WB-JEE-2017 (Mathematics)

1. Transforming to parallel axes through a point (p, q), the equation $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 1$. Then
   a. $p = -2, q = 3$
   b. $p = 2, q = -3$
   c. $p = 3, q = -4$
   d. $p = -4, q = 3$

2. Let A(2, −3) and B (−2, 1) be two angular points of ΔABC. If the centroid of the triangle moves on the line $2x + 3y = 1$, then the locus of the angular point C is given by
   a. $2x + 3y = 9$
   b. $2x - 3y = 9$
   c. $3x + 2y = 5$
   d. $3x - 2y = 3$

3. The point P (3, 6) is first reflected on the line y = x and then the image point Q is again reflected on the line y = − x to get the image point Q'. Then the circumcentre of the $\Delta PQQ'$ is
   a. (6, 3)
   b. (6, − 3)
   c. (3, −6)
   d. (0, 0)

4. Let d_1 and d_2 be the lengths of the perpendiculars drawn from any point of the line $7x - 9y + 10 = 0$ upon the lines $3x + 4y = 5$ and $12x + 5y = 7$ respectively. Then
   a. $d_1 > d_2$
   b. $d_1 = d_2$
   c. $d_1 < d_2$
   d. $d_1 > 2d_2$

5. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$ subtends at the origin an angle equal to
   a. $\frac{\pi}{3}$
   b. $\frac{\pi}{4}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{2}$

6. The locus of the mid-points of the chords of the circle x2 + y2 + 2x – 2y – 2 = 0 which make an angle of 90º at the centre is

a. $x^2 + y^2 - 2x - 2y = 0$
b. $x^2 + y^2 - 2x + 2y = 0$
c. $x^2 + y^2 + 2x - 2y = 0$
d. $x^2 + y^2 + 2x - 2y - 1 = 0$

7. Let P be the foot of the perpendicular from focus S of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on the line bx – ay = 0 and let C be the centre of hyperbola. Then the area of the rectangle whose sides are equal to that of SP and CP is

a. 2ab
b. ab
c. $\frac{(a^2 + b^2)}{2}$
d. $\frac{a}{b}$

8. B is an extremity of the minor axis of an ellipse whose foci are S and S'. If ∠SBS' is a right angle, then the eccentricity of the ellipse is

a. $\frac{1}{2}$
b. $\frac{1}{\sqrt{2}}$
c. $\frac{2}{3}$
d. $\frac{1}{3}$

9. The axis of the parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$ is

a. $x + y = 0$
b. $x + y - 1 = 0$
c. $x - y + 1 = 0$
d. $x - y = \frac{1}{\sqrt{2}}$

10. The line segment joining the foci of the hyperbola $x^2 - y^2 + 1 = 0$ is one of the diameters of a circle. The equation of the circle is

a. $x^2 + y^2 = 4$
b. $x^2 + y^2 = \sqrt{2}$
c. $x^2 + y^2 = 2$
d. $x^2 + y^2 = 2\sqrt{2}$

11. The equation of the plane through (1, 2, –3) and (2, –2, 1) and parallel to X-axis is

a. $y - z + 1 = 0$
b. $y - z - 1 = 0$
c. $y + z - 1 = 0$
d. $y + z + 1 = 0$

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12. Three lines are drawn from the origin O with direction cosines proportional to (1, −1, 1), (2 −3, 0) and (1, 0, 3). The three lines are

a. not coplanar
b. coplanar
c. perpendicular to each other
d. coincident

13. Consider the non-constant differentiable function f of one variable which obeys the relation $\frac{f(x)}{f(y)} = f(x-y)$ If f′(0) = p and f′(5)=q, then f′(−5) is

a. $\frac{p^2}{q}$
b. $\frac{q}{p}$
c. $\frac{p}{q}$
d. q

14. If f(x) = $\log_5 \log_3 x$, then f′(e) is equal t

a. e $\log_e 5$
b. b. e $\log_e 3$
c. $\frac{1}{e\log_e 5}$
d. $\frac{1}{e\log_e 3}$

15. Let F(x) = e^x, G(x) = e^{-x} and H(x) = G(F(x)), where x is a real variable. Then $\frac{dH}{dx}$ at x = 0 is

a. 1
b. −1
c. $-\frac{1}{e}$
d. −e

16. If f''(0) = k, k $\neq$ 0, then the value of $\lim_{x\to\infty}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

a. K
b. 2k
c. 3k
d. 4k

17. If $y = e^{m\sin^{-1}x}$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - ky = 0$

a. m^2
b. 2
c. -1
d. $-m^2$

18. The chord of the curve $y = x^2 + 2ax + b$, joining the points where $x = \alpha$ and $x = \beta$, is parallel to the tangent to the curve at abscissa x =

a. $\frac{a+b}{2}$

b. $\frac{2a+b}{3}$

c. $\frac{2\alpha+\beta}{3}$

d. $\frac{\alpha+\beta}{2}$

19. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$. Then $f(x) = 0$ has

a. 13 real roots

b. only one positive and only two negative real roots

c. not more than one real root

d. has two positive and one negative real r

20. Let f(x) = $\begin{cases} \frac{x^p}{(\sin x)^q}, & \text{If } 0 < x \le \frac{\pi}{2} \\ 0 & \text{If } x = 0 \end{cases}$, $,(p, q, \in R)$.Then Lagrange's mean value theorem is applicable to f(x) in closed interval [0, x]

a. for all p, q

b. only when p > q

c. only when p < q

d. for no value of p, q

21. $\lim_{x\to\infty}(\sin x)^{2\tan x}$

a. is 2

b. is 1

c. is 0

d. does not exist

22. $\int \cos(\log x)dx = F(x) + c$, where c is an arbitrary constant. Here F(x) =

a. $x[\cos(\log x)+\sin(\log x)]$

b. $x[\cos(\log x)-\sin(\log x)]$

c. $\frac{x}{2}[\cos(\log x)+\sin(\log x)]$

d. $\frac{x}{2}[\cos(\log x)-\sin(\log x)]$

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23. $\int \frac{x^2-1}{x^4+3x^2+1}dx\left(x>0\right)$ is

a. $\tan^{-1}\left(x+\frac{1}{x}\right)+c$

b. $\tan^{-1}\left(x-\frac{1}{x}\right)+c$

c. $\log_e\left(\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}-1}\right)+c$

d. $\log_e\left(\frac{x-\frac{1}{x}-1}{x-\frac{1}{x}+1}\right)+c$

24. Let $I=\int_{10}^{19}\frac{\sin x}{1+x^8}dx$. Then

a. $|I|<10^{-9}$

b. $|I|<10^{-7}$

c. $|I|<10^{-5}$

d. $|I|>10^{-7}$

25. Let $I_1=\int_0^n [x]dx$ and $I_2=\int_0^n \{x\}dx$, where [x] and {x} are integral and fractional parts of x and $n\in N-\{1\}$. Then I_1/I_2 is equal to

a. $\frac{1}{n-1}$

b. $\frac{1}{n}$

c. n

d. $n-1$

26. The value of $\lim_{x\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+......+\frac{1}{2n}\right]$ is

a. $\frac{n\pi}{4}$

b. $\frac{\pi}{4}$

c. $\frac{\pi}{4n}$

d. $\frac{\pi}{2n}$

27. The value of the integral $\int_0^1 e^{x^2}dx$

a. is less than 1

b. is greater than 1

c. is less than or equal to 1

d. lies in the closed interval [1,e]

28. $\int_{0}^{100} e^{x-[x]}dx =$

a. $\frac{e^{100}-1}{100}$

b. $\frac{e^{100}-1}{e-1}$

c. $100(e-1)$

d. $\frac{e-1}{100}$

29. Solution of $(x+y)^2\frac{dy}{dx}=a^2$ (a' being a constant) is

a. $\frac{(x+y)}{a}=\tan\frac{y+c}{a}$, c is an arbitrary constant

b. xy = a tan cx, c is an arbitrary constant

c. $\frac{x}{a}=\tan\frac{y}{c}$, c is an arbitrary constant

d. xy = tan(x+c), c is an arbitrary constant

30. The integrating factor of the first order differential equation

$x^2(x^2-1)\frac{dy}{dx}+x(x^2+1)=x^2-1$ is

a. e^x

b. $x-\frac{1}{x}$

c. $x+\frac{1}{x}$

d. $\frac{1}{x^2}$

31. In a G.P. series consisting of positive terms, each term is equal to the sum of next two terms. Then the common ratio of this G.P. series is

a. $\sqrt{5}$

b. $\frac{\sqrt{5}-1}{2}$

c. $\frac{\sqrt{5}}{2}$

d. $\frac{\sqrt{5}+1}{2}$

32. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

a. 125

b. 25

c. 5/3

d. 243

33. The expression $\frac{(1+i)^n}{(1-i)^{n-2}}$ equals

a. $-i^{n+1}$
b. $-i^{n+1}$
c. $-2i^{n+1}$
d. 1

34. Let z = x + iy, where x and y are real. The points (x, y) in the X-Y plane for which $\frac{z+i}{z-i}$ is purely imaginary lie on

a. a straight line
b. an ellipse
c. a hyperbola
d. a circle

35. If p, q are odd integers, then the roots of the equation $2px^2 + (2p + q)x + q = 0$ are

a. rational
b. irrational
c. non-real
d. equal

36. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

a. 210
b. 25200
c. 2520
d. 302400

37. The number of all numbers having 5 digits, with distinct digits is

a. 99999
b. $9 \times {}^{9}P_4$
c. ${}^{10}P_5$
d. ${}^{10}P_4$

38. The greatest integer which divides (p + 1)(p + 2)(p + 3)......(p + q) for all $p \in N$ and fixed $q \in N$ is

a. p!
b. q!
c. p
d. q

39. Let $((1 + x) + x^2)^9 = a_0 + a_1x + a_2x^2 + \dots + a_{18}x^{18}$. Then

a. $a_0 + a_2 + \dots + a_{18} = a_1 + a_3 + \dots + a_{17}$

b. $a_0 + a_2 + \dots + a_{18}$ is even

c. $a_0 + a_2 + \dots + a_{18}$ is divisible by 9

d. $a_0 + a_2 + \dots + a_{18}$ is divisible by 3 but not by 9

40. The linear system of equations $\left.\begin{array}{r} 8x-3y-5z=0 \\ 5x-8y+3z=0 \\ 3x+5y-8z=0 \end{array}\right\}$ has

a. only 'zero solution'

b. only finite number of non-zero solutions

c. no non-zero solution

d. infinitely many non-zero solutions

41. Let P be the set of all non-singular matrices of order 3 over □ and Q be the set of all orthogonal matrices of order 3 over □ .Then,

a. P is proper subset of Q

b. Q is proper subset of P

c. Neither P is proper subset of Q nor Q is proper subset of P

d. $P \cap Q = \phi$ the void set

42. Let $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}, B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$. Then all solutions of the equation det (AB) = 0 is

a. 1, −1, 0, 2

b. 1, 4, 0, −2

c. 1, −1, 4, 3

d. −1, 4, 0, 3

43. The value of det A, where $A = \left\{\begin{array}{ccc} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{array}\right\}$ lies

a. in the closed interval [1, 2]

b. in the closed interval [0, 1]

c. in the open interval (0, 1)

d. in the open interval (1, 2)

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44. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f is injective and $f(x)\, f(y) = f(x + y)$ for $\forall x, y \in \mathbb{R}$. If f(x), f(y), f(z) are in G.P, then x, y, z, are in

a. A.P always
b. G.P always
c. A.P depending on the value of x, y, z
d. G.P depending on the value of x, y, z

45. On the set $\mathbb{R}$ of real numbers we define xPy if and only if $xy \geq 0$. Then the relation P is

a. reflexive but not symmetric
b. symmetric but not reflexive
c. transitive but not reflexive
d. reflexive and symmetric but not transitive

46. On $\mathbb{R}$, the relation ρ be defined by '$x \rho y$ holds if and only if $x - y$ is zero or irrational'. Then

a. ρ is reflexive and transitive but not symmetric.
b. ρ is reflexive and symmetric but not symmetric
c. ρ is symmetric and transitive but not reflexive.
d. ρ is equivalence relation

47. Mean of n observations $x_1, x_2, \ldots\ldots, x_n$ is $\bar{x}$. If an observation x_q is replaced by x_q' then the new mean is

a. $\bar{x} - x_q + x_q'$
b. $\dfrac{(n-1)\bar{x} + x_q'}{n}$
c. $\dfrac{(n-1)\bar{x} - x_q'}{n}$
d. $\dfrac{n\bar{x} - x_q + x_q'}{n}$

48. The probability that a non leap year selected at random will have 53 Sundays is

a. 0
b. 1/7
c. 2/7
d. 3/7

49. The equation $\sin x (\sin x + \cos x) = k$ has real solutions, where k is a real number. Then

a. $0 \leq k \leq \dfrac{1+\sqrt{2}}{2}$
b. $2-\sqrt{3} \leq k \leq 2+\sqrt{3}$
c. $0 \leq k \leq 2-\sqrt{3}$
d. $\dfrac{1-\sqrt{2}}{2} \leq k \leq \dfrac{1+\sqrt{2}}{2}$

50. The possible values of x, which satisfy the trigonometric equation

$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ are

a. $\pm\frac{1}{\sqrt{2}}$

b. $\pm\sqrt{2}$

c. $\pm\frac{1}{2}$

d. ± 2

51. On set A = {1, 2, 3}, relations R and S are given by

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

S = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)} Then

a. $R \cup S$ is an equivalence relation

b. $R \cup S$ is reflexive and transitive but not symmetric

c. $R \cup S$ is reflexive and symmetric but not transitive

d. $R \cup S$ is symmetric and transitive but not reflexive

52. If one of the diameters of the curve $x^2 + y^2 - 4x - 6y + 9 = 0$ is a chord of a circle with centre (1, 1), the radius of this circle is

a. 3

b. 2

c. $\sqrt{2}$

d. 1

53. Let A (−1, 0) and B (2, 0) be two points. A point M moves in the plane in such a way that $\angle MBA = 2\angle MAB$. Then the point M moves along

a. a straight line

b. a parabola

c. an ellipse

d. a hyperbola

54. If $f(x) = \int_{-1}^{x} |t|\,dt$, then for any $x \ge 0$, f(x) is equal to

a. $\frac{1}{2}\left(1-x^2\right)$

b. $1-x^2$

c. $\frac{1}{2}\left(1+x^2\right)$

d. $1+x^2$

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55. Let for all x > 0, $f(x) = \lim_{x\to\infty} n\left(x^{\frac{1}{n}} - 1\right)$, then

a. $f(x) + f\left(\frac{1}{x}\right) = 1$

b. f(xy)=f(x)+f(y)

c. f(xy)=x f(y) + yf(x)

d. f(xy) = x f(x) + y f(y)

56. Let $I = \int_0^{100\pi} \sqrt{(1-\cos 2x)}$ dx, then

a. $I = 0$

b. $I = 200\sqrt{2}$

c. $I = \pi\sqrt{2}$

d. $I = 100$

57. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is

a. $\frac{4}{3}$ square units

b. $\frac{2}{3}$ square units

c. $\frac{3}{7}$ square units

d. $\frac{6}{7}$ square units

58. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the ends of both latus rectum. The area of the quadrilateral so formed is

a. 27 sq. units

b. $\frac{13}{2}$ sq. units

c. $\frac{15}{4}$ sq. units

d. 45 sq. units

59. The value of K in order that f(x) = sin x – cos x – Kx + 5 decreases for all positive real values of x is given by

a. $K < 1$

b. $K \geq 1$

c. $K > \sqrt{2}$

d. $K < \sqrt{2}$

60. For any vector $\vec{x}$,the value of $\left(\vec{x}\times\hat{i}\right)^2+\left(\vec{x}\times\hat{j}\right)^2+\left(\vec{x}\times\hat{k}\right)^2$ is equal to

a. $|\vec{x}|^2$

b. $2|\vec{x}|^2$

c. $3|\vec{x}|^2$

d. $4|\vec{x}|^2$

61. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

a. $\sqrt{2}$ units

b. 2 units

c. $\sqrt{3}$ units

d. $\sqrt{5}$ units

62. Let α and β be the roots of $x^2 + x + 1 = 0$. If n be positive integer, then $\alpha^n+\beta^n$ is

a. $2\cos\frac{2n\pi}{3}$

b. $2\sin\frac{2n\pi}{3}$

c. $2\cos\frac{n\pi}{3}$

d. $2\sin\frac{n\pi}{3}$

63. For real x, the greatest value of $\frac{x^2+2x+4}{2x^2+4x+9}$ is

a. 1

b. -1

c. $\frac{1}{2}$

d. $\frac{1}{4}$

64. Let $A=\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$. Then for positive integer n, A^n is

a. $\begin{pmatrix}1&n&n^2\\0&n^2&n\\0&0&n\end{pmatrix}$

b. $\begin{pmatrix}1&n&n\left(\frac{n+1}{2}\right)\\0&1&n\\0&0&1\end{pmatrix}$

c. $\begin{pmatrix}1&n^2&n\\0&n&n^2\\0&0&n^2\end{pmatrix}$

d. $\begin{pmatrix}1&n&2n-1\\0&\frac{n+1}{2}&n^2\\0&0&\frac{n+1}{2}\end{pmatrix}$

65. Let a, b, c be such that $b(a+c)\neq 0$

If $\begin{vmatrix}a&a+1&a-1\\-b&b+1&b-1\\c&c-1&c+1\end{vmatrix}+\begin{vmatrix}a+1&b+1&c-1\\a-1&b-1&c+1\\(-1)^{n+2}a&(-1)^{n+1}b&(-1)^{n}c\end{vmatrix}=0$, then the value of n is

a. any integer

b. zero

c. any even integer

d. any odd integer

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CATEGORY - III (Q66 to Q75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answer marked $\div$ actual number of correct answers.

66. Let f : R->R be twice continuously differentiable. Let f(0) = f(1) = f′(0) = 0. Then

    a. $f''(x) \neq 0$ for all x
    b. $f''(c) = 0$ for some $c \in R$
    c. $f''(x) \neq 0$ if $x \neq 0$
    d. f′(x) > 0 for all x

67. If f(x) = x^n, n being a non-negative integer, then the values of n for which $f'(\alpha+\beta) = f'(\alpha) + f'(\beta)$ for all $\alpha, \beta > 0$ is

    a. 1
    b. 2
    c. 0
    d. 5

68. Le f be a non-constant continuous function for all $x \geq 0$. Let f satisfy the relation f(x) f(a–x) = 1 for some a $\in$ R^+. Then $I = \int_0^a \frac{dx}{1+f(x)}$ is equal to

    a. a
    b. $\frac{a}{4}$
    c. $\frac{a}{2}$
    d. f(a)

69. If the line ax + by + c = 0, ab$\neq$ 0, is a tangent to the curve xy = 1–2x, then

    a. a > 0, b < 0
    b. a > 0, b > 0
    c. a < 0, b > 0
    d. a < 0, b < 0

70. Two particles move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity u and the second starts from rest with constant acceleration f. Then

a. they will be at the greatest distance at the end of time $\frac{u}{2f}$ from the start

b. they will be at the greatest distance at the end of time $\frac{u}{f}$ from the start

c. their greatest distance is $\frac{u^2}{2f}$

d. their greatest distance is $\frac{u^2}{f}$

71. The complex number z satisfying the equation $|z-i| = |z+1| = 1$ is

a. 0
b. $1 + i$
c. $-1 + i$
d. $1 - i$

72. On , the set of real numbers, a relation ρ is defined as '$a \rho b$' if and only if $1 + ab > 0$'. Then

a. ρ is an equivalence relation
b. ρ is reflexive and transitive but not symmetric
c. ρ is reflexive and symmetric but not transitive
d. ρ is only symmetric

73. If $a, b \in \{1, 2, 3\}$ and the equation $ax^2 + bx + 1 = 0$ has real roots, then

a. $a > b$
b. $a \leq b$
c. number of possible ordered pairs (a, b) is 3
d. $a < b$

74. If the tangent to $y^2 = 4ax$ at the point $(at^2, 2at)$ where $|t|>1$ is a normal to $x^2 - y^2 = a^2$ at the point $(a \sec \theta, a \tan \theta)$, then

a. $t = -\operatorname{cosec} \theta$
b. $t = -\sec \theta$
c. $t = 2 \tan \theta$
d. $t = 2 \cot \theta$

75. The focus of the conic $x^2 - 6x + 4y + 1 = 0$ is

a. (2, 3)
b. (3, 2)
c. (3, 1)
d. (1, 4)

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ANSWER KEY

1. (b)	2. (a)	3. (d)	4. (b)	5. (d)	6. (c)	7. (b)	8. (b)	9. (a)	10. (c)
11. (d)	12. (b)	13. (a)	14. (c)	15. (c)	16. (c)	17. (a)	18. (d)	19. (c)	20. (b)
21. (b, d)	22. (c)	23. (a)	24. (b)	25. (d)	26. (b)	27. (b)	28. (c)	29. (a)	30. (b)
31. (b)	32. (a)	33. (c)	34. (d)	35. (a)	36. (b)	37. (b)	38. (b)	39. (b)	40. (d)
41. (b)	42. (b)	43. (a)	44. (a)	45. (d)	46. (b)	47. (d)	48. (b)	49. (d)	50. (a)
51. (c)	52. (a)	53. (d)	54. (c)	55. (b)	56. (b)	57. (a)	58. (a)	59. (c)	60. (b)
61. (c)	62. (a)	63. (c)	64. (b)	65. (c)	66. (b)	67. (b, c)	68. (c)	69. (b,d)	70. (b, c)
71. (a, c)	72. (c)	73. (c, d)	74. (a, c)	75. (c)					

SOLUTIONS

1. (b)

Given $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ ...(i)

After transforming eq.(i) through (p, q) it becomes

$$2x^2 + 3xy + 4y^2 = 1 \quad \text{...(ii)}$$

Then, the point (p, q) satisfies (i) and (ii)

Differentiate (i) with respect to x and y respectively, we get

$4x + 3y + 1 = 0$ ...(iii)

$3x + 8y + 18 = 0$ ...(iv)

$\because$ (p, q) satisfies (i) and (ii)

$\therefore$ Point (p, q) satisfies (iii) and (iv)

So, we get

$4p + 3q + 1 = 0$ ...(v)

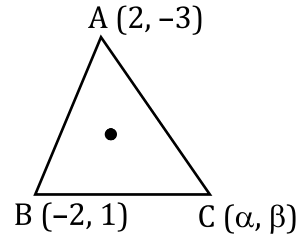
$3p + 8q + 18 = 0$ ...(vi)

Solving eq.(v) and (vi) simultaneously we get

$p = 2, q = -3$

2. (a)

Given: A(2, –3), B(–2, 1) of ΔABC



Centroid moves on 2x + 3y = 1

Let $C(\alpha, \beta)$ be the third point on ΔABC

Let $G \rightarrow$ Centroid of ΔABC.

Then, $G = \left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right) \Rightarrow \left(\frac{\alpha}{3}, \frac{\beta-2}{3}\right)$ ...(i)

Since, G moves on the line 2x + 3y = 1

Points on this line area of the $\left(x, \frac{1-2x}{3}\right)$

$\Rightarrow G = \left(t, \frac{1-2t}{3}\right)$

From eq.(i) $\frac{\alpha}{3} = t$ and $\frac{\beta-2}{3} = \frac{1-2t}{3}$

$\Rightarrow \alpha = 3t$ and $\beta = 3 - 2t$

$\therefore\ C \equiv (3t, 3 - 2t)$

Locus of point C is given by,

$\Rightarrow (3 - 2t)^2 + (3 - 2t + 3)^3 = (3t + 2)^2 + (3 - 2t - 1)^2$

$\Rightarrow 9t^2 + 4 - 12t + 36 + 4t^2 - 24t = 9t^2 + 4 + 12t + 4 + 4t^2 - 8t$

$\Rightarrow 40t = 32$

$\Rightarrow t = \frac{32}{40} = \frac{4}{5}$

$C \equiv \left(\frac{12}{5}, \frac{7}{5}\right)$ so the equation of Locus of C is given by 2x + 3y = d

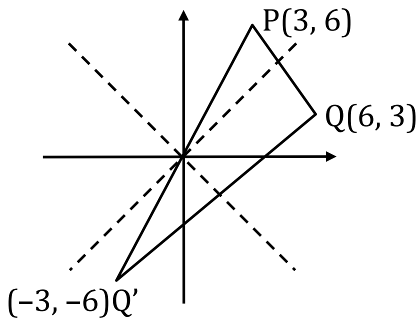
Substituting C in the given equation 2x + 3y = d we get d = 9.

