WBJEE -2019 (Mathematics)

1. P is the extremity of the latus rectum of ellipse $3x^2 + 4y^2 = 48$ in the first quadrant. The eccentric angle of P is

   a. $\frac{\pi}{8}$

   b. $\frac{3\pi}{4}$

   c. $\frac{\pi}{3}$

   d. $\frac{3\pi}{3}$

2. The direction ratios of the normal of the plane through the points (1,2,3), (–1,–2,1) and parallel to $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z}{4}$ is

   a. (2, 3, 4)

   b. (14, –8, –1)

   c. (–2, 0, –3)

   d. (1, –2, –3)

3. The equation of the plane. which bisects the line joining the points (1, 2, 3) and (3, 4, 5) at right angles is

   a. $x + y + z = 0$

   b. $x + y - z = 9$

   c. $x + y + z = 9$

   d. $x + y - z + 9 = 0$

4. The limit of the interior angle of a regular polygon of n sides as $n \to \infty$ is

   a. π

   b. $\frac{\pi}{3}$

   c. $\frac{3\pi}{2}$

   d. $\frac{2\pi}{3}$

5. Let f(x) > 0 for all x and f '(x)' exists for all x. If f is the inverse function of h and $h'(x) = \frac{1}{1+\log x}$. Then f '(x)' will be
   a. 1 + log (f(x))
   b. 1+ f(x)
   c. 1 – log (f(x))
   d. log f(x)

6. Consider the function f(x) = cosx^2. Then
   a. f is of period 2π
   b. f is of period $\sqrt{2\pi}$
   c. f is not periodic
   d. f is of periodic π

7. $\lim_{x \to 0^+} \left(e^x + x\right)^{1/x}$
   a. Does or exist finitely
   b. is 1
   c. is e^2
   d. is 2

8. Let f(x) be a derivable function, f '(x) > f(x) and f (0) = 0. Then
   a. f(x) > 0 for all x > 0
   b. f(x) < 0 for all x > 0
   c. no sign of f (x) can be ascertained
   d. f(x) is a constant function

9. Let f: [1, 3] $\to$ R be a continuous function that is differentiable in (1, 3) and f '(x) = $|f(x)|^2 + 4$ for all x $\in$ (1, 3). Then
   a. f(3) – f(1) = 5 is true
   b. f(3) – f(1) = 5 is false
   c. f(3) – f(1) = 7 is false
   d. f(3) – f(1) < 0 only at one point (1, 3)

10. $\lim\limits_{x\to 0^+}\left(x^n \ell n x\right), n>0$

a. does not exist

b. exists and is zero

c. exists and is 1

d. exists and is e^{-1}

11. If $\int \cos x \log\left(\tan\frac{x}{2}\right)dx = \sin x \log\left(\tan\frac{x}{2}\right) + f(x)$ then f(x) is equal to, (assuming c is a arbitrary real constant)

a. c

b. c – x

c. c + x

d. 2x + c

12. $y = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is an equation of a family of

a. straight lines

b. circles

c. ellipses

d. parabolas

13. The value of the integration $\int_{-\pi/4}^{\pi/4}\left(\lambda|\sin x| + \frac{\mu \sin x}{1+\cos x} + \gamma\right)dx$

a. is independent of λ only

b. is independent of μ only

c. in independent of γ only

d. depends on γ, μ and λ

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14. The value of $\lim\limits_{x\to 0}\frac{1}{x}\left[\int\limits_{y}^{a} e^{\sin^2 t}dt - \int\limits_{x+y}^{a} e^{\sin^2 t}dt\right]$ is equal to

a. $e^{\sin^2 y}$

b. $e^{2\sin y}$

c. $e^{|\sin y|}$

d. $e^{\operatorname{cosec}^2 y}$

15. If $\int 2^{2^x}.2^x dx = A.2^{2^x} + c$, then A =

a. $\frac{1}{\log 2}$

b. log 2

c. $(\log 2)^2$

d. $\frac{1}{(\log 2)^2}$

16. The value of the integral $\int\limits_{-1}^{1}\left\{\frac{x^{2015}}{e^{|x|}\left(x^2+\cos x\right)}+\frac{1}{e^{|x|}}\right\}dx$ is equal to

a. 0

b. $1 - e^{-1}$

c. $2e^{-1}$

d. $2(1 - e^{-1})$

17. $\lim\limits_{x\to\infty}\frac{1}{n}\left\{1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+.....+\sqrt{\frac{n}{n+3(n-1)}}\right\}$

a. does not exist

b. is 1

c. is 2

d. is 3

18. The general solution of the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{\frac{x}{y}}dy=0$ is (c is an arbitrary constant)

a. $x-ye^{\frac{x}{y}}=c$

b. $y-xe^{\frac{x}{y}}=c$

c. $x+ye^{\frac{x}{y}}=c$

d. $y+xe^{\frac{x}{y}}=c$

19. General solution of $(x+y)^2\frac{dy}{dx}=a^2$, $a\neq 0$ is (c is arbitrary constant)

a. $\frac{x}{a}=\tan\frac{y}{a}+c$

b. tan xy = c

c. tan (x + y) = c

d. $\tan\frac{y+c}{a}=\frac{x+y}{a}$

20. Let P(4, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at P intersects the x-axis at (16, 0), then the eccentricity of the hyperbola is

a. $\frac{\sqrt{5}}{2}$

b. 2

c. $\sqrt{2}$

d. $\sqrt{3}$

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21. If the radius of a spherical balloon increases by 0.1% then its volume increases approximately by

    a. 0.2%

    b. 0.3%

    c. 0.4%

    d. 0.05%

22. The three sides of a right-angled triangle are in G.P. (Geometrical Progression). If the two acute angles be α and β, then $\tan\alpha$ and $\tan\beta$ are:

    a. $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$

    b. $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$

    c. $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$

    d. $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

23. If $\log_2 6+\frac{1}{2x}=\log_2\left(2^{\frac{1}{x}}+8\right)$, then the value of x are

    a. $\frac{1}{4},\frac{1}{3}$

    b. $\frac{1}{4},\frac{1}{2}$

    c. $-\frac{1}{4},\frac{1}{2}$

    d. $\frac{1}{3},-\frac{1}{2}$

24. Let z be a complex number such that the principal value of argument, arg z > 0. Then arg z – arg(–z) is

a. $\frac{\pi}{2}$

b. $\pm\pi$

c. π

d. $-\pi$

25. The general value of the real angle θ, which satisfies the equation $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)..........(\cos n\theta + i\sin\theta) = 1$ is given by (assuming k is an integer)

a. $\frac{2k\pi}{n+2}$

b. $\frac{4k\pi}{n(n+1)}$

c. $\frac{4k\pi}{n+1}$

d. $\frac{6k\pi}{n(n+1)}$

26. Let a,b,c be real numbers such $a + b + c < 0$ and the quadratic equation $ax^2 + bx + c = 0$ has imaginary roots. Then

a. $a > 0, c > 0$

b. $a > 0, c < 0$

c. $a < 0, c > 0$

d. $a < 0, c < 0$

27. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B each containing 6 questions and he/she is not permitted to attempt more than 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions?
    a. 850
    b. 800
    c. 750
    d. 700

28. There are 7 greetings cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is:
    a. ${}^7C_3$
    b. $2.\,{}^7C_3$
    c. $3!\,{}^4C_4$
    d. $3!\,{}^7C_3\,{}^4C_3$

29. $7^{2n} + 16n - 1$ $(n \in N)$ is divisible by
    a. 65
    b. 63
    c. 61
    d. 64

30. The number of irrational terms in the expansion of $(3^{1/8} + 5^{1/4})^{84}$ is
    a. 73
    b. 74
    c. 75
    d. 76

31. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix $A - 3I_3$ is
    a. invertible
    b. orthogonal
    c. non-invertible
    d. real skew symmetric matrix

32. If M is any square matrix of order 3 over R and If M' be the transpose of M, then adj(M') – adj(M)' is equal to
    a. M
    b. M'
    c. null matrix
    d. identity matrix

33. If $A = \begin{pmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{pmatrix}$ and $|A^2| = 25$, then $|x|$ is equal to
    a. $\frac{1}{5}$
    b. 5
    c. 5^2
    d. 1

34. Let A and B be two square matrices of order 3 and $AB = O_3$, where O_3 denotes the null matrix of order 3. Then
    a. must be $A = O_3$, $B = O_3$
    b. if $A \neq O_3$, must be $B \neq O_3$
    c. if $A = O_3$, must be $B \neq O_3$
    d. may be $A \neq O_3$, $B \neq O_3$

35. Let P and T be the subsets of X-Y plane defined by

$P = \{(x,y): x > 0, y > 0 \text{ and } x^2 + y^2 = 1\}$

$T = \{(x,y): x > 0, y > 0 \text{ and } x^8 + y^8 < 1\}$

Then $P \cap T$ is

a. the void set ϕ

b. P

c. T

d. $P - T^C$

36. Let f: $R \to R$ be defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$ for all $x \in R$. Then

a. f is one-one but not onto mapping

b. f is onto but not one-one mapping

c. f is both one-one and onto

d. f is neither one-one nor onto

37. Let the relation ρ be defined on R as $a\rho b$ if $1 + ab > 0$. Then

a. ρ is reflexive only

b. ρ is equivalence relation

c. ρ is reflexive and transitive but not symmetric

d. ρ is reflexive and symmetric but not transitive

38. A problem in mathematics is given to 4 students whose chances of solving individually are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. Then probability that the problem will be solved at least by one student is

a. $\frac{2}{3}$

b. $\frac{3}{5}$

c. $\frac{4}{5}$

d. $\frac{3}{4}$

39. If X is a random variable such that $\sigma(X) = 2.6$, then $\sigma(1 - 4X)$ is equal to
    a. 7.8
    b. –10.4
    c. 13
    d. 10.4

40. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is
    a. 0
    b. 1
    c. 2
    d. 3

41. The angles of a triangle are in the ratio 2 : 3 : 7 and the radius of the circumscribed circle is 10 cm. The length of the smallest side is
    a. 2 cm
    b. 5 cm
    c. 7cm
    d. 10 cm

42. A variable line passes through a fixed point (x_1, y_1) and meets the axes at A and B. If the rectangle OAPB be completed, the locus of P is, (O being the origin of the system of axes)
    a. $(y - y_1)^2 = 4(x - x_1)$
    b. $\frac{x_1}{x} + \frac{y_1}{y} = 1$
    c. $x^2 + y^2 = x_1^2 + y_1^2$
    d. $\frac{x^2}{2x_1^2} + \frac{y^2}{2y_1^2} = 1$

43. A straight line through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If it intersects the X-axis, then its equation will be

a. $y + x\sqrt{3} + 2 + 3\sqrt{3} = 0$

b. $y - x\sqrt{3} + 2 + 3\sqrt{3} = 0$

c. $y - x\sqrt{3} - 2 - 2\sqrt{3} = 0$

d. $x - x\sqrt{3} + 2 - 3\sqrt{3} = 0$

44. A variable line passes through the fixed point (α, β). The locus of the foot of the perpendicular from the origin on the line is

a. $x^2 + y^2 - \alpha x - \beta y = 0$

b. $x^2 - y^2 + 2\alpha x + 2\beta y = 0$

c. $ax + by \pm \sqrt{(\alpha^2 + \beta^2)} = 0$

d. $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

45. if the point of intersection of the lines $2ax + 4ay + c = 0$ and $7bx + 3by - d = 0$ lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then ad : bc equals to

a. 2 : 3

b. 2 : 1

c. 1 : 1

d. 3 : 2

46. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is

a. $(x - p)^2 = 4qy$

b. $(x - q)^2 = 4py$

c. $(y - p)^2 = 4qx$

d. $(y - q)^2 = 4px$

47. If P(0, 0), Q(1, 0) and $R\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for which the lines PQ, QR and RP are the tangents is

a. $\left(\frac{1}{2}, \frac{1}{4}\right)$

b. $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$

c. $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$

d. $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

48. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains fixed when α varies?

a. Directrix

b. Vertices

c. foci

d. Eccentricity

49. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

a. $\frac{1}{4}$

b. $\frac{1}{3}$

c. $\frac{1}{2}$

d. $\frac{2}{3}$

50. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

a. $x = 3 \pm \sqrt{\frac{13}{6}}$

b. $x = 3 \pm \sqrt{\frac{6}{13}}$

c. $x = 6 \pm \sqrt{\frac{13}{3}}$

d. $x = 6 \pm \sqrt{\frac{3}{13}}$

51. The graphs of the polynomial $x^2 - 1$ and cos x intersect

a. at exactly two points

b. at exactly 3 points

c. at least 4 but at finitely many points

d. at infinitely many points

52. A point is in motion along a hyperbola $y = \frac{10}{x}$ so that its abscissa x increases uniformly at a rate of 1 unit per second. Then, the rate of change of its ordinate, when the point passes through (5, 2)

a. increases at the rate of $\frac{1}{2}$ unit per second

b. decreases at the rate of $\frac{1}{2}$ unit per second

c. decreases at the rate of $\frac{2}{5}$ unit per second

d. increases at the rate of $\frac{2}{5}$ unit per second

53. Let a = min$\{x^2 + 2x + 3 : x \in R\}$ and b = $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$. Then $\sum_{r=0}^{n} a^r b^{n-r}$ is

a. $\frac{2^{n+1}-1}{3.2^n}$

b. $\frac{2^{n+1}+1}{3.2^n}$

c. $\frac{4^{n+1}-1}{3.2^n}$

d. $\frac{1}{2}(2^n - 1)$

54. Let a > b >0 and $I(n) = a^{1/n} - b^{1/n}$, $J(n) = ((a - b)^{1/n}$ for all $n \geq 2$. then

a. I(n) < J(n)

b. I(n) > J(n)

c. I(n) = J(n)

d. I(n) + J(n) = 0

55. Let $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ be three unit vectors such that $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} \times \hat{\gamma})$ where $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma})$ $= (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

a. $\frac{5\pi}{6}$

b. $\frac{\pi}{6}$

c. $\frac{\pi}{3}$

d. $\frac{2\pi}{3}$

56. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + \lambda\hat{k}$ respectively. If the points A, B, C and D lie on a plane, the value of λ is

a. 0

b. 1

c. 2

d. – 4

57. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1, then it moves $\frac{1}{2}$ unit vertically up and reaches P_2, then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3, then it moves $\frac{1}{8}$ unit vertically down and reaches P_4, then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on. Let $P_n = (x_n, y_n)$ and $\lim_{n\to\infty} x_n = \alpha$ and $\lim_{n\to\infty} y_n = \beta$. Then (α, β) is

a. (2, 3)

b. $\left(\frac{4}{3}, \frac{2}{5}\right)$

c. $\left(\frac{2}{5}, 1\right)$

d. $\left(\frac{4}{3}, 3\right)$

58. For any non-zero complex number z, the minimum value of $|z| + |z - 1|$ is

a. 1

b. $\frac{1}{2}$

c. 0

d. $\frac{3}{2}$

59. The system of equations

$\lambda x + y + 3z = 0$

$2x + \mu y - z = 0$

$5x + 7y + z = 0$

Has infinitely many solutions in R. Then,

a. $\lambda = 2, \mu = 3$

b. $\lambda = 1, \mu = 2$

c. $\lambda = 1, \mu = 3$

d. $\lambda = 3, \mu = 1$

60. Let $f : X \longrightarrow Y$ and A, B are non-void subsets of Y, then (where the symbols have their usual interpretation)

a. $f^{-1}(A) - f^{-1}(B) \supset f^{-1}(A - B)$ but the opposite does not hold

b. $f^{-1}(A) - f^{-1}(B) \subset f^{-1}(A - B)$ but the opposite does not hold

c. $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$

d. $f^{-1}(A - B) = f^{-1}(A) \cup f^{-1}(B)$

61. Let S, T, U be three non-void sets and $f : S \to T$, $g : T \to U$ be so that $gof : S \to U$ is surjective. Then

a. g and f are both surjective

b. g is surjective, f may not be so

c. f is surjective, g may not be so

d. f and g both may not be surjective

62. The polar coordinate of a point P is $\left(2, -\frac{\pi}{4}\right)$. The polar coordinate of the point Q, which is such that the line joining PQ is bisected perpendicularly by the initial line, is

a. $\left(2, \frac{\pi}{4}\right)$

b. $\left(2, \frac{\pi}{6}\right)$

c. $\left(-2, \frac{\pi}{4}\right)$

d. $\left(-2, \frac{\pi}{6}\right)$

63. The length of conjugate axis of a hyperbola is greater than the length of transverse axis. Then the eccentricity e is

a. $= \sqrt{2}$

b. $> \sqrt{2}$

c. $< \sqrt{2}$

d. $\frac{1}{\sqrt{2}}$

64. The value of $\lim_{x \to 0^+} \frac{x}{p}\left[\frac{q}{x}\right]$ is

a. $\frac{[q]}{p}$

b. 0

c. 1

d. ∞

65. Let $f(x) = x^4 - 4x^3 + 4x^2 + c$, $c \in R$. Then

a. f(x) has infinitely many zeroes in (1, 2) for all c

b. f(x) has exactly one zero in (1, 2) if $-1 < c < 0$

c. f(x) has double zeroes in (1, 2) if $-1 < c < 0$

d. whatever be the value of c, f(x) has no zero in (1, 2)

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Category-III (Q. 66 to Q. 75)

Carry 2 marks each and on or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked + actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let f and g be differentiable on the interval I and let $a, b \in I$, $a < b$. Then
   a. If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ is solvable in (a, b)
   b. If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ may not be solvable in (a, b)
   c. If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ is solvable in (a, b), $k \in R$
   d. If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ may not be solvable in (a, b), $k \in R$

67. Consider the function $f(x) = \frac{x^3}{4} - \sin \pi x + 3$
   a. $f(x)$ does not attain value within the interval $[-2, 2]$
   b. $f(x)$ takes on the value $2\frac{1}{3}$ in the interval $[-2, 2]$
   c. $f(x)$ takes on the value $3\frac{1}{4}$ in the interval $[-2, 2]$
   d. $f(x)$ takes no value p, $1 < p < 5$ in the interval $[-2, 2]$

68. Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then
   a. a_1, a_2, a_3 are in G.P.
   b. b_1, b_2, b_3 are in A.P.
   c. c_1, c_2, c_3 are in H.P.
   d. a_1, a_2, a_3 are in A.P.

