WBJEE

BONG MOTION

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WBJEE - CHAPTER WISE: (PYQ)

Vector Algebra.

Q8.
The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}-3\hat{j}+2\hat{k}$, $\hat{i}-\hat{j}+2\hat{k}$ and $4\hat{i}-\hat{j}-\lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, the value of λ is- $\boxed{2019}$

(A) 0 (B) 1 (C) 2 (D) 4

Solution:

$\vec{AB} = -\hat{i}-\hat{j}+3\hat{k}$

$\vec{AC} = -2\hat{i}+\hat{j}+3\hat{k}$

$\vec{AD} = \hat{i}+\hat{j}+(1-\lambda)\hat{k}$

$[\vec{AB}\ \vec{AC}\ \vec{AD}] = 0$

$\Rightarrow \begin{vmatrix} -1 & -1 & 3 \\ -2 & 1 & 3 \\ 1 & 1 & (1-\lambda) \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 0 & 4-\lambda \\ -2 & 1 & 3 \\ 1 & 1 & (1-\lambda) \end{vmatrix} = 0 \quad (R_1' = R_1 + R_3)$

$\Rightarrow (4-\lambda)\begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$\Rightarrow -3(4-\lambda) = 0 \Rightarrow \lambda = 4$

$\vec{a}\cdot(\vec{b}\times\vec{c})$
$= [\vec{a}\ \vec{b}\ \vec{c}]$
$= 0$

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Vector Algebra.

Q9.
Let $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ be three unit vectors, such that $\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma}) = \frac{1}{2}(\hat{\beta}+\hat{\gamma})$ where $\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma}) = (\hat{\alpha}\cdot\hat{\gamma})\hat{\beta} - (\hat{\alpha}\cdot\hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is —

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ $\boxed{2019}$

Solution: $|\hat{\alpha}| = |\hat{\beta}| = |\hat{\gamma}| = 1$

$$\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma}) = \frac{1}{2}(\hat{\beta}+\hat{\gamma})$$

$$\Rightarrow (\hat{\alpha}\cdot\hat{\gamma})\vec{\beta} - (\hat{\alpha}\cdot\hat{\beta})\vec{\gamma} = \frac{1}{2}\hat{\beta} + \frac{1}{2}\hat{\gamma}$$

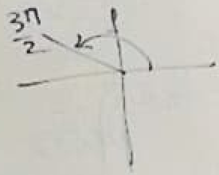
$$\therefore -(\hat{\alpha}\cdot\hat{\beta}) = \frac{1}{2}$$

$$\Rightarrow -|\hat{\alpha}||\hat{\beta}|\cos\theta = \frac{1}{2}$$

$$\Rightarrow -\cos\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = -\frac{1}{2} = \cos\left(\frac{3\pi}{2}\right)$$

$$\therefore \theta = \frac{3\pi}{2}.$$



WBJEE - CHAPTER WISE: (PYQ)

Vector Algebra.

Q10. If the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$ and $\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is —

$\boxed{2020}$

(A) 1 (B) 0 (C) −1 (D) 2

Solution: $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] \neq 0$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$$\Rightarrow \Delta \neq 0$$

এখন, $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \Delta \cdot (1 + abc) = 0$$

$$\therefore 1 + abc = 0 \Rightarrow abc = -1$$

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Vector Algebra.

Q11.
The unit vector in ZOX, making angles 45° and 60° respectively with $\vec{\alpha} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{j} - \hat{k}$ is—

(A) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ (B) ✓ $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$ (C) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$ (D) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$ $\boxed{2020}$

Solution: Let $\vec{\delta}$ be the unit vector in ZOX Plane

Let, $\vec{\delta} = a\hat{i} + 0\hat{j} + c\hat{k}$, $|\vec{\delta}| = 1$

$\vec{\alpha} \cdot \vec{\delta} = |\vec{\alpha}||\vec{\delta}|\cos 45^\circ = \sqrt{4+4+1} \cdot 1 \cdot \frac{1}{\sqrt{2}}$

