = e^{f(x)}(f(x)-1) + 1$

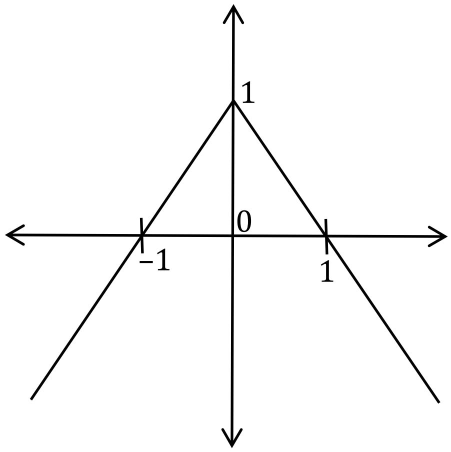
$y = (f(x)-1) + e^{-f(x)}$

$y + 1 = f(x) + e^{-f(x)}$

10. (c)

$f(x) = 1 - \sqrt{(x)^2}$

$f(x) = 1 - |x|$



$\therefore$ f(X) is maximum at x = 0 and is 1

11. (b)

$$x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x\right] dx$$

$$\sin\left(\frac{y}{x}\right) \frac{dy}{dx} = \frac{1}{x}\left[y \sin\left(\frac{y}{x}\right) - x\right]$$

$$\sin\left(\frac{y}{x}\right) \frac{dy}{dx} = \frac{y}{x} \sin\left(\frac{y}{x}\right) - 1$$

$$\frac{dy}{dx} = \frac{\frac{y}{x}\sin\left(\frac{y}{x}\right) - 1}{\sin\left(\frac{y}{x}\right)}$$

Let $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore v + x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v} - v$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1}{\sin v}$$

Integrate both side

$$\Rightarrow -\int \sin v\, dv = \int \frac{1}{x} dx$$

$\Rightarrow \cos v = \log x + c \Rightarrow \cos\frac{y}{x} = \log x + c$

$\{\because \text{ give } y(1) - \frac{\pi}{2} \Rightarrow x = 1, y = \frac{\pi}{2}\}$

$\therefore \cos\frac{\pi}{2} = \log 1 + c \Rightarrow c = 0$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \log x$

12. (a)

$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

Differentiate w.r.t. x

$f'(x) = 6x^2 - 18ax + 12a^2$ …….(1)

For extreme values

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 18ax + 12a^2 = 0$

$\Rightarrow x^2 - 3ax + 12a^2 = 0$

$\Rightarrow (x-a)\,(x-2a) = 0$

$\Rightarrow x = a, 2a$

Again differentiate equation (1) w.r.t. x

$f''(x) = 12\,x - 18\,a$

$x = a$

$f''(a) = 12a - 18a = -6a$

$f''(a) < 0$

So, maximum at $x = a = p$

$x = 2a$

$f''(2a) = 12(2a) - 18(a) = 6a$

$f''(2a) > 0$

So, minimum at $x = 2a = q$

Now,

$\Rightarrow P^2 = q$

$\Rightarrow (a)^2 = 2a$

$\Rightarrow a = 2$

13. (a)

Using the concept of A.M. $\geq$ G.M

A.M. $\geq$ G.M.

$$\Rightarrow \frac{\frac{6a}{5b}+\frac{10b}{3a}}{2} \geq \left(\frac{6a}{5b}\times\frac{10b}{3a}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{6a}{5b}+\frac{10b}{3a} \geq 2\ (4)^{\frac{1}{2}}$$

$$\Rightarrow \frac{6a}{5b}+\frac{10b}{3a} \geq 4$$

The least possible value of $\frac{6a}{5b}+\frac{10b}{3a}$ is 4

14. (a)

$2\log(x+1) - \log(x^2-1) = \log 2$

$\Rightarrow \log(x+1)^2 - \log(x^2-1) = \log 2$

$\Rightarrow \log\frac{(x+1)^2}{(x^2-1)} = \log 2$

$\Rightarrow \frac{(x+1)^2}{(x+1)(x-1)} = 2$

$\Rightarrow x + 1 = 2x - 2$

$\Rightarrow x = 3$

15. (c)

Let $p = x + iy$

Squaring both side

$p^2 = x^2 + i^2y^2 + 2ixy$

$p^2 = x^2 - y^2 + 2ixy$

Squaring both side

$p^4 = [(x^2 - y^2) + 2ixy]^2$

$p^4 = (x^2 - y^2)^2 + (2ixy)^2 + 4ixy(x^2 - y^2)$

$\because$ Imaginary part of p^4 is 0

$\therefore$ $xy(x^2 - y^2) = 0$

$\Rightarrow$ $x^3y - xy^3 = 0$

$\Rightarrow$ $x = \pm y$

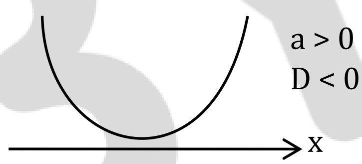
$\because$ $|p| = 1 \Rightarrow x^2 + y^2 = 1$

$2y^2 = 1$	$x = 0$
$y = \pm\frac{1}{\sqrt{2}}$	$y = \pm 1$
$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$	& $y = 0 \Rightarrow x \pm 1$

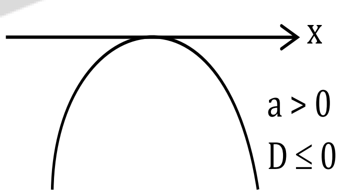
Total value of p is 8.

16. (b)
Given that
$z\bar{z} + (2-3i)z + (2+3i)\bar{z} + 4 = 0$
$\because$ centre and radius of $z\bar{z} + \bar{a}z + a\bar{z} + b = 0$
are $-a$ and $\sqrt{a\bar{a} - b}$
$\therefore$ radius $= \sqrt{(2-3i)(2+3i) - 4}$
$= \sqrt{13-4} = \sqrt{9} = 3$ unit

17. (c)
Given that
$ax^2 + bx + c$ has the same sign
Case-I: If $a > 0$, $ax^2 + bx + c > 0$, So $b^2 - 4ac < 0$



Case-II: If $a < 0$, $ax^2 + bx + c \leq 0$ So $D \leq 0$



18. (d)
The lift can stop at 12–1–1 = 10 floors (except the floor they enter and second floor)
Total number of ways = ${}^{10}P_3$

$= 10 \times 9 \times 8 = 720$

19. (b)

Set A = $\{a_1, a_2, a_3, a_4, \ldots\ldots, a_n\}$

From set of n element selecting a subset of m element = ${}^nC_m$

Now, a_4 is already selected.

$\therefore$ Total number of sets which contains a_n is ${}^{n-1}C_{m-1}$

Now, it is given that

${}^nC_m = K.\ {}^{n-1}C_{m-1}$

$$\Rightarrow \frac{n!}{m!(n-m)!} = K.\frac{(n-1!)}{(m-1)!(n-m)!}$$

$$\Rightarrow \frac{n}{m} = k$$

n = mk

20. (a)

Using the concept of A.M. $\geq$ G.M.

For J(n) A.m. > G. m.

$$\Rightarrow \frac{1+3+5\ldots\ldots+(2n-1)}{n} > (1.3.5.\ \ldots\ldots(2n-1))^{1/n}$$

$$\Rightarrow \frac{n^2}{n} > \left(J(n)\right)^{\frac{1}{n}}$$

$$\Rightarrow n > \left(J(n)\right)^{\frac{1}{n}}$$

$\Rightarrow n^n > J(n)$

I(n) > J(n)

21. (b)

$$\text{Let } S_n = \frac{{}^{15}C_1}{{}^{15}C_0} + \frac{2\,{}^{15}C_2}{{}^{15}C_1} + \frac{3\,{}^{15}C_3}{{}^{15}C_2} + \ldots\ldots + \frac{15\,{}^{15}C_{15}}{{}^{15}C_{14}}$$

$$S_n = \sum_{r=1}^{15} \frac{r.{}^{15}C_r}{{}^{15}C_{r-1}}$$

$$S_n = \sum_{r=1}^{15} \frac{r(15-r+1)}{r} \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$S_n = \sum_{r=1}^{15}(16-r)$

$S_n = 1 + 2 + 3 + 4 + \ldots\ldots + 15$

$S_n = \frac{15(16)}{2}$

$S_n = 120$

22. (d)

$A = \begin{bmatrix} 3-t & 1 & 0 \\ 1 & 3-t & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$|A| = -1\,[(-1)\,(3-t)-0]$

$\Rightarrow 5 = -1(t-3)$

$\Rightarrow 5 = 3-t$

$\Rightarrow t = -2$

23. (c)

$A = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix}$

$\because$ if matrix is not invertible $\Rightarrow |A| = 0$

$\therefore\ |A| = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix} = 0$

$\Rightarrow [-1(48-30) + 2\,(24-5x) + 3\,(72-24x)] = 0$

$\Rightarrow -18 + 48 - 10x + 216 - 72x = 0$

$-82x = -246$

$x = 3$

24. (a)

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1$

$\det A = ad - cb = 1 \ldots\ldots(1)$

Now,

$\Rightarrow A - \lambda I_2 = 0$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0$

$\Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$\Rightarrow (a - \lambda)(d - \lambda) - cb = 0$

$\Rightarrow \lambda^2 - \lambda(a + d) + ad - cb = 0$

$\Rightarrow \lambda^2 - \lambda(a + d) + 1 = 0$ {from eq. (1)}

$\because$ roots are imaginary

$\therefore D < 0$

$(a + d)^2 - 4(ad - cb) < 0$

$(a + d)^2 < 4$

25. (d)

$\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} a^2 & bc & c(c+a) \\ a(a+b) & b^2 & ca \\ ab & b(b+c) & c^2 \end{vmatrix}$

$\Rightarrow abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

$c_3 \to c_3 - c_1 - c_2$

$\Rightarrow (abc) \begin{vmatrix} a & c & 0 \\ a+b & b & -2b \\ b & b+c & -2b \end{vmatrix}$

$\Rightarrow (-2b)(abc) \begin{vmatrix} a & c & 0 \\ a+b & b & 1 \\ b & b+c & 1 \end{vmatrix}$

$\Rightarrow (-2b)(abc)(-2ac)$

$\Rightarrow 4a^2b^2c^2$

So, $4a^2b^2c^2 = ka^2b^2c^2$

$k = 4$

26. (d)

Given that

$f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$

$f\left[\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right] = 4 - 0 = 4$

$f\left[\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\right] = 4 - 0 = 4$

$\therefore$ not one-one function

As $O \in R$ but s does not contain any singular matrix so, f is not onto.

27. (b)

If $a - b = 0$ then $b - a = 0$

If $a - b$ is irrational then $b - a$ is irrational

$\therefore\ a\,\rho\,b \to b\,\rho\,a \Rightarrow$ symmetric

$\forall\ a \in R,\ a - a = 0,\ a\,\rho\,a \Rightarrow$ reflexive

If $a = 2,\ b = \sqrt{2},\ c = 3$ then

$a\,\rho\,b,\ b\,\rho\,c$ but $a\,\rho\,c$ is not true $\Rightarrow$ not transitive.

28. (b)

Given that

$\vec{\alpha} = 2\hat{i} + 2\hat{j} - \hat{k} \Rightarrow |\vec{\alpha}| = \sqrt{4+4+1} = 3$

$\vec{\beta} = \hat{j} - \hat{k} \Rightarrow |\vec{\beta}| = \sqrt{1+1} = \sqrt{2}$

Let the vector be $\vec{\gamma} = x\hat{i} + z\hat{k} \Rightarrow |\vec{\gamma}| = 1$

{Vector in zox plane}

$\vec{\gamma}.\vec{\alpha} = |\vec{\gamma}||\vec{\alpha}|\cos 45°$

$\Rightarrow (x\hat{i} + z\hat{k}).\ (2\hat{i} + 2\hat{j} - \hat{k}) = (3)\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow 2x - z = \frac{3}{\sqrt{2}}$ ......(1)

$\vec{\gamma}.\vec{\beta} = |\vec{\gamma}||\vec{\beta}|\cos 60°$

$\Rightarrow (x\hat{i}+z\hat{k}).(\hat{j}-\hat{k}) = (1)(\sqrt{2})\left(\frac{1}{2}\right)$

$\Rightarrow -z = \frac{1}{\sqrt{2}}$

$\Rightarrow z = -\frac{1}{\sqrt{2}}$ …… (2)

Comparing eq. (1) & eq. (2)

$x = \frac{1}{\sqrt{2}}$

$\therefore\ \vec{\gamma} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}} - \hat{k}$

29. (d)

A wins 1st attempt P(even number) = $\frac{1}{2}$

P(odd number) = $1-\frac{1}{2} = \frac{1}{2}$

P(A win) = P(A) + P($\bar{A}$) P(B) P($\bar{C}$) P(D) P(A) +…….

$\Rightarrow$ (A win) = $\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$ + …….

$= \frac{1}{2} + \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right) + \ldots\ldots$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^9 + \ldots\ldots.$

Using the concept of sum of infinite G.P.

P(A win) = $\frac{a}{1-r} = \frac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)^4}$

$= \frac{1}{2}\times\frac{16}{15} = \frac{8}{15}$

30. (b)

P (hitting a target) = $\frac{1}{10}$

$\therefore$ P (not hitting a target) = $1 - \frac{1}{10} = \frac{9}{10}$

$\therefore$ Let number of trials = n

Now, P (missing all) + P (hitting at least once) = 1

P(hitting at least once) = 1 – P(missing)

$= 1-\left(\frac{9}{10}\right)^n \geq \frac{1}{2}$

$\Rightarrow (0.9)^n \leq 0.5$

Now, n = 6 $\Rightarrow (0.9)^6 = 0.531441$

n = 7 $\Rightarrow (0.9)^7 = 0.4782969$

$\therefore$ Require at least 7 shots

31. (G)
    Bonus

32. (b)
    $y = e^x (A \cos x + B \sin x)$
    Differentiate w.r.t. x
    $\Rightarrow \frac{dy}{dx} = e^x(A \cos x + B \sin x) + e^x (-A \sin x + B \cos x)$
    $\Rightarrow \frac{dy}{dx} = y + e^x(-A \sin x + B \cos x)$
    Again differentiate w.r.t. x
    $\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x (-A \sin x + B \cos x) + e^x(-A \cos x - B \sin x)$
    $\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - e^x(A \cos x + B \sin x)$
    $\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

33. (d)
    $r \cos\left(\theta - \frac{\pi}{3}\right) = 2$
    $\{\therefore \cos (A - B) = \cos A \cos B + \sin A \sin B\}$
    $\therefore r\left[\cos\theta.\cos\frac{\pi}{3} + \sin\theta.\sin\frac{\pi}{3}\right] = 2$

$\Rightarrow r\left[\cos\theta\left(\frac{1}{2}\right)+\sin\theta\left(\frac{\sqrt{3}}{2}\right)\right] = 2$

$\Rightarrow \frac{r\cos\theta}{2}+\frac{\sqrt{3}r\sin\theta}{2} = 2$

$\Rightarrow r\cos\theta + \sqrt{3}r\sin\theta = 4$

{Let $r\cos\theta = x$, $r\sin\theta = y$}

$\Rightarrow x + \sqrt{3}y = 4$

34. (d)

Let $c_1 : x^2 + y^2 = a^2$

$c_2: (x-2a)^2 + y^2 = 4a^2$

Let, centre = (h, k) and radius = r for the variable circle

So using $c_1c_2 = r_1 + r_2$ for both cases we have:

$h^2 + k^2 = (r + a)^2$ .....(1)

And $(h-2a)^2 + k^2 = (r + 2a)^2$ ......(2)

Equation (2) – equation (1)

$r = \frac{a-4h}{2}$ ...... (3)

Substitute (3) in eq. (1) to get

$h^2 + k^2 = \left(\frac{a-4h}{2}+a\right)^2$

$12h^2 - 4k^2 - 24ah + 9a^2 = 0$

$\therefore$ locus: $12x^2 - 4y^2 - 24ax + 9a^2 = 0$ i.e. a hyperbola

35. (b)

Given that

$x^2 + 2xy + ay^2 = 0 \Rightarrow \left(\frac{x}{y}\right)^2+2\left(\frac{x}{y}\right) + a = 0$ ....(1)

$ax^2 + 2xy + y^2 = 0 \Rightarrow a\left(\frac{x}{y}\right)^2+2\left(\frac{x}{y}\right) + 1 = 0$ ....(1)

{Taking $\frac{x}{y}$ as a single variable}

$\because$ exactly one root in common

$\therefore (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$

$\Rightarrow (2-2a)(2-2a) = (1-a^2)^2$

$\Rightarrow (2-2a)^2 = (1-a^2)^2$

$\Rightarrow a = 1$ or -3

'a' cannot be 1 So, a = –3

1st equation $\Rightarrow \left(\frac{x}{y}\right)^2 + 2\left(\frac{x}{y}\right) + 1 = 0$

Roots: –1, –3

2nd equation $\Rightarrow -3\left(\frac{x}{y}\right)^2 + 2\left(\frac{x}{y}\right) + 1 = 0$

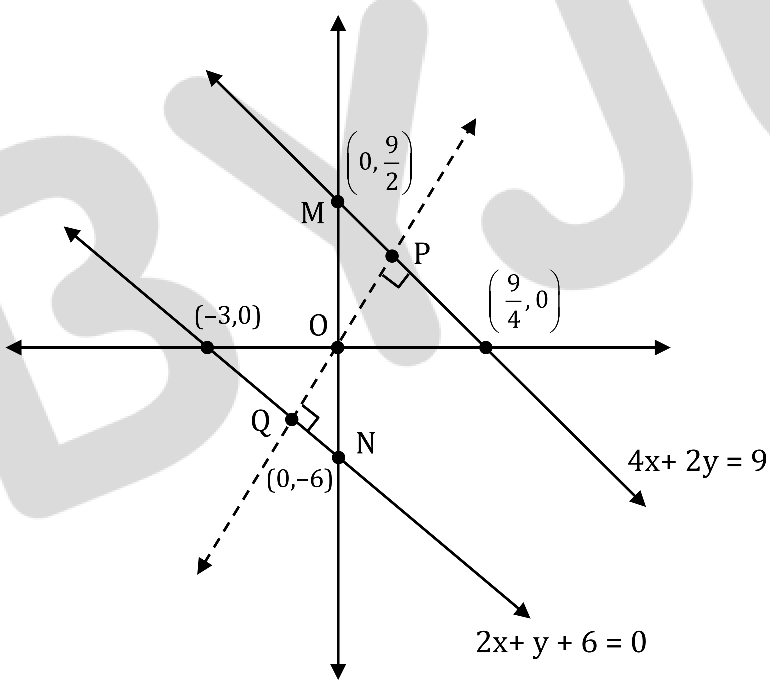
Roots: 1, $-\frac{1}{3}$

So other lines: $\frac{x}{y} = -3$ and $\frac{x}{y} = -\frac{1}{3}$

x = –3y and 3x = –y

x +3y = 0 and 3x + y = 0

36. (b)



$$\Delta OPM \sim \Delta OQN \Rightarrow \frac{OP}{OQ} = \frac{OM}{ON}$$

$$\Rightarrow \frac{OP}{OQ} = \frac{9/2}{6}$$

$$\Rightarrow \frac{OP}{OQ}=\frac{3}{4}$$

37. (a)

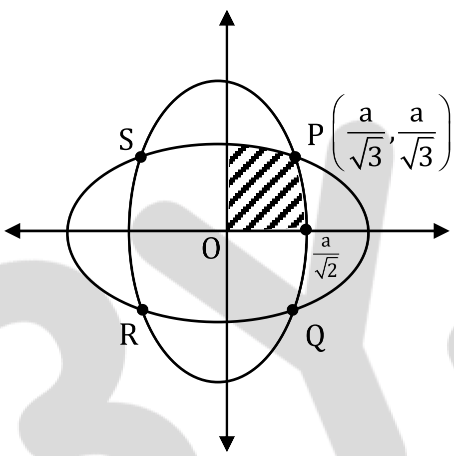
$C_1: x^2 + 2y^2 = a^2$

$C_2: 2x^2 + y^2 = a^2$

To find intersect point

$$P=\left(\frac{a}{\sqrt{3}},\frac{a}{\sqrt{3}}\right), \qquad Q=\left(\frac{a}{\sqrt{3}},\frac{-a}{\sqrt{3}}\right)$$

$$R=\left(\frac{-a}{\sqrt{3}},\frac{-a}{\sqrt{3}}\right), \qquad S=\left(\frac{-a}{\sqrt{3}},\frac{-a}{\sqrt{3}}\right)$$



$$\text{Area}=\frac{1}{\sqrt{2}}\int_{0}^{a/\sqrt{3}}\sqrt{a^2-x^2}\,dx+\int_{a/\sqrt{3}}^{a/\sqrt{2}}\sqrt{a^2-2x^2}\,dx$$

$$=\frac{1}{\sqrt{2}}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{a/\sqrt{3}}+\left[\frac{x}{2}\sqrt{a^2-2x^2}+\frac{a^2}{2\sqrt{2}}\sin^{-1}\frac{\sqrt{2}x}{a}\right]_{a/\sqrt{3}}^{a/\sqrt{2}}$$

$$=\frac{a^2}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$$

38. (a)