69. Two particles A and B move from rest along a straight line with constant accelerations f and h respectively. If A takes m seconds more than B and describes n units more than that of B acquiring the same speed, then

   a. $(f + h)m^2 = fhn$

   b. $(f - fh)m^2 = fhn$

   c. $(h - f)n = \frac{1}{2} fhm^2$

   d. $\frac{1}{2}(f + h)n = fhm^2$

70. The area bounded by $y = x + 1$ and $y = \cos x$ and the x-axis, is

   a. 1 sq. unit

   b. $\frac{3}{2}$ sq. unit

   c. $\frac{1}{4}$ sq. unit

   d. $\frac{1}{8}$ sq. unit

71. Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P. then ab equals

   a. $\frac{24}{5}$

   b. 64

   c. 16

   d. 8

72. If $q \in R$ and $\dfrac{1-i\cos\theta}{1+2i\cos\theta}$ is real number, then θ will be (when I : Set of integers)

a. $(2n+1)\dfrac{\pi}{2}, n \in I$

b. $\dfrac{3n\pi}{2}, n \in I$

c. $n\pi, n \in I$

d. $2n\pi, n \in I$

73. Let $A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. Then the roots of the equation $\det(A - \lambda I_3) = 0$

(Where I_3 is the identity matrix of order 3) are

a. 3, 0, 3

b. 0, 3, 6

c. 1, 0, –6

d. 3, 3, 6

74. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that AB = AC. The equation of line AC passing through (2, –7) is:

a. $x - y - 9 = 0$

b. $23x + 7y + 3 = 0$

c. $2x - y - 11 = 0$

d. $7x - 6y - 56 = 0$

75. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point (2, 8) is

a. $3x - y + 2 = 0$

b. $3x + y - 14 = 0$

c. $23x - 3y - 22 = 0$

d. $3x - 23y + 178 = 0$

WBJEE -2019 (Mathematics)

ANSWER KEYS

1. (c)	2. (b)	3. (c)	4. (a)	5. (a)	6. (c)	7. (c)	8. (a)	9. (b,c)	10. (b)
11. (b)	12. (d)	13. (b)	14. (a)	15. (d)	16. (d)	17. (c)	18. (c)	19. (d)	20. (b)
21. (b)	22. (b)	23. (b)	24. (c)	25. (b)	26. (d)	27. (a)	28. (b)	29. (d)	30. (b)
31. (c)	32. (c)	33. (a)	34. (d)	35. (b)	36. (d)	37. (d)	38. (c)	39. (d)	40. (a)
41. (d)	42. (b)	43. (b)	44. (a)	45. (b)	46. (a)	47. (c)	48. (c)	49. (c)	50. (a)
51. (a)	52. (c)	53. (c)	54. (a)	55. (d)	56. (d)	57. (b)	58. (a)	59. (c)	60. (c)
61. (b)	62. (a)	63. (b)	64. (a)	65. (b)	66. (a,c)	67. (b,c)	68. (b,d)	69. (c)	70. (b)
71. (b)	72. (a)	73. (b)	74. (b)	75. (a,c)					

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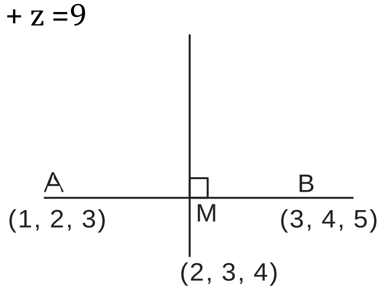
Solution

1. (c)
Given
Equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$
Let point P on curve $(4\cos\theta,\ 2\sqrt{3}\sin\theta)$
But we know point P is extremity of latus rectum of ellipse
So, P is (2, 3)
On comparing point P
$4\cos\theta = 2$ and $\quad 2\sqrt{3}\sin\theta = 3$
$\text{Cos}\theta = \frac{1}{2}$ and $\quad \sin\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$

2. (b)
Normal of the plane passing through the points (1, 2, 3), (–1, –2, 1) and parallel to given line
is $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1+1 & 2+2 & -3-1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 & 4 & -4 \end{vmatrix}$
$= -28\hat{i}+16\hat{j}+2\hat{k}$

Direction ratio's of the normal to plane is < 14, –8, –1>

3. (c)
Equation of plane is $(3-1)x+(4-2)y+(5-3)z = k$ .....(1)
This plane passes through $m \equiv \left(\frac{3+1}{2}, \frac{2+4}{2}, \frac{5+3}{2}\right)$
$\equiv (2, 3, 4)$
$\Rightarrow k = 2 \times 2 + 2 \times 3 + 2 \times 4 = 18$ (put in eq (1))
$\Rightarrow$ Eq. of plane is $x + y + z = 9$



4. (a)
We know,

One interior angle = $\frac{(n-2)\pi}{n}$

$\Rightarrow \lim\limits_{n\to\infty} \frac{(n-2)\pi}{n} = \pi$

5. (a)
Given

$\Rightarrow$ $f(x) = h^{-1}(x)$

$\Rightarrow$ $h(f(x)) = x$

On differentiating with respect to x

$\Rightarrow$ $h'(f(x)) . f'(x) = 1$

$\Rightarrow$ $f'(x) = \frac{1}{h'(f(x))} = 1 + \log(f(x))$

6. (c)
Given

$f(x) = \cos x^2$

Let, f(x) is periodic with period T

We know, $f(x + T) = f(x)$

$\Rightarrow \cos(x + T)^2 = \cos x^2$

$\Rightarrow \cos(x + T)^2 - \cos x^2 = 0$

$\Rightarrow -2\sin\left(\frac{(x+T)^2 - x^2}{2}\right)\sin\left(\frac{(x+T)^2 + x^2}{2}\right) = 0$

$\Rightarrow (x + T)^2 - x^2 = n\pi$ or $(x + T)^2 + x^2 = n\pi$

Which is not possible because these equation are quadratic equation not identity

$\Rightarrow$ f(x) is not periodic

7. (c)

$\lim\limits_{x\to 0^+}(e^x + x)^{1/x}$ (1^∞) form

$\lim\limits_{e^x\to 0^+}\left(\frac{e^x + x - 1}{x}\right)$

Using binomial expansion of $e^x = 1 + x + \frac{x^2}{2!} + \ldots..$

$\Rightarrow$ $e^{1+1} = e^2$

8. (a)

Given

$f'(x) > f(x)$

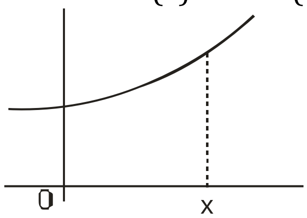
Multiply e^{-x} both sides

$\Rightarrow \quad f'(x) \cdot e^{-x} - f(x).e^{-x} > 0$

$\Rightarrow \quad (f(x).\, e^{-x})' > 0$

$\Rightarrow \quad e^{-x}.\, f(x)$ is increasing function

$\Rightarrow \quad e^{-x}.\, f(x) > e^{-0}.\, f(0)\ \forall\ \ x > 0$



$\Rightarrow \quad e^{-x} f(x) > 0 \qquad \because f(0) = 0$ (given)

$\Rightarrow \quad f(x) > 0\ \ \forall\ \ x > 0$

9. (b,c)

$f'(x) = |f(x)|^2 + 4$

Using LMVT theorem

$\Rightarrow \quad \dfrac{f(3) - f(1)}{3 - 1} = f'(c)$ for atleast one $c \in (1, 3)$

$\Rightarrow \quad f(3) - f(1) = 2(f(c))^2 + 8$

$\Rightarrow \quad f(3) - f(1) \geq 8$

$\Rightarrow \quad f(3) - f(1) = 5$ (false)

$\Rightarrow \quad$ Similarly $f(3) - f(1) = 7$ (false)

10. (b)

$$\lim_{x \to 0^+} (x^n \ell nx)$$

$$= \lim_{x \to 0^+} \left(\frac{\ell nx}{1/x^n}\right) \left(\frac{\infty}{\infty}\right) \text{form}$$

Using L-Hospital Rule

$$= \lim_{x \to 0^+} \frac{1/x}{\frac{-n}{x^{n+1}}}$$

$$= \lim_{x \to 0^+} \left(\frac{x^n}{-n}\right) = 0$$

11. (b)

$$\int \underset{\text{II}}{\cos x}\ \underbrace{\log\left(\tan\frac{x}{2}\right)}_{\text{I}} dx = \sin x \log\left(\tan\frac{x}{2}\right) + f(x) \text{ (given)}$$

Using integration by parts in R.H.S.

$\Rightarrow \log(\tan x/2).\sin x - \int \sin x.\dfrac{\sec^2\frac{x}{2}}{\tan\frac{x}{2}}.\dfrac{1}{2}dx$

$\Rightarrow \log\left(\tan\dfrac{x}{2}\right).\sin x - \int \sin x.\dfrac{1}{\sin x}dx$

$\Rightarrow \sin x.\log\left(\tan\dfrac{x}{2}\right) - x + c$ on comparing with given solution

$\Rightarrow f(x) = c - x$

12. (d)

$y = \int \cos\left\{2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right\}dx$

put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

$y = \int \cos\left\{2\tan^{-1}\sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}(-2\sin 2\theta)\,d\theta$

$y = \int \cos(2\tan^{-1}\tan\theta)(-2\sin 2\theta)\,d\theta$

$y = -\int 2\sin 2\theta.\cos 2\theta d\theta$

$y = -\int \sin 4\theta d\theta$

$y = \dfrac{\cos 4\theta}{4} + c'$

$y = \dfrac{2\cos^2 2\theta - 1}{4} + c'$ $\qquad \therefore x = \cos 2\theta$

$y = \dfrac{x^2}{2} + C$

$\Rightarrow$ Curve represents family of parabola

13. (b)

$$\int_{-\pi/4}^{\pi/4}\left(\lambda|\sin x|+\frac{u\sin x}{1+\cos x}+\gamma\right)dx$$

$$\Rightarrow \lambda\int_{-\pi/4}^{\pi/4}|\sin x|\,dx+\underbrace{\mu\int_{-\pi/4}^{\pi/4}\frac{\sin x dx}{1+\cos x}}_{\text{(odd function)}}+\int_{-\pi/4}^{\pi/4}\gamma dx$$

$$\Rightarrow 2\lambda\int_{0}^{\pi/4}\sin x+0+\frac{\gamma\pi}{2}$$

$$\Rightarrow 2\lambda\left(1-\frac{1}{\sqrt{2}}\right)+\frac{\gamma\pi}{2}$$

$\Rightarrow$ Independent of μ

14. (a)

$$\lim_{x\to 0}\frac{1}{x}\left[\int_{y}^{a}e^{\sin^2 t}dt-\int_{x+y}^{a}e^{\sin^2 t}dt\right]$$

$$=\lim_{x\to 0}\frac{1}{x}\left[\int_{y}^{a}e^{\sin^2 t}dt+\int_{a}^{x+y}e^{\sin^2 t}dt\right]$$

$$=\lim_{x\to 0}\frac{\int_{y}^{x+y}e^{\sin^2 t}dt}{x} \text{ (using L- Hospital rule)}$$

$$=\lim_{x\to 0}\frac{e^{\sin^2(x+y)}}{1}=e^{\sin^2 y}$$

15. (d)

$$\int 2^{2^x}.2^x\,dx=A.2^{2^x}+C \quad ....(1)$$

Put $2^{2^x}=t\Rightarrow\left(2^{2^x}.\ell n2\right)\left(2^x.\ell n2\right)dx=dt$

$$\Rightarrow\int\frac{dt}{(\ell n2)^2}$$

$$\Rightarrow\frac{t}{(\ell n2)^2}+C\Rightarrow\frac{2^{2^x}}{(\ell n2)^2}+C \text{ (on comparing with equation (1))}$$

$$\Rightarrow A=\frac{1}{(\ell n2)^2}$$

16. (d)

$$I = \int_{-1}^{1}\left(\frac{x^{2015}}{e^{|x|}(x^2+\cos x)}+\frac{1}{e^{|x|}}\right)dx$$

$$= \underbrace{\int_{-1}^{1}\left(\frac{x^{2015}}{e^{|x|}(x^2+\cos x)}\right)}_{\text{odd function}}dx + \int_{-1}^{1}\underbrace{\frac{1}{e^{|x|}}}_{\text{even function}}dx$$

$$I = 0+2\int_0^1 e^{-x}dx = 2\left(-e^{-x}\right)\Big|_0^1 = 2\left(\frac{-1}{e}+1\right)$$

$I = 2\,(1 - e^{-1})$

17. (c)

$$\lim_{n\to\infty}\frac{3}{n}\left\{1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+\ldots..+\sqrt{\frac{n}{n+3(n-1)}}\right\}$$

$$\Rightarrow \lim_{n\to\infty}\frac{3}{n}\left\{\sqrt{\frac{1}{1+3\left(\frac{0}{n}\right)}}+\sqrt{\frac{1}{1+3\left(\frac{1}{n}\right)}}+\sqrt{\frac{1}{1+3\left(\frac{2}{n}\right)}}+\ldots..+\sqrt{\frac{1}{1+3\left(\frac{n-1}{n}\right)}}\right\}$$

$$\Rightarrow \lim_{n\to\infty}\frac{3}{n}\sum_{r=0}^{n-1}\sqrt{\frac{1}{1+3.\left(\frac{r}{n}\right)}}$$

Put $\frac{1}{n}\to dx$

$\frac{r}{n}\to x$

Lower limit $\Rightarrow x = 0$

Upper limit $\Rightarrow x = 1$

$$\Rightarrow \quad 3\int_0^1\frac{1}{\sqrt{1+3}}dx = \left.\frac{3(1+3x)^{1/2}}{\frac{1}{2}\times 3}\right|_0^1 = 3\times\frac{2}{3}\times(4^{1/2}-1^{1/2})$$

$$= \frac{2}{3}\times 3 = 2$$

18. (c)

$$\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{x/y}dy=0$$

$$\frac{dy}{dx}=\frac{-(1+e^{x/y})}{\left(1-\frac{x}{y}\right)e^{x/y}} \quad ....(1)$$

Put $\frac{x}{y}=t \Rightarrow y=\frac{x}{t} \Rightarrow \frac{dy}{dx}=\frac{1}{t}-\frac{x}{t^2}\frac{dt}{dx}$ .....(2)

From equation (1) & (2)

$$\Rightarrow \frac{1}{t}-\frac{x}{t^2}\frac{dt}{dx}=\frac{-(1+e^t)}{(1-t)e^t}$$

$$\Rightarrow \frac{xdt}{dx}=\frac{t(t+e^t)}{(1-t)e^t}$$

$$\frac{dx}{x}=\frac{e^t(1-t)}{t(t+e^t)}dt$$

$$\Rightarrow \quad \frac{dx}{x}=\frac{(e^t+t)-t(e^t+1)}{t(t+e^+)}dt$$

$$\Rightarrow \quad \frac{dx}{x}=\left(\frac{1}{t}-\frac{(e^t+1)}{e^t+t}\right)dt$$

On integrating both sides

$$\Rightarrow \quad \ell nx=\ell nt-\ell n(e^t+t)+\ell nc \qquad \because \frac{x}{y}=t$$

$$\Rightarrow \quad \ell nx=\ell n\left(\frac{x/y}{e^{x/y}+x/y}\right)+\ell nc$$

$$\Rightarrow \quad x=\left(\frac{x/y.c}{e^{x/y}+x/y}\right)$$

$$\Rightarrow \quad ye^{x/y}+x=c$$

19. (d)

$(x+y)^2\frac{dy}{dx} = a^2 \Rightarrow \frac{dy}{dx} = \frac{a^2}{(x+y)^2}$ ....(1)

Put $x + y = t$ $\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$ ....(2)

$\Rightarrow$ From (1) & (2)

$\Rightarrow 1 + \frac{a^2}{t^2} = \frac{dt}{dx}$

$\Rightarrow dx = \frac{t^2}{t^2+a^2}dt$

$\Rightarrow dx = \left(1-\frac{a^2}{t^2+a^2}\right)dt$

Integrating both sides

$\Rightarrow x + c = t - a\tan^{-1} t/a$ $\because t = x + y$

$\Rightarrow x = x + y - a\tan^{-1}\left(\frac{x+y}{a}\right)+c$

$\Rightarrow a\tan^{-1}\left(\frac{x+y}{a}\right) = y + c \Rightarrow \frac{x+y}{a} = \tan\left(\frac{y+c}{a}\right)$

20. (b)

$\left.\frac{dy}{dx}\right|_{(4,3)} = \frac{-1}{m_N}$

$\Rightarrow \left.\frac{b^2x}{a^2y}\right|_{(4,3)} = -\left(\frac{4-16}{3-0}\right)$

$\Rightarrow \frac{4b^2}{3a^2} = \frac{12}{3}$

$\Rightarrow \frac{b^2}{a^2} = 3$

We know

$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+3} = 2$

21. (b)

Volume of spherical balloon V = $\frac{4}{3}\pi r^3$

$$\frac{\Delta V}{V} \times 100 = \frac{\left(\frac{4}{3}\pi\left(r+\frac{r}{10}\right)^3 - \frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3} \times 100$$

$$= \left(1+\frac{1}{10}\right)^3 - 1$$

$$= 1 + \frac{3}{10} + \frac{3}{100} + \frac{1}{1000} - 1 \quad \simeq 0.3\% \text{ approximately}$$

22. (b)

