Practice Set 1

Answer *with* Explanations

Physics

1. (a) Let the angle of incidence, angle of reflection and angle of refraction be i, r and r', respectively.

Now, as per the question,

$$90° - r + 90° - r' = 90°$$

$$\Rightarrow \quad r' = 90° - i \quad (\because i = r) \quad \dots(\text{i})$$

In case of refraction, according to Snell's law,

$$\sin i = \mu \sin r'$$

$$\Rightarrow \quad \sin i = \mu \sin(90° - i) \quad [\text{from Eq. (i)}]$$

$$\sin i = \mu \cos i$$

$$\Rightarrow \quad \tan i = \mu$$

$$\Rightarrow \quad i = \tan^{-1}[\mu]$$

$$= \tan^{-1}(1.62) \quad [\text{given}, \mu = 1.62]$$

$$= 58.31° \approx 58.3°$$

So, the angle of incidence is 58.3°.

2. (c) We can write, $\lambda_\alpha = \dfrac{1}{1620}$ per year

and $\lambda_\beta = \dfrac{1}{405}$ per year

We can write, $\lambda = \lambda_\alpha + \lambda_\beta = \dfrac{1}{324}$ per year

We know that,

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow \quad t = \frac{1}{\lambda}\log_e\left(\frac{N_0}{N}\right)$$

$$\Rightarrow \quad t = \frac{1}{\lambda}\log_e^4 = \frac{2}{\lambda}\log_e^2$$

$$\Rightarrow \quad t = 324 \times 2 \times 0.693 = 449 \text{ yr}$$

Therefore, ater 449 yr time, the 1/4th of the material remains after α and β-emission.

3. (c) $\therefore \lambda = \dfrac{n}{p} \Rightarrow \lambda \propto \dfrac{1}{p}$

Here, $p =$ momentum and $\lambda =$ wavelength of a proton.

$$\Rightarrow \quad \frac{\Delta p}{p} = -\frac{\Delta\lambda}{\lambda} \Rightarrow \left|\frac{\Delta p}{p}\right| = \left|\frac{\Delta\lambda}{\lambda}\right|$$

$$\Rightarrow \quad \frac{p_0}{p} = \frac{0.25}{100} = \frac{1}{400} \Rightarrow p = 400\, p_0$$

Therefore, the original momentum of the proton was 400 p_0.

4. (d) Radius of nth Bohr's orbit, $r_n = 0.53\, n^2$

$$r_{n+1} - r_n = r_{n-1}$$

Given,

$$0.53(n+1)^2 - 0.53\, n^2 = (0.53)\,(n-1)^2$$

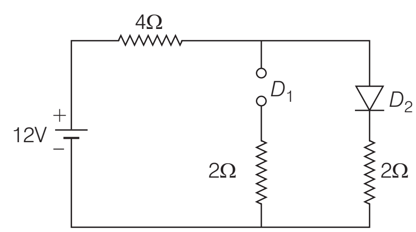
$$\Rightarrow \quad 0.53\,[(n+1)^2 - (n-1)^2] = 0.53\, n^2$$

$$[(n+1)^2 - (n-1)^2] = n^2$$

$$n^2 + 2n + 1 - n^2 + 2n - 1 = n^2$$

$$\Rightarrow \quad 4n = n^2 \Rightarrow n = 4$$

5. (b) In the given circuit, diode D_1 is reverse biased, while D_2 is forward biased. So, the circuit can be redrawn as



[$\because$ for ideal diodes, reverse biased means open and forward biased means short]

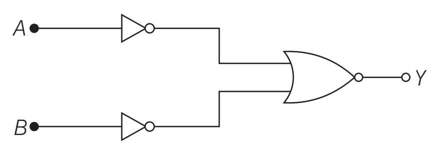
Apply KVL to get current flowing through the circuit, $-12 + 4i + 2i = 0$

$$\Rightarrow \quad i = \frac{12}{6} = 2\text{ A}$$

6. (c) In NOR gate, $Y = \overline{A + B}$

i.e. $\overline{0+0} = \bar{0} = 1, \overline{1+0} = \bar{1} = 0$

$\overline{0+1} = \bar{1} = 0$ and $\overline{1+1} = \bar{1} = 0$



$$Y = \overline{\bar{A} + \bar{B}}$$

According to Demorgan's theorem,

$$Y = \overline{\bar{A} + \bar{B}} = \bar{\bar{A}} \cdot \bar{\bar{B}} = AB$$

This is the output equation of AND gate.

7. (b) $\because$ Pressure $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{\text{Mass} \times \text{Acceleration}}{\text{Area}}$

$$= \frac{\text{Mass} \times \text{Distance}}{\text{Area} \times (\text{Time})^2}$$

$$\text{Pressure} = \frac{[\text{M}]\,[\text{L}]}{[\text{L}^2]\,[\text{T}^2]} = [\text{ML}^{-1}\text{T}^{-2}]$$

8. (d) The velocity of transverse wave in a string,

$$v = \sqrt{\frac{T}{M}} = \left[\frac{m'g}{M/l}\right]^{1/2} = \left[\frac{m'lg}{M}\right]^{1/2}$$

It follows from here,

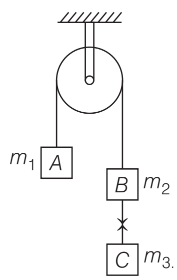
$$\frac{\Delta v}{v} = \frac{1}{2}\left[\frac{\Delta m'}{m} + \frac{\Delta l}{l} + \frac{\Delta M}{M}\right]$$

$$= \frac{1}{2}\left[\frac{0.1}{3.0} + \frac{0.01}{1.000} + \frac{0.1}{2.5}\right]$$

$$= \frac{1}{2}[0.03 + 0.001 + 0.04] = 0.036$$

Then, the percentage error in the measurement of velocity = 3.6

9. (b) Tension between m_2 and m_3 is given by



$$T = \frac{2m_1 m_2}{m_1 + m_2 + m_3} \times g = \frac{2 \times 2 \times 2}{2 + 2 + 2} \times 9.8 = 13\,\text{N}$$

10. (a) Given, $\sqrt{x} = t + 1$

Squaring on both sides, we get

$$x = (t+1)^2 = t^2 + 1 + 2t$$

Differentiating it w.r.t. t, we get

$$\frac{dx}{dt} = 2t + 2$$

$$\text{Velocity, } v = \frac{dx}{dt} = 2t + 2$$

So, it increase with time.

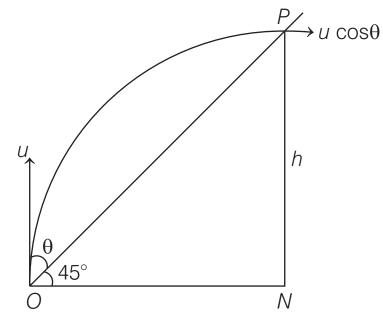
11. (b) Let the particle be projected from O with velocity u and strikes the plane at a point P after time t.

Let $ON = PN = h$, then $OP = \dfrac{PN}{\sin 45^\circ} = h\sqrt{2}$

If the particle strikes the plane horizontally, then it's vertical component of velocity at P is zero.

$$h = (u\cos\theta)(t) \quad \text{... (i)}$$

Along vertical direction, $0 = u\sin\theta - gt$



$\Rightarrow \quad u\sin\theta = gt \quad$ ... (ii)

and $\quad h = (u\sin\theta)t - \dfrac{1}{2}gt^2 \quad$ ...(iii)

Putting the value from Eqs. (i) and (ii) in Eq. (iii), we get

$$(u\cos\theta)(t) = (u\sin\theta)(t) - \frac{1}{2}(u\sin\theta)t$$

$$\tan\theta = 2$$

12. (b) $T_1 = T_{\text{earth}} = 1$ yr

and $T_2 = T_{\text{neptune}} = 165$ yr

Let R_1 and R_2 be the radii of the circular orbits of the earth and neptune, respectively.

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \frac{R_2^3}{R_1^3} = \frac{T_2^2}{T_1^2}$$

$\Rightarrow \quad R_2^3 = \dfrac{R_1^3 \times (165)^2}{1^2}$

$\Rightarrow \quad \dfrac{R_2^3}{R_1^3} = (165)^2$

$\Rightarrow \quad \dfrac{R_2}{R_1} = (165)^{2/3}$

$\Rightarrow \quad \dfrac{R_2}{R_1} = 30.08 \Rightarrow \dfrac{R_2}{R_1} \approx 30$

$\therefore \quad R_2 \approx 30R_1$

13. (b) Here, $l = 4\,\text{m} = 400\,\text{cm}$, $2r = 5\,\text{mm}$

or $\quad r = 2.5\,\text{mm} = 0.25\,\text{cm}$,

$F = 5\,\text{kg-wt} = 5000\,\text{g-wt} = 5000 \times 980\,\text{dyne}$,

$\Delta l = ?$, $Y = 2.4 \times 10^{12}\,\text{dyne cm}^{-2}$

$\therefore \quad Y = \dfrac{F}{\pi r^2} \times \dfrac{l}{\Delta l}$

$\Rightarrow \quad \Delta l = \dfrac{Fl}{\pi r^2 Y} = \dfrac{(5000 \times 980) \times 400}{(22/7) \times (0.25)^2 \times 2.4 \times 10^{12}}$

$$= 0.0041 \text{ cm}$$

Then, the increase in its length is 0.0041 cm.

14. (d) Given, terminal velocity, $v_t = 6.5 \times 10^{-2}\ \text{ms}^{-1}$
Radius of copper ball, $a = 2 \times 10^{-3}$ m
Gravitational acceleration, $g = 9.8\ \text{ms}^{-2}$,
density of copper, $\rho = 8.9 \times 10^3\ \text{kg m}^{-3}$
and density of oil, $\sigma = 1.5 \times 10^3\ \text{kg m}^{-3}$
Viscosity of the oil at 20°C, $\eta = \dfrac{2a^2 (\rho - \sigma)\, g}{9v_t}$

$$\eta = \frac{2}{9} \times \frac{(2 \times 10^{-3}) \times 9.8}{6.5 \times 10^{-2}} \times 7.4 \times 10^3\ \text{kgm}^{-3}$$

$\therefore \quad \eta = 9.9 \times 10^{-1}\ \text{kg m}^{-1}\text{s}^{-1}$

15. (a) According to Stefan's law, $E = \sigma T^4$

$\therefore \quad E_1 = \sigma (227 + 273)^4 = \sigma \times (500)^4$

and $\quad E_2 = \sigma (727 + 273)^4 = \sigma \times (1000)^4$

Hence, $\dfrac{E_2}{7} = \dfrac{(1000)^4}{(500)^4} = 16$ (Given, $E_1 = 7\ \text{cal / cm}^2\text{s}$)

$\therefore \quad E_2 = 16 \times 7 = 112\ \text{cal/cm}^2\text{s}$

Then, the rate of heat radiated in the same units will be 112 cal/cm^{2}s.

16. (a) The average kinetic energy (per molecule) of any (ideal) gas is always equal to $\frac{3}{2} k_B T$.

It depends only on temperature and is independent of the nature of the gas. Since, argon and chlorine both have the same temperature in the flask, the ratio of average kinetic energy (per molecule) of the two gases is 1 : 1.

Now,

$$\frac{1}{2} m v_{\text{rms}}^2 = \text{average kinetic energy per molecule}$$

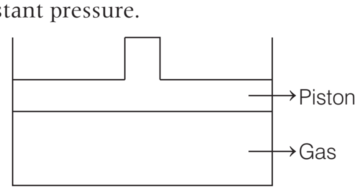
$$= \frac{3}{2} k_B T$$

$$\frac{(v_{\text{rms}}^2)_{\text{Ar}}}{(v_{\text{rms}}^2)_{\text{Cl}}} = \frac{(M)_{\text{Cl}}}{(M)_{\text{Ar}}} = \frac{70.9}{39.9} = 1.77$$

where, M denotes the molecular mass of the gas. Taking square root on both sides, we get

$$\frac{(v_{\text{rms}})_{\text{Ar}}}{(v_{\text{rms}})_{\text{Cl}}} = 1.33$$

17. (c) Given that gas is slowly heated, which means it remains in equilibrium (more or less) with the atmosphere, i.e. the process takes place at constant pressure.



From the equation of ideal gas law,

$$pV = nRT$$

For infinitesimal change,

$$pdV = nRdT$$

$\Rightarrow \quad p\Delta V = nR\Delta T$

Also, $\quad p\Delta V =$ work done by the gas $= \Delta W$

$\therefore \quad \Delta W = nR\Delta T$

Also, $\quad \Delta V \propto \Delta T$

$\therefore \quad \Delta T \propto \Delta V \propto V_2 - V_1$

Given, $V_2 = 4V_1$

$\therefore \quad \Delta T \propto 4V_1 - V_1 \propto 3V_1 \propto 3T_0$

Also, given $m = 2$ moles

The expression for work done becomes

$$\Delta W = nR\Delta T$$

$\Rightarrow \quad \Delta W = 2R3T_0 = 6RT_0$

18. (d) The capacitance of parallel plate air ($k = 1$) capacitor is

$$C = \frac{\varepsilon_0 A}{d} \quad \ldots \text{(i)}$$

The capacitance of a sphere of radius R is

$$C = 4\pi\varepsilon_0 R \quad \ldots \text{(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{\varepsilon_0 A}{d} = 4\pi\varepsilon_0 R$$

$\Rightarrow \quad d = \dfrac{A}{4\pi R}$

If r is radius of each circular plate capacitor, then

$$A = \pi r^2$$

$\because$ Separation distance between plates,

$$d = \frac{\pi r^2}{4\pi R} = \frac{r^2}{4R}$$

Here, $r = \dfrac{4}{2}\ \text{cm} = 2\ \text{cm} = 2 \times 10^{-2}$ m

$$R = \frac{20}{2} = 10\ \text{cm} = 10 \times 10^{-2}\ \text{m}$$

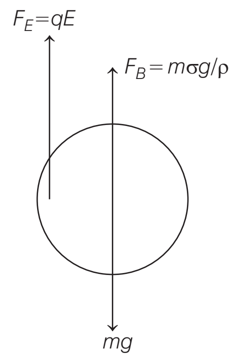
$\therefore \quad d = \dfrac{(2 \times 10^{-2})^2}{4 \times 10 \times 10^{-2}} = 1 \times 10^{-3}$ m

Then, the distance between the plates will be 1×10^{-3} m.