Given that

Centre on positive side of x-axis $\equiv (a, 0)$ $(a > 0)$

Radius = $\sqrt{17}$

$\therefore$ Equation of circle: $(x-a)^2 + (y-0)^2 = (\sqrt{17})^2$

$(x-a)^2 + y^2 = 17$

As, it passes through (0, 1)

$\therefore (0-a)^2 + (1)^2 = 17$

$\Rightarrow a^2 + 1 = 17$

$\Rightarrow a^2 = 16$

$\Rightarrow a = 4 \; \{\because a > 0\}$

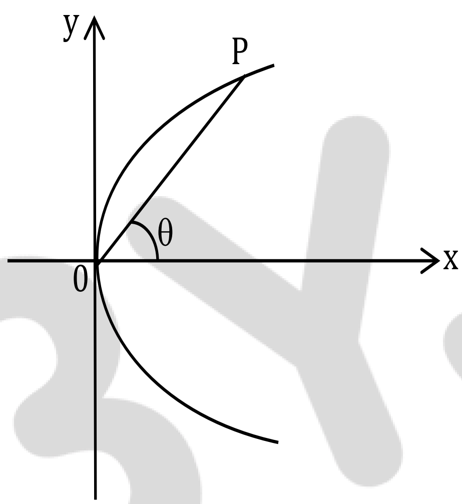
$\therefore$ Equation of circle is

$\Rightarrow (x-4)^2 + y^2 = 17$

$\Rightarrow x^2 - 8x + y^2 + 16 = 17$

$\Rightarrow x^2 + y^2 - 8x - 1 = 0$

39. (b)



Equation OP is: $(y-0) = \tan\theta\,(x-0)$

$y = x\tan\theta$

Solving with $y^2 = 4ax$, we get

$\Rightarrow (x\tan\theta)^2 = 4ax$

$x^2\tan^2\theta = 4ax$

$x\tan^2\theta = 4a$

$x = 4a\cot^2\theta$

Substituting, $y = (4a\cot^2\theta)(\tan\theta)$

$y = 4a\cot\theta$

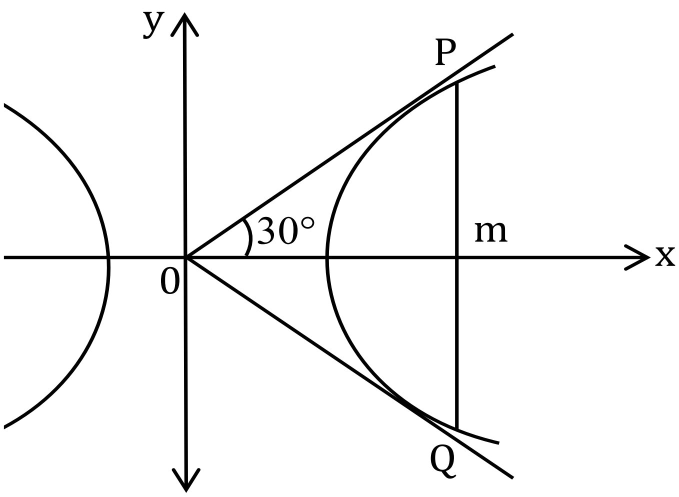
$\therefore P = (4a\cot^2\theta, 4a\cot\theta)$

So, OP = $\sqrt{(0-4a\cot^2\theta)^2 + (0-4a\cot\theta)^2}$

OP = $4a\cot\theta\,\text{cosec}\,\theta$

As $0° < \theta < 90°$ so $\cot\theta > 0$, $\text{cosec}\,\theta > 0$

40. (d)



Let P = (a sec θ, b tanθ) & Q ≡ (a sec θ, – b tan θ)

In ΔOPM

$\tan 30° = \dfrac{b\tan\theta}{a\sec\theta}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{b\sin\theta}{a}$

$\Rightarrow \dfrac{a}{b} = \sqrt{3}\sin\theta$

Eccentricity $e^2 = 1 + \dfrac{b^2}{a^2}$

$e^2 = 1+ \dfrac{1}{3\sin^2\theta}$

$e^2 > 1+ \dfrac{1}{3}$ $\{\because \max \sin^2\theta = 1\}$

$e^2 > \dfrac{4}{3}$

$e > \dfrac{2}{\sqrt{3}}$

41. (c)

Ellipse : $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

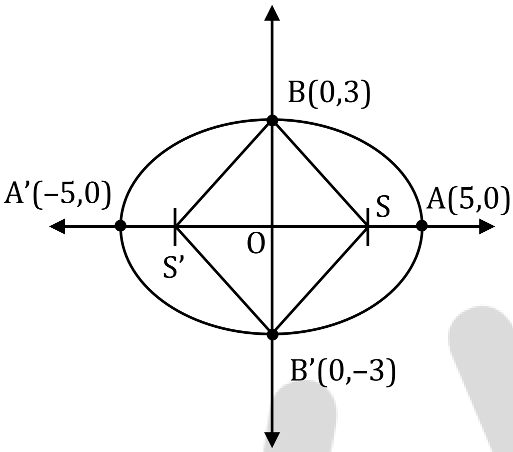
$a^2 = 25 \Rightarrow a = \pm 5$

$b^2 = 9 \Rightarrow b = \pm 3$

Eccentricity (e) = $\sqrt{1-\frac{b^2}{a^2}}$

$$e=\sqrt{1-\frac{9}{25}}$$

$$e=\frac{4}{5}$$



$\because$ S & S' foci of the ellipse

$$S=(ae,\,0)=\left(\pm5\times\frac{4}{5},\,0\right)=(\pm4,\,0)$$

Area of Rhombus = 4 × area of ΔBOS

$$= 4\times\left(\frac{1}{2}\times OS\times OB\right)$$

$$= 4\times\left(\frac{1}{2}\times4\times3\right)$$

$$= 24 \text{ sq. units}$$

42. (d)

Given that

$$\left.\begin{array}{l} L_1 : x\ +y=8 \\ L_2 : x+y=12 \end{array}\right\}\Rightarrow C_1=8,\ C_2=12,\ a=1,\ b=1$$

The distance between latus rectum and equation of tangent at vertex is 'a'.

Here $a = \left|\frac{C_1 - C_2}{\sqrt{a^2 + b^2}}\right|$

$$a = \left|\frac{8-12}{\sqrt{1+1}}\right|$$

$$a = \frac{4}{\sqrt{2}}$$

$$a = 2\sqrt{2}$$

Hence, length of latus rectum is

LR = 4a

$$= 4\left(2\sqrt{2}\right)$$

$$= 8\sqrt{2} \text{ units}$$

43. (G)
Bonus

44. (b)
Given that

Line: $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

$$\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Plane: 2x – 2y + z = 5

$\vec{n} = 2\hat{i} - 2\hat{j} + \hat{k} = 5$ (In Cartesian form)

Now,

$$\cos(90° - \theta) = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

$$= \left|\frac{\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right).\left(2\hat{i} - 2\hat{j} + \hat{k}\right)}{\left|3\hat{i} + 4\hat{j} + 5\hat{k}\right| \times \left|2\hat{i} - 2\hat{j} + \hat{k}\right|}\right|$$

$$=\left|\frac{6-8+5}{\sqrt{9+16+25}\sqrt{4+4+1}}\right|$$

$$=\left|\frac{3}{5\sqrt{2}.3}\right|$$

$$=\frac{\sqrt{2}}{10}$$

45. (c)

f(x) = sinx + cos ax

Period of sinx + cos ax is LCM of 1 and a but LCM of rational multiple of same irrational is defined.

46. (c)

$$f(x)=\sqrt{\left(\frac{1}{\sqrt{x}}-\sqrt{x-1}\right)}$$

$x > 0$, $x + 1 \geq 0 \Rightarrow x \geq -1$

$$\& \ \frac{1}{\sqrt{x}}-\sqrt{x+1}\geq 0$$

$$\Rightarrow \frac{1}{\sqrt{x}}\geq \sqrt{x+1}$$

Squaring both side

$$\Rightarrow \frac{1}{x}\geq x+1$$

$\Rightarrow x^2 + x - 1 \leq 0$

$$x=\frac{-1\pm\sqrt{1-4(1)(-1)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{5}}{2}$$

$$\therefore x\in\left(0,\frac{\sqrt{5}-1}{2}\right]$$

47. (c)

$\Delta x = -0.01$ {$\because$ changes from 2 to 1.99}

$f(x) = 2x^2 - 3x + 2$

Differentiate w.r.t. x

$f'(x) = 4x - 3$

Now,

$\Delta y = f'(x)\Delta x$

$\Delta y = f'(2)(-0.01)$

$\Delta y = [4(2) - 3]\ (-0.01)$

$\Delta y = -0.05$

48. (c)

Given

$$\lim_{x\to 0}\left[\frac{1+cx}{1-cx}\right]^{\frac{1}{x}} = 4$$

$\because$ 1^∞ form

$$\therefore \lim_{x\to 0} e^{\frac{1}{x}\left[\frac{1+cx}{1-cx}-1\right]} = 4$$

$$\Rightarrow \lim_{x\to 0} e^{\frac{1}{x}\left[\frac{1+cx-1+cx}{1-cx}\right]} = 4$$

$$\Rightarrow \lim_{x\to 0} e^{\frac{1}{x}\left[\frac{2cx}{1-cx}\right]} = 4$$

$$\Rightarrow \lim_{x\to 0} e^{\left[\frac{2c}{1-cx}\right]} = 4$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{2c}{1-cx}\right] = \log_e 4$$

$\Rightarrow 2c = 2\log_e 2$

$c = \log_e 2$

Now

$$L = \lim_{x\to 0}\left[\frac{1+2cx}{1-2cx}\right]^{\frac{1}{x}}$$

$\because$ 1^∞ form

$$\therefore \lim_{x\to 0} e^{\frac{1}{x}\left[\frac{1+2cx-1+2cx}{1-2cx}\right]}$$

$$L = \lim_{x\to 0} e^{\frac{1}{x}\left[\frac{4cx}{1-2cx}\right]}$$

$L = \lim_{x \to 0} e^{\left[\frac{4}{1-2cx}\right]}$

$L = e^{\frac{4c}{1}}$

$L = e^{4c}$

$L = e^{4\log_e 2}$

L = 16

49. (a)

Consider f(x) on [0, 1]

f(x) be the twice continuously differentiable function.

Applying Rolle's theorem on the interval [0, 1].

f'(a) = 0 for some a $\in$ (0, 1)

Now, applying Rolle's theorem to f'(x) on the interval [0, a]

f''(c) = 0 for some c $\in$ (0, a)

50. (b)

Given

$f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12$

Differentiate w.r.t. x

$f'(x) = 13x^2 + 11x^{10} + 9x^8 + 7x^6 + 5x^4 + 3x^2 + 1$

f'(x) = '+'

f'(x) > 0

i.e. monotonically increasing $\forall\ x \in R$

f(x) = 0 has exactly one real root

$\Rightarrow$ f(x) intersets x-axis at only one point

$\therefore$ exactly one solution.

51. (c)

Given that

$z^2 + pz + q = 0$

$\because$ z_1 & z_2 are the roots of given equation.

Sum of root = $z_1 + z_2 = -p$

Product of roots = $z_1 z_2 = q$

If z_1, z_2, z_3 are the vertices of an equilateral

Triangle then $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

If z_3 is origin then $z_1^2 + z_2^2 = z_1 z_2$

$\Rightarrow z_1^2 + z_2^2 + 2z_1z_2 = z_1z_2 + 2z_1z_2$

$\Rightarrow (z_1 + z_2) = 3z_1z_2$

$\Rightarrow (-p)^2 = 3q$

$p^2 = 3q$

52. (c)

$$\Rightarrow \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\because \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \quad \{\because \vec{a}, \vec{b} \;\&\; \vec{c} \text{ are three non-coplanar vector}\}$$

$\therefore$ abc + 1 = 0

abc = −1

53. (c)

Given that

C_1: $y = ax^2 + bx + c$

Differentiate w.r.t. x

$\frac{dy}{dx} = 2ax + b$ .....(1)

Given

Line: y = x

$\frac{dy}{dx} = 1$ .....(2)

$\because$ Slopes are equal

$\therefore$ 2ax + b = 1

at (1, 1)

2a + b = 1 .....(3)

Now, (1, 1) and (–1, 0) satisfies the curve

a + b + c = 1 .....(4)

a – b + c = 0 .....(5)

Equation (5) – equation (4)

$\Rightarrow$ a – b + c – a – b – c = 0 – 1

$\Rightarrow$ –2b = –1

$b = \frac{1}{2}$

$\therefore a = \frac{1}{4}$ {From equation (3)}

$c = \frac{1}{4}$

54. (b)

f(x) = cos x + x sin x – 1

Differentiate w.r.t. x

f'(x) = –sin x + sin x + x cos x > 0 ; $x \in \left(0, \frac{\pi}{2}\right)$

f(x) is increasing function

$\Rightarrow f(x) > f(0)$

$\cos x + x \sin x - 1 > 0$

$\cos x + x \sin x > 1$

55. (c)

Given that

$C_1: x^2 + y^2 = 1$

$C_2: x + y = 1$

To find intersection point

$\Rightarrow x^2 + (1 - x)^2 = 1$

$x^2 + 1 - 2x + x^2 = 1$

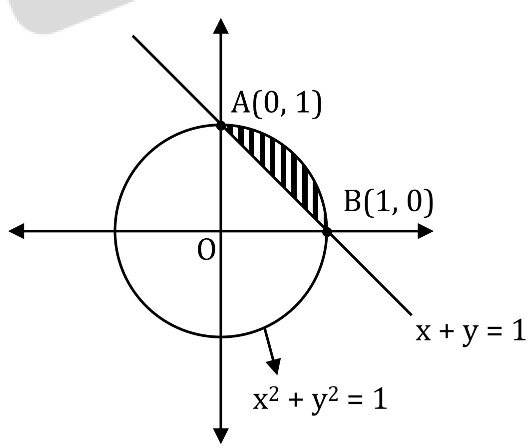
$2x^2 - 2x = 0$

$x^2 - x = 0$

$x(x - 1) = 0$

$x = 0, 1$

for $x = 0$	for $x = 1$
$\Rightarrow 0 + y = 1$	$\Rightarrow 1 + y = 1$
$y = 1$	$y = 0$
$A(0, 1)$	$B = (1, 0)$



Area = $\frac{1}{4}$ (area of circle) – area of ΔAOB

$= \frac{1}{4}\pi(1)^2 - \frac{1}{2}.1.1$

$= \frac{\pi}{4} - \frac{1}{2}$ sq. units

56. (a)

Given

$P(x) = ax^2 + bx + c$

$D_1 = b^2 - 4ac$

$Q(x) = -ax^2 + dx + c$

$D_2 = d^2 + 4ac$

$\Rightarrow D_1 + D_2$

$\Rightarrow b^2 - 4ac + d^2 + 4ac$

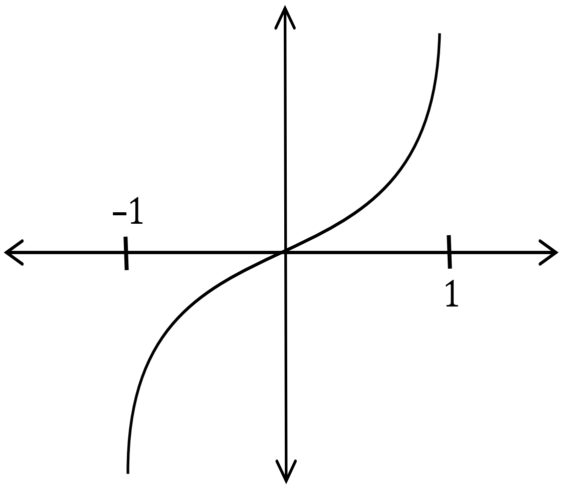
$\Rightarrow b^2 + d^2 > 0$

At least two real roots.

57. (d)

$f(x) = x\,|x|$

$$f(x) = \begin{cases} x^2 & x > 0 \\ 0 & x = 0 \\ -x^2 & x < 0 \end{cases}$$



$\because$ It is one-one and onto

$\therefore$ f(x) is bijective.

58. (d)

Given

$f(x) = \sqrt{x^2-3x+2}$

$g(x) = \sqrt{x}$ $\{\therefore x > 0\}$

$fog(x) = f[g(x)]$

$= f[\sqrt{x}]$

$= \sqrt{x-3\sqrt{x}+2}$

$\Rightarrow x - 3\sqrt{x} + 2 \geq 0$

$x + 2 \geq 3\sqrt{x}$

Squaring both side

$\Rightarrow (x + 2)^2 \geq (3\sqrt{x})^2$

$\Rightarrow x^2 + 4x + 4 \geq 9x$

$\Rightarrow x^2 - 5x + 4 \geq 0$

$\Rightarrow (x - 1)(x - 4) \geq 0$

$\therefore S = x \in (0,1] \cup [4,\infty)$

$gof(x) = g[f(x)]$

$= g(\sqrt{x^2-3x+2})$

$= \sqrt{\sqrt{x^2-3x+2}}$

$\Rightarrow x^2 - 3x + 2 \geq 0$

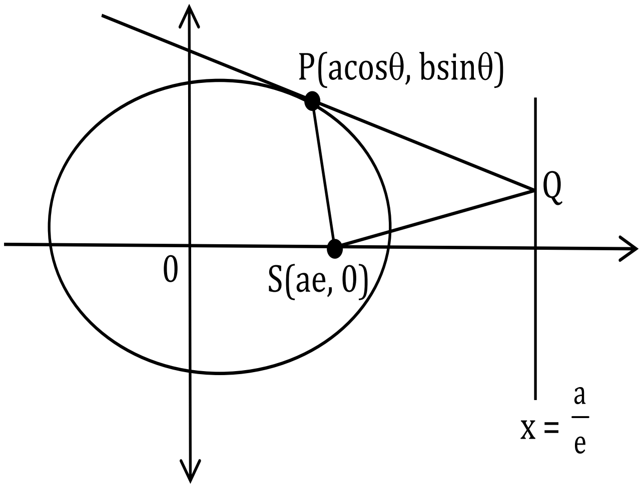
$\Rightarrow (x - 1)(x - 2) \geq 0$

$\therefore T = x \in (-\infty, 1] \cup [2, \infty)$

$\therefore S \cap T = (0, 1] \cup [4, \infty)$

59. (b)
Given ρ_1, ρ_2 are equivalence relations on S.
$\Rightarrow \rho_1, \rho_2$ are reflexive, symmetric and transitive.
Reflexive:
Let $x \in S$
$\Rightarrow (x, x) \in \rho_1$ and $(x, x) \in \rho_2$
$\Rightarrow (x, x) \in \rho_1 \cap \rho_2$
$\Rightarrow \rho_1 \cap \rho_2$ is reflexive.
Symmetric:
Let $(x,y) \in \rho_1 \cap \rho_2$
We have to show $(y, x) \in \rho_1 \cap \rho_2$
$(x, y) \in \rho_1 \cap \rho_2$
$\Rightarrow (x, y) \in \rho_1$ and $(x, y) \in \rho_2$
$\Rightarrow (y, x) \in \rho_1$ and $(y, x) \in \rho_2$
$\Rightarrow (y, x) \in \rho_1 \cap \rho_2$
$\Rightarrow \rho_1 \cap \rho_2$ is symmetric
Transitive:
Let $(x, y), (y, z) \in \rho_1 \cap \rho_2$
$\Rightarrow (x, y), (y, z) \in \rho_1$ and $(x, y), (y, z) \in \rho_2$
$\Rightarrow (x, z) \in \rho_1$ and $(x, z) \in \rho_2$
$\Rightarrow (x, z) \in \rho_1 \cap \rho_2$
$\Rightarrow \rho_1 \cap \rho_2$ is transitive.
Therefore $\rho_1 \cap \rho_2$ is equivalence relation.
$\rho_1 \cap \rho_2$ is always reflexive and symmetric but not transitive.
e.g. Let S = {1, 2, 3}
$\rho_1 = \{(1, 1), (2, 2)\ (3, 3), (1, 2), (2, 1)\}$
$\rho_2 = \{(1, 1), (2, 2)\ (3, 3), (2, 3), (3, 2)\}$
ρ_1, ρ_2 is equivalence relation.
But $\rho_1 \cup \rho_2$ is not transitive as $(1, 2), (2, 3) \in \rho_1 \cap \rho_2$ but $(1, 3) \notin \rho_1 \cup \rho_2$

60. (c)
Ellipse:



Equation of tangent at any point P(acosθ, bsinθ) on the ellipse is

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

Put $x=\frac{a}{e}$

$$\Rightarrow \frac{a\cos\theta}{ea}+\frac{y\sin\theta}{b}=1$$

$$\Rightarrow y=\left(1-\frac{\cos\theta}{e}\right)\frac{b}{\sin\theta}$$

$$\therefore Q=\left(\frac{a}{e},\left(1-\frac{\cos\theta}{e}\right)\frac{b}{\sin\theta}\right)$$

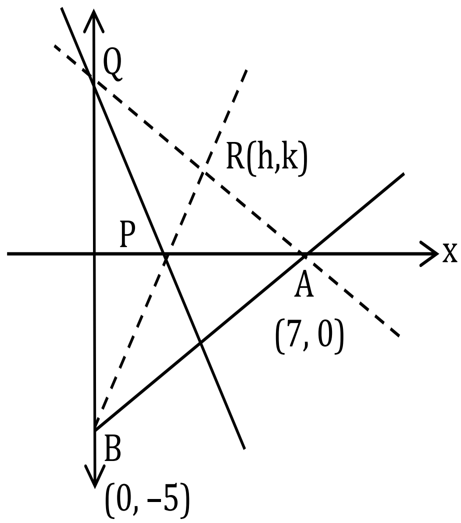
Now, slope of SQ × slope of PS

$$=\frac{\left(1-\frac{\cos\theta}{e}\right)\frac{b}{\sin\theta}}{\frac{a}{e}-ae}\times\frac{b\sin\theta}{a\cos\theta-ae}$$

$$=\frac{(e-\cos\theta)b}{a(1-e^2)\sin\theta}\times\frac{b\sin\theta}{a(\cos\theta-e)}=-1$$

So, the angle between the line PS and SQ is $\frac{\pi}{2}$.