Thus, the Locus of C is 2x + 3y = 9

3. (d)

Given P(3, 6) an y = x, Q $\rightarrow$ y = –x to get Q' Circumcenter of ΔPQQ' = ?

When the point P(3, 6) is reflected on the line y = x the x and y coordination interchange, we get point Q(6, 3). Again reflecting Q' on the line y = –x we get the image point Q'(–3, –6).



To find out the circumcenter, we need to solve any two bisector equation and then point out the intersection point.

So, midpoint of PQ' = $\left(\frac{3-3}{2}, \frac{6-6}{2}\right)$ = (0, 0)

Slope of PQ' is $m_1 = \frac{-6-6}{-3-3} = 2$

Slope of bisector is the negative reciprocal of the given slope so, slope (m_1) of bisector of PQ' is $-\frac{1}{2}$

Equation of PQ' with slope (m_1) $-\frac{1}{2}$ and point (0, 0) is given by

$\Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = -\frac{1}{2}(x - 0)$

$\Rightarrow y = -\frac{1}{2}x$

$\Rightarrow 2x + y = 0$ ...(1)

Now, mid-point of PQ = $\left(\frac{3+6}{2}, \frac{6+3}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$

Slope of PQ is $m_2 = \frac{3-6}{6-3} = \frac{-3}{3} = -1$

Slope of bisector of PQ is 1.

$\therefore$ Equation of PQ with slope (m_2) = 1 and point $\left(\frac{9}{2}, \frac{9}{2}\right)$ is given by

$$y - \frac{9}{2} = 1\left(x - \frac{9}{2}\right)$$

$\Rightarrow y = x$

$\Rightarrow x - y = 0$ ...(2)

Solving equation (1) & (2), we get

$$x = 0 \ \& \ y = 0$$

Therefore, circumcenter of $\Delta PQQ'$ is (0, 0)

4. (b)

Given: Perpendiculars drawn from any point of

$$7x - 9y + 10 = 0 \quad ...(i)$$

$$3x + 4y = 5 \quad ...(ii)$$

$$12x + 5y = 7 \quad ...(iii)$$

Any point on the line (i) will be of the form $\left(x, \frac{7x+10}{9}\right)$

d_1 and d_2 are the lengths at any point of eq.(i) upon eq.(ii) and eq.(iii) distance from $\left(a, \frac{7a+10}{9}\right)$ to eq.(ii) is

$$d_1 = \frac{\left|3a + 4\left(\frac{7a+10}{9}\right) - 5\right|}{\sqrt{3^2+4^2}}$$

$$= \frac{\left|\frac{27a+28a+40-45}{9}\right|}{\sqrt{25}}$$

$$= \frac{|11a-1|}{9}$$

Distance from $\left(a, \frac{7a+10}{9}\right)$ to eq.(iii) is

$$d_2 = \frac{\left|12a+5\left(\frac{7a+10}{9}\right)-7\right|}{\sqrt{12^2+5^2}}$$

$$= \frac{|143a-13|}{9\times 13}$$

$$= \frac{|11a-1|}{9}$$

So, $d_1 = d_2$

5. (d)

Given: Circles are $x^2 + y^2 - 4x - 4y = 0$ ...(a)

$2x^2 + 2y^2 = 32$ ...(b)

To find the common chord we need to find the intersection points of the circles.

So, let's subtract eq.(b) from eq.(a), we get

$\Rightarrow -4x - 4y = -16$

i.e. $x + y = 4$ ...(c)

Substitute $x = y - 4$ in (b), we get

$(y - 4)^2 + y^2 = 16$

$\Rightarrow y^2 - 8y + 16 + y^2 = 16$

$\Rightarrow y(y - 4) = 0$

$\Rightarrow y = 0, 4$

At $y = 0$, $x = 4$ and at $y = 4$, $x = 0$

So the points of intersection of these two circles is (4, 0) and (0, 4) thus, the chord drawn from (0, 4) to (4, 0).

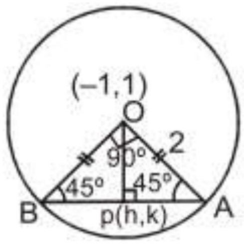
Thus, the angle subtended at the origin by this chord is a right angle.

Hence, the chord subtends an angle of $\frac{\pi}{2}$ at the origin.

6. (c)

Given Circles is $x^2 + y^2 + 2x - 2y - 2 = 0$

Which can be written as $(x + 1)^2 + (y - 1)^2 = 4$, so C(−1, 1) and radius is 2.



Let O be the center and AB be the chord, angle = 90°

Draw OA and OB

$\therefore\ \angle AOB = 90°$

Now OA and OB are radius of the circle.

$\therefore\ OA = OB$

$\Rightarrow \angle OAB = \angle OBA$ [Angle opposite to equal sides are equal]

But $\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 2\angle OAB = 180 - 90$

$\Rightarrow \angle OAB = 45° = \angle OBA$

OP⊥C so P(h, k) is the mid-point of chord AB.

Join OP which makes an angle of 90° with the chord AB.

Now, ΔAPO is a right angled triangle such that $\angle APO = 90°$

$$\Rightarrow \sin 45° = \frac{OP}{OA} = \frac{OP}{2} \Rightarrow OP = \sqrt{2}$$

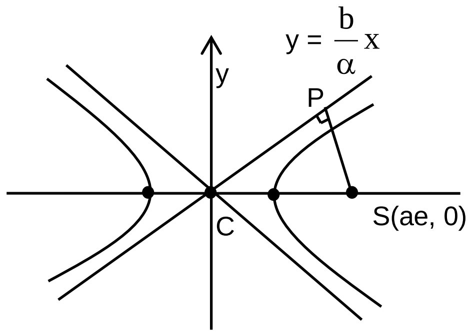
But $OP = \sqrt{(h+1)^2 + (k-1)^2}$ $\{\because \sqrt{(h+1)^2 + (k-1)^2} = \sqrt{2}$

$\Rightarrow h^2 + 2h + 1 + k^2 - 2k + 1 = 2$

$\Rightarrow h^2 + k^2 + 2h - 2k = 0$

Locus of P is $x^2 + y^2 + 2x - 2y = 0$

7. (b)



Given hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, Center (C) = (0, 0)

Let the coordinates of the focus s be (s, 0) and assume that a > b

$\therefore\ s = \sqrt{a^2 + b^2}$

Also, perpendicular from focus to asymptote [bx – ay = 0]

$y = \frac{b}{a} \times$ has slope $\frac{-a}{b}$ we have equation of normal

As $y = \frac{-a}{b}x + K$

$\Rightarrow by + ax = bk \Rightarrow a\sqrt{a^2 + b^2}$ (passing through the focus)

$$\Rightarrow SP = \sqrt{\left(\sqrt{a^2+b^2} - \frac{a^2}{\sqrt{a^2+b^2}}\right) + \left(0 - \frac{ab}{\sqrt{a^2+b^2}}\right)^2}$$

$$\Rightarrow SP = \sqrt{\frac{b^4}{a^2+b^2} + \frac{a^2b^2}{a^2+b^2}} = \sqrt{b^2\left(\frac{b^2+a^2}{a^2+b^2}\right)}$$

$\Rightarrow SP = b$

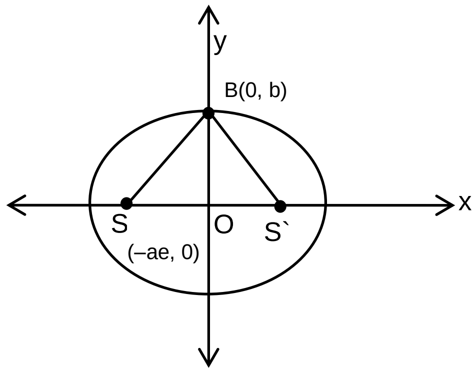
Also, $$CP = \sqrt{\left(0 - \frac{a^2}{\sqrt{a^2+b^2}}\right)^2 + \left(0 - \frac{ab}{\sqrt{a^2+b^2}}\right)^2}$$

$$CP = \sqrt{a^2\left(\frac{a^2+b^2}{a^2+b^2}\right)}$$

CP = a

Hence, area of rectangle with sides CP and SP is ab

8. b



Given: $\angle SBS'$ is a night angle

$\therefore$ By Pythagoras theorem

$\Rightarrow SS`^2 = SB^2 + S'B^2$

$\Rightarrow (ae + ae)^2 = (ae^2) + b^2 + (ae)^2 + b^2$

$\Rightarrow (2ae)^2 = 2(ae)^2 + 2b^2$

$\Rightarrow = 4(ae)^2 = 2(ae)^2 + 2b^2$

$\Rightarrow ae = b$

$\Rightarrow e = \frac{b}{a}$

But $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow e^2 = 1 - e^2$

$\Rightarrow 2e^2 = 1$

$\Rightarrow e = \sqrt{\frac{1}{2}}$

$\Rightarrow e = \frac{1}{\sqrt{2}}$

9. a.

Given parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$

$\Rightarrow (x + y)^2 = 5x - 5y + 5 \Rightarrow (x + y)^2 = 5(x - y + 1)$

For a parabola $(Ax + Cy)^2 + Dx + Ey + F = 0$

The axis is $Ax + Cy + \frac{AD+CE}{2(A^2+C^2)} = 0$

$\Rightarrow$ axis is $(x + y) + \frac{-5+5}{2(1+1)} = 0$

$\therefore$ axis is $x + y = 0$

10. c

Given hyperbola $x^2 - y^2 + 1 = 0$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 0$

Foci of the given hyperbola = $(0, \pm\sqrt{2})$

C (0, 0) diameter of the circle is the distance b + w foci

Diameter (D) = $\sqrt{(0-0)^2 + (\sqrt{2}+\sqrt{2})^2} = 2\sqrt{2}$

Centre of the circle will be same as that of hyperbola centre (c) = (0, 0) and Radius

(R) = $\sqrt{2}$

Equation of the circle is $x^2 + y^2 = 2$

11. d

Given equation plane (1, 2, −3) and (2, −2, 1) points

$$\Rightarrow \vec{u} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 4 \end{pmatrix}$$

A vector direction for the x-axis is

$$\Rightarrow \vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

A normal vector to the plane is

$\vec{n} = \vec{u} \times \vec{v}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 4 \\ 1 & 0 & 0 \end{vmatrix}$$

$\Rightarrow \hat{i}(0 - 0) - \hat{j}(0 - 4) + \hat{k}(0 + 4)$

$\Rightarrow (< 0, 4, 4 >)$

$\Rightarrow 0(x - 1) + 4(y - 2) + 4(z + 3) = 0$

$\Rightarrow 4y - 8 + 4z + 12 = 0$

$\Rightarrow 4y + 4z + 4 = 0$

$\Rightarrow y + z + 1 = 0$

12. b

Given: The lines are drawn from origin 0 with dcs proportional to (1, −1, 1) (2, −3, 0) and (1, 0, 3)

Consider $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \\ 1 & 0 & 3 \end{vmatrix}$

$\Rightarrow 1(-9 - 0) + 1\,(6 - 0) + 1\,(0 + 3)$

$\Rightarrow -9 + 9 = 0$

$\therefore\ \Delta = 0$ Hence, the lines are co-planar

13. a

$f(x) = a^{kx}$ is non-constant differentiable function.

Also, $\frac{f(x)}{f(y)} = \frac{a^{kx}}{a^{ky}} = a^{kx}\, a^{-ky} = a^{k(x-y)} = f\,(x - y)$

$\therefore\ f(x) = a^{kx}$ is the required function

$\Rightarrow f'(x) = ka^{kx}\ln a$

Given $f'(0) = P$ and $f'(5) = 9$

$\Rightarrow a^{sk} = \frac{9}{p}$

$\therefore\ f'(-5) = k.a^{-5k}\ln a \Rightarrow k\ln a^{-5k} = p \times \frac{p}{9} = \frac{p^2}{9}$

Hence, $f'(-5) = \frac{p^2}{9}$

14. c

Given $f(x) = \log_5 \log_3 x$

$\Rightarrow f(x) = \dfrac{\log\log_3 x}{\log 5}$

$= \dfrac{1}{\log 5} \times \log\log_3 x$

$= \dfrac{1}{\log 5} \times \log\left(\dfrac{\log x}{\log 3}\right)$

$= \dfrac{1}{\log 5} \times [\log(\log x) - \log(\log 3)]$

$\therefore f'(x) = \dfrac{1}{\log 5} \times \left[\dfrac{1}{\log x} \cdot \dfrac{1}{x} - 0\right]$

$\Rightarrow f'(x) = \dfrac{1}{x \log x \log 5}$

$\therefore f'(e) = \dfrac{1}{e \log e \log 5} = \dfrac{1}{e \log_e 5}$

Hence, the answer is $\dfrac{1}{e \log_e 5}$

15. c.

$f(x) = e^x$

$G(x) = e^{-x}$

$G(f(x) = e^{-(f(x))}$

$H(x) = e^{-ex}$

$\Rightarrow \log(H(x)) = \log_e e^{-e^x} = e^{-x}$ $\quad \Rightarrow \dfrac{d}{dx}\log(1 + (x)) = \dfrac{-d}{dx} e^x = -e^x$

$\dfrac{1}{H(x)} \dfrac{dH(x)}{dx} = -e^x$

$\dfrac{dH}{dx}(x) = -e^x e^{-ex}$

$\dfrac{d(H(0))}{dx} = -e^0 e^{-e^0}$

$\dfrac{d(H(0))}{dx} = -1e^{-1}$

$H'(0) = \dfrac{-1}{e}$

16. c

Given, $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$

$\Rightarrow \frac{2f(0)-3f(0)+f(0)}{0}=\frac{0}{0}$

Applying L-Hospital rule

$\Rightarrow \lim_{x\to 0}\frac{2f'(x)-3\times 2f'(2x)+4f'(4x)}{2x}$

$\Rightarrow \lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}=\frac{0}{0}$

Again L-Hospital

$\Rightarrow \lim_{x\to 0}\frac{2f'(x)-6.2f''(2x)+4.4f''(4x)}{2}$

$\Rightarrow \frac{2f''(0)-12f''(0)+16f''(0)}{2}$

$\Rightarrow \frac{4f''(0)+2f''(0)}{2}\Rightarrow\frac{6f''(0)}{2}=3f''(0)$ = 3 k.

17. a

Given, $y=e^{m\sin^{-1}x}$ then $(1 - x^2)\ \frac{d^2y}{dx^2} - x\ \frac{dy}{dx} - ky = 0$

Now we have y = $e^{m\sin^{-1}x}$ ……….(1)

Eq (1) apply chain rule then we get

$y'=e^{m\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$

$y'=m\frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}$ ……..(2)

Again differentiate equation (2) and applying the quotient rule, along with the chain rule we get.

$$y'' = \frac{\left(\sqrt{1-x^2}\right)\left(\frac{d}{dx} me^{m\sin^{-1}x}\right) - \left(me^{m\sin^{-1}x}\right)\left(\frac{d}{dx}\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\Rightarrow \frac{m^2 e^{m\sin^{-1}x} + \frac{mxe^{m\sin^{-1}x}}{\sqrt{1-x^2}}}{1-x^2}$$

$$\Rightarrow \frac{m^2 y + xy'}{1-x^2}$$

$\therefore$ $(1 - x^2)y'' = m^2y + xy'$

$\therefore$ $(1 - x^2)\, y'' - xy' = m^2y$

$k = m^2$

18. d

Given curve is $y = x^2 + 2ax + b$ differentiate above equation with respect to x we get.

$\Rightarrow \frac{dy}{dx} = 2x + 2a$ ....(1)

Equation (1) of tangent to the curve

But tangent to the curve is parallel to the chord of the curve which joints the points $x = \alpha$ and $x = \beta$

$\therefore$ Tangent to this curve $= (\alpha + \beta) + 2a$

$\therefore$ $2x + 2a = (\beta + \alpha) + 2a$

$$x = \frac{\alpha \times \beta}{2}$$

19. c

Given $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19 = 0$

$\Rightarrow f'(x) = 13x^{12} + 11x^{10} + 9x^8 + 7x^6 + 5x^4 + 3x^2 + 1$

Since $f'(x)$ consists of all even powers of x therefore

$f'(x) \geq 0$

That f(x) increasing function. Hence, it interests the x-axis at only one point. Therefore f(x) has atmost one real root.

20. b

Given $f(x) = \begin{cases} \dfrac{x^p}{(\sin x)^q} & ; \quad \text{if } 0 < x \le \dfrac{\pi}{2} \\ 0 & ; \quad \text{if } x = 0 \end{cases}$

Lagrange's mean theorem apply.

f, it is continuous at (a, b) and differential with in the closed interval [a, b]

i.e. $\lim\limits_{x \to a} f(x) = f(a)$

Here, f(a) = f(0) = 0

f(x) is differentiable at (0, x)

Consider $\lim\limits_{x \to 0} f(x)$

$$\Rightarrow \lim_{x \to 0} \frac{x^p}{(\sin x)^q}$$

$$\Rightarrow \lim_{x \to 0} \frac{x^p \cdot x^q \cdot x^{-q}}{(\sin x)^q} \Rightarrow \lim_{x \to 0} \frac{x^q \cdot x^{p-q}}{(\sin x)^q}$$

$$\Rightarrow \lim_{x \to 0} \frac{x^q}{(\sin x)^q} \times \lim_{x \to 0} x^{p-q} \Rightarrow \lim_{x \to 0} \left(\frac{x}{\sin x}\right)^q \times \lim_{x \to 0} x^{p-q}$$

$$\Rightarrow \lim_{x \to 0} x^{p-q} \ldots\ldots\ldots \lim_{x \to 0} \frac{x}{\sin x} = 1$$

Now $\lim\limits_{x \to 0} x^{p-q} = \begin{cases} 0 & \text{if} \quad p > q \\ \infty & \text{if} \quad p < q \\ 1 & \text{if} \quad p = q \end{cases}$

$\therefore \lim\limits_{x \to 0} f(x) = 0$ only when p > q

Hence, Lagrange's mean value theorem is applicable to f(x) in [0, x] only when p > q.

21. b, d

Given $y = (\sin x)^{2\tan x}$

$\Rightarrow \log y = 2\tan x.\log(\sin x)$

$$\Rightarrow \lim_{x\to 0} 2\tan x \log(\sin x) \Rightarrow \lim_{x\to 0} 2\frac{\log(\sin x)}{\frac{1}{\tan x}}$$

$$\Rightarrow \lim_{x\to 0} \frac{2.\frac{d}{dx}\log\sin x}{\frac{d}{dx}\cot x} \Rightarrow \lim_{x\to 0} \frac{2\frac{1}{\sin x}.\cos x}{-\operatorname{cosec}^2 x}$$

$$\Rightarrow \lim_{x\to 0} \frac{\frac{2}{\tan x}}{-\frac{1}{\sin^2 x}} \Rightarrow \lim_{x\to 0} -\frac{2\frac{d}{dx}\cot x}{\frac{d}{dx}\operatorname{cosec}^2 x}$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{-2\operatorname{cosec}^2 x}{-2\operatorname{cosec}^2 x \cot x}\right] = \tan x$$

$$\Rightarrow \lim_{x\to 0} \tan x$$

$\Rightarrow \tan x = 0$

$\log y = 0$

$y = e^o \Rightarrow 1$

$$\lim_{x\to 0} (\sin x)^{2\tan x} = 1$$

22. c

Given $\int \cos(\log x)dx$

Let $\log x = t \Rightarrow x = e^t$

$$\frac{1}{x}dx = dt \Rightarrow dx = xdt$$

$dx = e^t dt$

$I \Rightarrow \int \cos(\log x)dx \Rightarrow \int e^t \cos t dt$

$$\left[\because \begin{array}{l} e^{ax}\cos bx dx \\ \dfrac{e^{ax}}{a^2+b^2}(a\sin bx + b\cos bx) \end{array}\right]$$

a = 1, b = 1

$I = \dfrac{e^t}{1^2+1^2}(\sin t + \cos t) + c$

$I \Rightarrow \dfrac{x}{2}$ [sin(logx)+sin(logx)] + c

23. a

Given equation $\int \dfrac{x^2-1}{(x^4+3x^2+1)} dx$ (x > 0)

Dividing by x^2 numerator or denominator

$\Rightarrow \int \dfrac{\left(1-\dfrac{1}{x^2}dx\right)}{\left(x^2+\dfrac{1}{x^2}+3\right)}dx \qquad \Rightarrow \int \dfrac{\left(1-\dfrac{1}{x^2}\right)dx}{\left(x+\dfrac{1}{x}\right)^2+1}$

Let, $x+\dfrac{1}{x}=t \qquad \left(1-\dfrac{1}{x^2}\right)dx = dt$

$\Rightarrow \int \dfrac{dt}{t^2+1} = \tan^{-1}(x)+C$

$\Rightarrow \tan^{-1}\left(x+\dfrac{1}{x}\right) + C$

24. b

Given $I = \int_{10}^{19} \dfrac{\sin x}{1+x^8}dx \qquad \therefore |\sin x| \le |$

$\Rightarrow \left|\dfrac{\sin x}{1+x^8}\right| \le \left|\dfrac{1}{1+x^8}\right| \qquad$ ……..(1)

$$\begin{Bmatrix} \Rightarrow 10 \le x \le 19 \\ 10^8 \le x^8 \le 19^8 \\ 1+x^8 \ge x^8 \ge 10^8 \end{Bmatrix}$$

$$\Rightarrow \frac{1}{1+x^8} < \frac{1}{x^8} \le \frac{1}{10^8} \Rightarrow \left|\frac{1}{1+x^8}\right| \le \left|\frac{1}{10^8}\right|$$

So, $\left|\frac{\sin x}{1+x^8}\right| \le \frac{1}{10^8}$

$$\Rightarrow \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx \le \int_{10}^{19} 10^{-8}dx$$

$$\Rightarrow \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx \le (19-10)\times 10^{-8}$$

$$\Rightarrow \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx \le (10-1)\times 10^{-8}$$

$$\Rightarrow \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx \le 10^{-7} - 10^{-8}$$

$$\Rightarrow \int_{10}^{19}\left|\frac{\sin x}{1+x^2}\right|dx < 10^{-7} \Rightarrow |I| < 10^{-7}$$

Approximately $[10^{-8}]$

25. d

$$I_1 \int_0^n [x]dx = \int_0^1 o dx + \int_1^2 1dx + \int_2^3 2dx + \int_3^4 3dx + \ldots\ldots + \int_{n-1}^n (n-1)dx$$

$\Rightarrow$ 0 + 1 (2 – 1) + 2 [3– 2] + 3[4–3]+……+ n –1[n–(n–1)]

$\Rightarrow$ 1 + 2 + 3 + 4 + ………+ (n – 1)

$I_1 = \frac{(n-1)(n)}{2} = \frac{n(n-1)}{2}$

$I_2 = \int_0^n \{x\}dx \Rightarrow \int_0^n (x-[x])dx$

$$I_2 = \int_0^n x\,dx - \int_0^n [x]\,dx$$

$$I_2 = \left[\frac{x^2}{2}\right]_0^n - \frac{n(n-1)}{2}$$

$$I_2 = \frac{n^2}{2} - \frac{n(n-1)}{2} \Rightarrow \frac{n}{2}[n-(n-1)] = \frac{n}{2}$$

$$\frac{I_1}{I_2} = \frac{\frac{n(n-1)}{2}}{\frac{n}{2}} \Rightarrow (n-1)$$

26. b

$$\lim_{n\to\infty}\frac{1}{n}\left[\frac{n^2}{n^2+1^2}+\frac{n^2}{n^2+2^2}+.....+\frac{n}{2n}\right]$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\left[\frac{n^2}{n^2+1^2}+\frac{n^2}{n^2+2^2}+.....+\frac{n^2}{n^2+n^2}\right]$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{\frac{n^2+1^2}{n^2}}+\frac{1}{\frac{n^2+2^2}{n^2}}+.....+\frac{1}{\frac{n^2+n^2}{n^2}}\right]$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{1+\left(\frac{1}{n}\right)^2}+\frac{1}{1+\left(\frac{2}{n}\right)^2}+.....+\frac{1}{1+\left(\frac{n}{n}\right)^2}\right]$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\left(\frac{r}{n}\right)^2} \qquad \begin{cases}\frac{1}{n}=h\\ nh=1\\ h\to\infty\\ n\to 0\end{cases}$$

$$\Rightarrow \lim_{n\to 0} h\sum_{r=1}^{n}\frac{1}{1+(rh)^2} \qquad \Rightarrow \int_0^1 \frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1}x\Big|_0^1 = \tan^{-1}1 - \tan^{-1}0 \qquad = \frac{\pi}{4}-0=\frac{\pi}{4}$$

