Compound Angle

1. The minimum value of $\cos\theta+\sin\theta+\dfrac{2}{\sin 2\theta}$ for $\theta\in(0,\pi/2)$, is

(A) $2+\sqrt{2}$ (B) 2 (C) $1+\sqrt{2}$ (D) $2\sqrt{2}$

2. If $\theta\varepsilon\left(\dfrac{\pi}{2},\dfrac{3\pi}{2}\right)$, then the value of

$\sqrt{4\cos^4\theta+\sin^2 2\theta}+4\cot\theta\cos^2\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)$ is

(A) $-2\cot\theta$ (B) $2\cot\theta$ (C) $2\cos\theta$ (D) $2\sin\theta$

3. The sum of the series $\sum_{n=1}^{\infty}\sin\left(\dfrac{n!\pi}{720}\right)$ is

(A) $\sin\left(\dfrac{\pi}{180}\right)+\sin\left(\dfrac{\pi}{360}\right)+\sin\left(\dfrac{\pi}{540}\right)$

(B) $\sin\left(\dfrac{\pi}{6}\right)+\sin\left(\dfrac{\pi}{30}\right)+\sin\left(\dfrac{\pi}{120}\right)+\sin\left(\dfrac{\pi}{360}\right)$

(C) $\sin\left(\dfrac{\pi}{6}\right)+\sin\left(\dfrac{\pi}{30}\right)+\sin\left(\dfrac{\pi}{120}\right)+\sin\left(\dfrac{\pi}{360}\right)+\sin\left(\dfrac{\pi}{720}\right)$

(D) $\sin\left(\dfrac{\pi}{180}\right)+\sin\left(\dfrac{\pi}{360}\right)$

4. The value of $\cos^2 75° + \cos^2 45° + \cos^2 15° - \cos^2 30° - \cos^2 60°$ is

(A) 0 (B) 1 (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{4}$

5. If α,β are the roots of the quadratic equation $x^2 + ax + b = 0$, $(b \neq 0)$, then the quadratic equation whose roots are $\alpha-\dfrac{1}{\beta},\beta-\dfrac{1}{\alpha}$, is

(A) $ax^2 + a(b-1)x + (a-1)^2 = 0$ (B) $bx^2 + a(b-1)x + (b-1)^2 = 0$

(C) $x^2 + ax + bv = 0$ (D) $abx^2 + bx + a = 0$

6. If $b_1b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2, are all real numbers then at least one of the equations $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ has

(A) real roots
(B) purely imaginary roots
(C) roots of the from a + ib ($a,b \in R$, $ab \neq 0$)
(D) rational roots

7. If p,q are odd integers, then the roots of the equation $2px^2 + (2p + q)x + q = 0$ are

(A) rational (B) irrational (C) non-real (D) equal

8. For real x, the greatest value of $\frac{x^2+2x+4}{2x^2+4x+9}$ is

(A) 1 (B) –1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

9. If α and β are the roots of $x^2 - px + 1 = 0$ and γ is a root of $x^2 + px + 1 = 0$, then $(\alpha + \gamma)(\beta + \gamma)$ is

(A) 0 (B) 1 (C) –1 (D) p

10. The least value of

$$2x^2 + y^2 + 2xy + 2x - 3y + 8 \text{ for real number x and y, is}$$

(A) 2 (B) 8 (C) 3 (D) –1/2

11. Let $f(x) = 2x^2 + 5x + 1$. If we write f(x) as $f(x) = a(x + 1)(x - 2) + b(x - 2)(x - 1) + c(x - 1)(x + 1)$ for real numbers, a, b, c then

(A) there are infinite number of choices for a, b, c
(B) only one choice for a but infinite number of choices for b and c
(C) exactly one choice for each of a, b, c
(D) more than one but finite number of choices for a, b, c

12. Let p, q be real numbers. If α is the root of $x^2 + 3p^2x + 5q^2 = 0$, β is a root of $x^2 + 9p^2x + 15q^2 = 0$ and $a < \alpha < \beta$, then the equation $x^2 + 6p^2x + 10q^2 = 0$ has a root γ that always satisfies

(A) $\gamma = \frac{\alpha}{4} + \beta$ (B) $\beta < \gamma$ (C) $\gamma = \frac{\alpha}{2} + \beta$ (D) $\alpha < \gamma < \beta$

13. Let p(x) be a quadratic polynomial with constant term 1. Suppose p(x), when divided by x–1 leaves remainder 2 and when divided by x + 1 leaves remainder 4. Then, the sum of the roots of p(x) = 0 is

(A) –1 (B) 1 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

14. If $(\alpha + \sqrt{\beta})$ and $(\alpha - \sqrt{\beta})$ are the roots of the equation $x^2 + px + q = 0$, where α, β p and q are real, the roots of the equation $(p^2 - 4q)(p^2x^2 + 4px) - 16q = 0$ are

(A) $\left(\frac{1}{\alpha}+\frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\alpha}-\frac{1}{\sqrt{\beta}}\right)$ (B) $\left(\frac{1}{\sqrt{\alpha}}+\frac{1}{\beta}\right)$ and $\left(\frac{1}{\sqrt{\alpha}}-\frac{1}{\beta}\right)$

(C) $\left(\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\sqrt{\alpha}}-\frac{1}{\sqrt{\beta}}\right)$ (D) $\left(\sqrt{\alpha}+\sqrt{\beta}\right)$ and $\left(\sqrt{\alpha}-\sqrt{\beta}\right)$

15. The quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign. Then,

(A) $a \le 0$ (B) $0 < a < 4$ (C) $4 \le a < 8$ (D) $a \ge 8$

16. Let α, β be the roots of $x^2 - x - 1 = 0$ and $S_n = \alpha^n + \beta^n$, for all integers $n \ge 1$. Then, for every integer $n \ge 2$.

(A) $S_n + S_{n-1} = S_{n+1}$ (B) $S_n - S_{n-1} = S_{n+1}$ (C) $S_{n-1} = S_{n+1}$ (D) $S_n + S_{n-1} = 2S_{n+1}$

17. The equation $x^{(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5} = 3\sqrt{3}$ has

(A) at least one real root (B) exactly one real root

(C) exactly one irrational root (D) complex roots

18. If $\sin\alpha$, $\cos\alpha$ be the roots of the equation $x^2 - bx + c = 0$. Then, which of the following statements is/are correct ?