$\Rightarrow 2a - c = \frac{3}{\sqrt{2}}$ —— (1)

$\vec{\beta} \cdot \vec{\delta} = |\vec{\beta}||\vec{\delta}|\cos 60^\circ = \sqrt{2} \cdot 1 \cdot \frac{1}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow -c = \frac{1}{\sqrt{2}}$

$\Rightarrow c = -\frac{1}{\sqrt{2}}$

From (1) $2a + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$\Rightarrow 2a = +\frac{2}{\sqrt{2}}$

$\Rightarrow a = \frac{1}{\sqrt{2}}$

$\vec{\delta} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$.

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Vector Algebra.

Q12.
If $a(\vec{\alpha}\times\vec{\beta}) + b(\vec{\beta}\times\vec{\gamma}) + c(\vec{\gamma}\times\vec{\alpha}) = \vec{0}$, where a, b, c are non-zero scalars, then the vectors $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are —

(2021)

(A) parallel (B) non-parallel ✓(C) coplanar (D) mutually perpendicular

Solution:

$$a(\vec{\alpha}\times\vec{\beta}) + b(\vec{\beta}+\vec{\gamma}) + c(\vec{\gamma}\times\vec{\alpha}) = \vec{0}$$

$$\Rightarrow a(\vec{\alpha}\times\vec{\beta}) + b(\vec{\beta}\times\vec{\gamma}) = -c(\vec{\gamma}\times\vec{\alpha})$$

$$\Rightarrow (a\vec{\alpha}\times\vec{\beta}) - (b\vec{\gamma}\times\vec{\beta}) = -c(\vec{\gamma}\times\vec{\alpha})$$

$$\Rightarrow (a\vec{\alpha} - b\vec{\gamma})\times\vec{\beta} = c(\vec{\alpha}\times\vec{\gamma})$$

$$\Rightarrow \vec{\beta}\cdot[(a\vec{\alpha} - b\vec{\gamma})\times\vec{\beta}] = \vec{\beta}\cdot c(\vec{\alpha}\times\vec{\gamma})$$

$$\Rightarrow [\vec{\beta}\ (a\vec{\alpha} - b\vec{\gamma})\ \vec{\beta}] = c[\vec{\beta}\ \vec{\alpha}\ \vec{\gamma}]$$

$$\Rightarrow 0 = -c[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]$$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = 0$, so $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are coplanar.

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Vector Algebra.

Q13.
Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{\alpha}+3\vec{\beta}$ is collinear with $\vec{\gamma}$ and $\vec{\beta}+2\vec{\gamma}$ is collinear with $\vec{\alpha}$, then $\vec{\alpha}+3\vec{\beta}+6\vec{\gamma}$ is —

(2021)

(A) $\vec{\gamma}$ (B) $\vec{0}$ (C) $\vec{\alpha}+\vec{\gamma}$ (D) $\vec{\alpha}$

Solution: $(\vec{\alpha}+3\vec{\beta}) = m\vec{\gamma}$ $\Rightarrow \vec{\alpha}+3\vec{\beta}+6\vec{\gamma} = m\vec{\gamma}+6\vec{\gamma} = (m+6)\vec{\gamma}$

$(\vec{\beta}+2\vec{\gamma}) = n\vec{\alpha}$

$\Rightarrow 3\vec{\beta}+6\vec{\gamma} = 3n\vec{\alpha}$ $\Rightarrow \vec{\alpha}+3\vec{\beta}+6\vec{\gamma} = \vec{\alpha}+3n\vec{\alpha} = (1+3n)\vec{\alpha}$

$(m+6)=0$ and $(1+3n)=0$

$$\vec{\alpha}+3\vec{\beta}+6\vec{\gamma} = 0\cdot\vec{\gamma} = \vec{0}$$

WBJEE - CHAPTER WISE: (PYQ)

Vector Algebra.