27. b

$$\int_0^1 e^{x^2} dx$$

$$\left\{ m(b-a) < \int_0^1 e^{x^2} dx < m(b-a) \right. \qquad \ldots\ldots.(1)$$

a = 0, b = 1

$f(x) = e^{x^2}$

$m = f(0) = e^0 = 1$

$m\ f(1) = e^{1^2} = e^1 = e$

So, equation (1)

$$1(1-0) < \int_0^1 e^{x^2} dx < e(1-0)$$

$$\Rightarrow 1 < \int_0^1 e^{x^2} dx < e$$

$$\Rightarrow \int_0^1 e^{x^2} dx \text{ lies in } (l, e)$$

28. c

$$\int_0^{100} e^{x-[x]} dx \Rightarrow \int_0^{100} e^{[x]} dx \qquad \begin{cases} x = [x] + \{x\} \\ x - [x] = \{x\} \end{cases}$$

$0 \le \{x\} < 1$ period $= 1$

$$\int_0^{mT} f(x) dx = m \int_0^T f(x) dx \qquad T \to \text{time period}$$

$$\Rightarrow \int_0^{100} e^x dx \Rightarrow 100[e^x]_0^1$$

$$\Rightarrow 100\ (e^1 - e^0) \Rightarrow 100\ (e-1)$$

29. a

$$(x+y)^2 \frac{dy}{dx} = a^2$$

$$\text{Let} \begin{cases} x+y=t \\ 1+\frac{dy}{dx}=\frac{dt}{dx} \\ \frac{dy}{dx}=\frac{dt}{dx}-1 \end{cases}$$

$$\Rightarrow t^2\left(\frac{dt}{dx}-1\right)=a^2$$

$$\Rightarrow t^2\frac{dt}{dx}-t^2=a^2$$

$$\Rightarrow t^2\frac{dt}{dx}=a^2+t^2$$

$$\Rightarrow \int\frac{t^2+a^2-a^2}{t^2+a^2}dt=\int dx$$

$$\Rightarrow \int\left(1-\frac{a^2}{(t^2+a^2)}\right)dt=x+c$$

$$\Rightarrow \int dt-\int\frac{a^2}{(t^2+a^2)}dt=x+c$$

$$\Rightarrow t-a^2\int\frac{dt}{t^2+a^2}dt=x+c$$

$$\Rightarrow t-a^2\left[\frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)\right]=x+c$$

$$\Rightarrow x+y-a\tan^{-1}\left(\frac{x+y}{a}\right)=x+c$$

$$\Rightarrow \frac{y-c}{a}=\tan^{-1}\left(\frac{x+y}{a}\right)$$

$$\Rightarrow \frac{x+y}{a}=\tan\left(\frac{y-c}{a}\right)$$

30. b

$$x^2(x^2-1)\frac{dy}{dx}+x(x^2+1)y = x^2-1$$

$$\Rightarrow \frac{dy}{dx}+\frac{(x^2+1)}{x(x^2-1)}y=\frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx}+py=\theta$$

$$\Rightarrow \text{I.F.} = e^{\int pdx}=e^{\int\frac{(x^2+1)}{x(x^2-1)}dx}$$

$$\Rightarrow \text{I} = \int\frac{x^2+1}{x(x^2-1)}dx=\int\frac{(x^2-1)+1+1}{x(x^2-1)}dx$$

$$\Rightarrow \text{I} = \int\frac{1}{x}dx=\int\frac{2}{x(x-1)(x+1)}dx \qquad \begin{cases} A=\dfrac{2}{(-1)(1)}=-2 \\ B=\dfrac{2}{1\times 2}=1 \\ C=\dfrac{2}{(-1)(-2)}=1 \end{cases}$$

$$\Rightarrow \text{I} = \ell nx + \int\left(\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}\right)dx$$

$$\Rightarrow \text{I} = \ell nx + \int\left(\frac{-2}{x}+\frac{1}{x-1}+\frac{1}{x+1}\right)dx$$

$$\Rightarrow \text{I} = \ell nx - 2\,\ell nx + \ell n\,(x-1) + \ell n(x+1)$$

$$\Rightarrow \ell nx - \ell nx^2 + \ell n(x-1) + \ell n\,(x+1)$$

$$\Rightarrow \ell n\,x\,\frac{(x^2-1)}{x^2} \Rightarrow \ell n\frac{x^2-1}{x}$$

$$\text{I.F.} = e^{\ell n_e}\left(\frac{x^2-1}{x}\right) \Rightarrow \frac{x^2-1}{x}$$

$$\text{I.F.} = x-\frac{1}{x}$$

31. b

Each term is sum of next two terms

$\Rightarrow t_n = t_{n+1} + t_{n+2}$

$\Rightarrow ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow 1 = r + r^2$

$\Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1-4(-1)}}{2(1)}$$

$$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$$

$$r = \frac{\sqrt{5}-1}{2} \text{ or } r = \frac{-\sqrt{5}-1}{2}$$

But common ration can be negative

$$r = \frac{\sqrt{5}-1}{2}$$

32. a

$$(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$$

$$\Rightarrow \left(\frac{\log x}{\log 5}\right)\left(\frac{\log 3x}{\log x}\right)\left(\frac{\log y}{\log 3}\right) = \frac{\log x^3}{\log x}$$

$$\Rightarrow \frac{\log x \cdot \log 3x \cdot \log y}{\log 5 \cdot \log x \cdot \log 3x} = \frac{3\log x}{\log x}$$

$$\Rightarrow \frac{\log y}{\log 5} = 3$$

$\Rightarrow \log y = 3\log 5$

$y = 5^3 = 125$

33. c

$$\frac{(l+i)^n}{(1-i)^{n-2}} \Rightarrow \frac{(l+i)^n}{\frac{(l-i)^n}{(l-i)^2}}$$

$$\Rightarrow \frac{(l+i)^n(1-i)^2}{(1-i)^n} \Rightarrow \frac{\left(\frac{l}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^n(\sqrt{2})^n(l-i)^2}{\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^n(\sqrt{2})^n}$$

$$\Rightarrow \frac{\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^n(1+i^2-2i)}{\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)^n} \qquad \therefore \begin{cases}\cos\theta+i\sin\theta=e^{i\theta}\\ \cos\theta-i\sin\theta=e^{-i\theta}\end{cases}$$

$$\Rightarrow \frac{\left(e^{i\pi/4}\right)^n(1-1-2i)}{(e^{-i\pi/4})^n} \Rightarrow \frac{e^{in\pi/4}(-2i)}{e^{-in\pi/4}}$$

$$\Rightarrow e^{in\,\pi/4}\cdot e^{in\,\pi/4}(-2i)$$

$$\Rightarrow e^{in\left(\frac{\pi}{4}+\frac{\pi}{4}\right)}(-2i)$$

$$\Rightarrow e^{in\,\pi/2}(-2i)$$

$$\Rightarrow \left[\cos\left(\frac{n\pi}{2}\right)+i\sin\left(\frac{n\pi}{2}\right)\right](-2i)$$

34. d

Let z = x + iy

$$\therefore \left(\frac{z+i}{z-i}\right)=\frac{x+iy+i}{x+iy-i}$$

$$\Rightarrow \frac{(x+i(y+1))}{(x-i(1-y))}\times\frac{(x+i(1-y))}{(x+i(1-y))}$$

$$\Rightarrow \frac{x^2+(y^2-1)+2ix}{x^2+(1-y)^2}$$

Since, $\frac{z+i}{z-i}$ should be purely imaginary

$\therefore R_e\left(\frac{z+i}{z-i}\right)=0$

$\Rightarrow \frac{x^2+(y^2-1)}{x^2+(1-y)^2}=0$

$\Rightarrow x^2 + y^2 - 1 = 0$

$x^2 + y^2 = 1$ is Circle.

35. a

Given equation $2px^2 + (2p + q)x + q = 0$ .....(1)

Now $D = (2p + q)^2 - 8pq$

$\Rightarrow 4p^2 + 4pq + q^2 - 8pq$

$\Rightarrow 4p^2 - 4pq + q^2$

$\Rightarrow (2p - q)^2$ always a perfect square

$\therefore$ Roots are given by

$$x = \frac{-(2p-q)+\sqrt{(2p+q)^2-8pq}}{4p}$$

$$x = \frac{-2p-q\pm(2p-q)}{4p}$$

$$x = \frac{-2p-q+2p-q}{4p} = -\frac{q}{2p}$$ (rational p, q are odd integers)

Or $$x = \frac{-2p-q-2p+q}{4p} = \frac{-4p}{4p} = -1$$ which is also rational.

36. b

No of ways of selecting = $7C_3 \times 4C_2 = 210$

No. of group, each heavy 3 consonants and 2 vowels = 210

Each group contains 5 letters.

No of ways of arranging 5 letters = 5! = 210

Required no. of ways = 210 × 120 = 25200

37. b

5 digits 9 9 8 7 6

Total no. (1 to 9)

Formula = ${}^{n}p_{r} = \frac{n!}{(n-n)!}$ $\because$ 0 can't be used at first place

$9\times\frac{(9\times8\times7\times6\times5!)}{5!} \Rightarrow 9\times\frac{9!}{5!}$

Apply formula $\frac{9\times9!}{(9-4)!} = 9\times{}^{9}p_{4}$

38. b

(p + 1), (p + 2), (p + 3).....(p + q) are q consecutive numbers.

(p + 1) (p + 2) (p + 3) .... (p + q) is product of 'q' consecutive natural numbers

The product will be divisible by all its factors and since it is a product of consecutive natural numbers, so it will always be divisible by q!

Also q! is the greatest integer amongst all their divisors.

Hence, the greatest integer which divides the product is q!

39. b

Given, $((1+x)+x^2)^9 = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots + a_{18}x^{18}$ ....(1)

Put x = 1 in eq. (1), we get.

$\Rightarrow (1+1+1)^9 = a_0 + a_1 + a_2 + \ldots\ldots + a_{18}$

$\Rightarrow (3)^9 = a_0 + a_1 + a_2 + \ldots\ldots + a_{18}$ ....(2)

Put x = –1 in eq. (1) we get.

$\Rightarrow (1)^9 = a_0 - a_1 + a_2 - a_3 + \ldots\ldots a_{17} + a_{18}$ ....(3)

Adding eq. (2) & (3), we get.

$\Rightarrow 3^9 + 1 = a_0 + a_1 + \ldots + a_{18} + a_0 - a_1 + a_2 - \ldots\ldots - a_{17} + a_{18}$

$\Rightarrow 2a_0 + 2a_2 + 2a_4 + \ldots\ldots + 2a_{18}$

$\Rightarrow 2(a_0 + a_2 + a_4 + \ldots\ldots + a_{18}) = 3^9 + 1$

$\Rightarrow a_0 + a_2 + \ldots\ldots a_{18} = \frac{3^9+1}{2} \rightarrow$ even

Hence, $a_0 + a_2 + a_4 + \ldots\ldots\ldots + a_{18}$ is even

40. d

Given $8x - 3y - 5z = 0$

$5x - 8y + 3z = 0$

$3x + 5y - 8z = 0$

Use Cramer's rule

$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta}, z = \frac{\Delta z}{\Delta}$

$\Delta x = \Delta y = \Delta z$

$$D = \begin{vmatrix} 8 & -3 & -5 \\ 5 & -8 & 3 \\ 3 & 5 & -8 \end{vmatrix}$$

$D = (512 - 125 - 27) - (120 + 120 + 120)$

$D = 360 - 360 = 0$

So $\frac{\Delta x}{\Delta} = \frac{0}{0}, \frac{\Delta y}{\Delta} = \frac{0}{0}, \frac{0z}{\Delta} = \frac{0}{0}$

So infinitely many non-zero solutions

41. b

$P = \{P_1, P_2, P_3 \ldots\ldots P_n\}$ (non –singular matrix)

$Q = \{Q_1, Q_2, Q_3 \ldots\ldots Q_n\}$ (orthogonal matrix)

$Q, Q^T = I$

$\text{Det}(Q) = \pm 1$

P contains matrix whose value is non zero
Q is proper sub-sets of P.

42. b

Given $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}, B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$

$\det|AB| = \begin{vmatrix} x^2+17x & 3x^2+6x \\ 8x+10 & (x+2)^2 \end{vmatrix} = 0$

$\Rightarrow x(x+2)(x-4)(x-1) = 0$
$\Rightarrow x = 0, -2, 1, 4$

43. a

$$\text{Given } A = \begin{pmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{pmatrix}$$

$\Rightarrow R_1 \to R_1 + R_3$

$$|A| \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$$

$|A| \Rightarrow 1\ (\cos^2\theta + 1)$

$$\begin{cases} -1 \le \cos\theta \le 1 \\ 0 \le \cos^2\theta \le 1 \\ 1 \le \cos^2\theta + 1 \le 2 \end{cases}$$

$|A| \Rightarrow [1, 2]$

44. (a)

Given: $f : R \to R$ injective

$f(x)f(y) = f(x + y)$

Let $f(x) = a^x$ then $f(x)\ f(y) = a^x\ a^y = a^{x+y}$

Also $f(x) = f(y) \Rightarrow a^x = a^y$

$\Rightarrow a^{x-y} = a^0$

$\Rightarrow x - y = 0 \quad \Rightarrow x = y$

$\therefore$ a^n is injective

Now a^x, a^y, a^z are in G.P.

$\Rightarrow a^y = a^x \,.\, r$ where r is common ratio

Also $a^z = a^y r$

$$\Rightarrow \frac{a^y}{a^z} = \frac{a^x}{a^y} \Rightarrow a^{2y} = a^{x+z}$$

$$y = \frac{x+z}{2}$$

$\therefore$ x, y, z are in A.p.

45. d

Let p be the relation on the set real number R such that xpy if an only if xy ≥ 0

(a) we know that, for any real number x, $x^2 \geq 0$

$\Rightarrow xx \geq 0 \Rightarrow xpx$

$\therefore$ p 15 reflexive

(b) let $(x, y) \in p$ i.e. xpy

$\Rightarrow xy \geq 0 \;\; yx \geq 0 \Rightarrow ypx$

$\therefore$ p is symmetric

(c)Let xpy and ypz

$\Rightarrow xy \geq 0$ and $yz \geq 0$

But from this, we can't conclude x Z ≥ 0

For example

(−1, 0), (0, 2) satisfies the relation xy ≥ 0 but (−1, 2)

Doesn't satisfy relation $xy \geq 0$

Thus p is not transitive

Hence p is reflexive symmetric but not transitive

46. B

$xRy \Rightarrow x - y$ is zero or irrational.

$xRx \Rightarrow 0 \;\therefore$ Reflective

if $xRy \Rightarrow x - y$ is zero or irrational.

$\Rightarrow y - x$ is zero or irrational

$\therefore$ yR x symmetric xRy

$\Rightarrow x - y$ is 0 or irrational yRz

$\Rightarrow y - z$ is 0 or irrational then $(x - y)+(y-z)=(x-2)$ may be rational

$\therefore$ it is not transitive

47. d

Given: mean of n observation x_1, x_2 …….x_n is $\bar{x}$

Mean = $\frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}$ ……..(1)

New mean = $\frac{x_1 + ...x_{q-1} + x_{q^1} + x_{q+1} + ...x_n}{n}$

$= \frac{x_1 + ...x_{q^1} + ... + x_n + x_q - x_q}{n}$

$= \frac{\sum_{i=1}^{n} x_i - x_q + x_q'}{n}$

$= \frac{n\bar{x} - x_q + x_q'}{n}$ ……..from eq. (1)

48. b

We know non leap year – 365 days.

365 days, number of weeks = 52 and 1 day remaining 52 weeks, there will be 52 Sundays, 1 day remaining can be either Sunday, Mon, Tue, Wed, Thus, Fri & Saturday out of these 7 total out comes, the favorable outcomes are 1.

Hence the probability of getting 53 Sundays = $\frac{1}{7}$

49. d

Given: sin × (sin x + cos x) = k

$\Rightarrow \sin^2 x + \sin x \cos x = k$

$\Rightarrow \frac{1-\cos 2x}{2} + \frac{\sin 2x}{2} = k$

$\Rightarrow 1 - \cos 2x + \sin 2x = 2k$

$\Rightarrow \sin 2x - 2\cos 2x = 2k - 1$

$\Rightarrow -\sqrt{2} \le 2k - 1 \le \sqrt{2}$......$[\because -\sqrt{2} \le \sin 2x - \cos 2x \le \sqrt{2}]$

$\Rightarrow 1-\sqrt{2} \le 2k \le 1+\sqrt{2}$ $\qquad \Rightarrow \frac{1-\sqrt{2}}{2} \le k \le \frac{1+\sqrt{2}}{2}$

50. a

Given: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}.\frac{x+1}{x+2}}\right)=\frac{\pi}{4}$ {using formula of $\tan^{-1} a + \tan^{-1} b$}

$\Rightarrow \left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}.\frac{x+1}{x+2}}\right)=\tan\frac{\pi}{4}$

$\Rightarrow \frac{x-1}{x-2}+\frac{x+1}{x+2}=1-\frac{x-1}{x-2}\ \frac{x+1}{x+2}$ ....$\therefore$ $\{\tan\frac{\pi}{4}=1\}$

$\Rightarrow \frac{x^2+x-2+x^2-x-2}{x^2-4}=1-\frac{x^2-1}{x^2-4}$

$\Rightarrow 2x^2 - 4 = x^2 - 4 - x^2 + 1$

$\Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

51. (c)

Given A = {1, 2, 3}
R = {(1,1), (2,2), (3,3), (1,2), (2,1)}
S = {(1,1), (2,2), (3,3), (1,3), (3,1)}
(i) Check reflexive
Them (a,a) $\in$ R for every a $\in$ {1, 2, 3}
Since R$\cup$S = {(1,1), (2,2), (3,3), (1,2), (2,1), (1, 3), (3,1)}
R is reflexive
(ii) Check symmetric
If (a,b) $\in$ R & S, then (b, a) $\in$ R & S
Here (1,2) $\in$ R, but (2, 1) $\in$ R
$\therefore$ (1, 3) $\in$ R, but (3, 1) $\in$ S
$\therefore$ R & S symmetric
(iii) Check transitive
IF (a,b) $\in$ R & S & (b, c) $\in$ R & S, then (a, c) $\in$ R & S
Here R & S not transitive
R$\cup$S is reflexive and symmetric but not transitive.

52. (a)

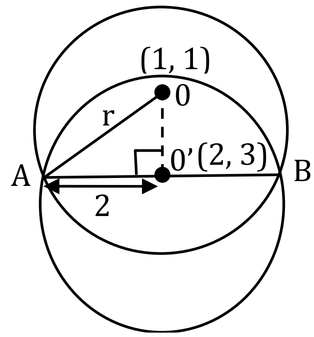
Given equation of circle is

$x^2 + y^2 - 4x - 6y + 9 = 0$

$\{ x^2 + y^2 + 2gx + 2fy + c = 0\}$

So, $g = -2, f = -3, c = 9$

Centre $(-g, -f) \Rightarrow (2, 3)$



Radius $r = \sqrt{f^2 + g^2 - c}$

$r = \sqrt{(-3)^2 + (-2)^2 - 9}$

$r = \sqrt{9 + 4 - 9}$

$r = \sqrt{4} = 2$

Chord $(0', 0) = \sqrt{(1-2)^2 + (1-3)^2}$

$= \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

We apply Pythagoras theorem

In $\Delta OO'A \Rightarrow O'O^2 + O'A^2 = OA^2 = R^2$

$R = \sqrt{(\sqrt{5})^2 + (2)^2} = \sqrt{5+4}$

$R = 3$

53. (d)

Given $A(-1, 0)$, $B(2, 0)$

We know that $\angle MBA = 2\angle MAB$

Also $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

Where m_1 & m_2 are the slopes of two lines and θ is the angle between two line.

Given $\theta = 2\phi \Rightarrow \tan\theta\ 2\phi$

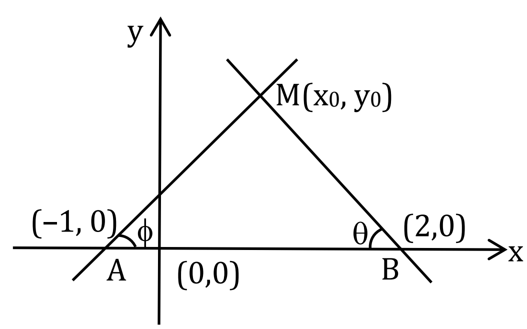
$\Rightarrow \tan\theta = \dfrac{2\tan\phi}{1-\tan^2\phi} \Rightarrow \dfrac{-y_0}{x_0 - 2} = \dfrac{2y_0(x_0+1)}{(x_0+1)^2 - y_0^2}$

$\Rightarrow \dfrac{-1}{x_0 - 2} = \dfrac{2(x_0+1)}{(x_0+1)^2 - y_0^2}$

$\Rightarrow 3x_o^2 - y_o^2 = 3$

$\therefore$ Locus of M is hyperbola $3x^2 - y^2 = 3$

Now verify alternatives.



54. (c)

Given $f(x) = \int_{-1}^{x} |t|\,dt,\ x \geq 0$

$$\left\{\begin{array}{c}\text{Formula apply} \\ \int_a^b f(x)\,dx \\ \Rightarrow \int_a^c f(x)\,dx + \int_c^b f(x)\,dx\end{array}\right\}$$

$$\Rightarrow f(x) = \int_{-1}^{0} -t\,dt + \int_{0}^{x} t\,dt$$

$$\Rightarrow f(x) = \left[\frac{-t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_{0}^{x}$$

$$f(x) \Rightarrow 0 - \left(-\frac{1}{2}\right) + \frac{x^2}{2} - 0 \Rightarrow \frac{1}{2} + \frac{x^2}{2} = \frac{1}{2}(1 + x^2)$$

55. (b)

$$f(x) = \lim_{x\to\infty} n\left(x^{\frac{1}{x}} - 1\right) \qquad \text{Let } n = \frac{1}{t} \quad \begin{array}{c} n\to\infty \\ t\to 0\end{array}$$

$$\Rightarrow \lim_{t\to 0} \frac{1}{t}(x^t - 1)$$

$$\Rightarrow \lim_{t\to 0} \frac{x^t - 1}{t} = \frac{x^0 - 1}{0} = \frac{0}{0}$$

L-Hospital

$$\lim_{t\to 0} \frac{x^t \ln x - 0}{1} = x^0 \ln x = 1 \ln x$$

$f(x) = \ln x$

$\Rightarrow f(xy) = \ln(xy) = \ln x + \ln y$

$\Rightarrow f(xy) = f(x) + f(y)$

56. (b)

Given I = $\int_0^{100\pi} \sqrt{(1-\cos 2x)dx}$

cos 2x = 1 – 2sin^{2}x

1–cos2x = 2sin^{2}x

$\Rightarrow \int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$

I = $\int_0^{100\pi} \sqrt{2\sin^2 xdx}$

$\Rightarrow 100\int_0^{\pi} \sqrt{2}\sin xdx$

$\Rightarrow 100\sqrt{2}\int_0^{\pi} \sin xdx$

$\Rightarrow 100\sqrt{2}[-\cos x]_0^{\pi}$

$\Rightarrow 100\sqrt{2}[-\cos x + \cos 0]$

$\Rightarrow 100\sqrt{2}[1+1]$

$\Rightarrow 200\sqrt{2}$

57. (a)

x = – 2y^2 .....(1)

x = 1 – 3y^2 .....(2)

Equation (1) & (2)

$\Rightarrow -2y^2 = 1-3y^2$

$\Rightarrow -2y^2 + 3y^2 = 1$

$\Rightarrow y^2 = 1$

$\Rightarrow y = \pm 1$

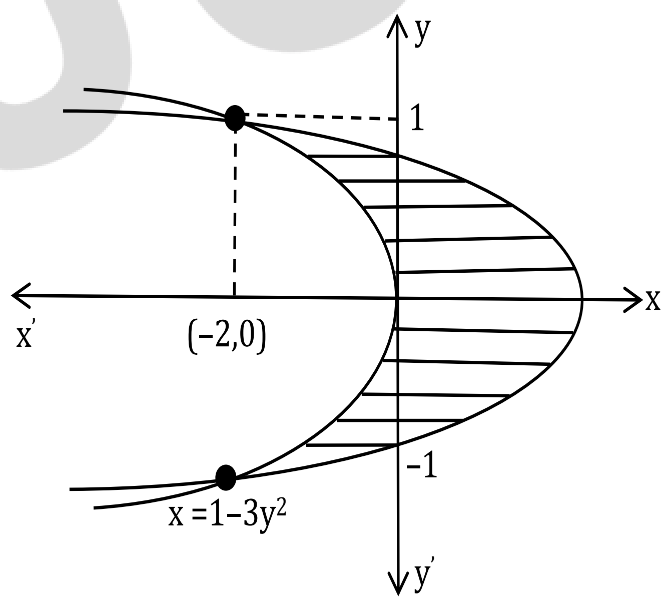
So area of region bounded by

A = > 2 $\int_0^1 [(1-3y^2)-(-2y^2)]dy$

$\Rightarrow 2\int_0^1 (1-y^2)dy$

$\Rightarrow 2\left[y-\frac{y^3}{3}\right]_0^1 \Rightarrow 2\left[\left(1-\frac{1}{3}\right)-\left(0-\frac{0}{3}\right)\right]$

$\Rightarrow 2\left[\frac{1}{1}-\frac{1}{3}\right] \Rightarrow 2\left[\frac{2}{3}\right] = \frac{4}{3}$ square units



58. (a)

$$\frac{x^2}{9}+\frac{y^2}{5}$$

Now $e^2 - 1 - \frac{5}{9} = \frac{4}{9} \therefore e = \frac{2}{3}$

Equation of tangent at [2, (5/3)] is given as

$\Rightarrow \left(\frac{2x}{9}\right)+\left(\frac{4}{3}\right) = 1$

$\Rightarrow$ Let F and f' be foci

$\Rightarrow$ area of $\Delta CPQ = \left(\frac{1}{2}\right)\times\left(\frac{9}{2}\right)\times 3 = \frac{27}{4}$

$\Rightarrow$ area of $\square$PQRS = $4 \times \frac{27}{4}$ = 27 sq. units.

