WB-JEE-2016 (Mathematics)

1. Let A and B be two events such that $P(A \cap B) = \frac{1}{6}$, $P(A \cup B) = \frac{31}{45}$ and $P(\overline{B}) = \frac{7}{10}$ then
   a. A and B are independent
   b. A and B are mutually exclusive
   c. $P\left(\frac{A}{B}\right) < \frac{1}{6}$
   d. $P\left(\frac{B}{A}\right) < \frac{1}{6}$

2. The value of $\cos 15° \cos 7\frac{1°}{2} \sin 7\frac{1°}{2}$ is
   a. $\frac{1}{2}$
   b. $\frac{1}{8}$
   c. $\frac{1}{4}$
   d. $\frac{1}{16}$

3. The smallest positive root of the equation $\tan x - x = 0$ lies in
   a. $(0, \pi/2)$
   b. $(\pi/2, \pi)$
   c. $\left(\pi, \frac{3\pi}{2}\right)$
   d. $\left(\frac{3\pi}{2}, 2\pi\right)$

4. If in a triangle ABC, AD, BE and CF are the altitudes and R is the circumradius, then the radius of the circumcircle of ΔEF is
   a. $\frac{R}{2}$
   b. $\frac{2R}{3}$
   c. $\frac{1}{3}R$
   d. None of these

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5. The points $(-a, -b)$, (a, b), $(0, 0)$ and (a^2, ab), $a \neq 0$, $b \neq 0$ always lie on this line. Hence, collinear
   a. collinear
   b. vertices of a parallelogram
   c. vertices of a rectangle
   d. lie on a circle

6. The line AB cuts off equal intercepts 2a from the axes. From any point P on the line AB perpendiculars PR and PS are drawn on the axes. Locus of mid-point of RS is
   a. $x - y = \frac{a}{2}$
   b. $x + y = a$
   c. $x^2 + y^2 = 4a^2$
   d. $x^2 - y^2 = 2a^2$

7. $x + 8y - 22 = 0$, $5x + 2y - 34 = 0$, $2x - 3y + 13 = 0$ are the three sides of a triangle. The area of the triangle is
   a. 36 square unit
   b. 19 square unit
   c. 42 square unit
   d. 72 square unit

8. The line through the points (a, b) and $(-a, -b)$ passes through the point
   a. $(1, 1)$
   b. $(3a, -2b)$
   c. (a^2, ab)
   d. (a, b)

9. The locus of the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = K$ and $\frac{x}{a} - \frac{y}{b} = \frac{1}{k}$, where k is a non-zero real variable, is given by
   a. a straight line
   b. an ellipse
   c. a parabola
   d. a hyperbola

10. The equation of a line parallel to the line $3x + 4y = 0$ and touching the circle $x^2 + y^2 = 9$ in the first quadrant is
    a. $3x + 4y = 15$
    b. $3x + 4y = 45$
    c. $3x + 4y = 9$
    d. $3x + 4y = 27$

11. A line passing through the point of intersection of $x+y=4$ and $x-y=2$ makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the x-axis. It intersects the parabola $y^2=4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1-x_2|$ is equal to
    a. $\frac{16}{9}$
    b. $\frac{32}{9}$
    c. $\frac{40}{9}$
    d. $\frac{80}{9}$

12. Then equation of auxiliary circle of the $16x^2+25y^2+32x-100y=284$ is
    a. $x^2 + y^2 + 2x - 4y - 20 = 0$
    b. $x^2 + y^2 + 2x - 4y = 0$
    c. $(x+1)^2+(y-2)^2 = 400$
    d. $(x+1)^2+(y-2)^2=225$

13. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOPQ is equilateral, O being the centre. Then the eccentricity e satisfies
    a. $1 < e < \frac{2}{\sqrt{3}}$
    b. $e = \frac{2}{\sqrt{2}}$
    c. $e = \frac{\sqrt{3}}{2}$
    d. $e > \frac{2}{\sqrt{3}}$

14. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines; $x+y=3$ and $2x+2y-1=0$ then
    a. $2<a<4$
    b. $-\frac{1}{2}<a<2$
    c. $0<a<2$
    d. $-\frac{1}{2}<a<\frac{3}{2}$

15. A straight line joining the points (1,1,1) and (0,0,0) intersects the plane $2x+2y+z=10$ at
    a. (1, 2, 5)
    b. (2, 2, 2)
    c. (2, 1, 5)
    d. (1, 1, 6)

16. Angle between the planes $x+y+2z=6$ and $2x-y+z=9$ is
    a. $\frac{\pi}{4}$
    b. $\frac{\pi}{6}$
    c. $\frac{\pi}{3}$
    d. $\frac{\pi}{2}$

17. If $y=(1+x)(1+x^2)(1+x^4)........(1+x^{2n})$ then the value of $\left(\frac{dy}{dx}\right)$ at $x = 0$ is
    a. 0
    b. −1
    c. 1
    d. 2

18. If f(x) is an odd differentiable function defined on $(-\infty,\infty)$ such that $f'(3) = 2$, then $f'(-3)$ equal to
    a. 0
    b. 1
    c. 2
    d. 4

19. $\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{(1-\sqrt{x})}{(1-x)}}$

a. is 1
b. does not exist
c. is $\sqrt{\frac{2}{3}}$
d. is/n 2

20. If $f(x)=\tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right]+\tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$ then the value of f'' (x) is

a. x^2
b. x
c. 1
d. 0

21. $\int\frac{\log\sqrt{x}}{3x}\,dx$ is equal to

a. $\frac{1}{3}\left(\log\sqrt{x}\right)^2+c$
b. $\frac{2}{3}\left(\log\sqrt{x}\right)^2+c$
c. $\frac{2}{3}(\log x)^2+c$
d. $\frac{1}{3}(\log x)^2+c$

22. $\int 2^x(f'(x)+f(x)\log 2)$ is equal to

a. $2^x f'(x)+c$
b. $2^x\log 2+c$
c. $2^x f(x)+c$
d. 2^x+c

23. $\int_0^1 \log\left(\frac{1}{x}-1\right)dx =$

a. 1
b. 0
c. 2
d. None of these

24. The value of $\underset{n\to\infty}{\mathrm{lt}}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+\ldots+\sqrt{2n-1}}{n^{\frac{3}{2}}}\right\}$ is

a. $\frac{2}{3}\left(2\sqrt{2}-1\right)$
b. $\frac{2}{3}\left(\sqrt{2}-1\right)$
c. $\frac{2}{3}\left(\sqrt{2}+1\right)$
d. $\frac{2}{3}\left(2\sqrt{2}+1\right)$

25. If the solution of the differential equation $x\frac{dy}{dx}+y=xe^x$ be, $xy = e^x\varphi(x) + c$ then $\varphi(x)$ is equal to

a. x+1
b. x–1
c. 1–x
d. x

26. The order of the differential equation of all parabolas whose axis of symmetry along x-axis is

a. 2
b. 3
c. 1
d. None of these

27. The line $y = x + \lambda$ is tangent to the ellipse $2x^2 + 3y^2 = 1$. Then λ is
    a. -2
    b. 1
    c. $\sqrt{\frac{5}{6}}$
    d. $\sqrt{\frac{2}{3}}$

28. The area enclosed by $y = \sqrt{5 - x^2}$ and $y = |x-1|$ is
    a. $\left(\frac{5\pi}{4} - 2\right)$ sq.units
    b. $\frac{5\pi - 2}{2}$ sq.units
    c. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq.units
    d. $\left(\frac{\pi}{2} - 5\right)$ sq.units

29. Let S be the set of points whose abscissas and ordinates are natural numbers. Let $P \in S$ such that the sum of the distance of P from (8,0) and (0,12) is minimum among all elements in S. Then the number of such points P in S is
    a. 1
    b. 3
    c. 5
    d. 11

30. Time period T of a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. If the length is increased by 2%, then an approximate change in the time period is
    a. 2%
    b. 1%
    c. $\frac{1}{2}$%
    d. None of these

31. The cosine of the angle between any two diagonals of a cube is

a. $\frac{1}{3}$

b. $\frac{1}{2}$

c. $\frac{2}{3}$

d. $\frac{1}{\sqrt{3}}$

32. If x is a positive real number different from 1 such that $\log_a x$, $\log_b x$, $\log_c x$ are in A.P., then

a. $b = \frac{a+c}{2}$

b. $b = \sqrt{ac}$

c. $c^2 = (ac)^{\log_a b}$

d. None of (A), (B), (C) are correct

33. If a, x are real numbers and $|a| < 1$, $|x| < 1$, then $1 + (1+a)x + (1+a+a^2)x^2 + \ldots.\infty$ is equal to

a. $\frac{1}{(1-a)(1-ax)}$

b. $\frac{1}{(1-a)(1-x)}$

c. $\frac{1}{(1-x)(1-ax)}$

d. $\frac{1}{(1+ax)(1-a)}$

34. if $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

a. $(2, \infty)$

b. $(1, 2)$

c. $(-2, -1)$

d. None of these

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35. The value of $\sum_{n=1}^{13}(i^n + i^{n+1}), i = \sqrt{-1}$, is
   a. i
   b. i−1
   c. 1
   d. 0

36. If z_1, z_2, z_3 are imaginary numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ is
   a. Equal to 1
   b. Less than 1
   c. Greater than 1
   d. Equal to 3

37. If p, q are the roots of the equation $x^2 + px + q = 0$, then
   a. $p = 1, q = -2$
   b. $p = 0, q = 1$
   c. $p = -2, q = 0$
   d. $p = -2, q = 1$

38. The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) are
   a. Three
   b. Two
   c. Infinitely many
   d. No values of k satisfies the requirement

39. The number of ways in which the letters of the word ARRANGE can be permuted such that the R's occur together is
   a. $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$
   b. $\frac{\lfloor 7}{\lfloor 2}$
   c. $\frac{\lfloor 6}{\lfloor 2}$
   d. $\lfloor 5 \times \lfloor 2$

40. If $\frac{1}{^5C_r}+\frac{1}{^6C_r}=\frac{1}{^4C_r}$, then the value of r equals to

a. 4

b. 2

c. 5

d. 3

41. For +ve integer n, $n^3 + 2n$ is always divisible

a. 3

b. 7

c. 5

d. 6

42. In the expansion of (x – 1) (x – 2) .... (x – 18), the coefficient of x^{17} is

a. 684

b. –171

c. 171

d. –342

43. $1+ {}^nC_1 \cos\theta + {}^nC_2 \cos 2\theta+.......+ {}^nC_n \cos n\theta$ equals

a. $\left(2\cos\frac{\theta}{2}\right)^n \cos\frac{n\theta}{2}$

b. $2\cos^2\frac{n\theta}{2}$

c. $2\cos^{2n}\frac{\theta}{2}$

d. $\left(2\cos 2\frac{\theta}{2}\right)^n$

44. If x, y and z be greater than 1, then the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

a. log x. logy. log z

b. log x + logy + log z

c. 0

d. 1 – {(log x).(logy).(logz)}

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45. Let A is a 3 × 3 matrix and B is its adjoint matrix. If |B| = 64, then |A| =
    a. ±2
    b. ±4
    c. ±8
    d. ±12

46. Let $Q = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ then Q^3x is equal to
    a. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
    b. $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
    c. $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
    d. $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

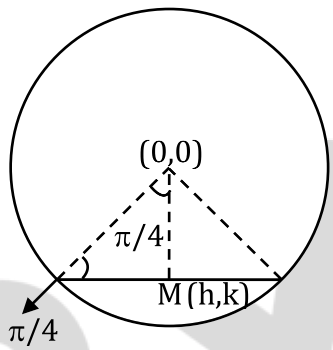
47. Let R be a relation defined on the set Z of all integers and xRy when x + 2y is divisible by 3. Then
    a. R is not transitive
    b. R is symmetric only
    c. R is an equivalence relation
    d. R is not an equivalence relation

48. If $A = \{5^n - 4n - 1 : n \in N\}$ and $B = \{16(n-1) : n \in N\}$, then
    a. A = B
    b. $A \cap B = \phi$
    c. $A \subseteq B$
    d. $B \subseteq A$

49. If the function f : R→R is defined by $f(x) = (x^2+1)^{35} \forall x \in R$, then f is
   a. one-one but not onto
   b. onto but not one-one
   c. neither one-one nor onto
   d. both one-one and onto

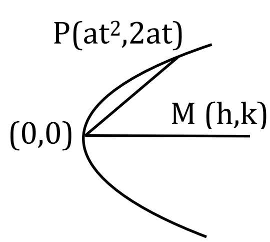
50. Standard Deviation of n observations $a_1, a_2, a_3 .....a_n$ is σ. Then the standard deviation of the observations $\lambda a_1, \lambda a_2 ...... \lambda a_n$ is
   a. $\lambda\sigma$
   b. $-\lambda\sigma$
   c. $|\lambda|\sigma$
   d. $\lambda^n\sigma$

51. The locus of the midpoints of chords of the circle $x^2+y^2 =1$ which subtends a right angle at the origin is



   a. $x^2 + y^2 = \frac{1}{4}$
   b. $x^2 + y^2 = \frac{1}{2}$
   c. $xy = 0$
   d. $x^2 - y^2 = 0$

52. The locus of the midpoints of all chords of the parabola $y^2 = 4ax$ through its vertex is another parabola with directrix



   a. $x = -a$
   b. $x = a$
   c. $x = 0$
   d. $x = -\frac{a}{2}$

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53. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2[x]dx$ equals

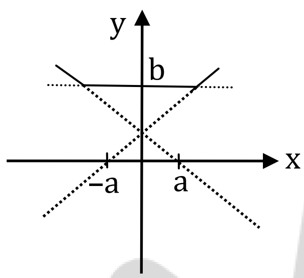
   a. $\frac{5}{3}$

   b. $\frac{7}{3}$

   c. $\frac{8}{3}$

   d. $\frac{4}{3}$

54. The number of points at which the function f(x) = max {a − x, a + x, b}, $-\infty < x < \infty$, $0 < a < b$ cannot be differentiable



   a. 0
   b. 1
   c. 2
   d. 3

55. For non-zero vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

   a. Collinear

   b. Perpendicular to each other

   c. Inclined at an acute angle

   d. Inclined at an obtuse angle

56. General solution of $y\frac{dy}{dx} + by^2 = a\cos x$, $0 < x < 1$ is Here c is an arbitrary constant

   a. $y^2 = 2a(2b\sin x + \cos x) + ce^{-2bx}$

   b. $(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$

   c. $(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{2bx}$

   d. $y^2 = 2a(2b\sin x + \cos x) + ce^{-2bx}$

57. The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abscissa increases is/are given by

a. $\left(3, \frac{16}{3}\right) \& \left(-3, \frac{-16}{3}\right)$

b. $\left(3, \frac{-16}{3}\right) \& \left(-3, \frac{16}{3}\right)$

c. $\left(\frac{1}{16}, \frac{1}{9}\right) \& \left(-\frac{1}{16}, \frac{1}{9}\right)$

d. $\left(\frac{1}{16}, \frac{1}{9}\right) \& \left(-\frac{1}{16}, \frac{1}{9}\right)$

58. The letters of the word COCHIN are permuted and all permutation are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

a. 96
b. 48
c. 183
d. 267

59. If the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$, then $A^n = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{pmatrix}$, $n \in N$ where

a. $a = 2n, b = 2^n$
b. $a = 2^n, b = 2n$
c. $a = 2^n, b = n2^{n-1}$
d. $a = 2^n, b = n2^n$

60. The sum of n terms of the following series; $1^3 + 3^3 + 5^3 + 7^3 + \ldots$ is

a. $n^2(2n^2 - 1)$
b. $n^3(n - 1)$
c. $n^3 + 8n + 4$
d. $2n^4 + 3n^2$

61. If α and β are roots of $ax^2 + bx + c = 0$ then the equation whose roots are α^2 and β^2 is

a. $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$
b. $a^2x^2 + (b^2 - ac)x + c^2 = 0$
c. $a^2x^2 + (b^2 + ac)x + c^2 = 0$
d. $a^2x^2 + (b^2 + 2ac)x + c^2 = 0$

62. If ω is an imaginary cube root of unity, then the value of
$(2-\omega)(2-\omega^2) + 2(3-\omega)(3-\omega^2) + \ldots\ldots + (n-1)(n-\omega)(n-\omega^2)$ is

a. $\frac{n^2}{4}(n+1)^2 - n$

b. $\frac{n^2}{4}(n+1)^2 + n$

c. $\frac{n^2}{4}(n+1)^2$

d. $\frac{n^2}{4}(n+1) - n$

63. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$ then the value of ${}^nC_8$ is

a. 10

b. 7

c. 9

d. 8

64. In a group 14 males and 6 females, 8 and 3 of the males and females respectively are aged above 40 years. The probability that a person selected at random from the group is aged above 40 years, given that the selected person is female, is

a. $\frac{2}{7}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{5}{6}$

65. The equation $x^3 - yx^2 + x - y = 0$ represents

a. a hyperbola and two straight lines

b. a straight line

c. a parabola and two straight lines

d. a straight line and a circle

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66. If the first and the $(2n+1)^{th}$ terms of an AP, GP and HP are equal and their n^{th} terms are respectively a, b, c then always
    a. $a = b = c$
    b. $a \geq b \geq c$
    c. $a + c = b$
    d. $ac - b^2 = 0$

67. The coordinates of a point on the line $x + y + 1 = 0$ which is at a distance $\frac{1}{5}$ unit from the line $3x + 4y + 2 = 0$ are
    a. $(2, -3)$
    b. $(-3, 2)$
    c. $(0, -1)$
    d. $(-1, 0)$

68. If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ unit on the line $y - 2x = 1$ then a is equal to
    a. 1
    b. -2
    c. -1
    d. 2

69. if f(x) is a function such that $f'(x) = (x-1)^2(4-x)$, then
    a. $f(0) = 0$
    b. f(x) is increasing in (0, 3)
    c. $x = 4$ is a critical point of f(x)
    d. f(x) is decreasing in (3, 5)

70. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are
    a. $\left(\frac{2}{5}, \frac{1}{5}\right)$
    b. $\left(-\frac{2}{5}, \frac{1}{5}\right)$
    c. $\left(-\frac{2}{5}, -\frac{1}{5}\right)$
    d. $\left(\frac{2}{5}, -\frac{1}{5}\right)$

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71. If $\int_{-3000}^{3000} \varphi(t) = \begin{cases} 1, \text{for } 0 \le t < 1 \\ 0 \text{ otherwise} \end{cases}$ then $\int_{-3000}^{3000}\left(\sum_{-3000}^{2016} \varphi(t-r')\varphi(t-2016)\right)dt =$
   a. a real number
   b. 1
   c. 0
   d. does not exist

72. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots $x = a$ and $y = \beta$ then
   a. $3 \le x \le 7$
   b. $3 \le y \le 7$
   c. $-2 \le y \le 2$
   d. $-2 \le x \le 2$

73. If $z = \sin\theta - i\cos\theta$ then for any integer n,
   a. $z^n + \frac{1}{z^n} = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$
   b. $z^n + \frac{1}{z^n} = 2\sin\left(\frac{n\pi}{2} - n\theta\right)$
   c. $z^n - \frac{1}{z^n} = 2i\cos\left(\frac{n\pi}{2} - n\theta\right)$
   d. $z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$

74. Let $f : X \to X$ be such that $f(f(x)) = x$ for all $x \in X$ and $X \subseteq R$, then
   a. f is one-to-one
   b. f is onto
   c. f is one-to-one but not onto
   d. f is onto but not one-to-one

75. If A, B are two events such that $P(A \cup B)^3 \frac{3}{4}$ and $\frac{3}{8} \le P(A \cap B) \le \frac{3}{8}$ then
   a. $P(A) + P(B) \le \frac{11}{8}$
   b. $P(A).P(B) \le \frac{3}{8}$
   c. $P(A) + P(B) \ge \frac{7}{8}$
   d. None of these