Let sides are a, ar, ar^2 $(r > 1)$

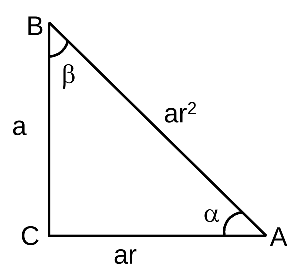
Using Pythagoras theorem

$\Rightarrow a^2 + a^2r^2 = a^2 r^4$

$\Rightarrow r^4 + r^2 - 1 = 0$

$\Rightarrow r^2 = \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ (not possible)

$\Rightarrow r = \sqrt{\frac{-1+\sqrt{5}}{2}}$



$\tan\alpha = \frac{1}{r}$

$= \sqrt{\frac{2}{\sqrt{5}-1}} = \sqrt{\frac{\sqrt{5}+1}{2}}$

$\tan\beta = r = \sqrt{\frac{\sqrt{5}-1}{2}}$

23. (b)

$\log_2 6 + \frac{1}{2x} = \log_2(2^{1/x} + 8)$

$\Rightarrow \log_2\left(\frac{2^{1/x}+8}{6}\right) = \frac{1}{2x}$

On taking anti log

$\Rightarrow \frac{2^{1/x}+8}{6} = 2^{1/2x}$

$\Rightarrow \left(2^{\frac{1}{2x}}\right)^2 - 6.2^{1/2x} + 8 = 0$

$\Rightarrow t^2 - 6t + 8 = 0$ Let $2^{\frac{1}{2x}} = t$

$\Rightarrow t = 4, 2$

$\Rightarrow 2^{\frac{1}{2x}} = 4, 2$

$\Rightarrow 2x = 1, \frac{1}{2}$

$\Rightarrow x = \frac{1}{2}, \frac{1}{4}$

24. (c)

$\Rightarrow \arg z - \arg(-z)$

$\Rightarrow \arg z - [\arg z - \arg(-1)]$ $\because \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

$\Rightarrow \arg z - [\arg z - \arg(-1)]$ $\because \arg(-1) = \pi$

$\Rightarrow \pi$

25. (b)

$\Rightarrow (\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)............(\cos n\theta + i\sin n\theta) = 1$

$\Rightarrow e^{i\theta}.e^{i2\theta}..........e^{in\theta} = 1$ (using Euler's formula)

$\Rightarrow e^{i\theta(1+2+.......+n)} = 1$

$\Rightarrow e^{i\theta\left(\frac{n(n+1)}{2}\right)} = 1$

$\Rightarrow \frac{n(n+1)\theta}{2} = 2k\pi$

$\Rightarrow \theta = \frac{4k\pi}{n(n+1)}$

26. (d)

Given

$\Rightarrow ax^2 + bx + c = 0$ has imaginary roots $\Rightarrow D < 0$

$\Rightarrow a + b + c < 0 \Rightarrow f(1) < 0$

$\Rightarrow f(x) < 0 \ \forall \ x \in R$ and $a < 0$

$\Rightarrow f(0) < 0 \Rightarrow c < 0$

27. (a)

$\Rightarrow {}^6C_2 \times {}^6C_4 + {}^6C_3 \times {}^6C_3 + {}^6C_4 \times {}^6C_2$

$\Rightarrow (15)^2 + (20)^2 + (15)^2$

$\Rightarrow 225 + 400 + 225 = 850$

28. (b)

$\Rightarrow {}^7C_4 \times$ (De-arrangement of 3 things)

$\Rightarrow 35 \times 2 = 70 = {}^7C_3 \times 2$

29. (d)

$\Rightarrow 7^{2n} + 16n - 1$

$\Rightarrow (8-1)^{2n} + 16n - 1$

$\Rightarrow \left[{}^{2n}C_0 8^{2n} - {}^{2n}C_1 8^{2n-1} + \dots\dots - {}^{2n}C_{2n-1} 8^1 + {}^{2n}C_{2n} \right] + 16n - 1$

$\Rightarrow [64\lambda - 2n.8 + 1] + 16n - 1$

$\Rightarrow 64\lambda - 16n + 1 + 1\,6n - 1$

$\Rightarrow 64\lambda$

30. (b)

$\Rightarrow \frac{84-n}{8}$ = rational $\Rightarrow$ n = 4, 12 ……………, 76

$\Rightarrow \frac{n}{4}$ = rational $\Rightarrow$ n = 4, 8, 12………………., 84

$\Rightarrow$ n can take total 11 terms

$\Rightarrow$ Total number of rational terms = 11

Irrational terms = total terms – rational terms = 85 – 11 = 74

31. (c)

$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}; I_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow A - 3I_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$

$\Rightarrow$ det. $(A - 3I_3) = -2(3) - 1(-3) + 1(3) = 0$

$\Rightarrow A - 3I_3$ is non invertible matrix

32. (c)

We know, for square matrix adj(M′) = (adj M)′

So, adj (M′) – (adj M)′ = 0 = null matrix

33. (a)

$$A = \begin{vmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{vmatrix} \text{(given)}$$

$\Rightarrow \quad |A^2| = 25\text{(given)}$

$\Rightarrow \quad |A| = \pm 5$

$$\Rightarrow \quad |A| = \begin{vmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{vmatrix} = \pm 5$$

$\Rightarrow \quad 25x = \pm 5$

$\Rightarrow \quad x = \pm \frac{1}{5}$

$\Rightarrow \quad |x| = \frac{1}{5}$

34. (d)

$$\text{Let } A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \text{ \& } B = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Hence AB = 0 but $A \neq 0$, $B \neq 0$

So, if AB = 0 then may be $A \neq 0$, B = 0

35. (b)

Let (h, k) satisfies $x^2 + y^2 = 1$ then $h^2 + k^2 = 1$

Now $h^8 + k^8 = h^8 + (1 - h^2)^4 = 2h^8 - 4h^6 - 4h^2 + 1$

$= 2h^2 (h^2 - 1) (h^4 - h^2 + 2) + 1$

$= -2h^2k^2 (h^4 - h^2 + 2) + 1 < 1 \quad \forall \quad h > 0, k > 0$

$\Rightarrow$ All solution of $x^2 + y^2 = 1$ satisfies $x^8 + y^8 < 1$

$\Rightarrow P \cap T = P$

36. (d)

$f(x) = x^2 - \frac{x^2}{1+x^2}$

$\Rightarrow f(x) = \frac{x^4}{1+x^2}$

$\because f(-x) = f(x)$

$\therefore$ f(x) is an even function, hence it is many one.

Also, $f(x) \geq 0 \quad \forall \quad x \in R$, hence it is into function

$\Rightarrow$ f(x) is neither one –one nor onto

37. (d)

$(a, a) \in \rho$ because $1 + a^2 > 0$ $\Rightarrow$ ρ is reflexive

If $1 + ab > 0$ then $1 + ba < 0$ $\Rightarrow$ if $(a, b) \in \rho$ then $(b, a) \in \rho$

$\Rightarrow$ ρ is symmetric

Now $\left(-4, \frac{1}{8}\right) \in \rho$ and $\left(\frac{1}{8}, 5\right) \in \rho$

But $(-4, 5) \notin \rho$, hence ρ is not transitive

38. (c)

Probability that no student solves problem is

$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right) = \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{1}{5}$

$\Rightarrow$ Probability that the problem will be solved by at least one student is $= 1 - \frac{1}{5} = \frac{4}{5}$

39. (d)

Given

$\sigma(x) = 2.6$

We know

$\sigma(ax + b) = |a|\,(\sigma(x))$

So $s(-4x + 1) = |-4|(\sigma(x)) = 4 \times 2.6 = 10.4$

40. (a)

$e^{\sin x} - e^{-\sin x} = 4$

Let $e^{\sin x} = t$

$\Rightarrow$ $t^2 - 4t - 1 = 0$

$\Rightarrow$ $t = 2 + \sqrt{5},\ 2 - \sqrt{5}$

We know that t is real positive number

$\Rightarrow t = 2 + \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow [e^{-1}, e^{1}] \notin 2 + \sqrt{5}$

$\Rightarrow$ hence no solution exist

41. (d)

Let, angles are – 2x, 3x, 7x

We know

$\Rightarrow 2x + 3x + 7x = 180° \Rightarrow 12x = 180° \Rightarrow x = 15°$

$\Rightarrow$ angles are – 30°, 45°, 75°

$\Rightarrow$ length of smallest side a $\Rightarrow \frac{a}{\sin A} = 2R$

$\Rightarrow a = 2R\sin A$

$\Rightarrow a = 2 \times 10 \times \sin 30° = 10$

42. (b)

Let P(h, k) then A is (h, 0) & B is (0, k)

Equation of AB is $\frac{x}{h}+\frac{y}{k}=1$ which passes through (x_1, y_1) is

$\Rightarrow \frac{x_1}{h}+\frac{y_1}{k}=1$

$\Rightarrow$ Required locus is $\frac{x_1}{x}+\frac{y_1}{y}=1$

43. (b)

L_1 : $\sqrt{3}x+y=1 \Rightarrow m_1 = \sqrt{3}$

$\theta = 60°$ (given)

$\Rightarrow \tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\Rightarrow \pm\sqrt{3} = \frac{-\sqrt{3}-m_2}{1-\sqrt{3}m_2}$

$\Rightarrow m_2 = \sqrt{3}, 0$ (not possible because lines are not parallel)

$\Rightarrow m_2 = \sqrt{3}$

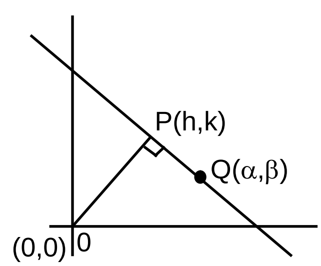
So, equation of line passing through (3, –2) and her slope $\sqrt{3}$ is

$\Rightarrow \quad y + 2 = \sqrt{3}(x - 3)$

$\Rightarrow \quad y - \sqrt{3}x+2+3\sqrt{3}=0$

44. (a)

$M_{PQ} \cdot M_{OP} = -1$



$\Rightarrow \left(\frac{k-\beta}{h-\alpha}\right)\cdot\left(\frac{k}{h}\right)=-1$

$\Rightarrow k^2 - k\beta = -(h^2 - h\alpha)$

$\Rightarrow h^2 + k^2 - h\alpha - k\beta = 0$

$\Rightarrow$ Required locus is $- x^2 + y^2 - \alpha x - \beta y = 0$

45. (b)

Let point on 4th quadrant which is equidistant from both the axis is $(\alpha, -\alpha)$

$\Rightarrow L_1 : 2a\alpha - 4a\alpha + c = 0 \Rightarrow \alpha = \dfrac{c}{2a}$ ....(1)

$\Rightarrow L_2: 7b\alpha - 3b\alpha - d = 0 \Rightarrow \alpha = \dfrac{d}{4b}$ ....(2)

From equation (1) & (2)

$\Rightarrow \dfrac{c}{2a} = \dfrac{d}{4b}$

$\Rightarrow \quad \dfrac{4}{2} = \dfrac{ad}{cb}$

$\Rightarrow \quad ad : bc = 2 : 1$

46. (a)

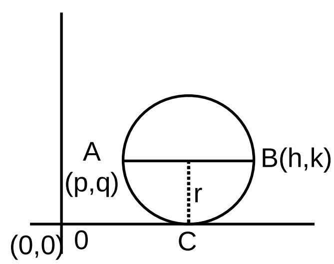
Let other and is (h, k) then centre equal to $\left(\dfrac{p+h}{2}, \dfrac{q+k}{2}\right)$

Because circle teaches x-axis hence radius = $\left|\dfrac{q+k}{2}\right|$

We know

$\Rightarrow$ (AB) Diameter = 2r

$\Rightarrow \sqrt{(h-P)^2 + (k-q)^2} = 2\left|\dfrac{q+k}{2}\right|$



On squaring both sides

$\Rightarrow (h-p)^2 + (k-q)^2 = (q+k)^2$

$\Rightarrow (h-p)^2 = (k+q)^2 - (k-q)^2$

$\Rightarrow$ Required locus is $= (x-p)^2 = 4qy$

47. (c)

ΔPQR is equilateral triangle

So, incentre is same as centroid

$\Rightarrow$ incentre = centroid = centre of circle = $\left(\dfrac{1+\frac{1}{2}+0}{3}, \dfrac{0+0+\frac{\sqrt{3}}{2}}{3}\right) = \left(\dfrac{1}{2}, \dfrac{1}{2\sqrt{3}}\right)$

48. (c)

Focus are $\left(\pm\sqrt{a^2+b^2},0\right) = \left(\pm\sqrt{\cos^2\alpha+\sin^2\alpha},0\right)$

$\Rightarrow$ focus are $\left(\pm1,0\right)$ which is independent of α

$\Rightarrow$ focus are fixed

49. (c)

S = (ae, 0)
T = (–ae, 0)
B = $\left(0,b\right)$

$\Rightarrow \quad (ST)^2 = (SB)^2$

$\Rightarrow \quad (2ae)^2 = (ae)^2 + b^2$

$\Rightarrow \quad 3(ae)^2 = b^2$

$\Rightarrow \quad b^2 = 3(a^2 - b^2) \Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$

e = $\sqrt{1-\frac{b^2}{a^2}}$

e = $\sqrt{1-\frac{3}{4}}$

e = $\frac{1}{2}$

50. (a)

Equation of hyperbola is $3(x^2 - 6x) - 3(y^2 - 4y) + 2 = 0$

$\Rightarrow (x-3)^2 - (y-2)^2 = \frac{13}{3}$

$\Rightarrow \frac{(x-3)^2}{\sqrt{\frac{13}{3}}} - \frac{(y-2)^2}{\sqrt{\frac{13}{3}}} = 1 \Rightarrow e = \sqrt{2}$

We know equation of directrix is $X = \pm\frac{a}{e}$

$\Rightarrow \quad x - 3 = \pm\frac{\sqrt{13/3}}{\sqrt{2}}$

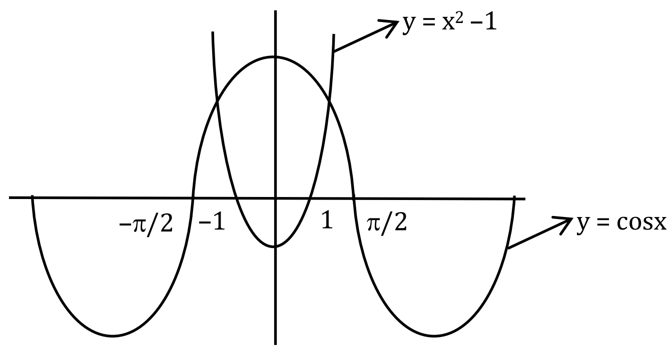
$\Rightarrow \quad x = 3 \pm\frac{\sqrt{13/3}}{\sqrt{2}}$

51. (a)

$y = x^2 - 1$ & $y = \cos x$

intersect at exactly

two points



52. (c)

Given

$\frac{dx}{dt} = 1$ unit per second

$y = \frac{10}{x}$ ....(1)

differentiating with respect to x

$\Rightarrow \left.\frac{dy}{dx}\right|_{(5,2)} = \frac{-10}{x^2}\frac{dx}{dt}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(5,2)} = \frac{-10}{25}\times 1$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(5,2)} = \frac{-2}{5}$

$\Rightarrow$ Ordinate decreases at rate $\frac{2}{5}$ unit per second.