(A) $c \le \frac{1}{2}$ (B) $b \le \sqrt{2}$ (C) $c > \frac{1}{2}$ (D) $b \le \sqrt{2}$

Complex Number

19. The equation $z\bar{z} + (2-3i)z + (2+3i)\bar{z} + 4 = 0$ represents a circle of radius

(A) 2 unit (B) 3 unit (C) 4 unit (D) 6 unit

20. If z_1, z_2, z_3 are imaginary numbers such that

$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

(A) equal to 1 (B) less than 1 (C) grater than 1 (D) equal to 3

21. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64} + \left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{64}$ is

(A) 0 (B) –1 (C) 1 (D) i

22. Let z_1, z_2 be two fixed complex numbers in the argand plane and z be an arbitrary point satisfying $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$. Then the locus of z will be

(A) an ellipse
(B) a straight line joining z_1 and z_2
(C) a parabola
(D)a bisector of the line segment joining z_1 and z_2

23. If z = x + iy, where x and y are real numbers and $i = \sqrt{-1}$, then the points (x,y) for which $\frac{z-1}{z-i}$ is real, lie on

(A) an ellipse
(B) a circle
(C) a parabola
(D) a straight line

24. If $\frac{z-1}{z+1}$ is purely imaginary, then

(A) $|z|\frac{1}{2}$
(B) $|z| = 1$
(C) $|z| = 2$
(D) $|z| = 3$

Sequence series

25. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1, then it moves $\frac{1}{2}$ unit vertically up and reaches P_2, then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3, then it moves $\frac{1}{8}$ unit vertically down and reaches P_4, then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on, Let $P_n = (x_n, y_n)$ and $\lim_{n\to\infty} x_n = \alpha$ and $\lim_{n\to\infty} y_n = \beta$. Then, (α, β) is

(A) (2, 3)
(B) $\left(\frac{4}{3}, \frac{2}{5}\right)$
(C) $\left(\frac{2}{5}, 1\right)$
(D) $\left(\frac{4}{5}, 3\right)$

26. Given that n numbers of arithmetic means are inserted between two sets of numbers a,2b and 2a, b where $a, b \in R$. Suppose further that the mth means between these sets of numbers are same, then the ratio a : b equals

(A) n – m + 1 : m
(B) n – m + 1 : n
(C) n : n – m + 1
(D) m : n – m + 1

27. If x is a positive real number different from 1 such that $\log_a x$, $\log_b x$, $\log_c x$ are in AP, then

(A) $b = \frac{a+c}{2}$
(B) $b = \sqrt{ac}$
(C) $c^2 (ac)^{\log_a b}$
(D) None of these

28. The value of

$$1000\left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+....+\frac{1}{999\times 1000}\right]$$ is

(A) 1000
(B) 999
(C) 1001
(D) $\frac{1}{999}$

29. If $x = 1 + \frac{1}{2 \times 1!} + \frac{1}{4 \times 2!} + \frac{1}{8 \times 3!} + ...$ and $y = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + ...$

Then, the value of $\log_e y$ is

(A) e (B) e^2 (C) 1 (D) $\frac{1}{e}$

30. The value of the infinite series $\frac{1^2 + 2^2}{3!} + \frac{1^2 + 2^2 + 3^2}{4!} + \frac{1^2 + 2^2 + 3^2 + 4^2}{5!} + ...$ is

(A) e (B) 5e (C) $\frac{5e}{6} - \frac{1}{2}$ (D) $\frac{5e}{6}$

31. The value of $\sum_{r=2}^{\infty} \frac{1 + 2 + \quad + (r-1)}{r!}$

(A) e (B) 2e (C) $\frac{e}{2}$ (D) $\frac{3e}{2}$

32. The sum of the infinite series $1 + \frac{1}{3} + \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{3 \cdot 6 \cdot 9 \cdot 12} + ...$

is equal to

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{\frac{3}{2}}$ (D) $\sqrt{\frac{1}{3}}$

33. If 64,27,36 are the Pth Qth and Rth terms of a GP, then P + 2Q is equal to

(A) R (B) 2R (C) 3R (D) 4R

Set

34. If A = $\{5^n - 4n - 1 : n \in N\}$ and B = $\{16(n - 1) : n \in N)$, then

(A) A = B (B) $A \cap B = \phi$ (C) $A \subseteq B$ (D) $B \subseteq A$

35. Let the number of elements of the sets A and B be p and q, respectively. Then, the number of relations from the set A to the set B is

(A) 2^{p+q} (B) 2^{pq} (C) p + q (D) pq

Relation

36. Let ρ be a relation defined on N, the set of natural number, as

$\rho = \{(x,y) \in N \times N : 2x + y = 41\}$. Then

(A) ρ is an equivalence relation

(B) ρ is only reflexive relation

(C) ρ is only symmetric relation

(D) ρ is not transitive

37. In the set of all 3 × 3 real matrices a relation is defined as follows. A matrix A is related to a matrix B, if and only if there is a non-singular 3 × 3 matrix P, such that $B = P^{-1} AP$. This relation is

(A) reflexive, symmetric but not transitive

(B) reflexive, transitive but not symmetric

(C) symmetric, transitive but not reflexive

(D) an equivalence relation.

Limit

38. If $\lim_{x\to 0}\left(\frac{1+cx}{1-cx}\right)^{1/x} = 4$, then $\lim_{x\to 0}\left(\frac{1+2cx}{1-2cx}\right)^{1/x}$ is

(A) 2 (B) 4 (C) 16 (D) 64

39. Let for all x > 0, f (x) = $\lim_{x\to\infty} n\left(x^{1/n}-1\right)$, then

(A) $f(x)+f\left(\frac{1}{x}\right)=1$

(B) f(xy) = f(x) + f(y)

(C) f(xy) = xf(y) + yf(x)

(D) f(xy) = xf(x) + yf(y)

40. Let $x_n = \left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2\left(1-\frac{1}{10}\right)^2 \ldots\ldots \left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2, n \geq 2$, Then, the value of $\lim_{n\to\infty} x_n$ is

(A) 1/3 (B) 1/9 (C) 1/81 (D) 0

41. Let f : R → R be differentiable at x = 0. If f(0) = 0 and f'(0) = 2, then the value of

$\lim_{x\to 0}\frac{1}{x}\left[f(x)+f(2x)+f(3x)+\ldots+f(2015x)\right]$ is

(A) 2015 (B) 0 (C) 2015 × 2016 (D) 2015 × 2014

42. The limit of $\left[\frac{1}{x^2}+\frac{(2013)^x}{e^x-1}-\frac{1}{e^x-1}\right]$ as $x \to 0$

(A) approaches $+\infty$

(B) approaches $-\infty$

(C) is equal to $\log_e(2013)$

(D) does not exist

43. The limit of $x \sin\left(e^{\frac{1}{x}}\right)$ as $x \to 0$

(A) is equal to 0

(B) is equal to 1

(C) is equal to $\frac{e}{2}$

(D) does not exist

Continuity & Differentiability

44. Let [x] denotes the greatest integer less than or equal to x. Then, the value of α for which the function f(x) = $\begin{cases} \dfrac{\sin\left[-x^2\right]}{\left[-x^2\right]}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$ is continuous at x = 0, is

(A) $\alpha = 0$ (B) $\alpha = \sin(-1)$ (C) $\alpha = \sin(1)$ (D) $\alpha = 1$

45. Let f : R $\to$ R be defined as

$$f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$$

Then, which of the following is true?