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ANSWER KEYS

1. (a)	2. (b)	3. (c)	4. (a)	5. (a)	6. (b)	7. (b)	8. (c,d)	9. (d)	10. (a)
11. (b)	12. (a)	13. (d)	14. (b)	15. (b)	16. (c)	17. (c)	18. (c)	19. (c)	20. (d)
21. (a)	22. (c)	23. (b)	24. (a)	25. (b)	26. (a)	27. (c)	28. (c)	29. (b)	30. (b)
31. (a)	32. (c)	33. (c)	34. (a)	35. (b)	36. (a)	37. (a)	38. (c)	39. (c)	40. (b)
41. (a)	42. (b)	43. (a)	44. (c)	45. (c)	46. (c)	47. (c)	48. (c)	49. (c)	50. (c)
51. (b)	52. (d)	53. (b)	54. (c)	55. (d)	56. (b)	57. (a)	58. (a)	59. (d)	60. (a)
61. (a)	62. (a)	63. (c)	64. (b)	65. (b)	66. (b,d)	67. (b,d)	68. (a,b)	69. (b,c)	70. (b,d)
71. (a,b)	72. (a,c)	73. (a,c)	74. (a,b)	75. (a,c)					

Solution

1. (a)

$$P(A\cap B)=\frac{1}{6},\qquad P(A\cup B)=\frac{31}{45}\qquad P(\overline{B})=\frac{7}{10}\text{(given)}$$

$P(B) = 1-\frac{7}{10}=\frac{3}{10}$

$\Rightarrow P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{31}{45}=P(A)+\frac{3}{10}-\frac{1}{6}\Rightarrow P(A)=\frac{5}{9}$

$\therefore P(A)\times P(B) = \frac{5}{9}\times\frac{3}{10}=\frac{1}{6}=P(A\cap B)$

$\Rightarrow$ A, B are independent events

2. (b)

$\text{Cos } 15° \cos 7\frac{1°}{2}\sin 7\frac{1°}{2}$

$\Rightarrow \dfrac{2\cos 7\frac{1^{o}}{2}\sin 7\frac{1^{o}}{2}\cos 15^{o}}{2}$

$\Rightarrow \dfrac{\sin 15^{o}\cos 15^{o}}{2}\times\dfrac{2}{2}$

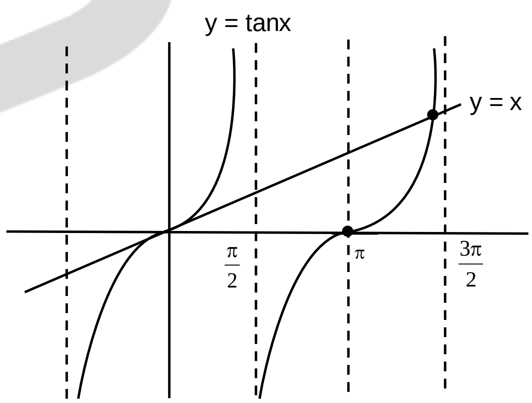
$\Rightarrow \dfrac{\sin 30^{o}}{4}=\dfrac{1}{8}$

3. (c)

$\Rightarrow \tan x = x$

$y = \tan x$ ……….(1)

$y = x$ ………….. (2)

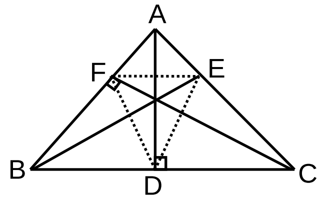


It is clearly visible that solution lies in $\left(\pi,\frac{3\pi}{2}\right)$

4. (a)

Here, DEF is a pedal triangle of ΔABC

We know



EF = a cosA = R sinA (side of pedal Δ)

$\angle FDE = 180^\circ - 2A$

Let, circumradius of ΔDEF be R'.

Now by sine rule in ΔDEF

$$2R' = \frac{EF}{\sin \angle FDE} = \frac{R \sin 2A}{\sin(180^\circ - 2A)} \Rightarrow R' = \frac{R}{2}$$

5. (a)

$$\Delta = \frac{1}{2}\begin{vmatrix} -a & -b \\ a & b \\ 0 & 0 \\ a^2 & ab \\ -a & -b \end{vmatrix}$$

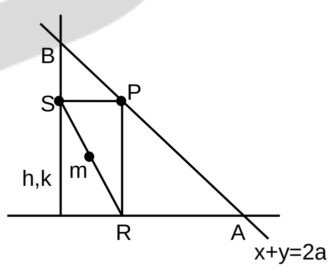
$$\Delta = \frac{1}{2}\left(-ab + ab + 0 - 0 + 0 - 0 - a^2b + a^2b\right)$$

$\Delta = 0$

$\Rightarrow$ Hence, points are collinear

6. (b)

Equation of line which cuts equal intercepts from axes is x + y = 2a



Let, co-ordinates of the midpoint be m (h, k).

So, R and S are (2h, 0) and (0, 2k)

Therefore, p must be (2h, 2k)

$\because$ P lies on AB

$\therefore$ 2h + 2k = 2a

$\Rightarrow$ x + y = a

7. (b)

L_1: x + 8y = 22

L_2: 5x + 2y = 34

L_3: 2x – 3y = –13

On solving L_1, L_2 & L_3 we get

A = (–2, 3), B = (6, 2) and C = (4, 7)

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} -2 & 3 \\ 6 & 2 \\ 4 & 7 \\ -2 & 3 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}(-4-18+42-8+12+14)\right| = 19 \text{ square units}$$

8. (c,d)

Equation of line passes through points (a, b) & (–a, –b) is

$\Rightarrow (y - b) = \frac{-2b}{-2a}(x - a)$

$\Rightarrow ay - ab = bx - ab$

$\Rightarrow ay = bx$

Now check options $\Rightarrow$ (C) & (D) are correct

9. (d)

$L_1 = \frac{x}{a} + \frac{y}{b} = k$

$L_2 = \frac{x}{a} - \frac{y}{b} = \frac{1}{k}$

Let point of intersection be (α, β)

So, $\frac{\alpha}{a} + \frac{\beta}{b} = k$ and $\frac{\alpha}{a} - \frac{\beta}{b} = \frac{1}{k}$

$\Rightarrow \left(\frac{\alpha}{a} + \frac{\beta}{b}\right)\left(\frac{\alpha}{a} - \frac{\beta}{b}\right) = 1$

$\Rightarrow \frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1$

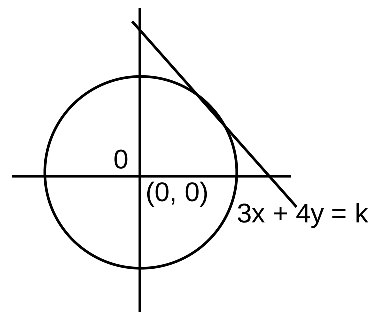
Locus: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which is hyperbola

10. (a)

Let, equation of line which is parallel to given line is

$3x + 4y = k$

This line is tangent to circle



$\Rightarrow d = r$

$\Rightarrow \left|\frac{0+0-k}{5}\right| = 3$

$\Rightarrow k = \pm 15$

So, equation of tangent in first quadrant is $3x + 4y = 15$

11. (b)

$L_1: x + y = 4$

$L_2: x - y = 2$

$\Rightarrow$ line passing through the point of intersection is

$\Rightarrow L = L_1 + \lambda L_2 = 0$

$\Rightarrow L = (x + y - 4) + \lambda (x - y - 2) = 0$

$\Rightarrow L = x(1 + \lambda) + y(1 - \lambda) - 4 - 2\lambda = 0$

$\Rightarrow M_L = \frac{3}{4}$ (given)

$\Rightarrow \frac{1+\lambda}{-1+\lambda} = \frac{3}{4}$

$\Rightarrow \lambda = -7$

Equation of line is $L = -6x + 8y + 10 = 0$

$y = \frac{3x-5}{4}$ (put in equation of parabola)

$\Rightarrow \left(\frac{3x-5}{4}\right)^2 = 4(x-3) \Rightarrow 9x^2 - 94x + 217 = 0$

$\Rightarrow x_1 + x_2 = \frac{94}{9}, x_1x_2 = \frac{217}{9}$

$\Rightarrow |x_1 - x_2| = \sqrt{(x_1+x_2)^2 - 4x_1x_2} = \frac{32}{9}$

12. (a)

$\Rightarrow 16x^2 + 25y^2 + 32x - 100y = 284$

$\Rightarrow 16(x^2 + 2x) + 25(y^2 + 4y) = 284$

$\Rightarrow 16\,(x + 1)^2 + 25(y - 2)^2 = 284 + 16 + 100$

$\Rightarrow 16\,(x + 1)^2 + 25(y - 2)^2 = 400$

$\Rightarrow \dfrac{(x+1)^2}{25} + \dfrac{(y-2)^2}{16} = 1$

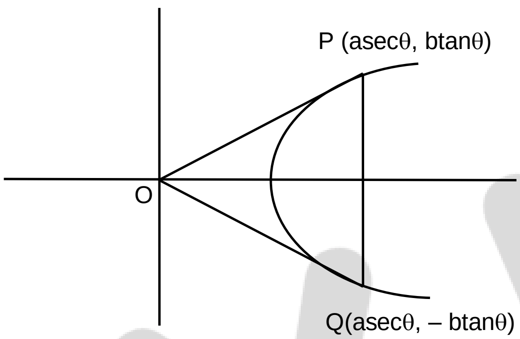
So, the auxiliary circle is $(x + 1)^2 + (y - 2)^2 = 25$

$\Rightarrow x^2 + y^2 + 2x - 4y - 20 = 0$

13. (d)

$\because \Delta OPQ$ is equilateral

$(OP)^2 = (PQ)^2$



$\Rightarrow a^2\sec^2\theta + b^2\tan^2\theta = (2b\tan\theta)^2$

$\Rightarrow a^2\sec^2\theta = 3b^2\tan^2\theta$

$\Rightarrow \sin^2\theta = \dfrac{a^2}{3b^2}$

$\Rightarrow$ Now, $\sin^2\theta < 1$

$\Rightarrow \dfrac{a^2}{3b^2} < 1$

$\Rightarrow \dfrac{b^2}{a^2} > \dfrac{1}{3}$

On adding 1 both sides

$\Rightarrow 1 + \dfrac{b^2}{a^2} > 1 + \dfrac{1}{3}$

$\Rightarrow e^2 > \dfrac{4}{3}$

On taking root both sides

$\Rightarrow e > \dfrac{2}{\sqrt{3}}$

14. (b)

$\Rightarrow y^2 - 4y = 4x - 4a$

$\Rightarrow y^2 - 4y + 4 = 4x - 4a + 4$

$\Rightarrow (y - 2)^2 = 4(x-(a-1))$

$\therefore$ Vertex: (a–1, 2)

Vertex lies in between lines

$\therefore L_1 \times L_2 < 0$

$\Rightarrow (a - 1 + 2 - 3) \times (2(a-1) + 4 - 1) < 0$

$\Rightarrow (a - 2)(2a + 1) < 0$

$\Rightarrow a \in \left(-\frac{1}{2}, 2\right)$

15. (b)

Equation of line joining points (1, 1, 1) & (0, 0, 0) is

$\Rightarrow L: \frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$

i.e. L : x = y = z = k (let) ……..(1)

Any point on L is p(k, k, k)

Since line & plane intersect at a point so, P lies on 2x + 2y + z = 10

$\Rightarrow 2k + 2k + k = 10$

$\Rightarrow k = 5$

So, point is (2, 2, 2)

16. (c)

Angle between planes = $\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$

$\Rightarrow \cos\theta = \frac{1\times 2 + 1\times(-1) + 2\times 1}{\sqrt{1^2 + 1^2 + 2^2} \cdot \sqrt{2^2 + (-1)^2 + 1^2}}$

$\Rightarrow \cos\theta = \frac{3}{6}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

17. (c)

$y = (1 + x)(1 + x^2)(1 + x^4) \ldots\ldots\ldots.(1 + x^{2n})$

Taking log both sides

$\log y = \log(1 + x) + \log(1 + x)^2 + \ldots\ldots\ldots + \log(1 + x^{2n})$

On differentiating both sides

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \ldots\ldots\ldots + \frac{2n\,x^{2n-1}}{1+x^{2n}}$$

$$\frac{dy}{dx} = y\left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \ldots\ldots\ldots + \frac{2n\,x^{2n-1}}{1+x^{2n}}\right) \because \text{ at } x = 0 \Rightarrow y = 1$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 1\cdot 1 = 1$$

18. (c)

$\because$ f(x) is odd differentiable

$\therefore$ $f(-x) = -f(x)$

On differentiating both sides

$\Rightarrow -f'(-x) = -f'(x)$

$\Rightarrow f'(x) = f'(-x)$ ....(1)

$\Rightarrow$ put $x = 3$ in eq. (1)

$\Rightarrow f'(3) = f'(-3)$

$\Rightarrow f'(-3) = 2 \quad \because f'(3) = 2$ (given)

19. (c)

$$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\left(\frac{1-\sqrt{x}}{1-x}\right)}$$

$$\Rightarrow \lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\left(\frac{1-\sqrt{x}}{(1-\sqrt{x})(1-\sqrt{x})}\right)}$$

$$\Rightarrow \lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\left(\frac{1}{1+\sqrt{x}}\right)}$$

$$\Rightarrow \left(\frac{1+1}{2+1}\right)^{\frac{1}{1+1}}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{1}{2}}.$$

20. (d)

$$f(x) = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$f(x) = \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-3.2\log x}\right)$$

$$f(x) = \tan^{-1} 1 - \tan^{-1}(2\log x) + \tan^{-1} 3 + \tan^{-1}(2\log x)$$

$f(x) = \tan^{-1}1 + \tan^{-1}3 = \text{constant}$

On differentiating with respect to x

$f'(x) = 0$

Again differentiating with respect to x

$f''(x) = 0$

21. (a)

$$I = \int \frac{\log\sqrt{x}}{3x}dx$$

Let $\log\sqrt{x} = t$

$$\Rightarrow \frac{1}{\sqrt{x}}.\frac{1}{2\sqrt{x}}dx = dt \Rightarrow \frac{dx}{x} = 2dt$$

$$\Rightarrow I = \int \frac{t}{3}2dt$$

$$\Rightarrow I = \frac{2}{3}\left(\frac{t^2}{2}\right) + c \;\Rightarrow I = \frac{1}{3}\left(\log\sqrt{x}\right)^2 + c$$

22. (c)

$$I = \int 2^x (f'(x) + f(x)\log 2)\,dx$$

Let $g(x) = 2^x f(x) \Rightarrow g'(x) = 2^x(f(x) + f'(x)\ell n2)$

$$I = \int g'(x)\,dx$$

$I = g(x) + C$

$I = 2^x f(x) + C$

23. (b)

$$I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$$

$$\Rightarrow I = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

On applying king property

$$\Rightarrow I = \int_0^1 \log\left(\frac{x}{1-x}\right)dx$$

$$= -\int_0^1 \log\left(\frac{1-x}{x}\right)dx = -I$$

$\Rightarrow$ I = – I

$\Rightarrow$ 2I = 0

$\Rightarrow$ I = 0

24. (a)

$$\lim_{n\to\infty}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+.......+\sqrt{2n-1}}{n^{3/2}}\right\}$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\left\{\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+.....+\sqrt{1+\frac{n-1}{n}}\right\}$$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n}\sqrt{1+\frac{r}{n}}$$

$$\Rightarrow \int_0^1 \sqrt{1+x}\,dx \qquad \because \text{ upper limit } = \lim_{n\to\infty}\frac{n-1}{n} = 1$$

$$\Rightarrow \frac{2}{3}(1+x)^{3/2}\Big|_0^1 \qquad \text{lower limit } = \lim_{n\to\infty}\frac{1}{n} = 0$$

$$\Rightarrow \frac{2}{3}\left(2\sqrt{2}-1\right)$$

25. (b)

$$x\frac{dy}{dx} + y = xe^x$$

$$\frac{dy}{dx} + \frac{y}{x} = e^x$$ …… linear differential equation

I.F = $e^{\int \frac{1}{x}dx} = e^{\ell nx} = x$

Solution of L.D.E

$\Rightarrow$ y. x = $\int e^x.x\,dx$

$\Rightarrow$ y . x = $xe^x - \int e^x dx$

$\Rightarrow$ y.x = $xe^x - e^x + c$

$\Rightarrow$ xy = $e^x(x-1) + c$

On comparing with given relation $xy = e^x \phi(x) + c$

$\phi(x) = (x-1)$

26. (a)

Let, equation of parabola $y^2 = 4a(x-b)$ ……..(1)

On differentiating with respect to x

$\Rightarrow 2yy' = 4a$

Again differentiating with respect to x

$\Rightarrow 2yy'' + 2(y')^2 = 0$

Order of differential equation = 2

27. (c)

E: $\frac{x^2}{3} + \frac{y^2}{2} = \frac{1}{6}$ ……….(1)

L: $y = x + \lambda$ …………(2)

Line is tangent to ellipse

$\Rightarrow 2x^2 + 3(x+\lambda)^2 = 1$

$\Rightarrow 2x^2 + 3(x^2 + 2x\lambda + \lambda^2) = 1$

$\Rightarrow 5x^2 + 6x\lambda + 3\lambda^2 - 1 = 0$

$\Rightarrow D = 0$

$\Rightarrow 36\lambda^2 - 20(3\lambda^2 - 1) = 0$

$\Rightarrow -24\lambda^2 + 20 = 0 \Rightarrow 6\lambda^2 - 5 = 0 \Rightarrow \lambda = \pm\sqrt{\frac{5}{6}}$

28. (c)

$y = \sqrt{5 - x^2}$ ...... (1)

$y = |x - 1|$ ...... (2)

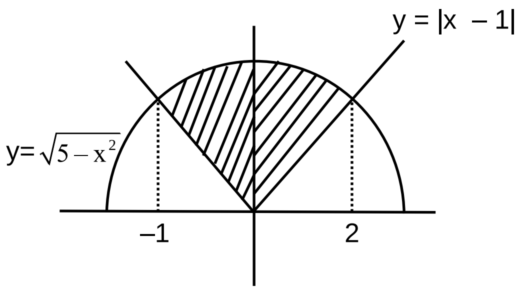
From equation (1) & (2)

$\Rightarrow 5 - x^2 = (x - 1)^2$

$\Rightarrow 5 - x^2 = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow x = -1, 2$



$$\text{Required area} = A = \int_{-1}^{2}\left(\sqrt{5-x^2} - |x-1|\right)dx$$

$$= \int_{-1}^{2}\left(\sqrt{5-x^2}\right)dx - \int_{-1}^{1}(1-x)\,dx - \int_{1}^{2}(x-1)\,dx$$

$$= 2 + \frac{5\pi}{4} - \frac{5}{2}$$

$$= \frac{5\pi}{4} - \frac{1}{2}$$

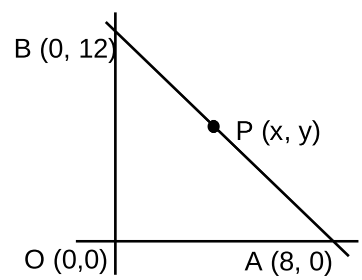
29. (b)

Equation of line AB =

$\Rightarrow \frac{x}{8} + \frac{y}{12} = 1$

$\Rightarrow 3x + 2y = 2y$

$\because$ Sum of distance of P is minimum,
so P will be on line AB



AB: 3x + 2y = 24

	x	y
Possible points	2	9
	4	6
	6	3

$\therefore (x, y) \equiv (2, 9), (4, 6), (6, 3)$

30. (b)