59. (c)

Given: f(x) = sin x – cos x – kx + 5

f(x) decrease for all x if f'(x) < 0

f'(x) = cos x + sin x – k < 0

$\therefore$ k > cos x + sin x

We know, cos x + sin x = $\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)$

$\Rightarrow \sqrt{2}\left(\sin\frac{\pi}{4}\cos x+\cos\frac{\pi}{4}\sin x\right) = \sqrt{2}\left(\sin\left(\frac{\pi}{4}+x\right)\right)$ and

$-1 \le \sin\left(\frac{\pi}{4}+x\right) \le 1 \Rightarrow -\sqrt{2} \le \sqrt{2}\sin\left(\frac{\pi}{4}+x\right) \le \sqrt{2}$

$\therefore -\sqrt{2} \le \cos x + \sin x \le \sqrt{2}$

Max (cos x +sin x) = $\sqrt{2}$

$\therefore k > \sqrt{2}$

60. (b)

Let $x = x_1i + x_2j + x_3k$

Then $x \times i = -x_2k + x_3j$

$x \times j = x_1k - x_3i$

$x \times k = -x_1j + x_2i$

$\therefore\ (x \times i)^2 + (x \times j)^2 + (x \times k)^2$

$\Rightarrow\ x_2^2 + x_3^2 + x_1^2 + x_3^2 + x_1^2 + x_2^2$

$\Rightarrow 2\ (x_1^2 + x_2^2 + x_3^2) \Rightarrow 2x^2$

$\Rightarrow 2|x|^2$

61. (c)

Let $\vec{a}, \vec{b}$ be two unit vectors whose sum is also

a unit vector $\vec{c}$

$\vec{a} + \vec{b} = \vec{c}$ .....(1)

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Now, magnitude of sum of $\vec{a}$ and $\vec{b}$ is

$\Rightarrow |\vec{a} + \vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 (\vec{a}.\vec{b})$

$\Rightarrow 1 = 1 + 1 + 2 (\vec{a}.\vec{b})$ ......(2)

$\Rightarrow \vec{a}.\vec{b} = \frac{1}{2}$

Magnitude of their difference is given by

$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\ \vec{b}$

$= 1 + 1 + 2 \times \frac{1}{2} = 3$

$\Rightarrow |a - b| = \sqrt{3}$

62. (a)

$x^2 + x + 1 = 0$ ....(1)

Roots of this equation is

$$x = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

α and β are the roots of eq.(1)

So $\alpha = \frac{-1+\sqrt{3}i}{2}$ and $\beta = \frac{-1-\sqrt{3}i}{2}$

Consider, $\alpha^n + \beta^n$

$$= \left(\frac{-1+\sqrt{3}i}{2}\right)^n + \left(\frac{-1-\sqrt{3}i}{2}\right)^n$$

$$= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^n + \left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)^n$$

$$= \left(e^{i\frac{2\pi}{3}}\right)^n + \left(e^{-i\frac{2\pi}{3}}\right)^n \ldots\ldots \left\{\begin{array}{c} \therefore \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \text{ and} \\ \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2} \end{array}\right\}$$

$$= e^{i\frac{2n\pi}{3}} + e^{-i\frac{2n\pi}{3}}$$

$$= \cos\left(\frac{2n\pi}{3}\right) + i\sin\left(\frac{2n\pi}{3}\right) + \cos\left(\frac{2n\pi}{3}\right) - i\sin\left(\frac{2n\pi}{3}\right)$$

$$= 2\cos\left(\frac{2n\pi}{3}\right)$$

Hence $\alpha^n + \beta^n = 2\cos\left(\frac{2n\pi}{3}\right)$

63. (c)

$y = \dfrac{x^2+2x+y}{2x^2+4x+9}$

$\Rightarrow 2x^2y + 4xy + 9y = x^2 + 2x + 4$

$\Rightarrow x^2(2y-1) + x\,(4y-2) + 9y-4 = 0$

$\Delta \geq 0$

$\Rightarrow (4y-2)^2 - 4\,(2y-1)\,(9y-4) \geq 0$

$\Rightarrow 4(2y-1)^2-4(2y-1)\,(9y-4) \geq 0$

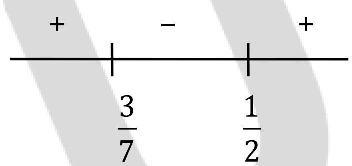
$\Rightarrow 4(2y-1)\,[2y-1-9y+4] \geq 0$

$\Rightarrow 4(2y-1)\,(3-7y) \geq 0$

$\Rightarrow 4(2y-1)\,(7y-3) \leq 0$

$\Rightarrow \dfrac{3}{7} \leq y \leq \dfrac{1}{2}$

$[y_{\text{greatest}} = \dfrac{1}{2}]$



64. (b)

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & n & n\left(\dfrac{n+1}{2}\right) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ The statement is demoted by P(n)

$P(n) : A^n = \begin{bmatrix} 1 & n & \dfrac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore\ P(1) : A^1 \begin{bmatrix} 1 & 1 & 1\frac{(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ P(1) is true

$$\text{Again } P(2) : A^2 = A \times A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & \frac{2(2+1)}{2} \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ P(2) is true.

Let P(m) b true

$$P(m) : A^m = \begin{bmatrix} 1 & m & m(m+1) \\ 0 & 1 & m \\ 0 & 0 & 1 \end{bmatrix}$$

$$P\,(m+1) = A^{m+1} = A^m\,.A = \begin{bmatrix} 1 & m & \frac{m(m+1)}{2} \\ 0 & 1 & m \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1+m & 1+m+\frac{m(m+1)}{2} \\ 0 & 1 & 1+m \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & (m+1) & \frac{(m+1)(m+2)}{2} \\ 0 & 1 & m+1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{m+1} = \begin{bmatrix} 1 & (m+1) & \frac{(m+1)\{(m+1)+1\}}{2} \\ 0 & 1 & (m+1) \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ P(m + 1) is true

$\Rightarrow$ P(1) & P(2) is true and p(m + 1) is true when p(m) is true

$\Rightarrow$ By mathematical induction method, we can say that p(n) is also true i.e.

$$A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

65. (d)

The 2[nd] det $D_2 = \begin{bmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^n a & -(-1)^n b & (-1)^n c \end{bmatrix}$

$$= (-1)^n \begin{bmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{bmatrix}$$

$= (-1)^n D$

$\therefore R_{13}, R_{23}$ and taking transpose

$\therefore$ $(1+(-1)^n)D_1 = 0$ for any odd integer

$\because$ $D_1 \neq 0$ since $b(a + c) \neq 0$

66. (b)

f(x) is continuous and differentiable

f(0) = f(1) = 0 by Rolle's theorem

$f'(a) = 0$, a belongs to (0, 1)

Given $f'(0) = 0$ by Rolle's theorem

$f''(0) = 0$ for some, c belongs to (0, a)

Therefore answer is $f''(c) = 0$ for some $C \in R$

67. (b, c)

We have $f(x) = x^n$

$\Rightarrow f'(x) = nx^{n-1}$

$\therefore f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$ for all $\alpha, \beta > 0$

$\Rightarrow n(\alpha + \beta)^{n-1} = n\alpha^{n-1} + n\beta^{n-1}$ for all $\alpha, \beta > 0$

$\Rightarrow (\alpha + \beta)^{n-1} = \alpha^{n-1} + \beta^{n-1}$ for all $\alpha, \beta > 0$

$\Rightarrow n - 1 = 1 \Rightarrow n = 2$

Also, for n = 0, we have

$f(x) = 1$ for all x
$\Rightarrow f'(x) = 0$ for all x
$\therefore f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$
Hence, n = 0, 2

68. (c)
Given: $f(x) = f(a - x) = 1$

Let $I = \int_0^a \frac{dx}{1+f(x)}$ $\qquad \Rightarrow \int_0^a \frac{f(x)dx}{1+f(x)}$

$\Rightarrow \int_0^a \frac{1+f(x)-1}{1+f(x)}dx$ $\qquad \Rightarrow \int_0^a dx - \int_0^a \frac{dx}{1+f(x)}$

$\Rightarrow 2I = \int_0^a dx$

$\Rightarrow 2I = |x|_0^a$

$\Rightarrow I = \frac{a}{2}$

69. (b, d)

Given: ax + by + c = 0

By = – ax – c

$y = -\frac{a}{b}x - \frac{c}{b}$

m = slope = $-\frac{a}{b}$

Tangent slope $\left(\frac{dy}{dx}\right)$

$-\frac{a}{b} = -\frac{1}{x^2}$

$x^2 = \frac{b}{a}$ $\quad\quad$ $x^2 > 0$

$\frac{b}{a} > 0$ $\quad$ if (1) $\quad$ b > 0, a > 0

& (2) $\quad$ b < 0, a < 0

$xy = 1 - 2x \rightarrow xy + 2x = 1$	
$x\frac{dy}{dx} + y = -2$	$x(y+2) = 1$
$x\frac{dy}{dx} = -(y+2)$	$y + 2 = \frac{1}{x}$
$\frac{dy}{dx} = \frac{-1}{x^2}$	

70. (b, c)

Let S $\rightarrow$ distance at time t

Particle 1 is $x_1 = u$

Particle 2 is $x_2 = \frac{1}{2}ft^2$

Then $S = x_1 - x_2$

$= u + (-\frac{1}{2}ft^2)$ $\quad\quad$ ……… (1)

For s being maximum

$\frac{ds}{dt} = 0$ $\quad \Rightarrow u - ft = 0 \quad \Rightarrow t = \frac{u}{f}$

(b) Option correct.

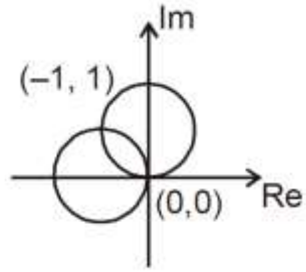
Substituting t in $eq^n(1)$ we get

$S = \frac{u}{2f}$ this is greatest distance

(c option correct)

71. (a, c)

Given: $|Z - i| = |Z + 1| = 1$



Let $Z = x + iy$

$|x + iy - i| = 1$

$\Rightarrow x^2 + (y - 1)^2 = 1$

$\Rightarrow x^2 + y^2 - 2y + 1 = 1$

$x^2 + y^2 - 2y + 1 = 1$

$x^2 + y^2 - 2y = 0$ ....(1)

Also $|z + 1| = 1$

$\Rightarrow |x + iy + 1| = 1$

$\Rightarrow (x + 1)^2 + y^2 = 1$

$\Rightarrow x^2 + y^2 + 2x = 0$ ......(2)

Eq.(1) and (2) we get

$-y = x$

For $y = 1, x = 1$

$\therefore z = -1 + i$ (c option is correct)

Also $y = 0 \Rightarrow x = 0$

$\therefore z = 0$ (a option is correct)

72. (c)

Reflexive: if$(x, x) \in R$, where x is domain

Relation = $\{1 + ab > 0$ where $a, b \in R\}$

For any $a \in R$, $a^2 \geq 0$

So, $1 + a^2 \geq 1$

Or, $1 + a^2 > 0$

$\Rightarrow 1 + a.a > 0$

$\Rightarrow (a,a) \in R$ its relation is Reflexive

Symmetric: if $(y,x) \in R$ where $(x,y) \in R$.

Relation $(1 + ab > 0$ where $a,b \in R$

Let $(a,b) \in R$ $a,b \in R$

So, 1 + ab > 0

$\Rightarrow$ 1 + ba > 0

$\Rightarrow (b,a) \in R$ is symmetric

Transitive :- if $(x,z) \in R$, where $(x,y) \in R$ and $(y,z) \in R$

Relation {1 + ab > 0 where $a, b \in R$ }

Let's take (–2, 0) and (0, 1) such that 1 + – 2 × 0 > 0

And 1 + 0 × 1 > 0 so, (–2, 0) and (0, 1) belongs to R.

But (–2, 1) doesn't belongs to R because 1 + – 2 × 1 < 0

This doesn't follow transitive.

Relation is not transitive.

73. (c, d)

$ax^2 + bx + 1 = 0$

a = a, b = b, c = 1

$D = b^2 - 4a$

$D \geq 0$

$b^2 - 4a \geq 0$ $\{\therefore$ a, b $\in$ (1, 2, 3)$\}$

$b^2 \geq 4a$

(a, b) = (1, 2) (2, 3) (1, 3) possible order pairs 3

And a < b

74. (a, c)

$y^2 = 4ax$

Tangent $y = \frac{x}{t} + at$ and normal x cosθ + ycotθ = 2a

So (at^2, 2at) must satisfy equation of normal

$\Rightarrow t^2\cos\theta + 2 + x\cot\theta - 2 = 0$

So, the roots are t = –cosecθ and t = 2tanθ

75. (c)

Given conic is $x^2 - 6x + 4y + 1 = 0$

$\Rightarrow x^2 - 6x + 9 - 9 + 4y + 1 = 0$

$\Rightarrow x^2 - 3x - 3x + 9 - 9 + 4y + 1 = 0$

$\Rightarrow x(x - 3) - 3(x - 3) + 4y - 8 = 0$

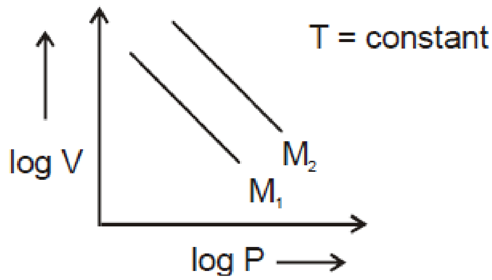
$\Rightarrow (x - 3)^2 + 4y - 8 = 0$

$\Rightarrow (x - 3)^2 = 4(-1)(y - 2)$

Thus, the focus is (3, 1)

WBJEE 2017 (Chemistry)

1. For same mass of two different ideal gases of molecular weights M_1 and M_2, plots of log V vs log P at a given constant temperature are shown. Identify the correct option



a. $M_1 > M_2$
b. $M_1 = M_2$
c. $M_1 < M_2$
d. Can be predicted only if temperature is known

2. Which of the following has the dimension of ML^0T^{-2} ?
a. Coefficient of viscosity
b. Surface tension
c. Vapour pressure
d. Kinetic energy

3. If the given four electronic configurations
(i) n = 4, l = 1 (ii) n = 4, l = 0 (iii) n = 3, l = 2 (iv) n = 3, l = 1
are arranged in order of increasing energy, then the order will be
a. (iv) < (ii) < (iii) < (i)
b. (ii) < (iv) < (i) < (iii)
c. (i) < (iii) < (ii) < (iv)
d. (iii) < (i) < (iv) < (ii)

4. Which of the following sets of quantum numbers represents the 19th electron of Cr (Z = 24)?
a. $\left(4,1-1,+\frac{1}{2}\right)$
b. $\left(4,0,0,+\frac{1}{2}\right)$
c. $\left(3,2,0,-\frac{1}{2}\right)$
d. $\left(3,2,-2,+\frac{1}{2}\right)$

5. 0.126 g of an acid is needed to completely neutralize 20 ml 0.1 (N) NaOH solution. The equivalent weight of the acid is
   a. 53
   b. 40
   c. 45
   d. 63

6. In a flask, the weight ratio of CH_4 (g) and SO_2(g) at 298 K and 1 bar is 1 : 2. The ratio of the number of molecules of SO_2(g) and CH_4(g) is
   a. 1 : 4
   b. 4 : 1
   c. 1 : 2
   d. 2 : 1

7. $C_6H_5F_{18}$ is a F18 radio-isotope labelled organic compound. F18 decays by positron emission. the product resulting on decay is :
   a. $C_6H_5O_{18}$
   b. $C_6H_5Ar_{19}$
   c. $B_{12}C_5H_5F$
   d. $C_6H_5O_{16}$

8. Dissolving NaCN in de-ionized water will result in a solution having
   a. pH < 7
   b. pH = 7
   c. pOH = 7
   d. pH > 7

9. Among Me_3N, C_5H_5N and MeCN (Me = methyl group) the electronegativity of N is in the order
   a. MeCN > C_5H_5N > Me_3N
   b. C_5H_5N > Me_3N > MeCN
   c. Me_3N > MeCN > C_5H_5N
   d. Electronegativity same in all

10. The shape of XeF_5^- will be
    a. Square pyramid
    b. Trigonal bipyramidal
    c. Planar
    d. Pentagonal bipyramid

11. The ground state magnetic property of B_2 and C_2 molecules will be
    a. B_2 paramagnetic and C_2 diamagnetic
    b. B_2 diamagnetic and C_2 paramagnetic
    c. Both are diamagnetic
    d. Both are paramagnetic

12. The number of unpaired electrons in $[NiCl_4]^{2-}$, $Ni(CO)_4$ and $[Cu(NH_3)_4]^{2+}$ respectively are
    a. 2, 2, 1
    b. 2, 0, 1
    c. 0, 2, 1
    d. 2, 2, 0

13. Which of the following atoms should have the highest 1st electron affinity?
    a. F
    b. O
    c. N
    d. C

14. $PbCl_2$ is insoluble in cold water. Addition of HCl increases its solubility due to
    a. Formation of soluble complex anions like $[PbCl_3]^-$
    b. Oxidation of Pb(II) to Pb (IV)
    c. Formation of $[Pb(H_2O)_6]^{2+}$
    d. Formation of polymeric lead complexes

15. Of the following compounds, which one is the strongest Bronsted acid in aqueous solution?
    a. $HClO_3$
    b. $HClO_2$
    c. $HOCl$
    d. $HOBr$

16. The correct basicity order of the following lanthanide ions is
    a. $La^{3+} > Lu^{3+} > Ce^{3+} > Eu^{3+}$
    b. $Ce^{3+} > Lu^{3+} > La^{3+} > Eu^{3+}$
    c. $Lu^{3+} > Ce^{3+} > Eu^{3+} > La^{3+}$
    d. $La^{3+} > Ce^{3+} > Eu^{3+} > Lu^{3+}$

17. When $BaCl_2$ is added to an aqueous salt solution, a white precipitate is obtained. The anion among CO_3^{2-} , SO_3^{2-} and SO_4^{2-} that was present in the solution can be :
    a. CO_3^{2-} but not any of the other two
    b. SO_3^{2-} but not any of the other two
    c. SO_4^{2-} but not any of the other two
    d. Any of them

18. In the IUPAC system, $PhCH_2CH_2CO_2H$ is named as
    a. 3-phenylpropanoic acid
    b. benzylacetic acid
    c. carboxyethylbenzene
    d. 2-phenylpropanoic acid

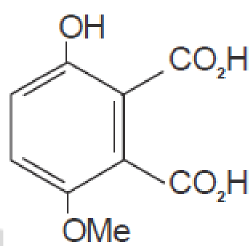
19. The isomerisation of 1-butyne to 2-butyne can be achieved by treatment with
    a. hydrochloric acid
    b. ammonical silver nitrate
    c. ammonical cuprous chloride
    d. ethanolic potassium hydroxide

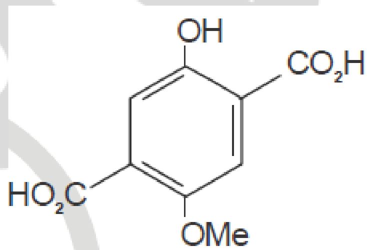
20. The correct order of acid strengths of benzoic acid (X), peroxybenzoic acid (Y) and p-nitrobenzoic acid (Z) is
    a. Y > Z > X
    b. Z > Y > X
    c. Z > X > Y
    d. Y > X > Z

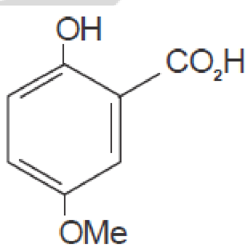
21. The yield of acetanilide in the reaction (100% conversion) of 2 moles of aniline with 1 mole of acetic anhydride is

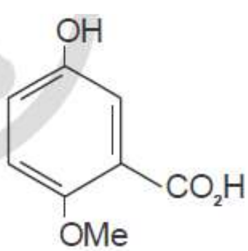
a. 270 g
b. 135 g
c. 67.5 g
d. 177 g

22. The structure of the product P of the following reaction is

a.


b.


c.


d.


23. ADP and ATP differ in the number of

 a. phosphate units

 b. ribose units

 c. adenine base

 d. nitrogen atom

24. The compound that would produce a nauseating smell/odour with a hot mixture of chloroform and ethanolic potassium hydroxide is

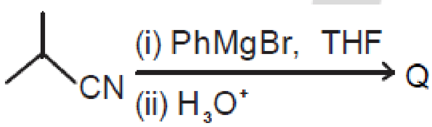
 a. $PhCONH_2$

 b. $PhNHCH_3$

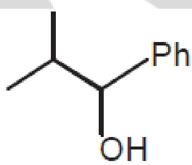
 c. $PhNH_2$

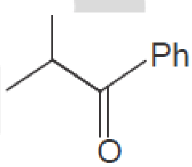
 d. PhOH

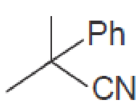
25. For the reaction below

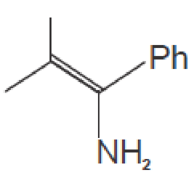


the structure of the product Q is

a. 

b. 

c. 

d. 