61. (c)



P is orthocenter of ΔABQ

$m_{BR} \times m_{AR} = -1$

$$\Rightarrow \left(\frac{k+5}{h-0}\right)\times\left(\frac{k-0}{h-7}\right) = -1$$

$$\Rightarrow \frac{k(k+5)}{h(h-7)} = -1$$

$\Rightarrow k^2 + 5k = -h(h-7)$

$\Rightarrow k^2 + 5k = -h^2 + 7h$

$\Rightarrow h^2 + k^2 + 5k - 7h = 0$

$\Rightarrow x^2 + y^2 + 5y - 7x = 0$

62. (b)
(Let)

$$L = \lim_{n\to\infty}\sum_{k=1}^{n}\int_{1/(k+\beta)}^{1/(k+\alpha)}\frac{dx}{1+x}$$

$$L = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\ell n(1+x)\right]_{\frac{1}{k+\beta}}^{\frac{1}{k+\alpha}}$$

$$L = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\ell n\left(1+\frac{1}{k+\alpha}\right)-\ell n\left(1+\frac{1}{k+\beta}\right)\right]$$

$$L = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\ell n\left(\frac{k+\alpha+1}{k+\alpha}\right)-\ell n\left(\frac{k+\beta+1}{k+\beta}\right)\right]$$

$$L = \lim_{n\to\infty}\sum_{k=1}^{n}\ell n\left[\left(\frac{k+\beta}{k+\alpha}\right)\left(\frac{k+\alpha+1}{k+\beta+1}\right)\right]$$

$$L = \ell n\left[\frac{\beta+1}{\alpha+1}\times\frac{\alpha+2}{\beta+2}\times\frac{\beta+2}{\alpha+2}\times\frac{\alpha+3}{\beta+3}\times......\right]$$

$$L = \log_e\left(\frac{1+\beta}{1+\alpha}\right)$$

63. (c)

Let $L = \lim_{x\to 1}\left[\frac{1}{\ell n x}-\frac{1}{(x-1)}\right]$

$\Rightarrow L = \lim_{x\to 1}\left[\frac{x-1-\ell n x}{(x-1).\ell n x}\right]$

0/0 form, using L Hospital's rule

$\Rightarrow L = \lim_{x\to 1}\left(\frac{1-0-\frac{1}{x}}{\ell n x+\frac{x-1}{x}}\right)$

$\Rightarrow L = \lim_{x\to 1}\left(\frac{x-1}{x\ell nx+x-1}\right)$

0/0 form, using L Hospital's rule

$\Rightarrow L = \lim_{x\to 1}\left(\frac{1}{\ell nx+1+1}\right)$

$\Rightarrow L = \left(\frac{1}{0+2}\right)$

$\Rightarrow L = \frac{1}{2}$

64. (b)

$y = \frac{1}{1+x+\ell nx}$

Differentiate w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x+\ell nx)^2}\left(1+\frac{1}{x}\right)$

$\Rightarrow \frac{dy}{dx} = -y^2\left(\frac{x+1}{x}\right)$

$\Rightarrow x\frac{dy}{dx} = -y^2\ (x+1)$

$\Rightarrow x\dfrac{dy}{dx} = -y^2\left(\dfrac{1}{y} - \ell nx\right)$

$\Rightarrow x\dfrac{dy}{dx} = -y + y^2\,\ell nx$

$\Rightarrow x\dfrac{dy}{dx} = y\,(y\,\ell nx - 1)$

65. (b)
Given
$y = be^{-x/a}$
Differentiate w.r.t. x
$\dfrac{dy}{dx} = \dfrac{-b}{a}e^{-x/a}$

Tangent: $\dfrac{x}{a} + \dfrac{y}{b} = 1$

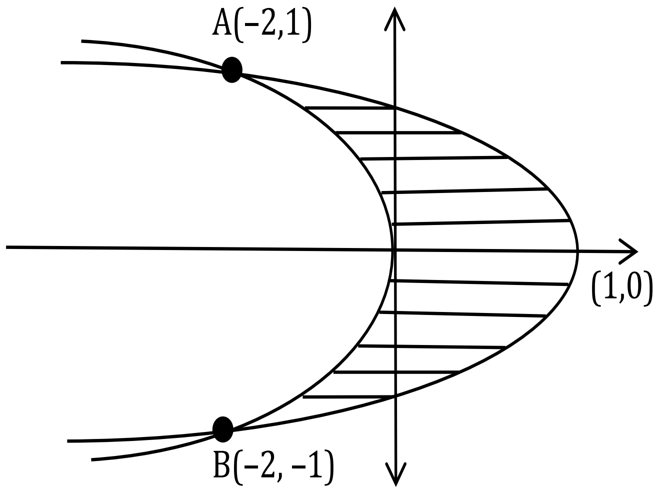
Slope of the tangent $\dfrac{1}{a} + \dfrac{1}{b}\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{-b}{a}$

So, $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is tangent to the curve at the point where $x = 0 \Rightarrow y = b$

So, $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is a tangent to the curve at the point $(0, b)$

66. (b)
$C_1: x = -2y^2$
$C_2: x = 1 - 3y^2$
To find intersecting point
$\Rightarrow -2y^2 = 1 - 3y^2$
$\Rightarrow y^2 = 1$
$y = \pm 1$
So, $x = -2$



$$\text{Area} = 2\left[\int_0^1 [1-3y^2-(2y^2)dy]\right]$$

$$= 2\left[\int_0^1 (1-y^2)dy\right]$$

$$= 2\left[y-\frac{y^3}{3}\right]_0^1$$

$$= 2\left[1-\frac{1}{3}\right] = \frac{4}{3} \text{ square units.}$$

67. (a,d)

If the initial velocity is u ft/sec then time taken for the entire motion is: $t = \frac{2u}{g}$

$g = 32$ ft/sec^2 and $t = 12$ second

$t = \frac{2u}{g}$

$\Rightarrow 12 = \frac{2u}{32}$

$\Rightarrow u = 192$

Greatest height attained

$H = \frac{u^2}{29}$

$\Rightarrow H = \frac{(192)^2}{2(32)}$

$\Rightarrow H = 576$

68. (a,c)

$$x^{(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5} = 3\sqrt{3}$$

Let $\log_3 x = t \Rightarrow x = 3^t$

$$\therefore\ (3^t)^{t^2 - \frac{9}{2}t + 5} = 3\sqrt{3}$$

$$\Rightarrow (3)^{t^3 - \frac{9}{2}t^2 + 5t} = (3)^{\frac{3}{2}}$$

Comparing

$$\Rightarrow\ t^3 - \frac{9}{2}t^2 + 5t = \frac{3}{2}$$

$\Rightarrow 2t^3 - 9t^2 + 10t = 3$

$\Rightarrow 2t^3 - 9t^2 + 10t - 3 = 0$

$\Rightarrow (2t-1)\ (t-1)\ (t-3) = 0$

$t = \frac{1}{2}, 1, 3$

$t = \frac{1}{2}$	$t = 1$	$t = 3$
$\therefore\ x = 3^{\frac{1}{2}}$	$x = 3^1$	$x = 3^3$
$x = \sqrt{3}$	$x = 3$	$x = 27$

$\therefore\ x = \sqrt{3}$, 3, 27 are the roots of the given equation.

69. (b)

The number of students answering exactly i
$(1 \le I \le n - 1)$ questions wrongly is $2^{n-i} - 2^{n-i-1}$
Thus, the total number of wrong answer is
$1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + \ldots\ldots + (n-1)\ (2^1 - 2^0) + n(2^0) + \ldots\ldots$
$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots.. + 2^0 = 2^n - 1$
Thus, $2^n - 1 = 2047$
$\Rightarrow 2^n = 2048$
$\Rightarrow 2^n = 2^{11}$
$\Rightarrow n = 11$

70. (c,d)

$$P(A' \cap B') = \frac{3}{5}$$

$$\Rightarrow 1 - P\ (A \cup B) = \frac{3}{5}$$

$$\Rightarrow P(A \cup B) = \frac{2}{5}$$

$$\Rightarrow P(A) + P(B) - P(A)\ P(B) = \frac{2}{5}$$

$\Rightarrow P(A) + P(B) = \frac{2}{5} + \frac{1}{20}$ $\{\because P(A)P(B) = \frac{1}{20} \Rightarrow P(B) = \frac{1}{20P(A)}\}$

$\Rightarrow P(A) + P(B) = \frac{9}{20}$

$\Rightarrow P(A) + \frac{1}{20P(A)} = \frac{9}{20}$

$\Rightarrow 20[P(A)]^2 + 1 = 9\,P(A)$

$\Rightarrow 20[P(A)]^2 - 9P(A) + 1 = 0$

$\Rightarrow (4P(A)-1)\,(5\,P(A)-1) = 0$

$\Rightarrow P(A) = \frac{1}{4}, \frac{1}{5}$

71. (a,b)

Equation of a line making an x-intercept = 'a' units and y – intercept = 'b' units is given by

$\frac{x}{a} + \frac{y}{b} = 1$ ....(1)

Also, $a + b = -1$ .....(2)

And equation (1) passes through point (4, 3)

$\therefore\ \frac{4}{a} + \frac{3}{b} = 1$ .....(3)

From (2), $b = -a-1$ .....(4)

Substituting eq.(4) in eq. (3) we get

$\Rightarrow \frac{4}{a} + \frac{3}{-a-1} = 1$

$\Rightarrow \frac{4}{a} - \frac{3}{a+1} = 1$

$\Rightarrow 4a + 4 - 3a = a\,(a + 1)$

$\Rightarrow a + 4 = a^2 + a$

$\Rightarrow a = \pm 2$

$\therefore$ a = 2   a = –2

b = –3   b = 1

$\therefore$ the probable equation will be

$\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{-x}{2} + \frac{y}{1} = 1$

72. (c)

Given

$f(x) = \frac{x}{3}\sin x - (1-\cos x)$

Now,

$$\lim_{x\to 0}\frac{f(x)}{x^k} = \lim_{x\to 0}\frac{\frac{x}{3}\sin x - 3(1-\cos x)}{x^k}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{x\sin x - 3(1-\cos x)}{x^k}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{2x\sin\frac{x}{2}\cos\frac{x}{2} - 6\sin^2\frac{x}{2}}{x^k}$$

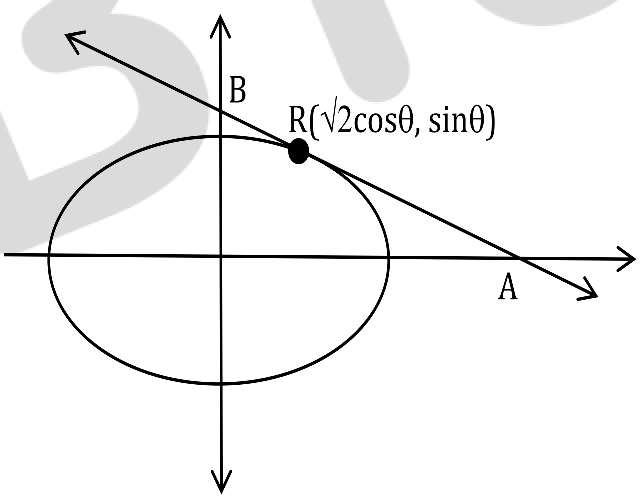
$$= \frac{1}{3}\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)\lim_{x\to 0}\left(\frac{2x\cos\frac{x}{2} - 6\sin\frac{x}{2}}{2x^{k-1}}\right)$$

$$= \frac{1}{3}\lim_{x\to 0}\left(\frac{x\cos\frac{x}{2} - 3\sin\frac{x}{2}}{x^{k-1}}\right) \neq 0$$

$\Rightarrow k - 1 = 1$

$\Rightarrow k = 2$

73. (d)



Tangent at R ($\sqrt{2}\sin\theta$, $\cos\theta$)

$$\Rightarrow \frac{x\sqrt{2}\cos\theta}{2} + \frac{y\sin\theta}{1} = 1 \quad \left\{\because \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1\right\}$$

$A = \left(\frac{\sqrt{2}}{\cos\theta}, 0\right)$, $B = \left(0, \frac{1}{\sin\theta}\right)$

Let P(h, k) be the locus of the mid point

$\therefore$ (h, k) = $\left(\frac{\sqrt{2}}{2\cos\theta}, \frac{1}{2\sin\theta}\right)$

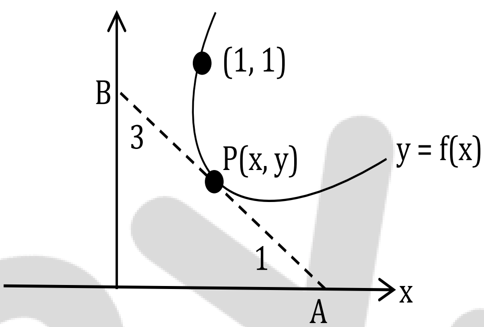
$\therefore \cos\theta = \frac{1}{\sqrt{2}h}, \sin\theta = \frac{1}{2k}$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$

$\therefore$ Locus of (h, k) is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

74. (a,c)



Since, BP: AP = 3 : 1, then equation of tangent is

Y – y = f′x (X–x)

The intercept on the coordinate axes are

$A = \left(x - \frac{y}{f'(x)} - 0\right)$ and B(0, y – x f′(x))

Since, P is internally intercepts a line AB

$\therefore x = \frac{3\left(x - \frac{y}{f'(x)}\right) + 1 \times 0}{3+1}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{-3x}$

$\Rightarrow \frac{dy}{dy} = -\frac{1}{3x}dx$

On integrating both sides, we get $xy^3 = c$

Since, curves passes through (1, 1), then c = 1

$\therefore xy^3 = 1$

At $x = \frac{1}{8} \Rightarrow y = 2$

75. (a)

$$y = \frac{x^2}{(x+1)^2(x+2)}$$

Using partial fraction concept

$$y = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$$

$$\Rightarrow \frac{x^2}{(x+1)^2(x+2)} = \frac{A(x+1)(x+2)+B(x+2)+C(x+1)^2}{(x+1)^2(x+2)}$$

$$\Rightarrow \frac{x^2}{(x+1)^2(x+2)} = \frac{x^2(A+C)+x(3A+B+2C)+(2A+2B+C)}{(x+1)^2(x+2)}$$

Comparing

A = −3, B = 1, C = 4

$$\therefore y = \frac{-3}{(x+1)} + \frac{1}{(x+1)^2} + \frac{4}{(x+2)}$$

Differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{3}{(x+1)^2} - \frac{2}{(x+1)^3} + \frac{4}{(x+2)^2}$$

Again differentiate w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-6}{(x+1)^3} + \frac{6}{(x+1)^4} + \frac{8}{(x+2)^3}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\left[\frac{-3}{(x+1)^3} + \frac{3}{(x+1)^4} + \frac{4}{(x+2)^3}\right]$$

WBJEE 2020 (Chemistry)

1. What will be the mass of one atom of ^{12}C:

a. 1 amu
b. 1.9923×10^{-23}g
c. 1.6603×10^{-22} g
d. 6 amu

2. Bond order of He_2, He_2^+ and He_2^{2+} are respectively.

a. 1, $\frac{1}{2}$, 0
b. 0, $\frac{1}{2}$, 1
c. $\frac{1}{2}$, 1, 0
d. 1, 0, $\frac{1}{2}$

3. To a solution of a colourless efflorescent sodium salt, when dilute acid is added, a colourless gas is evolved along with formation of a white precipitate. Acidified dichromate solution turns green when the colourless gas is passed through it. The sodium salt is

a. Na_2SO_3
b. Na_2S
c. $Na_2S_2O_3$
d. $Na_2S_4O_6$

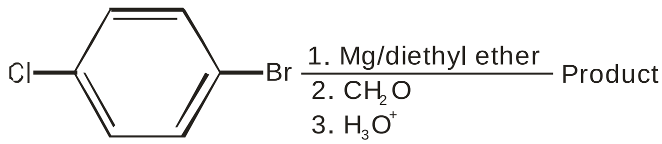
4. The reaction for obtaining the metal (m) from its oxide M_2O_3 ore is given by

$M_2O_3(s) + 2Al(l) \xrightarrow{\text{Heat}} Al_2O_3(l) + 2M(s)$,(s = solid, l = liquid) in the case, M is

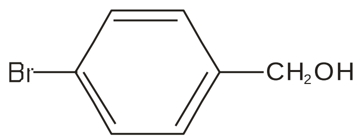
a. Copper
b. Calcium
c. Iron
d. Zinc

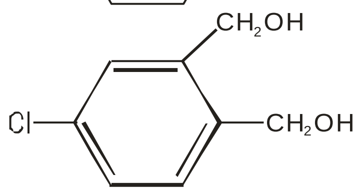
5. In the extraction of Ca by electro reduction of molten $CaCl_2$ some CaF_2 is added to the electrolyte for the following reason :

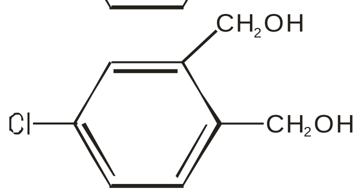
a. To keep the electrolyte in liquid state at temperature lower than the melting point of $CaCl_2$
b. To effect precipitation of Ca
c. To effect the electrolysis at lower voltage
d. To increase the current efficiency

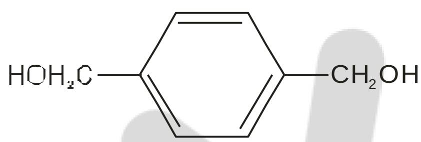
6. 

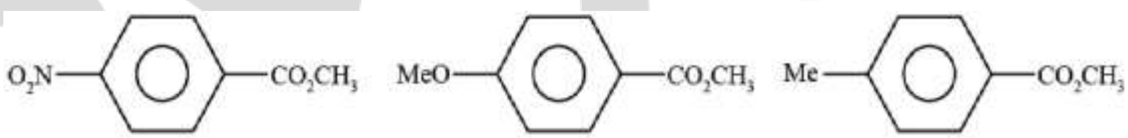
This product in the above reaction is

a. 

b. 

c. 

d. 

7. 

(I) (II) (III)

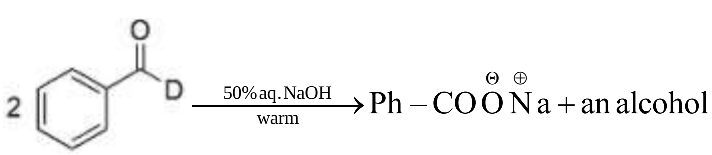
For the above three esters, the order of rates of alkaline hydrolysis is

a. I > II > III

b. II > III > I

c. I > III > II

d. III > I > II

8. 