Wwe have

$T = 2\pi\sqrt{\frac{\ell}{g}}$

Taking log both sides

$\log T = \log 2\pi + \frac{1}{2}(\log \ell - \log g)$

Differentiating both sides we get

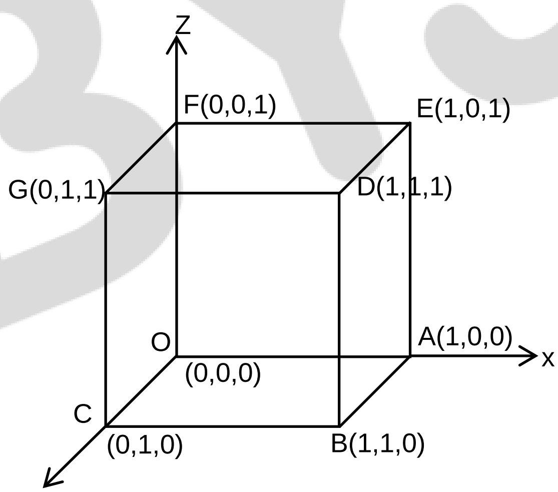
$\Rightarrow \frac{1}{T}dT = \frac{1}{2}\cdot\frac{1}{\ell}d\ell$

$\Rightarrow \frac{dT}{T}\times 100 = \frac{1}{2}\left(\frac{d\ell}{\ell}\times 100\right) = \frac{1}{2}\times 2 = 1\%$

31. (a)

Direction cosine of diagonal OD = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \frac{1}{\sqrt{3}}\right)$

Direction cosine of diagonal FB = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \frac{-1}{\sqrt{3}}\right)$



$\cos\theta = \frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\left(\frac{-1}{\sqrt{3}}\right) = \frac{1}{3}$

32. (c)

$\log_a x$, $\log_b x$, $\log_c x$ $\rightarrow$ A.P

Then,

$\Rightarrow 2\log_b x = \log_a x + \log_c x$

$$\Rightarrow \frac{2}{\log_x b} = \frac{1}{\log_x a} + \frac{1}{\log_x c}$$

$\Rightarrow 2\log_x a \log_x c = \log_x ac \cdot \log_x b$

$\Rightarrow \log_x c^2 = \log_x ac \cdot \log_x b$

$\Rightarrow c^2 = (ac)^{\log_a b}$

33. (c)

$1 + (1 + a)x + (1 + a + a^2)x^2 + \ldots\ldots\ldots$

Multiply & divide by $(1 - a)$ we get

$$= \frac{1}{1-a}\left[(1-a) + (1-a^2)x + (1-a^3)x^2 + \ldots\ldots\ \infty\right]$$

$$= \frac{1}{1-a}\left[(1 + x + x^2 + \ldots\ldots\ldots\infty) - (a + a^2x + a^3x^2 + \ldots\ldots\ldots\infty)\right]$$

$$= \frac{1}{1-a}\left[\frac{1}{1-x} - \frac{a}{1-ax}\right]$$

$$= \frac{1}{(1-a)}\left[\frac{1-ax-a+ax}{(1-x)(1-ax)}\right]$$

$$= \frac{1}{(1-x)(1-a\,x)}$$

34. (a)

$\log_{.3}(x-1) < \log_{.09}(x-1)$ $\qquad \because x - 1 > 0 \Rightarrow x > 1$ $\qquad$ ....(1)

$\Rightarrow \log_{.3}(x-1) < \log_{(.3)^2}(x-1)$

$\Rightarrow 2\log_{.3}(x-1) < \log_{.3}(x-1)$

$\Rightarrow \log_{.3}(x-1)^2 < \log_{.3}(x-1)$

$\Rightarrow (x - 1)^2 > (x - 1)$

$\Rightarrow x^2 - 3x + 2 > 0$

$x \in (-\infty, 1) \cup (2, \infty)$ $\qquad$ ....(2)

From (1) & (2)

$x \in (2, \infty)$

35. (b)

$\sum_{n=1}^{13}(i^n + i^{n+1})$

$\because$ we know,

$i + i^2 + i^3 + i^4 = 0$

$\Rightarrow \sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$

$\Rightarrow i^{13} + i^{14}$

$\Rightarrow i + i^2$

$\Rightarrow i - 1$

36. (a)

$\left|\frac{1}{z} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1 \qquad \because z\bar{z} = |z|^2 = 1 \Rightarrow \bar{z} = \frac{1}{z}$

$\Rightarrow \left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 1$

$\Rightarrow |\overline{z_1 + z_2 + z_3}| = 1 \qquad \because |\bar{z}| = |z|$

$\Rightarrow |z_1 + z_2 + z_3| = 1$

37. (a)

$x^2 + px + q = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$

$\Rightarrow p + q = -p \quad \Rightarrow \quad 2p + q = 0 \qquad .....(1)$

$\Rightarrow pq = q \quad \Rightarrow \quad q(p-1) = 0 \qquad ... \quad (2)$

From equation (1) & (2)

$\Rightarrow (-2p)(p-1) = 0$

$\Rightarrow \quad p = 0 \quad$ or $\quad p = 1$

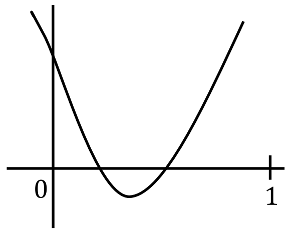
$q = 0 \quad$ or $\quad q = -2$

38. (c)

(1) $D > 0$

$9 - 4k > 0$

$k < \frac{9}{4}$ .....(1)



(2) $f(0) > 0$

$\Rightarrow k > 0$ ...... (2)

(3) $f(1) > 0$

$\Rightarrow 1 - 3 + k > 0$

$\Rightarrow k > 2$ .....(3)

From (1), (2) & (3)

$k \in \left(2, \frac{9}{4}\right)$

39. (c)

A $\boxed{\text{RR}}$ ANGE

Number of ways when R's occurs together = $\frac{\lfloor 6}{\lfloor 2}$

$= \frac{120.6}{2}$

$= 360$

40. (b)

$\Rightarrow \frac{1}{{}^5C_r} + \frac{1}{{}^6C_r} = \frac{1}{{}^4C_r}$

$\Rightarrow \frac{r!(5-r)!}{5!} + \frac{r!(6-r)!}{6!} = \frac{r!(4-r)!}{4!}$

$\Rightarrow 6(5 - r) + (6 - r)(5 - r) = 6 \times 5$

$\Rightarrow 30 - 6r + r^2 - 11r + 30 = 30$

$\Rightarrow r^2 - 17r + 30 = 0$

$\Rightarrow r = 2, 15$ (not possible)

$\therefore r = 2$

41. (a)

$n^3 + 2n$, where n is positive integer i.e. n = 1, 2, 3

When

$n = 1, 1^3 + 2.1 = 3 = 3 \times 1$

$n = 2, 2^3 + 2.2 = 12 = 3 \times 4$

$n = 3, 3^3 + 2\times 3 = 33 = 3 \times 11$

$n = 4, 4^3 + 2\times 4 = 72 = 3 \times 24$

$n = 5, 5^3 + 2\times 5 = 135 = 3 \times 45$

Hence, $n^3 + 3n$ is divisible by 3

Option (a) is correct

42. (b)

$(x-1)(x-2)(x-3) \ldots\ldots (x-18)$

$(x-1)(x-2) = x^2 - 3x + 2$, coefficient of $x = -3 = -(1+2)$

$(x-1)(x-2)(x-3) = (x^2-3x+2)(x-3)$

$= x^3 - 6x^2 + 11x - 6$, coefficient of $x^2 = -6 = -(1+2+3)$

As, similarly,

Coefficient of $x^{17} = -(1+2+3+\ldots\ldots+18)$

$$= -\left(\frac{18(18+1)}{2}\right)$$

$= -9 \times 19 = -171$

Hence option (b) is correct

43. (a)

$= 1 + {}^nC_1 \cos\theta + {}^nC_2 \cos 2\theta + \ldots\ldots + {}^nC_n \cos n\theta$

$\because (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots.. + {}^nC_n x^n$

Put $x = e^{i\theta}$

$(1+ e^{i\theta}) = 1+ {}^nC_1 e^{i\theta} + {}^nC_2 (e^{i\theta})^2 + \ldots..\ {}^nC_n (e^{i\theta})^n$

$= 1+ {}^nC_1 e^{i\theta} + {}^nC_2 e^{2i\theta} + \ldots..\ {}^nC_n (e^{ni\theta})$

$(\because e^{i\theta} = \cos\theta + i\sin\theta)$

$\Rightarrow (1+\cos\theta + i\sin\theta)^n = 1 + {}^nC_1 e^{i\theta} + {}^nC_2 e^{2i\theta} + {}^nC_3 e^{3i\theta} + \ldots.. + {}^nC_n e^{ni\theta}$

$\Rightarrow \left(1+2\cos^2\frac{\theta}{2}-1+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n = 1 + {}^nC_1 (\cos\theta + i\sin\theta) + {}^nC_2 (\cos 2\theta + i\sin 2\theta)+\ldots.+{}^nC_n (\cos n\theta + i\sin n\theta)$

$\Rightarrow \left(2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n = (1 + {}^nC_1 \cos\theta + {}^nC_2\cos 2\theta +\ldots.. + {}^nC_2 \cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2 \sin 2\theta +\ldots.+ {}^nC_n \sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2} (\cos\frac{\theta}{2} + i \sin\frac{\theta}{2})^n = (1 + {}^nC_1 \cos\theta + {}^nC_2\cos 2\theta +\ldots.. + {}^nC_n \cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2 \sin 2\theta +\ldots.+ {}^nC_n \sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(e^{i\frac{\theta}{2}}\right)^n = (1 + {}^nC_1 \cos\theta + {}^nC_2\cos 2\theta +\ldots.. + {}^nC_n \cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2 \sin 2\theta +\ldots.+ {}^nC_n \sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right) = (1 + {}^nC_1 \cos\theta + {}^nC_2\cos 2\theta +\ldots.. + {}^nC_n \cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2 \sin 2\theta +\ldots.+ {}^nC_n \sin n\theta)$

$\Rightarrow 2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right) = (1 + {}^nC_1 \cos\theta + {}^nC_2\cos 2\theta +\ldots.. + {}^nC_n \cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2 \sin 2\theta +\ldots.+ {}^nC_n \sin n\theta)$

$\Rightarrow \left(2\cos\frac{\theta}{2}\right)^n.\cos\frac{n\theta}{2} + i\left(2\cos\frac{\theta}{2}\right)^n.\sin\frac{n\theta}{2} = (1 + {}^nC_1 \cos\theta + {}^nC_2\cos 2\theta +\ldots.. + {}^nC_n \cos n\theta) + i({}^nC_1\sin\theta + {}^nC_2 \sin 2\theta + {}^nC_3 \sin 3\theta + \ldots.. + {}^nC_n \sin n\theta)$

Comparing both side with real part we get

$\left(2\cos\frac{\theta}{2}\right)^n \cos\frac{n\theta}{2} = 1 + {}^nC_1\cos\theta + {}^nC_2 \cos 2\theta + \ldots.. + {}^nC_n \cos n\theta)$

Hence option (a) is correct.

44. (c)

$$= \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log_x x & \log_x y & \log_x z \\ \log_y x & \log_y y & \log_y z \\ \log_z x & \log_z y & \log_z z \end{vmatrix} \qquad \left(\because \log_a^a = 1\right)$$

$$= \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix} \begin{array}{l} \leftarrow R_1 \\ \leftarrow R_2 \\ \leftarrow R_3 \end{array}$$

Taking $\frac{1}{\log x}, \frac{1}{\log y}, \frac{1}{\log z}$ Common R_1, R_2, R_3 we get

$$= \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

$= 0$

Since all rows are identical.

45. (c)

$\because$ B is adjoint matrix of A

$\Rightarrow B = \text{adj}(A)$

$\Rightarrow |B| = |\text{adj}(A)|$

$\Rightarrow 64 = |\text{adj}(A)|$ ($\because$ $|B| = 64$)

$\Rightarrow |A|^{(n-1)} = 64$ ($\because n = 3$)

$\Rightarrow |A|^{(3-1)} = 64$

$\Rightarrow |A|^2 = 8^2$

$\Rightarrow |A| = \pm 8$

Hence option (c) is correct.

46. (c)

Given Q = $\begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$

Let Q(θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Q(θ). Q(θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

= $\begin{bmatrix} \cos^2\theta - \sin^2\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$

$Q^2(\theta)$ = $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

Now $Q^2(\theta)$. Q(θ) = $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} . \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

= $\begin{bmatrix} \cos 2\theta.\cos\theta - \sin 2\theta.\sin\theta & -\cos 2\theta\sin\theta - \sin 2\theta.\cos\theta \\ \cos\theta.\sin 2\theta + \cos 2\theta.\sin\theta & -\sin\theta.\sin 2\theta + \cos 2\theta.\cos\theta \end{bmatrix}$

$\Rightarrow Q^3(\theta)$ = $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

$\Rightarrow Q^3(\pi/4)$ = $\begin{bmatrix} \cos\left(3\frac{\pi}{4}\right) & -\sin\left(3\frac{\pi}{4}\right) \\ \sin\left(3\frac{\pi}{4}\right) & \cos\left(3\frac{\pi}{4}\right) \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

Now $Q^3(\pi/4)$. x $\equiv \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} \end{bmatrix}$

= $\begin{bmatrix} -\frac{1}{2} - \frac{1}{2} \\ -\frac{1}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

Hence option (c) is correct

47. (c)

$R = \{x, y : x, y \in z, x + 2y \text{ is divisible by } 3\}$

Reflexive: Let $x, y \in z$

$x = y$

$x + 2x = 3x$

it is divisible by 3

$(x, x) \in R$

So it is reflexive

Symmetric: If $x R y \Rightarrow x + 2y$ is divisible by 3.

Now, $y + 2x = 3x + 3y - (x + 2y)$ is divisible by 3.

$\Rightarrow y R x$

i.e. it is symmetric.

Transitive: $x R y \Rightarrow x + 2y$ is divisible by 3.

$y R z \Rightarrow y + 2z$ is divisible by 3.

$\Rightarrow x + 2y + y + 2z$ is divisible by 3.

$\Rightarrow x + 3y + 2z$ is divisible by 3.

$\Rightarrow x + 2z$ is divisible by 3.

$\Rightarrow x R z$

Hence transitive

Therefore, R is equivalence relation.

48. (c)

$A = \{5^n - 4n-1 : n \in N\}$

When $n = 1, 5^1 - 4 \times 1-1 = 0$

$n = 2, 5^2 - 4 \times 2-1 = 16$

$n = 3, 5^3 - 4 \times 3-1 = 112$

$n = 4, 5^4 - 4 \times 4-1 = 608$

. . . .

$\Rightarrow A = \{0, 16, 112, 608\}$

While, $B = \{16(n-1), n \in N\}$

$B = \{0, 16, 32, 48……\}$

Hence it is clear that $A \subseteq B$

Hence option (c) is correct.

49. (c)

$f(x) = (x^2 + 1)^{35}. \forall x \in R$

$f(-x) = ((-x)^2 + 1)^{35}$

$= (x^2 + 1)^{35} = f(x)$

Hence f(x) is an even function

So its range $\neq R$ $(f(x) > 0 \ \forall x \in R)$

And also it is not one-one and not onto.

Hence option (c) is correct.

50. (c)

Observations are $a_1, a_2, a_3 \ldots\ldots a_n$

Mean $(\bar{x}) = \dfrac{a_1 + a_2 + \ldots + a_n}{n}$

$\because \sigma = \sqrt{\dfrac{\Sigma x_i^{\ 2}}{n} - (\bar{x})^2}$

$$\sigma = \sqrt{\frac{a_1^2 + a_2^2 + a_3^2 + \ldots a_n^2}{n} - \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^2}$$

$\lambda a_1, \lambda a_2, \lambda a_3, \ldots\ldots, \lambda a_n$

$$\sigma_1 = \sqrt{\frac{(\lambda a_1)^2 + (\lambda a_2)^2 + \ldots + (\lambda a_n)^2}{n} - \left(\frac{\lambda a_1 + \lambda a_2 + \ldots + \lambda a_n}{n}\right)^2}$$

$$= \sqrt{\lambda^2\left(\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n}\right) - \lambda^2\left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^2}$$

$$= |\lambda| \sqrt{\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n} - \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^2}$$

$\sigma_1 = |\lambda| \ \sigma$

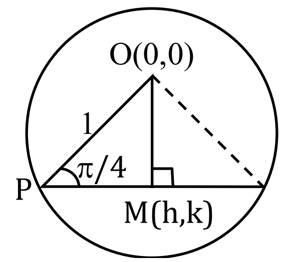
Hence option (c) is correct.

51. (b)

$OM = \sqrt{(0-h)^2+(0-k)^2}$

$OM = \sqrt{h^2+k^2}$

$\therefore$ In ΔOPM



$\sin\left(\frac{\pi}{4}\right) = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{h^2+k^2}}{1}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{h^2+k^2}}{1}$

Squaring both side, we get

$\Rightarrow \frac{1}{2} = \frac{h^2+k^2}{1}$

$\Rightarrow 2(h^2 + k^2) = 1$

$(h, k) \to (x, y)$

$2(x^2+y^2) = 1$

$\Rightarrow x^2+y^2 = \frac{1}{2}$

Hence option (b) is correct.

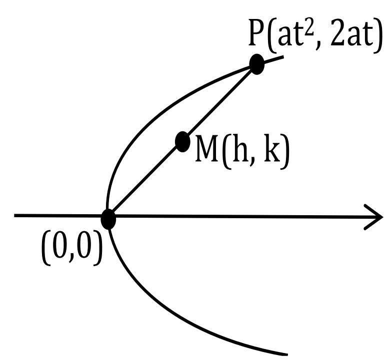
52. (d)

Let midpoint of chord = (h, k)

$\therefore h = \frac{at^2+0}{2} \Rightarrow at^2 = 2h$ .....(1)

$K = \frac{2at-0}{2} \Rightarrow at = k$

$\Rightarrow t = \frac{k}{a}$



Putting value of t in (1) we get

$\Rightarrow a\left(\frac{k}{a}\right)^2 = 2h$

$\Rightarrow k^2 = 2ah \Rightarrow y^2 = 2ax$

Directrix of parabola $y^2 = 2ax$ is

$x = \frac{-a}{2}$

Hence option (d) is correct

53. (b)

Let I = $\int_0^2 x^2[x]dx$

I = $\int_0^1 x^2[x]dx + \int_1^2 x^2[x]dx$

$\because \quad 0 \le x < 1 \Rightarrow [x] = 0$

$1 \le x < 2 \Rightarrow [x] = 1$

$\therefore$ I = $\int_0^1 x^2.0dx + \int_1^2 x^2.1dx$

= $0 + \int_1^2 x^2 dx$

= $0 + \left(\frac{x^3}{3}\right)^2$

= $\frac{2^3}{3} - \frac{1^3}{3}$

= $\frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

Hence option (b) is correct.

54. (c)

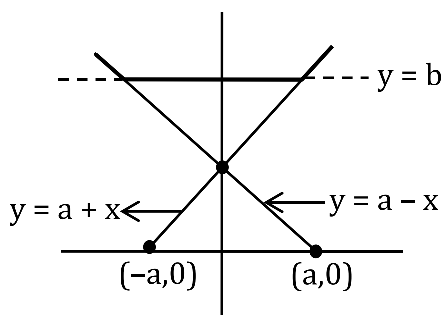
f(x) = $\{a - x, a + x.b\}$

$-a < x < \infty, a < a < b$

$y = a - x$

$y = a + x$

$y = b$



Possible graph of f(x) is shown.