26. You are supplied with 500 ml each of 2N HCl and 5N HCl. What is the maximum volume of 3M HCl that you can prepare using only these two solutions?
   a. 250 ml
   b. 500 ml
   c. 750 ml
   d. 1000 ml

27. Which one of the following corresponds to a photon of highest energy?
   a. $\lambda = 300$ nm
   b. $\nu = 3 \times 108\ s^{-1}$
   c. $\nu = 30\ cm^{-1}$
   d. $\varepsilon = 6.626 \times 10^{-27}$ J

28. Assuming the compounds to be completely dissociated in aqueous solution, identify the pair of the solutions that can be expected to be isotonic at the same temperature :
   a. 0.01 M Urea and 0.01 M NaCl
   b. 0.02 M NaCl and 0.01 M Na_2SO_4
   c. 0.03 M NaCl and 0.02 M $MgCl_2$
   d. 0.01 M Sucrose and 0.0_2 M glucose

29. How many faradays are required to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} in acid medium?
   a. 2
   b. 3
   c. 5
   d. 6

30. Equilibrium constants for the following reactions at 1200 K are given :
   $2H_2O(g) \rightleftharpoons 2H_2(g) + O_2(g)$; $K_1 = 6.4 \times 10^{-8}$
   $2CO_2(g) \rightleftharpoons 2CO(g) + O_2(g)$; $K_2 = 1.6 \times 10^{-6}$
   The equilibrium constant for the reaction
   $H_2(g) + CO_2(g) \rightleftharpoons CO(g) + H_2O(g)$ at 1200 K will be
   a. 0.05
   b. 20
   c. 0.2
   d. 5.0

31. In a close-packed body-centred cubic lattice of potassium, the correct relation between the atomic radius (r) of potassium and the edge-length (a) of the cube is

a. $r = \frac{a}{\sqrt{2}}$

b. $r = \frac{a}{\sqrt{3}}$

c. $r = \frac{\sqrt{3}}{2}a$

d. $r = \frac{\sqrt{3}}{4}a$

32. Which of the following solutions will turn violet when a drop of lime juice is added to it?

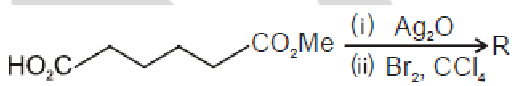
a. A solution of NaI

b. A solution mixture of KI and $NaIO_3$

c. A solution mixture of NaI and KI

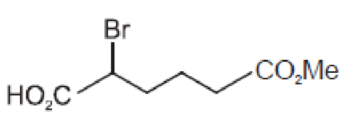
d. A solution mixture of KIO_3 and $NaIO_3$

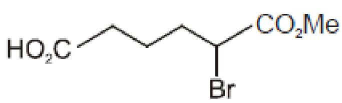
33. The reaction sequence given below gives product R



The structure of the product R is

a. 

b. 

c. 

d. 

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34. Reduction of the lactol S  S with sodium borohydride gives

a. 

b. 

c. 

d. 

35. What will be the normality of the salt solution obtained by neutralizing x ml y (N) HCl with y ml x (N) NaOH, and finally adding (x + y) ml distilled water?

a. $\frac{2(x+y)}{xy}N$

b. $\frac{xy}{2(x+y)}N$

c. $\left(\frac{2xy}{x+y}\right)N$

d. $\left(\frac{x+y}{xy}\right)N$

36. During electrolysis of molten NaCl, some water was added. What will happen?

a. Electrolysis will stop

b. Hydrogen will be evolved

c. Some amount of caustic soda will be formed

d. A fire is likely

37. The role of fluorspar, which is added in small quantities in the electrolysis reduction of alumina dissolved in fused cryolite, is

a. as a catalyst

b. to make fused mixture conducting

c. to lower the melting temperature of the mixture

d. to decrease the rate of oxidation of carbon at anode

38. The reduction of benzenediazonium chloride to phenyl hydrazine can be accomplished by

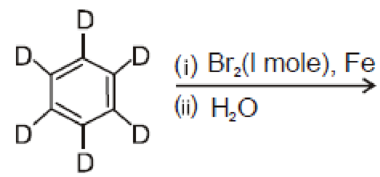
a. $SnCl_2$, HCl

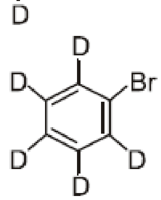
b. Na_2SO_3

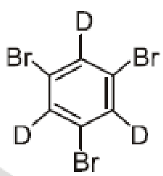
c. CH_3CH_2OH

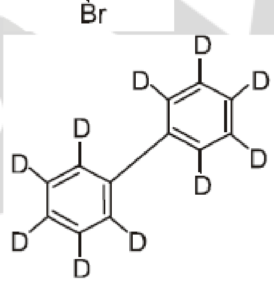
d. H_3PO_2

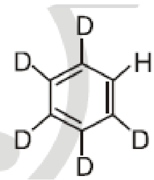
39. The major product(s) obtained from the following reaction of 1 mole of hexadeuterobenzene is/are



a. 

b. 

c. 

d. 

40. Identify the correct statement(s) :
The findings from the Bohr model for H-atom are

a. Angular momentum of the electron is expressed as integral multiples of $\frac{h}{2\pi}$

b. The first Bohr radius is 0.529A°

c. The energy of the n-th level E_n is proportional to $\frac{1}{n^2}$

d. The spacing between adjacent levels increases with increase in 'n'

WBJEE 2017 (Chemistry)

ANSWER KEYS

1. (a)	2. (b)	3. (a)	4. (b)	5. (d)	6. (c)	7. (a)	8. (d)	9. (a)	10. (c)
11. (a)	12. (b)	13. (a)	14. (a)	15. (a)	16. (d)	17. (d)	18. (a)	19. (d)	20. (c)
21. (b)	22. (c)	23. (a)	24. (c)	25. (b)	26. (c)	27. (a)	28. (c)	29. (d)	30. (d)
31. (d)	32. (b)	33. (d)	34. (c)	35. (b)	36. (b,c,d)	37. (b,c)	38. (a,b)	39. (a)	40. (a,b,c)

WBJEE 2017 (Chemistry)

Solution

1. (a)

Ideal gas equation PV = nRT ....(1)

Mole(x) = $\frac{wt}{Mw} = \frac{W}{M}$

So we can write from eq. (1) PV = $\frac{W}{M}$RT .....(2)

$\because$ here w, R & T $\rightarrow$ constant

So let w RT = k (new constant)

Now from equation (2) we get

PV = $\frac{K}{M}$ ....(3)

Taking log of both side log (PV) = log $\left(\frac{K}{M}\right)$

$\Rightarrow$ log P + log V = log $\frac{K}{M}$

$\Rightarrow$ log V = –log P + log $\left(\frac{K}{M}\right)$ .....(4) [y = mx + c]

Here + log $\frac{K}{M}$ $\rightarrow$ intercept

Now we can write the intercept for ideal gas (1) & (2) by the help of equation (1)

Ideal gas Having M_1 Molar wt. $\longrightarrow$ Intercept = log $\left(\frac{K}{M_1}\right)$

Ideal gas having M_2 mw $\rightarrow$ intercept = + log $\left(\frac{K}{M_2}\right)$

From the graph we see, log $\left(\frac{K}{M_2}\right) > \log\left(\frac{K}{M_1}\right)$

$\Rightarrow \frac{K}{M_2} > \frac{K}{M_1}$

$\Rightarrow M_1 > M_2$

Hence correct option is (a)

2. (b)

Surface tension = Work done per unit area

$= \dfrac{dw}{dA} = \dfrac{F.dx}{dA}$ .....(1) {$\because$ dw = fdx

dx $\rightarrow$ distance /length $\rightarrow$ m

dA $\rightarrow$ Area $\rightarrow$ m^2

$\because$ F = Ma = $\dfrac{kg \times m}{sec^2}$ ....(2)

Eq. (2) $\rightarrow$ Eq. (1)

Surface tension = $\dfrac{\frac{kg \times m}{sec^2} \times m}{m^2}$ = kg sec^{-2}

Therefore the dimension of the surface tension is

$[M^1L^0T^{-2}]$

Hence correct option (b).

3. (a)

According n + ℓ rule, n + ℓ value increases, energy of orbital also increases

Hence (1) n + ℓ = 4 + 1 = (5) $\Rightarrow$ 4p orbital

(2) n + ℓ = 4 + 0 = (4) $\Rightarrow$ 4s orbital

(3) n + ℓ = 3 + 2 = (5) $\Rightarrow$ 3d orbital

(4) n + ℓ = 3 + 2 = (5) $\Rightarrow$ 3d orbital

If for any two cases n + ℓ values are equal, then whose orbital which have lower n value, their orbital energy also lower.

$\rightarrow$ Hence overall order of energy = 1 > 3 > 2 > 4

So correct option is (a)

4. (b)

Electronic configuration of Cr (Atomic number 24)

$$\Rightarrow \underbrace{\underset{2}{\underline{1s^2}}\ \ \underset{8}{\underline{2s^2\ 2p^6}}\ \ \underset{8}{\underline{3s^2\ 3p^6}}}_{18e^{\ominus}}\ \ \underbrace{\underline{4s^2}}_{19e^{\ominus}}\quad 3d^5$$

Or $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^5\,4s^1 \rightarrow$ n = 4, ℓ = 0, s = + $\dfrac{1}{2}$

$\rightarrow$ Here $19e^{\ominus}$ enter in 4s orbital because 4s orbital have lower eng compared to 3d orbital

Hence correct option is (b)

5. (d)

Number of eq. of acid = No. of eq. of base

$$= \frac{N \times V}{1000} = \frac{0.1 \times 20}{1000} = 2 \times 10^{-3} \left\{ \begin{array}{c} N = \dfrac{\text{No.of eq.of solute}}{V(\text{Liter})} \\ \text{or} \\ N = \dfrac{\text{No.of eq.} \times 1000}{V(\text{me})} \end{array} \right.$$

$\Rightarrow 2 \times 10^{-3}$ equivalents have mass = 0.126 g

Hence, mass of 1 equivalent = $\frac{0.126}{2 \times 10^{-3}} = 63$ g

So correct option is (d)

6. (c)

Wt. ratio $W_{CH_4} : W_{SO_2} = 1:2$

Moles (n) = $\frac{\text{wt.}}{\text{Mw}}$ $\left\{ \begin{array}{l} \text{Mw of } SO_2 = 64 \text{ g mol}^{-1} \\ \text{Mw of } CH_4 = 16 \text{ g mol}^{-1} \end{array} \right.$

$$\frac{n_{SO_2}}{n_{CH_4}} = \frac{W_{SO_2}}{M_{SO_2}} \times \frac{M_{CH_4}}{W_{CH_4}} = \frac{2}{64} \times \frac{16}{1} = \frac{1}{2}$$

So mole ratio of SO_2: CH_4 = 1 : 2

Hence correct option is c.

7. (a)

Positron emission by ${}_9F^{18}$ follows as

${}_9F^{18} \longrightarrow {}_8O^{18} + {}_{+1}e^0$

$\rightarrow$ When an element emit a positron [Beta particle (β^+)] decreases the no. of proton by one. and increases the neutron by one. While mass number A remains the same.

So resulting product is as $\Rightarrow C_6 H_6 O^{18}$

Hence correct option is (a)

8. (d)

On dissolving NaCN

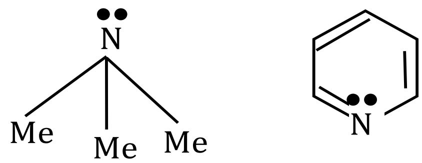
$$NaCN + H_2O \rightarrow \underset{(\text{Strong base})}{NaOH} + HCN \rightarrow \text{Weak acid}$$

Hence pH > 7

So correct option is (d)

9. (a)

$\rightarrow$ As the % S ($\uparrow$) E.N. ($\uparrow$)



Me C $\equiv$ N:

$\Rightarrow$ sp^3 | sp^2 | sp

% S $\Rightarrow$ (25%) | (33.33 %) | (50 %)

Character

Hence order of E.N. MeCN > C_5H_5N > Me_3N

So correct option is (A)

10. (c)

Shape of $XeF_5^{\ominus}$:

No. of electron pair = $\frac{8+5+1}{2} = 7 \rightarrow sp^3d^3$ hybridisation

$\Rightarrow np + \ell p = 7$

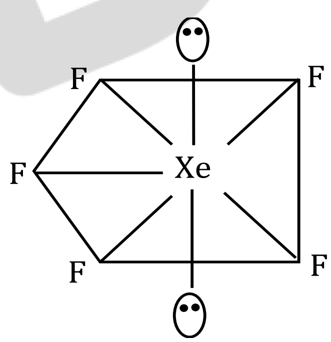
$\ell p = 7 - 5 = 2$

$\Rightarrow \ell p = 2$

<u>Shape of molecule which have sp^3d^3 hybridisation and 2 ℓp</u> –

$\Rightarrow$ ℓp present at axial to minimize the repulsion

$\Rightarrow$ Hence shape is planar & geometry pentagonal bipyramidal

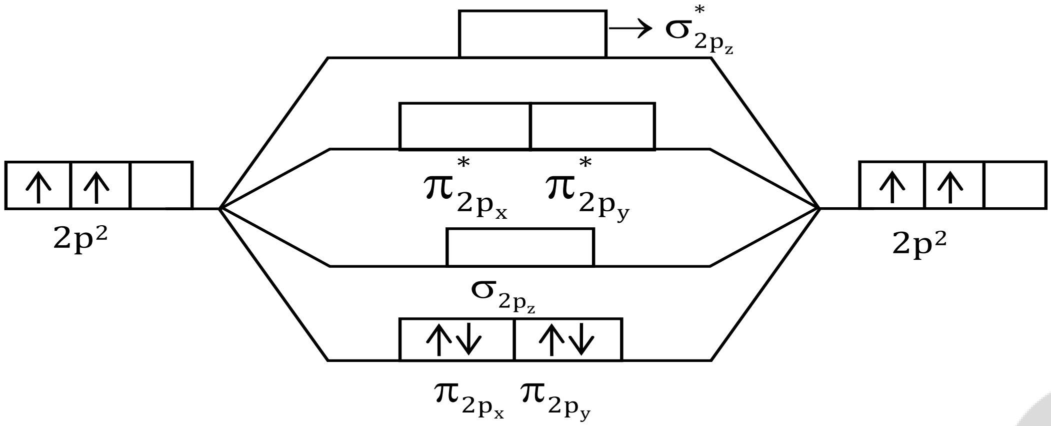


Geometry $\rightarrow$ Pentagonal bipyramidal

Shape $\rightarrow$ planar

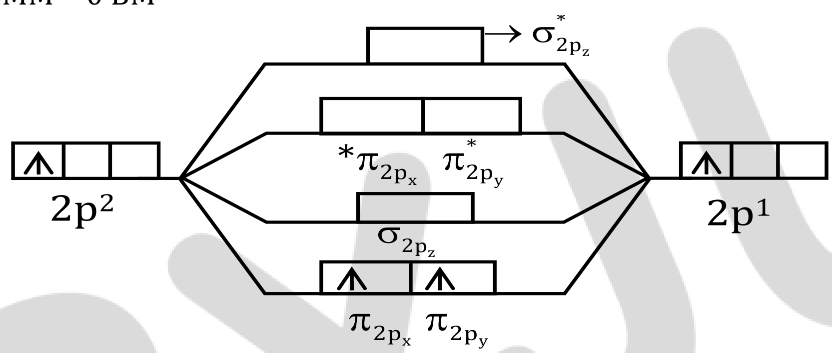
11. (a)

(1) MOT diagram of C_2 molecule $\Rightarrow {}_6C \rightarrow 1s^2 2s^2 2p^2$



Unpaired $e^\Theta = 0$

MM = 0 BM



$\rightarrow$ No. of unpaired $e^\Theta = 2$

MM = $\sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8}$

M.M = 2.83 BM

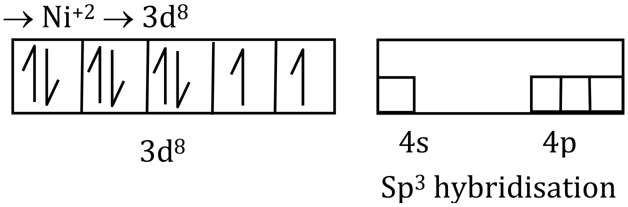
Hence C_2 diamagnetic & B_2 paramagnetic

So correct option is (a)

12. (b)

(1)$[NiCl_4]^{2-} \rightarrow Ni^{+2} \rightarrow 3d^8$

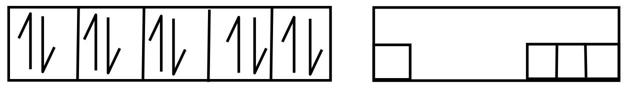
$[NiCl_4]^{2-} \rightarrow$



$\because$ Cl^Θ is a weak ligand, so there is no pairing of electrons

$\rightarrow$ $\boxed{\text{No.of unpaired electron} = 2}$

(2) $[Ni(Co)_4] \rightarrow Ni(0) \rightarrow [Ar]3d^8 4s^2$

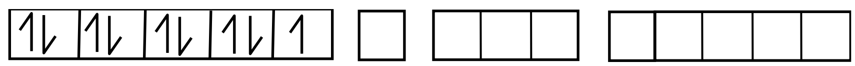
$[Ni(Co)_4]$ 

3d 4s 4p

Sp^3 hybridisation

→ Co is a strong ligand, so there is pairing of electrons occurs

→ No. of unpaired electrons = 0

3. $[Cu(NH_3)_4]^{2+}$: 

3d 4s 4p 4d

$\downarrow$

Cu^{2+}

$\downarrow$

$3d^9$

Sp^2d hybridisation

$\boxed{\text{No. of unpaired electron} = 1}$

Hence correct option is (b)

13. (a)

Electro affinity ⇒ E.A. of an atom is defined as the amount of energy released when an electron is attached to a neutral atom or molecule in the gaseous state to form a negative ion.

$x(g) + e^{\ominus} \rightarrow xg^{\ominus} + \text{energy}$

E.A. of "F" is 2nd highest amongst periodic table after "Cl"

14. (a)

→ Solubility of $PbCl_2$ increases in cold water on the addition of HCl due to the formation of soluble complex like $[PtCl_3]^{\ominus}$ (aq.).

→ $PbCl_2$ react with HCl as follows

$PbCl_2(s) + Cl^{\ominus} \rightarrow [PbCl_3]^{\ominus}$ (aq)

$PbCl_2(s) + 2Cl^{\ominus} \rightarrow [PbCl_4]^{2-}$ (aq)

Thus the addition of excess amount of $Cl^{\ominus}$ ions change the $PbCl_2$ as soluble complex of $[PbCl_4]^{2-}$.

Hence solubility increases

15. (a)

→ As the oxidation state of Central atom increases Electron negativity increases and acidity increases.

→ I effect increases, acidity increases

$\boxed{\text{Acidity} \propto -R > -H > -I}$

$HClO_3$	$HClO_2$	$HOCl$	$HOBr$
+5	+3	+1	+1

Hence correct option is (a)

16. (d)

Due to the lanthanoid contraction, increase in atomic number the basicity decrease from La^{3+} to Lu^{3+}.

So correct order of basicity follow as $La^{3+} > Ce^{3+} > E_4^{3+} > Lu^{3+}$.

Hence correct option is (d)

17. (d)

Reaction of $BaCl_2$ with CO_3^{2-}, SO_3^{2-} & SO_4^{2-} follow as

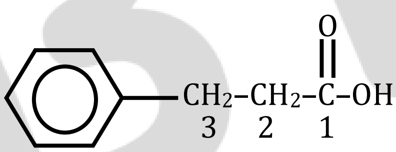
$$BaCl_2 + CO_3^{2-} \rightarrow \underset{\text{ppt}}{BaCO_3} + 2Cl^{\ominus}$$

$$BaCl_2 + SO_3^{2-} \rightarrow \underset{\text{ppt}}{BaSO_3} + 2Cl^{\ominus}$$

$$BaCl_2 + SO_4^{2-} \rightarrow \underset{\text{ppt}}{BaSO_4} + 2Cl^{\ominus}$$

Hence correct option is (d)

18. (a)



3-phenyl propanoic acid

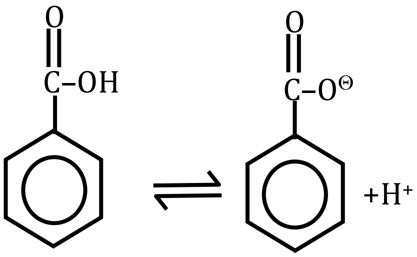
Hence correct option (A)

19. (d)

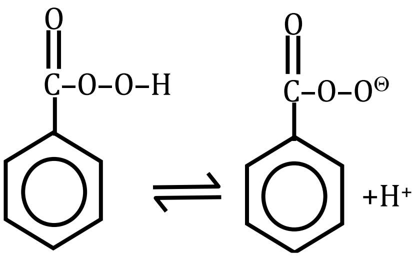
Isomerisation of 1-butyne to 2-butyne can be achived by treatment with ethanolic KOH

$$\underset{\text{1-butyne}}{CH_3-CH_2-C\equiv CH} \xrightarrow[\text{isomerisation}]{\text{alc.KOH}} \underset{\text{2-butyne}}{CH_3-C\equiv C-CH}$$

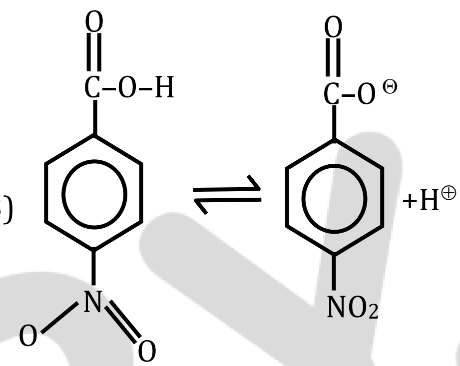
20. (c)

(1) 

(Stabilized via resonance)

(2) 

[Not resonance stabilized because it is not show any resonating structure]

(3) 

–R &–I effect

[Most acidic due to –R & –I effect of $-NO_2$ group]

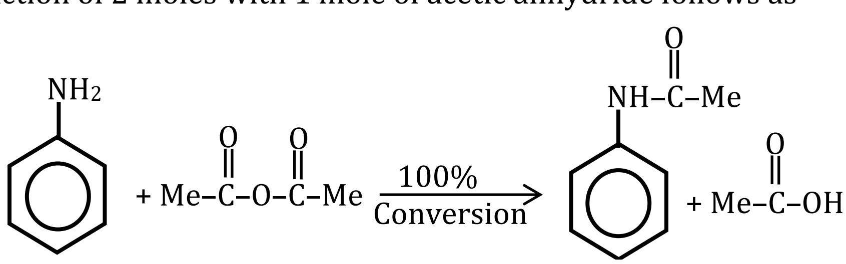
$\boxed{\text{Acidity} \propto -R > -H > -I}$

Hence correct order of acidity is 3 > 1 > 2

So correct option is (c)

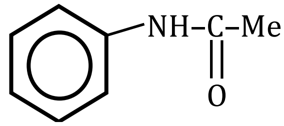
21. (b)

The reaction of 2 moles with 1 mole of acetic anhydride follows as



Initial	2	1	0	0
Final	2–1 =1	1–1 = 0	1	1

(Limiting reagent)

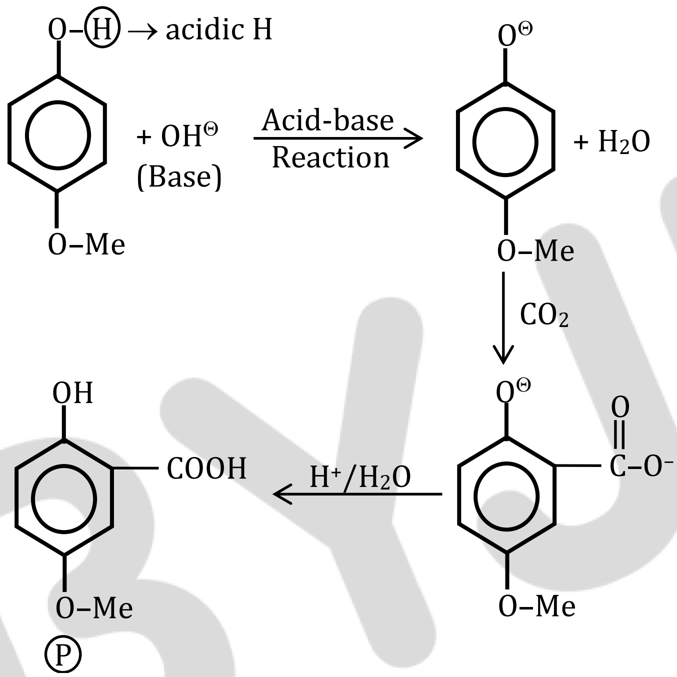
⇒ Mw of  Mean C_8H_9ON = 135 g mol^{-1}

Wt. of acetanilide = moles × Mw = 1 × 135

Wt. = 135 g → Hence correct option is (b)

22. (c)

Reaction mechanism follows as



23. (a)

ADP ⇒ Adenosine diphosphate ⇒ 2 phosphate group

ADP ⇒ Adenosine Triphosphate ⇒ 3 phosphate group

⇒ So difference only one phosphate group

Hence correct option is (a)

24. (c)