19. (d) Mass of the copper ball,

$$m = \frac{4}{3} \pi r^3 \rho$$

The ball is immersed in oil and a uniform electric field passes through the oil upwards. Now, the ball is acted on by three forces.



For the equilibrium of the copper ball,

$$mg = F_E + F_B = qE + \frac{m}{\rho}\sigma g$$

$$\Rightarrow \quad q = \frac{mg}{E}\left(1 - \frac{\sigma}{\rho}\right)$$

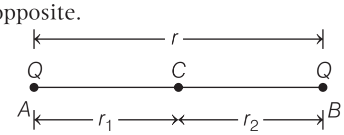
$$= \frac{4}{3}\frac{\pi r^3 \rho g}{E}\left(1 - \frac{\sigma}{\rho}\right)$$

$$= \frac{4\pi(0.5\times 10^{-2})^3(8600\times 9.8)}{3\times 36000\times 100}\left(1 - \frac{800}{8600}\right)$$

$$= 1.11\times 10^{-8}\ \text{C}$$

Therefore, the charge of the ball is 1.11×10^{-8} C, if in a homogeneous electric field, it is suspended in oil.

20. (b) For the charge q to be in equilibrium, the force exerted on it by the charge A and B should be equal and opposite.



$$F_{AC} = F_{BC}$$

i.e. $$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2^2} \text{ or } r_1 = r_2$$

Now, for the charge B, to be in equilibrium, the force exerted on it by charge $+Q$ at A should be equal to the force exerted on it by charge q at C equilibrium $F_{AB} = F_{BC}$

i.e. $$\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2^2}$$

i.e. $$Q^2 = 4Qq \qquad (\text{where, } r_2 = r/2)$$

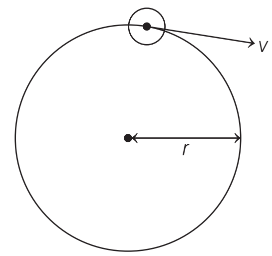
or $\quad q = Q/4$

Hence, the charge $q = -Q/4$ and it is located exactly midway between A and B.

21. (a) Magnetic moment, $M = NiA$

where, N = number of turns of the current loop

and $\quad i$ = current.



Since, the orbiting electron behaves as a current loop of current i, we can write

$$i = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$$

and $\quad A$ = area of the loop $= \pi r^2$

$$\Rightarrow \quad M = (1)\left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

Hence, the magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v is equal to $\frac{evr}{2}$.

22. (b) From the figure, magnetic field due to AB,

$$\mathbf{B}_1 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right] = \frac{\mu_0 I}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right]$$

Magnetic field due to circular loop,

$$\mathbf{B}_2 = \frac{\mu_0 I}{2R}$$

Magnetic field due to straight wire BC,

$$\mathbf{B}_3 = \frac{\mu_0 I}{4\pi R}[\sin\pi/2 + \sin\pi/4] = \frac{\mu_0 I}{4\pi R}\left[1 + \frac{1}{\sqrt{2}}\right]$$

$\therefore$ Resultant magnetic field,

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3$$

$$= \left(\frac{\mu_0 I}{2R} + \frac{2\mu_0 I}{4\pi R}\frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{2R}\left[1 + \frac{1}{\pi\sqrt{2}}\right]$$

23. (b) If the velocity v is perpendicular to the magnetic field B, the magnetic force is perpendicular to both v and B; and acts like a centripetal force. It has a magnitude qvB.

So, net force on a particle = centripetal force

$$\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{Bq} = \sqrt{\frac{2mE}{qB}}$$

$$\therefore \quad r_p : r_d : r_\alpha = \sqrt{\frac{m_p}{q_p}} : \sqrt{\frac{m_d}{q_d}} : \sqrt{\frac{m_\alpha}{q_\alpha}}$$

$$= \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$$

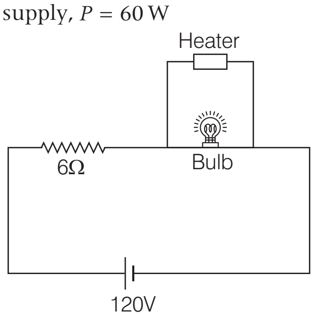
Then ratio of radii of circular paths travelled by these particle is $1 : \sqrt{2} : 1$.

24. (a) A vertical wire is carrying current in upward direction. So, the magnetic field produced will be anti-clockwise (according to right hand thumb rule). As the electron beam is sent horizontally towards the wire, the direction of the current will be horizontally away from the wire (direction of conventional current is opposite to the direction of the negative charge). According to Fleming's left hand rule, the force will act in upward direction, deflecting the beam in the same direction.

25. (d) Given, supply voltage, $V = 120\,\text{V}$

Resistance of lead wire, $R = 6\Omega$

Power supply, $P = 60\,\text{W}$



Resistance of bulb $= \dfrac{V^2}{P} = \dfrac{120 \times 120}{60} = 240\,\Omega$

and resistance of heater $= \dfrac{120 \times 120}{240} = 60\,\Omega$

Voltage across bulb before heater is switched ON,

then $V_1 = \dfrac{120}{246} \times 240 = 117.07$

and voltage across bulb after heater is switched ON,

$V_2 = \dfrac{120}{54} \times 48 = 106.66$

$\therefore$ Decrease in the voltage,

$$V_1 - V_2 = 117.07 - 106.66$$
$$= 10.41 \simeq 10.04\,\text{V}$$

26. (a) The given Wheatstone's bridge is balanced. No current flows through galvanometer G. The resistance of arm $ABC = P + R = 10 + 15 = 25\Omega$ is parallel with the resistance of arm

$$ADC = Q + S = 20 + 30 = 50\Omega.$$

The effective resistance of circuit,

$$R_{\text{eff}} = \frac{25 \times 50}{25 + 50} = \frac{50}{3}\,\Omega$$

$\therefore$ Current, $i = \dfrac{V}{R_{\text{eff}}} = \dfrac{6}{(50/3)} = 0.36\,\text{A}$

Hence, the current passing through battery of negligible resistance is 0.36 A.

27. (b) Given, inductance of a coil, $L = 300\,\text{mH}$

$$= 300 \times 10^{-3}\,\text{H}$$

Resistance of a coil, $R = 2\Omega$

Source voltage, $E = 2\,\text{V}$

Current, $I_0 = \dfrac{E}{R} = \dfrac{2}{2} = 1\,\text{A}$

From $I = I_0 \left(1 - e^{-\frac{Rt}{L}}\right)$

$\Rightarrow \quad \dfrac{1}{2} I_0 = I_0 \left(1 - e^{-\frac{2}{0.3}t}\right) = e^{-\frac{2}{0.3}t} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{-2t}{0.3} \log_e e = \log_e 1 - \log_3 2$

$$= 0 - 0.6931$$

$\Rightarrow \quad t = \dfrac{0.6931 \times 0.3}{2}$

$\therefore \quad t = 0.1\,\text{s}$

The current reaches half of its steady state value in 0.1 s.

28. (c) Distance between first dark fringes on either side of central bright fringes = width of central maximum

$$= \frac{2\lambda D}{a} = \frac{2 \times (600 \times 10^{-9}) \times 2}{10^{-3}}$$
$$= 24 \times 10^{-4}\,\text{m}$$
$$= 2.4\,\text{mm}$$

Here, $I = \dfrac{I_0}{4} = I_1 = I_2$ and if ϕ is the phase difference, then $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\therefore \quad \dfrac{I_0}{4} = \dfrac{I_0}{4} + \dfrac{I_0}{4} + 2\sqrt{\dfrac{I_0}{4} + \dfrac{I_0}{4}}\cos\phi$

or $1/4 = 1/4 + 1/4 + 2\left(\dfrac{1}{4}\right)\cos\phi$

or $\cos\phi = \dfrac{1}{2}$ or $\phi = \dfrac{2\pi}{3}$

If Δx the path difference between two waves, then

$$\frac{\phi}{2\pi} = \frac{\Delta x}{\lambda} \Rightarrow \Delta x = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi} \times \frac{2\pi}{3} = \frac{\lambda}{3}$$

For path difference Δx, if angular separation is θ, then

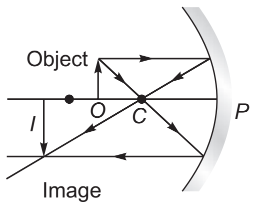
$$d \sin\theta \Delta x = \frac{\lambda}{3}$$

or $\quad \sin\theta = \dfrac{\lambda}{3d}$

or $\quad \theta = \sin^{-1}\left(\dfrac{\lambda}{3d}\right)$

29. (b) Given, focal length,

$$f = \frac{-15}{2} = -7.5 \text{ cm}$$



The distance of object from concave mirror, $u = -10$ cm, then from mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{(-10)} = \frac{1}{-7.5}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{7.5} + \frac{1}{10}$$

$$\Rightarrow \quad v = \frac{10 \times 7.5}{-2.5} = -30 \text{ cm}$$

The image is 30 cm from the mirror on the same side as the object.

Also, magnification, $m = -\frac{v}{u} = -\frac{(-30)}{(-10)} = -3$

Hence, image is magnified, real and inverted.

30. (*c*) Given, wavelength, $\lambda_1 = 660 \text{ nm}$

Position of minima in diffraction pattern is given by

$$a \sin \theta = n\lambda$$

For first minima of λ_1, we get

$$a \sin \theta_1 = 1\lambda_1 \text{ or } \sin \theta_1 = \frac{\lambda_1}{a}$$

For the first maxima approximately of wavelength λ_2,

$$a \sin \theta_2 = \frac{3}{2}\lambda_2, \quad \sin \theta_2 = \frac{3\lambda_2}{2a}$$

The two will be considered, if

$$\theta_1 = \theta_2$$

$$\text{or} \quad \sin \theta_1 = \sin \theta_2$$

$$\frac{\lambda_1}{a} = \frac{3\lambda_2}{2a}$$

$$\text{or} \quad \lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660 \text{ nm}$$

$$\therefore \quad \lambda_2 = 440 \text{ nm}$$

31. (b) As inductances obey laws similar to grouping of resistances,

Given, $\quad L_1 + L_2 = 10 \text{H}$

and $\quad \frac{L_1 L_2}{(L_1 + L_2)} = 2.4 \text{ H}$

Substituting the value of $(L_1 + L_2)$ from first expression into second,

$$L_1 L_2 = (2.4)(L_1 + L_2) = 2.4 \times 10 = 24$$

So, $\quad (L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1 L_2$

i.e. $\quad L_1 - L_2 = [(10)^2 - 4 \times 24]^{1/2} = 2\text{H}$

and as $L_1 + L_2 = 10\text{H}$, $L_1 = 6\text{H}$

and $\quad L_2 = 4\text{H}$

32. (b) Water is considered to be perfectly incompressible. Volume covered by the movement of smaller piston inwards is equal to the volume moved outwards due to longer piston.

$$L_1 A_1 = L_2 A_2$$

$$\Rightarrow \quad L_2 = \frac{A_1}{A_2} L_1 = \frac{\pi\left(\frac{1}{2} \times 10^{-2}\right)^2}{\pi\left(\frac{3}{2} \times 10^{-2}\right)^2} \times 6 \times 10^{-2}$$

$$= 0.67 \times 10^{-2} \text{ m} \approx 0.67 \text{ cm}$$

Note Atmospheric pressure is common to both pistons and has been ignored.

33. (d) Given, mass of metal,

$$m = 0.20 \text{ kg} = 200 \text{g}$$

Fall in temperature of metal,

$$\Delta T = 150 - 40 = 110°\text{C}$$

If c is specific heat of the metal, then heat lost by the metal,

$$\Delta Q = mc\Delta T = 200\, s \times 110 \quad \ldots \text{(i)}$$

Volume of water $= 150$ cc

Mass of water, $m' = 150$ g

Water equivalent of calorimeter,

$$w = 0.025 \text{ kg} = 25 \text{g}$$

Rise in temperature of water in calorimeter,

$$\Delta T' = 40 - 27 = 13°\text{C}$$

Heat gained by water and calorimeter,

$$\Rightarrow \quad \Delta Q' = (m' + w)\,\Delta T'$$

$$= (150 + 25) \times 13$$

$$\Rightarrow \quad \Delta Q' = 175 \times 13 \quad \ldots \text{(ii)}$$

As $\quad \Delta Q = \Delta Q'$

$\therefore$ From Eqs. (i) and (ii), we get

$$200 \times s \times 100 = 175 \times 13$$

$$\Rightarrow \quad s = \frac{175 \times 13}{200 \times 110} \approx 0.1$$

If some heat is lost to the surroundings, value of s so, obtained will be less than the actual value of s.