There are two sharp turn. Hence f(x) is not differentiable at two points.

55. (d)

$|\vec{a} + \vec{b}| < |\vec{a} - \vec{b}|$

Squaring both sides we get

$|\vec{a}+\vec{b}|^2 < |\vec{a}-\vec{b}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|.|\vec{b}|\cos\alpha < |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}|.|\vec{b}|\cos\alpha$

(α is the angle between $\vec{a}$ & $\vec{b}$)

$\Rightarrow 2\,|\vec{a}||\vec{b}|\cos\alpha < -2\,|\vec{a}|\,|\vec{b}|\cos\alpha$

$\Rightarrow 4\,|\vec{a}||\vec{b}|\cos\alpha < 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\alpha < 0$

$\Rightarrow \cos\alpha < 0$

$\Rightarrow \alpha$ is an obtuse angle.

Hence (d) option is correct.

56. (b)

$y\dfrac{dy}{dx} + by^2 = a\cos x,\ 0 < x < 1$

Put $y^2 = t \Rightarrow 2y\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$y\dfrac{dy}{dx} = \dfrac{1}{2}\dfrac{dt}{dx}$

$\dfrac{1}{2}\dfrac{dt}{dx} + bt = a\cos x$

$\Rightarrow \dfrac{dt}{dx} + 2bt = 2a\cos x$

It is linear in 't' we get

$\therefore$ IF $= e^{\int 2b\,dx} = e^{2bx}$

Solution is

$t.\ \text{IF} = \int 2a\cos x\,.\,\text{IF}\,dx$

$\Rightarrow t.\ e^{2bx} = \int 2a\cos x\,.\,e^{2bx}\,dx$

$\Rightarrow t.e^{2bx} = \dfrac{2a}{4b^2+1}(\sin x + 2b\cos x).\ e^{2bx} + c$

$\Rightarrow y^2.\ e^{2bx} = \dfrac{2a(\sin x + 2b\cos x)}{4b^2+1}\ e^{2bx} + c$

$\Rightarrow (4b^2+1)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$

Hence option (b) is correct.

57. (a)

Given, $16x^2 + 9y^2 = 400$

$$\Rightarrow \frac{16x^2}{400} + \frac{9y^2}{400} = 1$$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{\frac{400}{9}} = 1$$

Here $a^2 = 25 \Rightarrow a = 5$

$b^2 = \frac{400}{9} \Rightarrow b = \frac{20}{3}$

Any point on ellipse is $(a\cos\theta, b\sin\theta)$

$$\equiv \left(5\cos\theta, \frac{20}{3}\sin\theta\right)$$

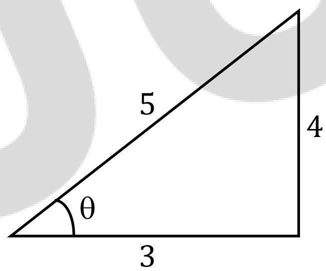
$x = 5\cos\theta$, $y = \frac{20}{3}\sin\theta$

$$\Rightarrow \frac{dx}{d\theta} = -5\sin\theta, \quad \frac{dy}{d\theta} = \frac{20}{3}\cos\theta$$

$\Rightarrow \frac{dx}{d\theta} = -\frac{dy}{d\theta}$ (Since ordinate is decreasing)

$$\Rightarrow -5\sin\theta = -\frac{20}{3}\cos\theta$$

$$\Rightarrow \tan\theta = \frac{4}{3}$$



$\cos\theta = \frac{3}{5}$ or $-\frac{3}{5}$

$\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

$\therefore$ Points are $\left(3, \frac{16}{3}\right)$ and $\left(-3, \frac{-16}{3}\right)$

Hence option (a) is correct.

58. (a)

Arranging in alphabetical order $\rightarrow$ C, C, H, I, N, O

Number of words that appear before the word COCHIN is

CC …….. $\rightarrow$ 4!

CH …….. $\rightarrow$ 4!

CI …….. $\rightarrow$ 4!

CN …….. $\rightarrow$ 4!

COCHIN …….. $\rightarrow$ 1

$\therefore$ No. of words before …….. COCHIN = 4! + 4! + 4! + 4! = 96

Hence option (a) is correct.

59. (d)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 8 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 2^2 & 0 \\ 2.2^2 & 0 & 0 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 8 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 24 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 2^3 & 0 & 0 \\ 0 & 2^3 & 0 \\ 3.2^3 & 0 & 2^3 \end{bmatrix}$$

. . . .

$$A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ n.2^n & 0 & 2^n \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{bmatrix}$$

$\therefore$ On comparing both matrix we get

$a = 2^n$, $b = n.2^n$

Hence option (d) is correct.

60. (a)

Given series is

$Sn = 1^3 + 3^3 + 5^3 + 7^3 + \ldots\ldots..$

$t_r = (2r-1)^3 = 8r^3 - 1 - 3.2r.1(2r-1)$

$= 8r^3 - 1 - 12r^2 + 6r$

$$\therefore\ Sn = \sum_{r=1}^{n} tr = \sum_{r=1}^{n}(8r^3 - 1 - 12r^2 + 6r)$$

$$= 8\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} 1 - 12\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r$$

$$= 8\left(\frac{n(n+1)}{2}\right)^2 - n - 12\ \frac{n(n+1)(2n+1)}{6} + 6\ \frac{n(n+1)}{2}$$

$= 2n^2(n+1)^2 - n - 2n(n+1)(2n+1) + 3n(n+1)$

$= 2n^2(n^2+1+2n) - n - (2n^2+2n)(2n+1) + 3n^2 + 3n$

$= 2n^4 + 2n^2 + 4n^3 - n - 4n^3 - 4n^2 - 2n^2 - 2n + 3n^2 + 3n$

$= 2n^4 - n^2$

$S_n = n^2(2n^2-1)$

Hence option (a) is correct

61. (a)

$\because$ α & β are roots of $ax^2 + bx + c = 0$

Let $y = x^2 \Rightarrow x = \sqrt{y}$

Putting $\sqrt{y}$ in the given equation, we get

$\Rightarrow a(\sqrt{y})^2 + b(\sqrt{y}) + c = 0$

$\Rightarrow ay + b\sqrt{y} + c = 0$

$\Rightarrow b(\sqrt{y}) = -ay - c$

Squaring both sides we get

$\Rightarrow b^2y = a^2y^2 + c^2 + 2acy$

$\Rightarrow a^2y^2 - (b^2-2ac)\,y + c^2 = 0$

So the required equation is

$a^2x^2-(b^2-2ac)\,x + c^2 = 0$

Hence option (a) is correct

62. (a)

$\because$ $(2-\omega)(2-\omega^2) + 2(3-\omega)(3-\omega^2) + \ldots + (n-1)(n-\omega)(n-\omega^2)$

$$\Rightarrow T_r = \sum_{r=2}^{n}(r-1)(r-\omega)(r-\omega^2)$$

$$= \sum_{r=2}^{n}(r^2 - r\omega - r + \omega)(r-\omega^2)$$

$$= \sum_{r=2}^{n}\left(r^3 - r^2\omega - r^2 + r\omega - \omega^2 r^2 + r\omega^3 + r\omega^2 - \omega^3\right)$$

$$= \sum_{r=2}^{n}\left((r^3 - r^2(\omega + 1 + \omega^2) + r(\omega + \omega^3 + \omega^2) - \omega^3\right)$$

$(\because 1 + \omega + \omega^2 = 0,\ \omega^3 = 1)$

$$= \sum_{r=2}^{n}\left(r^3 - r^2 \times 0 + r\left(\omega + 1 + \omega^2\right) - 1\right)$$

$$= \sum_{r=2}^{n}(r^3 - 1)$$

$$= \sum_{r=2}^{n} r^3 - \sum_{r=2}^{n} 1$$

$$= \left[\left(\frac{n(n+1)^2}{2}\right) - 1\right] - (n-1)$$

$$= \frac{n^2(n+1)^2}{4} - n$$

Hence option (a) is correct.

63. (c)

Given ${}^nC_{r-1} = 36\ {}^nC_r = 84, {}^nC_{r+1} = 126$

$$\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}} \ldots\ldots(i), \quad \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = 84 \ldots\ldots(ii), \quad \frac{\underline{|n}}{\underline{|r+1}\,\underline{|n-r-1}} = 126 \ldots.(iii)$$

Dividing (i) ÷(ii) we get

$$\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}} \div \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = \frac{36}{84}$$

$$\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}} \times \frac{\underline{|r}\,\underline{|n-r}}{\underline{|n}} = \frac{3}{7}$$

$$\frac{r\underline{|r-1}\,\underline{|n-r}}{\underline{|r-1}(n-r+1)\underline{|n-r}} = \frac{3}{7}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{7}$$

$7r = 3n - 3r + 3$

$10r - 3n = 3$ .....(iv)

Now dividing eq. (ii) ÷ (iii) we get

$$\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} \div \frac{\underline{|n}}{\underline{|r+1}\,\underline{|n-r-1}} = \frac{84}{126}$$

$$\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} \times \frac{\underline{|r+1}\,\underline{|n-r-1}}{\underline{|n}} = \frac{84}{126}$$

$$\frac{r+1}{n-r} = \frac{2}{3} \Rightarrow 5r - 2n = -3 \ldots.(v)$$

Solving eq. (iv) & (v) we get $r = 3$ & $n = 9$

$\therefore\ {}^nC_8 = {}^9C_8 = 9$

Hence option (c) is correct.

64. (b)

Since there are total 14 males and 6 females in a group. In which 8 males and 3 females are aged above 40 years.

Here out of 6 females 3 are above 40 and 3 are aged below 40. So probability of person aged above 40 given female person = $\frac{1}{2}$

65. (b)

Given equation, $x^3 - yx^2 + x - y = 0$

$\Rightarrow x^2(x - y) + (x - y) = 0$

$\Rightarrow (x - y)(x^2 + 1) = 0$

So only possibility is $x - y = 0$ or $x^2 + 1 = 0$

$\Rightarrow x = y$ or $x^2 + 1 = 0$ (not possible)

Hence, given equation represents straight line. Hence option (B) is correct.

66. (b, d)
There is mistake in question.
If there are $(2n - 1)^{th}$ terms instead of $(2n + 1)$ terms then n^{th} terms of the AP, GP and HP are the AM, GM & HM of the 1^{st} and the last terms.
So, $a \geq b \geq c \,\&\, ac - b^2$ (B, D)
Otherwise if there are $(2n + 1)$ terms then the n^{th} terms should be in decreasing order of AP. GP & HP.
i.e. $a \geq b \geq c$ (B)

67. (b,d)

Let any parametric point on the line $x + y + 1 = 0$ is $(t, -1 -t)$.

Distance of $(t, -1 - t)$ from $3x + 4y + 2 = 0$ is

$$\Rightarrow \left|\frac{3\times t + 4(-1-t)+2}{\sqrt{3^2+4^2}}\right| = \frac{1}{5}$$

$$\Rightarrow \left|\frac{3t - 4 - 4t + 2}{5}\right| = \frac{1}{5}$$

$$\Rightarrow \left|\frac{-t-2}{5}\right| = \frac{1}{5}$$

$\Rightarrow |t+2| = 1$

$\Rightarrow t + 2 = \pm 1$

$\Rightarrow t + 2 = \pm 1$

$\Rightarrow t + 2 = 1$ or $t + 2 = -1$

$t = -1$ or $t = -3$

$\therefore$ Possible points on the line $x + y + 1 = 0$

$(t, -1-t) \equiv (-1, -1 + 1) = (-1, 0)$

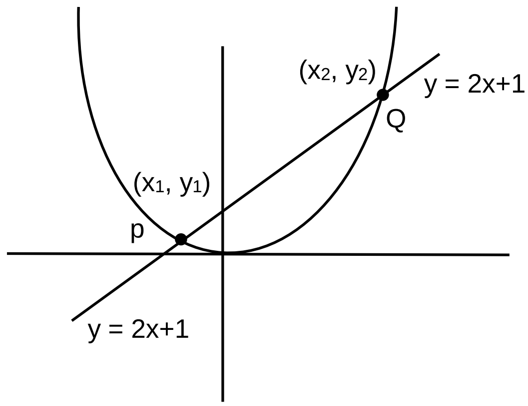
$(t, -1-t) \equiv (-3, -1 + 3) = (-3, 2)$

Hence option (B, D) are correct

68. (a, b)

Parabola: $x^2 = ay$

Line: $y - 2x = 1$



Solving parabola and line

$x^2 = a(1 + 2x)$

$\Rightarrow x^2 = a + 2ax$

$\Rightarrow x^2 - 2ax - a = 0$

Let x_1 and x_2 are roots.

$\therefore\ x_1 + x_2 = 2a$

$x_1 x_2 = -a$

$\therefore\ (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

$= (2a)^2 - 4(-a)$

$= 4a^2 + 4a$

$(x_1 - x_2)^2 = 4a(a + 1)$

Point (x_1, y_1) lie on line $y = 2x + 1$

$\therefore\ y_1 = 2x_1 + 1$

Also point (x_2, y_2) line on line $y = 2x + 1$

$\therefore\ y_2 = 2x_2 + 1$

$\therefore\ y_1 - y_2 = 2(x_1 - x_2)$

$\Rightarrow (y_1 - y_2)^2 = 4\ (x_1 - x_2)^2 = 4.\ 4a(a + 1)$

$(y_1 - y_2)^2 = 16\ a(a + 1)$

Length PQ $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{4a(a+1) + 16a(a+1)}$

PQ $= \sqrt{20a(a+1)}$

$\Rightarrow\ \sqrt{40} = \sqrt{20\,a(a+1)}$

Squaring both sides we get

$\Rightarrow\ 40 = 20\ a(a + 1)$

$\Rightarrow\ 2 = a(a + 1)$

$\Rightarrow\ a^2 + a - 2 = 0$

$\Rightarrow\ (a + 2)\ (a - 1) = 0$

$\Rightarrow\ a = -2, 1$

Hence option (A, B) are correct.

69. (b, c)

$\because f'(x) = (x-1)^2 (4-x)$

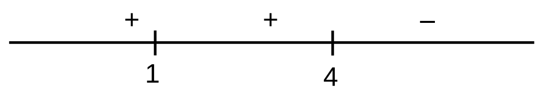
For critical point put $f'(x) = 0$

$\Rightarrow (x-1)^2(4-x) = 0$

$\Rightarrow x = 1, 4$

Therefore $x = 1$ & 4 are critical point of $f(x)$

Now sign scheme for $f'(x)$



$\therefore$ $f(x)$ is increasing in the interval $(-\infty, 4)$

Hence also increasing in the interval $(0, 3)$

And $f(x)$ is decreasing in the interval $(4, \infty)$

Hence (B, C) option is correct.

From $f'(x)$, we can't determine $f(x)$ uniquely so $f(o)$ can't be predicted.

70. (b,d)

Given ellipse is $4x^2 + 9y^2 = 1$

$$\frac{x^2}{\frac{1}{1/4}} + \frac{y^2}{1/9} = 1$$

Here, $a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$

$b^2 = \frac{1}{9} \Rightarrow b = \frac{1}{3}$

$\therefore$ Any point on ellipse is $(a\cos\theta, b\sin\theta)$

$\therefore$ Point on ellipse is $\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{3}\right)$

$\therefore$ Equation of tangent at point $\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{3}\right)$ is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\Rightarrow \frac{x\frac{\cos\theta}{2}}{\frac{1}{4}} + \frac{y\frac{\sin\theta}{3}}{\frac{1}{9}} = 1$$

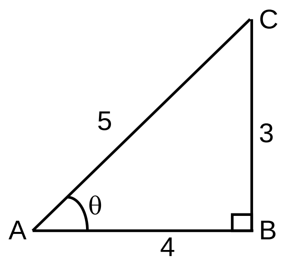
$$\Rightarrow \frac{2x\cos\theta}{1}+\frac{3y\sin\theta}{1}=1$$

$\Rightarrow 2x\cos\theta + 3y\sin\theta = 1$ ...... (1)

Slope of line $9y = 8x \Rightarrow y = \frac{8}{9}x$ .... (2)

Equation slope of (1) and (2) we get

$$\Rightarrow \frac{-2\cos\theta}{3\sin\theta}=\frac{8}{9}\Rightarrow \tan\theta=\frac{-3}{4}$$



Either, $\cos\theta = \frac{-4}{5}, \sin\theta = \frac{3}{5}$

Or

$\cos\theta = \frac{4}{5}, \sin\theta = -\frac{3}{5}$

So, a point on ellipse is $(a\cos\theta, b\sin\theta) = \left(\frac{1}{2}\cos\theta, \frac{1}{3}\sin\theta\right)$

$$\equiv \left(\frac{1}{2}\left(\frac{-4}{5}\right),\frac{1}{3}\left(\frac{3}{5}\right)\right)\text{or}\left(\frac{1}{2}\left(\frac{4}{5}\right),\frac{1}{3}\left(\frac{-3}{5}\right)\right)$$

$$\equiv \left(\frac{-4}{10},\frac{1}{5}\right)\text{or}\left(\frac{4}{10},\frac{-1}{5}\right)$$

$$=\left(-\frac{2}{5},\frac{1}{5}\right)\text{ or }\left(\frac{2}{5},-\frac{1}{5}\right)$$

Hence option (B, D) are correct

71. (a,b)

Given $\phi(t) = \begin{cases}1, & 0\le t<1\\ 0, & \text{otherwise}\end{cases}$

So $\int_{-3000}^{3000}\left(\sum_{r'=2014}^{2016}\phi(t-r')+\phi(t-2016)\right)dt$

$= \int_{-3000}^{3000}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)\,dt$

$= \int_{-3000}^{2016}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)\,dt$

$+ \int_{2016}^{2017}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)\,dt$

$+ \int_{2017}^{3000}[\phi(t-2014)+\phi(t-2015)+\phi(t-2016)]\phi(t-2016)]\phi(t-2016)\,dt$

$= \int_{3000}^{2016} o\,dt + \int_{2016}^{2017}(0+0+1).1\,dt + \int_{2017}^{3000} o\,dt$

$= 0 + \int_{2016}^{2017} dt + 0$

$= [t]_{2016}^{2017}$

$= 2017 - 296$

$= 1.$

Hence option (A, B) is correct

72. (a,c)

Given equation is $x^2 + y^2 - 10x + 21 = 0$

$\Rightarrow x^2 - 10x + (y^2 + 21) = 0$ have roots

$x = a$ and $y = b$

$\therefore$ for real roots $D \geq 0$

$\Rightarrow (-10)^2 - 4\cdot1\cdot(y^2 + 21) \geq 0$

$\Rightarrow 100 - 4y^2 - 84 \geq 0$

$\Rightarrow -4y^2 + 16 \geq 0$

$\Rightarrow y^2 \leq 4$

$\Rightarrow -2 \leq y \leq 2$

Hence option (c) is correct

Now, $y^2 = -x^2 + 10x - 2$

For real roots of y

$\Rightarrow -x^2 + 10x - 21 \geq 0$

$\Rightarrow x^2 + -10x + 21 \leq 0$

$\Rightarrow x^2 - 7x - 3x + 21 \leq 0$

$\Rightarrow x(x-7) - 3(x-7) \leq 0$

$\Rightarrow (x - 7)(x - 3) \leq 0$

$\Rightarrow 3 \leq x \leq 7$

Option (A) is correct

Hence option (A, C) are correct

73. (a,c)

$\therefore z = \sin\theta - i\cos\theta$

$= \cos\left(\theta - \frac{\pi}{2}\right) + i\sin\left(\theta - \frac{\pi}{2}\right)$

$= z\ e^{i(\theta - \frac{\pi}{2})}$

So, $z^n = \left(e^{i(\theta-\frac{\pi}{2})}\right)^n = e^{\left(in\left(\theta-\frac{\pi}{2}\right)\right)}$

$$z^n = \cos\left(n\left(\theta - \frac{\pi}{2}\right)\right) + i\sin n\left(\theta - \frac{\pi}{2}\right) \qquad .....(1)$$

Now $\frac{1}{z^n} = \frac{1}{(z)^n} = \frac{1}{\left(e^{i\left(\theta-\frac{\pi}{2}\right)}\right)^n} = \frac{1}{e^{ni\left(\theta-\frac{\pi}{2}\right)}}$

$$\frac{1}{z^n} = e^{-ni(\theta-\frac{\pi}{2})} = \cos(-n(\theta - \frac{\pi}{2})) + i\sin(-n(\theta - \frac{\pi}{2}))$$

$$= \cos\left(n\left(\theta - \frac{\pi}{2}\right)\right) - i\sin\left(n\left(\theta - \frac{\pi}{2}\right)\right) \qquad .....(2)$$

Subtracting

$Z^n - \frac{1}{z^n} = \cos n(\theta - \frac{\pi}{2}) + i\sin(n(\theta - \frac{\pi}{2})) - \cos(n(\theta - \frac{\pi}{2})) + i\sin(n(\theta - \frac{\pi}{2})$

$= 2\,i\sin n\left(\theta - \frac{\pi}{2}\right)$

$z^n - \frac{1}{z^n} = 2\,i\sin\left(n\theta - \frac{n\pi}{2}\right)$ (option (c))

Adding (1) & (2) we get

$Z^n + \frac{1}{z^n} = \cos\left(n\left(\theta-\frac{\pi}{2}\right)\right) + i\sin\left(n\left(\theta-\frac{\pi}{2}\right)\right) + \cos\left(n\left(\theta-\frac{\pi}{2}\right)\right) - i\sin\left(n\left(\theta-\frac{\pi}{2}\right)\right)$

$= 2\cos\left(n\left(\theta - \frac{\pi}{2}\right)\right)$

$= 2\cos\left(n\theta - \frac{n\pi}{2}\right)$ option (A)

Hence option (A, C) are correct

74. (a,b)

$f(f(x)) = x\ \forall x \in X\ \&\ x \subseteq R$

So, $f(x) = f^{-1}(x)$

$\Rightarrow$ f(x) is self inverse

Hence f(x) is one-one and onto

Therefore, option (A, B) is correct.