(A) f is discontinuous for all x

(B) f is continuous for all x

(C) f is discontinuous at x = kπ, where k is an integer

(D) f is continuous at x = kπ, where k is an integer

46. Let f : R $\to$ R be such that f(2x – 1) = f(x) for all x$\in$R. If f is continuous at x = 1 and f(1) = 1, then

(A) f(2) = 1 (B) f(2) = 2

(C) f is continuous only at x = 1 (D) f is continuous at all points

Method of differentiation

47. Let y = $\dfrac{x^2}{(x+1)^2(x+2)}$ = Then, $\dfrac{d^2y}{dx^2}$ is

(A) $2\left[\dfrac{3}{(x+1)^4} - \dfrac{3}{(x+1)^3} + \dfrac{4}{(x+2)^3}\right]$ (B) $3\left[\dfrac{2}{(x+1)^3} + \dfrac{4}{(x+1)^2} - \dfrac{5}{(x+2)^3}\right]$

(C) $\dfrac{6}{(x+1)^3} - \dfrac{4}{(x+1)^2} + \dfrac{3}{(x+1)^3}$ (D) $\dfrac{7}{(x+1)^3} - \dfrac{3}{(x+1)^2} + \dfrac{2}{(x+1)^3}$

48. If f(x) = $\tan^{-1}\left[\dfrac{\log\left(\dfrac{e}{x^2}\right)}{\log\left(ex^2\right)}\right] + \tan^{-1}\left[\dfrac{3+2\log x}{1-6\log x}\right]$ then the value of f”(x) is equal to

(A) x^2 (B) x (C) 1 (D) 0

Application of derivative

49. If the tangent to the curve $y^2 = x^3$ at (m^2, m^3) is also a normal to the curve at (M^2, M^3), then the value of mM is

(A) $-\frac{1}{9}$ (B) $-\frac{2}{9}$ (C) $\frac{-1}{3}$ (D) $\frac{-4}{9}$

50. The equation x log x = 3 – x

(A) has no root in (1,3) (B) has exactly one root is (1,3)

(C) xlogx – (3 – x) > 0 in[1,3] (D) xlogx – (3 – x) < 0 in [1,3]

51. Let f(x) be a differentiable function in [2, 7]. If f(2) = 3 f'(x) ≤ 5 for all x in (2, 7), then the maximum possible value of f(x) at x = 7 is

(A) 7 (B) 15 (C) 28 (D) 14

52. For the curve $x^2 + 4xy + 8y^2 = 64$ the tangents are parallel to the x-axis only at the points

(A) $\left(0,2\sqrt{2}\right)$and$\left(0,-2\sqrt{2}\right)$ (B) (8, – 4) and (–8, 4)

(C) $\left(8\sqrt{2},-2\sqrt{2}\right)$ and $\left(-8\sqrt{2},2\sqrt{2}\right)$ (D) (9,0) and (–8, 0)

53. If $f(x) = e^x(x - 2)^2$, then

(A) f is increasing in (–∝,0) and (2, ∝) and decreasing in (0,2)

(B) f is increasing in (–∝,0) and decreasing in (0,∝)

(C) f is increasing in (2,∝) and decreasing in (–∝,0)

(D) f is increasing in (0, 2) and decreasing in (–∝,0) and (2, ∝)

54. Let exp (x) denote the exponential function e^x. If $f(x) = \exp\left(x^{\frac{1}{x}}\right)$, x > 0, then the minimum value of f in the interval [2,5] is

(A) $\exp\left(e^{\frac{1}{e}}\right)$ (B) $\exp\left(2^{\frac{1}{2}}\right)$ (C) $\exp\left(5^{\frac{1}{5}}\right)$ (D) $\exp\left(3^{\frac{1}{3}}\right)$

55. If f(x) = x $\left(\frac{1}{x-1}+\frac{1}{x}+\frac{1}{x+1}\right), x > 1$. Then,

(A) f(x) ≤ 1 (B) 1 < f(x) ≤ 2 (C) 2 < f(x) ≤ 3 (D) f(x) > 3

56. Maximum value of the function f(x) = $\frac{x}{8}+\frac{2}{x}$ on the interval [1,6] is

(A) 1 (B) $\frac{9}{8}$ (C) $\frac{13}{12}$ (D) $\frac{17}{8}$

57. If f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, then
(A) f(x) must be an increasing function (B) f(x) must be a decreasing function
(C) |f(x)| must be an increasing function (D) |f(x)|must be a decreasing function.

58. Rolle's theorem is applicable in the interval [–2,2] for the function
(A) $f(x) = x^3$ (B) $f(x) = 4x^4$ (C) $f(x) = 2x^3 + 3$ (D) $f(x) = \pi|x|$

59. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$, then assuming k as an integer,
(A) f(x) increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$
(B) f(x) decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$
(C) f(x) decreases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$
(D) f(x) increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

60. Let f be any continuously differentiable function on [a,b] twice differentiable on (a,b) such that $f(a) = f'(a) = 0$ and $f(b) = 0$. Then,
(A) $f''(a) = 0$ (B) $f'(x) = 0$ for some $x \in (a,b)$
(C) $f''(x) \neq 0$ for some $x \in (a,b)$ (D) $f'''(x) = 0$ for some $x \in (a,b)$

Indefinite Integration

61. If $\int e^{\sin x} \cdot \left[\frac{x\cos^3 x - \sin x}{\cos^2 x}\right] dx = e^{\sin x} f(x) + c$, where c is constant of integration, then f(x) is equal to
(A) sec x – x (B) x – sec x (C) tan x – x (D) x – tan x

62. $\int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx \, (x > 0)$ is
(A) $\tan^{-1}\left(x + \frac{1}{x}\right) + C$ (B) $\tan^{-1}\left(x - \frac{1}{x}\right) + C$
(C) $\log_e\left|\frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1}\right| + C$ (D) $\log_e\left|\frac{x - \frac{1}{x} - 1}{x - \frac{1}{x} + 1}\right| + C$

63. The value of $\int \frac{(x-2)}{\left\{(x-2)^2(x+3)^7\right\}^{1/3}}dx$ is

(A) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3}+C$ (B) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{3/4}+C$

(C) $\frac{5}{12}\left(\frac{x-2}{x+3}\right)^{4/3}+C$ (D) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{5/3}+C$

Definite Integration

64. The value of

$\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$ is equal to

(A) 27 (B) 54 (C) –54 (D) 0

65. The value of the integral $I=\int_{1/2014}^{2014}\frac{\tan^{-1}x}{x}dx$ is

(A) $\frac{\pi}{4}\log 2014$ (B) $\frac{\pi}{2}\log 2014$ (C) π log 2014 (D) $\frac{1}{2}\log 2014$

66. The value of $\lim_{x\to\infty}\frac{1}{n}\left\{\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+...+\sec^2\frac{n\pi}{4n}\right\}$ is

(A) $\log_e 2$ (B) $\frac{\pi}{2}$ (C) $\frac{4}{\pi}$ (D) e

67. The value of

$\lim_{x\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+...+\frac{1}{2n}\right]$ is

(A) $\frac{n\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{4n}$ (D) $\frac{\pi}{2n}$

68. The value of $\lim_{n\to\infty}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+.....+\sqrt{2n-1}}{n^{3/2}}\right\}$ is