34. (c) The magnitudes of the electric and magnetic fields in the electromagnetic wave are related as

$$B = \frac{E}{c}$$

where, c is speed of light.

Given, electric field, $E = 6.3 \text{ V/m}$

Speed of light, $c = 3 \times 10^8$ m/s

$\therefore$ Magnetic field, $B = \dfrac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8}$ T

To find the direction, we note that E is along y-direction and the wave propagates along X-axis. Therefore, B should be in a direction perpendicular to both X and Y-axes. Using vector algebra, $\mathbf{E} \times \mathbf{B}$ should be along x-direction. Since, $(+\hat{\mathbf{j}}) \times (+\hat{\mathbf{k}}) = \hat{\mathbf{i}}$, B is along the z-direction. Thus, $B = 2.1 \times 10^{-8}\,\hat{\mathbf{k}}$T

35. (a) As, reactive inductance, $X_L = 2\pi fL$

$$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8\Omega$$

Reactive capacitance, $X_C = \dfrac{1}{2\pi fC}$

$$= \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4\Omega$$

Phase difference, $\phi = \tan^{-1} \dfrac{X_C - X_L}{R}$

$$\phi = \tan^{-1}\left(\frac{4-8}{3}\right) = -53.1^\circ$$

Since, ϕ is negative, the current in the circuit lags the voltage across the source.

Therefore, the angle between the voltage across the source and the current is $-53.1°$.

36. (a,c,d) The saturation current depends upon the intensity of the incident light and not on energy of the incident light. The intercept of straight line. In a graph between E_k and ν or negative energy axis given the value of would function of cathode metal. The point where the straight line out the frequency gives the value of threshold frequency whereas the slope of straight line can help to find the Planck's constant.

37. (a,c) In the given one complete cycle, $1 \to 2 \to 3 \to 1$, the system returns to its initial state.

$\therefore$ $dU = 0$ and $dQ = dW$, i.e. heat is completely converted into mechanical energy, which is not possible in such a process. Further, the two adiabatic curves (2 3) and (3 1) cannot intersect each other.

38. (a,c) Horizontal range is same when angle of projection is θ and $(90° - \theta)$

$$\therefore \quad R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$$

When angle of projection is θ, then $T_1 = \dfrac{2u\sin\theta}{g}$

When angle of projection is $(90° - \theta)$, then

$$T_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad T_1 T_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2}$$

$$\Rightarrow \quad = \left(\frac{2u^2\sin\theta\cos\theta}{g}\right)\left(\frac{2}{g}\right) = \frac{2R}{g}$$

$$\therefore \quad T_1 T_2 \propto R$$

and $\dfrac{T_1}{T_2} = \dfrac{2u\sin\theta/g}{2u\cos\theta/g} = \tan\theta$

39. (a,c) For charging a battery, the external voltage generator must provide an output voltage V greater than emf of battery E.

In charging state $I = \left(\dfrac{V - E}{R + r}\right)$

40. (a) As the field due to an arc at the centre is given by $B = \dfrac{\mu}{4\pi}\dfrac{i\theta}{r}$

So, $B_0 = \dfrac{\mu_0}{4\pi}\dfrac{i_1\theta}{r} + \dfrac{\mu_0}{4\pi}\dfrac{i_2(2\pi - \theta)}{r} \otimes$

But, $(V_A - V_B) = i_1R_1 = i_2R_2$

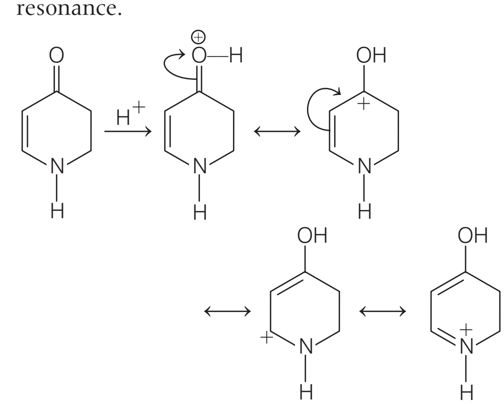
i.e. $i_2 = \dfrac{i_1R_1}{R_2} = \dfrac{i_1l_1}{l_2}$ or $i_2 = i_1\dfrac{\theta}{(2\pi - \theta)}$ (as $l = r\theta$)

So, $B_0 = \dfrac{\mu_0 i_1\theta}{4\pi r} + \dfrac{\mu_0 i_1 \pi\theta}{4\pi r} \otimes = 0$

Therefore, the magnetic field intensity B at the centre of the circular loop is zero.

Chemistry

41. ***(b)*** Oxygen is the better site of protonation, since it gives oxonium ion which is stabilised by resonance.



42. *(a)* Two positive charges present at the adjacent place, elevates the energy. thus, lowers the stability most.

43. *(d)* Nucleophilicity $\propto \dfrac{1}{\text{electronegativity of atom bearing negative charge}}$

Thus, correct order of nucleophilicity is

$$F^- < OH^- < NH_2^- < CH_3^-$$

44. *(c)* For d-subshell, $l = 2$

Orbital angular momentum

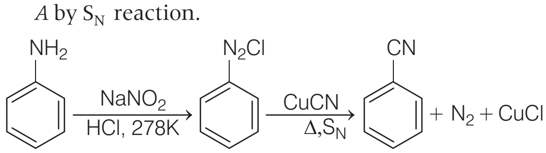
$$= \sqrt{l(l+1)}\,\frac{h}{2\pi}\ \text{B.M}$$
$$= \sqrt{2(2+1)}\ \hbar\ \text{B.M} = \sqrt{6}\hbar\ \text{B.M.}$$

45. *(b)* $[Cr(en)_3]^{3+}$ exists in d-and l-forms due to the absence of the symmetry element(s). So it will give racemic mixture when d- and l-form are mixed in 1 : 1 molar ratio.

(a) $[Ni(dmg)_2]$ has square planar geometry and thus has mirror plane so, optically inactive.

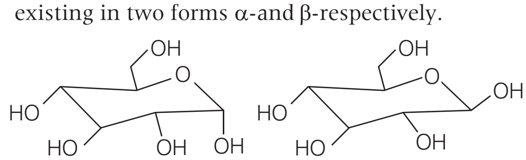
(c) $cis-[Cu(gly)_2]$ has square planar geometry and thus, have mirror plane so, optically inactive.

46. *(b)* Formation of A is by diazotisation and B from A by S_N reaction.



Benzene diazonium chloride 'A'   Benzonitrile 'B'

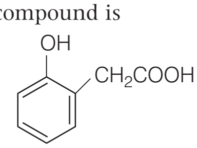
47. *(d)* Anomers of glucose are cyclic diastereomers (epimers) differing in configuration at C-1 existing in two forms α-and β-respectively.



α-form   β-form

48. *(d)* The compound liberated CO_2 with $NaHCO_3$, so, it contain — COOH group and gives colour with neutral $FeCl_3$ solution, it also contains a — OH group directly attached to the benzene ring (i.e. phenol).

Hence, the compound is



49. *(c)* We know that,

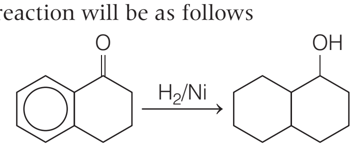
$$\log\frac{k_2}{k_1} = \frac{\Delta H}{2.303R}\left[\frac{T_1 - T_1}{T_1T_2}\right]$$
$$\log\frac{9.25}{18.5} = \frac{\Delta H}{2303\times 8.314}\times\frac{75}{925\times 1000}$$
$$-0.3010 = \frac{\Delta H \times 75}{2.303\times 8.314\times 925\times 1000}$$
$$\Delta H = -\ 71080.57\ \text{Jmol}^{-1}$$
$$\Delta H = -\ 71.08\ \text{kJmol}^{-1}$$

50. *(b)* 5.0×10^{-2} M NaOH $\equiv [OH^-] = 5\times 10^{-2}$ M

$$[H^+][OH^-] = 1\times 10^{-14}$$
$$[H^+]\times 5\times 10^{-2} = 1\times 10^{-24}$$
$$[H^+] = \frac{1\times 10^{-14}}{5\times 10^{-2}} = 2\times 10^{-13}$$
$$pH = -\log[H^+] = -\log\ (2\times 10^{-13})$$
$$= 12.69 \approx 12.70$$

51. *(a)* H_2 / Ni is a powerful reducing agent. It will destroy the aromaticity of benzene ring by reducing it and also will convert >C= O (carboxyl) to >CHOH (alcoholic) group.

So, reaction will be as follows



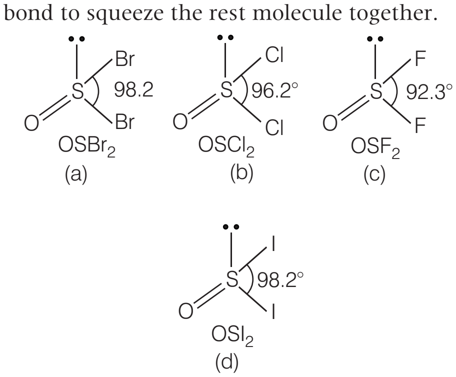
52. *(d)* H_2O (l) and H_2O (g) both exist together at same temperature and pressure.

$$H_2O(l) \rightleftharpoons H_2O\ (g)$$

In the state of equilibrium, $\Delta G = ve$ and conversion of liquid into gas increases disorderness.

Hence, entropy $\Delta S = +$ ve .

53. *(c)* In OSF_2, F shows strongest attraction for electron in S—F bonds. This reduces electron-electron repulsion near the S-atom enhancing the ability of lone-pair and double bond to squeeze the rest molecule together.



54. (a) Variation of K with temperature is given by

$$\log K = \frac{\Delta S^\circ}{R} - \frac{\Delta H^\circ}{RT}$$

Given, $$\log K = 4.0 - \frac{2000}{T}$$

On comparing, $\frac{\Delta S^\circ}{R} = 4$ or $\Delta S^\circ = 4R$

Hence, option (a) is correct.

55. (c) In van der Waals' equation of state for a non-ideal gas, the intermolecular force are given by the term $\left(p + \frac{a}{V^2}\right)$.

Thus, the option (c) is correct.

56. (a) The electronic arrangement of option (a) is absurd because, for each value of l, m is $-l$ to $+l$ for $l = 2$, $m \neq -3$.

57. (d)

(a) $[Fe(H_2O)_6]Cl_3 \rightleftharpoons \underbrace{3Cl^- + [Fe(H_2O)_6]^{3+}}_{\text{4 ions}}$

$i = (1 + 3x)$

(b) $[Fe(H_2O)_5Cl]Cl_2 \rightleftharpoons 2Cl^- + [Fe(H_2O)_5Cl]^{2+}$,

$i = (1 + 2x)$

(c) $[Fe(H_2O)_4Cl_2]Cl \rightleftharpoons Cl^- + [Fe(H_2O)_4Cl_2]^+$,

$i = (1 + x)$

(d) $[Fe(H_2O)_3Cl_3] \rightleftharpoons$ No ionisation, $i = 1$

$$\Delta T_f \propto i$$

and freezing point $= 0 - \Delta T_f$

Thus, greater the value of ΔT_f, similar the freezing point hence, maximum freezing point is of (d).

58. (b) The correct order of increasing ionic character is

$$BeCl_2 < MgCl_2 < CaCl_2 < BaCl_2$$

59. (b) Among O, S and Se the electron affinity of O (oxygen) is smallest due to inter-electronic repulsions present in its relatively compact $2p$-subshell. Electron affinity of Se is however, smaller the S due to its larger size.

Thus, the correct order of electron affinity is

$$S > Se > O$$

60. (c) For second ionisation energy (IE) compare $C^+ (2s^2 2p^1)$, $N^+ (2s^2 2p^2)$, $O^+ (2s^2 2p^3)$ and $F^+ (2s^2 2p^4)$

Thus, the order of IE among these unipositive species is $O^+ > F^+ > N^+ > C^+$

So, the option (c) is correct.

61. (c) $\underset{\text{1 mol}}{N_2} + \underset{\text{3 mol}}{3H_2} \longrightarrow \underset{\text{2 mol}}{2NH_3}$

1 mol of N_2 reacts with 3 moles of H_2 to produce 2 moles of NH_3 which occupies a volume

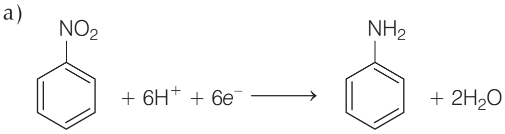
$= 2 \times 22.4 = 44.8$ L at S.T.P.