Carbyl – amine reaction

→ When $CHCl_3$ + KOH (alcoholic), React with 1° amine

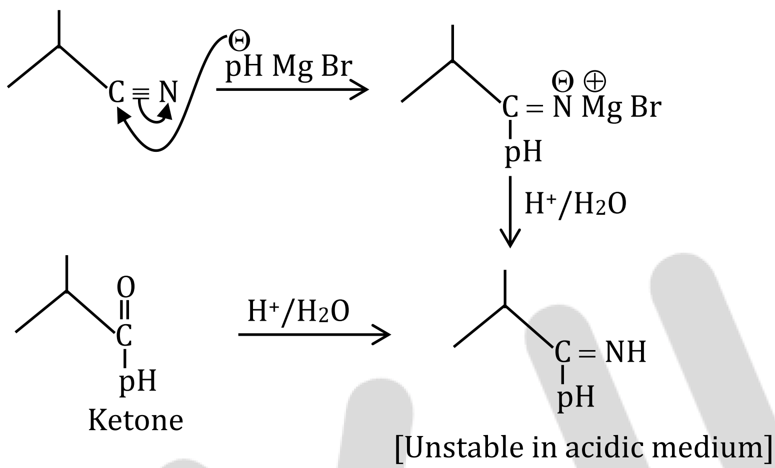
Or aromatic amine: Isocyanide product will form, reaction follow as –

$$PH - NH_2 + CHCl_3 + \text{alcoholic KOH} \rightarrow \underset{\text{(Nauseating smell)}}{PH - NC}$$

Hence correct option is (c)

25. (b)

Reaction mechanism follow as :



26. (c)

→ We have to prepare maximum volume solution of 3N. {3 M is same as 3 N for HCl due to valency factor = 1}

Hence we take 500 ml of 2N solution & x ml of 5N solution

$N_1V_1 + N_2V_2 = N_3V_3$

$\Rightarrow 500 \times 2 + x \times 5 = 3\,(x + 500)$

$1000 + 5x = 3x + 1500$

$x = 250$ ml

$\therefore$ Final volume of the solution = x + 500 = 250 + 500 = 750 ml

27. (a)

Energy of photon (E) = $h\nu = \dfrac{hc}{\lambda}$

(1) $E = \dfrac{hc}{\lambda} = \dfrac{6.626\times10^{-34}\,\text{J.S}\times3\times10^{8}\,\text{m sec}^{-1}}{300\times10^{-9}\,\text{m}}$

$\boxed{E = 6.626\times10^{-19}\,\text{J}}$

(2) $E = h\nu = 6.626 \times 10^{-34}$ J. sec $\times 3 \times 10^{8}$ sec^{-1}

$\boxed{E = 1.9878 \times 10^{-25} J}$

n 30 cm^{-1} = 30 × 10^2 m^{-1}

(3) E = chν = 6.626 × 10^{-34} J sec × 30 × 10^2 m^{-1} × 3 × 10^8 m sec^{-1}

= 5.9634 × 10^{-22} J

(4) E = 6.626 × 10^{-27} J

Among these, maximum energy = 6.626 × 10^{-19} J

Hence correct option is (A)

28. (c)

For isotonic at some temperature $\pi_1 = \pi_2$

i_1c_1 RT = ℓ_2c_2RT .....(1)

Option (c) 0.03 M NaCl & 0.02 M $MgCl_2$

(1) $\underset{1}{NaCl} \longrightarrow \underbrace{Na^+ + Cl^\ominus}_{2}$ $\quad i = \frac{2}{1} = 2$

(2) $\frac{MgCl_2}{1} \longrightarrow \underbrace{Mg^{2+} + 2Cl^\ominus}_{3}$ $\quad i = \frac{3}{1} = 2$

Now from equation (1)

2 × 0.03 $\quad$ 3× 0.02

= 0.06 $\quad$ = 0.06

$\boxed{\therefore (i_1C_1 = i_2C_2) \rightarrow \text{Hence, It is isotonic}}$

29. (d)

$$\underset{\substack{(+6)\\1\,mol}}{Cr_2O_7^{2-}} \xrightarrow{6electron} \underset{(+3)}{2Cr^{3+}}$$

→ Hence reduction of 1 mol $Cr_2O_7^{2-}$ required 6 electron

So no. of the Faraday's = 6F

30. (d)

(1)$2H_2O(g) \rightleftharpoons 2H_2(g) + O_2(g)$: $K_1 = 6.4 \times 10^{-8}$

$6.4 \times 10^{-8} = K_1 = \frac{[H_2]^2[O_2]}{[H_2O]^2}$

Or $\sqrt{K_1} = K' = \frac{[H_2][O_2]^{\frac{1}{2}}}{[H_2O]}$ .....(1)

(2) $2CO_2(g) \rightleftharpoons 2CO(g) + O_2(g)$, $K_2 = 1.6 \times 10^{-6}$

$K_2 = \dfrac{[CO]^2[O_2]}{[CO_2]^2}$

$\Rightarrow \sqrt{K_2} = K'' = \dfrac{[CO][O_2]^{\frac{1}{2}}}{[CO_2]}$ .....(2)

(3) $H_2(g) + CO_2(g) \rightleftharpoons CO(g) + H_2O(g)$

$K_3 = \dfrac{[CO]\ [H_2O]}{[H_2][CO_2]}$ .....(3)

$\Rightarrow K'' \times \dfrac{1}{K'} = \dfrac{[CO][O_2]^{\frac{1}{2}}}{[CO_2]} \times \dfrac{[H_2O]}{[H_2][O_2]^{\frac{1}{2}}}$

$K'' \times \dfrac{1}{K'} = \dfrac{[CO][H_2O]}{[CO_2][H_2]} = K_3$

$K_3 = \dfrac{1}{K'} \times K'' = \dfrac{\sqrt{1.6 \times 10^{-6}}}{\sqrt{6.4 \times 10^{-8}}} = \sqrt{\dfrac{1}{4} \times 10^2}$

$K_3 = \dfrac{1}{2} \times 10^1 = 5$

$K_3 = 5$

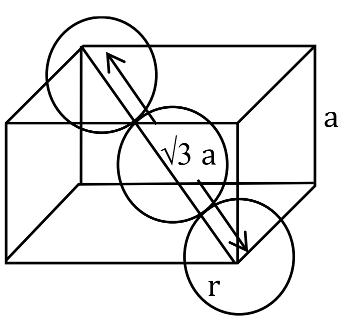
Hence correct option is (d)

31. (d)

In BCC lattice all atoms each other

$\therefore\ \sqrt{3}a = 4r \rightarrow r = \dfrac{\sqrt{3}\,a}{4}$

(Where a = edge length, r = radius of lattice sphere)



32. (b)

Lime Juice is acidic in nature as it contains "citric acid" the citric acid present in lime juice will liberate I_2 from the iodide which will give purple or violet colour.

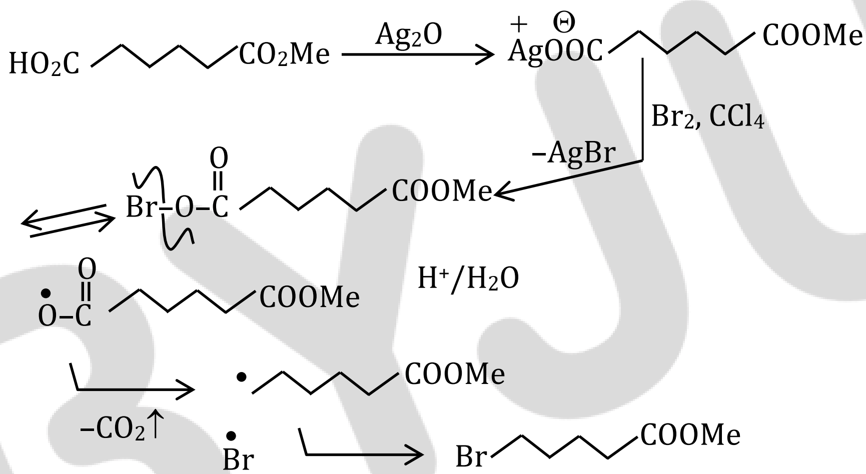
Reaction follow as –

$$I^{\ominus} + IO_3^{\ominus} + H^{+} \rightarrow \underset{\left(\begin{array}{c}\text{violet}\\\text{colour}\end{array}\right)}{I_2} + H_2O$$

33. (d)

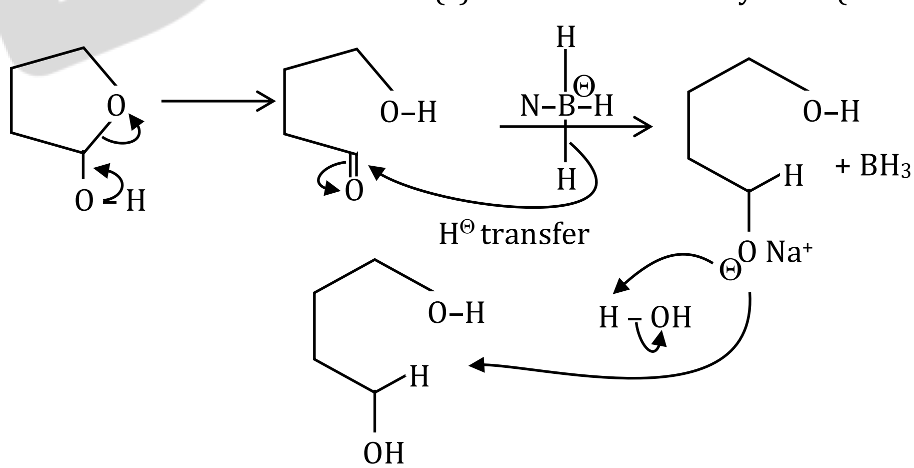
→ Reaction mechanism of the reaction follows as

→ This reaction is an example of "Borodine" Huns Diecker reaction



34. (c)

The reduction reaction of lactol (s) with sodium borohydride ($NaBH_4$)



Hence correct option is (c)

35. (b)

Normality = $\frac{\text{wt} \times 1000}{\text{eq.wt} \times \text{v(ml)}} = \frac{\text{No.of M.eq}}{\text{volume of solution(ml)}}$

→ Reaction between NaOH & HCl follow as

→ It is an acid-base reaction hence final product is salt & H_2O

$$NaOH + HCl \rightarrow \underset{\text{Salt}}{NaCl} + H_2O$$

	NaOH	HCl	NaCl	H_2O
No. of M. eq. added	xy	xy	0	0
After reaction No. of M. eq left	0	0	xy	xy

Normality of the solution = $\frac{\text{No.of m.eq}}{\text{volume of solution(ml)}} = \frac{xy}{(x+y)+(x+y)} = \frac{xy}{2(x+y)}$

36. (b,c,d)

Electrolysis of molten NaCl

Molten NaCl (An electrolyte) means free Na^+ ions & $Cl^\ominus$ ion, so it is conducts current with the help of ions. As electric current is passed in the cell, $Cl^\ominus$ ions are attracted to the anode (positive) & Na^+ ions attracted to cathode (negative).

Anode to cathode follow as :

Cathode $Na^+ + e^\ominus \rightarrow Na(a)$

Anode $Cl^\ominus \rightarrow \frac{1}{2} Cl_2 + e^\ominus$

Overall reaction $Na^+ + Cl^\ominus \rightarrow Na + \frac{1}{2} Cl_2$

If we add some water then, H_2 will liberate & NaOH will form in the solution and reaction follows as

→ Exothermic reaction

$$Na(s) + H_2O \rightarrow NaOH + \frac{1}{2} H_2 + Q \text{ (energy)}$$

Hence correct option is (b,c,d)

37. (b,c)

→ The electrolysis of Al_2O_3 is carried out in the steel tank linked inside with graphite. The graphite lining serves as cathode. The anode is also made of graphite rods hanging in the molten mass the electrolyte consist of alumina dissolved in fused cryolite (Na_3AlF_6) & CaF_2.

→ Cryolite lower the melting point of alumina to 950°C & CaF_2 increase the fluidity of the mass, so that the liberated aluminium metal may sink at the bottom of the cell.

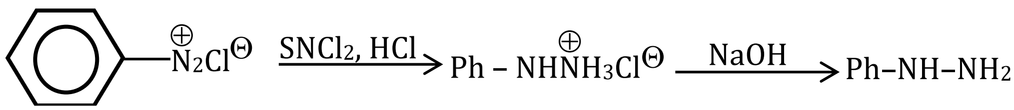
Therefore, it makes the fused mixture very conducting & lower the fusion temperature of the metal.

→ When an electric current is passed through this mixture the Al is collected at the cathode in the molten state & sinks at the bottom & it is tapped off.

Hence correct option is (b,c)

38. (a,b)

Reaction diazonium salt with following reagent follow as :

(1) 

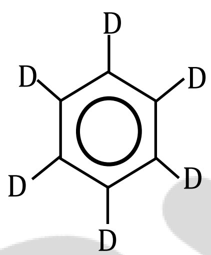
(2) $Ph-\overset{+}{N}Cl^{\ominus} \xrightarrow{Na_2SO_3} Ph-NH-NH_2$

(3) $Ph-\overset{\oplus}{N}Cl^{\ominus} \xrightarrow{CH_3CH_2-O-H} Ph-H+N_2 + HCl + CH_3CHO$

(4) $Ph-\overset{+}{N}Cl^{\ominus} \xrightarrow{H_3PO_2} Ar-H+H_3PO_3 + N_2 + HCl$

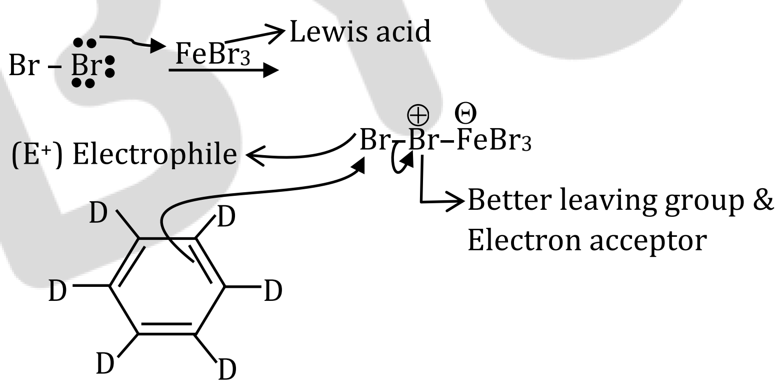
Hence correct option are (a,b)

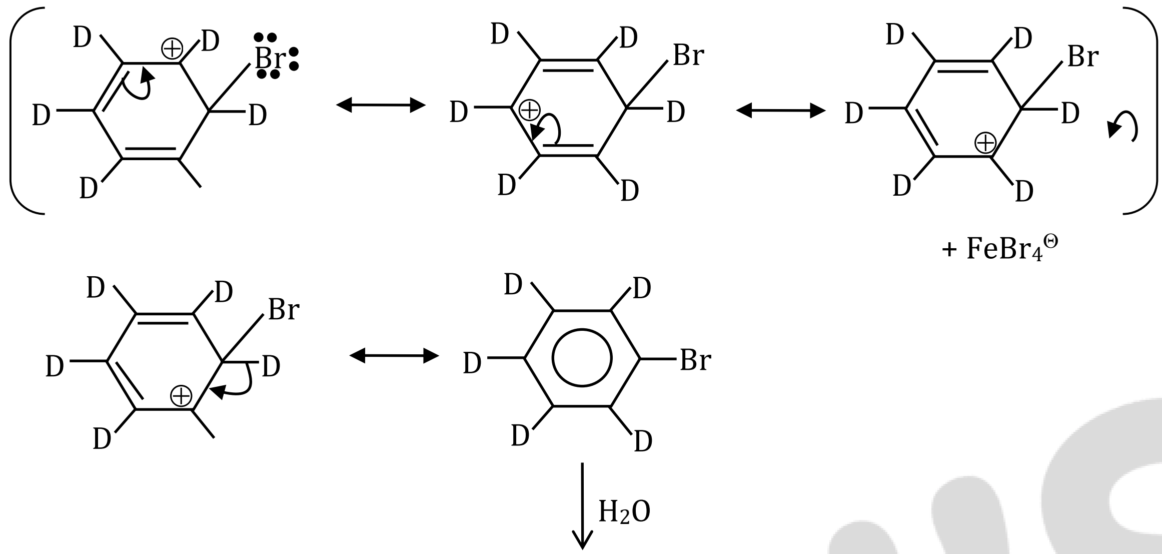
39. (a)



$\rightarrow$ Deuterated benzene, $Fe + Br_2 \rightarrow FeBr_3$

Mechanism





Insoluble C_6D_5–Br $\rightarrow$ ppt

Hence correct option are (a)

40. (a,b,c)

(A) Angular momentum - $\frac{nh}{2\pi}$ where [n = 1, 2, 3]

(B) $r_n = 0.529 \times \frac{n^2}{z}$ Å

For H, z = 1

$\Rightarrow$ n = 1 (according question)

r = 0.529Å

(C) $E_n = -13.6 \times \frac{z^2}{n^2}$ eV/atom

$E_n \propto \frac{1}{n^2}$

(D) From the above reaction

$E_1 = -13.6$ eV/atom

$E_2 = -13.6 \times \frac{z^2}{(2)^2} = \frac{-13.6}{4} = -3.4$ eV/atom

$E_3 = \frac{-13.6}{9} = 1.51$ ev atom

$E_2 - E_1 = 10.2$ ev/atom, $E_3 - E_2 = 0.89$ ev /atom

$\Rightarrow (E_2 - E_1) > (E_3 - E_2) > (E_4 - E_3)$ ……

As the difference between the successive energy level decreases, “n” increases.

Hence, correct option is (a,b,c)

WBJEE 2017 (Physics)

1. The velocity of a particle executing a simple harmonic motion is 13 ms^{-1}, when its distance from the equilibrium position (Q) is 3 m and its velocity is 12 ms^{-1}, when it is 5 m away from Q. The frequency of the simple harmonic motion is

   a. $\frac{5\pi}{8}$

   b. $\frac{5}{8\pi}$

   c. $\frac{8\pi}{5}$

   d. $\frac{8}{5\pi}$

2. A uniform string of length L and mass M is fixed at both ends while it is subject to a tension T. It can vibrate at frequencies (υ) given by the formula (where n = 1, 2, 3, ...)

   a. $\upsilon = \frac{n}{2}\sqrt{\frac{T}{ML}}$

   b. $\upsilon = \frac{n}{2L}\sqrt{\frac{T}{M}}$

   c. $\upsilon = \frac{1}{2n}\sqrt{\frac{T}{ML}}$

   d. $\upsilon = \frac{n}{2}\sqrt{\frac{TL}{M}}$

3. A uniform capillary tube of length l and inner radius r with its upper end sealed is submerged vertically into water. The outside pressure is p_0 and surface tension of water is γ. When a length x of the capillary is submerged into water, it is found that water levels inside and outside the capillary coincide. The value of x is

   a. $\frac{l}{\left(1+\frac{p_0 r}{4\gamma}\right)}$

   b. $l\left(1-\frac{p_0 r}{4\gamma}\right)$

   c. $l\left(1-\frac{p_0 r}{2\gamma}\right)$

   d. $\frac{l}{\left(1+\frac{p_0 r}{2\gamma}\right)}$

4. A liquid of bulk modulus k is compressed by applying an external pressure such that its density increases by 0.01%. The pressure applied on the liquid is

a. $\frac{k}{10000}$

b. $\frac{k}{1000}$

c. 1000 k

d. 0.01k

5. Temperature of an ideal gas, initially at 27ºC, is raised by 6°C. The rms velocity of the gas molecules will,

a. increase by nearly 2%

b. decrease by nearly 2%

c. increase by nearly 1%

d. decrease by nearly 1%

6. 2 moles of an ideal monoatomic gas is carried from a state (P_0, V_0) to a state ($2P_0$, $2V_0$) along a straight line path in a P-V diagram. The amount of heat absorbed by the gas in the process is given by

a. $3P_0V_0$

b. $\frac{9}{2}P_0V_0$

c. $6P_0V_0$

d. $\frac{3}{2}P_0V_0$

7. A solid rectangular sheet has two different coefficients of linear expansion α_1 and α_2 along its length and breadth respectively. The coefficient of surface expansion is (for $\alpha_1 t << 1$, $\alpha_2 t << 1$)

a. $\frac{\alpha_1 + \alpha_2}{2}$

b. $2(\alpha_1 + \alpha_2)$

c. $\frac{4\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$

d. $\alpha_1 + \alpha_2$

8. A positive charge Q is situated at the centre of a cube. The electric flux through any face of the cube is (in SI units)

a. $\frac{Q}{6\varepsilon_0}$

b. $4\pi Q$

c. $\frac{Q}{4\pi\varepsilon_0}$

d. $\frac{Q}{6\pi\varepsilon_0}$

9. Three capacitors of capacitance 1.0, 2.0 and 5.0 μF are connected in series to a 10V source. The potential difference across the 2.0 μF capacitor is

a. $\frac{100}{17}$V

b. $\frac{20}{17}$V

c. $\frac{50}{17}$V

d. 10 V

10. A charge of 0.8 coulomb is divided into two charges Q_1 and Q_2. These are kept at a separation of 30 cm. The force on Q1 is maximum when
    a. $Q_1 = Q_2 = 0.4C$
    b. $Q_1 \approx 0.8C$, Q_2 negligible
    c. Q_1 negligible, $Q_2 \approx 0.8C$
    d. $Q_1 = 0.2\ C$, $Q_2 = 0.6\ C$

11. The magnetic field due to a current in a straight wire segment of length L at a point on its perpendicular bisector at a distance r (r >> L)
    a. decreases as $\frac{l}{r}$
    b. decreases as $\frac{l}{r^2}$
    c. decreases as $\frac{l}{r^3}$
    d. approaches a finite limit as $r \to \infty$

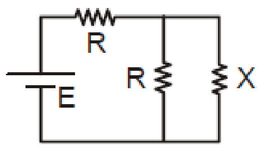
12. The magnets of two suspended coil galvanometers are of the same strength so that they produce identical uniform magnetic fields in the region of the coils. The coil of the first one is in the shape of a square of side a and that of the second one is circular of radius $a/\sqrt{\pi}$. When the same current is passed through the coils, the ratio of the torque experienced by the first coil to that experienced by the second one is
    a. $l : \frac{1}{\sqrt{\pi}}$
    b. 1 : 1
    c. $\pi : 1$
    d. $1 : \pi$

13. A proton is moving with a uniform velocity of 10^6 ms^{-1} along the Y-axis, under the joint action of a magnetic field along Z-axis and an electric field of magnitude 2×10^4 Vm^{-1} along the negative X-axis. If the electric field is switched off, the proton starts moving in a circle. The radius of the circle is nearly (given : $\frac{e}{m}$ ratio for proton = 10^8 Ckg^{-1})
    a. 0.5 m
    b. 0.2 m
    c. 0.1m
    d. 0.05 m

14. When the frequency of the AC voltage applied to a series LCR circuit is gradually increased from a low value, the impedance of the circuit
    a. monotonically increases
    b. first increases and then decreases
    c. first decreases and then increases
    d. monotonically decreases

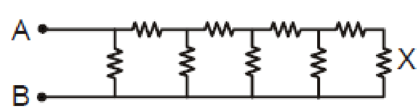
15. Six wires, each of resistance r, are connected so as to form a tetrahedron. The equivalent resistance of the combination when current enters through one corner and leaves through some other corner is
    a. r
    b. 2r
    c. $\frac{r}{3}$
    d. $\frac{r}{2}$

16. A Consider the circuit shown in the figure. The value of the resistance X for which the thermal power generated in it is practically independent of small variation of its resistance is



    a. X = R
    b. $X = \frac{R}{3}$
    c. $X = \frac{R}{2}$
    d. X = 2R

17. Consider the circuit shown in the figure where all the resistances are of magnitude 1kΩ. If the current in the extreme right resistance X is 1 mA, the potential difference between A and B is

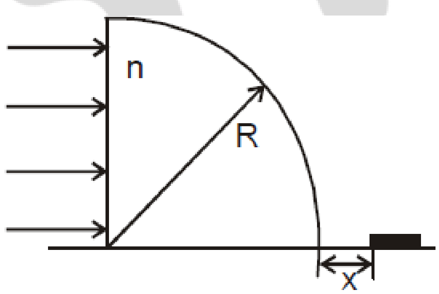


    a. 34 V
    b. 21 V
    c. 38 V
    d. 55 V

18. The ratio of the diameter of the sun to the distance between the earth and the sun is approximately 0.009. The approximate diameter of the image of the sun formed by a concave spherical mirror of radius of curvature 0.4 m is
    a. 4.5×10^{-6} m
    b. 4.0×10^{-6} m
    c. 3.6×10^{-3} m
    d. 1.8×10^{-3} m

19. Two monochromatic coherent light beams A and B have intensities L and $\frac{L}{4}$ respectively. If these beams are superposed, the maximum and minimum intensities will be
    a. $\frac{9L}{4}, \frac{L}{4}$
    b. $\frac{5L}{4}, 0$
    c. $\frac{5L}{2}, 0$
    d. $2L, \frac{L}{2}$

20. A point object is held above a thin equiconvex lens at its focus. The focal length is 0.1 m and the lens rests on a horizontal thin plane mirror. The final image will be formed at
    a. infinite distance above the lens
    b. 0.1 m above the center of the lens
    c. infinite distance below the lens
    d. 0.1 m below the center of the lens

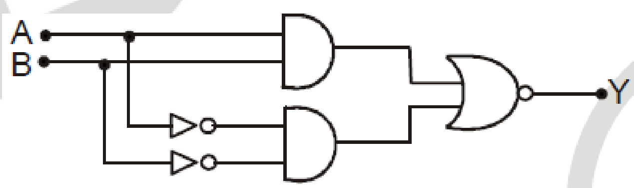
21. 