(A) $\frac{2}{3}\left(2\sqrt{2}-1\right)$ (B) $\frac{2}{3}\left(\sqrt{2}-1\right)$ (C) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (D) $\frac{2}{3}\left(2\sqrt{2}+1\right)$

69. Let f : R $\rightarrow$ R be a continuous function which satisfies $f(x)=\int_0^x f(t)\,dt$ Then, the value of $f(\log_e 5)$ is

(A) 0 (B) 2 (C) 5 (D) 3

70. $\lim\limits_{x\to\infty}\dfrac{\sqrt{1}+\sqrt{2}+....+\sqrt{n-1}}{n\sqrt{n}}$ is equal to

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) 0

71. If $I=\int_0^2 e^{x^4}(x-\alpha)dx=0,$ then α lies in the interval

(A) (0, 2) (B) (–1, 0) (C) (2, 3) (D) (–2, –1)

72. The value of the integral

$\int_{-1}^{1}\left\{\dfrac{x^{2013}}{e^{|x|}\left(x^2+\cos x\right)}+\dfrac{1}{e^{|x|}}\right\}dx$ is equal to

(A) 0 (B) $1 - e^{-1}$ (C) $2e^{-1}$ (D) $2(1 - e^{-1})$

73. The value of $I=\int_0^{\frac{\pi}{4}}\left(\tan^{n+1}x\right)dx+\dfrac{1}{2}\int_0^{\frac{\pi}{2}}\tan^{n-1}\left(\dfrac{x}{2}\right)dx$ is

(A) $\dfrac{1}{n}$ (B) $\dfrac{n+2}{2n+1}$ (C) $\dfrac{2n-1}{n}$ (D) $\dfrac{2n-3}{3n-2}$

74. If $[\alpha]$ denote the greatest integer which is less than or equal to a. Then, the value of the integer $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[\sin x\cos x]\,dx$ is

(A) $\dfrac{\pi}{2}$ (B) π (C) $-\pi$ (D) $-\dfrac{\pi}{2}$

75. The value of the integral $\int_0^{\pi/2}\dfrac{1}{1+(\tan x)^{101}}dx$ is equal to

(A) 1 (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{8}$ (D) $\dfrac{\pi}{4}$

76. The value of the integral $\int_0^{\pi/4}\dfrac{\sin x+\cos x}{3+\sin 2x}dx$

(A) $\log_e 2$ (B) $\log_e 3$ (C) $\dfrac{1}{4}\log_e 2$ (D) $\dfrac{1}{4}\log_e 3$

77. The value of $\lim\limits_{n\to\infty}\dfrac{(n!)^{\frac{1}{n}}}{n}$ is

(A) 1 (B) $\dfrac{1}{e^2}$ (C) $\dfrac{1}{2e}$ (D) $\dfrac{1}{e}$

78. The value of the integral

$$\int_1^5 [|x-3|+|1-x|]dx$$ is equal to

(A) 4 (B) 8 (C) 12 (D) 16

79. Let [x] denote the greatest integer less than or equal to x, then the value of the integral

$\int_{-1}^{1}(|x|-2[x])\,dx$ is equal to

(A) 3 (B) 2 (C) –2 (D) –3

Area Under Curve

80. The area of the region bounded by the curves $y = x^2$ and $x = y^2$ is

(A) 1/3 (B) 1/2 (C) 1/4 (D) 3

81. The area of the region bounded by the parabola $y = x^2 - 4x + 5$ and the straight line $y = x + 1$ is

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) $\frac{9}{2}$

82. The area of the region, bounded by the curves
$y = \sin^{-1} x + x(1 - x)$ and
$y = \sin^{-1} x - x(1 - x)$ in the first quadrant, is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Differential Equation

83. Let f be a differentiable function with $\lim\limits_{x\to\infty} f(x) = 0$. If $y' + yf'(x) - f(x)f'(x) = 0$, $\lim\limits_{x\to\infty} y(x) = 0$, then (where $y' = \frac{dy}{dx}$)

(A) $y + 1 = e^{f(x)} + f(x)$ (B) $y - 1 = e^{f(x)} + f(x)$
(C) $y + 1 = e^{-f(x)} + f(x)$ (D) $y - 1 = e^{-f(x)} + f(x)$

84. The general solution of the differential equation $\left(1+e^{\frac{x}{y}}\right)dx + \left(1-\frac{x}{y}\right)e^{x/y}dy = 0$ is (C is an arbitrary constant)

(A) $x - ye^{\frac{x}{y}} = C$ (B) $y - xe^{\frac{x}{y}} = C$

(C) $x + ye^{\frac{x}{y}} = C$ (D) $y + xe^{\frac{x}{y}} = C$

85. The integrating factor of the first order differential equation

$x^2(x^2-1)\dfrac{dy}{dx}+x(x^2+1)y=x^2-1$ is

(A) e^x (B) $x-\dfrac{1}{x}$ (C) $x+\dfrac{1}{x}$ (D) $\dfrac{1}{x^2}$

86. The solution of the differential equation

$\dfrac{dy}{dx}+\dfrac{y}{x\log_e x}=\dfrac{1}{x}$

under the condition y = 1 when x = e is

(A) $2y=\log_e x+\dfrac{1}{\log_e x}$ (B) $y=\log_e x+\dfrac{2}{\log_e x}$

(C) $y\log_e x=\log_e x+1$ (D) $y=\log_e x+e$

87. The general solution of the differential equation $\dfrac{dy}{dx}=\dfrac{x+y+1}{2x+2y+1}$ is

(A) $\log_e|3x+3y+2|+3x+6y=C$ (B) $\log_e|3x+3y+2|-3x+6y=C$

(C) $\log_e|3x+3y+2|-3x-6y=C$ (D) $\log_e|3x+3y+2|+3x-6y=C$

88. The integrating factor of the differential equation $3x\log_e x\dfrac{dy}{dx}+y=2\log_e x$ is given by

(A) $(\log_e x)^3$ (B) $\log_e(\log_e x)$ (C) $\log_e x$ (D) $(\log_e x)^{1/3}$

89. Let y be the solution of the differential equation $x\dfrac{dy}{dx}=\dfrac{y^2}{1-y\log x}$ satisfying y(1) = 1. Then, y satisfies

(A) $y=x^{y-1}$ (B) $y=x^y$ (C) $y=x^{y+1}$ (D) $y=x^{y+2}$

Matrix and Determinant

90. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix or order 3. Then, the matrix $A-3I_3$ is

(A) invertible (B) orthogonal

(C) non-invertible (D) real Skew Symmetric matrix

91. If the following three linear equations have a non-trivial solution, then

$x+4ay+az=0$

$x+3by+bz=0$

$x+2cy+cz=0$

(A) a,b,c are in AP (B) a,b,c are in GP

(C) a,b,c are in HP (D) a + b + c = 0

92. If the polynomial

$$f(x)=\begin{vmatrix}(1+x)^a & (2+x)^b & 1\\ 1 & (1+x)^a & (2+x)^b\\ (2+x)^b & 1 & (1+x)^a\end{vmatrix},$$ then the constant term of f(x) is