75. (a,c)

$P\left(A\cup B\right)\geq\frac{3}{4}$ and $\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$

We know that

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A) + P(B) = P(A\cup B) + P(A\cap B)$ ……..(1)

$\therefore\ \frac{3}{4}\leq P(A\cup B)\leq 1$

$\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$

$\Rightarrow\ \frac{3}{4}+\frac{1}{8}\leq P(A\cup B)+P(A\cap B)\leq 1+\frac{3}{8}$

$\Rightarrow\ \frac{7}{8}=\frac{6+1}{8}\leq P\left(A\cup B\right)+P(A\cap B)\leq\frac{11}{8}$

From (1)

$\frac{7}{8}\leq P\left(A\right)+P(B)\leq\frac{11}{8}$

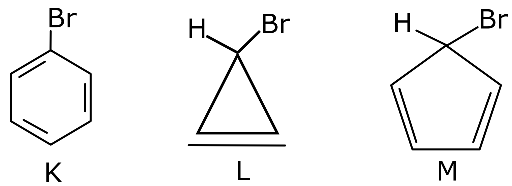
Hence $P(A) + P(B) \geq\frac{7}{8}$ option (c)

$P(A) + P(B) \leq\frac{11}{8}$ option (A)

Hence option (A, C) are correct

WBJEE 2016 (Chemistry)

1. Consider the following compounds:



Which one of the following statements is correct?

a. Only K forms a precipitate on treatment with alcoholic $AgNO_3$ solution.

b. Only L forms a precipitate on treatment with alcoholic $AgNO_3$ solution.

c. Only M forms a precipitate on treatment with alcoholic $AgNO_3$ solution.

d. K, L, and M form precipitates with alcoholic $AgNO_3$ solution.

2. Consider the following nuclear reactions : ${}^{238}_{92}M \rightarrow {}^{Y}_{X}N + 2\alpha; N \rightarrow {}^{A}_{B}L + 2\beta^{+}$; The number of neutrons in the element L is

a. 142

b. 144

c. 140

d. 146

3. Of the following atoms, which one has the highest n/p ratio?

a. Ne^{16}

b. O^{16}

c. F^{16}

d. N^{16}

4. The spin-only magnetic moment of $[CrF_6]^{4-}$ (atomic number of Cr is 24) is

a. 0

b. 1.73 BM

c. 2.83 BM

d. 4.9 BM

5. Among the following groupings, which one represents the set of iso-electronic species?

   a. NO^+, C_2^{2-}, O_2^-, CO

   b. N_2, C_2^{2-}, CO, NO

   c. CO, NO^+, CN^-, C_2^{2-}

   d. NO, CN^-, N_2, O_2^-

6. In the complex ion $[Cu(CN)_4]^{3-}$ the hybridization state, oxidation state and number of unpaired electrons of copper are respectively

   a. dsp^2, +1,1
   b. sp^3, +1, zero
   c. sp^3, +2,1

   d. dsp^3, +2, zero

7. The maximum number of 2p electrons with electronic spin $= -\frac{1}{2}$ are

   a. 6 b. 0

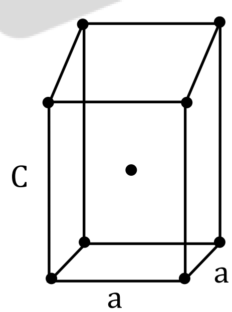
   c. 2 d. 3

8. For $N^{3-} > O^{2-} > F^-$ and Na^+, the order in which their ionic radii varies is

   a. $N^{3-} > O^{2-} > F^- > Na^+$ b. $N^{2-} > Na^+ > O^{2-} > F^-$

   c. $Na^+ > O^{2-} > N^{3-} > F^-$ d. $O^{2-} > F^- > Na^+ > N^{3-}$

9. Assign the Bravais lattice type of the following unit cell structure.



   a. Cubic I b. Orthorhombic I

   c. Tetragonal I d. Monoclinic

10. The equilibrium constant for the reaction $N_2 + 3H_2 \rightleftharpoons 2NH_3$ is 'K'. Then, the equilibrium constant for the reaction $2N_2 + 6H_2 \rightleftharpoons 4NH_3$ will be

a. K
b. K^2
c. $\sqrt{K}$
d. 2K

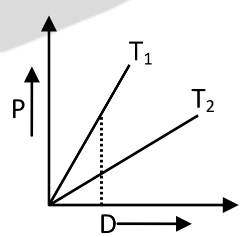
11. Which of the following is the correct option for free expansion of an ideal gas under adiabatic condition?

a. $q = 0, \Delta T \neq 0, W = 0$
b. $q = 0, \Delta T \neq 0, W \neq 0$
c. $q = 0, \Delta T = 0, W = 0$
d. $q = 0, \Delta T = 0, W \neq 0$

12. 75% of a first order reaction was completed in 32 min. When would 50% of the reaction complete?

a. 24 min
b. 16 min
c. 8 min
d. 64 min

13. Pressure (P) vs. density (D) curve for an ideal gas at two different temperatures T_1 and T_2 is shown below.



Identify the correct statement about T_1 and T_2:

a. $T_1 > T_2$
b. $T_1 < T_2$
c. $T_1 = T_2$
d. Cannot be said

14. Which of the following compounds is least effective in precipitating $Fe(OH)_3$ sol.?

a. $K_4[Fe(CN)_6]$
b. K_2CrO_4
c. KBr
d. K_2SO_4

15. Which statement is incorrect?

a. Borazine has a 3D-layer structure like that of graphite
b. Boric acid has a hydrogen bonded layer structure in the solid state
c. Borazine molecule is $(BN)_3$
d. $[Al_6O_{18}]^{18-}$ contains a non-planar Al_6O_6 -ring

16. Which one of the following does not produce O_2 as the only gaseous product on heating?
a. Lead Nitrate
b. Potassium Chlorate
c. Mercuric Oxide
d. Potassium Nitrate

17. Which of the following is true in respect of adsorption?
a. $\Delta G < 0$; $\Delta > 0$; $\Delta H < 0$
b. $\Delta G < 0$;$\Delta S < 0$; $\Delta H < 0$
c. $\Delta G > 0$; $\Delta S > 0$; $\Delta H < 0$
d. $\Delta G < 0$;$\Delta S < 0$;$\Delta H > 0$

18. Which property that polyacetylene exhibits is unusual for an organic polymer?
a. Electrical conductivity
b. Flexibility
c. High boiling point
d. Solubility

19. Which statement is incorrect about complexes formed by the lanthanoids?
a. Hard donor ligands are favoured
b. High coordination numbers (more than six) are often observed
c. The 4f atomic orbitals do not play a significant part in metal-ligand bonding
d. Aqua ions are typically 6-coordinate

20. In the alumino-thermite process, aluminium acts as
a. a reducing agent
b. an oxidizing agent
c. an additive agent
d. a flux

21. Consider the following reaction: $6NaOH + 3Cl_2 \rightarrow 5NaCl + A + 3H_2O$ what is the oxidation number of chlorine in A?
a. +5
b. −1
c. +3
d. +1

22. A sudden large difference between the values of second and third ionization energies of elements would be associated with which of the following electronic configurations?
a. $1s^2\ 2s^2\ 2p^6\ 2s^1$
b. $1s^2\ 2s^2\ 2p^6\ 3s^2$

c. $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^1$
d. $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$

23. Na_2O_2 is produced in reaction between H_2O_2 and NaOH. Here, the role of H_2O_2 is
a. as an oxidising agent
b. as an acid
c. as a base
d. as a reducing agent

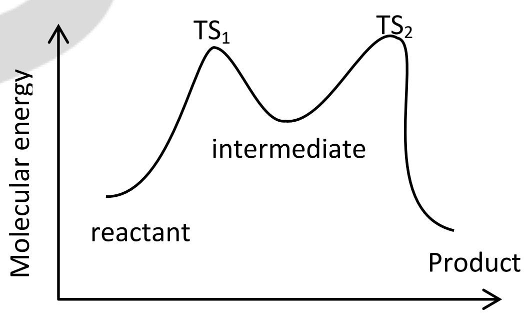
24. Amongst the following compounds, the one which would not form a white precipitate with ammonical silver nitrate solution is
a. $HC \equiv CH$
b. $H_3C - C \equiv C - CH_3$
c. $H_3C - C \equiv CH$
d. $CH_3CH_2CH_2C \equiv CH$

25. m-dinitrobenzene can be converted to m-nitroaniline by reduction with :
a. Raney Nickel
b. $LiAlH_4$
c. $(NH_4)_2S$
d. Na/C_2H_5OH

26. The correct IUPAC name of $H_3C - C(CH_3)_2 - CH = CH_2$ is :
a. 3, 3, 3– trimethylprop–1–ene
b. 1, 1,1 trimethyl-α–propene
c. 3, 3–dimethylbut–1–ene
d. 2, 2–dimethylbut–3–ene

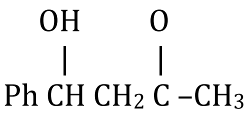
27. Which combination of reagents used in the indicated order will give m-nitropropylbenzene from benzene?
a. 1) conc. HNO_3/conc. H_2SO_4
2) $CH_3CH_2CH_2/AlCl_3$
b. 1)$CH_3CH_2CH_2Cl/AlCl_3$
2) conc. HNO_3/conc. H_2SO_4
c. 1) $CH_3CH_2COCl/AlCl_3$
2) conc. HNO_3/conc. H_2SO_4
3) H_2NNH_2/NaOH
d. 1) conc. HNO_3/conc. H_2SO_4
2) $CH_3CH_2COCl/AlCl_3$
3) H_2NNH_2/NaOH

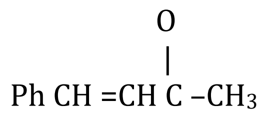
28. Which of the statement (A) - (D) about the reaction profile below is false?

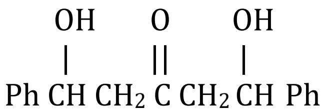


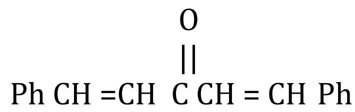
a. The product is more stable than the reactant
b. The second step is rate determining
c. The reaction is exothermic
d. The equilibrium constant is greater than 1 if the molar entropy change is negligible

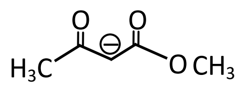
29. Which of the following is the major product when one mole of propanone and two moles of benzaldehyde react in presence of catalytic amount of NaOH?

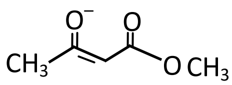
a. 

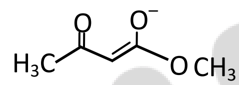
b. 

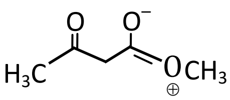
c. 

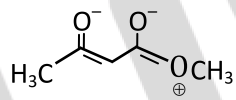
d. 

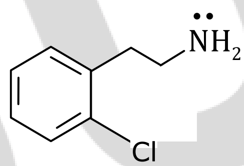
30. For the following anion,  the resonance structure that contributes most is

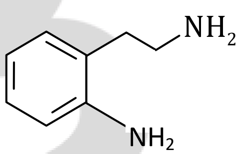
a. 

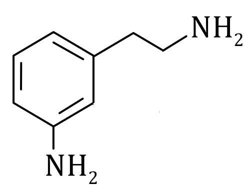
b. 

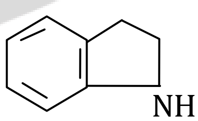
c. 

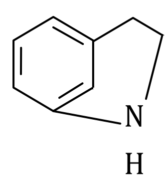
d. 

31. The major product obtained upon treatment of  with $NaNH_2$ and liquid NH_3 is

a. 

b. 

c. 

d. 

32. Which structure for XeO_3 and XeF_4 are consistent with the VSEPR model?

a. XeO_3, trigonal pyramidal; XeF_4, square planar

b. XeO_3, trigonal planar; XeF_4 square planar

c. XeO_3, trigonal pyramidal; XeF_4, tetrahedral

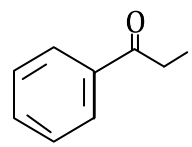
d. XeO_3, trigonal planar; XeF_4, tetrahedral

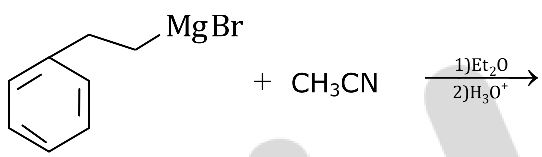
33. If CO_2 gas is passed through 500 ml of 0.5 (M) $Ca(OH)_2$, the amount of $CaCO_3$ produced is of the value

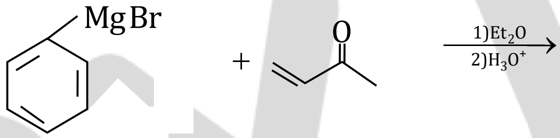
a. 10 g
b. 20 g
c. 50 g
d. 25 g

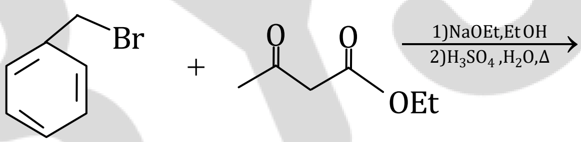
34. The emf of a Daniel cell at 298 K is E1. The cell is $Zn|ZnSO_4$ (0.01M) || $CuSO_4$ (1M) | Cu When the concentration of $ZnSO_4$ is changed to 1M and that of $CuSO_4$ to 0.01M, the emf changes to E_2. The relationship between E_1 and E_2 will be
a. $E_1 - E_2 = 0$
b. $E_1 < E_2$
c. $E_1 > E_2$
d. $E_1 = 10^2 E_2$

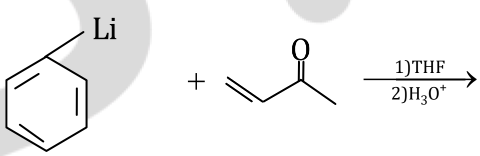
35. Which reaction is not appropriate for the synthesis of the following?



a.


b.


c.


d.


36. Which of the following statements are correct with reference to isoelectric point of alanine?
a. At the isoelectric point, alanine bears no net charge
b. At the isoelectric point, the concentration of zwitter ion is maximum
c. It is not the average of pKa_1 and pKa_2 value
d. Alanine will have a net positive charge at pH below the isoelectric point

37. Consider the proposed mechanism for the destruction of ozone in the stratosphere?

$O_3 + Cl \rightarrow ClO + O_2$ $\qquad$ $ClO + O_3 \rightarrow Cl + 2O_2$

Which of the statements about the mechanism is/are correct?

a. Cl is a catalyst

b. O_2 is an intermediate

c. Equal amounts of Cl and ClO are present at any time

d. The number of moles of O_2 produced equals the number of moles of O_3 consumed

38. Which of the following statements(s) is (are) correct?

a. The electronic configuration of Cr (at. no: 24) is [Ar] $3d^5\,4s^1$.

b. The magnetic quantum number may have a negative value.

c. In Ag (at. no: 47), 23 electrons have spins of one type and 24 electrons have spins of opposite type.

d. The oxidation state of nitrogen in HN_3 is – 3

39. Equal quantities of electricity are passed through 3 voltameters containing $FeSO_4$, $Fe_2(SO_4)_3$ and $Fe(NO_3)_3$. Consider the following statement:

(1) The amounts of iron deposited in $FeSO_4$ and $Fe_2(SO_4)_3$ are equal

(2) The amount of iron deposited in $Fe(NO_3)_3$ is $2/3^{rd}$ of the amount deposited in $FeSO_4$

(3) The amount of iron deposited in $Fe_2(SO_4)_3$ and $Fe(NO_3)_3$ are equal

a. (1) is correct

b. (2) is correct

c. (3) is correct

d. Both (1) and (2) are correct

40. Which of the following statements are correct for the following isomeric compounds I and II?

a. I and II are enantiomers

d. I and II are both optically active

c. I is D-alanine while II is L-alanine

d. I and II are diastereomers

WBJEE 2016 (Chemistry)

ANSWER KEY

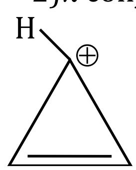
1. (b)	2. (b)	3. (d)	4. (d)	5. (c)	6. (b)	7. (d)	8. (a)	9. (c)	10. (b)
11. (c)	12. (b)	13. (a)	14. (c)	15. (a,c,d)	16. (a)	17. (b)	18. (a)	19. (d)	20. (a)
21. (a)	22. (b)	23. (b)	24. (b)	25. (c)	26. (c)	27. (c)	28. (b)	29. (d)	30. (a)
31. (c)	32. (a)	33. (d)	34. (c)	35. (d)	36. (a,b,d)	37. (a,c)	38. (a,b,c)	39. (b,d)	40. (a,b,c)

SOLUTIONS

1. (b)

Those compounds will easily form a precipitate with $AgNO_3$ in which the conjugate formed is stable.

In option L, we can see that the compound formed is aromatic in nature i.e. contains $(4n + 2)\pi$ conjugation so is stable.

K =  this compound is stable since it is aromatic.