This alcohol is

a. Ph – CHD – OH

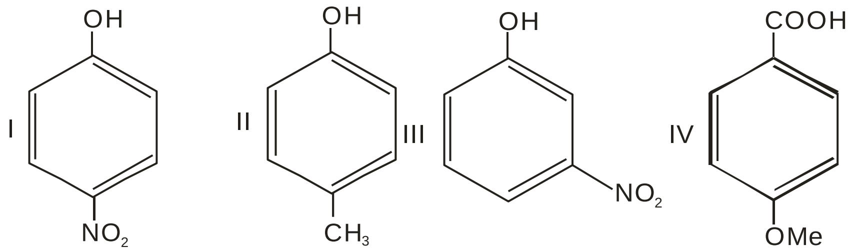
b. Ph – CHD – OD

c. Ph – CD_2 – OH

d. Ph – CD_2 – OD

WBJEE 2020 (Chemistry)

9. The correct order of acidity for the following compound is :



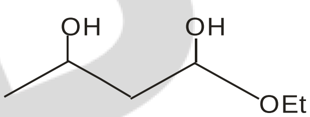
a. II < IV < III < I
b. II < III < I < IV
c. II < III < IV < I
d. III < II < I < IV

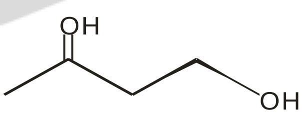
10. For the following carbocations the correct order of stability is

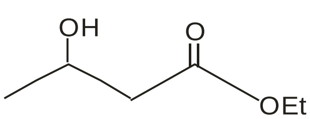
I : ${}^{\oplus}CH_2-COCH_3$ II : ${}^{\oplus}CH_2-OCH_3$ III : ${}^{\oplus}CH_2-CH_3$

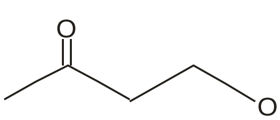
a. III < II < I
b. II < I < III
c. I < II < III
d. I < III < II

11. The reduction product of ethyl-3-oxobutanoate by $NaBH_4$ in methanol is

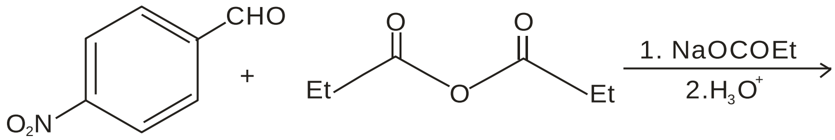
a. 

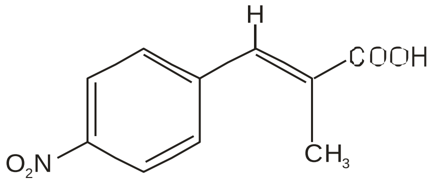
b. 

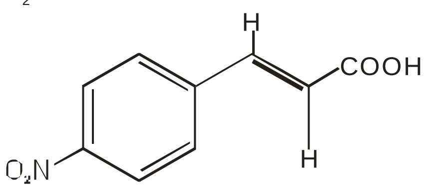
c. 

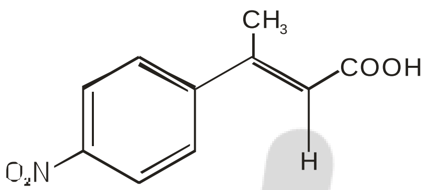
d. 

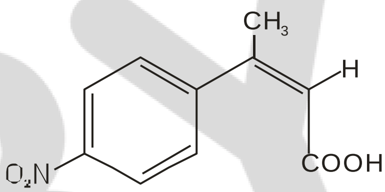
12. What is the major product of the following reaction?



a. 

b. 

c. 

d. 

13. The maximum number of electrons in an atom in which the last electron filled has the quantum number n = 3, I = 2 and m = −1 is

a. 17

b. 27

c. 28

d. 30

14. In the face-centred cubic lattice structure of gold the closest distance between gold atoms is : ('a' being the edge length of the cubic unit cell)

a. $a\sqrt{2}$

b. $\frac{a}{\sqrt{2}}$

c. $\frac{a}{2\sqrt{2}}$

d. $2\sqrt{2}$

15. The equilibrium constant for the following reaction are given at 25°C

$2A \rightleftharpoons B + C, K_1 = 1.0$

$2B \rightleftharpoons C + D, K_2 = 16$

$2C + D \rightleftharpoons 2P, K_3 = 25$

The equilibrium constant for the reactions $P \rightleftharpoons A + \frac{1}{2}B$ at $25^\circ C$ is

a. $\frac{1}{20}$
b. 20
c. $\frac{1}{42}$
d. 21

16. Among the following, the ion which will be more effective for flocculation of $Fe(OH)_3$ sol is :

a. PO_4^{3-}
b. SO_4^{2-}
c. SO_3^{2-}
d. NO_3^{2-}

17. The mole fraction of ethanol in water is 0.08. Its molality is :

a. 6.32 $molkg^{-1}$
b. 4.83 $molkg^{-1}$
c. 3.82 $molkg^{-1}$
d. 2.84 $molkg^{-1}$

18. 5 ml of 0.1 M $Pb(NO_3)_2$ is mixed with 10 ml of 0.02 M K I. The amount of PbI_2 precipitated will be about

a. 10^{-2} mol
b. 10^{-4} mol
c. 2×10^{-4} mol
d. 10^{-3} mol

19. At 273 K temperature and 76 cm Hg pressure, the density of a gas is 1.964 gL^{-1} . The gas is

a. CH_4
b. CO
c. He
d. CO_2

20. Equal masses of ethane and hydrogen are mixed in an empty container at 298K. The fraction of total pressure exerted by hydrogen is

a. 15 : 16
b. 1 : 1
c. 1 : 4
d. 1 : 6

21. An ideal gas expands adiabatically against vacuum. Which of the following is correct for the given process?

a. $\Delta S = 0$
b. $\Delta T = -$ ve
c. $\Delta U = 0$
d. $\Delta P = 0$

22. K_f (water) = 1.86 K kg mol^{-1}. The temperature at which ice begins to separate from a mixture of 10 mass % ethylene glycol is

a. – 1.86 °C
b. – 3.72 °C
c. – 3.3 °C
d. – 3 °C

23. The radius of the first Bohr orbit of a hydrogen atom is 0.53×10^{-8} cm. The velocity of the electron in the first Bohr orbit is:
   a. 2.188×10^8 cms^{-1}
   b. 4.376×10^8 cms^{-1}
   c. 1.094×10^8 cms^{-1}
   d. 2.188×10^9 cms^{-1}

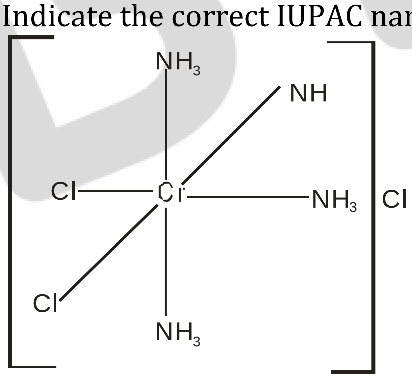
24. Which of the following statement is not true for the reaction :
   $2F_2 + 2H_2O \rightarrow 4HF + O_2$?
   a. F_2 is more strongly oxidising than O_2
   b. F – F bond is weaker than O = O bond
   c. H – F bond is stronger than H – O bond
   d. F is less electronegative than O

25. The number of unpaired electrons in the uranium ($_{92}U$) atom is :
   a. 4
   b. 6
   c. 3
   d. 1

26. The difference between orbital angular momentum of an electron in a 4f orbital and another electron in a 4s orbitals is :
   a. $2\sqrt{3}$
   b. $3\sqrt{2}$
   c. $\sqrt{3}$
   d. 2

27. Which of the following has the largest number of atoms?
   a. 1 g of Ag
   b. 1g of Fe
   c. 1g of Cl_2
   d. 1g of Mg

28. Indicate the correct IUPAC name of the co-ordination compound shown in the figure.



   a. Cis-dichlorotetraminochromium (III) chloride
   b. Trans-dichlorotetraminochromium (III) chloride
   c. Trans-tetraminedichlorochromium (III) chloride
   d. Cis-tetraamminedichlorochromium (III) chloride

WBJEE 2020 (Chemistry)

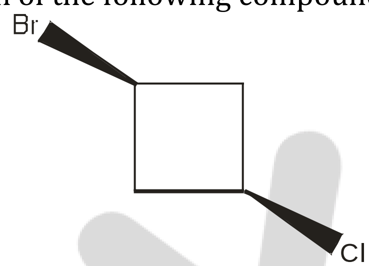
29. A homonuclear diatomic gas molecule shows 2 electrons magnetic moment. The-electron and two-electron reduced species obtained from above gas molecule can act as both oxidizing and reducing agent. When the gas molecule is one-electron oxidized the bond length decreases compared to the neutral molecule. The gas molecule is :

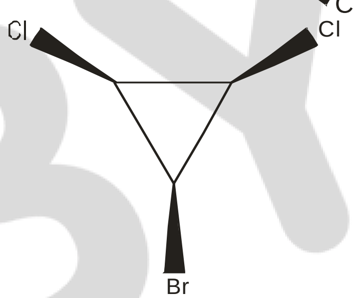
a. N_2
b. Cl_2
c. O_2
d. B_2

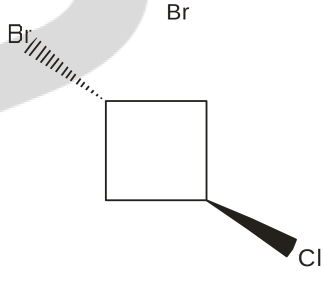
30. $CH_3 - O - CH_2 - Cl \xrightarrow[\Delta]{aq.^{-}OH} CH_3 - O - CH_2 - OH$ which information below regarding this reaction is applicable?

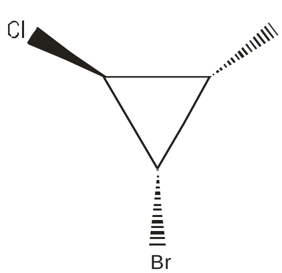
a. It follows S_N2 pathway, because it is a primary alkyl chloride
b. It follows S_N1 pathway, because the intermediate carbocation is resonance stabilized
c. $S_N1$pathway is not followed, because the intermediate carbocation is destabilised by –I effect of oxygen.
d. A mixed $S_N1$and $S_N2$pathway is followed

31. Which of the following compound is asymmetric?

a.


b.


c.


d.


WBJEE 2020 (Chemistry)

32. For a reaction 2A + B $\rightarrow$ P, when concentration of B alone is doubled, $t_{\frac{1}{2}}$ does not change and when concentrations of both A and B is doubled, rate increases by a factor of 4. The unit of rate constant is,

a. s^{-1}
b. $L\ mol^{-1}\ s^{-1}$
c. $Mol\ L^{-1}\ s^{-1}$
d. $L^2\ mol\ L^{-2}\ s^{-1}$

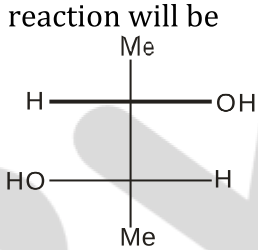
33. A solution is saturated with $SrCO_3$ and SrF_2. The $[CO_3^{2-}]$ is found to be 1.2×10^{-3} M. If the value of solubility product of $SrCO_3$ and SrF_2 are 7.0×10^{-10} and 7.9×10^{-9} respectively. The concentration of F^- in the solution would be

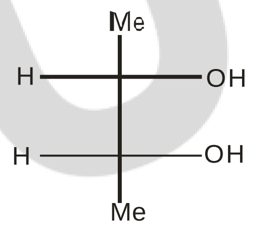
a. 3.7×10^{-6} M
b. 3.2×10^{-3} M
c. 5.1×10^{-7} M
d. 3.7×10^{-2} M

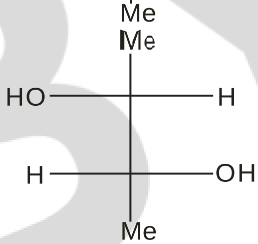
34. SiO_2 is attacked by which one/ones of the following?

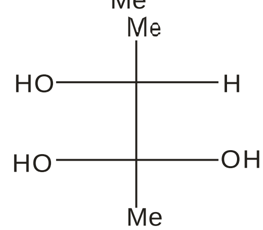
a. HF
b. Conc.HCl
c. hot NaOH
d. Fluorine

35. $Me - C \equiv C - Me \xrightarrow[\text{EtOH},-33°C]{Na/NH_3(liq.)} \underset{=}{X} \xrightarrow{\text{dil.alkaline}\,KMnO_4}$ product (s). The product (s) from the above reaction will be

a. 

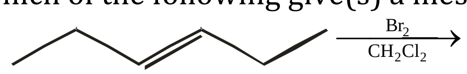
b. 

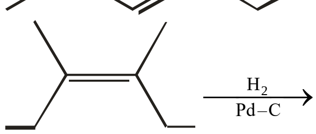
c. 

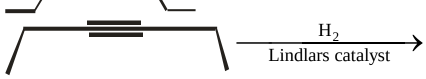
d. 

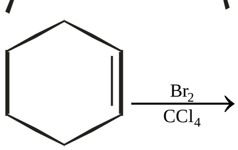
e.

36. Which of the following give(s) a meso-compound as the main product?

a. 

b. 

c. 

d. 

37. For spontaneous polymerization, which of the following is (are) correct?

    a. ΔG is negative
    b. ΔH is negative
    c. ΔS positive
    d. ΔS is negative

38. Which of the following statement(s) is/are incorrect:

    a. A sink of SO_2 pollutant is O_3 in the atmosphere.
    b. FGD is a process of removing NO_2 from atmosphere.
    c. NO_2 in fuel gases can be removed by alkaline scrubbing.
    d. The catalyst used to convert CCl_4 to CF_4 by HF is SbF_6.

39. The total number of alkyl bromides (including stereoisomers) formed in the reaction $Me_3 - C - CH = CH_2 + HBr \rightarrow$ will be

    a. 1
    b. 2
    c. 3
    d. No bromide forms

40. How and why does the density of liquid water change on prolonged electrolysis?

    a. Decreases, as the proportion of H_2O increases
    b. Remains unchanged
    c. Increases as the proportion of D_2O increases
    d. Increases, as the volume decreases

WBJEE 2020 (Chemistry)

ANSWER KEYS

1. (b)	2. (b)	3. (c)	4. (c)	5. (a)	6. (c)	7. (c)	8. (c)	9. (b)	10. (d)
11. (c)	12. (a)	13. (b)	14. (b)	15. (a)	16. (a)	17. (b)	18. (b)	19. (d)	20. (a)
21. (c)	22. (c)	23. (a)	24. (d)	25. (a)	26. (a)	27. (d)	28. (d)	29. (c)	30. (b)
31. (d)	32. (b)	33. (d)	34. (a,c)	35. (a,c)	36. (a,b)	37. (a,b,d)	38. (b,c)	39. (c)	40. (a)

WBJEE 2020 (Chemistry)

Solution

1. (b)
Mole concept
1 mole of element = atomic mass of atom
1 mole of element = 6.022×10^{23} atoms
So, mass of one mole ^{12}C = 12g
One mole of ^{12}C contains = 6.022×10^{23} atoms
Mass of 1 atom of ^{12}C = $\frac{12g \times 1\ ^{12}C\ \text{atom}}{6.022 \times 10^{23}\ ^{12}C\ \text{atom}}$
= 1.9923×10^{-23}g

2. (b)
Bond order = $\frac{N_{BMO} - N_{ABMO}}{2}$
Where N_{BMO} = number of Bonding molecular orbital's electrons
N_{ABMO} = number of antibonding molecular orbital's electrons.
Electronic configuration of He_2 = $(\sigma 1s)^2\ (\sigma^* 1s)^2$
From the above electronic configuration,
Bond order = $\frac{N_b - N_a}{2}$ = $\frac{2-2}{2} = \frac{0}{2} = 0$
Same as for other molecules
Electronic configuration of $He_2^+ : (\sigma 1s)^2 (\sigma^* 1s)^1$
Bond order = $\frac{2-1}{2} = \frac{1}{2}$
Electronic configuration of $He_2^{2+} : (\sigma 1s)^2 (\sigma^* 1s)^0$
Bond order = $\frac{2-0}{2} = 1$
So, Bond order of He_2, He_2^+, He_2^{2+} are respectively 0, $\frac{1}{2}$, 1

3. (c)
When the salt reacts dilute acid, sulphur dioxide gas is liberated.

$$Na_2S_2O_{3(aq)} + 2HCl_{(aq.)} \rightarrow \underset{\text{White ppt}}{S_{(s)}} \downarrow + SO_{2\,(g)} \uparrow + H_2O_{(\ell)} + 2NaCl_{(aq.)}$$

Na_2SO_3 and $Na_2S_4O_6$ both produce SO_2 on treatment with HCl but does not give white precipitate of sulphur.

Acidified dichromate solution turns green on passing SO_2 gas.

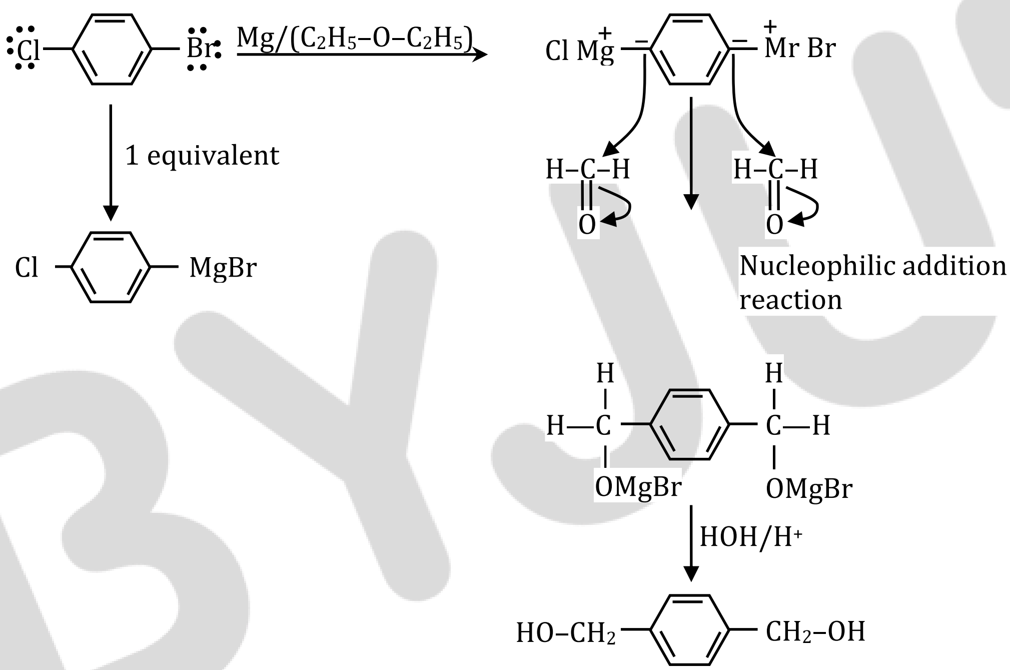
$$3SO_{2(g)} + Na_2Cr_2O_{7(g)} + H_2SO_{4(aq.)} \longrightarrow \underset{\text{(dark green)}}{Cr_2(SO_4)_{3(aq.)}} + K_2SO_4(aq.) + H_2O_{(\ell)}$$

4. (c)
Thermite reaction: A thermite reaction is demonstrated by igniting a mixture of aluminium and iron oxide generating molten iron and aluminium oxide.

This reaction is an oxidation-reduction reaction, a single replacement reaction.

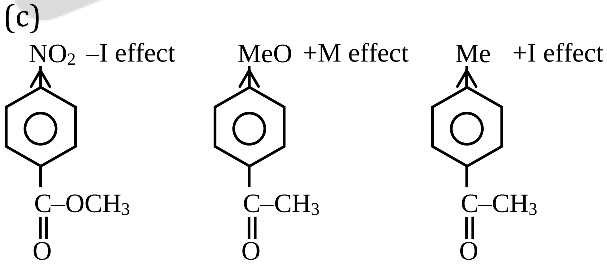
5. (a)
In the extraction of calcium by electro-reduction of molten Calcium chloride ($CaCl_2$), some amount of CaF_2 is added to keep the electrolyte in liquid state at temperature lower than the melting point of $CaCl_2$. As it is known, the electrolyte can be decomposed if the potential applied is high enough.

6. (c)



Final Product

7. (c)



(I) (II) (III)

Under alkaline condition, hydroxide attacks on carbonyl carbon of esters to furnish respective acids.
So, when the partial positive charge on carbonyl carbon increases, attack of ^{-}OH becomes more favorable and rate of hydrolysis increases.

$- \text{I effect} \propto \dfrac{1}{\text{positive charge}}$

$+\text{M effect} \propto \text{positive charge}$

$+\text{I effect} \propto \text{positive charge}$

+M effect is dominant than +I effect.

Hence, order of rate of alkaline hydrolysis I > III > II.

8. (c)

Cannizaro reaction is a redox reaction in which two molecules of aldehyde react to give oxidation product which is a salt of a carboxylic acid and the reduction product which is an alcohol, under basic condition.

$$2Ph-\overset{\overset{\large O}{\|}}{C}-D \xrightarrow[\text{warm}]{\text{50\% aq. NaOH}} Ph - CD_2OH + Ph - \overset{\overset{\large O}{\|}}{C} - ONa$$

9. (b)

Acidity $\propto$ resonance

Acidity $\propto$ –I effect

Anything which stabilizes the conjugate base will increase the acidity. Resonance increases the stability of conjugate base because the negative charge can be delocalized i.e. COO^- group.