(A) $2-3\cdot 2^b+2^{3b}$
(B) $2+3\cdot 2^b+2^{3b}$
(C) $2+3\cdot 2^b-2^{3b}$
(D) $2-3\cdot 2^b-2^{3b}$

[a and b are positive integers]

93. Let $A=\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$ Then, for positive integer n, A^n is

(A) $\begin{pmatrix}1&n&n^2\\0&n^2&n\\0&0&n\end{pmatrix}$

(B) $\begin{pmatrix}1&n&n\left(\frac{n+1}{2}\right)\\0&1&n\\0&0&1\end{pmatrix}$

(C) $\begin{pmatrix}1&n^2&n\\0&n&n^2\\0&0&n^2\end{pmatrix}$

(D) $\begin{pmatrix}1&n&2n-1\\0&\frac{n+1}{2}&n^2\\0&0&\frac{n+1}{2}\end{pmatrix}$

94. Let A be a 3 × 3 matrix and B be its adjoint matrix. If |B| = 64, then |A| is equal to

(A) ±2 (B) ±4 (C) ±8 (D) ±12

95. The value of λ such that the system of equations
$2x - y - 2z = 2$; $x - 2y + z = -4$;
$x + y + \lambda = 4$, has no solution, is

(A) 3 (B) 1 (C) 0 (D) –3

96. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ equals

(A) 2AB (B) 2BA (C) A + B (D) AB

97. If ω is an imaginary cube root of unity, then the value of the determinant

$$\begin{vmatrix}1+\omega & \omega^2 & -\omega\\ 1+\omega^2 & \omega & -\omega^2\\ \omega+\omega^2 & \omega & -\omega^2\end{vmatrix}$$ is

(A) -2ω (B) $-3\omega^2$ (C) –1 (D) 0

98. For a matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$, If U_1, U_2 and U_3 are 3 × 1 column matrices satisfying

$AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and U is 3 × 3 matrix whose columns are U_1,U_2 and U_3,

Then, sum of the elements of U^{-1} is

(A) 6 (B) 0 (C) 1 (D) 2/3

99. If $p = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $X = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ Then, P^3X is equal to

(A) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ (C) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (D) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

100. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ $Q = PP^T$ then the value of the determinant of Q is

(A) 2 (B) –2 (C) 1 (D) 0

101. Consider the system of equations

$x + y + z = 0 \quad \alpha x + \beta y + \gamma z = 0$

$\alpha^2 x + \beta^2 y + \gamma^2 z = 0$

Then, the system of equation has

(A) a unique solution for all values of α,β and γ

(B) infinite number of solutions, if any two of α,β,γ are equal

(C) a unique solution, if α,β and γ are distinct.

(D) more than one, but finite number of solutions depending on value of α,β, and γ.

Permutation & Combination

102. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B, each containing 6 questions and he/she is not permitted to attempt more then 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions ?

(A) 850 (B) 800 (C) 750 (D) 700

103. There are 7 greeting cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is,

(A) ${}^{7}C_3$ (B) $2\ {}^{7}C_3$ (C) $3!\ {}^{4}C_4$ (D) $3!\ {}^{7}C_3\ {}^{4}C_3$

104. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

(A) 210 (B) 25200 (C) 2520 (D) 302400

105. The number of ways in which the letters of the word ARRANGE can be permuted such that the R's occur together, is

(A) $\frac{7!}{2!2!}$ (B) $\frac{7!}{2!}$ (C) $\frac{6!}{2!}$ (D) $5! \times 2!$

106. A vehicle registration number consists of 2 letters of English alphabet followed by 4 digits, where the first digit is not zero. Then, the total number of vehicles with distinct registration number is

(A) $26^2 \times 10^4$ (B) ${}^{26}P_2 \times {}^{10}P_4$ (C) ${}^{26}P_2 \times 9 \times {}^{10}P_3$ (D) $26^2 \times 9 \times 10^3$

107. On the occasion Dipawali festival each student of a class sends greeting cards to others. If there are 20 student in the class, the number of cards sends by students is

(A) ${}^{20}C_2$ (B) ${}^{20}P_2$ (C) $2 \times {}^{20}C_2$ (D) $2 \times {}^{20}P_2$

Binomial Theorem

108. If $c_0, c_1, c_2, \ldots\ldots, c_{15}$ are the binomial coefficients in the expansion fo $(1 + x)^{15}$, then the value of $\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \ldots.. + 15\frac{c_{15}}{c_{14}}$ is

(A) 1240 (B) 120 (C) 124 (D) 140

109. Let $(1+ x + x^2)^9 = a_0 + a_1 x + a_2x^2 + \ldots.. + a_{18}x^{18}$. Then,

(A) $a_0 + a_2 + \ldots.+ a_{18} = a_1 + a_3 + \ldots..+ a_{17}$

(B) $a_0 + a_2 + \ldots + a_{18}$ is even

(C) $a_0 + a_2 + \ldots\ldots.+ a_{18}$ is divisible by 9

(D) $a_0 + a_2 + \ldots.+ a_{18}$ is divisible by 3 but not by 9

110. If n is even positive integer, then the condition that the greatest term in the expansion of $(1 + x)^n$ may also have the greatest coefficient, is

(A) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (B) $\frac{n}{n+1} < x < \frac{n+1}{n}$

(C) $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$ (D) $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

111. $1 + {}^nC_1 \cos\theta + {}^nC_2 \cos 2\theta + \ldots + {}^nC_n \cos n\theta$ equals

(A) $\left(2\cos\frac{\theta}{2}\right)^n \cos\frac{n\theta}{2}$ (B) $2\cos^2\frac{n\theta}{2}$

(C) $2\cos^{2n}\frac{\theta}{2}$ (D) $\left(2\cos^2\frac{\theta}{2}\right)^n$

112. The number of irrational terms in the binomial expansion of $(3^{1/5} + 7^{1/3})^{100}$ is

(A) 90 (B) 88 (C) 93 (D) 95

113. If x and y are digits such that 17 ! = 3356xy428096000, then x + y equals

(A) 15 (B) 6 (C) 12 (D) 13

114. The sum of the series $1+\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2+\ldots+\frac{1}{n+1}{}^nC_n$ is equal to

(A) $\frac{2^{n+1}-1}{n+1}$ (B) $\frac{3\left(2^n-1\right)}{2n}$ (C) $\frac{2^n+1}{n+1}$ (D) $\frac{2^n+1}{2n}$

115. Let $(1 + x)^{10} = \sum_{r=0}^{10} c_r x^r$ and $(1 + x)^7 = \sum_{r=0}^{7} d_r x^r$. If $P = \sum_{r=0}^{5} c_{2r}$ and $Q = \sum_{r=0}^{3} d_{2r+1}$ then $\frac{P}{Q}$ is equal to

(A) 4 (B) 8 (C) 16 (D) 32

Probability

116. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game. What is the probability that A wins if A begins ?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D) $\frac{8}{15}$

117. The probability that a non-leap year selected at random will have 53 Sunday is

(A) 0 (B) 1/7 (C) 2/7 (D) 3/7

118. In a Certain town, 60% of the families own a car, 30% own a house and 20% own both car and house. if a family is randomly chosen, then what is the probability that this family owns a car or a house but not both?