62. (d) $4AgNO_3 + 2H_2O + H_3\overset{+1}{P}O_2 \longrightarrow 4Ag + \underset{(X)}{4HNO_3} + \underset{(Y)}{H_3\overset{+5}{P}O_4}$

63. (b) I. $(CH_3)_2CO \xrightarrow{NH_2OH} \underset{(X)}{(CH_3)_2C{=}N{-}OH} \xrightarrow{LiAlH_4} \underset{\text{isopropyl amine}}{(CH_3)_2CHNH_2}$

II. $(CH_3)_2CHOH + PCl_5 \longrightarrow \underset{(X)}{(CH_3)_2CHCl} \xrightarrow{NH_3} \underset{\text{isopropyl amine}}{(CH_3)_2CHNH_2}$

64. (a)



(Nitrobenzene) (Aniline)

123 gm (1 mol) nitrobenzene requires

$= 6 \times 96500$ C

12.3 gm nitrobenzene requires

$$= \frac{6 \times 96500}{123} \times 12.3 = 57900 \text{ C}$$

65. (a) The complex of dichloro *bis* (ethylene diamine) cobalt (III) chloride is $[Co(en)_2Cl_2]Cl$

$[Co(en)_2Cl_2]Cl \longrightarrow [Co(en)_2Cl_2]^+ + Cl^-$

Only one Cl^-, which is precipitated as AgCl

100 ml of 0.024 M complex = 2.4 millimol

= 0.024 mol

66. (a) $4FeCr_2O_4 + 8Na_2CO_3 + \underset{\text{(air)}}{7O_2} \xrightarrow{\text{Heat}} 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

$Na_2CrO_4 \xrightarrow{H^+/NH_4Cl} (NH_4)_2Cr_2O_7 \xrightarrow{\text{Heat}} Cr_2O_3 + N_2\uparrow$

$Cr_2O_3 \xrightarrow{Al} Cr + Al_2O_3$

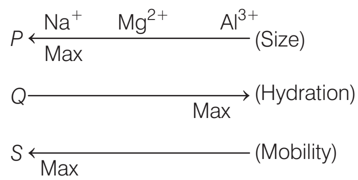
67. (b) $2FeSO_4 \xrightarrow{\Delta} \underset{\text{Blackish brown}}{Fe_2O_3} + SO_2 + SO_3$

$Fe_2O_3 + 6HCl \longrightarrow \underset{\text{Yellow solution}}{2FeCl_3} + 3H_2O$

$FeCl_3 + 3CNS^- \longrightarrow \underset{\text{Blood red coloured solution}}{[Fe(CNS)_3]} + 3Cl^-$

68. (c) Smaller is the size of ion (P) $\rightarrow$ more is its hydration (Q). So, lesser is its ionic mobility (s)

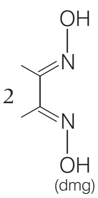
K^+ Na^+ Li^+



Thus, (c) is the correct answer.

69. (d) $CO^{2+}(aq) + 4SCN^-(aq) \longrightarrow \underset{(X)}{[CO(SCN)_4]^{2-}}$

IUPAC name of X : tetrathiocyanato-s-cobaltate (II)

$Ni^{2+}(aq)$ + 2  $\longrightarrow \underset{(Y)}{[Ni(dmg)_2]}$

IUPAC name of Y : *bis* (dimethyl glyoximato) nickel (II)

70. (d) $k_2 = \dfrac{2.303}{t_2} \log \dfrac{100}{50}$ for 50% of B reacted

$$k_1 = \frac{2.303}{t_1} \log \frac{100}{6} \text{ for 94\% of } A \text{ reacted}$$

$$\therefore \frac{k_2}{k_1} = \frac{t_1}{t_2} \times \frac{0.3010}{1.2218}$$

Since, $t_2 = t_1$, because 50% B has reacted when, 94% A has reacted.

$$\therefore \frac{k_2}{k_1} = \frac{0.3010}{1.2218} = 0.246 \text{ and } \frac{k_1}{k_2} = 4.06$$

71. (c) $Cu^{2+} \xrightarrow[E_1\,,\,\Delta G_1]{e^-} Cu^+ \xrightarrow[E_2\,,\,\Delta G_2]{e^-} Cu$

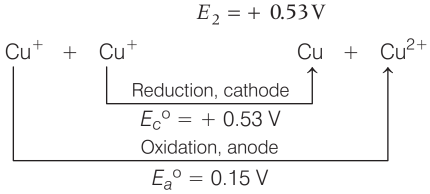
$$\underset{E_3, \Delta G_3}{\xrightarrow{2e^-}}$$

$$\Delta G_1 + \Delta G_2 = \Delta G_3$$

$$-n_1FE_1 - n_2FE_2 = -n_3FE_3$$

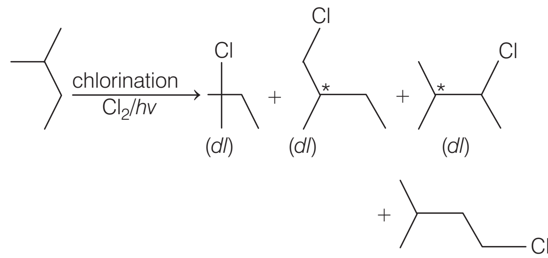
$$1 \times 0.15 + 1 \times E_2 = 2 \times 0.34$$

$$E_2 = +0.53\,V$$

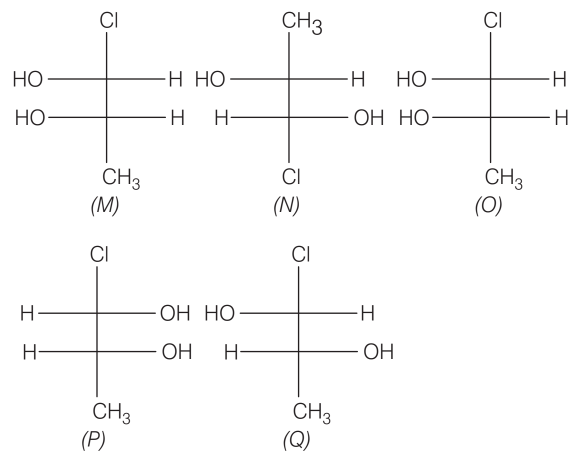


$$\therefore \quad E^\circ_{cell} = E^\circ_c - E^\circ_a = +0.53 - 0.15 = 0.38\,V$$

72. (b) On chlorination of 2-methyl butane 4 optically active isomers are formed.



73. (d) Converting all of them into Fischer projection.



M and N have — OH on same side and opposite side respectively, they cannot be mirror image, they are diastereomers.

M and O are identical.

M and P are non-superimposable mirror images, hence, enantiomers. M and Q are non-identical, they are distereomers.

74. (b) Equilibrium constant for the reaction,

$$SO_2(g) + \frac{1}{2} O_2(g) \rightleftharpoons SO_3(g)$$

$$K_c = \frac{1}{4.9 \times 10^{-2}}$$

and for $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$$K_c = \left(\frac{1}{4.9 \times 10^{-2}}\right)^2$$

$$= \frac{10^4}{(4.9)^2} = 416.490 \approx 416$$

75. (c) I. $CH_3-\overset{\overset{O}{\|}}{C}-OEt \xrightarrow[\text{(ii) } H^+]{\text{(i) } CH_3MgBr \text{ (excess)}}$

$$CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{OH}{|}}{C}}-CH_3$$
(3° alcohol)

II. $H-\overset{\overset{O}{\|}}{C}-OEt \xrightarrow[\text{(ii) } H^+]{\text{(i) PhMgBr (excess)}} H-\underset{\underset{Ph}{|}}{\overset{\overset{OH}{|}}{C}}-Ph$
(2° alcohol)

III. $H-\overset{\overset{O}{\|}}{C}-OEt \xrightarrow[\text{(ii) } H^+]{\text{(i) } CH_3MgBr \text{ (excess)}} H-\underset{\underset{CH_3}{|}}{\overset{\overset{OH}{|}}{C}}-CH_3$

IV. $ClCH_2COOEt \xrightarrow[\text{(ii) } H^+]{\text{(i) } CH_3MgBr \text{ (excess)}}$

$$Cl-CH_2-\underset{\underset{CH_3}{|}}{\overset{\overset{OH}{|}}{C}}-CH_3$$
(3° alcohol)

∴ Hence, the option (c) is correct.

76. (b, d) (b) For the reaction $X \longrightarrow P$

$$t = \frac{2.303}{r} \log \frac{[x]_0}{[X]}$$

$$\log [X] = \frac{-k}{2.303} t + \log [x]_0$$

As such a graph between $\log [X]$ and time t is linear

(d) At constant temperature,

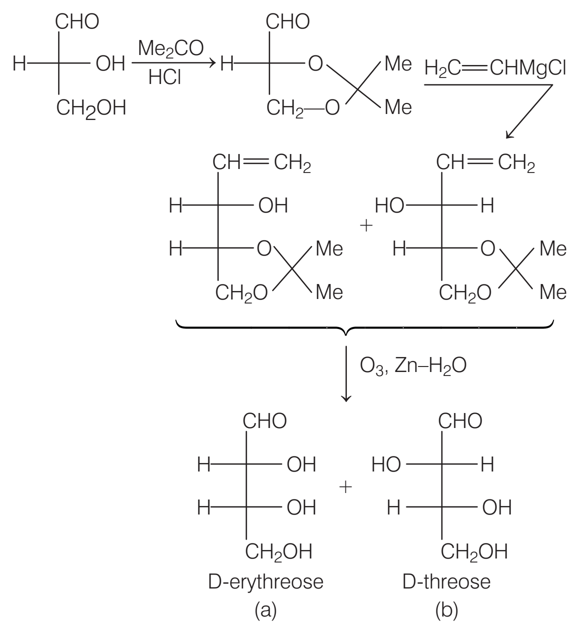
$$V = \frac{K}{P} \text{(Boyle's law)}$$

or $$P = K / V$$

As such a graph between P and $\frac{1}{V}$ is linear at constant temperature.

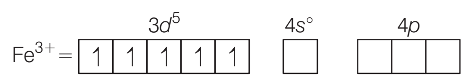
[**Note :** A graph between $\log k$ and $\frac{1}{T}$ is linear and not between $\log K_p$ and $\frac{1}{T}$]

77. (a, d)

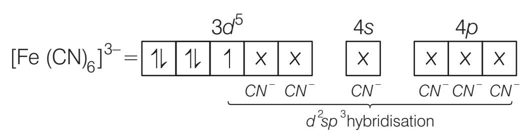


78. (b, c) Only $(NH_2)_2C{=}S$ and $p-NH_2C_6H_4SO_3H$ contain both N and S which are required for observing blood red colouration in Lassaigne's test.

79. (a, d) In $[Fe(CN)_6]^{3-}$, Fe is present as Fe^{3+}



CN^- being strong field ligand pair up these unpaired electrons, so, that now the complex have only one unpaired electron as



Now, the two $3d$, one $4s$ and three $4p$-orbitals hybridise to give six d^2sp^3 hybridised orbitals which are occupied by electrons of CN^- since, the complex contain one unpaired electron, it is paramagnetic in nature.

80. (a, c) $CH_3-\underset{\underset{CH_3}{|}}{C}=CHCH_3 \xrightarrow[\Delta]{KMnO_4}$

$$CH_3-\underset{\underset{O}{\|}}{C}-CH_3 + CH_3COOH$$

When, $CH_3-\underset{\underset{CH_3}{|}}{C}=CHCH_3$ reacts with

$NaIO_4$ or boiling $KMnO_4$ produces both

$CH_3-\underset{\underset{O}{\|}}{C}-CH_3$ (acetone)

and $CH_3-\underset{\underset{O}{\|}}{C}-OH$ (carboxylic acid) are produced.

Mathematics

1. (c) Since, $2\cos^2 x + \sin x \le 2$

and $\quad \dfrac{\pi}{2} \le x \le \dfrac{3\pi}{2}$

$\Rightarrow \quad 2 - 2\sin^2 x + \sin x \le 2$

$\Rightarrow \quad 2\sin^2 x - \sin x \ge 0$

$\Rightarrow \quad \sin x(2\sin x - 1) \ge 0$

$\Rightarrow \quad \dfrac{\pi}{2} \le x \le \dfrac{5\pi}{6}$ or $\pi \le x \le \dfrac{3\pi}{2}$

$\therefore \quad A \cap B = \left\{x : \dfrac{\pi}{2} \le x \le \dfrac{5\pi}{6} \text{ or } \pi \le x \le \dfrac{3\pi}{2}\right\}$

2. (a) Given, $z = \dfrac{(\sqrt{3} + i)^{4n+1}}{(1 - i\sqrt{3})^{4n}} = \dfrac{(2e^{\frac{i\pi}{6}})^{4n+1}}{(2e^{\frac{-i\pi}{3}})^{4n}}$

$$= \frac{2^{4n+1} \cdot e^{i(4n+1)\frac{\pi}{6}}}{2^{4n} \cdot e^{\frac{-i4n\pi}{3}}} = 2 \cdot e^{i(12n+1)\frac{\pi}{6}}$$

$$= 2 \cdot e^{2n\pi i} \cdot e^{\frac{\pi i}{6}} \quad (\text{As}, e^{2n\pi} = 1)$$

$$= 2 \cdot e^{\frac{\pi i}{6}}$$

So, $\quad \arg z = \dfrac{\pi}{6}$

3. (b) Number of occurrence of 10 tails $= {}^{10}C_{10} = 1$

Number of occurrence of 9 tails $= {}^{10}C_9 = 10$

Number of occurrence of 8 tails $= {}^{10}C_8 = 45$

Total possible outcomes $= 2^{10}$

Let T be the even that atleast 8 tails occur

So, $P(T) = \dfrac{1 + 10 + 45}{2^{10}} = \dfrac{7}{128}$

4. (b) Given, $n_1 = 9$, $\bar{x}_1 = 100$, $n_2 = 6$, $\bar{x}_2 = 80$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{9 \times 100 + 6 \times 80}{9 + 6} = 92$$