Electron withdrawing substituent's ($-NO_2$ group) can increase acidity of a nearby atom.

$-NO_2$ groupis ortho/para directing group that shows maximum acidity at para position.

CH_3 group shows +I effect which destabilize conjugate base.

So, compound I is more acidic than compound II.

Hence, the acidity order is II < III < I < IV.

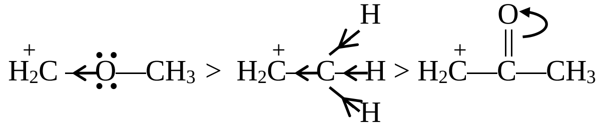
10. (d)

Stability of carbocations depends on following factors:

Stability of carbocation $\propto$ + M or +I effect

Stability of carbocation $\propto$ Hyper conjugation

Stability of carbocation $\propto \dfrac{1}{-\text{I effect}}$

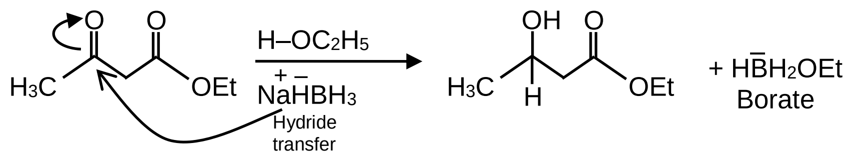


+M effect II   Hyperconjugation III   –I effect I

11. (c)

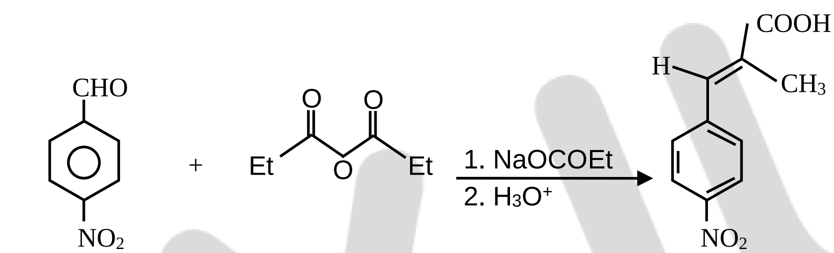
$NaBH_4$ is most selective reducing agent which only reacts with aldehydes or ketones due to its milder nature. So, keto group of Ethyl-3-oxobutanoate reduce only not ester group.

The reaction is as follows:



12. (a)

Perkin reaction is used to convert an aromatic aldehyde and an anhydride to an α, β-unsaturated carboxylic acid using sodium acetate, and an acid work up.



13. (b)

Principle Quantum number (n) = 3 → shell

Azimuthal Quantum number (ℓ) = 2

ℓ= 0, 1, 2

s, p, d

It means last electron enters in 3d sub shell

Value of magnetic Quantum number (m) = –1

We know, m can be +2, +1, 0, –1,–2.

While filling electron's in d-orbital, we can find out values of m for last electron.

For atomic number 27, values of m can be +1/–1.

For atomic number 28, value of m is 0.

For atomic number 30, values of m can be +2/–2.

So, by analyzing the data we can say that for atomic number 27 only m can be +1/–1.

So, electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 1 4s^2 3d^7$.

Maximum number of electrons in Co: 27

14. (b)

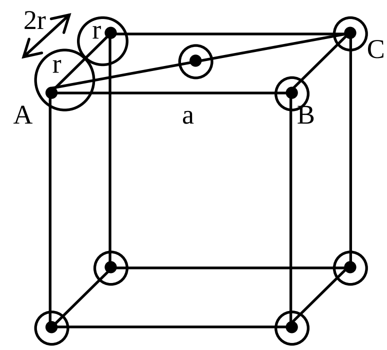
In FCC lattice, the closest distance is 2r.
Edge length of the cubic unit cell = a
In ΔABC
According to Pythagoras theorem:
$AC^2 = AB^2 + BC^2$
$= a^2 + a^2$
$AC^2 = 2a^2$
$AC = \sqrt{2}.a$

$$d = \frac{AC}{2} = \frac{\sqrt{2}.a}{2} \{d = \text{distance}$$

$$d = \frac{a}{\sqrt{2}}$$

$$r\text{ (radius)} = \frac{d}{2} = \frac{a}{2\sqrt{2}}$$

$$2r = \frac{a}{\sqrt{2}}$$



In FCC closest distance between gold atoms is $\frac{a}{\sqrt{2}}$.

15. (a)
According to the question:

$$2A \rightleftharpoons B + C;\ K_1 = 1.0$$

$$K_1 = \frac{[B][C]}{[A]^2}$$

$$2B \rightleftharpoons C + D;\ K_2 = 16$$

$$K_2 = \frac{[C][D]}{[B]}$$

$$2C + D \rightleftharpoons 2P;\ K_3 = 25$$

$$K_3 = \frac{[P]^2}{[C]^2[D]}$$

Final reaction: $P \rightleftharpoons A + \frac{1}{2}B$

$$K = \frac{[A][B]^{1/2}}{[P]}$$

To get final equilibrium constant (K)

$$K = \sqrt{\frac{1}{K_3} \times \frac{1}{K_2} \times \frac{1}{K_1}}$$

$$K = \sqrt{\frac{1}{25} \times \frac{1}{16} \times \frac{1}{1.0}}$$

$$K = \frac{1}{20}$$

16. (a)
"According to Hardy Schulze rule, greater the valency of the active ion or flocculating ion, greater will be its coagulating power." thus effective flocculation of $Fe(OH)_3$ sol is:
$PO_4^{3-} > SO_4^{2-} \approx SO_3^{2-} > NO_3^-$

17. (b)
As we know,
Mole fraction of ethanol + mole fraction of water = 1
$(x_{C_2H_5OH})$ (x_{H_2O})
$x_{C_2H_5OH} = 0.08$
$x_{H_2O} = 1 - x_{C_2H_5OH}$
$x_{H_2O} = 0.92$
Mass of water = moles × molar mass
= 0.92 × 18
= 16.56 g = 0.01656 kg
Molality = $\frac{\text{moles of solute}}{\text{mass of solvent}(kg)}$
$m = \frac{0.08}{0.01656}$
m = 4.83 mol/kg
molality of the solution is 4.83 mol/kg.

18. (b)
Given,
Volume of $Pb(NO_3)_2$ = 5 ml
Concentration $[Pb(NO_3)_2]$ = 0.1 M
Moles of Pb $(NO_3)_2$ = C(Concentration) × V(Volume) = $\frac{5\times0.1}{1000} = 5\times10^{-4}$ mol
Volume of KI = 10 ml
Concentration of KI = 0.02 M
Moles of KI = Concentration × volume
$= \frac{0.02\times10}{1000}$
= 2×10^{-4}mol

	$Pb(NO_3)_2$ +	2KI	$\longrightarrow$ $PbI_2 + 2KNO_3$
Initial moles	5×10^{-4} mol	2×10^{-4} mol	
	$\frac{5\times10^{-4}}{1}$	$\frac{2\times10^{-4}}{2}$	
		10^{-4} Limiting reagent	

L.R. ⇒ Moles of reactant is divided by its stoichiometry coefficient. Lesser moles of reactant is limiting reagent.
Because of KI bing a limiting reagent, the amount of PbI_2 precipitated will be 10^{-4} mol.

19. (d)

Given,
Density (d) = 1.964 g/L
Pressure (P) = 76 cm Hg= 760 mm Hg or 1 atm
Temperature (T) = 273 K
R = 0.0821 atm/k mol
According to ideal gas equation:
PV = nRT

$PV = \frac{m \times R}{Mwt \times T}$ $\quad\quad \therefore\ n = \frac{mass(m)}{Moleculer\ weight\,(Mwt)}$

Molecular weight = $\frac{m \times R}{P \times V \times T}$ $\quad\quad \therefore density = \frac{mass(m)}{Volume(v)}$

$Mwt = \frac{d \times R}{P \times T}$

$= \frac{1.964 \times 0.0821}{1 \times 273}$

= 44g
44g is the molecular weight of CO_2 gas.

20. (a)

According to Raoult's law partial vapor pressure of each volatile component in the solution is directly proportional to its mole fraction.

$P_{H_2} \propto x_{H_2}$

Suppose x_g of both are mixed.

Moles of H_2 = $\frac{weight}{molecular\ weight} = \frac{x}{2}$

Moles of C_2H_5 = $\frac{x}{30}$

Mole fraction of H_2 = $\frac{\frac{x}{2}}{\frac{x}{2}+\frac{x}{30}} = \frac{\frac{x}{2}}{\frac{16x}{30}} = \frac{15}{16}$

So, fraction of total pressure exerted by hydrogen is $\frac{15}{16}$.

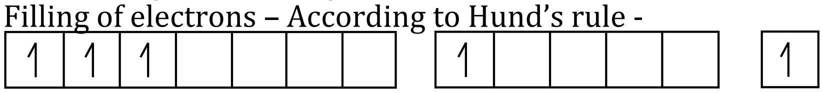
21. (c)

For an ideal gas expanding adiabatically,
$\Delta U = w$
$\therefore q = 0$
So, $W = -P_{ext}\Delta V = 0$
Expansion against $P_{ext} = 0$
For Adiabatic Process:
$\Delta U = q + W$
$\Delta U = 0$

22. (c)
As we know
$$\Delta T_f = \frac{K_f \times W_B \times 1000}{M_B \times W_A}$$
Given that
$K_f = 1.86k\ kg / mol$
10 mass% $\Rightarrow$ 10g ethylene glycol mixed with 90g ice $\Rightarrow$ 100 g solution
w_A = ice_{CH_2O} = 90g
w_B = ethylene glycol = 10g
Molecular weight of ethylene glycol (m_B) = 62
So, $\Delta T_f = \frac{1.86 \times 10 \times 1000}{62 \times 90}$
$\Delta T_f = 3.33°C$
$\Delta T_f = T_f° - T_f$
$T_f = \Delta T_f - T_f^{\circ}$
$T_f = -3.3°C$

23. (a)
Velocity of electron in first Bohr orbit of hydrogen atom
$mvr = \frac{nh}{2\pi}$
r = 0.53 × 10^{-8} m
n = 1
h = 6.626 × 10^{-26} m
m = 9.1 × 10^{-31} kg
$v = \frac{nh}{2\pi mr}$
On substituting the values velocity (v) = $\frac{1 \times 6.626 \times 10^{-26}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.53 \times 10^{-8}}$
= 2.18 × 10^6 m/s
= 2.18 × 10^8 cm/s

24. (d)
Oxygen is less electronegative element than the fluorine having electro negativity values 3.44 and 3.98 respectively.
Hence, the option D is incorrect.

25. (a)
General electronic configuration of Uranium ($_{92}U$) = [Rn]$5f^3\ 6d^1\ 7s^2$
Orbital diagram of filling electrons:
Filling of electrons – According to Hund's rule -

5f 6d 7s
So, the number of total unpaired electrons in uranium atom = 3 + 1 = 4

26. (a)

Expression for the orbital angular momentum

$$(\mu_\ell) = \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$$

For 4f electron, ℓ = 3 and

For 4s electron, ℓ = 0

So, $\mu_{\ell_{(4f)}} - \mu_{\ell_{(4s)}} = \sqrt{3+(3+1)}\frac{h}{2\pi} - \sqrt{0(0+1)}\frac{h}{2\pi}$

$= \sqrt{12}$

$= 2\sqrt{3}$

27. (d)

(A) 1g of Ag = $\frac{1}{\text{atomic weight}}$ mole atom of Au

$= \frac{1}{107} \times N_A$ atoms of Au

$= \frac{1}{107} \times 6.022 \times 10^{23}$ atoms of Au

Similarly,

(B) 1g of Fe = $\frac{1}{56}$ mole atom of Fe

1g of Fe = $\frac{1}{56}$ × 6.022 × 10^{23} atoms of Fe

(C) 1g of Cl_2 = $\frac{1}{71}$ mole molecules of Cl_2

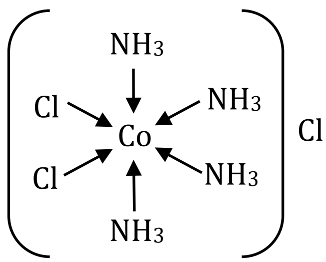
= $\frac{1}{71}$ × 6.022 × 10^{23} atoms of Cl_2

(D) 1g of Mg = $\frac{1}{24}$ mole atom of Mg

= $\frac{1}{24}$ × 6.022 × 10^{23} atoms of Mg

So, 1g magnesium has the largest number of atoms.

28. (d)



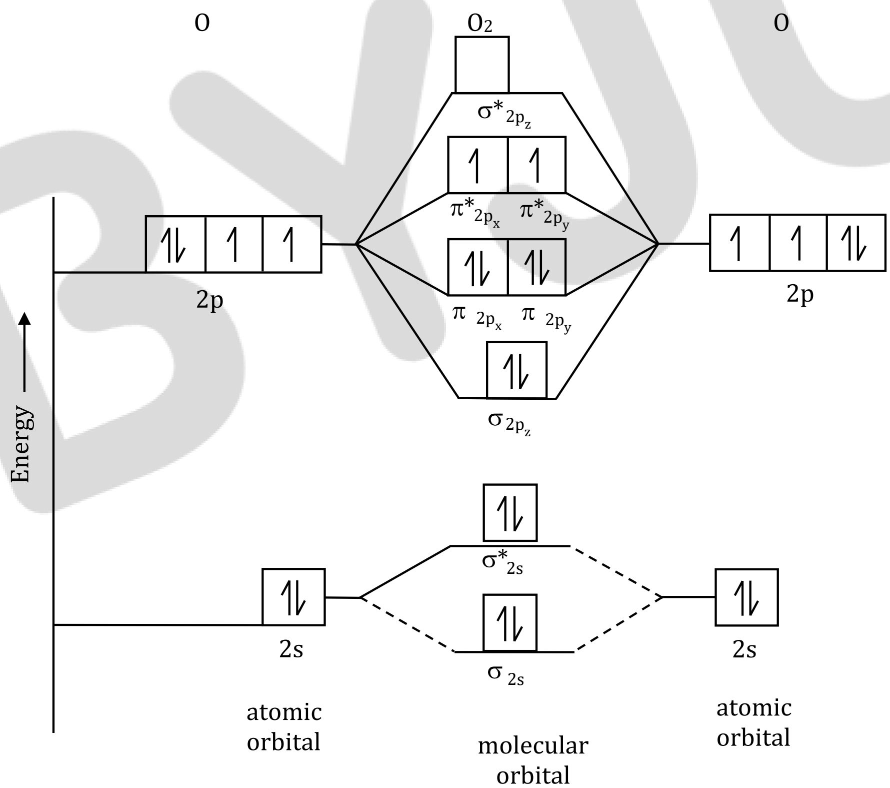
IUPAC name of Co-ordination compound is:Cis-tetraminedichlorochromium (III) chloride.

To write the correct IUPAC name of the compound first we write cationic part and then anionic part. The prefix 'tetra' indicates presence of four NH_3 ligands prefix 'di' indicates two chlorine ligand.

The name of ligands is written in alphabetical order because both the ammonia and the chlorine atoms are on same sides hence we use cis before the name of the compound.

29. (c)
The gas molecule must be O_2 because having two unpaired electrons in its outer most electronic configuration i.e.

$(\sigma_{1s})^2 (\sigma*_{1s})^2 (\sigma_{2s})^2 (\sigma_{2s}*)^2 \left(\pi_{2p_x}\right)^2 \left(\pi_{2p_y}\right)^2 \left(\pi*_{2p_x}\right)^1 \left(\pi*_{2p_y}\right)^1$



After removal of one electron (oxidized), bond length decreases as compared to neutral molecule.
O_2 behaves as oxidizing as well as reducing agent by either losingor gaining of an electron.

30. (b)

$$CH_3\text{–}O\text{–}CH_2\text{–}Cl \xrightarrow[\Delta]{aq.\,\bar{O}H} CH_2\text{–}O\text{–}CH_2\text{–}OH$$

Mechanism:

$$CH_3\text{–}O\text{–}\overset{+\delta}{C}H_2 \text{—} Cl + {}^{-}OH \longrightarrow CH_3\text{–}O\text{–}\overset{\oplus}{C}H_2 \longleftrightarrow CH_3\text{–}\overset{\oplus}{O}\text{=}CH_2$$

1° carbocation

stabilized by +M effect of OCH_3 group

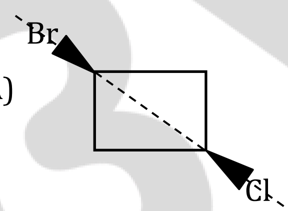
$$\downarrow \bar{O}H$$

$$CH_3\text{–}O\text{–}CH_2\text{–}OH$$

It is the Nucleophilic substitution reaction and follows S_N1 mechanism due to 1° stable carbocation.

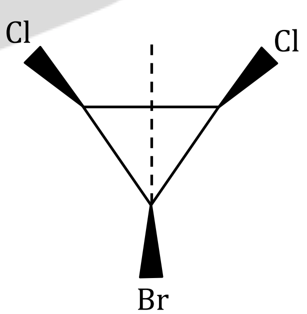
31. (d)

(A)



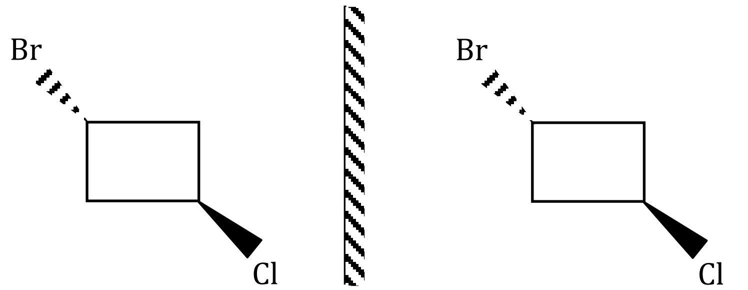
Plane of symmetry is present; hence it is a symmetric molecule.

(B)



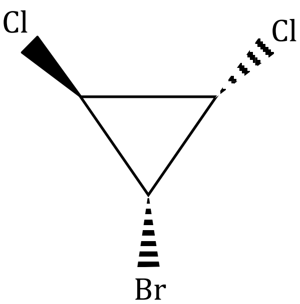
Plane of symmetry is present; hence it is a symmetric molecule.

(C)



superimposable mirror images

(D)



Plane of symmetry absent, has non superimposable mirror image (Diastereomer). Hence it is an Asymmetric compound.

32. (b)
For the reaction:

$$2A + B \longrightarrow P$$

Rate of reaction (r) = $k[A]^2[B]$

According to first order reaction, half-life $t_{1/2} = \frac{0.693}{K}$ is not affected by change in concentration.

So, order of the reaction with respect to b is one, similarly for A is also one.
Overall order of the reaction = 1 + 1 = 2
Hence, unit of rate constant is L/mol.

33. (d)
Given that solubility product of $SrCO_3 = 7.0 \times 10^{-10}$
Solubility product of $SrF_2 = 7.9 \times 10^{-9}$

Concentration of $\left[CO_3^{2-}\right] = 1.2 \times 10^{-3}\,M$

Solution is saturated with $SrCO_3$ and SrF_2.