(A) 0.5 (B) 0.7 (C) 0.1 (D) 0.9

119. A fair coin is tossed at a fixed number of times. If the probability of getting exactly 3 heads equals the probability of getting exactly 5 heads, then the probability of getting exactly one head is

(A) 1/64 (B) 1/32 (C) 1/16 (D) 1/8

120. If the letters of the word 'PROBABILITY' are written down at random in a row, then probability that two B's are together, is

(A) $\frac{2}{11}$ (B) $\frac{10}{11}$ (C) $\frac{3}{11}$ (D) $\frac{6}{11}$

121. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

(A) $\frac{3p}{4p+3}$ (B) $\frac{5p}{3p+2}$ (C) $\frac{5p}{4p+1}$ (D) $\frac{4p}{3p+1}$

122. Each of a and b can take values 1 or 2 with equal probability. The probability that the equation $ax^2 + bx + 1 = 0$ has real roots, is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

123. Cards are drawn one-by-one without replacement from a well shuffled pack of 52 cards. Then, the probability that a face card (jack, queen or king) will appear for the first time on the third turn is equal to

(A) $\frac{300}{2197}$ (B) $\frac{36}{85}$ (C) $\frac{12}{85}$ (D) $\frac{4}{51}$

124. An urn contains 8 red and 5 white balls. Three balls are drawn at random. Then, the probability that balls of both colour are drawn is.

(A) $\frac{40}{143}$ (B) $\frac{70}{143}$ (C) $\frac{3}{13}$ (D) $\frac{10}{13}$

Straight Line

125. Let each of the equations $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ represent two straight lines passing through the origin. If they have a common line, then the other two lines are given by

(A) $x - y = 0, x - 3y = 0$ (B) $x + 3y = 0, 3x + y = 0$
(C) $3x + y = 0, 3x - y = 0$ (D) $(3x - 2y) = 0, x + y = 0$

126. A straight line through the point (3 – 2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If it intersects the X-axis, then its equation will be

(A) $y + x\sqrt{3} + 2 + 3\sqrt{3} = 0$ (B) $y - x\sqrt{3} + 2 + 3\sqrt{3} = 0$

(C) $y - x\sqrt{3} - 2 - 2\sqrt{3} = 0$ (D) $x - x\sqrt{3} + 2 - 3\sqrt{3} = 0$

127. Let d_1 and d_2 be the lengths of the perpendicular drawn from any point of the line $7x - 9y + 10 = 0$ upon the lines $3x + 4y = 5$ and $12x + 5y = 7$, respectively. Then,

(A) $d_1 > d_2$ (B) $d_1 = d_2$ (C) $d_1 < d_2$ (D) $d_1 = 2d_2$

128. Let S be the set of points, whose abscissae and ordinates are natural numbers. Let $P \in S$, such that the sum of the distance of P from (8, 0) and (0, 12) is minimum among all elements in S. Then, the number of such points P in S is

(A) 1 (B) 3 (C) 5 (D) 11

129. The line AB cuts off equal intercepts 2a from the axes. From any point P on the line AB perpendiculars PR and PS are drawn on the axes. Locus of mid-point of RS is

(A) $x - y = \frac{a}{2}$ (B) $x + y = a$ (C) $x^2 + y^2 = 4a^2$ (D) $x^2 - y^2 = 2a^2$

130. The line through the points (a, b) and (–a, –b), passes through the point

(A) (1, 1) (B) (3a, – 2b) (C) (a^2, ab) (D) (a, b)

131. For the variable t, the locus of the points of intersection of lines $x - 2y = t$ and $x + 2y = \frac{1}{t}$ is

(A) the straight line x = y

(B) circle with centre at the origin and radius 1

(C) the ellipse with centre at the origin and one focus $\left(\frac{2}{\sqrt{5}}, 0\right)$

(D) the hyperbola with centre at the origin and one focus $\left(\frac{\sqrt{5}}{2}, 0\right)$

132. The number of lines which pass through the point (2,–3) and are at a distance 8 from the point (–1,2) is

(A) infinite (B) 4 (C) 2 (D) 0

133. The line joining $A(b\cos\alpha, b\sin\alpha)$ and $B(a\cos\beta, a\sin\beta)$, where $a \neq b$, is produced to the point M(x,y) so that AM : MB = b : a. Then, $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2}$ is equal to

(A) 0 (B) 1 (C) –1 (D) $a^2 + b^2$

134. Let P(2,–3), Q(–2,1) be the vertices of the ΔPQR. If the centroid of ΔPQR lies on the line $2x + 3y = 1$, then the locus of R is

(A) $2x + 3y = 9$ (B) $2x - 3y = 7$ (C) $3x + 2y = 5$ (D) $3x - 2y = 5$

135. If a straight line passes through the pint (α,β) and the portion of the line intercepted between the axes is divided equally at that point, then $\frac{x}{\alpha}+\frac{y}{\beta}$ is

(A) 0 (B) 1 (C) 2 (D) 4

136. A straight line through the point of intersection of the lines $x + 2y = 4$ and $2x + y = 4$ meets the coordinate axes at A and B. The locus of the mid-point of AB is

(A) $3(x + y) = 2xy$ (B) $2(x + y) = 3xy$
(C) $2(x + y) = xy$ (D) $x + y = 3xy$

137. The coordinates of a point on the line $x + y + 1 = 0$, which is at a distance $\frac{1}{5}$ unit from the line $3x + 4y + 2 = 0$, are

(A) (2, –3) (B) (–3, 2) (C) (0, –1) (D) (–1, 0)

Circle

138. A variable circle passes through the fixed point A(p, q) and touches X-axis. The locus of the other end of the diameter through A is

(A) $(x - p)^2 = 4qy$ (B) $(x - q)^2 = 4py$ (C) $(x - p)^2 = 4qx$ (D) $(y - q)^2 = 4px$

139. If one of the diameter of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is (2,–3), the radius of S is

(A) $\sqrt{41}$ unit (B) $3\sqrt{5}$ unit (C) $5\sqrt{2}$ unit (D) $2\sqrt{5}$ unit.