A parallel beam of light is incident on a glass prism in the shape of a quarter cylinder of radius R = 0.05 m and refractive index n = 1.5 placed on a horizontal table as shown in the figure. Beyond the cylinder, a patch of light is found whose nearest distance x from the cylinder is
    a. $(3\sqrt{3} - 4) \times 10^{-2}$m
    b. $(2\sqrt{3} - 2) \times 10^{-2}$m
    c. $(3\sqrt{5} - 5) \times 10^{-2}$m
    d. $(3\sqrt{2} - 3) \times 10^{-2}$m

22. The de Broglie wavelength of an electron is 0.4 × 10–10 m when its kinetic energy is 1.0 keV. Its wavelength will be 1.0×10^{-10} m, when its kinetic energy is
 a. 0.2 keV
 b. 0.8 keV
 c. 0.63 keV
 d. 0.16 keV

23. When light of frequency υ_1 is incident on a metal with work function W (where h υ_1 > W), the photocurrent falls to zero at a stopping potential of V_1. If the frequency of light is increased to υ_2, the stopping potential changes to V_2. Therefore, the charge of an electron is given by
 a. $\frac{W(\upsilon_2+\upsilon_1)}{\upsilon_1 V_2+\upsilon_2 V_1}$
 b. $\frac{W(\upsilon_2+\upsilon_1)}{\upsilon_1 V_1+\upsilon_2 V_2}$
 c. $\frac{W(\upsilon_2-\upsilon_1)}{\upsilon_1 V_2-\upsilon_2 V_1}$
 d. $\frac{W(\upsilon_2-\upsilon_1)}{\upsilon_2 V_2-\upsilon_1 V_1}$

24. Radon-222 has a half-life of 3.8 days. If one starts with 0.064 kg of Radon-222, the quantity of Radon-222 left after 19 days will be
 a. 0.002 kg
 b. 0.062 kg
 c. 0.032 kg
 d. 0.024 kg

25. 

In the given circuit, the binary inputs at A and B are both 1 in first case and both 0 in the next case. The respective outputs at Y in these two cases will be:
 a. 1, 1
 b. 0, 0
 c. 0, 1
 d. 1, 0

26. When a semiconducting device is connected in series with a battery and a resistance, a current is found to flow in the circuit. If, however, the polarity of the battery is reversed, practically no current flows in the circuit. The device may be
  a. a p-type semiconductor
  b. a n-type semiconductor
  c. an intrinsic semiconductor
  d. a p-n junction

27. The dimension of the universal constant of gravitation G is
  a. $[ML^2T^{-1}]$
  b. $[M^{-1}L^3T^{-2}]$
  c. $[M^{-1}L^2T^{-2}]$
  d. $[ML^3T^{-2}]$

28. Two particles A and B (both initially at rest) start moving towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is 2v, the speed of the centre of mass is
  a. Zero
  b. v
  c. $\frac{3v}{2}$
  d. $-\frac{3v}{2}$

29. Three vectors $\vec{A} = a\vec{i} + \vec{j} + \vec{k}$; $\vec{B} = \vec{i} + b\vec{j} + \vec{k}$ and $\vec{C} = \vec{i} + \vec{j} + c\vec{k}$ are mutually perpendicular ($\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along X,Yand Z axis respectively). The respective values of a, b and c are
  a. 0,0,0
  b. $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$
  c. 1, −1, 1
  d. $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

30. A block of mass 1 kg starts from rest at x = 0 and moves along the x-axis under the action of a force F=kt, where t is time and k =1 Ns^{-1}. The distance the block will travel in 6 seconds is
  a. 36 m
  b. 72 m
  c. 108 m
  d. 18 m

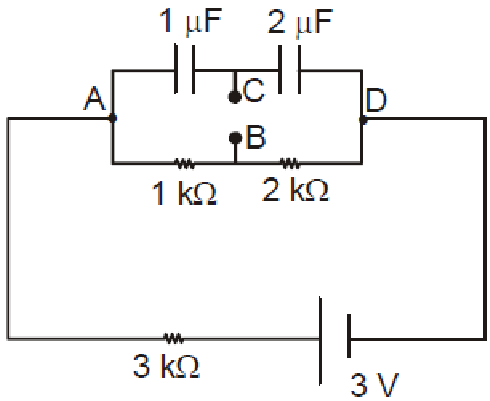
31. A particle with charge Q coulomb, tied at the end of an inextensible string of length R meter, revolves in a vertical plane. At the centre of the circular trajectory there is a fixed charge of magnitude Q coulomb. The mass of the moving charge M is such that $Mg = \frac{Q^2}{4\pi\varepsilon_0 R^2}$. If at the highest position of the particle, the tension of the string just vanishes, the horizontal velocity at the lowest point has to be
   a. 0
   b. $2\sqrt{gR}$
   c. $\sqrt{2gR}$
   d. $\sqrt{5gR}$

32. A bullet of mass 4.2×10^{-2} kg, moving at a speed of 300 ms^{-1}, gets stuck into a block with a mass 9 times that of the bullet. If the block is free to move without any kind of friction, the heat generated in the process will be
   a. 45 cal
   b. 405 cal
   c. 450 cal
   d. 1701 cal

33. A particle with charge e and mass m, moving along the X-axis with a uniform speed u, enters a region where a uniform electric field E is acting along the Y-axis. The particle starts to move in a parabola. Its focal length (neglecting any effect of gravity) is
   a. $\frac{2mu^2}{eE}$
   b. $\frac{eE}{2mu^2}$
   c. $\frac{mu}{2eE}$
   d. $\frac{mu^2}{2eE}$

34. A unit negative charge with mass M resides at midpoint of the straight line of length 2a adjoining two fixed charges of magnitude + Q each. If it is given a very small displacement $x(x << a)$ in a direction perpendicular to the straight line, it will
   a. come back to its original position and stay there
   b. execute oscillations with frequency $\frac{1}{2\pi}\sqrt{\frac{Q}{4\pi\varepsilon_0 Ma^3}}$
   c. fly to infinity
   d. execute oscillations with frequency $\frac{1}{2\pi}\sqrt{\frac{Q}{4\pi\varepsilon_0 Ma^2}}$

35.



Consider the circuit given here. The potential difference V_{BC} between the points B and C is

a. 1V
b. 0.5 V
c. 0
d. –1V

36. If the pressure, temperature and density of an ideal gas are denoted by P, T and ?, respectively, the velocity of sound in the gas is

a. Proportional to $\sqrt{P}$, when T is constant.
b. Proportional to $\sqrt{T}$
c. Proportional to $\sqrt{P}$, when r is constant.
d. Proportional to T.

37. Two long parallel wires separated by 0.1 m carry currents of 1 A and 2 A respectively in opposite directions. A third current-carrying wire parallel to both of them is placed in the same plane such that it feels no net magnetic force. It is placed at a distance of

a. 0.5 m from the 1^{st} wire, towards the 2^{nd} wire.
b. 0.2 m from the 1^{st} wire, towards the 2^{nd} wire.
c. 0.1 m from the 1^{st} wire, towards the 2^{nd} wire.
d. 0.2 m from the 1^{st} wire, away from the 2^{nd} wire.

38. If χ stands for the magnetic susceptibility of a substance, μ for its magnetic permeability and μ_0 for the permeability of free space, then

a. for a paramagnetic substance : $\chi > 0, \mu > 0$
b. for a paramagnetic substance : $\chi > 0, \mu > \mu_0$
c. for a diamagnetic substance : $\chi < 0, \mu < 0$
d. for a ferromagnetic substance : $\chi > 1, \mu < \mu_0$

39. Let v_n and E_n be the respective speed and energy of an electron in the nth orbit of radius r_n, in a hydrogen atom, as predicted by Bohr's model. Then

a. plot of $E_n r_n / E_1 r_1$ as a function of n is a straight line of slope 0.

b. plot of $r_n v_n / r_1 v_1$ as a function of n is a straight line of slope 1.

c. plot of $\ln\left(\frac{r_n}{r_1}\right)$ as a function of ln(n) is a straight line of slope 2.

d. plot of $\ln\left(\frac{r_n E_1}{E_n r_1}\right)$ as a function of ln(n) is a straight line of slope 4.

40. A small steel ball bounces on a steel plate held horizontally. On each bounce the speed of the ball arriving at the plate is reduced by a factor e (coefficient of restitution) in the rebound, so that

$V_{upward} = eV_{downward}$

If the ball is initially dropped from a height of 0.4 m above the plate and if 10 seconds later the bouncing ceases, the value of e is

a. $\sqrt{\frac{2}{7}}$

b. $\frac{3}{4}$

c. $\frac{13}{18}$

d. $\frac{17}{18}$

WBJEE 2017 (Physics)

ANSWER KEYS

1. (b)	2. (a)	3. (d)	4. (a)	5. (c)	6. (c)	7. (d)	8. (a)	9. (c)	10. (a)
11. (b)	12. (b)	13. (a)	14. (c)	15. (d)	16. (c)	17. (a)	18. (d)	19. (a)	20. (b)
21. (c)	22. (d)	23. (c)	24. (a)	25. (b)	26. (d)	27. (b)	28. (a)	29. (b)	30. (a)
31. (b)	32. (b)	33. (d)	34. (G)	35. (b)	36. (b,c)	37. (c)	38. (b,d)	39. (a,b,c,d)	40. (d)

WBJEE 2017 (Physics)

Solution

1. (b)

Velocity of particle at any x from mean-position executing SHM is:-

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

Given:-

(i) At x = 3 m ; v = 13 ms^{-1}

$$\frac{[13]^2}{\omega^2} + [3]^2 = A^2 \qquad .....(1)$$

(ii) At x = 5 m ; v = 12 ms^{-1}

$$\frac{[12]^2}{\omega^2} + [5]^2 = A^2 \qquad .....(2)$$

From (1) & (2):-

$$\frac{169}{\omega^2} + 9 = \frac{144}{\omega^2} + 25$$

$$\frac{169}{\omega^2} - \frac{144}{\omega^2} = 25 - 9$$

$$\frac{1}{\omega^2}[25] = 16$$

$$\omega^2 = \frac{25}{16}$$

$$\omega = \frac{5}{4}\ \frac{\text{Rad}}{\text{Sec}}$$

Frequency [f] :-

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{5}{8\pi}$$

2. (a)

If the string is vibrating in n segment and wavelength of wave is λ.

$$L = \frac{n\lambda}{2}$$

Velocity of transverse wave in string is:-

$$v = \sqrt{\frac{T}{\mu}}$$

T = Tension in the string

μ = mass per unit length = $\frac{M}{L}$

$$v = \sqrt{\frac{T}{\left[\frac{M}{L}\right]}} = \sqrt{\frac{TL}{M}}$$

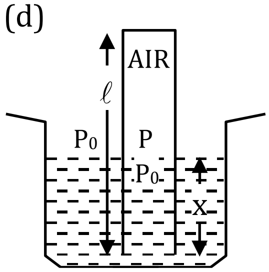
Velocity of a wave:-

$v = \upsilon\lambda$

$$\upsilon = \frac{v}{\lambda}$$

$$\upsilon = \frac{n}{2L}\sqrt{\frac{TL}{M}} \quad ; \qquad \upsilon = \frac{n}{2}\sqrt{\frac{T}{ML}}$$

3. (d)



Assumption:-

The temperature of Air in the capillary remains constant

[PV = nRT]

So for air inside capillary

$P_iV_i = P_fV_f$

$P_0A\ell = PA[\ell - x]$

$$P = \frac{P_0\ell}{\ell - x} \qquad \text{.....(i)}$$

[Angle of contact $\theta = 0°$]

At interface:- $P - P_0 = \frac{2\gamma}{r}$ .....(ii)

Put value of P from (i) in equation (ii):-

$$\frac{P_0\ell}{\ell - x} - P_0 = \frac{2\gamma}{r}$$

$$\frac{P_0 x}{\ell - x} = \frac{2\gamma}{r}$$

$$\frac{P_0 r}{2\gamma} = \frac{\ell - x}{x}$$

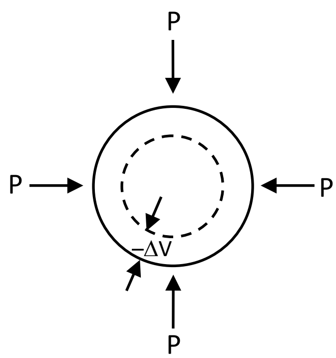
$$\frac{P_0 r}{2\gamma} = \frac{\ell}{x} - 1$$

$$\frac{P_0 r}{2\gamma} + 1 = \frac{\ell}{x}$$

$$x = \frac{\ell}{1 + \frac{P_0 r}{2\gamma}}$$

4. (a)

Bulk – modulus $[K] = \frac{-P}{\frac{\Delta V}{V}}$ .....(i)



Mass of liquid element is constant

$$m = \rho V$$

$$\frac{\Delta m}{m} = \frac{\Delta\rho}{\rho} + \frac{\Delta V}{V}$$

$$0 = \frac{\Delta\rho}{\rho} + \frac{\Delta V}{V}$$

$$-\frac{\Delta V}{V} = \frac{\Delta\rho}{\rho}$$

From (i) :-

$$P = -K\frac{\Delta V}{V}$$

$$P = +K\frac{\Delta\rho}{\rho}$$

$$\frac{\Delta\rho}{\rho} = \frac{0.01}{100} = \frac{1}{10000}$$

$$P = \frac{K}{10000}$$

5. (c)

The rms velocity of an ideal gas:-

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} \propto \sqrt{T}$$

For % change in rms velocity:-

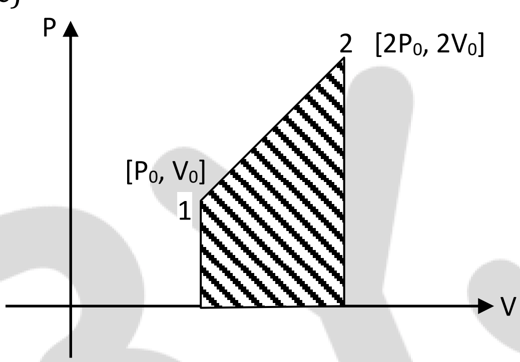
$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\frac{\Delta V}{V} = \frac{1}{2}\left[\frac{6}{300}\right]$$

$$\left[\frac{\Delta v}{v}\right] = \frac{1}{100}$$

$$\%\frac{\Delta v}{v} = 1\%$$

6. (c)



Given:-

n = 2 moles

[Ideal monoatomic gas] $C_v = \frac{3R}{2}$,

Work done by the gas = Area enclosed by curve on volume axis

W = $\frac{1}{2}\left[P_0 + 2P_0\right]\cdot v_0$

= $\frac{3P_0V_0}{2}$

$$\Delta U = nC_v\Delta T$$

$$= n\left[\frac{3R}{2}\right]\left[T_2 - T_1\right]$$

$$= n\left[\frac{3R}{2}\right]\left[\frac{2P_0 2V_0}{nR} - \frac{P_0V_0}{nR}\right]$$

$$= \frac{9P_0V_0}{nR}$$

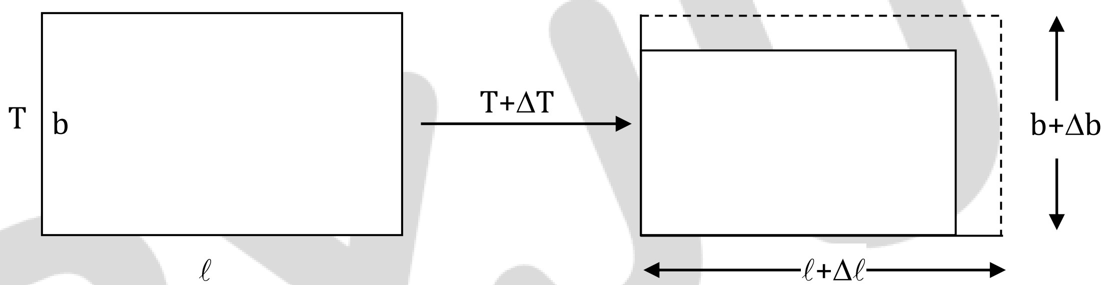
For a process:-

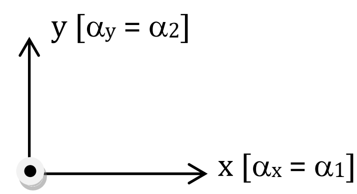
$$\Delta Q = W + \Delta U$$

$$\Delta Q = \frac{3P_0V_0}{2} + \frac{9P_0V_0}{2}$$

$$= 6P_0V_0$$

7. (d)





$$A = \ell b$$

$$\frac{\Delta A}{A} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}$$

$$\Delta \ell = \ell \alpha_1 \Delta T$$

$$\Delta b = b\alpha_2 \Delta T$$

$$\frac{\Delta A}{A} = \alpha_1 \Delta T + \alpha_2 \Delta T \quad .....(i)$$

If β is Arial co-efficient of solid then,

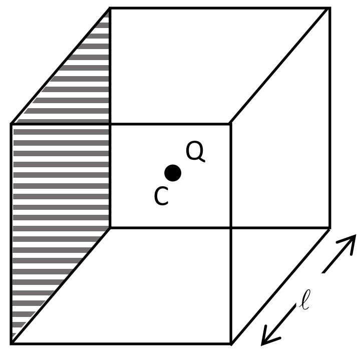
$$\Delta A = \beta A \Delta T$$

$$\beta \Delta T = \alpha_1 \Delta T + \alpha_2 \Delta T \quad .....(ii)$$

From (i) and (ii)

$$\beta = \alpha_1 + \alpha_2$$

8. (a)



From Gauss law electric flux through any closed surface is given by:-

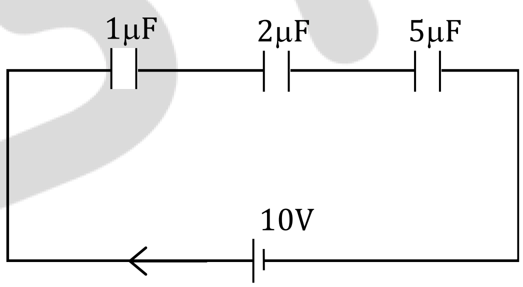
$\phi = \frac{qin}{\varepsilon_0}$

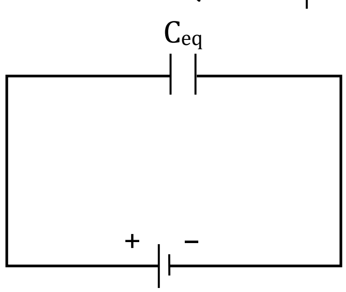
$\phi = \frac{Q}{\varepsilon_0}$

As charge is at the body centre of the cube hence, flux passing through each face is same according to symmetricity.

$\phi_{face} = \frac{Q}{6\varepsilon_0}$

9. (c)





$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{5}$$

$$\frac{1}{C_{eq}} = 1 + \frac{1}{2} + \frac{1}{5}$$

$$C_{eq} = \frac{10}{17}\mu F$$

Charge – flown by battery:-

$Q = C_{eq}V$

$= \left[\frac{10}{17}\right][10]$

$= \frac{100}{17}\mu C$

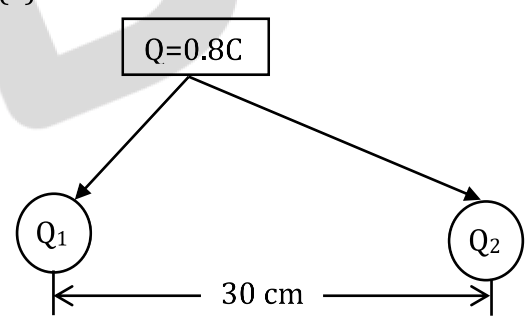
As all capacitors are connected in series charge is same across all capacitors.

For 2 μF $\quad q_2 = Q = C_2 V_2$

$$V_2 = \frac{Q}{C_2} = \frac{\left[\frac{100}{17}\right]\mu C}{2\mu F}$$

$$V_2 = \frac{50}{17}\text{volt}$$

10. (a)



$Q = Q_1 + Q_2$

$Q_2 = Q - Q_1$

Force on charge Q_1

$$F=\frac{KQ_1Q_2}{r^2}$$

$$F=\frac{KQ_1[Q-Q_1]}{r^2}$$

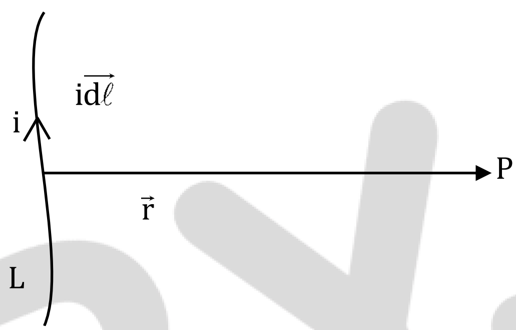
For F_{max}

$$\frac{dF}{dQ_1}=0$$

$$Q-2Q_1=0$$

$$Q_1=\frac{Q}{2}=0.4\text{ C}$$

11. (b)



By Biot – Savart law:-

$$\vec{dB}=\frac{\mu_0 i}{4\pi r^3}\left[\vec{d\ell}\times\vec{r}\right]$$

$$\vec{dB}\propto\frac{1}{r^2}$$

12. (b)

Torque $[\tau]$ = iAB

As i and B are same

so $\tau\propto A$

$$\frac{\tau_1}{\tau_2}=\frac{A_1}{A_2}$$

$$\frac{\tau_1}{\tau_2}=\frac{a^2}{\pi\left[\frac{a}{\sqrt{\pi}}\right]^2}$$

= 1 Ans.