53. (c)

$a = \min [x^2 + 2x + 3; x \in R]$

$a = \min [(x+1)^2+2]$

$\Rightarrow a = 2$

$$b = \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2^2}.\frac{2\sin^2\frac{\theta}{2}}{\frac{\theta^2}{2^2}} = \frac{1}{2}$$

$\Rightarrow b = \frac{1}{2}$

Now,

$$\Rightarrow \sum_{r=0}^{n} a^r b^{n-r} = a^0 b^n + ab^{n-1} + a^2b^{n-2} + \ldots\ldots\ldots\ldots + a^n b^0$$

$$= \left(\frac{1}{2}\right)^n + \frac{2}{2^{n-1}} + \frac{2^2}{2^{n-2}} + \ldots\ldots\ldots + 2^n$$

$$= \left(\frac{1}{2}\right)^n \left[1+4+4^2+\ldots\ldots\ldots+4^n\right]$$

$$= \frac{1}{2^n}\left(\frac{4^{n+1}-1}{4-1}\right)$$

$$= \frac{4^{n+1}-1}{3.2^n}$$

54. (a)

Given

$\Rightarrow a > b > 0$

$\Rightarrow a > b$

$\Rightarrow \frac{b}{a} < 1$

Now,

$I_{(n)} = a^{1/n} - b^{1/n}$

$J_{(n)} = (a - b)^{1/n}$

$$\frac{I_{(n)}}{J_{(n)}} = \frac{a^{1/n} - b^{1/n}}{(a-b)^{1/n}} = \frac{\frac{a^{1/n} - b^{1/n}}{a^{1/n}}}{\frac{(a-b)^{1/n}}{a^{1/n}}}$$

$$= \frac{1 - \left(\frac{b}{a}\right)^{1/n}}{\left(1 - \frac{b}{a}\right)^{1/n}}$$

Let $b = 16$
$a = 625$
$n = 4$

$$= \frac{1 - \left(\frac{16}{625}\right)^{1/4}}{\left(1 - \frac{16}{625}\right)^{1/4}} = \frac{1 - \frac{2}{5}}{\left(\frac{609}{625}\right)^{1/4}} = \frac{3}{4.96} < 1$$

$$= \frac{I_{(n)}}{J_{(n)}} < 1$$

$= I_{(n)} < J_{(n)}$

55. (d)

$|\hat{\alpha}| = |\hat{\beta}| = |\hat{\gamma}| = 1$ (given)

Now,

$\Rightarrow \hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$

On comparing both sides

$\Rightarrow -\hat{\alpha} \cdot \hat{\beta} = \frac{1}{2}$

$\Rightarrow \hat{\alpha} \cdot \hat{\beta} = \frac{-1}{2}$

$\Rightarrow |\hat{\alpha}|\,|\hat{\beta}| \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$

56. (d)

Given

A, B, C, & D are on a plane

$\because$ $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar

$\therefore [\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ -1 & -1 & -1-\lambda \end{vmatrix} = 0$$

$R_3 \to R_3 + R_1$

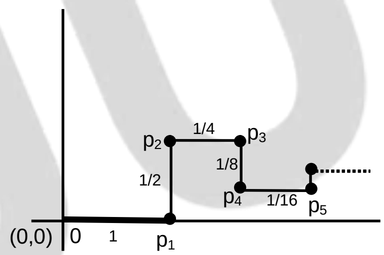
$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ 0 & 0 & -4-\lambda \end{vmatrix} = 0$$

$\Rightarrow (-4-\lambda)(-1+2) = 0$

$\Rightarrow \lambda = -4$

57. (b)

$\Rightarrow x_n = 1 + \frac{1}{4} + \frac{1}{16} + \ldots\ldots\ldots$



$$\Rightarrow \lim_{n\to\infty} x_n = \lim_{n\to\infty}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\ldots\right)$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3} = \alpha$$

$$\Rightarrow y_n = \frac{1}{2} - \frac{1}{8} + \frac{1}{32} + \ldots\ldots$$

$$\Rightarrow \lim_{n\to\infty} y_n = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{8} + \frac{1}{32} + \ldots\ldots\right)$$

$$= \frac{1/2}{1+\frac{1}{4}} = \frac{2}{5} = \beta$$

$$= (\alpha, \beta) \equiv \left(\frac{4}{3}, \frac{2}{5}\right)$$

58. (a)

Using inequality $|A| + |B| \geq |A - B|$

$\Rightarrow |z| + |z-1| \geq |z-(z-1)|$

$\Rightarrow |z| + |z-1| \geq 1$

$\Rightarrow$ minimum value of $|z| + |z-1|$ is 1

59. (c)

For infinitely many solution $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda\mu + 7\lambda - 7 + 42 - 15\mu = 0$

$\Rightarrow (\lambda - 15)(\mu + 7) + 140 = 0$

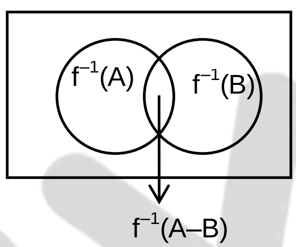
Now check from options

$\Rightarrow (\lambda, \mu) = (1, 3)$

60. (c)

We can see that pre image of A – B i.e. $f^{-1}(A - B)$ will be $f^{-1}(A) - f^{-1}(B)$

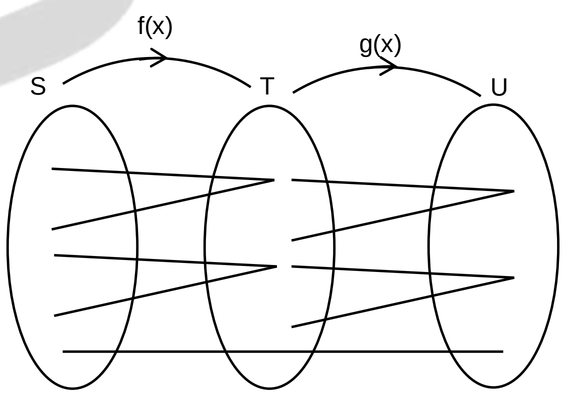
Therefore, $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$



61. (b)

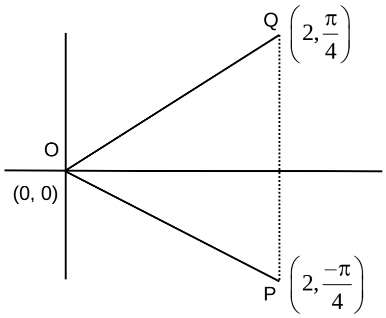
Obvious g is surjective other wise gof cannot be surjective but there is no need of f to be surjective.

Example.



Hence f(x) is not surjective still gof is surjective

62. (a)
line joining PQ is bisected by x –axis
So point Q is $\left(2, \frac{\pi}{4}\right)$



63. (b)
$\Rightarrow b > a$ (given)
On squaring both sides
$\Rightarrow \frac{b^2}{a^2} > 1$
On adding 1 in both sides
$\Rightarrow 1 + \frac{b^2}{a^2} > 2$
Taking root both sides
$\Rightarrow \sqrt{1+\frac{b^2}{a^2}} > \sqrt{2}$
$\Rightarrow e > \sqrt{2}$

64. (a)
$\Rightarrow \lim_{x\to 0^+} \frac{x}{p}\left[\frac{q}{x}\right]$
$\Rightarrow \lim_{x\to 0^+} \frac{x}{p}\left(\frac{q}{x} - \left\{\frac{q}{x}\right\}\right)$
$\Rightarrow \lim_{x\to 0^+} \frac{x}{p}.\frac{q}{x} - \lim_{x\to 0^+} \frac{x}{p}\left\{\frac{q}{x}\right\}$ $\quad \therefore 0 \le \left\{\frac{q}{x}\right\} < 1$
$\Rightarrow \frac{q}{p} - \lim_{x\to 0^+} \frac{x}{p}(\text{finite})$
$\Rightarrow \frac{q}{p} - 0 \times (\text{finite})$
$\Rightarrow \frac{q}{p} - 0 \Rightarrow \frac{q}{p}$

65. (b)

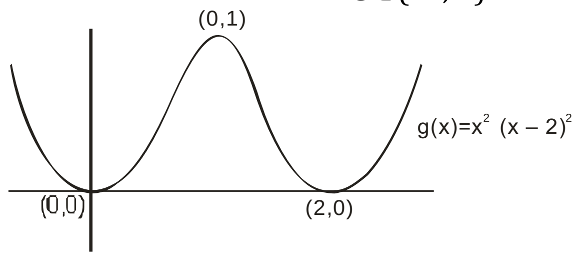
$f(x) = x^4 - 4x^3 + 4x^2 + c,\ c \in R$

Using IVT theorem

For atleast one root/zero $= f(1) . f(2) < 0$

$= (1 + c) c < 0$

$= c \in (-1, 0)$



$f(x) = x^2(x - 2)^2 + c$

$\Rightarrow$ f(x) has exactly one zero in (1, 2) if $c \in (-1, 0)$

66. (a,c)

$f(a) = 0 = f(b)$ $\Rightarrow f'(a)f'(b) < 0$

Let $h(x) = f'(x) + f(x) g'(x)$ …………..(1)

Put x = a in equation (1)

$h(a) = f'(a)$ $\because f(a) = 0$

Put x = b in equation (1)

$h(a) = f'(b)$ $\because f(b) = 0$

$\therefore$ h(a) . h(b) < 0 $\Rightarrow$ h(x) = 0 has roots between (a, b)

Similarly $g'(x) + kg(x) = 0$ has roots between (a, b) as $g(a) = 0 = g(b)$

67. (b,c)

$f(x) = \frac{x^3}{4} - \sin \pi x + 3$

$f(-2) = 1$

$f(2) = 5$

$\because$ function is continuous

By intermediate value theorem , f(x) takes all values between 1 to 5

68. (b,d)

$I_n = \int_0^1 x^n \tan^{-1} x\,dx$

Using Integration by parts

$\Rightarrow I_n = \tan^{-1} x \left.\frac{x^{n+1}}{n+1}\right|_0^1 - \int_0^1 \frac{x^{n+1}}{n+1}.\left(\frac{1}{1+x^2}\right)dx$

$\Rightarrow (n+1)\,I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{1+x^2}dx$ ....(1)

Put $n \to n+2$ in equation (1)

$\Rightarrow (n+3)\,I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+3}}{1+x^2}dx$ ....(2)

From equation(1) + (2)

$\Rightarrow (n+1)I_n + (n+3)\,I_{n+2} = \frac{\pi}{2} - \frac{1}{n+2}$

$\Rightarrow$ on comparing with given equation

$\Rightarrow a_n = n+1,\ b_n = n+3,\ c_n = \frac{\pi}{2} - \frac{1}{n+2}$

69. (c)

$S + n = \frac{1}{2}f(t+m^2)$ and $S = \frac{1}{2}ht^2$, $V = ht$

$\therefore \frac{1}{2}ht^2 + n = \frac{1}{2}f(t+m^2)$ ....(1)

Also $V = 0 + ht = 0 + f(t+m) \Rightarrow t + m = \frac{ht}{f}$ (put in equation (1))

from equation (1),

$\Rightarrow \frac{1}{2}ht^2 + n = \frac{1}{2}f\left(\frac{ht}{f}\right)^2$

$\Rightarrow t^2 = \frac{2hf}{h(h-f)}$

Also,

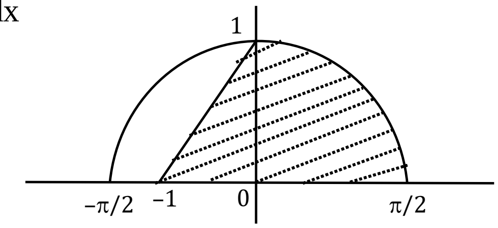
$ht = f(t+m) \Rightarrow t^2 = \frac{m^2f^2}{(h-f)^2}$

$\therefore \frac{2nf}{h(h-f)} = \frac{m^2f^2}{(h-f)^2}$

$\Rightarrow \quad n(h-f) = \frac{1}{2}f\,hm^2$

70. (b)

Area $= \frac{1}{2} \times 1 \times 1 + \int_0^{\pi/2} \cos x dx$



$= \frac{1}{2} + \sin x\Big|_0^{\pi/2}$

$= \frac{1}{2} + 1 = \frac{3}{2}$

71. (b)

$x^2 - 3x + a = 0 \begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}$ $\Rightarrow x_1 + x_2 = 3$; $x_1 \cdot x_2 = a$

$x^2 - 12x + b = 0 \begin{smallmatrix} x_3 \\ x_4 \end{smallmatrix}$ $\Rightarrow x_3 + x_4 = 12$; $x_3 \cdot x_4 = b$

$\because$ x_1, x_2, x_3, x_4 are in G.P, then

$\Rightarrow \frac{x_3 + x_4}{x_1 + x_2} = \frac{12}{3}$

$\Rightarrow \frac{Ar^2 + Ar^3}{A + Ar} = 4$

$\Rightarrow r^2 = 4 \quad \Rightarrow \quad x = 2$ ($\because$ G.P is increasing)

$\because x_1 + x_2 = 3$

$\Rightarrow A + Ar = 3$

$\Rightarrow A(3) = 3$

$\Rightarrow A = 1$

$\therefore ab = x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 1.2.\ 4.8 = 64$

72. (a)

$\Rightarrow \frac{1 - i\cos\theta}{1 + 2i\cos\theta}$

Using rationalization

$\Rightarrow \frac{1 - i\cos\theta}{1 + 2i\cos\theta} \times \frac{1 - 2i\cos\theta}{1 - 2i\cos\theta}$

$\Rightarrow \frac{(1 - 2\cos^2\theta) - 3i\cos\theta}{1 + 4\cos^2\theta}$ is real

$\Rightarrow \therefore \cos\theta = 0$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}, n \in I$

73. (b)

Let $(A - \lambda I_3) = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)^3 - 9(3-\lambda) = 0$

$\Rightarrow (3-\lambda)[(3-\lambda)^2 - 3^2] = 0$

$\Rightarrow 3-\lambda = 0$ or $3-\lambda-3 = 0$ or $3-\lambda+3 = 0$

$\Rightarrow \lambda = 0, 3, 6$

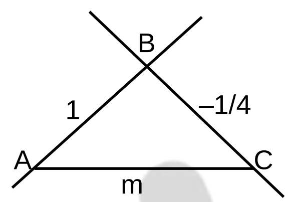
74. (b)

AB = AC (given)

$\Rightarrow \angle ABC = \angle BCA$

Let slope of AC is m.

$$\therefore \left|\frac{m+\frac{1}{4}}{1-\frac{m}{4}}\right| = \left|\frac{\frac{-1}{4}-1}{1-\frac{1}{4}}\right|$$



$\Rightarrow m = \frac{-23}{7}$, 1 (rejected)

$\therefore$ equation of line is $23x + 7y + 3 = 0$

75. (a,c)

$$\frac{x^2}{1} - \frac{y^2}{5} = 1$$

Let the tangent of hyperbola $y = mx \pm\sqrt{a^2m^2 - b^2}$

$\Rightarrow$ $y = mx \pm\sqrt{m^2-5}$ ......(1)

$\downarrow$Passes through (2, 8)

$\Rightarrow$ $(8-2m) = \pm\sqrt{m^2-5}$

On squaring both sides

$\Rightarrow$ $(8-2m)^2 = (m^2-5)$

$\Rightarrow$ $m = 3$ or $\frac{23}{3}$ (put in equation (1))

So, equation of tangent is $\Rightarrow y = 3x \pm 2$ or $y = \frac{23x}{3} \pm \frac{22}{3}$

$\Rightarrow y = 3x + 2,\ y = 3x - 2,\ 3y = 23x + 22,\ 3y = 23x - 22$

WB JEE-2019 (Chemistry)

Category - I (Q.41 to Q.70)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

41. The H–N–H angle in ammonia is 107.6°, while the H–P–H angle in phosphine is 93.5°. Relative to phosphine, the p-character of the lone pair on ammonia is expected to be:
    a. Less
    b. More
    c. Same
    d. Cannot be predicted

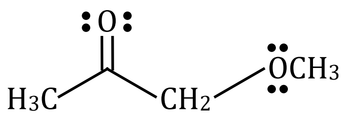
42. The reactive species in chlorine bleach is
    a. Cl_2O
    b. OCl^-
    c. ClO_2
    d. HCl

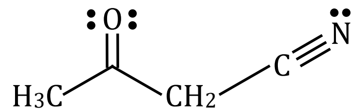
43. The conductivity measurement of a coordination compound of Cobalt (III) shows that it dissociates into 3 ions in solution. The compound is
    a. Hexaamminecobalt(III) chloride
    b. Pentaamminesulphatocobalt(III) chloride
    c. Pentaamminechloridocobalt(III) sulphate
    d. Pentaamminechloridocobalt(III) chloride

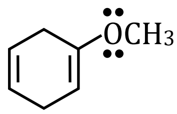
44. In the Bayer's process, the leaching of alumina is done by using
    a. Na_2CO_3
    b. NaOH
    c. SiO_2
    d. CaO

45. Which atomic species cannot be used as a nuclear fuel?
    a. $^{233}_{92}U$
    b. $^{235}_{92}U$
    c. $^{239}_{94}U$
    d. $^{238}_{92}U$

46. The molecule/molecules that has/have delocalised lone pair(s) of electrons is/are

(I) 

(II) 

(III) 

(IV) $CH_3CH=CHCH_2\ddot{N}HCH_3$

a. I, II and III

b. I, II and IV

c. I and III

d. only III

47. The conformations of n-butane, commonly known as eclipsed, gauche and anti-conformations can be interconverted by

a. rotation around C-H bond of a methyl group

b. rotation around C-H bond of a methylene group

c. rotation around C_1-C_2 linkage

d. rotation around C_2-C_3 linkage

48. The correct order of the addition reaction rates of halogen acids with ethylene is

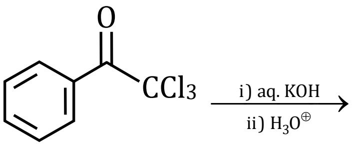
a. hydrogen chloride > hydrogen bromide > hydrogen iodide

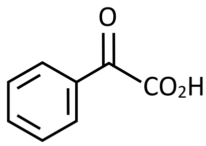
b. hydrogen iodide > hydrogen bromide > hydrogen chloride

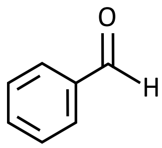
c. hydrogen bromide > hydrogen chloride > hydrogen iodide

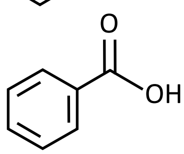
d. hydrogen iodide > hydrogen chloride > hydrogen bromide

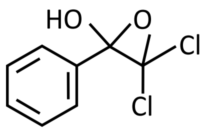
49. One of the products of the following reactions P.


P ; Structure of P is

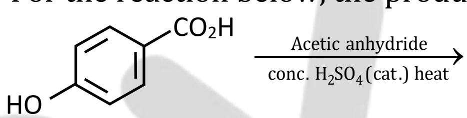
a. 

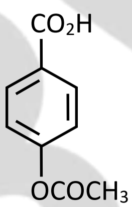
b. 

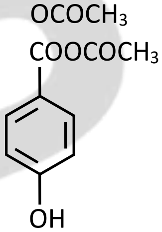
c. 

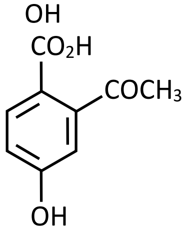
d. 

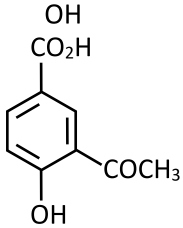
50. For the reaction below, the product is Q.


Q [$C_9H_8O_4$]

a. 

b. 

c. 

d. 