5. (a) As, $\quad 10^3 < 8721.3 < 10^4$

$\Rightarrow \quad 3 < \log_{10} 8721.3 < 4$

$\Rightarrow \quad [\log_{10} 8721.3] = 3$

6. (b) Let A be the area and θ (in radians) be the sector angel. Then,

$$A = \frac{1}{2} \times 60^2 \times \theta = 1800\,\theta \qquad \left[\because A = \frac{1}{2}r^2\theta\right]$$

$\Rightarrow \quad \dfrac{dA}{d\theta} = 1800$

Let $\Delta\theta$ be an error in θ and ΔA be the corresponding error in A. Then,

$$\Delta A = \frac{dA}{d\theta} \cdot \Delta\theta$$

$$\Delta A = 1800 \times \frac{\pi}{180°} \quad \left[\because 1° = \frac{\pi}{180°} \text{ radians}\right]$$

$$= 10\,\pi \text{ cm}^2$$

7. (a) When $x \to 0^-$, then

$[x^{2n+1}] = -1$ and $[x^{2n}] = 0$ for $n = 0, 1, 2 \dots$

$$\therefore \lim_{x \to 0^-} \frac{[x] + [x^2] + [x^3] + \dots\dots + [x^{2n+1}] + n + 1}{1 + [x^2] + |x| + 2x}$$

$$= \lim_{x \to 0^-} \frac{(-1) + 0 + (-1) + 0 + \dots + 0 + (-1) + n + 1}{1 + 0 - x + 2x}$$

$$= \lim_{x \to 0^-} \frac{0}{1 + x} = 0$$

8. (a) Given, $\displaystyle\int \sqrt{\sec x - 1}\, dx = \int \sqrt{\frac{1 - \cos x}{\cos x}}\, dx$

$$= \sqrt{2}\int \frac{\sin\frac{x}{2}}{\sqrt{2\cos^2\frac{x}{2} - 1}}\, dx$$

$$= -2\sqrt{2}\int \frac{dz}{\sqrt{2z^2 - 1}} = -2\int \frac{dz}{\sqrt{z^2 - \left(\frac{1}{\sqrt{2}}\right)^2}}$$

$$= -2\log\left[z + \sqrt{z^2 - \left(\frac{1}{\sqrt{2}}\right)^2}\right] + C$$

$$= -2\log\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C \qquad \left[\because z = \cos\frac{x}{2}\right]$$

9. (c) Let $\dfrac{3x-4}{3x+4}=t$

$$\Rightarrow \quad 3x-4=3xt+4t \Rightarrow x=\frac{4t+4}{3(1-t)}$$

$$\therefore \quad f(t)=\frac{4t+4}{3(1-t)}+2$$

$$\therefore \quad f(x)=\frac{4x+4}{3(1-x)}+2$$

$$=\frac{4(x-1)+8}{3(1-x)}+2$$

$$=2-\frac{4}{3}-\frac{8}{3(x-1)}$$

Hence, $\int f(x)\,dx=-\dfrac{8}{3}\ln|x-1|+\dfrac{2}{3}x+C$

10. (c) The word 'MEDITERRANEAN' has 2A, 3E, 1D, 1I, 1M, 2N, 2R, 1T.
Fixing E and R at the first and last places the rest can be arrange as :

(I) Both letters are of different kind i.e. ${}^8C_2$ ways.

number of words $={}^8C_2\times 2!=56$

(II) Both are of the same kind i.e., ${}^3C_2$ ways,

number of words $={}^3C_2\times\dfrac{2!}{2!}=3$

Therefore, total number of words $=56+3=59$.

11. (b) Given, $\dfrac{dy}{dx}=y\tan x-y^2\sec x$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$$

Putting $\dfrac{1}{y}=t \Rightarrow \dfrac{-1}{y^2}\dfrac{dy}{dx}=\dfrac{dt}{dx}$,

$$\therefore \quad \frac{dt}{dx}+\tan x\cdot t=\sec x$$

$\text{IF}=e^{\int \tan x\,dx}=e^{\log\sec x}=\sec x$

$\therefore$ The solution of the DE is

$$t\sec x=\int \sec^2 x\,dx+C$$

$$\Rightarrow \quad \sec x=y(C+\tan x)$$

12. (d) T_{r+1} in the expansion of $\left(\dfrac{3x^2}{2}-\dfrac{1}{3x}\right)^9$

$$={}^9C_r\left(\frac{3}{2}\right)^{9-r}x^{18-2r}(-1)^r\cdot\frac{1}{3^r}x^{-r}$$

$$={}^9C_r(-1)^r\left(\frac{3}{2}\right)^{9-r}\frac{1}{3^r}x^{18-3r}$$

Independent term = (coefficient of x^0 + coefficient of x^{-1} + 2 coefficient of x^{-4}) in the expansion of $\left(\dfrac{3}{2}x^2-\dfrac{1}{3x}\right)^9$

For coefficient of x°, $18-3r=0$

$$\Rightarrow \quad r=6$$

$\therefore$ coefficient of $x^0={}^9C_6\left(\dfrac{3}{2}\right)^3\cdot\dfrac{1}{3^6}=\dfrac{7}{18}$

For coefficient of x^{-1}, $18-3r=-1$

$$\Rightarrow \quad r=\frac{19}{3}$$

$\therefore$ coefficient of $x^{-1}=0$

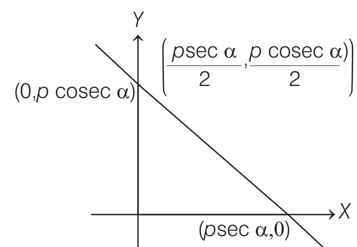
Similarly, coefficient of $x^{-4}=0$

Hence, term independent of $x=\dfrac{7}{18}$.

13. (d) If (h,k) is the mid-point, then

$$h=\frac{p}{2\cos\alpha},\ k=\frac{p}{2\sin\alpha}$$

So, $\left(\dfrac{p}{2h}\right)^2+\left(\dfrac{p}{2k}\right)^2=\cos^2\alpha+\sin^2\alpha=1$



$$\Rightarrow \quad \frac{1}{h^2}+\frac{1}{k^2}=\frac{4}{p^2}$$

$\therefore$ Locus of (h,k) is $\dfrac{1}{x^2}+\dfrac{1}{y^2}=\dfrac{4}{p^2}$

14. (b) Assume the equation of the hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

Given the line $2x+y=1$ touches the hyperbola

$$\therefore \quad (2)^2a^2-b^2=1$$

$$\Rightarrow \quad 4a^2-b^2=1$$

$$\Rightarrow \quad 4a^2+a^2(1-e^2)=1$$

$$\Rightarrow \quad 5a^2-a^2e^2=1 \qquad \ldots\text{(i)}$$

As the line passes through $\left(\dfrac{a}{e},0\right)$

$$\frac{2a}{e}+0=1 \Rightarrow a=\frac{e}{2} \qquad \ldots\text{(ii)}$$

Putting value of a in Eq. (i), we get

$$\frac{5e^2}{4} - \frac{e^4}{4} = 1$$

$\Rightarrow \quad e^4 - 5e^2 + 4 = 0$

$\Rightarrow \quad e^2 = 1 \text{ or } e^2 = 4$

$\Rightarrow \quad e = 2 \text{ as } e^2 \neq 1$

Hence, the required equation is

$$\frac{x^2}{1} - \frac{y^2}{3} = 1 \qquad \left[\because e + \sqrt{\frac{1-b^2}{a^2}}\right]$$

15. (c) $\Delta = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

$$= 10!\,11!\,12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 1 & 12 & 12\times 13 \\ 1 & 13 & 13\times 14 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 10!\,11!\,12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix}$$

$$= (10!\,11!\,12!)(50-48)$$
$$= 2(10!\,11!\,12!)$$

16. (a) We have,

$AB = A$ and $BA = B$

$\therefore \quad ABA = A^2 \Rightarrow A(BA) = A^2$

$\Rightarrow \quad AB = A^2 \Rightarrow A = A^2$

also, $\quad BA = B \Rightarrow BAB = B^2$

$\Rightarrow \quad BA = B^2 \Rightarrow B = B^2$

17. (d) Given, $f(x) = e^{x^3 - 3x + 2}$

Let $g(x) = x^3 - 3x + 2$

$\Rightarrow \quad g'(x) = 3x^2 - 3 = 3(x^2 - 1)$

$\Rightarrow \quad g'(x) \geq 0$ for $x \in (-\infty, -1]$

So, $g(x)$ is increasing function

$\therefore$ $f(x)$ is one-one.

Now, range of $f(x)$ is $(0, e^4]$, but co-domain is $(0, e^5]$

$\therefore$ $f(x)$ is into function.

18. (a) Given, $f(a) = 64a^3 + \frac{1}{a^3} = (4a)^3 + \frac{1}{a^3}$

$$= \left(4a + \frac{1}{a}\right)^3 - 3\cdot 4a \cdot \frac{1}{a}\left(4a + \frac{1}{a}\right)$$
$$= 3^3 - 12\cdot 3$$
$$= 27 - 36 = -9$$

$\left[\text{As } a, b \text{ are roots of } 4x + \frac{1}{x} = 3\right.$

$\left.\therefore\ 4a + \frac{1}{a} = 3\right]$

Similarly, $f(b) = -9$

$\therefore$ $f(a) = f(b) = -9$

19. (d) Let the components of line vector be a, b, c

$\therefore \quad a^2 + b^2 + c = (63)^2 \quad \ldots(i)$

also $\frac{a}{3} = \frac{b}{-2} = \frac{c}{6} = \lambda$ (say) then,

$a = 3\lambda,\ b = -2\lambda$ and $c = 6\lambda$

Putting values of a, b, c in Eq. (i)

$9\lambda^2 + 4\lambda^2 + 36\lambda^2 = (63)^2$

$49\lambda^2 = (63)^2$

$\Rightarrow \quad \lambda = \pm 9$

Since, $a = 3\lambda < 0$ as the given line makes an obtuse angle with the X-axis

$\therefore \quad \lambda = -9$

So, the components are $-27, 18, -54$.

20. (c) As, $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in GP, then

$\sin^2\beta = \sin\alpha\,\cos\alpha \quad \ldots(i)$

Given, $\quad x^2 + 2x\cot\beta + 1 = 0$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (2\cot\beta)^2 - 4$

$= 4(\text{cosec}^2\beta - 2)$

$= 4(\text{cosec}\,\alpha\sec\alpha - 2)$

[from Eq. (i)]

$= 4(2\,\text{cosec}\,2\alpha - 2) \geq 0$

Hence, roots are real.

21. (b) $\lim_{\alpha \to 0} \frac{f(1-\alpha) - f(1)}{\alpha^3 + 3\alpha}$

$$= \lim_{\alpha \to 0} \frac{(f)(1-\alpha) - f(1)}{-\{(1-\alpha)-1\}(\alpha^2+3)}$$

$$= -\lim_{\alpha \to 0} \frac{f(1-\alpha) - f(1)}{\{(1-\alpha)-1\}} \times \frac{1}{(\alpha^2+3)}$$

$$= -f'(1) \times \frac{1}{3} = -\frac{1}{3} f'(1)$$

Now, $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$\Rightarrow \quad f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$\Rightarrow \quad f'(1) = 30 - 56 + 30 - 63 + 6 = -53$

Hence, $\lim_{x \to 0} \frac{f(1-\alpha) - f(1)}{\alpha^3 + 3\alpha} = \frac{53}{3}$

22. (a) The equation of tangent at $P(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ is $\sqrt{3}x + y = 4$. Its slope is $-\sqrt{3}$

So, slope of a line perpendicular to PT is $\frac{1}{\sqrt{3}}$. The equation of tangents of slope $\frac{1}{\sqrt{3}}$ to the circle

$$(x-3)^2 + (y-0)^2 = 1^2 \text{ are}$$

$$y - 0 = \frac{1}{\sqrt{3}}(x-3) \pm \sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

or $\quad \sqrt{3}y = x - 3 \pm 2$

or $\quad x - \sqrt{3}y - 1 = 0$ and $x - \sqrt{3}y - 5 = 0$

23. (a) The total number of combinations which can be formed of 5 different green dyes, taking one or more $= 2^5 - 1 = 31$

and by taking one or more of 4 different red dyes

$$= 2^4 - 1 = 15$$

The number of combinations by taking three different red dyes, take none, one or more of them

$$= 2^3 = 8$$

Hence, required number of combinations

$$= 31 \times 15 \times 8 = 3720$$

24. (c) Given, $\quad \frac{x^2}{4} + \frac{y^2}{1} = 1 \quad \ldots(i)$

Its equation of tangent is

$$\frac{x}{2}\cos\theta + y\sin\theta = 1 \quad \ldots(ii)$$

The second ellipse can be rewritten as

$$\frac{x^2}{6} + \frac{y^2}{3} = 1 \quad \ldots(iii)$$

Suppose (ii) meets the ellipse (iii) at P and Q and the tangents at P and Q to the ellipse (iii) intersect at (h, k), then (ii) is the chord of contact of (h, k) with respect to ellipse (iii) so its eqution is

$$\frac{hx}{6} + \frac{ky}{3} = 1 \quad \ldots(iv)$$

As Eqs. (ii) and (iv) represent the same line

$$\frac{h/6}{\frac{(\cos\theta)}{2}} = \frac{\frac{k}{3}}{\sin\theta} = 1$$

$\Rightarrow \quad h = 3\cos\theta, k = 3\sin\theta$

$\therefore$ Locus of (h, k) is $x^2 + y^2 = 9$

25. (d) Given, $\frac{dy}{dx} + 1 = e^{x+y} \quad \ldots(i)$

Put $\quad x + y = t$

Differentiating w.r.t. x, we get

$$1 + \frac{dy}{dx} = \frac{dt}{dx} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{dt}{dx} = e^t$$

$\Rightarrow \quad e^{-t}dt = dx$

$\Rightarrow \quad -e^{-t} = x + c$

$\Rightarrow \quad x + e^{-t} = c$

$\Rightarrow \quad x + e^{-(x+y)} = c$

26. (b) Given, $49^n + 16n - 1 = (1+48)^n + 16n - 1$

$= 1 + {}^nC_1(48) + {}^nC_2(48)^2 + \ldots + {}^nC_n(48)^n + 16n - 1$

$= (48n + 16n) + {}^nC_2(48)^2 + {}^nC_3(48)^3 + \ldots + {}^nC_n(48)^n$

$= 64n + 8^2[{}^nC_2 6^2 + {}^nC_3 \cdot 6^3 \cdot 8 + {}^nC_4 \cdot 6^4 \cdot 8^2 + \ldots + {}^nC_n \cdot 6^n \cdot 8^{n-2}]$

$\therefore$ $49^n + 16n - 1$ is divisible by 64.