13. (a)

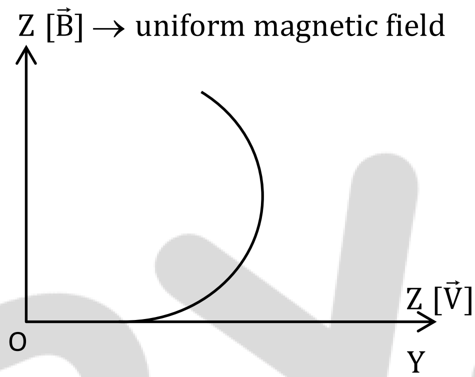
We know from EM waves:-

$\left|\vec{E}\right| = \left|\vec{B}\right| V$

$B = \dfrac{F}{V} = \dfrac{2\times10^{4}}{10^{6}}$

$B = 2\times10^{-2}$ T

Now when E is switched off



$\vec{F}_{net} = \overrightarrow{qE} + 9[\vec{V}\times\vec{B}] \qquad \because \left(\overrightarrow{qE} = 0\right)$

$\vec{F}_{net} = q[\vec{V}\times\vec{B}]$

For circular motion

$$\frac{mv^2}{R} = qVB$$

$$R = \frac{mv}{qB} \qquad \text{Given} \left[\frac{m}{a}\right] = 10^{-8}\,\text{Ckg}^{-1}$$

$$= \frac{10^{-8}\times10^{6}}{2\times10^{-2}}$$

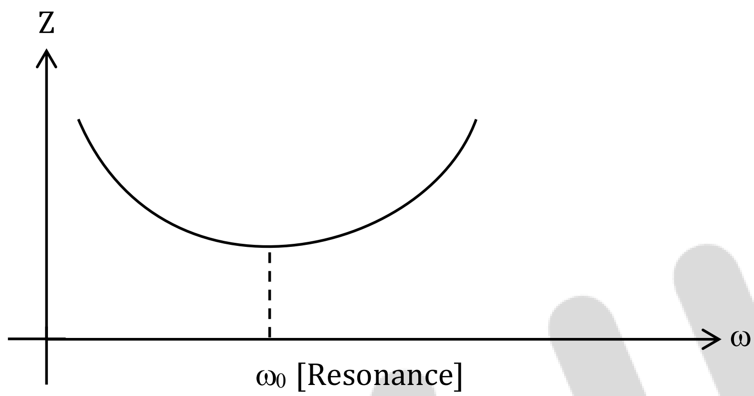
$= 0.5$ m

14. (c)

The impedance of the circuit [series LCR] is given by:-

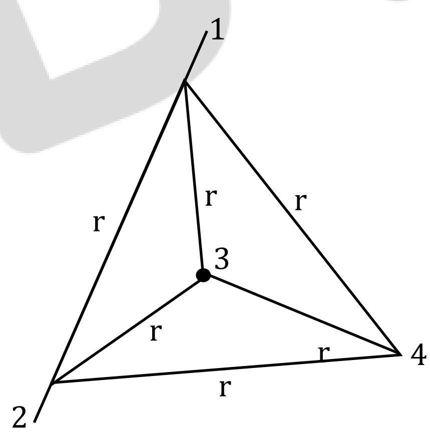
$$Z^2 = [X_L - X_C]^2 + R^2$$

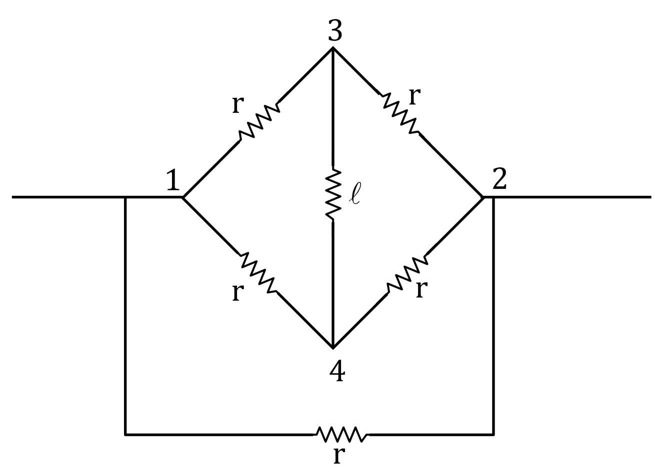
$$Z^2 = [\omega_L - \frac{1}{\omega c}]^2 + R^2$$



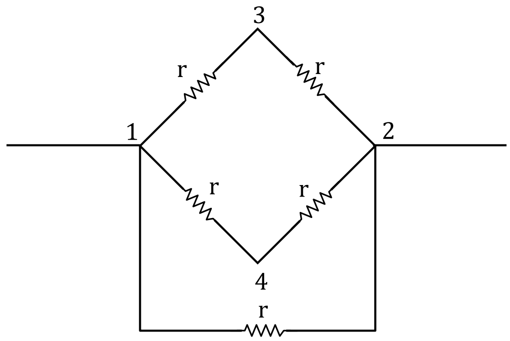
As we gradually increase frequency, z first decreases and then increases.

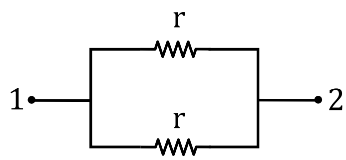
15. (d)





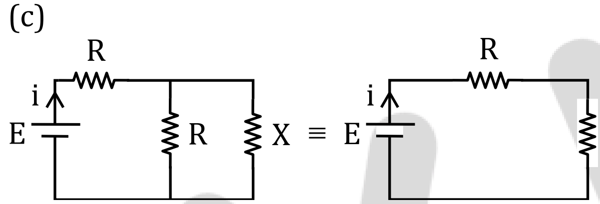
3 & 4 are equipotential point then,





$$Req = \frac{r}{2}$$

16. (c)



$$\frac{RX}{R+X} = R_1$$

$$i = \frac{E}{R + \frac{RX}{R+X}}$$

Voltage-drop across [R_1] :-

$$V_{R_1} = iR_1$$

$$= \left[\frac{E}{R + \frac{RX}{R+X}}\right]\left[\frac{RX}{R+X}\right]$$

$$V_{R_1} = \frac{EX}{R+2X}$$

$$P_X = \frac{V_{R_1}^2}{X}$$

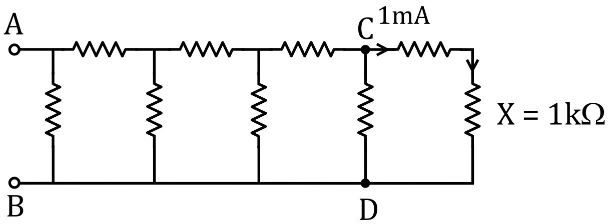
$$P_X = \frac{E^2X}{[R+2X]^2}$$

$$\frac{dP_X}{dx} = \frac{E^2[R-2X]}{[R+2X]^3}$$

[dP_X] will be zero for all dx if

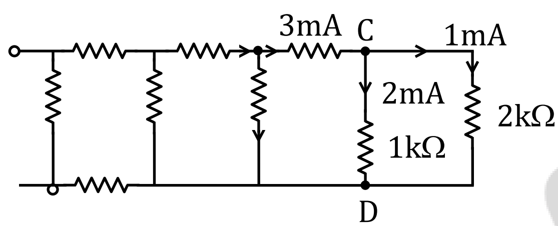
$$X = \frac{R}{2}$$

17. (a)

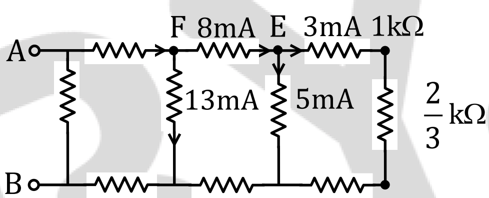


Across C & D same potential current will divide according to :-

$$i \propto \frac{1}{R}$$

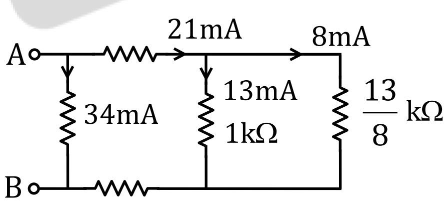


At C apply KCL



Apply KCL at E.

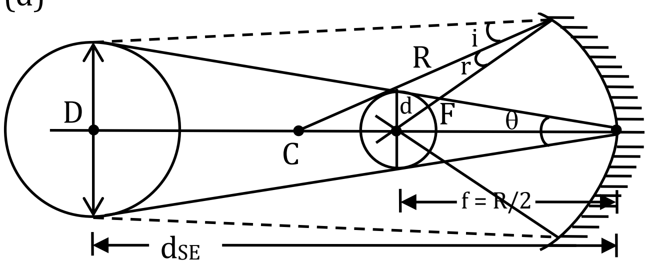
Apply KCl at F.



$V_{AB} = iR = 34 \times 10^{-3} A \times 1 \times 10^{3}\ \Omega$

$= 34\ V$

18. (d)



For concave-mirror if object is placed at infinity: image will be formed at focus [f]

$$\theta = \frac{D}{d_{SE}} = \frac{d}{f}$$

$d = \theta.f$

$d = 0.009 \times 0.2$ m

$d = 1.8 \times 10^{-3}$ m

19. (a)

$$I = \sqrt{I_1}^2 + \sqrt{I_2}^2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

For maximum intensity:-

$\cos\phi = 1$

$$I_{max} = \left[\sqrt{I_1} + \sqrt{I_2}\right]^2$$

For minimum intensity:-

$$I_{min} = \left[\sqrt{I_1} - \sqrt{I_2}\right]^2$$

$$I_{max} = \left[\sqrt{L} + \sqrt{\frac{L}{4}}\right]^2$$

$$= \frac{9L}{4}$$

$$I_{min} = \left[\sqrt{L} - \sqrt{\frac{L}{4}}\right]^2$$

$$= \frac{L}{4}$$

20. (b)

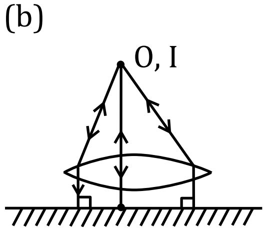
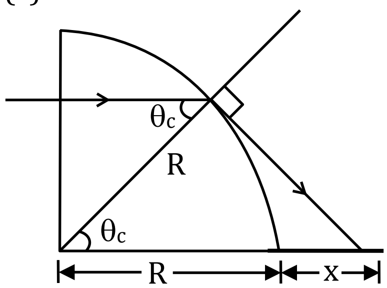


Image will be formed on object itself.

21. (c)



$$\sin\theta_c = \frac{1}{n} = \frac{2}{3}$$

$$\cos\theta_c = \frac{R}{R+x}$$

$$\sqrt{1-\frac{4}{9}} = \frac{R}{R+x}$$

$$\frac{\sqrt{5}}{3} = \frac{R}{R+x}$$

$$x = \left[3\sqrt{5}-5\right]\times 10^{-2}\ \text{m}$$

22. (d)

Wavelength:-

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{\sqrt{2mk}}$$

$$\lambda \propto \frac{1}{\sqrt{k}}$$

$$\sqrt{\frac{k_2}{k_1}} = \left[\frac{\lambda_1}{\lambda_2}\right]$$

$$\frac{k_2}{k_1} = \left[\frac{\lambda_1}{\lambda_2}\right]^2$$

$$k_2 = \left[\frac{0.4\times 10^{-10}\,\text{m}}{1.0\times 10^{-10}\,\text{m}}\right]^2 1\text{keV}$$

k_2 = 0.16 keV

23. (c)

We know for a photon:-

$KE_{max} = h\upsilon - h\upsilon_0$

$KE_{max} = h\upsilon - \phi_0$ .....(i)

$KEmax = qV_0$ .....(ii)

q = charge on electron

V_0 = stopping potential

ϕ_0 = work-function of a metal

From (i) & (ii)

$qV_0 = h\upsilon - \phi_0$

When light of intensity υ_1 falls :-

$eV_1 = h\upsilon_1 - w$

$h\upsilon_1 = w + eV_1$ .....(iii)

When light of intensity υ_2 falls

$eV_2 = h\upsilon_2 - w$

$h\upsilon_2 = eV_2 + w$ .....(iv)

(iii) ÷ (iv) :-

$$\frac{\upsilon_1}{\upsilon_2} = \frac{w + eV_1}{w + eV_2}$$

$w\upsilon_1 + e\upsilon_1V_2 = w\upsilon_2 + e\upsilon_2V_1$

$e\upsilon_1V_2 - e\upsilon_2V_1 = w[\upsilon_2 - \upsilon_1]$

$$e = \frac{w[\upsilon_2 - \upsilon_1]}{\upsilon_1V_2 - \upsilon_2V_1}$$

24. (a)

The equation of Radioactive-Decay :-

$N=N_0e^{-\lambda t}$

The radioactive constant:-

$$\lambda = \frac{\ell n2}{t_{1/2}}$$

$$= \frac{\ell n2}{3.8}$$

Quantity N at t = 19 day:-

$$N = 0.064e^{-\frac{\ell n2}{3.8}\times 19}$$

$N = 0.064\ e^{-5\ell n2}$

$N = 0.064\ e^{-\ell n32}$

N = 0.002 kg

25. (b)

$$Y=\overline{AB+\overline{A}\,\overline{B}}$$

For A = 1 ; B = 1

$Y = 0$

& For A = 0 ; B = 0

$Y = 0$

26. (d)

p–n junction Diode :-

- It is a one way device. It offers a low resistance when forward biased hence current easily flow.
- It offers high resistance when reverse biased and current almost becomes zero.

27. (b)

From Newton's law of gravitation:-

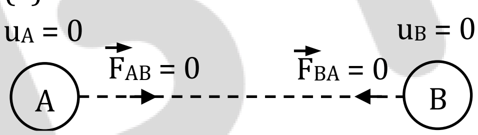
$$F=\frac{Gm_1m_2}{r^2}$$

$$[G]=\left[\frac{Fr^2}{m_1m_2}\right]$$

$$=\left[\frac{M^1L^1T^{-2}\cdot L^2}{M^2}\right]$$

$$= [M^{-1}L^3T^{-2}]$$

28. (a)



(initial condition at t = 0)

$$\vec{V}_{com}\Big|_{t=0}=0$$

$$\vec{F}_{ext}=m\vec{a}_{com}\Big|_{system}=m\frac{d\vec{V}_{com}\Big|_{sys}}{dt}$$

Considering A & B as a system

$$\vec{F}_{ext}\Big|_{sys}=0$$

$$\vec{V}_{com}\Big|_{sys}=0$$

Hence, at all instants centre of mass the system will be at rest.

29. (b)

There are three unknowns hence we require 3 equations accordingly. [use application of scalar product]

$\vec{A}\cdot\vec{B}=0$ $\quad$ [$\theta = 90°$]

$a + b + 1 = 0$ $\quad$ .....(i)

$\vec{B}\cdot\vec{C}=0$

$1 + b + c = 0$ $\quad$ .....(ii)

$\vec{C}\cdot\vec{A}=0$

$a + 1 + c = 0$ $\quad$ .....(iii)

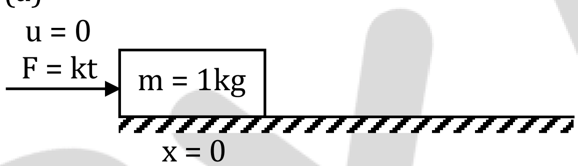
Adding (i), (ii) & (iii) :-

$2[a + b + c] + 3 = 0$

$a=\frac{-3-2[b+c]}{2}$; $\quad b=\frac{-3-2[a+c]}{2}$; $\quad c=\frac{-3-2[a+b]}{2}$

$a=-\frac{1}{2}$ $\quad b=-\frac{1}{2}$ $\quad c=-\frac{1}{2}$

30. (a)



$F = kt = ma$

$k = 1NS^{-1}$; $m = 1$

$t=\frac{ma}{k}=\frac{dv}{dt}$

$$\int_{u=0}^{v} dv=\int_{t=0}^{t} t\,dt$$

$$v=\frac{t^2}{2}$$

$$\frac{dx}{dt}=\frac{t^2}{2}$$

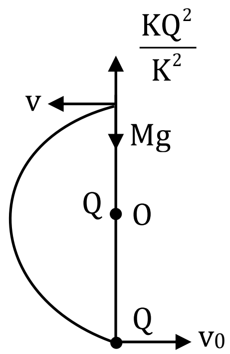
$$\int_{x=0}^{x} dx=\int_{t=0}^{t}\frac{t^2}{2}dt$$

$$x=\frac{t^3}{6}$$

At t = 6 second:-

$x = 36$ m

31. (b)



As $Mg = \frac{Q^2}{4\pi\varepsilon_0 R^2}$

At highest point tension in the string vanishes

T = 0

As T = 0

v = 0

Applying work-kinetic-energy theorem:-

WD|$_{\text{All-forces}}$ = ΔK.E

$$Mg[2R] = \frac{1}{2}mv_0^2 \ v_0 = 2\sqrt{gR}$$

32. (b)

Mass of Bullet = m

Velocity of Bullet = $300\frac{m}{s}$

When Bullet will get stuck inside Block, both will move with same velocity i.e. V

Applying conservation of linear – momentum:-

mv = [m + 9m] V

$$V = \frac{v}{10}$$

As there is no friction hence

Heat generated = change in K.E

$$= \frac{1}{2}mv^2 - \frac{1}{2}10mV^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}10m\left[\frac{v}{10}\right]^2$$

$$= \frac{9}{20}mv^2$$

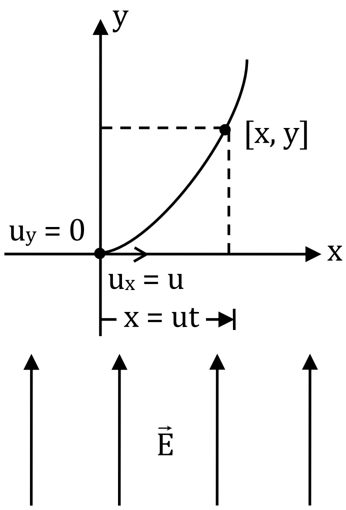
= 1701 J

Total heat generated in calories:-

$$= \frac{1701}{4.2}$$

= 405 cal

33. (d)



$$\vec{a}_y = \frac{\vec{F}_E}{m} = \left[\frac{q\vec{E}}{m}\right]$$

No effect of gravity is considered

For particle trajectory

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$y = \frac{1}{2}\left[\frac{qE}{m}\right] t^2$$

$$y = \frac{1}{2}\left[\frac{qE}{m}\right]\left[\frac{x}{u}\right]^2$$

$$y = \frac{Ee}{2mu^2} x^2$$

Now this parabola relates with

$$x^2 = 4ay$$

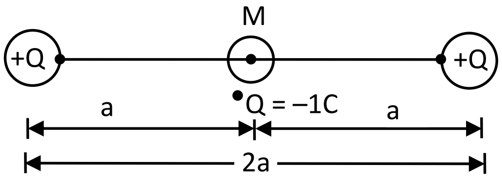
$$x^2 = \left[\frac{2mu^2}{Ee}\right] y$$

$$4a = \frac{2mu^2}{Ee}$$

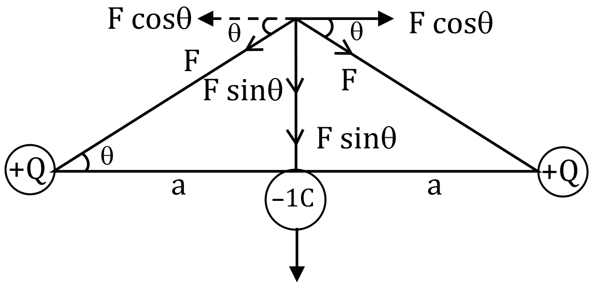
Hence

$$a = \frac{mu^2}{2Ee}$$

34. (G) Bonus



Its given small displacement



Mean-position [Equilibrium-position]
Restoring force towards mean position

$$F_{net} = -2F\sin\theta$$

$$F_{net} = -2\frac{KQ\times 1}{\left[y^2+a^2\right]}\times\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net} = -\frac{2KQy}{\left[y^2+a^2\right]^{3/2}}$$

$y <<< a$ small-displacement

$$F_{net} = -\frac{2KQy}{a^3}$$

$$\text{Frequency}\left[f\right] = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

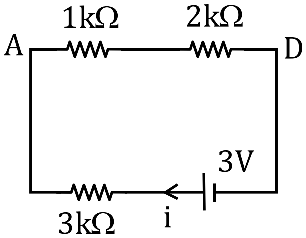
$$= \frac{1}{2\pi}\sqrt{\frac{2KQ}{Ma^3}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{Q}{2\pi\varepsilon_0 Ma^3}}$$

None of the option is correct.

35. (b)

At starting of the circuit [t = 0] capacitor can be replaced by a simple wire. Hence



$$i = \frac{V}{Req}$$

[Req = 6kΩ]

$$i = \frac{3}{6 \times 10^3}$$

$I = 0.5 \times 10^{-3}$ A

$V_{AD} = iR = 0.5 \times 10^{-3} \times 3 \times 10^3$

$= 1.5$ V

$$Q = \left[\frac{2}{3}\right] \times 1.5 = 1\mu C$$

Applying KVL from B to C

$$V_B - 0.5 \times 10^{-3} \times 2 \times 10^3 + \frac{1}{2} = V_C$$

$V_B - V_C = 0.5$ V

36. (b,c)

Velocity of sound:-

$$V = \sqrt{\frac{\gamma RT}{M}} \quad \text{.....(i)}$$

$PV = nRT$

$$PV = \frac{m}{M}RT$$

$$P = \left[\frac{m}{V}\right]\frac{RT}{M}$$

$$P = \frac{\rho RT}{M}$$

$$\frac{RT}{M} = \frac{P}{\rho}$$

Putting value in (i):-

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{.....(ii)}$$

Now from (i) & (ii)

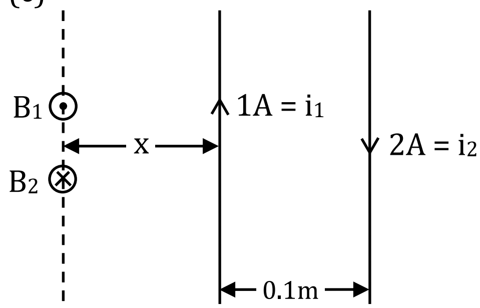
$$V \propto \sqrt{T}$$

When ρ = constant

$$V \propto \sqrt{P}$$

Option (b) & (c) are correct.

37. (c)



$B_1 = B_2$

$$\frac{\mu_0[1]}{2\pi x} = \frac{\mu_0[2]}{2\pi[0.1+x]}$$

$x = 0.1$ m

38. (b,d)

$\chi = \mu_r - 1$

$$\mu_r = \frac{\mu}{\mu_0}$$

For paramagnetic substance:- $\chi > 0$, $\mu_r > 1$; $\mu > \mu_0$

For diamagnetic substance:- $\chi < 0$, $\mu_r < 1$; $\mu < \mu_0$

For ferromagnetic substance:- $\chi >> 1$, $\mu >> \mu_0$

If $0 < \mu < \mu_0$ then substance will not be paramagnetic. Hence option (a) is incorrect.

39. (a,b,c,d)

$V_n \propto \frac{1}{n}$ $\qquad E_n \propto \frac{1}{n^2}$ $\qquad r_n \propto n^2$

$\because E_n r_n \propto n^\circ$ $\qquad \because E_n r_n \propto E_1 r_1$ $\qquad \frac{E_n r_n}{E_1 r_1} = \text{constant}$ [slope = 0]

$\because r_n V_n \propto n$ $\qquad \because \frac{r_n V_n}{r_1 V_1} = n$ $\qquad$ [Slope = 1]

$\because r_n \propto n^2$ $\qquad \because \frac{r_n}{r_1} = n^2$

$$\ell n\left[\frac{r_n}{r_1}\right] = 2\ell nn \text{ [slope = 2]}$$

$\because \frac{r_n}{E_n} \propto n^4$ $\qquad \because \frac{r_n E_1}{E_n r_1} = n^4$

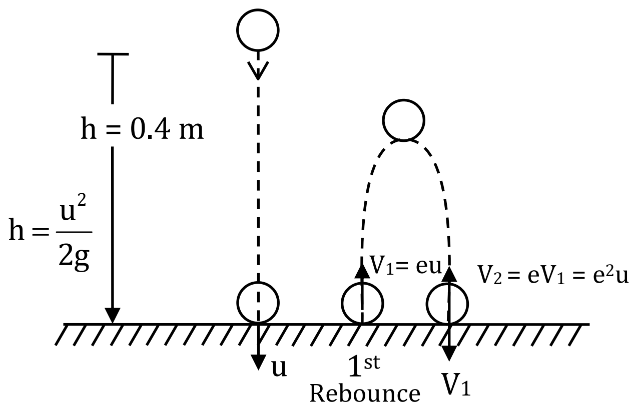
$$\ell n\left[\frac{r_n E_1}{r_1 E_n}\right] = 4\ell nn \text{ [Slope = 4]}$$

Option a, b, c & d are correct.

40. (d)

Total time of bouncing = 10 sec

$$t = \sqrt{\frac{2h}{8}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{8}} + ...$$



Maximum height achieved in 1^{st} rebounce :-

$$h_1 = \frac{V_1^2}{2g} = \frac{e^2u^2}{2g} = he^2$$

Time taken in 1^{st} rebounce :-

$$t_1 = 2\sqrt{\frac{2h_1}{g}} = 2\sqrt{\frac{he^2}{g}}$$

Time taken in 2^{nd} rebounce :-

$$t_2 = 2\sqrt{\frac{2he^4}{g}}$$

$$t = 2\sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2he^2}{g}} + 2\sqrt{\frac{2he^4}{g}} + ... = 10$$

$$= \sqrt{\frac{2h}{g}}\left[1 + 2e + 2e^2 + ...\right] = 10$$

$$= \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right] = 10$$

$$e = \frac{17}{18}$$