$SrF^2 \rightarrow Sr^{2+} + 2F^-$

$K_{sp}SrF_2 = [Sr^{2+}][F^-]^2$

$SrCO_3 \rightarrow Sr^{+2} + Co_3^{2-}$

$K_{sp}(SrCO_3) \rightarrow (Sr^{+2})[CO_3]^{2-}$

so,

$$\frac{K_{sp}(SrF_2)}{K_{sp}(SrCO_3)} = \frac{[Sr^{+2}][F^-]^2}{[Sr^{+2}][CO_3^{2-}]} = \frac{[F^-]^2}{[CO_3^{2-}]}$$

$$[F^-]^2 = \frac{[CO_3^{2-}] \times 7.9 \times 10^{-10}}{7 \times 10^{-10}}$$

$$[F^-]^2 = \frac{1.2 \times 10^{-3} \times 7.9}{7}$$

$[F^-]^2 = 13.5 \times 10^{-4}$

$[F^-] = \sqrt{13.5 \times 10^{-4}}$

$[F^-] = 3.7 \times 10^{-2}$ M

So, concentration of F^- in the solution would be 3.7×10^{-2} M

34. (a,c)

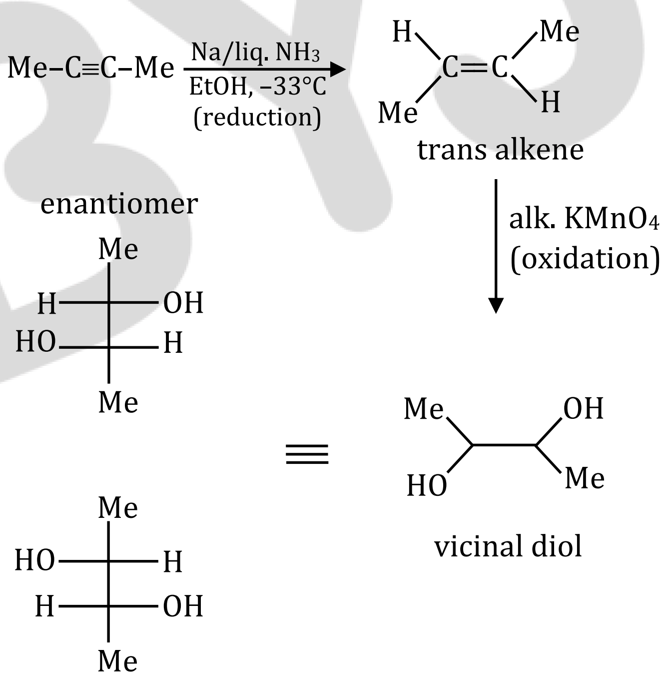
Reactions of SiO_2:

$SiO_2 + HF \longrightarrow H_2[SiF_6] + H_2O$

$SiO_2 + NaOH \longrightarrow Na_2SiO_3 + H_2O$

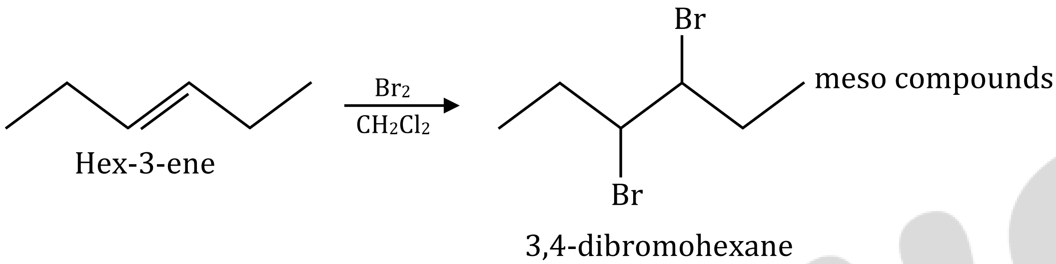
SiO_2 is an acid oxide. Therefore, it dissolves in alkaline solutions.

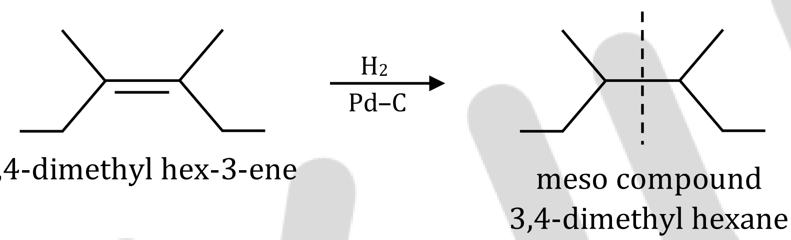
35. (a,c)

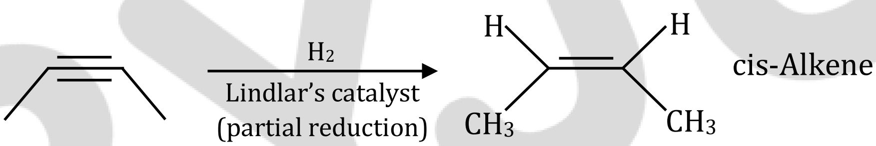


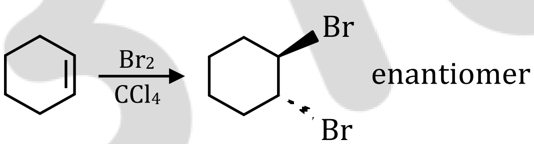
In the above reaction, presences of Na in liquid NH3 alkynes reduce to give trans alkene. Alkaline $KMnO_4$ gives vicinal diol as oxidation product. These diols are enantiomers of each other.

36. (a,b)

(A)


(B)


(C)


(D)


37. (a,b,d)

The free energy change for any polymerization will be,

$$\Delta G = G_{polymer} - G_{monomer}$$

So, $\Delta G = \Delta H - T\Delta S$

$$\Delta T = (H_{polymer} - H_{monomer}) - T(S_{polymer} - S_{monomer})$$

When the polymer has a lower free energy than the initial monomer, a polymerisation can occur spontaneously and the sign of ΔG is negative. ΔH is negative due to it's an exothermic process. Entropy change for polymerisation is also negative because, polymers are orderly arranged.

38. (b,c)

Fuel gas desulphurisation (FGD) process is used to remove sulphur from exhaust flue gases. In this system, the sulphur removed from the flue gas is converted into its elemental form or into sulphuric acid.

Alkaline scrubbing is used to remove SO_2 from combustion exhaust gas.

39. (c)

For chemical reaction:

$$Me_3{-}C{-}CH{=}CH_2 + H^+ Br^- \longrightarrow Me_3C{-}CH_2{-}\overset{\oplus}{C}H_2 \longrightarrow Me_3C{-}CH_2{-}CH_2{-}Br$$

1° less stable carbocation

$$\updownarrow$$

$$Me_3C{-}\overset{\oplus}{C}H{-}CH_3$$

2° more stable carbocation

$$\Big\downarrow Br^-$$

$$Me_3C{-}\overset{*}{C}H(Br){-}CH_3$$

(R/S)

Total number of stereo isomers possible for 4,4-dimethylpentene are three. As the molecule contain one chiral carbon atom.

Number of R and S isomers = $2^{n-1} = 2^{1-1} = 2$

So, total number of stereo isomers possible are = 2 + 1 = 3.

40. (a)

In electrolysis, water is decomposed in the presence of electricity, to produce hydrogen gas and oxygen gas.

The half reaction,

Reduction at cathode: $2H^+(aq.) + 2e^- \longrightarrow H_2(g)$

Oxidation at anode: $2H_2O(\ell) \longrightarrow O_2(g) + 4H^+(aq.) + 4e^-$

Over all reaction: $2H_2O(\ell) \longrightarrow 2H_2(g) + O_2(g)$

On electrolysis volume of gases produced increases.

Relation between density and volume

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

As the volume increases density decreases, so as the proportion of water molecules increases density decreases.

WB-JEE-2020 (Physics)

1. The bob of a swinging second's pendulum (one whose time period is 2s) has a small speed v_0 at its lowest point. Its height from this lowest point, 2.25s after passing through it is

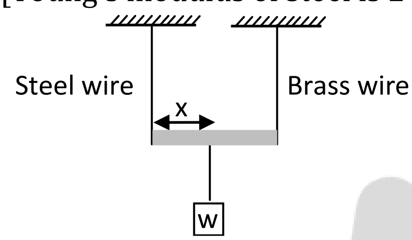
a. $\frac{v_0^2}{2g}$

b. $\frac{v_0^2}{g}$

c. $\frac{v_0^2}{4g}$

d. $\frac{gv_0^2}{4g}$

2. A steel and brass wire, each of length 50 cm and cross-sectional area 0.005 cm^2 hang from ceiling and are 15 cm apart. Lower ends of the wires are attached to a light horizontal bar. A suitable downward load is applied to the bar so that each of the wire extends in length by 0.1 cm. At what distance from the steel wire the load must be applied?
[Young's modulus of steel is 2×10^{12} dynes/cm^2 and that of brass is 1×10^{12}dynes/cm^2]



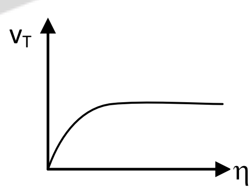
a. 75 cm

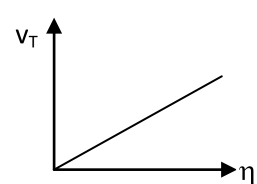
b. 5 cm

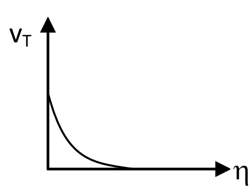
c. 10 cm

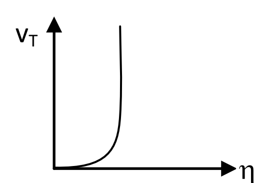
d. 3 cm

3. Which of the following diagrams correctly shows the relation between the terminal velocity V_T of a spherical body falling in a liquid and viscosity v of the liquid?

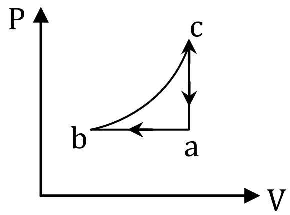
a. 

b. 

c. 

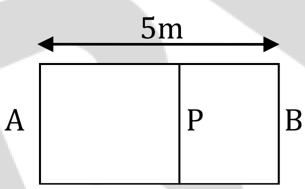
d. 

4. An ideal gas undergoes the cyclic process abca as shown in the given P.V diagram. It rejects 50 J of the heat during ab and absorbs 80J of heat during ca. During bc, there is no transfer of heat and 40J of work is done by the gas. What should be the area of the closed curve abca?



a. 30 J
b. 40 J
c. 10 J
d. 90 J

5. A container AB in the shape of a rectangular parallelepiped of length 5m is divided internally by a movable partition P as shown in the figure. The left compartment is filled with a given mass of an ideal gas of molar mass 32 while the right compartment is filled with an equal mass of another ideal gas of molar mass 18 at same temperature. What will be the distance of P from the left wall A when equilibrium is established?

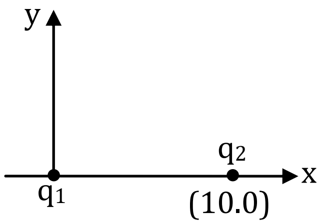


a. 2.5 m
b. 1.8 m
c. 3.2 m
d. 2.1 m

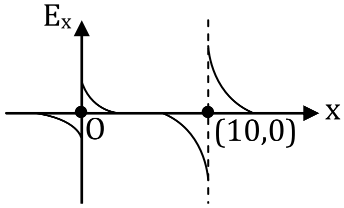
6. When 100 g of boiling water at 100°C is added into a calorimeter containing 300 g of cold water at 10°C, temperature of the mixture becomes 20°C. Then a metallic block of mass 1 kg at 10°C is dipped into the mixture in the calorimeter. After reaching thermal equilibrium, the final temperature becomes 19°C. What is the specific heat of the metal in C.G.S. units?

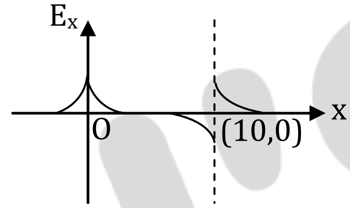
a. 0.01
b. 0.3
c. 0.09
d. 0.1

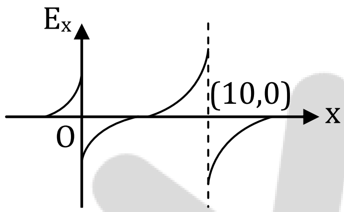
7.

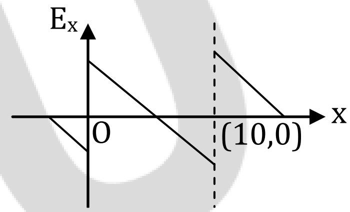


As shown in the figure, a point charge $q_1 = +1\times10^{-6}$ C is placed at the origin in x-y plane and another point charge $q_2 = +3\times10^{-6}$ is placed at the co-ordinate (10,0). In that case, which of the following graph(s) shown most correctly the electric field vector in E_x in x-direction?

a. 

b. 

c. 

d. 

8. Four identical point masses, each of mass m and carrying charge +q are placed at the corners of a square of sides 'a' on a frictionless plain surface. If the particles are released simultaneously, the kinetic energy of the system when they are infinitely far apart is

a. $\frac{q^2}{4\pi\in_0 a}(2\sqrt{2}+1)$

b. $\frac{q^2}{4\pi\in_0 a}(\sqrt{2}+2)$

c. $\frac{q^2}{4\pi\in_0 a}(\sqrt{2}+4)$

d. $\frac{q^2}{4\pi\in_0 a}(\sqrt{2}+1)$

9. A very long charged solid cylinder of radius 'a' contains a uniform charge density ρ. Dielectric constant of the material of the cylinder is k. What will be the magnitude of electric field at a radial distance 'x' (x < a) from the axis of the cylinder?

a. $\rho\frac{x}{\in_0}$

b. $\rho\frac{x}{2k\in_0}$

c. $\rho\frac{x^2}{2a\in_0}$

d. $\rho\frac{x}{2k}$

WB-JEE-2020 (Physics)

10. A galvanometer can be converted to a voltmeter of full-scale deflection V_0 by connecting a series resistance R_1 and can be converted to an ammeter of full – scale deflection I_0 by connecting a shunt resistance R_2. What is the current flowing through the galvanometer at its full scale deflection?

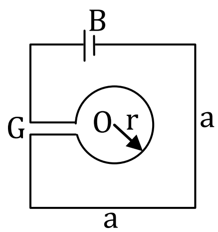
a. $\frac{V_0 - I_0R_2}{R_1 - R_2}$

b. $\frac{V_0 + I_0R_2}{R_1 + R_2}$

c. $\frac{V_0 - I_0R_1}{R_1 - R_2}$

d. $\frac{V_0 + I_0R_1}{R_1 + R_2}$

11.



As shown in the figure a single conducting wire is bent to form a loop in the form of a circle of radius 'r' concentrically inside a square of side 'a', where a:r = 8 : π. A battery B drives a current through the wire. If the battery B and the gap G are of negligible sizes, determine the strength of magnetic field at the common centre O.

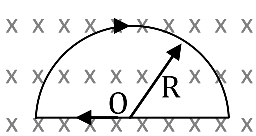
a. $\frac{\mu_0 I}{2\pi a}\sqrt{2}(\sqrt{2}-1)$

b. $\frac{\mu_0 I}{2\pi a}(\sqrt{2}+1)$

c. $\frac{\mu_0 I}{\pi a}2\sqrt{2}(\sqrt{2}+1)$

d. $\frac{\mu_0 I}{\pi a}2\sqrt{2}(\sqrt{2}-1)$

12. As shown in figure, a wire is bent to a D-shaped closed loop, carrying current I, where the curved part is a semi-circle of radius R. The loop is placed in a uniform magnetic field B, which is directed into the plane of the paper. The magnetic force felt by the closed loop is



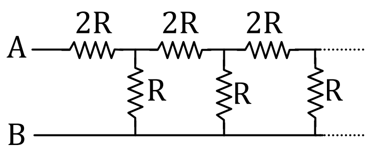
a. 0

b. IRB

c. 2IRB

d. $\frac{1}{2}$IRB

13. What will be the equivalent resistance between the terminals A and B of the infinite resistive network shown in figure?



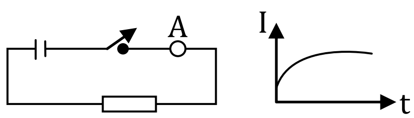
a. $\frac{(\sqrt{3}+1)}{2}$

b. $\frac{(\sqrt{3}-R)}{2}$

c. $3\frac{R}{2}$

d. $(\sqrt{3}+1)R$

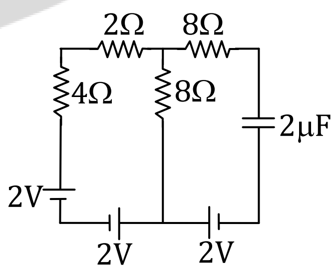
14.



When a DC voltage is applied at the two ends of a circuit kept in a closed box, it is observed that the current gradually increases from zero to a certain value and then remains constant. What do you think that the circuit contains?

a. A resistor alone
b. A capacitor alone
c. A resistor and a inductor in series
d. A resistor and a capacitor in series

15. Consider the circuit shown. If all the cells have negligible internal resistance, what will be the current through the 2Ω resistor when steady state is reached?



a. 0.66 A

b. 0.29 A

c. 0 A

d. 0.14 A

16. Consider a conducting wire of length L bent in the form of a circle of radius R and another conductor of length 'a' (a<<R) bent in the form of a square. The two loops are then placed in the same plane such that the square loop is exactly at the centre of the circular loop. What will be the mutual inductance between the two loops?

a. $\mu_0 \frac{\pi a^2}{L}$

b. $\mu_0 \frac{\pi a^2}{16L}$

c. $\mu_0 \frac{\pi a^2}{4L}$

d. $\mu_0 \frac{a^2}{4\pi L}$

17. An object is placed 60 cm in front of a convex mirror of focal length 30 cm. A plane mirror is now placed facing the object in between the object and the convex mirror such that it covers lower half of the convex mirror. What should be the distance of the plane mirror from the object so that there will be no parallax between the images formed by the two mirrors?

a. 40 cm

b. 30 cm.

c. 20 cm.

d. 15 cm.

18. A thin convex lens is placed just above an empty vessel of depth 80 cm. The image of a coin kept at the bottom of the vessel is thus formed 20 cm above the lens. If now, water is poured in the vessel up to a height of 64 cm, what will be the approximate new position of the image? Assume that refractive index of water is 4/3.

a. 21.33 cm above the lens

b. 6.67 cm below the lens

c. 33.67 cm above the lens

d. 24 cm above the lens

19. The intensity of light emerging from one of the slits in a Young's double slit experiment is found to be 1.5 times the intensity of light emerging from the other slit. What will be the approximate ratio of intensity of an interference maximum to that of an interference minimum?

a. 2.25

b. 98

c. 5

d. 9.9

20. In a Fraundhofer diffraction experiment, a single slit of width 0.5 mm is illuminated by a monochromatic light of wavelength 600 nm. The diffraction pattern is observed on a screen at a distance of 50 cm from the slit. What will be the linear separation of the first order minima?

a. 1.0 mm

b. 1.1 mm

c. 0.6 mm

d. 1.2 mm

WB-JEE-2020 (Physics)

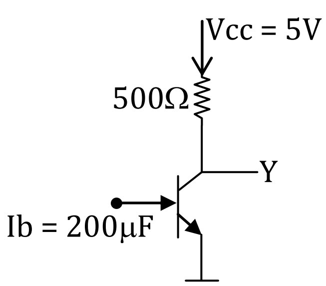
21. If R is the Rydberg constant in cm^{-1}, then hydrogen atom does not emit any radiation wavelength in the range of
   a. $\frac{1}{R}$ to $\frac{4}{3R}$ cm
   b. $\frac{7}{5R}$ to $\frac{19}{5R}$ cm
   c. $\frac{4}{R}$ to $\frac{36}{5R}$ cm
   d. $\frac{9}{R}$ to $\frac{144}{7R}$ cm

22. A nucleus X emits a beta particle to produce a nucleus Y. If their atomic masses are M_x and M_y respectively. The maximum energy of the beta particle emitted is (where m_e is the mass of an electron and c is the velocity of light)
   a. $(M_x - M_y - m_e)c^2$
   b. $(M_x - M_y + m_e)c^2$
   c. $(M_x - M_y)c^2$
   d. $(M_x - M_y + 2m_e)c^2$

23. For nuclei with mass number close to 119 and 238, the binding energies per nucleon are approximately 7.6 MeV and 8.6 respectively. If a nucleus of mass number 238 breaks into two nuclei of nearly equal masses, what will be the approximate amount of energy released in the process of fission?
   a. 214 MeV
   b. 119 MeV
   c. 2047 MeV
   d. 1142 MeV

24. A common emitter transistor amplifier is connected with a load resistance of $6k\Omega$. When a small a.c. signal of 15 mV is added to the base emitter voltage, the alternating base current is $20\mu A$ and the alternating collector current is 1.8 mA. What is the voltage gain of the amplifier?
   a. 90
   b. 640
   c. 900
   d. 720

25.