2. (b)

Nuclear reaction is:-

$${}^{238}_{92}M \longrightarrow {}^{y}_{x}N + 2\alpha\left(2\times {}^{4}_{2}He\right) \text{ or } {}^{230}_{88}N$$

$${}^{230}_{88}N \longrightarrow {}^{A}_{B}L + 2\ {}^{0}_{1}e \text{ or } {}^{230}_{96}L$$

$\therefore$ Number of neutrons in L = 230 – 36

= 144

3. (d)

Highest n/p ratio

$\longrightarrow {}_{10}Ne^{16} \Rightarrow \begin{matrix} p=10 \\ n=6 \end{matrix}$ So, $\frac{n}{p} = \frac{6}{10} = 0.6$

$\longrightarrow {}_{8}O^{16} \Rightarrow \begin{matrix} p=8 \\ n=8 \end{matrix}$ So, $\frac{n}{p} = \frac{8}{8}$

n/p = 1

$\longrightarrow {}_{9}F^{16} \Rightarrow \begin{matrix} p=9 \\ n=7 \end{matrix}$ n/p = 7/9

= 0.778

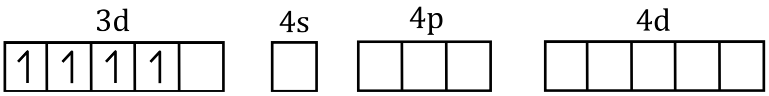
$\longrightarrow {}_{7}N^{16} \Rightarrow \begin{matrix} p=7 \\ n=9 \end{matrix}$ n/p = 9/7

n/p = 1.2857

So, highest n/p ratio is of N^{16}

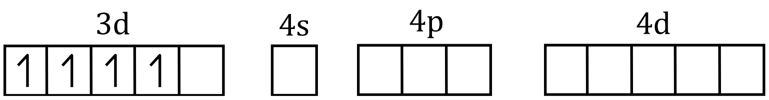
4. (d)

$_{24}Cr = [Ar]3d^5 4s^1$

Cr^{+2} 

F^- is a weak field ligand.

$\Delta_0 < P$, No pairing

$[CrF_6]^{4-}$ = 

Number of unpaired electron = 4

Spin only magnetic moment (μ) = $\sqrt{n(n+2)}$

$= \sqrt{4 \times 6}$

$= \sqrt{24}$

= 4.9 BM

5. (c)

Two or more molecular entities are as isoelectronic if they have same number of valance electrons and same structure i.e. number and connectivity of atoms.

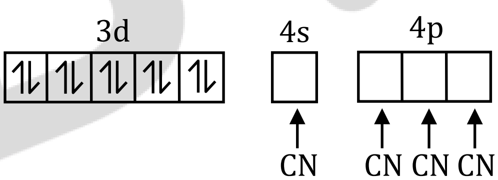
Here we can see that CO, NO^+, CN^-, C_2^{2-}

All have 14 electrons each.

$\rightarrow$ A carbon has 6 electrons, oxygen has 8 electrons, nitrogen has 7 electrons.

6. (b)

$\left[\overset{+1}{Cu}\ \ (\overset{-4}{CN})_4\right]^{3-}$

Cu^+ 

Hybridisation = sp^3

Oxidation state of Cu = +1

Number of unpaired electrons = 0.

7. (d)

The maximum electrons which can be accommodated in p-orbital is 6 electrons.

Three electrons have +1/2 spin and other three have –1/2 spin.

So, a maximum electron in 2p with spin –1/2 is 3.

8. (a)
N^{3-}, O^{2-}, F^-, Na^+ all have same number of valance electrons. So, these species are isoelectronic and we know, as atomic number increases. Nuclear attraction increase, hence the order of ionic radii will be
$N^{3-} > O^{2-} > F^- > Na^+$

9. (c)
By seeing given structure. We can conclude that a = b ≠ c and all angles are 90° i.e.
$\alpha = \beta = \gamma = 90°$
And it is body centered lattice i.e. the Bravais notation is I.
∴ Bravais lattice is Tetragonal-I.

10. (b)
Given: $N_2 + 3H_2 \leftrightharpoons 2NH_3$ equilibrium constant = k
Multiplying equation by 2, we get
$2N_2 + 6H_2 \leftrightharpoons 4NH_3$
If equation is multiplied by 2 then k will be squared K^2
Or
$N_2 + 3H_2 \leftrightharpoons 2NH_3$, K

$$K = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

Now,
$2N_2 + 6H_2 \leftrightharpoons 4NH_3$

$$K' = \frac{[NH_3]^4}{[N_2]^2[H_2]^6}$$

$= K^2$

11. (c)
For the free expansion of real gas, the opposing force is zero, and when opposing force is zero then work done = 0
We, know the equation
$\Delta U = q + w$
If w = 0 then $\Delta U = q$
But the process is adiabatic hence q = 0
∴ q = 0, T = 0, w = 0

12. (b)

The first order reaction is given as:-

$$K=\frac{2.303}{t}\log\frac{a_0}{a_0-x} \qquad ..... (i)$$

Also, half life $t_{1/2}=\frac{2.303\log 2}{K}$ .....(ii)

Equating equation (i) & (ii) as value of K are same

$$\frac{2.303}{t}\log\frac{a_0}{a_0-x}=\frac{2.303\log 2}{t_{1/2}}$$

$$\frac{2.303}{32}\log\frac{100}{100-75}=\frac{2.303\log 2}{t_{1/2}}$$

Solving, $\frac{\log 2}{t_{1/2}}=\frac{1}{32}2\log 2$

$t_{1/2}$ = 16 min.

13. (a)

As we know,

Ideal gas equation is PV = nRT

$P=\frac{wRT}{MV}$ or we can write

$=\frac{dRT}{M}$

So,

PM = dRT

$P \propto d.T$

So, $T_1 > T_2$

14. (c)

$Fe(OH)_3$ is positively charged sol. charge of Br^- anion is least and so KBr is least effective in coagulating $Fe(OH)_3$ sol.

Also, for coagulating positive charged sol. negatively charged sol. is required. More the negative charge, more easily the coagulation will take place.

15. (a,c,d)

Three options are incorrect.

→ Borazine is $B_3N_3H_6$ and its structure is not like that of graphite.

→ $[Al_6O_{18}]^{-18}$ contain AlO_4^{5-} tetrahedral units.

16. (a)

We can find out the answer by writing heating equations of compounds.

(A) $Pb(NO_3)_2(s) \xrightarrow{\Delta} PbO(s) + 2NO_2(g) + \frac{1}{2}O_2(g)$

(B) $KClO_3(s) \xrightarrow{\Delta} KCl(s) + \frac{3}{2}O_2(g)$

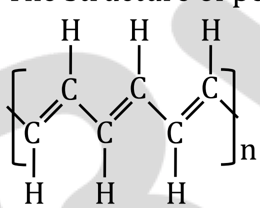
(C) $HgO(s) \xrightarrow{\Delta} Hg(\ell) + \frac{1}{2}O_2(g)$

(D) $KNO_3(s) \xrightarrow{\Delta} KNO_2(s) + \frac{1}{2}O_2(g)$

$\therefore$ We can observe that when lead nitrate is heated it will give NO_2 gas along with O_2 gas.

17. (b)
For adsorption process
$\Delta G < 0$, $\Delta H <$ and $\Delta S < 0$
$\rightarrow$ Adsorption of gases on solid surface is generally exothermic because entropy decreases.
$\rightarrow$ Adhering of gas molecules to the surface lowers randomness.

18. (a)
Due to the conjugation, it can conduct electricity.
The structure of polyacetylene is shown:



19. (d)
The statement aqua ions typically 6-coordinate is incorrect, for example the coordination number for $[Ln(H_2O)_n]^{3+}$ in aqueous solution is thought to be 9 for the early lanthanoids and for the later, smaller numbers of the series.

20. (a)
Some metal oxides cannot be reduced satisfactorily by carbon. For them aluminium, a more reactive metal is used. The process is called thermic process or alumino-thermic process since this process is only for those metals that can't be reduced by carbon. Therefore, aluminium is used in place of carbon atoms and it acts as an reducing agent.
Aluminium reduces oxides of metals like Fe/Cr/Mn at elevated temperature.
$2Al + Fe_2O_3 \rightarrow 2Fe + Al_2O_3$

WBJEE 2016 (Chemistry)

21. (a)

Sodium hydroxide react with chlorine gives sodium chloride, sodium chlorate and water molecules as the products. Chemical reaction can be written as:

$6NaOH + 3Cl_2 \rightarrow 5NaCl + 3H_2O + \underset{(A)}{NaClO_3}$

So, the compound A is $NaClO_3$

Let oxidation state of chlorine is x.

Oxidation state of Cl in $NaClO_3$

$$(+1) \times (x) \times 3(-2) = 0$$

$x = 6 - 1$

$x = +5$

Oxidation number of chlorine is +5.

22. (b)

After removal of two electrons from $1s^2$, $2s^2$, $2p^6$, $3s^2$ we get stable noble gas (Ne) configuration. On removing one more electron in this completely filled octet, high amount of energy required. So there is large difference between 2nd ionization energy and 3rd ionization energy for this configuration.

23. (b)

It is an acid base reaction in which hydrogen peroxide act as acid to react with sodium hydroxide (base) to produce sodium peroxide and water.

$\underset{Acid}{H_2O_2} + \underset{base}{NaOH} \rightarrow \underset{Salt}{Na_2O_2} + \underset{Water}{H_2O}$

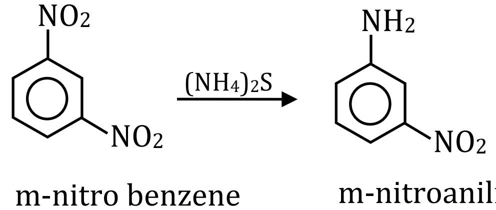
24. (b)

Only terminal alkynes having acidic hydrogen to react with ammonical silver nitrate solution known as Tollen's reagent.

In the given options, $CH_3{\equiv}CCH_3$ does not have any acidic hydrogen to react with an ammonical silver nitrate solution.

25. (c)

Selective reduction of m-nitro benzene to form m-nitroaniline is done by ammonium sulphide. In this reaction out of two nitro groups, only one $-NO_2$ group is reduced.

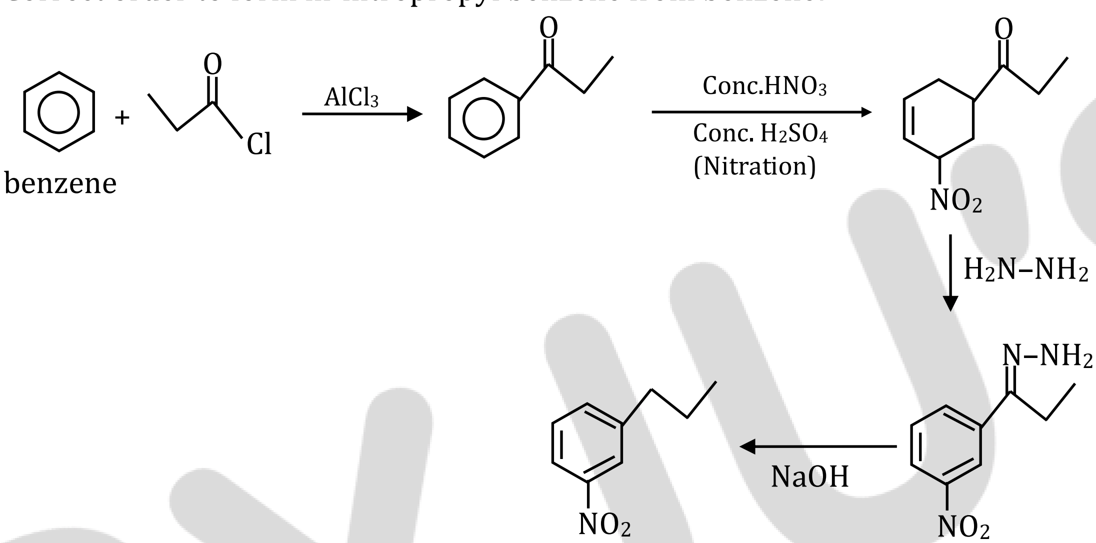


m-nitro benzene m-nitroaniline

26. (c)
The correct IUPAC name of $H_3C–C(CH_3)_2–CH=CH_2$ is 3,3-dimethyl but-1-ene.

```
       CH3
       |
H3C – C – CH = CH2
 4    3|   2      1
       CH3
```

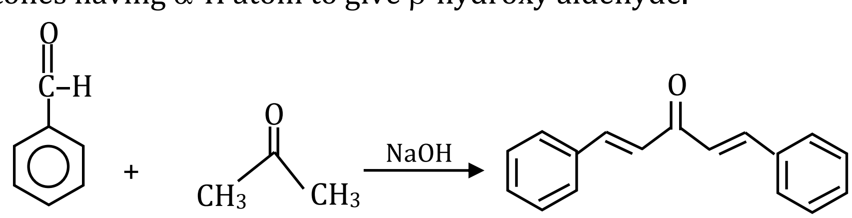
27. (c)
Correct order to form m-nitropropyl benzene from benzene:



m-nitropropyl benzene

28. (b)
The potential energy diagram shows the lower activation energy of a hypothetical reaction in second step.
The rate of reaction depends on the activation energy; lower activation energy means that larger number of molecules will have sufficient energy to undergo an effective collision.
Hence, second step is not rate determining step, it is fast step due to low activation energy.

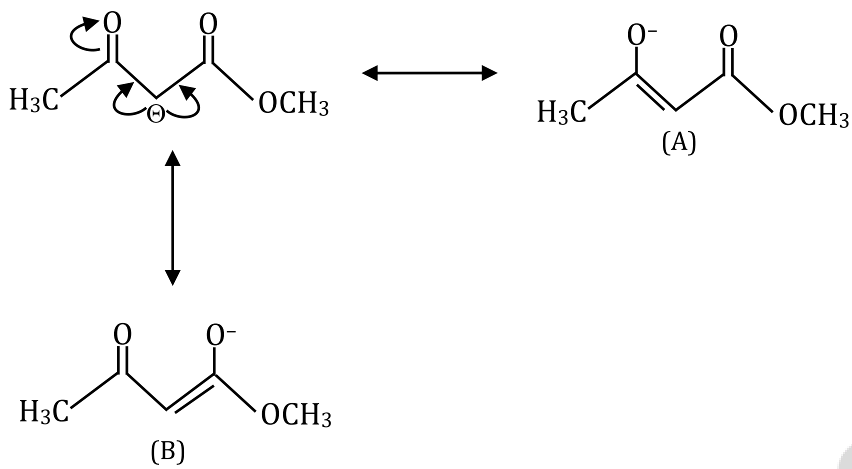
29. (d)
This reaction is an example of aldol condensation reaction and is given by aldehydes or ketones having α-H atom to give β-hydroxy aldehyde.



Benzaldehyde
(2 mole)

(Dibenzalacetone)

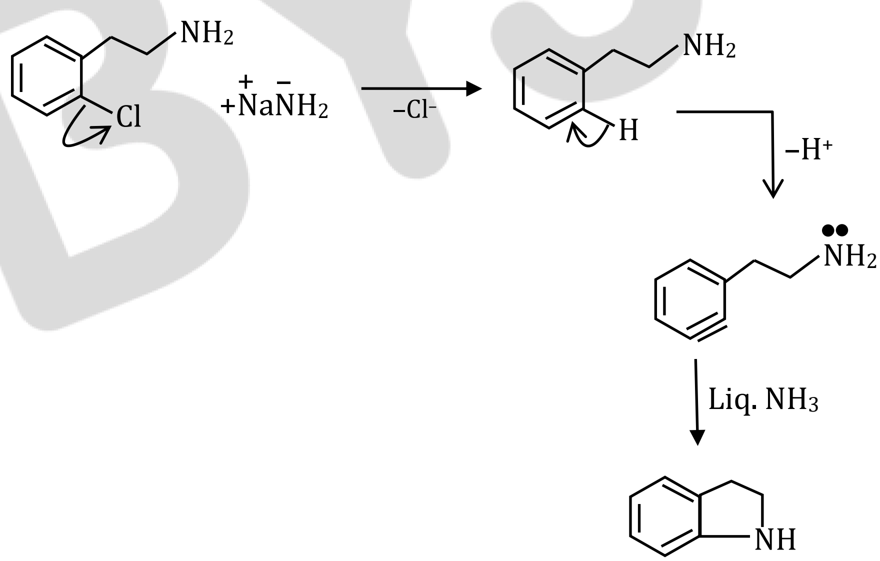
30. (a)



Resonance hybrid (A) is more stable than (B)

Option c and d is least stable because positive charge on electronegative oxygen atom.

31. (c)

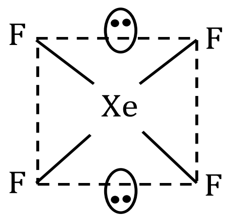


Major product

32. (a)

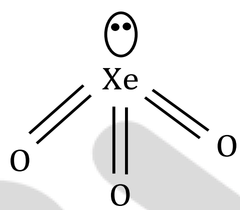
Xe has electronic configuration [Kr]$5s^2$ $5p^6$.

In case of XeF_4, the structure will be square planar. There are four pairs of bonding electrons and two lone pair in the molecule. Two lone pairs will be opposite each other to maintain symmetry.



XeF_4 (sp^3d^2 hybridization)

XeO_3 has trigonal pyramidal geometry with asymmetric charge distribution on central atom. There are three pairs of bonding electrons and one lone pair in the molecule.