140. The locus of the mid-points of the chords of the circle $x^2 + y^2 + 2x - 2y - 2 = 0$, which make an angle of 90° at the centre is

(A) $x^2 + y^2 - 2x - 2y = 0$ (B) $x^2 + y^2 - 2x + 2y = 0$
(C) $x^2 + y^2 + 2x - 2y = 0$ (D) $x^2 + y^2 + 2x - 2y - 1 = 0$

141. A point P lies on the circle $x^2 + y^2 = 169$. If Q = (5,12) and R = (–12,5), then the $\angle QPR$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

142. The equations of the circles, which touch both the axes and the line $4x + 3y = 12$ and have centres in the first quadrant, are

(A) $x^2 + y^2 + x - y + 1 = 0$ (B) $x^2 + y^2 - 2x - 2y + 1 = 0$
(C) $x^2 + y^2 -12x - 12y + 36 = 0$ (D) $x^2 + y^2 -6x - 6y + 36 = 0$

Parabola

143. Let P(at^2, 2at), Q,R(ar^2, 2ar) be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are parallel where the co-ordinates of K is (2a,0), then the value of r is

(A) $\frac{t}{1-t^2}$ (B) $\frac{1-t^2}{t}$ (C) $\frac{t^2+1}{t}$ (D) $\frac{t^2-1}{t}$

144. A line passing through the point of intersection of x + y = 4 and x – y = 2 makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the X-axis. It intersects the parabola $y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2), respectively. Then, $|x_1 - x_2|$ is equal to

(A) $\frac{16}{9}$ (B) $\frac{32}{9}$ (C) $\frac{40}{9}$ (D) $\frac{80}{9}$

145. The locus of the mid-points of all chords of the parabola $y^2 = 4ax$ through its vertex is another parabola with directrix

(A) x = – a (B) x = a (C) x = 0 (D) $x = -\frac{a}{2}$

146. If y = 4x + 3 is parallel to a tangent to the parabola $y^2 = 12x$ then its distance from the normal parallel to the given line is

(A) $\frac{213}{\sqrt{17}}$ (B) $\frac{219}{\sqrt{17}}$ (C) $\frac{211}{\sqrt{17}}$ (D) $\frac{210}{\sqrt{17}}$

147. The value of λ for which the curve $(7x + 5)^2 + (7y + 3)^2 = \lambda^2(4x + 3y - 24)^2$ represents a parabola is

(A) $\pm\frac{6}{5}$ (B) $\pm\frac{7}{5}$ (C) $\pm\frac{1}{5}$ (D) $\pm\frac{2}{5}$

148. The equation $y^2 + 4x + 4y + k = 0$ represents a parabola whose latusrectum is

(A) 1 (B) 2 (C) 3 (D) 4

Ellipse

149. Consider the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The portion of the tangent at any point of the curve intercepted between the point of contact and the directrix subtends at the corresponding focus an angle of

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

150. B is an extremity of the minor axis of an ellipse whose foci are S and S'. If $\angle SBS'$ is a right angle, then the eccentricity of the ellipse is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

151. The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abscissa increases is/are given by

(A) $\left(3,\frac{16}{3}\right)$ and $\left(-3-\frac{16}{3}\right)$ (B) $\left(3,-\frac{16}{3}\right)$ and $\left(-3\frac{16}{3}\right)$

(C) $\left(\frac{1}{16},\frac{1}{9}\right)$ and $\left(-\frac{1}{16},-\frac{1}{9}\right)$ (D) $\left(\frac{1}{16},-\frac{1}{9}\right)$ and $\left(-\frac{1}{16},\frac{1}{9}\right)$

152. Lines x + y = 1 and 3y = x + 3 intersect the ellipse $x^2 + 9y^2 = 9$ at the points P,Q and R. The area of the ΔPQR is

(A) $\frac{36}{5}$ (B) $\frac{18}{5}$ (C) $\frac{9}{5}$ (D) $\frac{1}{5}$

153. For the variable, the locus of the point of intersection of the line 3tx – 2y + 6t = 0 and 3x + 2ty – 6 = 0 is

(A) the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ (B) the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

(C) the ellipse $\frac{x^2}{4}-\frac{y^2}{9}=1$ (D) the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$

154. The locus of the mid-point of the chords of an ellipse $x^2 + 4y^2 = 4$ that are drawn from the positive end of the minor axis, is

(A) a circle with centre $\left(\frac{1}{2},0\right)$ and radius 1

(B) a parabola with focus $\left(\frac{1}{2},0\right)$ and directrix x = –1

(C) an ellipse with centre $\left(0,\frac{1}{2}\right)$, major axis 1 and minor axis $\frac{1}{2}$

(D) a hyperbola with centre $\left(0,\frac{1}{2}\right)$, transverse axis 1 and conjugate axis $\frac{1}{2}$

155. The eccentric angle in the first quadrant of a point on the ellipse $\frac{x^2}{10}+\frac{y^2}{8}=1$ at a distance 3 unit from the centre of the ellipse is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

156. If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ units on the line $y - 2x = 1$, then a is equal to

(A) 1 (B) –2 (C) –1 (D) 2

Hyperbola

157. A double ordinate PQ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is such that ΔOPQ is equilateral, O being the centre of the hyperbola. Then the eccentricity e satisfies the relation

(A) $1 < e < \frac{2}{\sqrt{3}}$ (B) $e = \frac{2}{\sqrt{3}}$ (C) $e = \frac{\sqrt{3}}{2}$ (D) $e > \frac{2}{\sqrt{3}}$

158. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

(A) $x = 3 \pm \sqrt{\frac{13}{6}}$ (B) $x = 3 \pm \sqrt{\frac{6}{13}}$ (C) $x = 6 \pm \sqrt{\frac{13}{3}}$ (D) $x = 6 \pm \sqrt{\frac{3}{13}}$

159. A hyperbola, having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

(A) $x^2\sin^2\theta - y^2\cos^2\theta = 1$ (C) $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$

(C) $(x^2 + y^2)\sin^2\theta = 1 + y^2$ (D) $x^2\operatorname{cosec}^2\theta = x^2 + y^2 + \sin^2\theta$

160. Let $16x^2 - 3y^2 - 32x - 12y = 44$ represents a hyperbola . Then,

(A) length of the transverse axis is $2\sqrt{3}$ (B) length of each latusrectum is $32/\sqrt{3}$

(C) eccentricity is $\sqrt{19/3}$ (D) equation of a directrix is $x = \frac{\sqrt{19}}{3}$

Vector

161. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} - \lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, the value of λ is

(A) 0 (B) 1 (C) 2 (D) –4

162. For non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} + \vec{b}| < |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(A) collinear (B) perpendicular to each other

(C) inclined at an acute angle (D) inclined at an obtuse angle

3D

163. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$ is

(A) $\frac{2\sqrt{3}}{5}$ (B) $\frac{\sqrt{2}}{10}$ (C) $\frac{4}{5\sqrt{2}}$ (D) $\frac{\sqrt{5}}{6}$

164. The equation of the plane through the point (2, –1, –3) and parallel to the lines $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is

(A) $8x+14y+13z+37=0$ (B) $8x-14y-13z-37=0$

(C) $8x-14y-13z+37=0$ (D) $8x-14y+13z+37=0$

165. The foot of the perpendicular drawn from the point (1,8,4) on the line joining the point (0,–11,4) and (2,–3,1) is

(A) (4,5,2) (B) (–4,5,2) (C) (4,–5,2) (D) (4,5,–2)

166. A straight line joining the points (1, 1, 1) and (0, 0, 0) intersects the plane $2x+2y+z=10$ at

(A) (1, 2, 5) (B) (2, 2, 2) (C) (2, 1, 5) (D) (1, 1, 6)

167. The value of λ for which the straight line $\frac{x-\lambda}{3}=\frac{y-1}{2+\lambda}=\frac{z-3}{-1}$ may lie on the plane $x-2y=0$, is