In the circuit shown, the value of β of the transistor is 48. If the base current supplied $200\mu A$, what is the voltage at the terminal Y?
   a. 0.2 V
   b. 0.5 V
   c. 4 V
   d. 4.8 V

WB-JEE-2020 (Physics)

26. The frequency v of the radiation emitted by an atom when an electron jumps from one orbit to another is given by v = kδE, where k is a constant and δE is the change in energy level due to the transition. Then dimension of k is
    a. ML^2T^{-2}
    b. The same dimension of angular momentum.
    c. ML^2T^{-1}
    d. $M^{-1}L^{-2}T$

27. Consider the vectors $\vec{A} = \hat{i} + \hat{j} - \hat{k}, \vec{B} = 2\hat{i} - \hat{j} + \hat{k}, \vec{C} = \frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j} + 2\hat{k})$. What is the value of $\vec{C}.(\vec{A} \times \vec{B})$?
    a. 1
    b. 0
    c. $3\sqrt{2}$
    d. $18\sqrt{5}$

28. A fighter plane, flying horizontally with a speed of 360 km/h at an altitude of 500 m drops a bomb for a target straight ahead of it on the ground. At what approximate distance, the bomb should be dropped ahead of the target, so that it hits the target? Assume that acceleration due to gravity g is 10 ms^{-2}. Also neglect air drag.
    a. 1000 m
    b. $50\sqrt{5}$ m
    c. $500\sqrt{5}$ m
    d. 866 m

29. A block of mass m rests on a horizontal table with coefficient of static friction μ. What minimum force must be applied on the block to drag it on the table?
    a. $\frac{\mu}{\sqrt{1+\mu^2}}mg$
    b. $\frac{\mu-1}{\mu+1}mg$
    c. $\frac{\mu}{\sqrt{1-\mu^2}}mg$
    d. μmg

30. A tennis ball hits the floor with a speed v at an angle θ with the normal to the floor. If the collision is inelastic and the coefficient of restitution is ε, what will be the angle of reflection?
    a. $\tan^{-1}\left[\frac{\tan\theta}{\varepsilon}\right]$
    b. $\sin^{-1}\left[\frac{\sin\theta}{\varepsilon}\right]$
    c. $\theta\varepsilon$
    d. $\theta\frac{2\varepsilon}{\varepsilon+1}$

31. A metallic block of mass 20 kg is dragged with a uniform velocity of 0.5 ms^{-1} on a horizontal table for 2.1 s. The coefficient of static friction between the block and the table is 0.10. When will be the maximum possible rise in temperature of the metal block, if the specific heat of the block is 0.1 C.G.S. unit? Assume g = 10 $ms^{-1}$and uniform rise in temperature throughout the whole block.[Ignore absorption of heat by the table]
   a. 0.0025^oC b. 0.0035^oC
   c. 0.001^oC d. 0.05^oC

32. Consider an engine that absorbs 130 cal of heat from a hot reservoir and delivers 30 cal heat to a cold reservoir in each cycle. The engine also consumes 2J energy in each cycle to overcome friction. If the engine works at 90 cycles per minute, what will be the maximum power delivered to the load?
   a. 816 W b. 819 W
   c. 627 W d. 630 W

33. Two pith balls, each carrying charge +q are hung from a hook by two strings. It is found that when each charge is tripled, angle between the strings double. What was the initial angle between the strings?
   a. 30^o b. 60^o
   c. 45^o d. 90^o

34. A conducting circular loop of resistance 20Ω and cross sectional area 20×10^{-2} m^2 is placed perpendicular to a spatially uniform magnetic field B, which varies with time t as B = 2 sin(50πt) T.
   Find the net charge flowing through the loop in 20 ms starting from t = 0.
   a. 0.5 C
   b. 0.2 C
   c. 0 C
   d. 0.14 C

35. A pair of parallel metal plates is kept with a separation'd'. One plate is at a potential +V and the other is at ground potential. A narrow beam of electrons enters the space between the plates with a velocity v_0 and in a direction parallel to the plates. What will be the angle of the beam with the plates after it travels an axial distance L?
   a. $\tan^{-1}\left[\frac{eVL}{mdv_0}\right]$ b. $\tan^{-1}\left[\frac{eVL}{mdv_0^2}\right]$
   c. $\sin^{-1}\left[\frac{eVL}{mdv_0}\right]$ d. $\sin^{-1}\left[\frac{eVL}{mdv_0^2}\right]$

36. A simple pendulum of length l is displaced so that its taut string is horizontal and then released. A uniform bar pivoted at one end is simultaneously released from its horizontal position. If their motions are synchronous, what is the length of the bar?

a. $\frac{3l}{2}$
b. l
c. $2l$
d. $\frac{2l}{3}$

37. A charged particle moves with constant velocity in a region where no effect of gravity is felt but an electrostatic field $\vec{E}$ together with a magnetic field $\vec{B}$ may be present. Then which of the following cases are possible?

a. $\vec{E} \neq 0, \vec{B} \neq 0$
b. $\vec{E} \neq 0, \vec{B} = 0$
c. $\vec{E} = 0, \vec{B} = 0$
d. $\vec{E} = 0, \vec{B} \neq 0$

38. A 400Ω resistor, a 250mH inductor and a 2.5μF capacitor are connected in series with AC source of peak voltage 5V and angular frequency 2kHz. What is the peak value of the electrostatic energy of the capacitor?

a. 2μJ
b. 2.5μJ
c. 3.3μJ
d. 5μJ

39. A point source of light is used in an experiment of photoelectric effects. If the distance between the source and the photo-electric surface is doubled, which of the following may result?

a. Stopping potential will be halved
b. Photoelectric current will decrease
c. Maximum kinetic energy of photo-electron will decrease
d. Stopping potential will increase slightly

40. Two metallic spheres of equal outer radii are found to have same moment of inertia about their respective diameters. Then which of the following statement(s) is/are true?

a. Two spheres have equal mass
b. The ratio of masses is nearly 1.67 : 1
c. The spheres are made of different materials
d. Their rotational kinetic energies will be equal when rotated with equal uniform angular speed about their respective diameters

WB-JEE-2020 (Physics)

ANSWER KEYS

1. (c)	2. (b)	3. (c)	4. (a)	5. (b)	6. (d)	7. (a)	8. (c)	9. (b)	10. (a)
11. (d)	12. (a)	13. (d)	14. (c)	15. (c)	16. (b)	17. (a)	18. (a)	19. (b)	20. (d)
21. (b)	22. (c)	23. (a)	24. (d)	25. (a)	26. (d)	27. (b)	28. (a)	29. (a)	30. (a)
31. (a)	32. (c)	33. (a)	34. (c)	35. (b)	36. (a)	37. (a,c,d)	38. (d)	39. (b)	40. (d)

WB-JEE-2020 (Physics)

Solution

1. (c)

Given:- Time period of swinging seconds pendulum is 2s.
It means it completes its one revolution in 2s.
So the bob will reach its lowest point after 2s.
As it is travelling further for ¼ sec. i.e. t = 2.5s

Hence, $v = A\omega\cos\omega t = v_0\cos\frac{\pi}{4} = \frac{v_0}{\sqrt{2}} \{\because A\omega = v_0 = v_{max}\}$

$$\left\{\begin{array}{l}\because \text{displacement } x = A\sin\omega t \\ \text{velocity } v = \frac{dx}{dt} = A\omega\cos\omega t\end{array}\right\}$$

From law of conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\frac{v_0^2}{2} + mgh$$

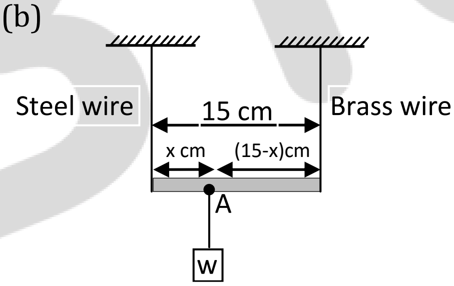
$$\frac{1}{2}v_0^2 - \frac{1}{2}\frac{v_0^2}{2} = gh$$

$$\frac{1}{2}v_0^2\left(1 - \frac{1}{2}\right) = gh$$

$$\frac{v_0^2}{4} = gh$$

$$\Rightarrow h = \frac{v_0^2}{4}$$

2. (b)



Given:-
Length of steel and Brass wire each =50 cm.
Cross sectional area = 0.005 cm^2
Distance between the wires = 15 cm.
Extension in wire = 0.1 cm.
(Y_s) Young's modulus of steel = 2×10^{12} dynes/cm^2
(Y_b) Young's modulus of brass = 1×10^{12} dynes/cm^2
As we know that Young's modulus is given by

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$\text{Stress} = \frac{\text{Force}}{\text{Area}}$, $\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$

$\therefore \quad Y = \frac{F/A}{\Delta\ell/\ell}$

$\Rightarrow \quad F = \frac{YA\Delta\ell}{\ell}$

$\because \quad A, \Delta\ell$ & ℓ are same for both the wires.

Taking moments about point A

$F_S(x) = F_b(15-x)$

$Y_S(x) = Y_b(15-x)$

Putting values of Y_s and Y_b

2×10^{12} (x) = (1×10^{12}) (15-x)

2x = (15 – x)

3x = 15

x = 5cm.

3. (c)

The force of viscosity acting on a smooth sphere in stream line motion can be expressed with Stock's formula -

$F = 6\pi\eta rv$

Where F = force (N)

η = viscosity of fluid

r = radius of sphere (m)

v = relative velocity between fluid and sphere (m/s) or terminal velocity.

$\therefore \quad v \propto \frac{1}{\eta}$

Velocity decreases for each terminal velocity.

4. (a)

From P – V diagram (given)

a b = Isobaric process

c a = Isochoric process

It is a cyclic process and for cyclic process abca, summation of heat will be equal to summation of work.

Now

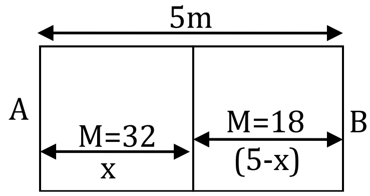
$$\Delta Q = Q_{ab} + Q_{bc} + Q_{ca}$$

$$= -50 + 0 + 80$$

$$\Delta Q = 30J$$

{From Joule's law, $\Delta W = \Delta Q$ }

5. (b)



For equilibrium pressure on both sides should be equal.

$\because$ PV = nRT

Where P = Pressure $\left\{\because\ n = \frac{m}{M}\right\}$

n = no. of moles

V = volume

R = gas constant

T = Temperature

$\therefore$ $PV = \frac{m}{M}RT$

$\Rightarrow$ $P = \frac{mRT}{MV}$

$\Rightarrow$ MV = Constant

Putting the values

$\Rightarrow$ 32 x A = 15 (5 – x) A

$\Rightarrow$ 50 x = 90

$\Rightarrow$ x = 1.8 m

6. (d)

Initially

By energy conservation we can say that

Heat loss by boiling water = heat gained by cold water

$\because$ $Q = ms\Delta\theta$

s = Specific heat capacity

Q = heat

$\Delta\theta$ = Temperature difference

$100 \times 1 \times 80 = (300 + W)\ 1 \times 10$

Where W is the water equivalent of calorimeter

$\Rightarrow$ 8000 = 3000 + 10W

$\Rightarrow$ 5000 = 10W

$\Rightarrow$ W = 500 g

When block of mass 1 kg added at 10°C then

Total mass of water = 100 + 500 + 300 = 900 g

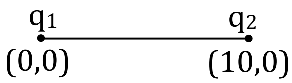
$\therefore$ $900\times1\times(20-19) = 1000 \times s \times (19 - 10)$

$9 \times 1 = 10 \times s \times 9$

1 = 10 s

$\Rightarrow$ s = 0.1 CGS unit

7. (a)



$q_1 = 1 \times 10^{-6}$ C

$q_2 = 3 \times 10^{-6}$ C

Coulomb force between these charges will be

$$F = \frac{kq_1q_2}{r^2}$$

$$F = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 3 \times 10^{-6}}{(10)^2}$$

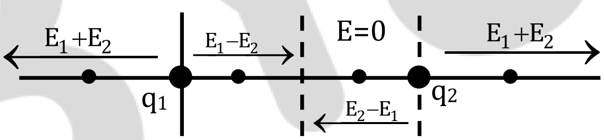
$$F = 27 \times 10^{-3-2} \quad = 27 \times 10^{-5} N$$

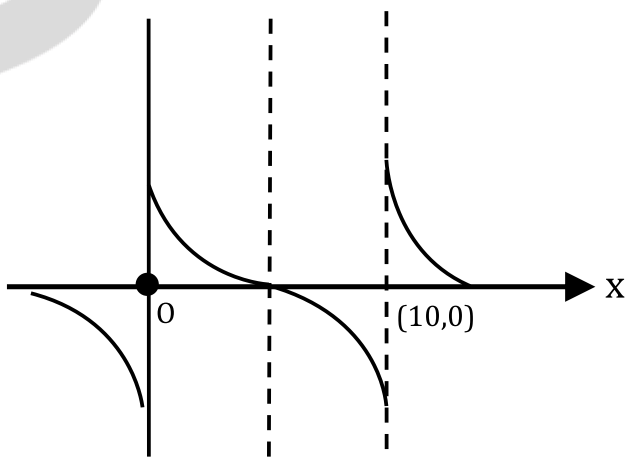
$\because$ Electric field $E = \frac{kq}{r^2}$

To the left of q_1 resultant $E_1 + E_2$ will be in –ve x-direction

To the right of q_2 resultant $E_1 + E_2$ will be in +ve x-direction

Also these is a neutral point in between the charges as they are both positive charges.





8. (c)

From energy conservation

$(K.E.)_i + (P.E.)_i = (K.E.)_f + (P.E.)_f$

$\because$ Electrostatic energy = $\frac{kq^2}{r}$

Here r = a

Initial electrostatic energy = $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\times 4 + \frac{q^2\times 2}{4\pi\varepsilon_0 a\sqrt{2}}$

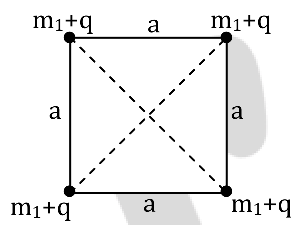
$= \frac{q^2}{4\pi\varepsilon_0 a}[4+\sqrt{2}]$

At infinity $U_\infty = 0$

(P.E. will be zero)

$\therefore$ ΔU = K.E. $\Rightarrow$ K.E. $\frac{q^2}{4\pi\varepsilon_0}[4+\sqrt{2}]$

It will be the kinetic energy of the system.



9. (b)

Using Gauss's law

$\oint E.dA = \frac{Q_{en}}{K\varepsilon_0}$ .... (1)

Q_{en}= enclosed charge

$Q_{en} = \rho\times\pi r^2\ell$

(Area) $A = 2\pi r\times\ell$

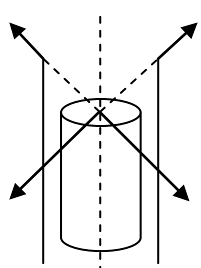
Here r = x

$\therefore$ $A = 2px\ell$

So from equation .... (1)

$E\,.\,2\pi x\ell = \frac{\rho\pi x^2\ell}{K\varepsilon_0}$

$\Rightarrow E = \frac{\rho x}{2K\varepsilon_0}$



10. (a)

When galvanometer is used or converted as a voltmeter, resistance is used in series with it

$I_g(G+R_1)=V_0$

$G+R_1=\frac{V_0}{I_g} \quad \Rightarrow V_0=I_g(G+R_1) \quad$ .... (1)

$R_1=\frac{V_0}{I_g}-G \quad$ .... (A)

When galvanometer is used or converted as an ammeter, shunt is used in parallel with galvanometer.

$\therefore \quad I_g(G)=(I_0-I_g)R_2$

$R_2=\frac{I_gG}{I_0-I_g} \quad$ .... (2)

$\Rightarrow \quad G=\frac{R_2(I_0-I_g)}{I_g} \quad$ ....(3)

From equation (1), (2) & (3)

$V = G\,I_g + R_1I_g$

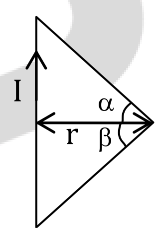
$V=R_2(I_0-I_g)+R_1I_g$

$\Rightarrow I_g=\frac{V-I_0R_2}{R_1-R_2}$

11. (d)

Magnetic field for square wire is given as –

$B=\frac{\mu_0 I}{4\pi r}(\sin\alpha+\sin\beta)$



$r=\frac{a}{2}$

$\therefore B=\frac{\mu_0 I}{4\pi\frac{a}{2}}\,(\sin 45^\circ+\sin 45^\circ)$ For one wire (out of the plane)

$\therefore B=\frac{\mu_0 I}{4\pi\frac{a}{2}}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$

$$B = \frac{\mu_0 I\sqrt{2}}{2\pi a}$$

$\therefore$ For four wires $B_1 = 4 \times \frac{\mu_0 I\sqrt{2}}{2\pi a}$

$$\Rightarrow B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

For circular loop, magnetic field is given as

$$B_2 = \frac{\mu_0 I}{2r}$$

Given that $\frac{a}{r} = \frac{8}{\pi} \Rightarrow r = \frac{\pi a}{8}$

$B_2 = \frac{\mu_0 I}{2\left(\frac{\pi a}{8}\right)} = \frac{4\mu_0 I}{\pi a}$ (Into the loop) (Into the plane)

Net magnetic field B = $B_2 - B_1$

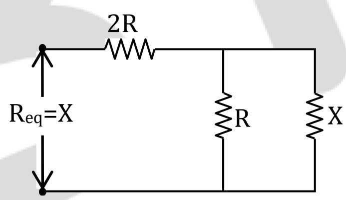
$$B = \frac{\mu_0 I}{\pi a} 2\sqrt{2}\left(\sqrt{2} - 1\right)$$

12. (a)

Figure forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

13. (d)

Equivalent circuit



From figure we can see that resistance x and R in parallel and then connected with R in series.

So

$$X = \frac{RX}{R+X} + 2R$$

$$X = \frac{RX + 2R(R+X)}{R+X}$$

$$X(R+X) = RX + 2R^2 + 2RX$$

$$XR + X^2 = RX + 2R^2 + 2RX$$

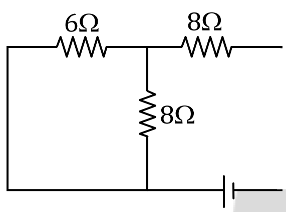
$$X^2 - 2RX - 2R^2 = 0$$

$$\Rightarrow X = \left(1 + \sqrt{3}\right)R$$

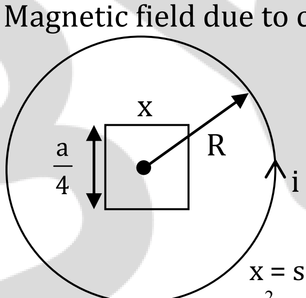
14. (c)

According to the graph, it is the condition of steady state. In this case the current gradually increases from zero to a certain value and then remains constant. It will occur in LR circuit transient.
In steady state, a capacitor behaves like an infinite resistance while an inductor will behave like a wire.
Hence current is constant with inductor and zero with capacitor.
Since DC current is given, that's why resistance in series is also necessary.

15. (c)
The equivalent emf of loop = 0
The capacitor is in steady state and in this state the capacitor draws no current from cell.
So the circuit is open. Hence no current passes through it.



16. (b)
Let i be the current flowing through the circular loop.
Magnetic field due to circular loop at the centre



x = side of the square loop.
x^2 = area of the square loop.

$B = \frac{\mu_0 i}{2R}$ .... (1)

Circumference of the loop $L = 2\pi R$

$\Rightarrow \quad R = \frac{L}{2\pi}$ .... (2)

Now flux through the square loop
$\phi = B\,A$
A = area of loop = x^2
$\therefore \quad \phi = B(x^2)$ .... (3)

Given $x = \frac{a}{4}$ .... (4)

$\because$ We know that $\phi = Mi$, where ϕ = flux, M = Mutual inductance, i = current

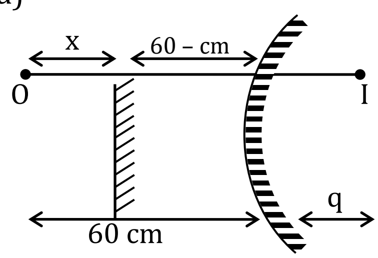
$\Rightarrow$ Mi = Bx^2 .... (5)

Putting the values from equation (1) , (2), (4) in equation (5)

$$\Rightarrow \quad Mi = \frac{\mu_0 i}{2\left(\frac{L}{2\pi}\right)}\left(\frac{a}{4}\right)^2$$

$$\Rightarrow \quad M = \frac{\mu_0 \pi a^2}{16L}$$

17. (a)



Let light is incident from left to right

Given focal length of convex mirror (f) = 30 cm.

Object distance u = 60 cm

Image distance = (v) = q

From mirror formula

$\frac{I}{f} = \frac{1}{v} + \frac{1}{u}$ .... (1)

f = 30 cm. (+ve), u = -60 (taking direction is opposite to the incident light)

v= q (+ve) (distance of the image, as we know convex mirror always forms virtual erect and diminished image)

Substituting the values in equation (1)

$$\frac{1}{30} = \frac{1}{q} - \frac{1}{60} \quad \Rightarrow \quad \frac{1}{q} = \frac{1}{30} + \frac{1}{60}$$

$$\Rightarrow \quad \frac{1}{q} = \frac{2+1}{60}$$

$$\Rightarrow \quad q = \frac{60}{3} = 20 \text{ cm}$$

Plane mirror forms image at the same distance as the object is placed.

Hence, for the condition of no parallax

x = (60-x) + q

$\Rightarrow$ x = 60 – x + 20

$\Rightarrow$ x = 40 cm

18. (a)

Using the lens formula

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

For convex lens

Focal length f = +ve

Object distance u = –ve

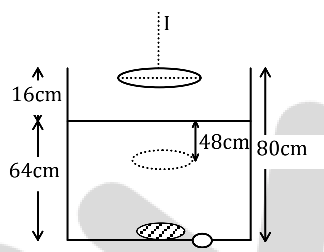
($\because$ +ve and –ve sign considered according to the direction of incident ray)

So, $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{20}+\frac{1}{80}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{f}=\frac{4+1}{80}$$

$$\Rightarrow \quad f = 16 \text{ cm}$$



Now when water is poured, the image will shift. Its distance from the surface of water

$$h'=\frac{h}{\mu}=\frac{64}{4/3}=\frac{64\times 3}{4}=48\text{cm}$$

Hence its distance from lens = 48 + 16 = 64 cm

This will be the new object distance.