WB JEE-2019 (Chemistry)

51. Cyclopentanol on reaction with NaH followed by CS_2 and CH_3I produces a/an
    a. ketone
    b. alkene
    c. ether
    d. xanthate

52. The compound, which evolves carbon dioxide on treatment with aqueous solution of sodium bicarbonate 25°C, is
    a. C_6H_5OH
    b. CH_3COCl
    c. CH_3CONH_2
    d. $CH_3COOC_2H_5$

53. The indicated atom is not a nucleophilic site in
    a. $\underset{\uparrow}{B}H_4^-$
    b. $\underset{\uparrow}{C}H_3MgI$
    c. $CH_3\underset{\uparrow}{O}H$
    d. $CH_3\underset{\uparrow}{N}H_2$

54. The charge carried by 1 millimole of M^{n+} ions is 193 coulombs. The value of n is
    a. 1
    b. 2
    c. 3
    d. 4

55. Which of the following mixtures will have the lowest pH at 298 K?
    a. 10 ml 0.05NCH_3COOH + 5 ml 0.1 NNH_4OH
    b. 5 ml 0.2NH_4Cl + 5 ml 0.2 N NH_4OH
    c. 5 ml 0.1N CH_3COOH + 10 ml 0.05 N CH_3COONa
    d. 5 ml 0.1N CH_3COOH + 5 ml 0.1 N NaOH

56. Consider the following two first order reactions occurring at 298 K with same initial concentration of A:
    (1) $A \rightarrow B$: rate constant, k = 0.693 min^{-1}
    (2) $A \rightarrow C$: half – life, $t_{1/2}$ = 0.693 min^{-1}
    Choose the correct option:
    a. Reaction (1) is faster than Reaction (2).
    b. Reaction (1) is slower than Reaction (2).
    c. Both reaction proceed at the same rate.
    d. Since two different products are formed, rates cannot be compared.

57. For the equilibrium $H_2O(\ell) \rightleftharpoons H_2O(g)$, which of the following is correct?
    a. $\Delta G = 0, \Delta H < 0, \Delta S < 0$
    b. $\Delta G < 0, \Delta H > 0, \Delta S > 0$
    c. $\Delta G > 0, \Delta H = 0, \Delta S > 0$
    d. $\Delta G = 0, \Delta H > 0, \Delta S > 0$

58. For a Vander Waal's gas, the term $\left(\frac{ab}{v^2}\right)$ represents some
    a. Pressure
    b. Energy
    c. Critical density
    d. Molar mass

59. In the equilibrium $H_2 + I_2 \rightleftharpoons 2HI$, if at a given temperature the concentrations of the reactants are increased, the value of the equilibrium constant, K_C, will:
    a. Increase
    b. Decrease
    c. Remain the same
    d. Cannot be predicted with certainty

60. If electrolysis of aqueous $CuSO_4$ solution is carried out using Cu-electrodes, the reaction taking place at the anode is
    a. $H^+ + e \rightarrow H$
    b. $Cu^{2+} (aq) + 2e \rightarrow Cu(s)$
    c. $SO_4^{2-} (aq) - 2e \rightarrow SO_4$
    d. $Cu(s) - 2e \rightarrow Cu^{2+} (aq)$

61. Which one of the following electronic arrangements is absurd?
    a. $n = 3, \ell = 1, m = -1$
    b. $n = 3, \ell = 0, m = 0$
    c. $n = 2, \ell = 0, m = -1$
    d. $n = 2, \ell = 1, m = 0$

62. The quantity hv/k_B corresponds to

a. Wavelength

b. Velocity

c. Temperature

d. Angular momentum

63. In the crystalline solid $MSO_4.nH_2O$ of molar mass 250 g mol^{-1}, the percentage of anhydrous salt is 64 by weight. The value of n is

a. 2
b. 3
c. 5
d. 7

64. At S.T.P., the volume of 7.5 g of a gas is 5.6 L. The gas is

a. NO

b. N_2O

c. CO

d. CO_2

65. The half–life period of ${}_{53}I^{125}$ is 60 days. The radioactivity after 180 days will be

a. 25 %

b. 12.5 %

c. 33.3 %

d. 3.0 %

66. Consider the radioactive disintegration

${}_{82}A^{210} \rightarrow B \rightarrow C \rightarrow {}_{82}D^{206}$

The sequence of emission can be

a. β, β, β

b. α, α, β

c. β, β, γ

d. β, β, α

67. The second Ionisation energy of the following elements follows the order

a. Zn > Cd < Hg

b. Zn > Cd > Hg

c. Cd > Hg < Zn

d. Zn < Cd < Hg

68. The melting points of (i) $BeCl_2$ (ii) $CaCl_2$ and (iii) $HgCl_2$ follows the order

a. i < ii < iii

b. iii < i < ii

c. i < iii < ii

d. ii < i < iii

69. Which of these species will have non-zero magnetic moment?

a. Na^+

b. Mg

c. F^-

d. Ar^+

70. The first electron affinity of C, N and O will be of the order

a. C < N < O

b. N < C < O

c. C < O < N

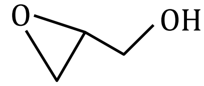
d. O < N < C

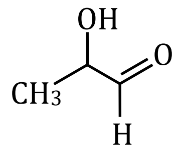
WB JEE-2019 (Chemistry)

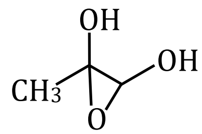
Category-II (Q.71 to Q75)

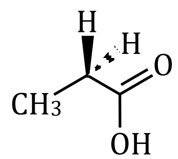
Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

71. Oxidation of allyl alcohol with a peracid gives a compound of molecular formula $C_3H_6O_2$, which contains an asymmetric carbon atom. The structure of the compound is

a. 

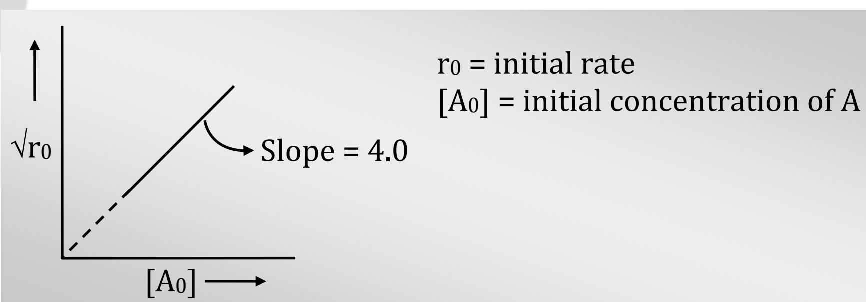
b. 

c. 

d. 

72. The total number of isomeric liner dipeptide which can be synthesized from racemic alanine is
   a. 1
   b. 2
   c. 3
   d. 4

73. The kinetic study of a reaction like vA→ P at 300 K provides the following curve. Where concentration is taken in mol dm^{-3} and time in min.



a. $n = 0, k = 4.0 mol\ dm^{-3}\ min^{-1}$
b. $n = 1/2, k = 2.0\ mol^{1/2}\ dm^{3/4} dm^{-3}\ min^{-1}$
c. $n = 1, k = 8.0\ min^{-1}$
d. $n = 2, k = 16.0\ dm^3\ mol^{-1}\ min^{-1}$

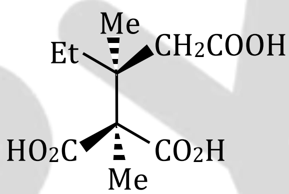
74. At constant pressure, the heat of formation of compound is not dependent on temperature, when
    a. $\Delta C_p = 0$
    b. $\Delta C_V = 0$
    c. $\Delta C_p > 0$
    d. $\Delta C_p < 0$

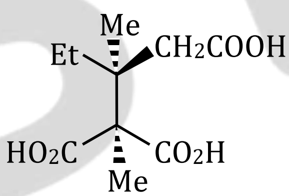
75. A copper coin was electroplated with Zn and then heated at high temperature until there is a change in colour. What will be the resulting colour?
    a. White
    b. Black
    c. Silver
    d. Golden

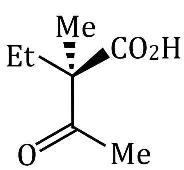
Category-III (Q.76 to Q.80)

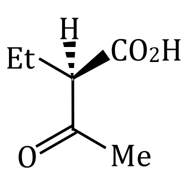
Carry 2 marks each one or more option(s) is/are correct. If all correct answer are not marked and also no incorrect answer is marked then score = 2 × number of correct answer marked + actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but three is no negative marking for the same and zero mark will be awarded.

76. The compounds(s), capable of producing achiral compound on heating at 100°C is/are:

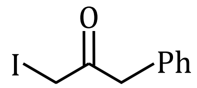
a. 

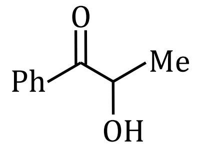
b. 

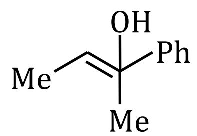
c. 

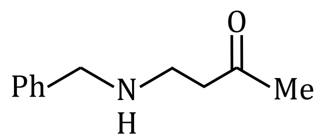
d. 

77. Haloform reaction with I_2 and KOH will be responded by

a. 

b. 

c. 

d. 

78. Identify the correct statement(s):

a. The oxidation number of Cr in CrO_5 is +6.

b. $\Delta H > \Delta U$ for the reaction $N_2O_4(g) \rightarrow 2NO_2(g)$. Provided both gases behave ideally.

c. pH of 0.1N H_2SO_4 is less than that of 0.1 N HCl at 25°C

d. $\left(\frac{RT}{F}\right) = 0.0591$ volt at 25°C.

79. Compounds with spin-only magnetic moment equivalent to five unpaired electrons are:

a. $K_4[Mn(CN)_6]$

b. $[Fe(H_2O)_6]Cl_3$

c. $K_3[FeF_6]$

d. $K_4[MnF_6]$

80. Which of the following chemical may be used to identify three unlabelled beakers containing conc. NaOH, conc. H_2SO_4 and water?

a. NH_4NO_3

b. NaCl

c. $(NH_4)_2CO_3$

d. HCOONa

WB JEE-2019 (Chemistry)

ANSWER KEYS

41. (a)	42. (b)	43. (d)	44. (b)	45. (d)	46. (d)	47. (d)	48. (b)	49. (c)	50. (a)
51. (d)	52. (b)	53. (a)	54. (b)	55. (c)	56. (b)	57. (d)	58. (b)	59. (c)	60. (d)
61. (c)	62. (c)	63. (c)	64. (a)	65. (b)	66. (d)	67. (a)	68. (b)	69. (d)	70. (b)
71. (a)	72. (d)	73. (d)	74. (a)	75. (b)	76. (c)	77. (a,b)	78. (a,b)	79. (b,c,d)	80. (a,c)

SOLUTION

41. (a)
As %s-character increases, bond angle increases
As % p-character increases, bond angle decreases.
∴ p-character order will be: $PH_3 > NH_3$
∴ Bond angle order: $PH_3 < NH_3$

42. (b)
OCl^- is used as a bleaching agent. Its components will be: $OCl^- \longrightarrow [O] + Cl^-$

43. (d)
The compound is
Pentaaminechloridecobalt (III) chloride
$[Co(NH_3)_5Cl]Cl_2$ □□ $[Co(NH_3)_5Cl^-]^{+2} + 2Cl^-$
∵ Gives 3 ions in aqueous solution.

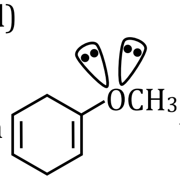
44. (b)
The separation of the alumina from impurities is usually accomplished by Bayer's process.
The reactions involved in this process are:

$$Al_2O_3.2H_2O + 2NaOH \xrightarrow[8\text{ atm}]{190^\circ C} \underset{(\text{Soluble})}{2NaAlO_2} + 3H_2O$$

$$NaAlO_2 + H_2O \longrightarrow NaOH + Al(OH)_3\downarrow$$

$$2Al(OH)_3 \longrightarrow Al_2O_3 + 3H_2O$$

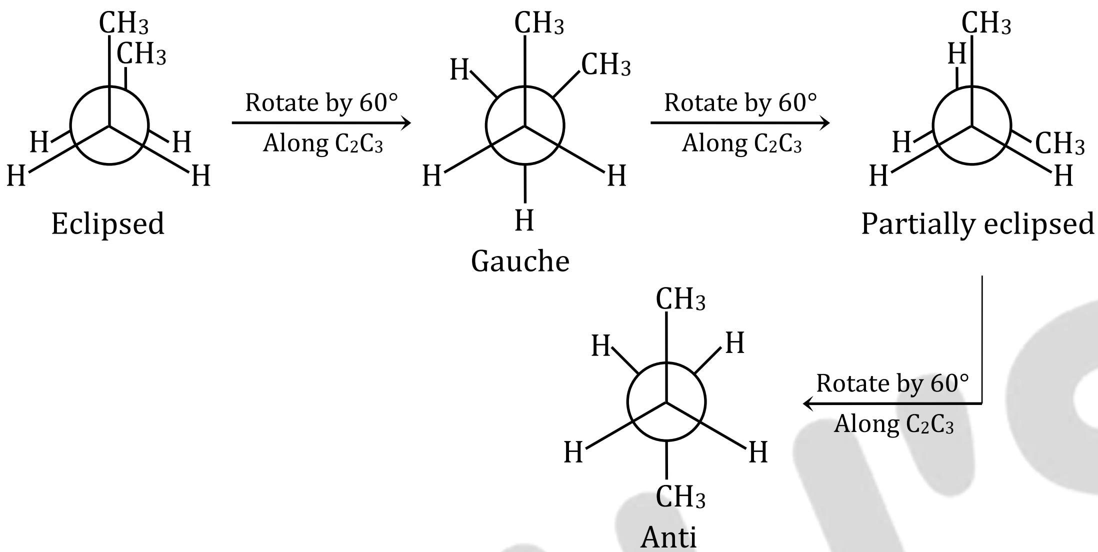
45. (d)
Since ${}^{238}_{92}U$ isotope of uranium does not participate in nuclear chain reaction, it cannot be used as nuclear fuel.

46. (d)
In 

there is π bond and lone pair conjugation so, the lone pair of oxygen get delocalized on π bond located on next carbon.

47. (d)

Rotation around $C_2–C_3$



48. (b)

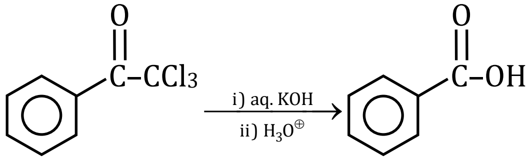
Reactivity order is

HI > HBr > HCl

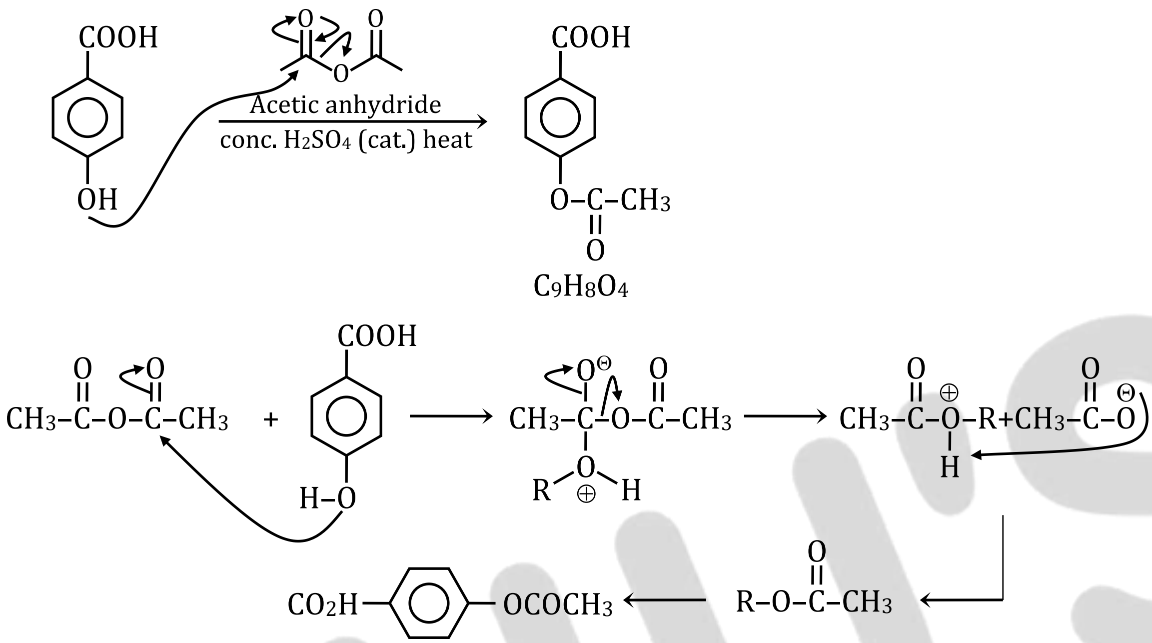
∵ Leaving group order is I^-> Br^-> Cl^-

The compound that has more stable leaving group will be more reactive with ethylene.

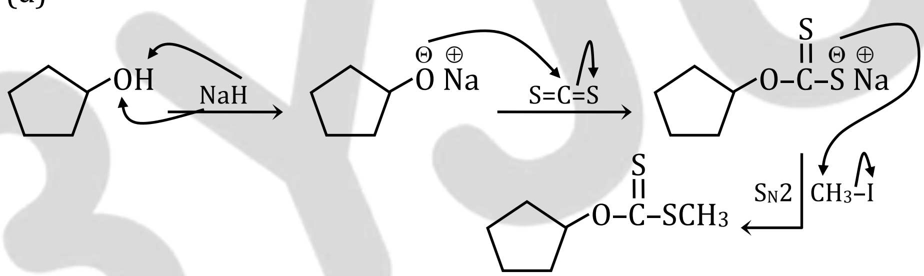
49. (c)



50. (a)



51. (d)



52. (b)
CH_3COCl hydrolyses to form CH_3COOH even at 25°C, which subsequently reacts with $NaHCO_3$ present in the same medium to form CO_2.

53. (a)
There is no lone pair on 'B' atom of $BH_4^{\ominus}$, hence it is not nucleophilic site.