XeO_3 (sp^3 hybridization)

33. (d)

$CO_2 + Ca(OH)_2 \longrightarrow CaCO_3 + H_2O$

Given, volume of $Ca(OH)_2$ = 500 mL

Concentration of $Ca(OH)_2$ = 0.5 M

Moles of $Ca(OH)_2$ = Volume(v) × concentration

$= \frac{500 \times 0.5}{1000} = 0.25$ mole

1 mole $Ca(OH)_2$ gives 1 mole $CaCO_3$

0.25 mole $Ca(OH)_2$ gives $= \frac{1}{1} \times 0.25$ mole $CaCO_3$ = 0.25 mole $CaCO_3$

Mass of $CaCO_3$ = mole × molar mass {$\therefore$ Molarmass of $CaCO_3$ is 100 g/mole}

= 0.25 mole × 100 g/mole = 25 g

34. (c)

$Zn|ZnSO_4(0.01m)||CuSO_4(1m)|Cu$

According to Nernst equation

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n} \log \frac{[anode]}{[cathode]}$$

$$= E^{\circ}_{cell} - \frac{0.059}{2} \log \frac{[Zn^{+2}]}{[Cu^{+2}]}$$

On substituting the concentration values

$$E_1 = E^{\circ}_{cell} - \frac{0.059}{2} \log \frac{0.01}{1}$$

$$E_1 = E^{\circ}_{cell} - \frac{0.059}{2} \log 10^{-2}$$

$$E_1 = E^{\circ}_{cell} - \frac{0.059}{2} \times (-2)$$

$$E_1 = E^{\circ}_{cell} + 0.059$$

On changing concentration of $ZnSO_4$ and $CuSO_4$, EMF of the cell is

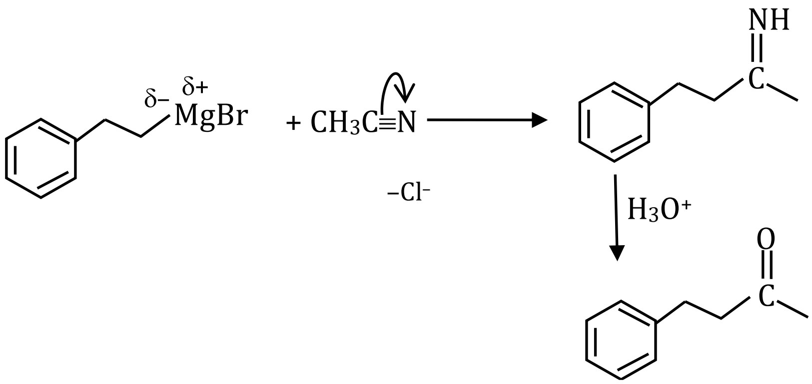
$$E_2 = E^{\circ}_{cell} - \frac{0.059}{2} \log \frac{1}{0.01}$$

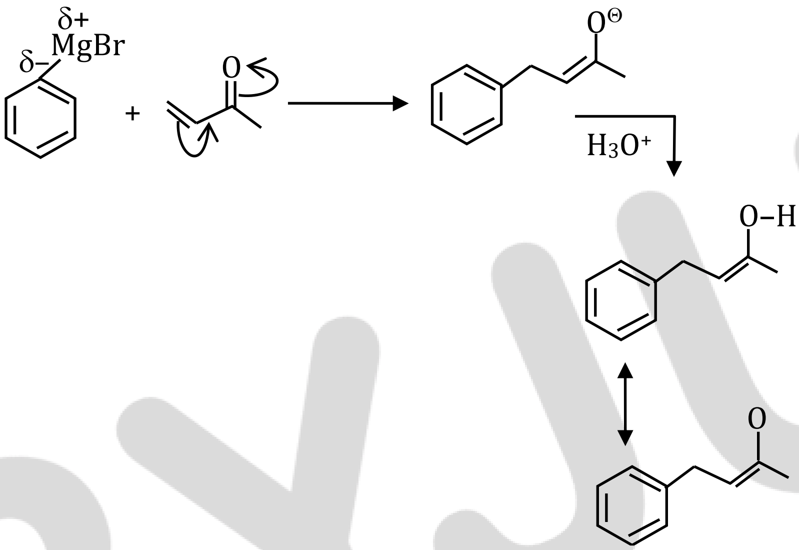
$$E_2 = E^{\circ}_{cell} - \frac{0.059}{2} \times 2$$

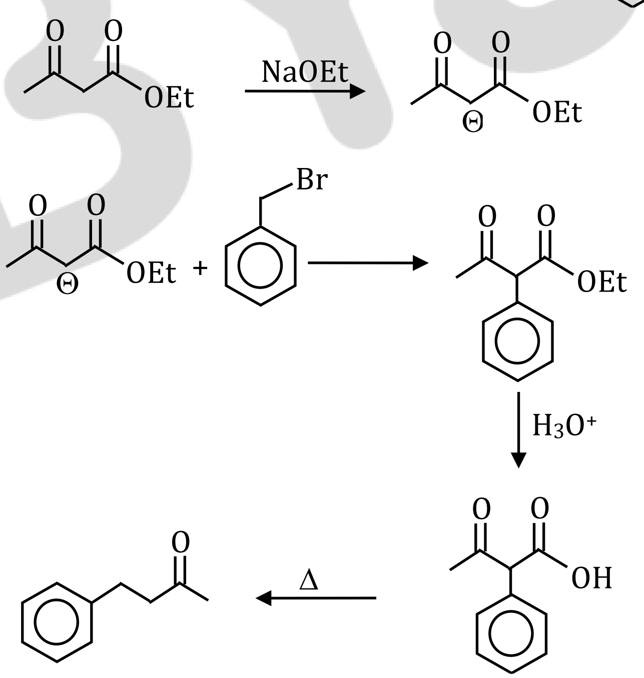
$$E_2 = E^{\circ}_{cell} - 0.059$$

Hence, $E_1 > E_2$

35. (d)

(A) 

(B) 

(C) 

(D) Phenyl lithium does not undergo 1, 4 addition reaction.

36. (a,b,d)

At the isoelectric point, the amino acid exists as zwitter ion to the maximum ion concentration.

A zwitter ion is dipolar ion, i.e.

$CH_3 - CH - COO^{\ominus}$
|
NH_3
$\oplus$

Zwitter ion(Alanine)

For amino acids that have no ionisable side chain, the p^I value is the average of its two p^{Ka}'s. If the amino acid has an ionisable side chain, the p^I value is the average of the p^{Ka}'s of similarly ionisable groups.

$CH_3 - CH - COOH$
|
NH_3
$\oplus$

$pI = pk_a + pK_{a2}$

Alanine has a net positive charge at pH below the isoelectric point.

37. (a,c)

(A) Electronic configuration of $_{24}Cr$: $[Ar]\ 3d^5\ 4s^1$, Due to extra stability.

(B) Magnetic quantum number may have negative value.

When $\ell = 2$, m = –2, –1, 0, +1, +2

(C) Silver atoms have atomic number 47.

38. (a,b,c)

Electronic configuration $_{47}Ag$: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6\ 4d^{10} 5s^1$

So, 23 electrons have a spin of one type and 24 of the opposite type.

In HN_3, oxidation state of N is $-\frac{1}{3}$

39. (b,d)

1. $FeSO_4 \rightarrow Fe^{2+} + SO_4^{2-}$

   $Fe^{2+} + 2e^- \rightarrow Fe(s)$

$\therefore$ 2F charge is needed for 1 mole deposition of Fe.

2. $Fe_2(SO_4)_3 \rightarrow 2Fe^{3+} + 3SO_4^{2-}$

   $Fe^{3+} + 3e^- \rightarrow Fe(s)$

   $3F \rightarrow 1$ mole

   2F charge is needed for $\frac{2}{3}$ mole deposition of Fe.

3. $Fe(NO_3)_3 \rightarrow Fe^{3+} + 3NO_3^-$

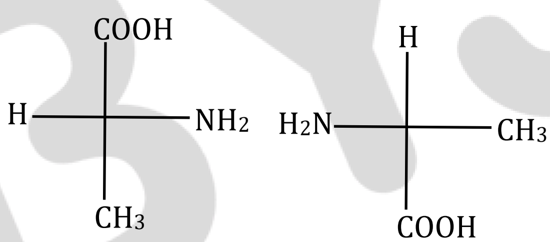
   $Fe^{3+} + 3e^- \rightarrow Fe(s)$

   $3F \rightarrow 1$ mole

2F charge is needed for 2/3 mole deposition of Fe.

Hence, the correct option is B & D.

40. (a,b,c)



R-form (I) S-form (II)

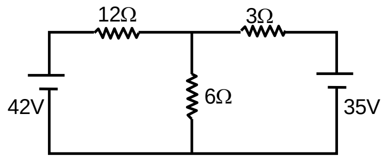
Enantiomers: Non-super imposable non mirror images are enantiomers. So, compound I and II are enantiomers.

Compound I and II are optically active due to absence of plane of symmetry.

Compound I is D-alanine while II is L-alanine.

WB-JEE-2016 (Physics)

1. The current flowing through the 3Ω resistor in the circuit is,



a. 4.2A
b. 3.0A
c. 3.5 A
d. 1.6A

2. A body attached to the lower end of a vertical spring oscillates with time period of 1 sec. The time period when two such springs are connected one below another is approximately

a. 0.7 sec
b. 1 sec
c. 1.4 sec
d. 2 sec

3. The minimum and maximum capacitances, which may be obtained by the combination of three capacitors each of capacitance 6μF are

a. 6 μF and 18μF
b. 2 μF and 18μF
c. 2 μF and 12μF
d. 6 μF and 12μF

4. A zener diode has break down voltage of 5.0 V. The resistance required to allow a current of 100 mA through the zener in reverse bias when connected to a battery of emf 12 V is

a. 50Ω
b. 70 Ω
c. 100 Ω
d. 150Ω

5. A series LCR circuit resonates at 10 kHZ. If the capacitor is 0.01 μF, the inductance used is approximately

a. 10 mH
b. 25 mH
c. 50 mH
d. 100 mH

6. Two charges of equal amount +Q are placed on a line. Another charge q is placed at the mid-point of the line. The system will be in equilibrium if the value of q is

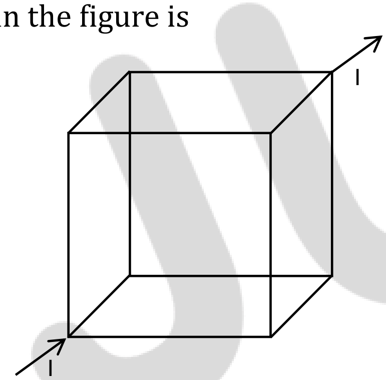
a. $-\frac{Q}{4}$
b. $-\frac{Q}{2}$
c. $+\frac{Q}{2}$
d. $+\frac{Q}{4}$

7. The increase in electrostatic potential energy of a dipole of moment p when it is taken from parallel to anti-parallel orientation in an electric field E is
   a. 2pE　　b. 3pE
   c. 5pE　　d. 6pE

8. An n-p-n transistor of current gain 80 in common emitter mode gives emitter current equal to 8.1 mA. The base current is
   a. 0.1μA　　b. 0.01mA
   c. 0.1mA　　d. 0.01μA

9. If a bar magnet of magnetic moment M is kept in a uniform magnetic field B, its time period of oscillation is T. In the same magnetic field, the time period of another magnet of same dimension and same mass but of moment M/4 is,
   a. T　　b. 2T
   c. T/2　　d. T/4

10. The magnetic field intensity at the centre of cubical cage of identical wires of length 'a' due to a current I flowing as shown in the figure is



   a. $\frac{2I}{a}$　　b. $\frac{\sqrt{2}I}{a}$
   c. 0　　d. I/2a

11. A micro-ammeter gives full scale deflection at 100 μA. Its resistance is 2kΩ. The resistance required to convert it to a voltmeter reading 1V is
   a. 1kΩ in series　　b. 8kΩ in series
   c. 1 kΩ in parallel　　d. 8kΩ in parallel

12. An electron of mass 'm' is accelerated by a potential difference V and the corresponding de-Broglie wavelength is λ. The de Broglie wavelength of a proton of mass M if it is accelerated by the same potential difference is ,
   a. $0\lambda\left(\frac{m}{M}\right)$　　b. $\lambda\left(\frac{M}{m}\right)$
   c. $\lambda\left(\frac{m}{M}\right)$　　d. $\lambda\sqrt{\frac{M}{m}}$

13. If I_1 and I_2 be the sizes of real images for two positions of a convex lens between object and screen, then the size of the object is

a. I_1/I_2
b. $I_1 I_2$
c. $\sqrt{I_1 I_2}$
d. $\sqrt{I_1 / I_2}$

14. Impedance of a coil having inductance 0.4 H at frequency of 50 Hz will be

a. $20\pi\Omega$
β. $40\pi\Omega$
c. $2\ \pi\Omega$
d. $4\pi\Omega$

15. 1000 drops of water of radius 1 cm each carry a charge of 10 esu combine to form a single drop. The capacitance increases thereby

a. 1 time
b. 10 times
c. 100 times
d. 1000 times

16. Light is an electromagnetic wave whose

a. $\vec{E}$ and $\vec{B}$ are parallel and both are perpendicular to the direction of propagation

b. $\vec{E}$ and $\vec{B}$ are mutually perpendicular and the direction of propagation is parallel to $\vec{E}$

c. $\vec{E}$ and $\vec{B}$ and the direction of propagation are mutually perpendicular

d. $\vec{E}$ and $\vec{B}$ are mutually perpendicular and the direction of propagation is parallel to $\vec{B}$

17. In Bohr model of an atom, two electrons move round the nucleus in circular orbits of radii in the ratio 1: 4. The ratio of their kinetic energies are

a. 1 : 4
b. 4 : 1
c. 8 : 1
d. 1 : 8

18. If a Young's double slit experiment were conducted inside water instead of air, the fringe width would

a. increase
b. decrease
c. remain same
d. become zero

19. A photon of energy 8eV is incident on a metal plate with threshold frequency 1.6×10^{15} Hz. The maximum kinetic energy of the emitted photo electrons is (given $h = 6 \times 10^{-34}$ Js)

a. 6 eV
b. 1.6eV
c. 1.2eV
d. 2 eV

20. The ratio of the masses of two planets is 2:3 and the ratio of their radii is 3:2. The ratio of acceleration due to gravity on these two planets is

a. 9 : 4
b. 4 :9
c. 2 : 3
d. 3 : 2

21. A mass m rests on another mass M. The co-efficient of static friction between the surfaces of m and M is μ. M rests on a smooth frictionless horizontal plane. The maximum force applied horizontally on M for which m will move along with M without slipping is,

a. mg + μmg
b. (M + m) μg
c. $\frac{mM}{M+m}\mu g$
d. μmg

22. The dimension of co-efficient of viscosity η is

a. $[MLT^{-2}]$
b. $[M^{-1}LT^{-1}]$
c. $[M^{-1}LT^{-2}]$
d. $[ML^{-1}T^{-1}]$

23. An explosive of mass 9 kg is divided in two parts. One part of mass 3 kg moves with velocity of 16 m/s. The kinetic energy of other part will be

a. 192 J
b. 162 J
c. 150 J
d. 200 J

24. The length of a wire is l_1 when tension is T_1 and is l_2 when tension is T_2. The length of the wire without any tension is

a. $\frac{I_2T_1 - I_1T_2}{T_I - T_2}$
b. $\frac{I_1T_1 - I_2T_2}{T_1 - T_2}$
c. $\frac{I_1T_1}{T_2 - T_1}$
d. $\frac{I_2T_2}{T_1 - T_2}$

25. A particle of mass m is moving in a circular orbit of radius r in a force field given by $\vec{F} = -\frac{k}{r^2}\hat{r}$. The angular momentum L of the particle about the centre varies as

a. $L \propto \sqrt{r}$
b. $L \propto r^{3/2}$
c. $L \propto \frac{1}{\sqrt{r}}$
d. $L \propto r^{1/3}$

WB-JEE-2016 (Physics)

26. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass, are initially heated to a temperature of 200°C. When left in air at room temperature, the following cools fastest.
    a. Sphere
    b. Cube
    c. Plate
    d. All of these

27. Two sound waves of wavelengths 50 cm and 51 cm produce 12 beats per second. The velocity of sound is
    a. $306 ms^{-1}$
    b. $331\ ms^{-1}$
    c. $340\ ms^{-1}$
    d. $360 ms^{-1}$

28. Air is expanded from 50 litre to 150 litre at 2 atmospheric pressure (1 atm pressure = 10^5 kgm^{-2}). The external work done is
    a. 200 J
    b. 2000 J
    c. 2×10^4 J
    d. 2×10^4 J

29. The Young's modulus of a metal is 2×10^{12} dyne/cm2 and its breaking stress is 11000 kg/cm^2. In case of longitudinal strain the maximum energy that can be stored per cubic metre of this metal is approximately (Assume $g = 10\ m/s^2$)
    a. 58.28×10^5 J
    b. 30.25×10^5 J
    c. 37.15×10^5 J
    d. 15.15×10^5 J

30. When a body moves in a circular orbit its total energy is
    a. positive
    b. negative
    c. Zero
    d. infinite

31. Equivalent resistance of the given infinite circuit is
    a. 4Ω
    b. 2Ω
    c. Infinity
    d. Greater than 4Ω but less than 6Ω

32. The ionization potential of hydrogen is −13.6 eV. The energy required to excite the electron from the first to the third orbit is approximately
    a. 10.2 J
    b. 12.09×10^{-6} J
    c. 19.94 J
    d. 19.34×10^{-19} J

33. Assume that the Earth rotates in a circular orbit round the Sun in 365 days. If the mass of the sun gets doubled but the radius of the orbit remains unchanged, the length of the year would be approximately
    a. 183 days
    b. 258 days
    c. 516 days
    d. 730 days

34. For the following set(s) of forces (in the same unit) the resultant can never be zero
   a. 10,10,10
   b. 10,10,20
   c. 10,20,30
   d. 10,20,40

35. A particle of mass 'm' and carrying a charge 'q' enters with a velocity 'v' perpendicular to a uniform magnetic field. The time period of rotation of the particle
   a. Decreases with increase of velocity v
   b. Increases with increase of radius of the orbit
   c. Depends only on magnetic field
   d. Depends on magnetic field and (q/m) of the particle

36. At any instant t current i through a coil of self-inductance 2 mH is given by $i = t^2e^{-t}$. The induced e.m.f. will be zero at time
   a. 1 sec
   b. 2 sec
   c. 3 sec
   d. 4 sec

37. A and B are two parallel sided transparent slabs of refractive indices n_1 and n_2 respectively. A ray is incident at an angle θ on the surface of separation of A and B, and after refraction from B into air grazes the surface of B. Then
   a. $\sin\theta = \frac{1}{n_2}$
   b. $\sin\theta = \frac{1}{n_1}$
   c. $\sin\theta = \frac{n_2}{n_1}$
   d. $\sin\theta = \frac{n_1}{n_2}$

38. The pair of parameters temperature T, pressure P, volume V and work W characterises the thermodynamic state of matter
   a. T, P
   b. T, V
   c. T, W
   d. P, W

39. Equation of a wave is given by $y = 10^{-4}\sin(60t + 2x)$, x and y in metre and t is in second. Then
   a. Wave is propagating along the negative x direction with velocity 30 m/s
   b. Wavelength is π metre
   c. Frequency is 30/π Hz
   d. Wave is propagating along positive x direction with velocity 60 m/s

40. An electric dipole is placed in a non-uniform electric field $\vec{E}$. The electric field is along x direction. The dipole will experience
   a. A torque when the dipole is parallel to $\vec{E}$
   b. A torque when the dipole makes an angle with $\vec{E}$
   c. A force perpendicular to $\vec{E}$
   d. A force when the dipole is parallel to $\vec{E}$

WB-JEE-2016 (Physics)

ANSWER KEY

1. (b)	2. (c)	3. (b)	4. (b)	5. (b)	6. (a)	7. (a)	8. (c)	9. (b)	10. (c)
11. (b)	12. (c)	13. (c)	14. (b)	15. (b)	16. (c)	17. (b)	18. (b)	19. (d)	20. (G)
21. (b)	22. (d)	23. (a)	24. (a)	25. (a)	26. (c)	27. (a)	28. (c)	29. (b)	30. (b)
31. (d)	32. (d)	33. (b)	34. (d)	35. (d)	36. (b)	37. (b)	38. (a, b)	9. (a, b, c)	40. (b, d)

WB-JEE-2016 (Physics)

SOLUTIONS

1. (b)

Applying Kvl in loop (1)

$-12i-6(i-i_1)+42=0$

In loop (2)

$-3i_1+6i_1-6i-35=0$

From loop (1) equation and (2) equation.