Now we have to calculate the value of v, again using lens formula

$$\frac{1}{v}-\frac{1}{(-64)}=\frac{1}{16}$$

$$\frac{1}{v}=\frac{1}{16}-\frac{1}{64}$$

$$\frac{1}{v}=\frac{4-1}{64}$$

$$\Rightarrow \; v=\frac{64}{3} = 21.33 \text{ cm above the lens.}$$

19. (b)

$I_1 = 1.5\ I_2$

$\Rightarrow \quad \frac{I_1}{I_2} = 1.5$

Or $\quad \frac{I_1}{I_2} = \frac{3}{2}$

$\because \quad \frac{I_{max.}}{I_{min.}} = \left[\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2 = \left[\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right]^2$

$= \frac{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}(\sqrt{2})}{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}(\sqrt{2})}$

$\frac{I_{max.}}{I_{min.}} = \frac{3+2+2\sqrt{6}}{3+2-2\sqrt{6}}$

$\frac{I_{max.}}{I_{min.}} = \frac{5+2\sqrt{6}}{5-2\sqrt{6}}$

$= \frac{5+2\sqrt{6}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$

$= \frac{\left(5+2\sqrt{6}\right)^2}{(5)^2-(2\sqrt{6})^2}$

$= \frac{25+24+20\sqrt{6}}{25-24}$

= 97.8

$\frac{I_{max.}}{I_{min.}} \approx 98$

20. (d)

Given d = 0.5 mm, D = 50 cm, λ = 600 nm

Let x be the distance between the first order minima's

$$x = \frac{2\lambda D}{d}$$

Substituting the values

$$x = \frac{2\times600\times10^{-9}\times50\times10^{-2}}{0.5\times10^{-3}} = 12\times10^{-4}$$

x = 1.2 mm

21. (b)

As we know that the wave number is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{n_f^2}-\frac{1}{n_i^2}\right]$$

For range of wavelength:-

n_i = 1,2,3 …………..for Lyman, Balmer, Paschen ……

$n_f = n_i + 1$ and $n_f = \infty$ for upper and lower range

Thus, Lyman: $\left[\frac{1}{R}\text{to}\frac{4}{3R}\right]$

Balmer: $\left[\frac{4}{R}\text{to}\frac{36}{5R}\right]$

Paschen: $\left[\frac{9}{R}\text{to}\frac{144}{36R}\right]$

Bracket: $\left[\frac{16}{R}\text{to}\frac{400}{9R}\right]$

Pfund: $\left[\frac{25}{R}\text{to}\frac{900}{11R}\right]$

So Option A belongs to Lyman series
Option C belongs to Balmer series
Option D belongs to Paschen series
Option B does not belongs to any transition series.

Therefore hydrogen atom does not emit any radiation of wavelength in this range.

22. (c)

$${}_zX^A \longrightarrow {}_{z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + Q$$

Let M_x be the mass of the nucleus Xand
M_y be the mass of the nucleus Y.

Since mass change= $M_x - M_y$

This will be the mass reduced to β-particle.

Hence, energy $E = \Delta mc^2$

$E = (M_x - M_y)\, c^2$

23. (a)

Energy released = Total binding energy of reactant – Total binding energy of products.

E = 238 × 8.6 – 119×2×7.6

E = 238 ×(1) (MeV)

$$\left\{\begin{matrix} 119 \longrightarrow 7.6\,\text{MeV} \\ 238 \longrightarrow 119+119 \end{matrix}\right\}$$

Since the energy released will also be transferred as kinetic energy of the daughter nuclei.

Therefore the answer closest to 238 MeV from option should be chosen.

24. (d)

Load resistance $R_L = 6k\Omega = 6000\Omega$

Base emitter voltage $V_{BE} = 15$ mV $= 15\times10^{-3}$V

Base current $I_b = 20\mu A = 20\times10^{-6}$A

Collector current I_c= 1.8 mA $= 1.8\times10^{-3}$ A

Input resistance

$$R_I = \frac{V_{BE}}{I_b} = \frac{15\times10^{-3}}{20\times10^{-6}} = \frac{3}{4}\times1000$$

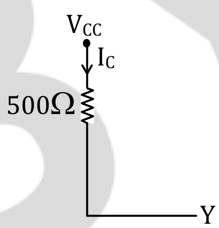
Current gain $\beta = \frac{\Delta I_c}{\Delta I_b} = \frac{1.8\times10^{-3}}{20\times10^{-6}} = 90$

$\because$ Voltage gain $(A_v) = \beta\frac{R_L}{R_I}$

$$= 90\times\frac{6000\times4}{3\times1000} = 720$$

25. (a)

$\beta = 48$(Given)



$I_B = 200\mu A = 200\times10^{-6}$ A

$\because \quad \beta = \frac{I_c}{I_b} \Rightarrow I_c = \beta I_B = 48\times200\times10^{-6} = 96\times10^{-4}$ A

Now $V_{CC} = I_C I_R + V_{CE}$

$\therefore \quad V_{CE} = 5 - (96\times10^{-4})\times500$

(Voltage at terminal y)

$V_y = V_{CE} = 5 - 4.8 = 0.2$ volt

26. (d)

$$v = k\delta E$$

$$\Rightarrow K = \left[\frac{\nu}{\delta E}\right] = \frac{[T^{-1}]}{[M^1L^2T^{-2}]}$$

$$\Rightarrow K = [M^{-1}L^{-2}T^1]$$

$\{\therefore$ Dimension of frequency = $\frac{1}{\text{sec}} = [T^{-1}]$

Dimension of energy = $= [M^1L^2T^{-2}]$

27. (b)

$$\vec{A} = \hat{i} + \hat{j} - \hat{k},\ \vec{B} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{C} = \frac{1}{\sqrt{5}}\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

We have to find the value of $\vec{C}.(\vec{A}\times\vec{B})$

Now

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}[1-(+1)] - \hat{j}[1+2] + \hat{k}[-1-2]$$

$$= -3\hat{j} - 3\hat{k}$$

Now

$$\vec{C}\cdot(\vec{A}\times\vec{B}) = \left[\frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j} + 2\hat{k})\right]\cdot\left[-3\hat{j} - 3\hat{k}\right]$$

$$= \frac{1}{\sqrt{5}}(6-6) = 0 \qquad \left\{\begin{matrix} \text{as } \hat{i}\cdot\hat{j} = 1 \\ \hat{j}\cdot\hat{j} = 1 \\ \hat{k}\cdot\hat{k} = 1 \end{matrix}\right\}$$

28. (a)

$$u = 360\frac{\text{km}}{\text{hr}} = 360\times\frac{5}{18} = 100 \text{ m/s}$$

h = 500 m

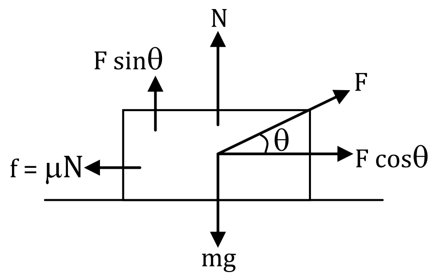
$$\because R = u\sqrt{\frac{2h}{g}}$$

$$\therefore R = 100\sqrt{\frac{2\times 500}{10}}$$

R = 100 ×10

R = 1000 m.

29. (a)



f = friction force

F cos θ = horizontal component of applied force

F sin θ = vertical component of applied force.

Now applying equilibrium in vertical direction

$F \sin\theta + N = mg$ ...... (1)

$N = mg - F \sin\theta$ ...... (2)

Applying equilibrium in horizontal direction

$F \cos\theta = \mu N$

$F \cos\theta = \mu (mg - F \sin\theta)$ {from equation (2)}

$F \cos\theta + \mu F \sin\theta = \mu mg$

$F (\cos\theta + \mu \sin\theta) = \mu mg$

$F = \dfrac{\mu mg}{\cos\theta + \mu \sin\theta}$ ...... (3)

For F minimum, $(\cos\theta + \mu \sin\theta)$ should be maximum

$\therefore \dfrac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$

$-\sin\theta + \mu\cos\theta = 0$

$\mu\cos\theta = \sin\theta$

$\mu = \tan\theta$ ...... (4)

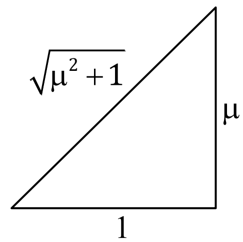
∴ from equation (3)

$$F_{min} = \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu.\mu}{\sqrt{\mu^2+1}}}$$

$$F_{min} = \frac{\mu mg}{\frac{\mu^2+1}{\sqrt{\mu^2+1}}}$$

$$F_{min} = \frac{\mu mg}{\sqrt{\mu^2+1}}$$

$\tan\theta = \mu$



$$\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}$$

$$\cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$

30. (a)

Velocity of approach in the given case is the normal component of velocity.

Hence $v_n = v\cos\theta$

By definition of coefficient of restitution, velocity of reparation will be,

$$v_n' = \in v_n = \in v\cos\theta$$

Tangential component of velocity will not change

$$v_t' = v_t = v\sin\theta$$

Speed of reflected ball is

$$v' = \sqrt{v_n^2 + v_t^2}$$

$$= \sqrt{(\in v\cos\theta)^2 + (v\sin\theta)^2}$$

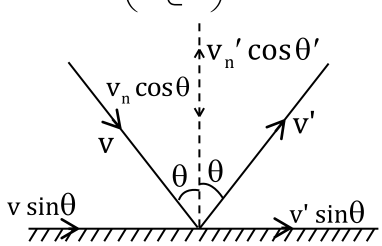
$$= v\sqrt{\in^2\cos^2\theta + \sin^2\theta}$$

Angle with normal is given by –

$$\theta' = \tan^{-1}\frac{v_t'}{v_n'}$$

$$\theta' = \tan^{-1}\left(\frac{v\sin\theta}{\in v\cos\theta}\right)$$

$$\theta = \tan^{-1}\left(\frac{\tan\theta}{\in}\right)$$



31. (a)

Mass $m = 20$ kg

Velocity $v = 0.5\ ms^{-1}$

Time = 2.1s, $\mu = 0.10$

Distance or displacement

$s = v \times t = 0.5 \times 2.1$

$s = 1.05$ m.

Friction force

$F = \mu mg$

$F = 0.1 \times 20 \times 10 = 20$

$\therefore$ Work done= F.s

$W = 20 \times 1.05 = 21$ J

This work done by friction is converted into heat energy.

$W = Q$

$W = ms\Delta T$

$\therefore$ $Q = ms\Delta T$

$21 = 20 \times 0.1 \times 4.2 \times 10^3 . \Delta T$

$$\Delta T = \frac{21}{20 \times 0.1 \times 4.2 \times 10^3}$$

$\Rightarrow$ $\Delta T = 0.0025^\circ C$

32. (c)

Absorb heat = 130 Calorie from hot reservoir

Delivered heat = 30 Calorie to cold reservoir

Consume heat = 2J energy in each cycle

Engine works at 90 cycles per minute.

Work done per cycle = $(130-30) \times 4.2 - 2$ {1 Calorie = 4.2 Joule}

$= 100 \times 4.2 - 2$

$= 420 - 2 = 418$ J

Total work done = 418×90

{$\because$ In each cycle work done = 418 J}

$$\text{Power} = \frac{\text{Workdone}}{\text{time}}$$

$$P = \frac{W}{t} = \frac{418 \times 90}{60} = \frac{418 \times 3}{2} = 627W$$

33. (a)

In first case

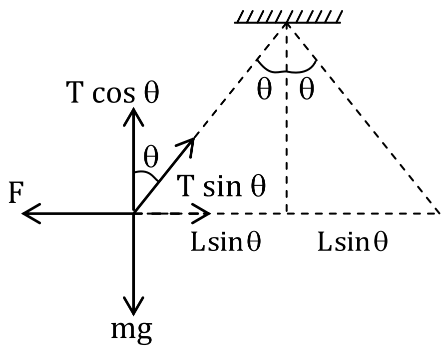
Applying equilibrium in horizontal direction

$T\sin\theta = F$ ……. (1)

Applying equilibrium in vertical direction

$T\cos\theta = mg$ ……. (2)

$\frac{\text{equation}(1)}{\text{equation}(2)}$ $\tan\theta = \frac{F}{mg}$ …… (3)



$\tan\theta = \frac{kq^2}{(2L\sin\theta)}mg$ …… (4) $\left\{\because \quad F = \frac{kq_1q_2}{r^2}\right.$

In second case $\rightarrow$ when each charge is tripled then q becomes 3q

$\tan 2\theta = \frac{kq\,q^2}{(2L\sin\theta)^2.mg}$ …… (5)

Equation (5) divided by equation (4)

$$\frac{\tan 2\theta}{\tan\theta} = \frac{9\sin^2\theta}{\sin^2 2\theta}$$

$$\frac{2}{1-\tan^2\theta} = \frac{9}{4\cos^2\theta} = \frac{9}{4}\sec^2\theta$$

$$\frac{2}{1-\tan^2\theta} = \frac{9}{4}(1-\tan^2\theta)$$

$\tan^2\theta = x,\qquad \frac{2}{1-x} = \frac{9}{4}(1+x)$

$\Rightarrow \quad 8 = 9 - 9x^2 \quad \Rightarrow \quad x = \frac{1}{3}$

$\Rightarrow \quad \tan^2\theta = \frac{1}{3} \quad \Rightarrow \quad \theta = 30^\circ$

$\therefore$ Initial angle between the strings = 30°

If it would have been asked final answer then angle would have been = 60°.

34. (c)

Given

R = 20Ω, A(area) = 20×10^{-2} m^2,

Magnetic field = B varies with time as B = 2 sin (50πt)T

q =?, time (t) = 20ms = 20×10^{-3}s.

Initial time t = 0

∵ Magnetic flux $\phi = B\cdot A$

emf $\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt} = A\frac{d}{dt}\left(2\sin(50\pi t)T\right)$

$\varepsilon = A\times100\pi\ \cos(50\pi t)$ ...... (1)

$\because$current $I=\frac{dq}{dt}=\frac{\varepsilon}{R}$

$\Rightarrow$ $dq = \frac{\varepsilon}{R}dt$ ...... (2)

From equation (1) and (2)

$$dq = \frac{A}{R}\left[100\pi\cos 50\pi t\right]dt$$

$$\int_0^q dq = \int_0^{20\times10^{-3}}\frac{A}{R}\left[100\pi\cos(50\pi t)dt\right.$$

$$= \frac{A}{R}100\pi\int_0^{20\times10^{-3}}\cos(50\pi t)dt$$

$$= \frac{A}{R}100\pi\left[\frac{\sin(50\pi t)}{50\pi}\right]_0^{20\times10^{-3}}$$

$$= \frac{A}{R}\frac{100\pi}{50\pi}\left[\sin\left(50\pi\times20\times10^{-3}\right)-0\right]$$

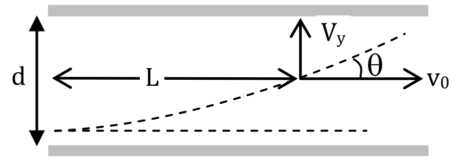
$$\frac{2A}{R}\left[\sin\pi-0\right]$$

q = 0 $\left\{\because\ [\sin\pi]=0\right\}$

35. (b)

$\because$ Time = $\dfrac{\text{distance}}{\text{speed}}$

$$t = \frac{L}{v_0} \qquad \text{...... (1)}$$



v_0 = velocity component along x – axis

v_y = velocity component along y – axis

$\because$ F = ma

F = qE = ma $\qquad \{\because F = qE\}$

Here q = e

$\Rightarrow \quad a = \dfrac{eE}{m}$

$\because \quad a = \dfrac{v}{t}$

$\therefore$ v = at

So $\quad v = \dfrac{eE}{m} t$

Substituting the value of t from equation (1)

$$v_y = \frac{eV}{md}\frac{L}{v_0} \qquad \left[\because E = \frac{V}{d}\right]$$

$$\tan\theta = \frac{\text{velocity comonent of y}}{\text{velocity component of x}} = \frac{v_y}{v_x} = \frac{e}{m}\frac{V}{d}\frac{L}{v_0}.\frac{1}{v_0}$$

$$\Rightarrow \quad \tan\theta = \frac{eVL}{mdv_0^2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$$

36. (a)
Time period of simple pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}},$$

ℓ = length of the string

g = acceleration due to gravity

For a compound pendulum, let L' be the length I be the moment of inertia about the pivot.

$\therefore$ Time period $\quad T' = 2\pi\sqrt{\frac{I}{Mgd}}$

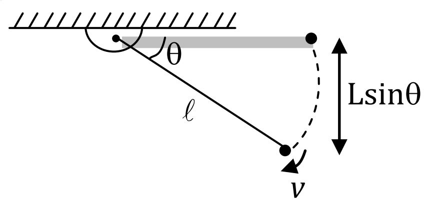
$$= 2\pi\sqrt{\frac{ML'^2}{3Mg\frac{L'}{2}}}, \qquad \text{where } d = \frac{3}{2}L'$$

$$T' = 2\pi\sqrt{\frac{2L'}{3g}}$$

$\because$ Motion is synchronous

$$T = T'$$

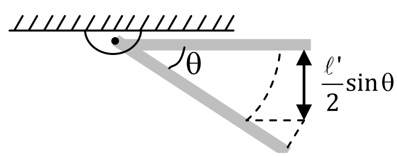
$$\Rightarrow \quad \sqrt{\frac{\ell}{g}} = \sqrt{\frac{2L'}{3g}}$$



Squaring both sides

$$\frac{\ell}{g} = \frac{2L'}{3g}$$

$$\frac{3}{2}\ell = L'$$

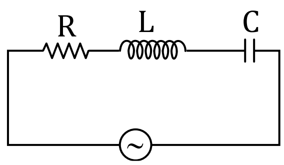


37. (a,c,d)
If a charged particle moves in a gravity free space without change in velocity then,
(1) Particle can move with constant velocity in any direction, so B = 0, E = 0

(2) Particle can move in a circle with constant speed. Magnetic force will provide the centripetal force that causes particle to move in a circle.

(3) If qE = qvB and magnetic & electric force in opposite direction, in this case also particle move with uniform speed.

38. (d)



Given values

$R = 400\Omega$, $L = 250$ mH, $C = 2.5\mu F$

$V_P = 5V$, $\omega = 2KH_z$

Inductive reactance, $X_L = \omega L$

$= 2\times10^3 \times250\times10^{-3}\Omega$

$X_L = 500\Omega$

Capacitive reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\times10^3\times2.5\times10^{-6}}\Omega$

$= 200\Omega$

Impedance $Z = \sqrt{R^2+\left(X_L-X_C\right)^2}$

$= \sqrt{(400)^2+(500-200)^2}$

$= 500\ \Omega$

Peak current, $I_P = \dfrac{V_P}{Z} = \dfrac{5}{500} = \dfrac{1}{100}A$

∴ Voltage (Peak) drop in C

$$V_C = I_P \times X_C = \frac{1}{100}\times200 = 2V$$

∴ Peak electrostatic energy of the capacitor

$$E_C = \frac{1}{2}CV_C^2 = \frac{1}{2}\times2.5\times10^{-6}\times2^2$$

$E_C = 5\mu J$

39. (b)

$\because$ Intensity $(I) \propto \frac{1}{d^2}$

Where d = distance between the source and surface.

So

If the distance between the source and surface is doubled i.e. d'=2d, then the intensity of light falling on the surface becomes one fourth i.e., $I' = \frac{1}{(2d)^2} = \frac{1}{4}I$ and

photoelectric current $\propto$ intensity

$\therefore$ current will decrease

The stopping potential (V_S) in photoelectric effect is related with the incident light frequency (ν) by the following equation.

$eV_s = h\nu - \phi$, where ϕ is the work function of the material. It does not depend on the distance of the light source from the emitting metal. So, the stopping potential will remain constant in this case.

40. (d)

To calculate kinetic energy when both the spheres are rotated with equal uniform angular speed about their diameters it will be given by

$$K.E. = \frac{1}{2}I\omega^2$$

$\because$ according to the question in option (d), ω is given same for both the spheres. In question it has already been mentioned that moment of inertia about their diameters is same. So we can say that their rotational kinetic energy will be constant.

As only outer radius is given same, so masses need not be equal as change in inner radius can cancel out effect of change in mass to maintain same moment of inertia of both spheres

As we already mentioned that in question only outer radius is given so to determine ratio of mass, inner radius is also required.

We can't comment on density of the sphere because information about inner radius not given.