Q14.
If $\vec{\alpha}$ is a unit vector, $\vec{\beta} = \hat{i}+\hat{j}-\hat{k}$, $\vec{\gamma} = \hat{i}+\hat{k}$, then the maximum value of $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]$ is —

(2022)

(A) 3 (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{6}$ ✓

Solution: $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \vec{\alpha}\cdot(\vec{\beta}\times\vec{\gamma})$

$= \vec{\alpha}\cdot\vec{\delta}$

$= |\vec{\alpha}||\vec{\delta}|\cos\theta.$

$= 1\cdot\sqrt{6}\cdot(\cos\theta)_{max}$

$= 1\cdot\sqrt{6}\cdot 1$

$= \sqrt{6}$

where, $\vec{\delta} = \vec{\beta}\times\vec{\gamma}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix}$$

$= \hat{i}(1-0) - \hat{j}(1+1) + \hat{k}(0-1)$

$= \hat{i} - 2\hat{j} - \hat{k}$

$|\vec{\delta}| = \sqrt{1+1+4} = \sqrt{6}$

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Vector Algebra.

Q15.
If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ is a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

(2022)

(A) $\pm \frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} + \hat{k})$ (B) $\pm \frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ (C) $\pm \frac{1}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ (D) $\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$.

Solution:

$\vec{d}$ perpendicular to a plane which is made by $\vec{a}$, $\vec{b}$ and $\vec{c}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(1-1) - \hat{j}(1+1) + \hat{k}(-1-1)$$

$$= -2\hat{j} - 2\hat{k}$$

$$= -2(\hat{j} + \hat{k})$$

$$\vec{b} \times \vec{a} = +2(\hat{j} + \hat{k})$$

$\vec{d}$ $||^{el}$ to $\pm 2(\hat{j} + \hat{k})$

$$\therefore \vec{d} = \frac{\pm 2(\hat{j} + \hat{k})}{\sqrt{8}} = \pm \frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$$

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Vector Algebra.

Q16. If the volume of the parallelopiped with $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}$ as coterminous edges is 9 cu. units., then the volume of the parallelopiped with $\underset{A}{(\vec{a}\times\vec{b})}\times\underset{B}{(\vec{b}\times\vec{c})}$, $\underset{B}{(\vec{b}\times\vec{c})}\times\underset{C}{(\vec{c}\times\vec{a})}$ and $\underset{C}{(\vec{c}\times\vec{a})}\times\underset{A}{(\vec{a}\times\vec{b})}$ as coterminous edges is — (2023)

(A) 9 cu. units (B) 729 cu. units (C) 81 cu. units (D) 243 cu. units.

Solution: $\vec{a}, \vec{b}, \vec{c}$, Volume $= [\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore [\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}] = 9$ cu. units $= [\vec{a}\ \vec{b}\ \vec{c}]^2$ — ①

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 3$ cu units.

$[A\ B\ C] = 9$

$\therefore [A\times B\quad B\times C\quad C\times A] = [A\ B\ C]^2 = 9^2$ cu. units.

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Vector Algebra.

Q 17.
The value of 'a' for which the scalar triple product formed by the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{\beta} = \hat{j} + a\hat{k}$ and $\vec{\gamma} = a\hat{i} + \hat{k}$ is maximum, is —

(2023)

(A) 3 (B) -3 (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

Solution: $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1(1-0) + a(a^2-1)$

$= 1 + a^3 - a = \Delta$

$\frac{d\Delta}{da} = 0 \Rightarrow 0 + 3a^2 - 1 = 0$

$\Rightarrow a^2 = \frac{1}{3}$

$\Rightarrow a = \pm\frac{1}{\sqrt{3}}$

$\frac{d^2\Delta}{da^2} < 0 \Rightarrow \frac{d}{da}(3a^2-1) < 0 \Rightarrow 6a < 0$

$\therefore a = -\frac{1}{\sqrt{3}}$