27. (d) $\lim_{x \to -\infty}\left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3}\right] = \lim_{y \to \infty}\frac{-y^4 \sin\frac{1}{y} + y^2}{1 + y^3}$

[putting $x = -y$: as $x \to -\infty$, $y \to \infty$]

$$= \lim_{y \to \infty}\frac{-\left(\frac{\sin\frac{1}{y}}{\frac{1}{y}}\right) + \frac{1}{y}}{1 + \frac{1}{y^3}} = \frac{-1+0}{1+0} = -1$$

28. (a) Mid-point of line joining points $(4, -5)$ and $(-2, 9)$ is $\left(\frac{4-2}{2}, \frac{-5+9}{2}\right)$ or $(1, 2)$

$\therefore$ Inclination is $m = \frac{2-6}{1+3} = \frac{-4}{4} = -1$

$\Rightarrow \quad \tan\theta = -1 \Rightarrow \theta = \frac{3\pi}{4}$

29. (a) Period of

$$\sin\left(\frac{\pi x}{n!}\right) = \frac{2\pi}{\frac{\pi}{n!}} = 2n!$$

Period of $\cos\left(\frac{\pi x}{(n+1)!}\right) = \frac{2\pi}{\frac{\pi}{(n+1)!}}$

$$= 2 \times (n+1)!$$

$\therefore \quad$ Period of $f(x) = \text{LCM of } \{2n!, (2n+1)!\}$

$$= 2(n+1)!$$

30. (c) Given, the distance $\left(\frac{2}{m}, 2\right)$ between $\left(\frac{6}{m}, 6\right)$ is less than 5.

$$\Rightarrow \left(\frac{2}{m} - \frac{6}{m}\right)^2 + (2-6)^2 < 25$$

$$\Rightarrow \frac{16}{m^2} < 9$$

$$\Rightarrow m^2 > \frac{16}{9}$$

$$\Rightarrow m > \frac{4}{3} \text{ or } m < -\frac{4}{3}$$

31. (d) Let $A = \lim_{n \to \infty} \frac{(n!)^{1/n}}{n}$

Taking log on both sides, we get

$$\log A = \lim_{n\to\infty} \log\left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\ldots\frac{n}{n}\right)^{1/n}$$

$$= \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \log\left(\frac{r}{n}\right)$$

$$= \int_0^1 \log x \, dx = [x\log x - x]_0^1$$

$$\Rightarrow \log A = -1$$

$$\Rightarrow A = e^{-1}$$

32. (b) Using $C_1 \to C_1 - C_2 - C_3$, we get

$$\Delta(x) = \begin{vmatrix} 0 & \cos x & 1-\cos x \\ 0 & \cos x & 1+\sin x - \cos x \\ -1 & \sin x & 1 \end{vmatrix}$$

$$= (-1)\begin{vmatrix} \cos x & 1-\cos x \\ \cos x & 1+\sin x-\cos x \end{vmatrix}$$

$$= -\cos x \sin x = \frac{-1}{2}(\sin 2x)$$

$$\therefore \int_0^{\frac{\pi}{2}} \Delta(x)\,dx = -\frac{1}{2}\left(-\frac{1}{2}\right)[\cos 2x]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}(\cos\pi - \cos 0) = -\frac{1}{2}$$

33. (b) Let the third vertex be (a, b),

Then $\quad 9a + 7b = 28 \quad \ldots(\text{i})$

If the coordinates of centroid are (x, y), then

$$x = \frac{a+4+6}{3},\ y = \frac{b+7+1}{3}$$

$$\Rightarrow a = 3x - 10,\ b = 3y - 8$$

Putting in Eq. (i) we get,

$$9(3x-10) + 7(3y-8) = 28$$

$$\Rightarrow 9x + 7y - 58 = 0$$

34. (d) $\lim_{n\to\infty} {}^nC_x \left(\frac{m}{n}\right)^x \left(1 - \frac{m}{n}\right)^{n-x}$

$$= \lim_{n\to\infty} \frac{n!}{x!\ (n-x)!}\frac{m^x}{n^x}\cdot\frac{\left(1-\frac{m}{n}\right)^n}{\left(1-\frac{m}{n}\right)^x}$$

$$= \lim_{n\to\infty} \frac{m^x}{x!}\frac{\left(1-\frac{m}{n}\right)^n}{n^x}\left[\frac{n!}{(n-x)!\left(1-\frac{m}{n}\right)^x}\right]$$

$$= \lim_{n\to\infty} \frac{m^x}{x!}\left(1-\frac{m}{n}\right)^n \cdot \frac{n(n-1)(n-2)\ldots(n-x+1)}{n^x\left(1-\frac{m}{n}\right)^x}$$

$$= \lim_{n\to\infty} \frac{m^x}{x!}\left(1-\frac{m}{n}\right)^n \cdot \frac{1\cdot\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1+\frac{1-x}{n}\right)}{\left(1-\frac{m}{n}\right)^x}$$

$$\Rightarrow \frac{m^x}{x!}\frac{e^{-m}\cdot 1}{1^x} = \frac{m^x\cdot e^{-m}}{x!}$$

35. (d) Given, $\quad D_f = (0, 1) \Rightarrow 0 < x < 1$

Now, $\quad h(x) = f(e^x) + f(\ln|x|)$

$\therefore \quad 0 < e^x < 1$

and $\quad 0 < \ln|x| < 1 \Rightarrow -\infty < x < 0$

and $\quad 1 < |x| < e$

$x \in (-\infty, 0) \cup (1, \infty)$ and $x \in (-e, -1) \cup (1, e)$

$\therefore \quad x \in (-e, -1)$

36. (d) ${}^nC_n\left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$

$$\therefore (-1)^n\cdot\left(\frac{1}{2}\right)^{\frac{n}{2}} = \left(\frac{1}{3^{5/3}}\right)^{\log_3 8} = 3^{-5/3\cdot 3\log_3 2}$$

$$= 3^{\log_3 2^{-5}} = 2^{-5} = \left(\frac{1}{2}\right)^5$$

$\therefore \quad n = 10$

The fifth term

$${}^{10}C_4\cdot(2^{1/3})^6\cdot\left(-\frac{1}{\sqrt{2}}\right)^4 = {}^{10}C_4 = {}^{10}C_6$$

37. (a) Applying, $C_1 \to C_1 - C_3$, we get

$$\begin{vmatrix} x-5 & 2 & 5 \\ 0 & x & 3 \\ 5-x & 4 & x \end{vmatrix} = 0$$

$$\Rightarrow \quad (x-5)\begin{vmatrix} 1 & 2 & 5 \\ 0 & x & 3 \\ -1 & 4 & x \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_3$, we get

$$\Rightarrow \quad (x-5)\begin{vmatrix} 0 & 6 & 5+x \\ 0 & x & 3 \\ -1 & 4 & x \end{vmatrix} = 0$$

$$\Rightarrow \quad (x-5)[18 - x(5+x)] = 0$$

$$\Rightarrow \quad x^2 + 5x - 18 = 0$$

Sum of roots $= -\frac{5}{1} = -5$

38. (a) The lines are coplanar if

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1$ and $C_3 \to C_3 + C_1$

$$\Rightarrow \quad \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & k+1 \end{vmatrix} = 0$$

$$\Rightarrow 2(k+1) - (1-k)(k+2) = 0 \Rightarrow k = 0 \text{ or } -3$$

39. (b) Normal at L : $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$$\Rightarrow \quad \frac{a^2x}{ac} - \frac{b^2y}{b^2/a} = a^2 - b^2$$

Since, $(0, -b)$ lies on line

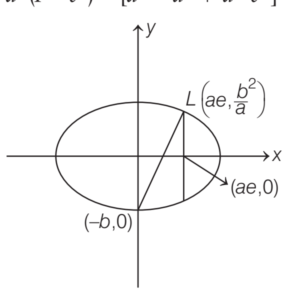
$$\Rightarrow \quad 0 + ab = a^2 - b^2$$

$$\Rightarrow \quad (ab)^2 = (a^2 - b^2)^2$$

$$\because \quad e^2 = 1 - \frac{b^2}{a^2}$$

$$\therefore \quad a^2 \cdot a^2(1 - e^2) = [a^2 - a^2(1 - e^2)]^2$$

$$\Rightarrow \quad a^4(1 - e^2) = [a^2 - a^2 + a^2e^2]^2$$



$$\Rightarrow \quad a^4(1 - e^2) = a^4 e^4$$

$$\Rightarrow \quad 1 - e^2 = e^4$$

$$\Rightarrow \quad e^2 + e^4 = 1$$

40. (a) The family of lines

$$(x + y - 1) + \lambda(2x + 3y - 5) = 0$$

passes through a point such that

$$x + y - 1 = 0$$

$$2x + 3y - 5 = 0$$

i.e. $(-2, 3)$ and family of lines

$(3x + 2y - 4) + \mu(x + 2y - 6) = 0$

Passes through a point such that

$$3x + 2y - 4 = 0$$

and $\quad x + 2y - 6 = 0$

i.e. $\left(-1, \frac{7}{2}\right)$

$\therefore$ Equation of the required straight line is

$$y - 3 = \frac{\frac{7}{2} - 3}{-1 + 2}(x + 2)$$

$$\Rightarrow \quad y - 3 = \frac{x+2}{2}$$

$$\Rightarrow \quad x - 2y + 8 = 0$$

41. (d) As u, v, w, z are all positive real numbers

$$\therefore \quad m > 0$$

Now, AM $\geq$ GM

$$\Rightarrow \quad \frac{(u + v) + (w + z)}{2} \geq \sqrt{(u + v)(w + z)}$$

$$\Rightarrow \quad \frac{2}{2} \geq \sqrt{m}$$

or $\quad m \leq 1$

Hence, we get

$$0 < m \leq 1$$

42. (a) $I = \int (\cos x)^{-2005} \operatorname{cosec}^2 x \, dx - 2005 \int \frac{dx}{\cos^{2005} x}$

$(\cos x)^{-2005}(-\cot x) - \int [(-2005)(\cos x)^{-2006}$

$[(-\sin x)(-\cot x)] dx - 2005 \int \frac{dx}{\cos^{2005} x}$

$= (\cos x)^{-2005}(-\cot x) + 2005 \int \frac{dx}{\cos^{2005} x}$

$-2005 \int \frac{dx}{\cos^{2005} x}$

$$I = -\frac{\cot x}{(\cos x)^{2005}} + C$$

43. (c) For function to be defined,

we must have $-1 \leq 2 - 4x^2 < 2$

$\therefore \quad 2-4x^2<2 \Rightarrow x \in R-\{0\}$

and $\quad 2-4x^2 \geq -1 \Rightarrow x^2 \leq \frac{3}{4}$

$\Rightarrow \quad x \in \left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]-\{0\}$

44. (b) Given, $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$

$\Rightarrow \quad \frac{(x-a)^2-b^2}{b(x-a)}+\frac{(x-b)^2-a^2}{a(x-b)}=0$

$\Rightarrow \quad (x-a-b)\left\{\frac{x-a+b}{b(x-a)}+\frac{x-b+a}{a(x-b)}\right\}=0$

$\Rightarrow (x-a-b)\{a(x-a)(x-b)+ab(x-b)$
$\quad +b(x-a)(x-b)+ab(x-a)\}=0$

$\Rightarrow x(x-a-b)\{a(x-a)+b(x-b)\}=0$

$\Rightarrow \quad x=0, a+b, \frac{a^2+b^2}{a+b}$

So, $\quad x_1=a+b, x_2=\frac{a^2+b^2}{a+b}, x_3=0$

Now, $x_1-x_2-x_3=c \Rightarrow c=\frac{2ab}{a+b}$

$\Rightarrow a, c, b$ are in HP

45. (c) Given, $y=x^2$ and the line $y=2x-4$

On solving both Eqs. we get

$$x^2-2x+4=0$$

Let $\quad a=x^2-2x+4$

$\therefore \quad a'=2x-2$

For least value, $a'=0$

$\Rightarrow \quad 2x-2=0 \Rightarrow x=1$

Also a' is positive at $x=1$

$\therefore$ It is minimum, putting $x=1$ in the original equation of parabola, we get $y=1$