54. (b)
Given: Charge on 1 milimole of Mn^{2+} ions = 193 C

So, $193 = \dfrac{n \times 96500}{1000}$

Rearranging, $n = \dfrac{193 \times 1000}{96500}$

So, n = 2

55. (c)

(A) CH_3COOH + $NaOH$ $\longrightarrow$ $CH_3COO\ NH_4$

0.05 N   0.1 N   WAWB salt

10 ml   5 ml

0.5   0.5

$$pH = 7 - \frac{1}{2}pK_b + \frac{1}{2}pK_a \simeq 7$$

(B) ($NH_4Cl + NH_4OH$) Basic buffer solution

$$pOH = pK_b + \log\frac{CA}{B}$$

$pH > 7$

(D) CH_3COOH + $NaOH$ $\longrightarrow$ $CH_3COO\ Na$ (WAWB salt)

$$pH = 7 + \frac{1}{2}pK_b = \frac{1}{2}\log C$$

$pH > 7$

(C) CH_3COOH + CH_3COONa

	CH_3COOH	CH_3COONa
	0.1 N	0.05 N
	5 ml	10 ml
milli eq.	0.5	0.5

It is acidic buffer solution $pH = pK_a + \log\frac{CH_3COO^{\ominus}}{CH_3COOH}$

$pH = pK_a$

This solution will have lower pH.

56. (b)

For 1st order reaction

Rate constant $K = \frac{0.693}{t_{1/2}}$ and Rate = K(A)'

For (I) reaction K = 0.6930 minute^{-1}

For (II) reaction $K = \frac{0.693}{t_{1/2}} = \frac{0.693}{0.693} = 1\ min^{-1}$

$\because$ given $t_{1/2} = 0.693$

So, $K_I < K_{II}$

Then, Rate I < Rate II

57. (d)

For equilibrium $H_2O(l) \rightleftharpoons H_2O(g)$

$\Delta G = 0 \quad \because$ equilibrium

$\Delta H > 0$ (+ve), for $T = \frac{\Delta H}{\Delta S}$ the process will be spontaneous above the temperature of T

$\Delta S > 0$ (+ve)

$$\Delta S_{sys} = \underbrace{nC_V \ln\frac{T_2}{T_1}}_{0} + nR\ln\frac{V_2}{V_1}$$

$\Delta S_{sys} = nR\ln\frac{V_2}{V_1}$ at constant temperature.

58. (b)

Vander Waal's equation is given as:

$$\left[P\left(1+\frac{an^2}{V^2}\right)\right](V - nb) = nRT$$

$\frac{ab}{v^2}$ represent energy by dimensional analysis.

Unit of a (Vander Waal's constant) = atm.ℓ^2/mole^2

Unit of b (Vander Waal's constant) = l/mole

So, $\frac{ab}{v^2} = \frac{\frac{atm.\ell^2}{mole^2}\times\frac{\ell}{mole}}{\ell^2/mole^2} = \frac{atm.\ell}{mole}$

It is unit of energy per mole.

59. (c)
Equilibrium constant doesn't depend on the molar concentration of reactants. It is only affected by the change in temperature.

60. (d)

On electrolysis of aq. solution of $CuSO_4$ on using Cu electrode. According to SOP values at anode.

$2H_2O \longrightarrow O_2 + 4H^+ + 4e^- \qquad E^0 = -1.23$ v

$Cu \longrightarrow Cu^{2+} + 2e^- \qquad E^0 = -0.34$ v

So, reaction carried out on anode, which has high SOP value.

61. (c)
Quantum number set n = 2, l = 0, m = –1 is not possible (not valid) since value of m ≤ +l to –l.

62. (c)
$\frac{h\nu}{k_B} = \frac{3}{2}T$ (it represents temperature)

63. (c)
Mass of anhydrous MSO_4 salt = $250 \times \frac{64}{100}$ = 160 gm/mole
Total mass of H_2O is $MSO_4.nH_2O$
= 250 – 160 = 90 gm/mole
So value of n = $\frac{90}{18}$ = 5

64. (a)
(A) For finding which gas is it, we have to find molar mass.
∵ At STP weight of 5.6 L gas = 7.5 gm (given)
At STP weight of 22.4 L gas = $\frac{7.5}{5.6} \times 22.4$ = 30 gm/mole
And we know, molar mass of NO is 30 gm/mole
So answer (A).

65. (b)
$t_{1/2}$ = 60 days (given)
Radioactivity after t time $N_t = \frac{N_o}{(2)^n}$ ... (1)
& we know, $n = \frac{t}{t_{1/2}}$
So, $n = \frac{180}{60}$; n = 3
Putting value of n in eq. (1)
$N_t = \frac{N_o}{(2)^3} = \frac{N_o}{8} = 0.125\ N_o$
So, radioactivity after 180 days = 12.5 %

66. (d)
$_{82}A^{210} \xrightarrow{\beta} {}_{83}B^{210} \xrightarrow{\beta} {}_{64}C^{210} \xrightarrow{\alpha} {}_{82}C^{206}$
$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A}$ + β-particle
$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4}$ + α-particle

67. (a)

2nd I.E. order: Cd < Zn < Hg

So, Zn > Cd < Hg

Element	2nd I.E. (kJ/mole)
Zn	1734
Cd	1631
Hg	1809

68. (b)

According to covalent character

Melting points $\propto \frac{1}{\text{Covalent character}}$

Melting points $HgCl_2 = 276°C$

$BeCl_2 = 399°C$

$CaCl_2 = 775°C$

Melting point order = $HgCl_2 < BeCl_2 < CaCl_2$

69. (d)

By writing configurations we can find number of unpaired electrons.

If number of unpaired electrons is 0

Then, magnetic moment will also be 0

	Number of unpaired e^-
$_{11}Na^+ \to 1s^2 2s^2 2p^6$	0
$_{12}Mg \to 1s^2 2s^2 2p^6 3s^2$	0
$_9F^- \to 1s^2 2s^2 2p^6$	0
$_{18}Ar^+ \to 1s^2 2s^2 2p^6 3s^2 3p^5$	1

70. (b)

1^{st} electron affinity order: N < C < O

According to electronic configuration,

N = $1s^2\ 2s^2\ 2p^3$ Half filled orbitals are more stable.

E.A. kJ/mole [C = 121.77, N = –6.8, O = 140]

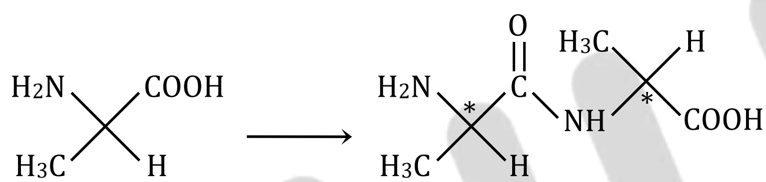
71. (a)

$CH_2{=}CH{-}CH_2{-}OH \xrightarrow[RCO_3H]{[O]} \underset{\text{(O bridge)}}{CH_2{-}CH}{-}CH_2{-}OH$

$(C_3H_6O_2)$

An epoxide ring formation will take place.

72. (d)



R + S SS, SR, RR, RS

Dipeptide has two chiral carbons and both side unsymmetrical. Hence RR, RS, SR, SS is possible.

73. (d)

Rate = $k(A)^n$

According to graph (n = 2)

Slope = $\dfrac{(\text{Rate})^{1/2}}{(A)} = 4$

$K = \dfrac{\text{Rate}}{(A)^2}$; $\dfrac{\text{Rate}}{(A)^2} = (4)^2$

$\dfrac{\text{Rate}}{(A)^2} = 16$

74. (a)

For reaction: (According to Kirchhoff's equation)

$\Delta H_2 = \Delta H_1 + \Delta C_p(\Delta T)$

When $\Delta C_p = 0$

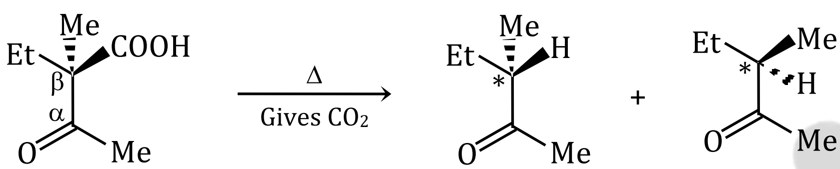
Then, ΔH_f will not depend on temperature.

75. (b)

If these coins are heated, the zinc will diffuse into the copper layer, producing a surface alloy of zinc and copper. These alloys are brasses. Copper also oxidizes when heated in air, producing a black layer of copper-oxide (CuO).

76. (c)

On heating the compound CO_2 gas will evolve out.

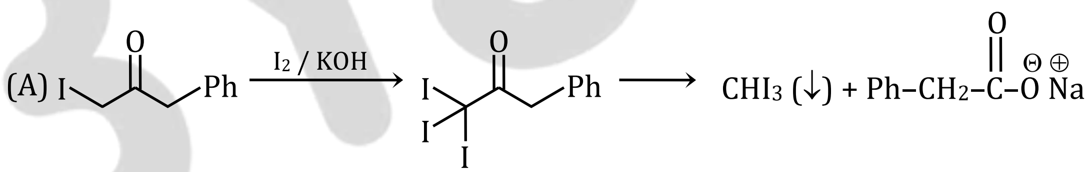


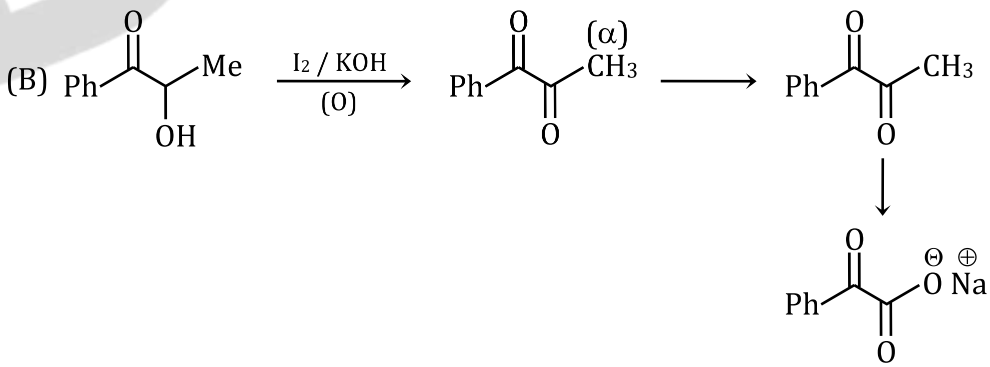
Achiral mixture

In product chiral carbon atom is present hence, R and S structure will be there.

77. (a,b)

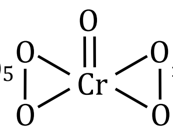
I_2/KOH reacts on methyl ketones or 2° alcohols.





2° alcohols can be converted to ketones.

78. (a,b)

(A) Oxidation number of Cr of CrO_5  $\Rightarrow +6$

(B) $N_2O_4\,(g) \longrightarrow 2NO_2\,(g)$ $\quad\quad \Delta H = \Delta U + \Delta ngRT$

$\Delta ng = 2 - 1 = 1$

$\Delta n = +ve$

So, $\Delta H > \Delta U$

(C) pH of 0.1 N $H_2SO_4 \Rightarrow [H^+]$ 0.1 N

$pH = -\log[H^+] = \log(10^{-1})$

pH of 0.1 N HCl $= [H^+] = 0.1$ N

$pH = \log\,[H^+]$

$\quad = -\log(10^{-1}) \Rightarrow 1$

(D) $\dfrac{RT}{F} = \dfrac{8.314 \times 298}{96500} = 0.0256$

79. (b,c,d)

(A) In $K_4[Mn(CN)_6]$, Mn has +20.5 and $Mn^{2+} = [Ar]\,3d^5 4s^0$, and CN^- is a strong field ligand pairing takes place and the complex has only one unpaired electron.

(B) In $[Fe(H_2O)_6]Cl_3$, Fe has +30.5 and $Fe^{3+} = [Ar]3d^5 4s^0$, the number of unpaired electron = 5. (H_2O is a weak field ligand thus no pairing)

(C) In $K_3[FeF_6]$, Fe has +30.5 and $Fe^{3+} = [Ar]3d^5 4s^0$, thus number of unpaired electron = 5. (F^- is a weak field ligand thus no pairing).

(D) In $K_4[MnF_6]$, Mn has 0.5 of +2 thus $Mn^{2+} = [Ar]3d^5 4s^0$ is number of unpaired electron = 5.

F^- is a weak field ligand thus no pairing.

80. (a,c)

(A) NH_4NO_3 will evolve NH_3, pungent smelling gas with NaOH, NO_2 brown gas with conc. H_2SO_4 and no reaction in water, thus NH_4NO_3 can label all three beakers.

$$NH_4NO_3 \xrightarrow{\text{conc. NaOH}} NH_3\uparrow + NaNO_3 + H_2O$$

$$NH_4NO_3 \xrightarrow{\text{conc. } H_2SO_4} (NH_4)_2SO_4 + NO_2(\uparrow) + O_2 + H_2O$$

(C) $(NH_4)_2CO_3$ will evolve NH_3 pungent smelling gas with NaOH, effervescence of CO_2 with conc. H_2SO_4 and no reaction with water, thus $(NH_4)_2CO_3$ can label all three beakers.

$$(NH_4)_2CO_3 \xrightarrow{\text{conc. NaOH}} NH_3\uparrow + Na_2CO_3 + H_2O$$

$$(NH_4)_2CO_3 \xrightarrow{\text{conc. } H_2SO_4} (NH_4)_2SO_4 + CO_2(\uparrow) + H_2O$$

WB JEE-2019 (Physics)

1. A point object is placed on the axis of a thin convex lens of focal length 0.05 m at a distance of 0.2m from the lens and its image is formed on the axis. If the object is now made to oscillate along the axis with small amplitude of A cm, then what is the amplitude of oscillation of the image?

$$\left[\text{you may assume } \frac{1}{1+x} \approx 1-x, \text{where } x << 1\right]$$

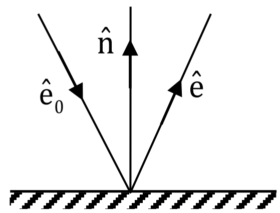
   a. $\frac{4A}{9} \times 10^{-2}$ m
   b. $\frac{5A}{9} \times 10^{-2}$ m
   c. $\frac{A}{3} \times 10^{-2}$ m
   d. $\frac{A}{9} \times 10^{-2}$ m

2. In Young's double slit experiment, d is the separation between the slits and D is the distance between the plane of the slits and the screen. If D is increased by 0.5% and d is decreased by 0.3% then, for a given light of a certain wavelength, which one of the following options is correct?
"He fringe width ...."
   a. increases by 0.8%
   b. decreases by 0.8%
   c. increases by 0.2%
   d. decreases by 0.2%

3. When the frequency of the light used is changed from 4×10^{14} $s^{-1}$to 5×10^{14} s^{-1}, the angular width of the principal (central) maximum in a single slit Fraunhoffer diffraction pattern changes by 0.6 radian. What is the width of the slit (assume that the experiment is performed in vacuum)
   a. 1.5×10^{-7} m
   b. 3×10^{-7}m
   c. 5×10^{-7}m
   d. 6×10^{-7}m

WB JEE-2019 (Physics)

4. A ray of light is reflected by a plane mirror $\hat{e}_0, \hat{e}$ and $\hat{n}$ be the unit vectors along the incident ray, reflected ray and the normal to the reflecting surface respectively. Which of the following gives an expression for $\hat{e}$?



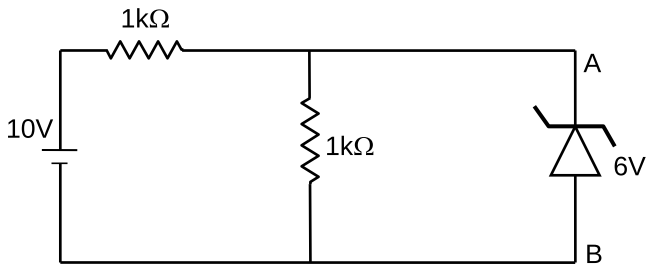
   a. $\hat{e}_0 + 2(\hat{e}_0.\hat{n})\hat{n}\ 0$
   b. $\hat{e}_0 - 2(\hat{e}_0.\hat{n})\hat{n}$
   c. $\hat{e}_0 - (\hat{e}_0.\hat{n})\hat{n}$
   d. $\hat{e}_0 + (\hat{e}_0.\hat{n})\hat{n}$

5. A parent nucleus X undergoes α-decay with a half-life of 75000 years. The daughter nucleus Y undergoes β-decay with a half-life of 9 months. In a particular sample, it is found that the rate of emission of β-particles is nearly constant (over several months) at 10^7/hour. What will be the number of α–particles emitted in an hour?
   a. 10^2
   b. 10^7
   c. 10^{12}
   d. 10^{14}

6. A proton and an electron initially at rest are accelerated by the same potential difference. Assuming that a proton is 2000 times heavier than an electron, what will be the relation between the de Broglie wavelength of the proton (λp) and that of electron (λc)?
   a. $\lambda_p = 2000\lambda_e$
   b. $\lambda_p = \frac{\lambda_e}{2000}$
   c. $\lambda_p = 20\sqrt{5}\lambda_e$
   d. $\lambda_p = \frac{\lambda_e}{20\sqrt{5}}$

7. To which of the following, the angular velocity of the electron in the n-th Bohr orbit is proportional?
   a. n^2
   b. $\frac{1}{n^2}$
   c. $\frac{1}{n^{3/2}}$
   d. $\frac{1}{n^3}$

8. In the circuit shown, what will be the current through the 6V zener?



a. 6mA, from A to B
b. 2mA, from A to B
c. 2 mA, from B to A
d. zero

9. Each of the two inputs A and B can assume values either 0 or 1. Then which of the following will be equal to $\bar{A}.\bar{B}$. ?

a. A + B
b. $\overline{A+B}$
c. $\overline{A.B}$
d. $\bar{A}+\bar{B}$

10. The correct dimensional formula for impulse is given by

a. ML^2T^{-2}
b. MLT^{-1}
c. ML^2T^{-1}
d. MLT^{-2}

11. The density of the material of a cube can be estimated by measuring its mass and the length of one of its sides. If the maximum error in the measurement of mass and length are 0.3% and 0.2% respectively, the maximum error in the estimation of the density of the cube is approximately

a. 1.1%
b. 0.5%
c. 0.9%
d. 0.7%

12. Two weights of the mass $m_1$and m_2(> m1) are joined by an inextensible string of negligible mass passing over a fixed frictionless pulley. The magnitude of the acceleration of the loads is
   a. g
   b. $\frac{m_2 - m_1}{m_2}g$
   c. $\frac{m_1}{m_2 + m_1}g$
   d. $\frac{m_2 - m_1}{m_2 + m_1}g$