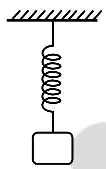
$$\Rightarrow \quad \begin{array}{l} -18i+6i+42=0 \\ -18i-27i-105=0 \\ \hline -21i \quad = \quad 63 \end{array}$$

i = – 3A

2. (c)

Time period of oscillation is given by

$T = 2\pi\frac{\sqrt{m}}{K}$



Where

m = mass

K = spring constant

First case:-

So according to question $T = 2\pi\sqrt{\frac{m}{K}}$

Second case:-

When two springs are connected one below another then they will be in series therefore

$$\frac{1}{K_{eq}}=\frac{1}{K}+\frac{1}{K}$$

$$\frac{1}{K_{eq}}=\frac{K}{2}$$

$$T'=\sqrt{\frac{m}{K_{eq}}}$$

$$K_{eq}=\frac{K}{2}$$

$$T'=\sqrt{2}\ T=1.414T$$

3. (b)

The minimum capacitance can be obtained by connecting all capacitor in series it can be calculated as follows:

$$\frac{1}{C}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1}{2}$$

$$\frac{1}{C}=\frac{3}{6}$$

C_{min} = 2μF

The maximum capacitance can be obtained by connecting all capacitors in parallel it can be calculated as follows:

C_{max} = 6 + 6 + 6 = 18μF

Therefore, the correct answer is (b)

4. (b)

Hence here the voltage remaining across the zener

= 12V – 5V = 7V

Therefore the resistance required to allow correct =

V = IR

$$R=\frac{V}{I} \quad \Rightarrow \quad \frac{7}{100\times10^{-3}}$$

$$R = 7 \times 10 \quad \Rightarrow \quad R = 70\Omega$$

5. (b)

Since Resonant frequency = $F=\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ ...... (1)

Given

E = 10 K Hz = 10^4Hz

C = 0.01 μF = 0.01 × 10^{-6} F

C = 1 × 10^{-8} F

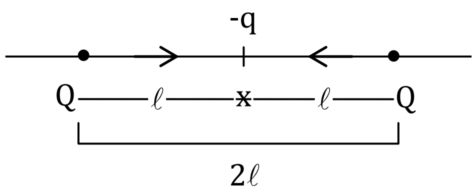
Substituting there values in equation (1)

$$10^4=\frac{1}{2\pi}\times\frac{1}{\sqrt{\cdot L\times10^{-8}}}$$

$$L \approx \frac{1}{40}H$$

$$L \approx 25mH$$

6. (a)



$$F_{QQ} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{(2\ell)^2} \qquad \text{...... (1)}$$

$$F_q Q = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{\ell^2} \qquad \text{...... (2)}$$

From eq. (1) and eq. (2)
Equilibrium condition eq.1 = 2

$$\frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{4\ell^2} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{\ell^2}$$

$q = \frac{-Q}{4}$ $\Rightarrow$ Therefore the correct answer is (a)

7. (a)

$U = -pE\cos\theta$

Dipole moment = p

Electric field = E

Now find the value of U_1 and U_2

$U_1 = -pE\cos 0$ [cos 0° = 1]

$U_1 = -pE \times 1$

$U_1 = -pE$

And

$U_2 = -pE \times \cos 180^\circ$

$U_2 = -pE \times (-1)$

$U_2 = +pE$

$\Delta U = (U_2 - U_1)$

$\Delta U \Rightarrow pE + pE = 2pE$

Increase in potential energy $\Delta U = 2pE$

8. (c)

$$B = \frac{I_C}{I_B} = 80 \quad \ldots\ldots (1)$$

$$I_e = 8.1\,mA$$

For CE configuration

$$I_E = I_C$$

$$I_C = 8.1\,mA$$

$$I_B = \frac{I_C}{B} = \frac{8.1\times10^{-3}\,A}{80}$$

$$I_B = 0.1\,mA$$

Therefore, the correct answer is (c)

9. (b)

Given value

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \ldots\ldots (1) \qquad \text{[I = moment of inertia]}$$

$$T' = 2\pi\sqrt{\frac{I}{\frac{M}{4}B}} = 2\times2\pi\sqrt{\frac{I}{MB}}$$

From eq. (1) $2\pi\sqrt{\frac{I}{MB}} = T$

Now

$$T' = 2T$$

Therefore, the correct answer is (b)

10. (c)

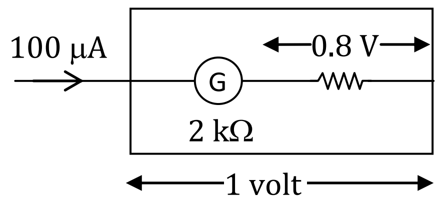
For symmetrical current distribution, magnetic field at the centre due to all current carrying conductors will get cancelled.

Hence $B_{net} = 0$

11. (b)

Since we know that to convert a Galvanometer in to voltmeter a resistance is connected in series with it. (Galvanometer)

$\because$ V = IR



I = 100μA (Given)

R = 2kΩ(Given)

$\therefore$ V = $100\times10^{-6}\times2\times10^{3}$

V = 0.2 volt

Resultant voltage across the resistance which is connected in series with galvanometer (or micro ammeter) will be = 1 – 0.2 = 0.8 volt.

$\therefore$ Now resistance required to convert in voltmeter.

$$I\times R=(0.8V)$$

$$10^{-4}\times R=0.8$$

$$R=(8\times10^{3})\Omega \text{ in series.}$$

12. (c)

$$K.E.=\frac{1}{2}mv^2$$

$$\Rightarrow \quad m\times\frac{1}{2}mv^2=v\times m$$

$$p^2=2vm$$

$$p=\sqrt{2vm}$$

De-Broglie wave length formula

$$\lambda=\frac{h}{p}$$

$$\lambda=\frac{h}{\sqrt{2vm}}$$

Same potential difference

$$\lambda\propto\frac{1}{\sqrt{m}}$$

Now

$$\frac{\lambda}{\lambda_p}=\frac{\sqrt{M}}{\sqrt{m}} \quad \Rightarrow \quad \lambda_p=\frac{\lambda\sqrt{m}}{\sqrt{M}}$$

$$\lambda_p=\sqrt{\frac{m}{M}}$$

Therefore, the correct answer is (c)

13. (c)

$\frac{I_1}{O} = \frac{v}{u}$ ...... (1)

$\frac{I_2}{O} = \frac{u}{v}$ ...... (2)

From eq. (1) and (2)

$\Rightarrow \quad \frac{I_1}{O} = \frac{O}{I_2}$

$\Rightarrow \quad O^2 = I_1 I_2$

$\Rightarrow \quad O = \sqrt{I_1 I_2}$

Therefore, the correct answer is (c)

14. (b)

$Z = \sqrt{(X_L - X_C)^2 + R^2}$ $\begin{bmatrix} X_C & = 0 \\ R & = 0 \end{bmatrix}$

Now

$Z = X_L \;= (\omega_L)$

$\omega = 2\pi F$

$Z = 2 \times \pi \times F \times L$

$Z = 2 \times \pi \times 50 \times 0.4$

$Z = 40\pi\Omega$

Therefore, the correct answer is (b)

15. (b)

$1\text{esu} = 3.33 \times 10^{-10} C$

Volume of single big drop = vol. of 1000 small drops

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times 1000$

$R^3 = \sqrt{1000\ r^3}$

$R = 10r$

Electric potential on surface of drop

$V_S = K\frac{q}{r}$ [K = Coulomb's constant]

Comparing with $Q = C \cdot V$

$C = \frac{q}{V_s} = \frac{q}{\frac{Kq}{r}}$

$C = \frac{r}{K}$ ...... (1)

Similarly capacitance of big drop

$C_b = \frac{R}{K}$ …… (2)

From eq. (1) and (2)

$\frac{C_b}{C} = \frac{R}{r}$

$\frac{C_b}{C} = \frac{10r}{r}$

$C_b = 10_C$ $\therefore$ Capacitance is 10 times

Therefore, the correct answer is (b)

16. (c)

As we know that the electromagnetic waves are the waves in which electric field $\vec{E}$, magnetic field $\vec{B}$ and the direction of propagation of electromagnetic wave are mutually perpendicular light is electromagnetic waves so it will also contains these properties.

17. (b)

Since Kinetic energy

$E \propto \frac{1}{r}$ where = r = radius

$E_2 r_2 = E_1 r_1$

$\frac{E_1}{E_2} = \frac{r_2}{r_1} \Rightarrow \frac{4}{1}$

Therefore, the correct answer is (b)

18. (b)

In water, speed of light decreases

Since frequency remains same, therefore the wavelength decrease

The formula for fringe width is $\frac{\lambda D}{d}$

Thus, fringe width decreases in water because wavelength of light decreases.

Therefore, the correct answer is (b)

19. (d)

v_0 = threshold frequency

$\phi = uv_0 = \frac{6\times10^{-34}\times1.6\times10^{15}}{1.6\times10^{-19}}ev$

$= 6\times10^{-34}\times10^{34}$

= 6ev

$K_{max} = E - \phi$

$= 8ev - 6ev = 2ev$

Therefore, the correct answer is (d)

20. (G)

Bonus

Acceleration due to gravity on the first planet

$$g_1 = \frac{GM_1}{R_1^2}$$

Acceleration due to gravity on the second planet

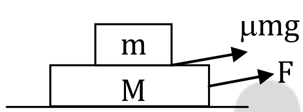
$$g_2 = \frac{GM_2}{R_2^2}$$

$$\frac{g_1}{g_2} = \frac{M_1}{M_2} \times \frac{R_2^2}{R_1^2}$$

$$\Rightarrow \quad \frac{2}{3} \times \left(\frac{2}{3}\right)^2 \quad \Rightarrow \quad \frac{8}{27}$$

Therefore, none of the options are correct

21. (b)



$$F - \mu N^1 = (m + M)a$$

$$a = \frac{F - \mu(m + M)g}{(m + M)}$$

$$a = \frac{f}{(m + M)} - \mu g \qquad \ldots\ldots (1)$$

Maximum value of acceleration

$$ma = \mu_1 mg$$

$$a = \mu_1 g$$

Value of 'a' put in eq. (1)

$$\frac{F}{m + M} - \mu_2 g = \mu_1 g$$

$$f_{max} = (\mu_1 g + \mu_2 g) \cdot (m + M) \qquad [\mu_1 + \mu_2 = a]$$

$$f_{max} = \mu g(m + M)$$

Therefore, the correct answer is (b)

22. (d)

Coefficient of viscosity $(\eta) = \frac{f_r}{A_v}$ ...... (1)

Where;
F = tangential force
r = distance between the layers,
v = velocity

Dimensional formula of $\Rightarrow$ Force $= M^1L^1T^{-2}$
Area $= M^0L^2T^0$
Distance $= M^0L^1T^0$
Velocity $= M^0L^1T^{-1}$

Put all these values in eq. (1)
We get,

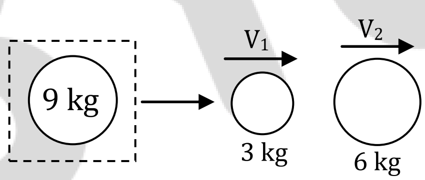
$$(\eta)=\frac{[M^1L^1T^{-2}][M^0L^1T^0]}{[M^0L^2T^0][M^0L^1T^{-1}]}$$

$$(\eta) \Rightarrow [M^1L^{-1}T^{-1}]$$

SI unit = Pascal second
Therefore the correct answer is (d)

23. (a)

Given mass of explosive = M = 9kg
Initial value of explosive = 0 m/s
Mass of smaller fragment = 3kg $\rightarrow m_1$
Mass of bigger fragment = 6kg $\rightarrow m_2$



$$3v_1 = 6v_2$$

$$v_2 = \frac{3v_1}{6}$$

$$= \frac{3\times16}{6} = 8\,m/s$$

$$(KE)6\,kg = \frac{1}{2}\times6\times(8)^2$$

=192 J

Therefore, the correct answer is (a)

24. (a)

Change in length in the first case = (l_1 -l)
Change in length in second case = (l_2 - l)

Now, young modulus $= \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

$$\gamma = \frac{T/A}{\Delta l / l}$$

Where
γ = Young's Modulus
T = Tension
A = Area
ΔL = Change in length
l = Original length

$\therefore \quad \gamma = \frac{T_1}{A} \times \frac{l}{(l_1 - l)}$ for first case

$\gamma = \frac{T_2}{A} \times \frac{l}{(l_1 - l)}$ for second case

Since young's modulus remains the same,
So,

$$\frac{T_1}{A} \times \frac{l}{(l_1 - l)} = \frac{T_2}{A} \times \frac{l}{(l_1 - l)}$$

$$T_1(l_2 - l) = T_2(l_2 - l)$$

$$l(T_2 - T_1) = T_2 l_1 - T_1 l_2$$

$$l = \frac{T_1 l_2 - T_2 I_1}{T_1 - T_2}$$

25. (a)

$$mv^2 r^2 = kr$$

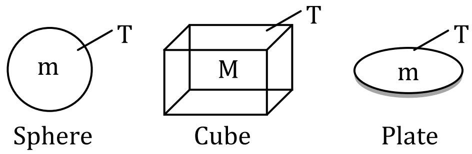
$$\Rightarrow \quad \frac{mv^2}{r} = \frac{k}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{mr}}$$

$$L = mvr = \sqrt{mrk}$$

Thus we get = $L \propto \sqrt{r}$

26. (c)



$$\rho = \frac{\text{mass}}{\text{volume}}$$

$\Rightarrow$ ρ = Same for all objects

As volume and mass same for 3 objects. Thickness of plate $\rightarrow$ Least $\rightarrow$surface area = Max.

According to Stefan's loss heat law $H \propto AT^4$

$H_{sphere} = H_{cube} = H_{plate} = A_{sphere} : A_{cube} : A_{plate}$

$\therefore$ Plate will cool fast

Sphere cools slowest

Therefore, the correct answer is (c)

27. (a)

Wavelength of first wave $\lambda_1 = 0.5\text{m}$

Wavelength of second wave $\lambda_2 = 0.51\text{m}$

Frequency of beat = 12

We know that

$$f = \frac{v}{\lambda_1} = \frac{u}{\lambda_2}$$

$$12 = v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$12 = v\left(\frac{1}{0.5} - \frac{1}{0.51}\right)$$

$$v = \frac{12 \times 0.50 \times 0.51}{0.51 - 0.50}$$

$= v = 306\text{m/s}$

Therefore, the correct answer is (a)

28. (c)

v_1 = 50 litre = 50 × 10^{-3}

v_2 = 150 litre = 150 ×10^{-3}

W = PΔV $\Rightarrow$ P ($V_2 - V_1$)

P = 2 atm = 2×10^5 N^{-2}

W = 2×10^5 (150-50) ×10^{-3}

W = 2×10^5 × 100 ×10^{-3}

W = 2 × 10^4 J

Therefore, the correct answer is (c)

29. (b)

Energy density, $\frac{du}{dv} = \frac{(\text{Stress})^2}{2y}$

Stress $= \frac{11000\text{ g}}{10^{-4}} \text{N/m}^2$

Now $\Rightarrow$ $\frac{\left(\frac{11000\text{g}}{10^{-4}}\right)^2}{2 \times 2 \times 10^{12} \times \frac{10^{-5}}{10^{-4}}}$

Stress = $\frac{121 \times 10^6 \times 10^2}{4 \times 10^{12} \times 10^{-1} \times 10^{-8}}$

$\Rightarrow \frac{30.25 \times 10^8}{10^{-8} \times 10^{12} \times 10^1}$

$\Rightarrow \frac{30.25 \times 10^{17}}{10^{12}}$

Stress $\Rightarrow 30.25 \times 10^5$J

Therefore, the correct answer is (b)

30. (b)

Suppose an object with mass m doing a circular orbit around a much heavier object with mass M.

Now we know its potential energy, it's

$U = -\frac{GMm}{R}$ ......(1)

As we know that the relation between linear velocity and angular velocity is given by

$\nu = \omega R$

$\nu^2 = \omega^2 R^2 = G\frac{M}{R}$ ......(2)

$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{R}$ ......(3)

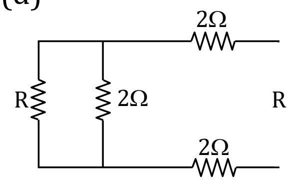
$\left[K = \frac{U}{2}\right]$

$E = K + U = \frac{U}{2} = -\frac{GMm}{2R}$ ......(4)

So the total energy is always negative

Therefore, the correct answer is (b)

31. (d)



$$R = 2 + 2 + \frac{R \times 2}{R + 2}$$

$\Rightarrow$ $R = 4 + \frac{2R}{2 + R}$

$\Rightarrow$ $2R + R^2 = 8 + 4R + 2R$

$\Rightarrow$ $R^2 - 4R - 8 = 0$

$\Rightarrow$ $R = 5.46\ \Omega$

Therefore, the correct answer is (d)

32. (d)

$E = \frac{-13.6}{n^2} = \frac{-13.6}{9} = -1.5\text{ev}$

We know that energy required

$\Delta E = E_2 - E_1$

$\Delta E = 1.51 + 13.6 \ = 12.08\text{ev}$

So, $\Delta E = 12.08\text{ev}$

$= 12.08 \times 1.6 \times 10^{-19}\text{J}$

$\Delta E = 19.34 \times 10^{-19}\text{J}$

Therefore, the correct answer is (d)

33. (b)

Since time period of revolution

$\because$ $T = 2\pi\sqrt{\frac{R}{g}}$

We know

$g = \frac{GM}{R^2}$

So, $T = 2\pi\sqrt{\frac{R^3}{GM}}$

$T \propto \frac{1}{\sqrt{M}}$

$\frac{T_1}{T_2} = \frac{\sqrt{2M}}{\sqrt{M}}$

$T_2 = \frac{T_1}{\sqrt{2}}$

$T_2 = \frac{360}{\sqrt{2}} = 258.13$

$T_2 \simeq 258.13$

34. (d)
Option (a)
a = 10, b = 10, c = 10
The resultant force is (a + b) < c
20 < 10
The resultant force is zero

Option (b)
a = 10, b = 10, c = 20
The resultant force is (a + b) < c
20 < 20
The resultant force is zero
Option (c)
a = 10, b = 20, c = 23
The resultant force is (a + b) < c
30 < 23
The resultant force is zero

Option (d)
a = 10, b = 20, c = 40
The resultant force is (a + b) < c
30 < 40
The resultant force is not zero

Therefore, the correct answer is (d)

35. (d)
$$T = \frac{2\pi m}{qB}$$
Depends on B and $\left(\frac{a}{m}\right)$ of particle
Therefore, the correct answer is (d)

36. (b)
$L = 2\times10^{-3}$ H,
$I = t^2 e^{-t}$
t = –?
At induced emf zero
$$\varepsilon = \frac{-LdI}{dt} = \frac{Ld(t^2e^{-t})}{dt} \quad [\varepsilon = 0]$$
$\varepsilon = -L[2t\, e^{-t} - t^2e^{-t}]$
$\Rightarrow \quad 2te^{-t} = t^2e^{-t}$
$\Rightarrow \quad \tau = 2$sec.
Therefore, the correct answer is (b)

37. (b)

$$n_1 \sin\theta = n_2 \sin\theta$$

Also

$$n_2 \sin\theta = 1 \times \sin 90$$

$$\Rightarrow \quad \sin\theta = \frac{1}{n_2}$$

$$n_1 \sin\theta = \frac{n_2}{n_2} = 1$$

$$\sin\theta = \frac{1}{n_1}$$

38. (a,b)

We know that

The relation between the thermodynamics variable (P,V,T) of the system is called equation of state where in –

PV = nRT

(P,V,T) represent thermodynamic state of matter work does not represent thermodynamic state of matter.

Therefore, the correct answer is (a,b)

39. (a,b,c)

$$V = \frac{dy}{dt} = 10 \times 60 \times \cos(60t + 2x)$$

$$\text{Velocity} = \frac{w}{K} = \frac{60}{2} = 30\,\text{m/s}$$

y is the form $A\sin(\omega t + kx)$, velocity is negative

K = 2, $\lambda = \frac{2\pi}{K}$

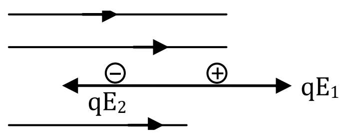
$$\Rightarrow \quad \lambda = \pi m$$

$$f = \frac{\omega}{2\pi} = \frac{60}{2\pi} = \frac{30}{\pi}\text{Hz}$$

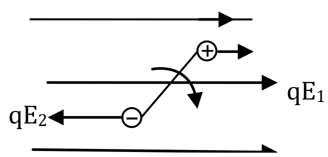
Therefore, the correct answer is (a,b,c)

40. (b,d)

Case – 1



Case – 2



In the question the electric field is in x-direction and a dipole is placed in this electric field.
Here we can consider 2 cases.

Case – 1 and Case – 2

In Case – 1

Dipole is placed parallel to the electric field so dipole will experience a net force and in this case the line of force is same. So there will be no torque for the dipole but as electric field is non uniform so there will be a net force on the dipole i.e.,

$F = q(E_1 - E_2) \neq 0$

In Case – 2

We placed the dipole in electric field (non-uniform) making an angle with electric field. So there will be definitely a torque working and that will not be equal to zero.

$E_1 = E_2$ or $E_1 \neq E_2$

$\tau \neq 0$