$\therefore$ The required point are $(1, 1)$

46. (a) Given, $\quad \frac{x^2}{4}+\frac{y^2}{\frac{7}{4}}=1$

Here, $\quad a^2=4, b^2=\frac{7}{4}$

$\therefore \quad b^2=a^2(1-e^2)$

$\Rightarrow \quad \frac{7}{4}=4(1-e^2) \Rightarrow e=\frac{3}{4}$

$\therefore \quad$ Foci $\left(\pm\frac{3}{2}, 0\right)$

Radius $=\sqrt{\left(\frac{3}{2}-\frac{1}{2}\right)^2+(2-0)^2}$

$=\sqrt{5}$

47. (c) Putting $1+e^x=v^2$

The integral reduces to $2\int \log(v^2-1)dv$

$=2\left[v\log(v^2-1)-2\int\left(1+\frac{1}{v^2-1}\right)dv\right]$

$=2\left[v\log(v^2-1)-2\left(v+\frac{1}{2}\log\frac{v-1}{v+1}\right)\right]+C$

$=2x\sqrt{1+e^x}-4\sqrt{1+e^x}-2\log\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}+C$

Hence, $f(x)=2(x-2)$, $g(x)=\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$

48. (c) $\lim\limits_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} \quad \left(\frac{0}{0}\text{ form}\right)$

$=\lim\limits_{x\to\alpha}\frac{(2ax+b)\sin(ax^2+bx+c)}{2(x-\alpha)}$

(using L' Hospital Rule)

$=\lim\limits_{x\to\alpha}\frac{(2ax+b)\sin[a(x-\alpha)(x-\beta)]}{2(x-\alpha)}$

$=\lim\limits_{x\to\alpha}\frac{(2ax+b)\sin[a(x-\alpha)(x-\beta)]}{2a(x-\alpha)(x-\beta)}a(x-\beta)$

$=\frac{a^2}{2}\left(2\alpha+\frac{b}{a}\right)(\alpha-\beta)=\frac{a^2}{2}(2\alpha-\alpha-\beta)(\alpha-\beta)$

$=\frac{a^2(\alpha-\beta)^2}{2}$

49. (b) Given, the graph is symmetric about $x=k$.

then $f(k-x)=f(k+x)$

$\Rightarrow a(k-x)^3+b(k-x)^2+c(k-x)+d$
$=a(k+x)^3+b(k+x)+c(k+x)+d$

$\Rightarrow 2ax^3-(6ak^2+4bk+2c)x=0$

Which is true for all x if $a=0$

and $6ak^2+4bk+2c=0$

i.e. $a=0$ and $k=-\frac{c}{2b} \Rightarrow a+k=\frac{-c}{2b}$

50. (c) The centroid of the equilateral triangle is the centre of its circumcentre and its radius is the distance of any vertex from the centroid.

Distance of centroid from vertex

$=\frac{2}{3}(\text{Median})=\frac{2}{3}\times 6a=4a$

Hence, equation of the circle is

$$(x-0)^2+(y-0)^2=(4a)^2$$
$$\Rightarrow \quad x^2+y^2=16a^2$$

51. (c) Given,
$$0 \le |a+b+c+d|^2 = |a|^2+|b|^2+|c|^2+|d|^2 + 2a\cdot b+b\cdot c+c\cdot a+a\cdot d+c\cdot d+b\cdot d$$
$$=4+2a\cdot b+b\cdot c+c\cdot a+a\cdot d+c\cdot d+b\cdot d$$
So, $a\cdot b+b\cdot c+c\cdot a+a\cdot d+c\cdot d+b\cdot d \ge -2$

Now, $|a-b|^2+|b-c|^2+|c\cdot d|^2 + |d-a|^2+|c-a|^2+|b-d|^2$
$$=2(|a|^2|+|b|^2+|c|^2+|d|^2 - (a\cdot b+c\cdot d+d\cdot a+a\cdot c+b\cdot d+b\cdot c))$$
$\le 2|4+2| \le 12$

52. (b) Since, $|z-1|=1$,

$\therefore$ Let $z-1=\cos\theta+i\sin\theta$

Then $z-2=\cos\theta+i\sin\theta-1$
$$=-2\sin^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$
$$=2i\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right) \quad \ldots(\text{i})$$
and $\quad z=1+\cos\theta+i\sin\theta$
$$=2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$
$$=2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right) \quad \ldots(\text{ii})$$
From Eqs. (i) and (ii), we get
$$\frac{z-2}{2}=i\tan\frac{\theta}{2}=i\tan(arg\, z) \quad \left(\because arg\, z=\frac{\theta}{2}\right)$$

53. (b) Let $t_m=t_n$
$$\Rightarrow \quad 1+(m-1)10=31+(n-1)5$$
$$\Rightarrow \quad 10m-9=5n+26$$
$$\Rightarrow \quad 10m=5(n+7)$$
$$\Rightarrow \quad m=\frac{n+7}{2}=\lambda$$
$\therefore$ $m=\lambda$ and $n=2\lambda-7$, where $m\le 100$ and $n\le 100$

$\therefore \quad \lambda \le 100$ and $\lambda \le 53\frac{1}{2} \Rightarrow 1 \le \lambda \le 53$

$\Rightarrow$ Number of common terms = 53

Where largest common term is,
$$t_{53}=1+52(10)=521$$

54. (b) Given, $\quad Q=PAP^T$
$$\Rightarrow \quad P^TQ=AP^T \quad (\because \text{as } P^TP=I=PP^T)$$
$$\therefore \quad P^TQ^{2005}P=AP^TQ^{2004}P$$
$$=A^2P^TQ^{2003}P=A^3P^TQ^{2002}P$$
$$=A^{2004}P^T(QP)$$
$$=A^{2004}P^T(PA)$$
$$(Q=PAP^T \Rightarrow QP=PA)=A^{2005}=\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$$

55. (c) $f(x)=y \Rightarrow 2^{x(x-1)}=y \Rightarrow x(x-1)\log_2 2=\log_2 y$
$$\Rightarrow x(x-1)=\log_2 y = x^2-x-\log_2 y=0$$
$$\Rightarrow \quad x=\frac{1\pm\sqrt{1+4\log_2 y}}{2}$$
$$\therefore \quad x=\frac{1+\sqrt{1+4\log_2 y}}{2}$$
$$\therefore \quad f^{-1}(x)=\frac{1}{2}(1+\sqrt{1+4\log_2 x})$$

56. (a) Given, $x^2+y^2+2x-4y+6=0$

Putting $x=x'+\alpha$, $y=y'+\beta$, we get
$$x'^2+y'^2+x'(2\alpha+2)+y'(2\beta-4) + (\alpha^2+\beta^2+2\alpha-4\beta+6)=0$$
To eliminate linear terms, we must have
$$2\alpha+2=0 \text{ and } 2\beta-4=0$$
$$\Rightarrow \quad \alpha=-1, \beta=2$$
$$\therefore \quad (\alpha,\beta)\equiv(-1,2)$$

57. (a) $t_r=\frac{r}{r^4+r^2+1}=\frac{r}{(r^2+1)^2-r^2}$
$$=\frac{1}{2}\left[\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right]$$
$$=\frac{1}{2}\left[\frac{1}{r(r-1)+1}-\frac{1}{(r+1)r+1}\right]$$
$$\therefore \quad \sum_{r=1}^{n}t_r=\sum_{r=1}^{n}\frac{1}{2}[f(r)-f(r+1)]$$
where, $f(r)=\frac{1}{r(r-1)+1}=\frac{1}{2}[f(1)-f(n+1)]$
$$=\frac{1}{2}\left[1-\frac{1}{(n+1)n+1}\right]\to\frac{1}{2} \text{ as } n\to\infty$$

58. (d) The general equation of any circle is written as
$$x^2+y^2+2gx+2fy+c=0$$
For intercept made by the circle on X-axis, put $y=0$

$\therefore$ we get, $\quad x^2+2gx+c=0$

If x_1 and x_2 are roots of this equation,

Then length of the intercept is
$$|x_1-x_2|=\sqrt{(x_1+x_2)^2-4x_1x_2}=2\sqrt{g^2-c}$$
Similarly, $|y_1-y_2|=2\sqrt{f^2-c}$

Since, the lengths of these intercepts are equal

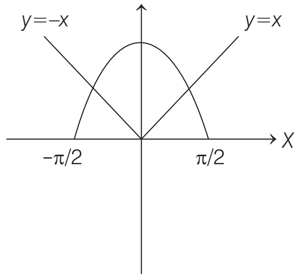
$\therefore \quad \sqrt{g^2 - c} = \sqrt{f^2 - c}$

$\Rightarrow \quad g^2 = f^2 = (-g)^2 = (-f)^2$

Therefore, centre lies on $x^2 - y^2 = 0$

59. (b) Given $|x| = \cos x$

Thus, for finding the value of x for which both curves have point of intersections



Clearly, there are two point of intersection of $y = |x|$ and $y = \cos x$.

Hence, there are two real solutions.

60. (d) Consider the function,

$$f(x) = \frac{x^2}{(x^3 + 200)}$$

$$f'(x) = \frac{x(400 - x^3)}{(x^3 + 200)^2} = 0$$

when, $\quad x = (400)^{1/3} \quad (\because x \neq 0)$

$$x = (400)^{1/3} - h \Rightarrow f'(x) > 0$$

$$x = (400)^{1/3} + h \Rightarrow f'(x) < 0$$

$f(x)$ has maxima at $x = (400)^{1/3}$

Since, $7 < (400)^{1/3} < 8$, either a_7 or a_8 is the greatest term.

61. (c) As a, b, c are non-coplanar, $b \times c$, $c \times a$, $a \times b$ are also non-coplanar.

So, any vector can be expressed as a linear combination of these vectors.

Let $\mathbf{a} = \lambda \mathbf{b} \times \mathbf{c} + \mu \mathbf{c} \times \mathbf{a} + \gamma \mathbf{a} \times \mathbf{b}$

$\therefore \mathbf{a} \cdot \mathbf{a} = \lambda [\mathbf{b}\ \mathbf{c}\ \mathbf{a}]$, $\mathbf{a} \cdot \mathbf{b} = \mu [\mathbf{c}\ \mathbf{a}\ \mathbf{b}]$,

$\mathbf{a} \cdot \mathbf{c} = \gamma [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

$$\therefore \quad \mathbf{a} = \frac{(\mathbf{a} \cdot \mathbf{a})\, \mathbf{b} \times \mathbf{c}}{[\mathbf{b}\ \mathbf{c}\ \mathbf{a}]} + \frac{(\mathbf{a} \cdot \mathbf{b})\, \mathbf{c} \times \mathbf{a}}{[\mathbf{c}\ \mathbf{a}\ \mathbf{b}]} + \frac{(\mathbf{a} \cdot \mathbf{c})\, \mathbf{a} \times \mathbf{b}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$$

$\therefore (\mathbf{a} \cdot \mathbf{a})\, \mathbf{b} \times \mathbf{c} + (\mathbf{a} \cdot \mathbf{b})\, \mathbf{c} \times \mathbf{a} + (\mathbf{a} \cdot \mathbf{c})\, \mathbf{a} \times \mathbf{b} = [\mathbf{b}\ \mathbf{c}\ \mathbf{a}]\, \mathbf{a}$

62. (a) $\because \left[x^2 - \frac{1}{2}\right] = \left[x^2 + \frac{1}{2} - 1\right] = \left[x^2 + \frac{1}{2}\right] - 1$

$$\therefore \quad f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left(\left[x^2 + \frac{1}{2}\right] - 1\right)$$

$$\because \quad \left(x^2 + \frac{1}{2}\right) \geq \frac{1}{2}$$

$$\therefore \quad \left[x^2 + \frac{1}{2}\right] = 0 \text{ or } 1$$

If $\left[x^2 + \frac{1}{2}\right] = 0$

Then, $f(x) = \sin^{-1}(0) + \cos^{-1}(0 - 1) = 0 + \pi = \pi$

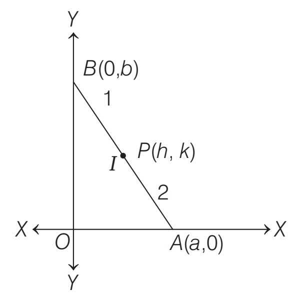
and for $\left[x^2 + \frac{1}{2}\right] = 1$

$$f(x) = \sin^{-1} 1 + \cos^{-1}(1 - 1)$$

$$= \sin^{-1} 1 + \cos^{-1} 0 = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Therefore, range of $f(x)$ is $\{\pi\}$.