13. Body starts from rest, under the action of an engine working at a constant power and moves along a straight line. The displacement S is given as a function of time (t) as
   a. $S = at + bt^2$, a, b are constants
   b. $S = bt^2$, b is a constant
   c. $S = at^{3/2}$a is a constant
   d. S = at, a is a constant

14. Two particles are simultaneously projected in the horizontal direction from a point P at a certain height. The initial velocities of the particles are oppositely directed to each other and have magnitude v each. The separation between the particles at a time when their position vectors (drawn from the point P) are mutually perpendicular, is
   a. $\frac{v^2}{2g}$
   b. $\frac{v^2}{g}$
   c. $\frac{4v^2}{g}$
   d. $\frac{2v^2}{g}$

15. Assume that the earth moves around the sun in a circular orbit of radius R and there exists a planet which also moves around the sun in a circular orbit with an angular speed twice as large as that of the earth. The radius of the orbit of the planet is
   a. $2^{-2/3R}$
   b. $2^{2/3}R$
   c. $2^{-1/3}R$
   d. $\frac{R}{\sqrt{2}}$

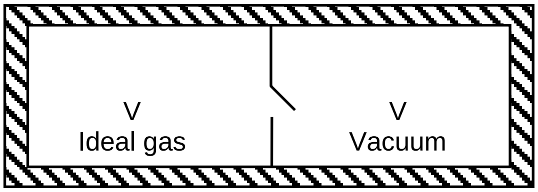
16. A compressive force is applied to a uniform rod of rectangular cross-section so that its length decreases by 1%. If the Poisson's ratio for the material of the rod be 0.2, which of the following statements is correct?
''The volume approximately …….. ''
    a. decreases by 1%
    b. decreases by 0.8%
    c. increases by 0.6%
    d. increases by 0.2%

17. A small spherical body of radius r and density ρ moves with the terminal velocity v in a fluid of coefficient of viscosity η and densityσ. What will be the net force on the body?
    a. $\frac{4\pi}{3}r^3(\rho-\sigma)g$
    b. $6\pi\eta rv$
    c. Zero
    d. Infinity

18. Two black bodies A and B have equal surface areas and are maintained at temperatures 27°C and 177°Crespectively. What will be the ratio of the thermal energy radiated per second by A to that by B?
    a. 4 : 9
    b. 2 : 3
    c. 16 : 81
    d. 27 : 177

19. What will be the molar specific heat at constant volume of an ideal gas consisting of rigid diatomic molecules?
    a. $\frac{3}{2}R$
    b. $\frac{5}{2}R$
    c. R
    d. 3R

20. Consider the given diagram. An ideal gas is contained in a chamber (left) of volume V and is at an absolute temperature T. It is allowed to rush freely into the right chamber of volume V which is initially vacuum. The whole system is thermally isolated. What will be the final temperature of the system after the equilibrium has been attained?



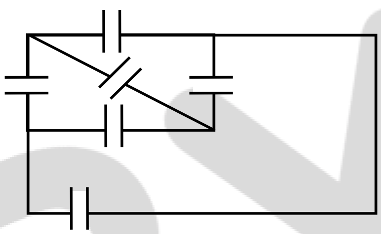
a. T

b. $\frac{T}{2}$

c. 2T

d. $\frac{T}{4}$

21. Five identical capacitors, of capacitance 20μF each, are connected to a battery of 150V, in combinations shown in the diagram. What is the total amount of charge stored?



a. 15×10^{-3}C

b. 12×10^{-3} C

c. 10×10^{-3} C

d. 3×10^{-3} C

22. Eleven equal point charges, all of them having a charge +Q, are placed at all the hour positions of a circular clock of radius r, except at the 10 hour position. What is the electric field strength at the centre of the clock?

a. $\frac{Q}{4\pi\varepsilon_0 r^2}$ from the centre towards the mark 10

b. $\frac{Q}{4\pi\varepsilon_0 r^2}$ from the mark 10 towards the centre

c. $\frac{Q}{4\pi\varepsilon_0 r^2}$ from the centre towards the mark 6

d. Zero

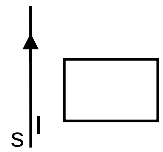
23. A negative charge is placed at the midpoint between two fixed equal positive charges, separated by a distance 2d. If the negative charge is given a small displacement x (x << d) perpendicular to the line joining the positive charges, how the force (F) developed on it will approximately depend on x?
   a. $F \propto x$
   b. $F \propto \frac{1}{x}$
   c. $F \propto x^2$
   d. $F \propto \frac{1}{x^2}$

24. To which of the following quantities, the radius of the circular path of a charged particle moving at right angles to a uniform magnetic field is directly proportional?
   a. energy of the particle
   b. magnetic field
   c. charge of the particle
   d. Momentum of the particle.

25. An electric current 'I' enters and leaves a uniform circular wire of radius r through diametrically opposite points. A particle carrying a charge q moves along the axis of the circular wire with speed v. What is the magnetic force experienced by the particle when it passes through the centre of the circle?
   a. $qv\frac{\mu_0 i}{a}$
   b. $qv\frac{\mu_0 i}{2a}$
   c. $qv\frac{\mu_0 i}{2\pi a}$
   d. zero

26. A current 'I' is flowing along an infinite, straight wire, in the positive Z-direction and the same current is flowing along a similar parallel wire 5 m apart, in the negative Z-direction. A point P is at a perpendicular distance 3 m from the first wire and 4 m from the second. What will be magnitude of the magnetic field Bat P?
   a. $\frac{5}{12}(\mu_0 l)$
   b. $\frac{7}{24}(\mu_0 l)$
   c. $\frac{5}{24}(\mu_0 l)$
   d. $\frac{25}{288}(\mu_0 l)$

27. A square conducting loop is placed near an infinitely long current carrying wire with one edge parallel to the wire as shown in the figure. If the current in the straight wire is suddenly halved. Which of the following statements will be true?



''The loop will ……… ''

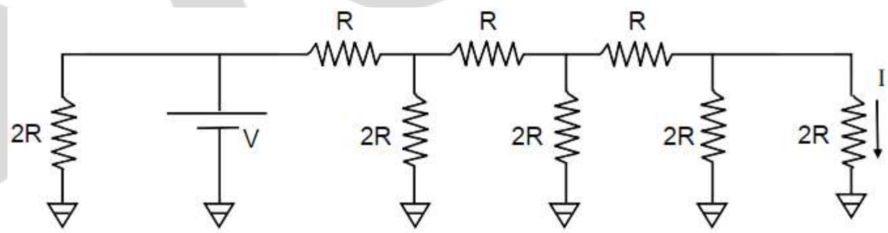
a. stay stationary.

b. move towards the wire.

c. move away from the wire.

d. move parallel to the wire.

28. What is the current I shown in the given circuit?



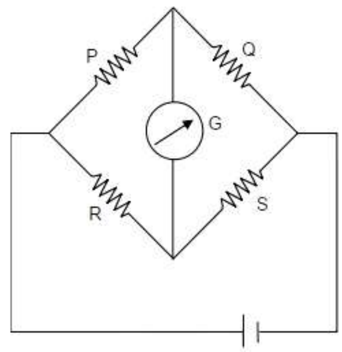
a. $\frac{V}{2R}$

b. $\frac{V}{R}$

c. $\frac{V}{16R}$

d. $\frac{V}{8R}$

29. When the value of R in the balanced wheatstone bridge, shown in the figure, is increased from 5Ω to 7Ω, the value of S has to be increased by 3Ω in order to maintain the balance. What is the initial value of S?



a. 2.5Ω
b. $3\ \Omega$
c. $5\ \Omega$
d. $7.5\ \Omega$

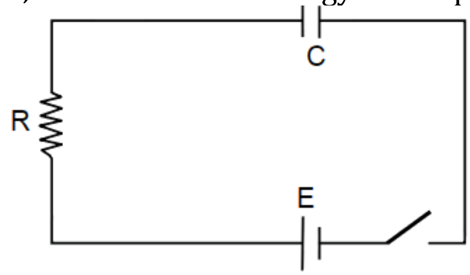
30. When a 60 mH inductor and a resistor are connected in series with an AC voltage source, the voltage leads the current by 60°. If the inductor is replaced by a 0.5 μF capacitor, the voltage lags behind the current by 30°. What is the frequency of the AC supply?

a. $\frac{1}{2\pi}\times 10^4\,\text{Hz}$
b. $\frac{1}{\pi}\times 10^4\,\text{Hz}$
c. $\frac{3}{2\pi}\times 10^4\,\text{Hz}$
d. $\frac{1}{2\pi}\times 10^8\,\text{Hz}$

31. A parallel plate capacitor in series with a resistance of 100Ω, an inductor of 20 mH and an AC voltage source of variable frequency shows resonance at a frequency of $\frac{1250}{\pi}\,\text{Hz}$. If this capacitor is charged by a DC voltage source to a voltage 25V, what amount of charge will be stored in each plate of the capacitor?

a. 0.2 μC
b. 2 mC
c. 0.2 mC
d. 0.2 C

32. A capacitor of capacitance C is connected in series with a resistance R and a DC source of emf E through a key. The capacitor starts charging when the key is closed. By the time the capacitor has been fully charged, what amount of energy is dissipated in the resistance R ?



a. $\frac{1}{2}CE^2$
b. 0
c. CE^2
d. $\frac{E^2}{R}$

33. A horizontal fire hose with a nozzle of cross-sectional area $\frac{5}{\sqrt{21}} \times 10^{-3} m^2$ delivers a cubic metre of water in 10s. What will be the maximum possible increase in the temperature of water while it hits a rigid wall (neglecting the effect of gravity)?
a. 1°C
b. 0.1°C
c. 10°C
d. 0.01°C

34. Two identical blocks of ice move in opposite directions with equal speed and collide with each other. What will be the minimum speed required to make both the blocks melt completely, if the initial temperatures of the blocks were –8°C each?
a. 840 ms^{-1}
b. 420 ms^{-1}
c. 8.4 ms^{-1}
d. 84 ms^{-1}

35. A particle with charge q moves with a velocity v in a direction perpendicular to the directions of uniform electric and magnetic fields, E and B respectively, which are mutually perpendicular to each other. Which one of the following gives the condition for which the particle moves undeflected in its original trajectory?
a. $V = \frac{E}{B}$
b. $v = \frac{B}{E}$
c. $v = \sqrt{\frac{E}{B}}$
d. $v = q\frac{B}{E}0$

WB JEE-2019 (Physics)

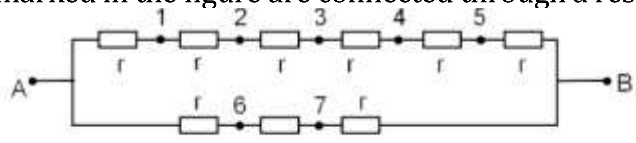
36. A metallic loop is placed in a uniform magnetic field B with the plane of the loop perpendicular to B. Under which conditions(s) given below an emf will be induced in the loop?
"If the loop is ...."
    a. Moved along the direction of B
    b. Squeezed to a smaller area.
    c. Rotated about its axis
    d. Rotated about one of its diameters.

37. Electrons are emitted with kinetic energy T from a metal plate by an irradiation of light of intensity J and frequency v. Then which of the following will be true?
    a. $T \propto J$
    b. T linearly increasing with v
    c. $T \propto$ time of irradiation
    d. Number of electrons emitted $\propto J$

38. The initial pressure and volume of a given mass of an ideal gas (with $C_p/C_{v=\gamma}$), taken in a cylinder fitted with a piston, are $P_0$and V_0respectively. At this stage the gas has the same temperature as that of the surrounding medium which is T_0. It is adiabatically compressed to a volume equal to $V_{0/2}$. Subsequently the gas is allowed to come to thermal equilibrium with the surroundings. What is the heat released to the surroundings?
    a. 0
    b. $\left(2^{r-1}-1\right)\frac{P_0V_0}{r-1}$
    c. $\gamma P_0V_0 In2$
    d. $\frac{P_0V_0}{2(\gamma-1)}$

39. A projectile thrown with an initial velocity of $10ms^{-1}$ at an angle α with the horizontal, has a range of 5m.Taking g = $10ms^{-2}$ and neglecting air resistance, what will be the estimated value of $\underline{\alpha}$?
    a. 15°
    b. 30°
    c. 45°
    d. 75°

40. In the circuit shown in the figure all the resistances are identical and each has the value r Ω. The equivalent resistance of the combination between the points A and B will remain unchanged even when the following pairs of point A and B will remain unchanged even when the following pairs of point s marked in the figure are connected through a resistance R.



    a. 2 and 6
    b. 3 and 6
    c. 4 and 7
    d. 4 and 6

WB JEE-2019 (Physics)

ANSWER KEYS

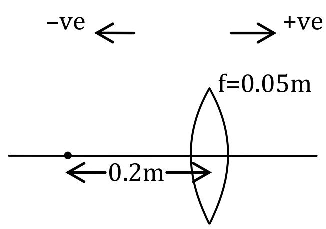
1. (d)	2. (a)	3. (c)	4. (b)	5. (b)	6. (d)	7. (d)	8. (d)	9. (b)	10. (b)
11. (c)	12. (d)	13. (c)	14. (c)	15. (a)	16. (c)	17. (c)	18. (c)	19. (b)	20. (a)
21. (d)	22. (a)	23. (a)	24. (d)	25. (d)	26. (G)	27. (b)	28. (c)	29. (d)	30. (a)
31. (c)	32. (a)	33. (a)	34. (a)	35. (a)	36. (b,d)	37. (b,d)	38. (b)	39. (a,d)	40. (a,c)

* G – Indicates GRACE MARK awarded for the question number

WB JEE-2019 (Physics)

Solution

1. (d)



$\therefore$ u = -0.2 m

f = 0.05 m

By using lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{100}{5} - \frac{10}{2}$$

$$\frac{1}{v} = \frac{1}{15} m$$

Let A_{image} be the amplitude of the image.

$$\Rightarrow \frac{-dv}{v^2} = -\frac{du}{u^2}$$

$$\therefore \quad dv = du \times \frac{v^2}{u^2}$$

$$\therefore \quad A_{image} = A.\frac{v^2}{u^2}$$

Substituting values,

$$A_{image} = A \times \frac{25}{225}$$

$$A_{image} = \frac{A}{9} m$$

$$A_{image} = \frac{A}{9} \times 10^{-2} m$$

$\therefore$ (d)

2. (a)

The fringe width is given by: $\beta = \frac{\lambda D}{d}$

Where, D $\rightarrow$ distance of screen from slits

d $\rightarrow$ width of the slit

Now, $\frac{\Delta\beta}{\beta}\times 100 = \frac{\Delta D}{D}\times 100 - \frac{\Delta d}{d}\times 100$

= 0.5 –(–0.3)

= 0.8%

Hence, the fringe width increases by 0.8%.

∴ (a)

3. (c)

Change in angular width, $\Delta\theta$ = 0.6 rad.

Wavelength of light is given by,

$$\lambda = \frac{c}{\nu}$$

c $\rightarrow$ Speed of light

$\nu \rightarrow$ Frequency of light

∴ Change in wavelength;

$$\Delta\lambda = \frac{c}{\nu_1} - \frac{c}{\nu_2}$$

$$= 3\times 10^8\left[\frac{1}{4\times 10^{14}} - \frac{1}{5\times 10^{14}}\right]$$

$$= 1.5\times 10^{-7}\,\text{m}$$

∴ Angular width of central maxima is,

$$\theta = \frac{2\lambda}{d}$$

$$\Delta\theta = \frac{2\Delta\lambda}{d}$$

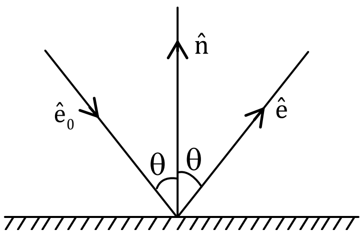
∴ $d = \frac{2\Delta\lambda}{\Delta\theta}$

∴ $d = \frac{2\times 1.5\times 10^{-7}}{0.6}$

∴ d = 5×10^{-7}m

∴ (c)

4. (b)



$\hat{e}_0 \cdot \hat{n} = 1 \times 1 \times \cos(180 - \theta)$

$\hat{e}_0 \cdot \hat{n} = -\cos\theta$ ...... (1)

$\hat{e} \cdot \hat{n} = 1 \times 1 \times \cos\theta$ ...... (2)

Equation (2) – (1)

$\hat{e} \cdot \hat{n} - \hat{e}_0 = 2\cos\theta$

$(\hat{e} - \hat{e}_0)\hat{n} = 2(-\hat{e}_0\hat{n})$

$(\hat{e} - \hat{e}_0)\hat{n} \cdot \hat{n} = -2(\hat{e}_0 \cdot \hat{n}) \cdot \hat{n}$

$\hat{e} = \hat{e}_0 - 2(\hat{e}_0 \cdot \hat{n})\hat{n}$

$\therefore$ (b)

5. (b)

When emission of β - particles is nearly constant then the number of nucleus of y at that duration is constant.

So, $\frac{dN_y}{dt} = 0$

$N_y \rightarrow$ Number of daughter nuclei at timed.