63. (b)



From figure, $h = \frac{1(a) + 2 \cdot 0}{2 + 1}$ and $k = \frac{1 \cdot 0 + 2 \cdot b}{2 + 1}$

$$\Rightarrow \quad h = \frac{a}{3} \text{ and } k = \frac{2b}{3}$$

$$\Rightarrow \quad a = 3h \text{ and } b = \frac{3k}{2}$$

$$\therefore \quad 9h^2 + \frac{9k^2}{4} = l^2 \Rightarrow 36h^2 + 9k^2 = 4l^2$$

Hence, locus of the point is $36x^2 + 9y^2 = 4l^2$

64. (c) $\lim_{x \to \infty} \left[\frac{1^{\frac{1}{x}} + 2^{\frac{1}{x}} + 3^{\frac{1}{x}} + \ldots\ldots n^{\frac{1}{x}}}{n}\right]^{nx}$

$$= \lim_{y \to 0} \left[\frac{1^y + 2^y + 3^y + \ldots\ldots n^y}{n}\right]^{n/y}$$

$$= e^{\lim_{y \to 0} \frac{n}{y}\left[\frac{1^y + 2^y + 3^y + \ldots\ldots n^y}{n} - 1\right]}$$

$$= e^{\lim_{y \to 0}\left[\frac{1^y + 2^y + 3^y + \ldots\ldots n^y - n}{y}\right]}$$

$$= e^{\lim_{y\to 0}\left[\frac{(1^y-1)}{y}+\frac{(2^y-1)}{y}+\frac{(3^y-1)}{y}+\dots+\frac{(n^y-1)}{y}\right]}$$

$$= e^{(\log 1+\log 2+\log 3+\dots+\log n)}$$

$$= e^{\log(1\cdot 2\cdot 3\dots n)} = n!$$

65. (a) Given, $17^{1995}+11^{1995}-7^{1995}$

$$= (7+10)^{1995}+(1+10)^{1995}-7^{1995}$$

$$= [7^{1995}+{}^{1995}C_1\cdot 7^{1994}\cdot 10^1+{}^{1995}C_2\cdot 7^{1993}\cdot 10^2+\dots+{}^{1995}C_{1995}\cdot 10^{1995}]$$
$$+[{}^{1995}C_0+{}^{1995}C_1\cdot 10^1+{}^{1995}C_2\cdot 10^2+\dots+{}^{1995}C_{1995}\cdot 10^{1995}]-7^{1995}$$

$$= [{}^{1995}C_1\cdot 7^{1994}\cdot 10^1+\dots+10^{1995}]+[{}^{1995}C_1\cdot 10^1+\dots+{}^{1995}C_{1995}\cdot 10^{1995}]+1$$

$= [\text{multiple of } 10]+1$

Thus, the digit at units place is 1.

66. (b,c,d) We have,

$$f(x)=\frac{\sin(x+a)}{\sin(x+b)}$$

$$\Rightarrow\quad f'(x)=\frac{\sin(x+b)\cos(x+a)-\sin(x+a)\cos(x+b)}{\sin^2(x+b)}$$

$$=\frac{\sin(b-a)}{\sin^2(x+b)}$$

if $\sin(b-a)=0$

$\Rightarrow\quad f'(x)=0$

$\Rightarrow f(x)=$ constant function

Now, $\quad b-a=n\pi$ or $b-a=(2n+1)\pi$

or $b-a=2n\pi$, $n\in I$ then $f(x)$ has no maxima or minima.

67. (b,c) We have, $f(xy)=f(x)+f(y)$

Now, $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f\left(x\left(1+\frac{h}{x}\right)\right)-f(x\cdot 1)}{h}$$

$$=\lim_{h\to 0}\frac{\left(f(x)+f\left(1+\frac{h}{x}\right)\right)-(f(x)+f(1))}{h}$$

$$=\lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)-f(1)}{\left(1+\frac{h}{x}\right)-1}\cdot\frac{1}{x}=f'(1)\cdot\frac{1}{x}=\frac{2}{x}$$

$\Rightarrow\quad f(x)=2\log|x|+c$

$\Rightarrow\quad f(1)=c$

$\Rightarrow\quad c=0 \qquad (\because f(1)=0)$

$\therefore\quad f(x)=2\log|x|+c$

$\Rightarrow\quad f(x)=2\log|x|$

$\therefore\quad A=\int_1^2 2\log|x|\,dx$

$$=2\int_1^2\log x\,dx=2(x\log x-x)_1^2$$

$$=2[(2\log 2-2)-(0-1)]$$

$$=2(2\log 2-1)$$

$$=4\log 2-2$$

$$=4\left(\log 2-\frac{1}{2}\right)$$

$$=4(\log 2-\log\sqrt{e})$$

$$=4\log\left(\frac{2}{\sqrt{e}}\right)$$

68. (a,b,d) $A^2=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}=\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}$

We have, $A^2-4A-5I_3$

$$\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}-4\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}-5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=0$$

$\Rightarrow\quad 5I_3=A^2-4A=A(A-4I_3)$

$\Rightarrow\quad I_3=A\left[\frac{1}{5}(A-4I_3)\right]$

$\Rightarrow\quad A^{-1}=\frac{1}{5}(A-4I_3)$

Since, $|A|=5$

$\therefore\quad |A^3|=|A|^3=5^3\neq 0\Rightarrow A^3$ is invertible

Similarly, A^2 is invertible.

69. (c, d) Let $\quad b=a+p$

$\quad c=a+2p$

and $\quad d=a+3p$

then $$\frac{\frac{1}{a}+\frac{1}{d}}{\frac{1}{b}+\frac{1}{c}}=\frac{\frac{1}{a}+\frac{1}{a+3p}}{\frac{1}{a+p}+\frac{1}{a+2p}}$$

$$=\frac{(a+p)(a+2p)}{a(a+3p)}$$

$$= \frac{a^2 + 3ap + 2p^2}{a^2 + 3ap} > 1$$

$$\Rightarrow \quad \frac{1}{a} + \frac{1}{d} > \frac{1}{b} + \frac{1}{c}$$

Now, $\left(\frac{1}{b} + \frac{1}{c}\right)(a + d) = \left(\frac{1}{a+p} + \frac{1}{a+2p}\right)(a + a + 3p)$

$$= \frac{(2a + 3p)^2}{a^2 + 3ap + 2p^2}$$

$$= 4 + \frac{p^2}{a^2 + 3ap + 2p^2} > 4$$

70. (a,c,d) We have, $z^2 + az + a^2 = 0$

$\Rightarrow z = a\omega, a\omega^2$(where, '$\omega$' is a not real root of cube unity)

$\Rightarrow$ locus of z is a pair of straight lines and

$$\arg(z) = arg(a) + qrg(\omega)$$

or $\quad arg(a) + arg(\omega^2)$

$$\Rightarrow \quad arg(z) = \pm \frac{2\pi}{3}$$

Also, $\quad |z| = |a||\omega| \text{or } |a||\omega^2|$

$$\Rightarrow \quad |z| = |a|$$

71. (b,d) We have,

$$\sin\beta = \sqrt{\sin\alpha \cos\beta}$$

$$\Rightarrow \quad \sin^2\beta = \sin\alpha \cos\alpha \quad \ldots(\text{i})$$

Now, $\quad \cos 2\beta = 1 - 2\sin^2\beta$

$$= 1 - 2\sin\alpha \cos\alpha \quad (\text{using Eq. (i)})$$

$$= (\sin\alpha - \cos\alpha)^2$$

$$= 2\sin^2\left(\frac{\pi}{4} - \alpha\right)$$

or $2\cos^2\left(\frac{\pi}{4} + \alpha\right)$

72. (a,b,c,d) Equation of the line is

$$\frac{x-3}{\cos\theta} = \frac{y-4}{\sin\theta} = r \quad \ldots(\text{i})$$

Where r is the distance of any point on the line from P.

$\therefore$ Coordinates of any point on the line is

$$(3 + r\cos\theta, 4 + r\sin\theta) \quad \ldots(\text{ii})$$

If Eq. (ii) represents R, then

$$3 + r\cos\theta = 6$$

$$\Rightarrow \quad r = \frac{3}{\cos\theta} = PR$$

If Eq. (ii) represents S,

then $\quad 4 + r\sin\theta = 8$

$$\Rightarrow \quad r = \frac{4}{\sin\theta} = PS$$

Hence, $PR = 3\sec\theta$, $PS = 4 \operatorname{cosec}\theta$

$$\therefore PR + PS = \frac{3\sin\theta + 4\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{2(3\sin\theta + 4\cos\theta)}{\sin 2\theta}$$

and $\left(\frac{3}{PR}\right)^2 + \left(\frac{4}{PS}\right)^2 = \cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \quad \frac{q}{(PR)^2} + \frac{16}{(PS)^2} = 1$$

73. (a,b,c,d) Let

$$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$

$$S_1 \equiv x^2 + y^2 - 4 = 0$$

$$S_2 \equiv x^2 + y^2 - 6x - 8y + 10 = 0$$

$$S_3 \equiv x^2 + y^2 + 2x - 4y - 2 = 0$$

$\therefore$ Common chords are

$$S - S_1 \equiv 2gx + 2fy + c + 4 = 0 \quad \ldots(\text{i})$$

$$S - S_2 \equiv (2g + 6)x + (2f + 8)y + c - 10 = 0 \quad \ldots(\text{ii})$$

$$S - S_3 \equiv (2g - 2)x + (2f + 4)y + c + 2 = 0 \quad \ldots(\text{iii})$$

For cutting the extremities of diameter, chords (i), (ii) and (iii) pass through the centres of S_1, S_2 and S_3 respectively

then

$$c + 4 = 0, (2g + 6)3 + (2f + 8)4 + c - 10 = 0$$

and $(2g - 2)(-1) + (2f + 4)(2) + c + 2 = 0$

after solving we get,

$$c = -4, g = -2, f = -3$$

Now, $\quad gf = 6$

$$g + f = -2 - 3 = -5$$

$$= c - 1 = -4 - 1 = -5$$

and $(-2)^2 + (-3)^2 + 4 = 17$

74. (c) $f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} + \frac{1}{x^2 - 36}$

$\Rightarrow f(x) = f_1 + f_2$

For domain of f_1

$$f_1 = \frac{1}{x^2 - 36}$$

$$x^2 - 36 \neq 0$$

$$\Rightarrow \quad x \neq \pm 6$$

For domain of f_2

$$f_2 = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$$

It is defined, if

$$\log_{0.4}\left(\frac{x-1}{x+5}\right) \geq 0 = \log_{0.4}(1)$$

and $\quad \dfrac{x-1}{x+5} > 0$

$\Rightarrow \quad 0 < \dfrac{x-1}{x+5} \leq 1$

i.e., $\quad \dfrac{x-1}{x+5} > 0$

$\Rightarrow \quad x \in (-\infty, -5) \cup (1, \infty)$

and $\quad \dfrac{x-1}{x+5} - 1 \leq 0$

$\Rightarrow \quad \dfrac{-6}{x+5} \leq 0 \Rightarrow \dfrac{6}{x+5} \geq 0$

$\Rightarrow \quad x + 5 \geq 0 \Rightarrow x \geq -5$

$\therefore$ Domain of $f_2 = (1, \infty)$

Hence, domain of $f(x) = (1, \infty) - \{6, -6\}$

75. (b, c) We have,

$$f(1) = f(-1) = 1$$

Let $x > 1$, then $0 < \dfrac{1}{x^2} < 1$

$\Rightarrow \quad \left[\dfrac{1}{x^2}\right] = 0$

$\Rightarrow \quad f(x) = 0 \; \forall \; x > 1$

Also, if $x < -1$, then $x^2 > 1$

$\Rightarrow \quad 0 < \dfrac{1}{x^2} < 1$

$\Rightarrow \quad f(x) = 0 \; \forall \; x < -1$

Hence, $f(1) = 1$, $f(-1) = 1$ and $f(x) = 0$, if $|x| > 1$

$\therefore$ $f(x)$ cannot be continuous at $x = 1$ and $x = -1$

Again,

Let $\quad \dfrac{1}{2} < x^2 < 1 \Rightarrow 1 < \dfrac{1}{x^2} < 2$

$\Rightarrow \quad \left[\dfrac{1}{x^2}\right] = 1$

$\Rightarrow \quad \dfrac{1}{2} < x^2\left[\dfrac{1}{x^2}\right] < 1$

$\therefore \quad \left[x^2\left[\dfrac{1}{x^2}\right]\right] = 0$

$\Rightarrow \quad f(x) = 0$ if $x \in \left(-1, \dfrac{1}{\sqrt{2}}\right) \cup \left(\dfrac{1}{\sqrt{2}}, 1\right)$

Next, Let $\dfrac{1}{3} < x^2 < \dfrac{1}{2}$

$\Rightarrow \quad 2 < \dfrac{1}{x^2} < 3$

$\Rightarrow \quad \left[\dfrac{1}{x^2}\right] = 2$

$\Rightarrow \quad \dfrac{2}{3} < x^2\left[\dfrac{1}{x^2}\right] < 1$

$\therefore \quad \left[x^2\left[\dfrac{1}{x^2}\right]\right] = 0$

$\Rightarrow f(x) = 0$, if

$$x \in \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}\right)$$

At $x = \pm\dfrac{1}{\sqrt{2}}, x^2 = \dfrac{1}{2}, \dfrac{1}{x^2} = 2$

$\Rightarrow \quad f(x) = 1$

Similarly, at $x = \dfrac{1}{\sqrt{3}}, \dfrac{1}{2}, \dfrac{1}{\sqrt{5}}$ ....

$\therefore$ $f(x)$ is discontinuous at infinite number of points given by

$$x \in \left\{\pm\frac{1}{\sqrt{n}}, n \in N\right\}$$