$\Rightarrow \quad \lambda_x N_x - \lambda_y N_y = \frac{dN_y}{dt}$

$\lambda_x N_x \rightarrow$ rate of α–particle emission

$\lambda_y N_y \rightarrow$ rate of β–particle emission

$\lambda_x N_x - \lambda_y N_y = 0$

$\lambda_x N_x = \lambda_y N_y = 10^7$/hours

$\therefore$ (b)

6. (d)

De Broglie wavelength

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2m(KE)}}$$

$$\lambda = \frac{h}{\sqrt{2mqv}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

$$\therefore \frac{\lambda_{proton}}{\lambda_{electron}} = \sqrt{\frac{m_{electron}}{m_{proton}}}$$

$$\frac{\lambda_p}{\lambda_e} = \sqrt{\frac{1}{2000}} = \frac{1}{20\sqrt{5}}$$

$$\therefore \lambda_p = \frac{1}{20\sqrt{5}} \times \lambda_e$$

$\therefore$ (d)

7. (d)

Angular momentum:

$$L = mr^2\omega = \frac{nh}{2\pi}$$

$$\Rightarrow \omega \propto \frac{n}{r^2}$$

$$\therefore \omega \propto \frac{n}{(r_0 n^2)^2}$$

$$\Rightarrow \omega \propto \frac{1}{n^3}$$

$\therefore$ (d)

8. (d)

The voltage across zener diode will be,

$$V = 10 \times \left(\frac{1}{1+1}\right) \Rightarrow V = 5V$$

As, the breakdown voltage of zener diode is 6V and the potential difference across the diode is only 5V, therefore it will not conduct.

$\therefore$ No current will flow

$\therefore$ (d)

9. (b)

By using Demorgen's law

$$\overline{A}\cdot\overline{B}=\overline{A+B}$$

$\therefore$ (b)

10. (b)

Impulse = charge in momentum

$\Rightarrow I=\Delta P$

$\Rightarrow [I]=[\Delta P]=[MLT^{-1}]$

$\therefore$ (b)

11. (c)

The density of the cube is

$$\rho=\frac{M}{V}$$

$M \rightarrow$ mass of cube

$V \rightarrow$ Volume of cube

Also, $V = L^3$

$$\frac{\Delta V}{V}=\frac{3\Delta L}{L}$$

$\therefore$ Maximum error in density

$$\left(\frac{\Delta\rho}{\rho}\right)_{max}=\left(\frac{\Delta m}{m}\right)+\left(\frac{\Delta V}{V}\right)$$

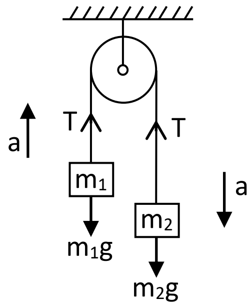
$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{max}=\left(\frac{\Delta m}{m}\right)+3\left(\frac{\Delta L}{L}\right)$$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{max}\times100=[0.3+3(0.2)]\%$$

$= 0.9\%$

$\therefore$ (c)

12. (d)



$m_2 g - T = m_2 a$ ...... (1)

$T - m_1 g = m_1 a$ ...... (2)

On solving equation (1) & (2)

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

$\therefore$ (d)

13. (c)

$$P = \frac{W}{t}$$

$P \rightarrow$ Power (Constant)

$W \rightarrow$ Work done

$t \rightarrow$ time taken

The work done is equal to change in kinetic energy

$$\therefore \; P = \frac{\Delta KE}{\Delta t} = \frac{1/2 mv^2}{t} = \frac{mv^2}{2t}$$

$$\Rightarrow V \propto \sqrt{t}$$

$$\Rightarrow \frac{ds}{dt} \propto \sqrt{t}$$

$\therefore$ $S \propto t^{3/2}$

$\therefore$ (c)

14. (c)

Let $\vec{r}_1$ and $\vec{r}_2$ be the position vector of two particles which are given by,

$$\vec{r}_1 = vt\hat{i} - \frac{1}{2}gt^2\hat{j}$$

$$\vec{r}_2 = vt(-\hat{i}) - \frac{1}{2}gt^2\hat{j}$$

$$\vec{r}_1 \cdot \vec{r}_2 = 0$$

$$\Rightarrow \quad -v^2t^2 + \frac{1}{4}g^2t^4 = 0$$

$$\therefore \quad v^2 = \frac{1}{4}g^2t^2$$

$$\therefore \quad v = \frac{gt}{2} \Rightarrow \quad t = \frac{2v}{g}$$

$\therefore$ Separation between then is

$$\Delta x = 2vt$$

$$\Delta x = 2v \times \frac{2v}{g}$$

$$\Delta x = \frac{4v^2}{g}$$

$\therefore$ (c)

15. (a)

$$T^2 \propto r^3$$

$T \rightarrow$ Time period

$r \rightarrow$ radius of orbit

$$\Rightarrow \frac{r_E}{r_p} = \left(\frac{T_E}{T_p}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{r_E}{r_p} = \left(\frac{\omega_P}{\omega_E}\right)^{\frac{2}{3}} \quad [\because \ T \propto \omega]$$

$$\Rightarrow \quad \frac{R}{r_P} = (2)^{\frac{2}{3}}$$

$$\therefore \quad r_P = (2)^{-\frac{2}{3}} R$$

$\therefore$ (a)

16. (c)

$V = A\ell$

$\Rightarrow V = ab\ell$

$A \rightarrow$ area of cross section of rectangle of sides a and b.

$\ell \rightarrow$ length

Poison's ratio is given by

$$\sigma = -\frac{\Delta a / a}{\Delta \ell / \ell}$$

$$\sigma = -\frac{\Delta b / b}{\Delta \ell / \ell}$$

$$\therefore \frac{\Delta a}{a} = \frac{\Delta b}{b}$$

$$\Rightarrow \frac{\Delta v}{v} = \frac{2\Delta a}{a} + \frac{\Delta \ell}{\ell}$$

$$= -2\sigma \frac{\Delta \ell}{\ell} + \frac{\Delta \ell}{\ell}$$

$$= \frac{\Delta \ell}{\ell}(1 - 2\sigma)$$

$$= -1(1 - 2 \times 0.2)$$

$$= -0.6$$

$\therefore$ Volume decreases by 0.6%

$\therefore$ (c)

17. (c)

Body is moving with terminal velocity, it means net force acting on the body is zero.

18. (c)

Thermal energy radiated per second by a block body is,

$$Q = \sigma T^4$$

$$\therefore \frac{Q_1}{Q_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$= \left(\frac{273 + 27}{273 + 177}\right)^4$$

$$= \left(\frac{300}{450}\right)^4$$

$$= \left(\frac{2}{3}\right)^4$$

$$= \frac{16}{81}$$

$\therefore$ (c)

19. (b)

Molar heat capacity of a gas; $C_V = \frac{fR}{2}$

$R \rightarrow$ Universal gas constant

$f \rightarrow$ Degree of freedom

for diatomic gas; f = 5

$\therefore$ $C_v = \frac{5R}{2}$

$\therefore$ (b)

20. (a)

When there is free expansion.

$\Delta u = 0$

As $\Delta u = nC_V \Delta T$

$\Rightarrow$ $\Delta T = 0$

$\Rightarrow$ $T_i = T_f = T$

$\therefore$ (a)

21. (d)

Given condition is of balanced wheat stone bridge.

$\therefore$ $C_{eq} = 20\mu F$

$\therefore$ Amount of stored charge is

$Q = C_{eq}. V$

$= 20 \times 10^{-6} \times 150$

$= 3 \times 10^{-3} C$

$\therefore$ (d)

22. (a)

Had all the hour positions were occupied by +Q charges, then the net electric field strength at the centre of clock would have been zero, as due to symmetry of charge positions they would have cancelled electric field due to each other.

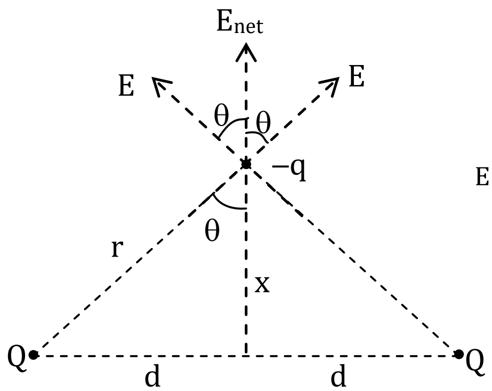
$\therefore$ $E_1 + E_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots E_{11} + E_{12} = 0$

$\therefore$ $E_1 + E_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots E_{11} + E_{12} = -E_{10}$

$\therefore$ The electric field due to point charge at 10th hour positions will be $\frac{Q^2}{4\pi\varepsilon_0}$ and the direction is from the centre towards the mark 10

$\therefore$ (a)

23. (a)



$$E = \frac{KQ}{r^2}$$

$$E_{net} = \frac{2KQ}{r^2}\cos\theta$$
$$= \frac{2KQ}{r^2} \times \frac{x}{r}$$
$$= \frac{2KQx}{r^3}$$

$\therefore$ Force; $\quad F = \frac{-2KQqx}{r^3} \quad \left\{r = \sqrt{d^2 + x^2}\right\}$

$\therefore \; F = \frac{-2KQqx}{(d^2 + x^2)^{3/2}}$

As; $\quad x <<< d;$

$\Rightarrow F = \frac{-2KQqx}{d^3}$

$\therefore \; F \propto x$

$\therefore$ (a)

24. (d)

The radius R of the circular path of a charged particle q in uniform magnetic field B is given by

$\therefore \; R = \frac{mv}{qB} = \frac{p}{qB} \quad \{\because \; p = mv\}$

$m \rightarrow$ mass of charged particle

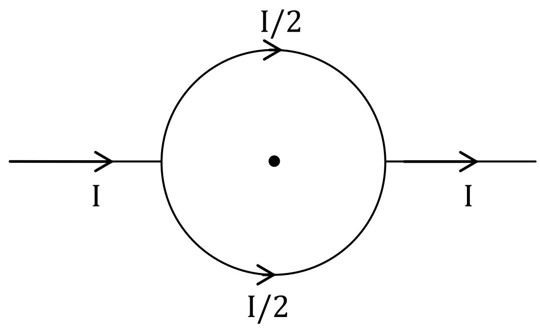
$v \rightarrow$ velocity of charged particle

$p \rightarrow$ momentum of charged particle

$\therefore \; R \propto p$

$\therefore$ (d)

25. (d)



Net magnetic field at the centre will be zero, as the magnitude of the magnetic field due to each half circle will be same but opposite in direction.

Now; $\vec{F} = q(\vec{V} \times \vec{B})$

$\vec{F} = q(\vec{V} \times 0)$

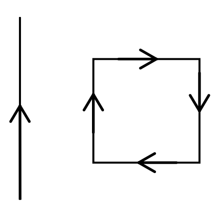
$\therefore \ \vec{F} = 0$

$\therefore$ (d)

26. (G)

Bonus

27. (b)

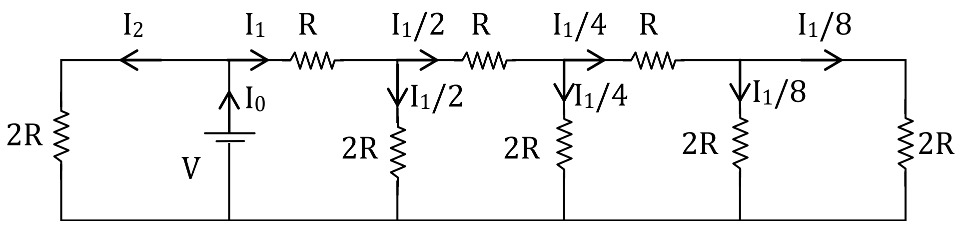


Induced current in the loop will be in clockwise direction, so it will move towards the wire.

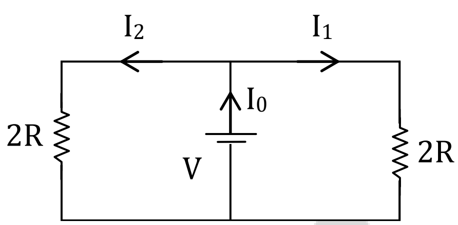
$\therefore$ (b)

28. (C)

The given circuit can be redrawn as:



Above circuit can be redrawn as:



$$I_1 = \frac{V}{2R}$$

$$\therefore \quad \frac{I_1}{8} = \frac{V}{16R}$$

$\therefore$ (c)

29. (d)

For a balanced wheat stone bridge

$$\frac{P}{Q} = \frac{R}{S}$$

As the values of P& Q are kept constant so, their ratio will remain constant

$$\therefore \quad \frac{P}{Q} = \frac{R}{S} = \frac{5}{S} = \frac{7}{S+3}$$

$\therefore$ 5S + 15 = 7S

$\therefore$ 2S = 15

$\therefore$ S = 7.5Ω

$\therefore$ (d)

30. (a)
When inductor is connected with resistance

$\therefore$ $\tan 60^o = \dfrac{\omega L}{R}$ ......(1)

When capacitor is connected with resistance

$\therefore$ $\tan 30^o = \dfrac{1/\omega c}{R} = \dfrac{1}{\omega c R}$ ......(2)

$\therefore$ Dividing equation (1) by (2)

$$\frac{\tan 60^o}{\tan 30^o} = \frac{\omega L}{R} \times \frac{\omega c R}{1}$$

$$\Rightarrow \frac{\sqrt{3}}{1/\sqrt{3}} = \omega^2 Lc$$

$$3 = \omega^2 \times 60 \times 10^{-3} \times 0.5 \times 10^{-6}$$

$$\omega = 10^4$$

$\therefore$ $f = \dfrac{\omega}{2\pi}$

$\therefore$ $f = \dfrac{10^4}{2\pi}\,\text{Hz}$

$\therefore$ (a)

31. (c)
Angular frequency at resonance is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow c = \frac{1}{\omega_0^2 L}$$

Also $\omega_0 = 2\pi f$

$$\Rightarrow \omega_0 = 2\pi \times \frac{1250}{\pi}$$

$$\Rightarrow \omega_0 = 2500\,\text{rad}$$

$$\Rightarrow C = \frac{1}{2500 \times 2500 \times 20 \times 10^{-3}}$$

$\Rightarrow C = 8 \times 10^{-6}$ F

$\therefore$ Charge stored in each plate

$Q = CV$

$Q = 8 \times 10^{-6} \times 25 = 0.2 \times 10^{-3}$ C

$\therefore$ $Q = 0.2$ mC

$\therefore$ (c)

32. (a)

Energy stored in capacitor = $\frac{1}{2}CE^2$

Energy supplied by the source of emf = CE^2

$\therefore$ Energy dissipated in resistance $= CE^2 - \frac{1}{2}CE^2$

$= \frac{1}{2}CE^2$

$\therefore$ (a)

33. (a)

$\frac{dV}{dt} = A \cdot v$

$A \rightarrow$ Area of cross section of the nozzle

$V \rightarrow$ Volume

$v \rightarrow$ velocity

$\therefore \quad \frac{1}{10} = \frac{5}{\sqrt{21}} \times 10^{-3}.V$

$\Rightarrow \quad V = 20\sqrt{21}$ m/s

Now, $\frac{1}{2}mv^2 = ms\Delta T$

$\Rightarrow \quad \Delta T = \frac{V^2}{2S}$

$S \rightarrow 4.2 \times 10^3$ J/kg

$\Rightarrow \quad \Delta T = 1^{o}C$

$\therefore$ (a)

34. (a)

By energy conservation

$2 \times \frac{1}{2}mu^2 = 2 \times [ms\Delta T + mL]$

$\Rightarrow \quad u^2 = 2[S\Delta T + L]$

$S \rightarrow$ Specific heat capacity of ice = 2100 J/kgK

$L \rightarrow$ Latent heat of fusion of ice = 336 KJ/kg

$\therefore \quad u^2 = 2[2100 \times (8) + 336000]$

$\therefore$ u = 840 m/s

$\therefore$ (a)

35. (a)

Let the charge on the particle is q.

$\therefore$ Force on particle due to electric field,

$F_E = qE$

& force on particle due to magnetic field,

$F_B = qVB$

$\Rightarrow$ As the particle moved undeflected in its original trajectory then

$qE = qVB$

$\therefore \quad V = \frac{E}{B}$

$\therefore$ (a)

36. (b,d)

Emf is induced when, there is a change in flux with respect to time. Out of the given options. Only in case of option. B & D flux will charge.

$\therefore$ (b,d)

37. (b,d)

Kinetic energy of photoelectrons emitted.

$T = h\nu - \phi$

$\Rightarrow T \propto \nu$

Also, $J = \frac{dN}{dt}(h\nu)$

$\Rightarrow J \propto \nu$

$\Rightarrow T \propto J$

$\therefore$ (b,d)

38. (b)

As the process is adiabatic

$\therefore\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\therefore\ T_0V_0^{\gamma-1} = T\left(\frac{V_0}{2}\right)^{\gamma-1}$

$\therefore\ T = T_0(2)^{\gamma-1}$

Also change in internal energy in this process is,

$\Delta Q = nCv\Delta T$

Also, $\Delta u = n\frac{R}{r-1}\left(2^{\gamma-1}T_0 - T_0\right)$

$\therefore\ \Delta u = \frac{nRT_0}{r-1}\left(2^{\gamma-1} - 1\right)$

$\therefore\ \Delta U = \frac{P_0V_0}{r-1}\left(Z^{\gamma-1} - 1\right)$

As the piston is fixed so there is no work done

$\therefore$ Energy released to surrounding will be equal to Δu.

$\therefore\ \Delta Q = \Delta u = \frac{P_0V_0}{r-1}\left(Z^{\gamma-1} - 1\right)$

$\therefore$ (b)

39. (a,d)

Range, $R = \frac{u^2 \sin 2\alpha}{g}$

$\Rightarrow 5 = \frac{100 \times \sin 2\alpha}{10}$

$\Rightarrow \sin(2\alpha) = \frac{1}{2}$

$\Rightarrow 2\alpha = 30^o$

$\therefore\ \alpha = 15^o$

Also, $\sin(2\alpha) = \frac{1}{2}$

$\Rightarrow\quad 2\alpha = 150^o$

$\therefore\ \alpha = 75^o$

$\therefore$ (a,d)

40. (a,c)

For balanced Wheat stone bridge ratio of adjacent side should be same

$\therefore$ (a,c)