(A) 2 (B) 0 (C) $-\frac{1}{2}$ (D) there is no such λ

SOT

168. Let p,q and r be the altitudes of a triangle with area S and perimeter 2t. Then, the value of $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ is

(A) $\frac{S}{t}$ (B) $\frac{t}{S}$ (C) $\frac{S}{2t}$ (D) $\frac{2S}{t}$

Trigonometric Equation

169. The general value of the real angle θ, which satisfies the equation,

$(\cos\theta + i\sin\theta)(\cos2\theta + i\sin2\theta)\ldots\ldots$
$(\cos n\theta + i\sin n\theta) = 1$ is given by, (asuming k is an integer)

(A) $\frac{2k\pi}{n+2}$ (B) $\frac{4k\pi}{n(n+1)}$ (C) $\frac{4k\pi}{n+1}$ (D) $\frac{6k\pi}{n(n+1)}$

170. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is

(A) 0 (B) 1 (C) 2 (D) 3

171. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then general value of θ is

(A) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (B) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$

(C) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (D) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

Function

172. If $f : S \to R$, where S is the set of all non-singular matrices of order 2 over R and $f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$, then

(A) f is bijective mapping (B) f is one-one but not onto

(C) f is onto but not one-one (D) f is neither one-one nor onto

173. The domain of definition of $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$ is

(A) $(-\infty, -1) \cup (2, \infty)$ (B) $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$

(C) $(-\infty, 1) \cup (2, \infty)$ (D) $[-1, 1] \cup (2, \infty)$

Here $(a,b) \equiv \{x : a < x < b\}$ and

$[a,b] \equiv \{x : a \le x \le b\}$

174. Consider the function y = $\log_a\left(x + \sqrt{x^2+1}\right)$ $a > 0, a \ne 1$. The inverse of the function

(A) does not exist (B) is $x = \log_{1/a}\left(y + \sqrt{y^2+1}\right)$

(C) is x = sinh (y log a) (D) is x = $\cosh\left(-y\log\frac{1}{a}\right)$

175. The total number of injection (one-one into mappings) from $\{a_1,a_2,a_3,a_4\}$ to $\{b_1,b_2,b_3,b_4,b_5,b_6,b_7\}$ is

(A) 400 (B) 420 (C) 800 (D) 840

176. Let R be the set of real numbers and the function $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$. Then the value of x for which $f(g(x)) = g(f(x))$ is

(A) –1 (B) 0 (C) 1 (D) 2

ITF

177. If $0 \le A \le \frac{\pi}{4}$, then

$\tan^{-1}\left(\frac{1}{2}\tan 2A\right)$ + $\tan^{-1}$ (cot A) + $\tan^{-1}$ ($\cot^3$ A) is equal to

(A) $\frac{\pi}{4}$ (B) π (C) 0 (D) $\frac{\pi}{2}$

178. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + ...\right) = \frac{\pi}{6}$, where |x| < 2, then the value of x is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$

179. Which of the following real valued functions is are not even functions?

(A) $f(x) = x^3 \sin x$

(B) $f(x) = x^2 \cos x$

(C) $f(x) = e^x x^3 \sin x$

(D) f(x) = x – [x], where [x] denotes the greatest integer less than or equal to x.

Statistics

180. Mean of n observations $x_1, x_2, ...\ x_n$ is $\bar{x}$. If an observation x_q is replaced by x_q' then the new mean is

(A) $\bar{x} - \bar{x}_q + x_q'$ (B) $\frac{(n-1)\bar{x} + x_q'}{n}$ (C) $\frac{(n-1)\bar{x} - x_q'}{n}$ (D) $\frac{n\bar{x} - x_q + x_q'}{n}$

181. Standard deviation of n observations $a_1, a_2, a_3, ..., a_n$ is σ. Then, the standard deviation of the observations $\lambda a_1, \lambda a_2, ...\lambda a_n$ is

(A) $\lambda\sigma$ (B) $-\lambda\sigma$ (C) $|\lambda|\sigma$ (D) $\lambda^n\sigma$

182. The variance of first 20 natural number is

(A) 133/4 (B) 279/12 (C) 133/2 (D) 399/4

Logarithm

183. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$ then x lies in the interval

(A) $(2, \infty)$ (B) (1, 2) (C) (–2, –1) (D) None of these

Mathematical Induction

184. let a, b, c and d be any four real numbers Then, $a^n + b^n = c^n + d^n$ holds for any natural number n, if

(A) $a + b = c + d$

(B) $a - b = c - d$

(C) $a + b = c + d, a^2 + b^2 = c^2 + d^2$

(D) $a - b = c - d, a^2 - b^2 = c^2 - d^2$

Answer Key

1.	(A)	**2.**	(B)	**3.**	(C)	**4.**	(C)	**5.**	(B)
6.	(A)	**7.**	(A)	**8.**	(C)	**9.**	(A)	**10.**	(D)
11.	(C)	**12.**	(D)	**13.**	(D)	**14.**	(A)	**15.**	(B)
16.	(A)	**17.**	(AC)	**18.**	(A,B)	**19.**	(B)	**20.**	(A)
21.	(B)	**22.**	(A)	**23.**	(D)	**24.**	(B)	**25.**	(B)
26.	(D)	**27.**	(C)	**28.**	(B)	**29.**	(A)	**30.**	(C)
31.	(C)	**32.**	(B)	**33.**	(C)	**34.**	(C)	**35.**	()
36.	(D)	**37.**	(D)	**38.**	(C)	**39.**	(B)	**40.**	(B)
41.	(C)	**42.**	(A)	**43.**	(A)	**44.**	(C)	**45.**	(D)
46.	(C)	**47.**	(A)	**48.**	(D)	**49.**	(D)	**50.**	(B)
51.	(C)	**52.**	(B)	**53.**	(A)	**54.**	(C)	**55.**	(D)
56.	(D)	**57.**	(D)	**58.**	(B)	**59.**	(AC)	**60.**	(BC)
61.	(B)	**62.**	(A)	**63.**	(D)	**64.**	(A)	**65.**	(B)
66.	(C)	**67.**	(B)	**68.**	(A)	**69.**	(A)	**70.**	(C)
71.	(A)	**72.**	(D)	**73.**	(A)	**74.**	(D)	**75.**	(D)
76.	(D)	**77.**	(D)	**78.**	(C)	**79.**	(A)	**80.**	(A)
81.	(D)	**82.**	(C)	**83.**	(C)	**84.**	(C)	**85.**	(B)
86.	(A)	**87.**	(D)	**88.**	(D)	**89.**	(B)	**90.**	(C)
91.	(C)	**92.**	(A)	**93.**	(B)	**94.**	(C)	**95.**	(D)
96.	(C)	**97.**	(B)	**98.**	(B)	**99.**	(C)	**100.**	(A)
101.	(B,C)	**102.**	(A)	**103.**	(B)	**104.**	(B)	**105.**	(C)
106.	(D)	**107.**	(BC)	**108.**	(B)	**109.**	(B)	**110.**	(A)
111.	(A)	**112.**	(*)	**113.**	(A)	**114.**	(A)	**115.**	(B)
116.	(D)	**117.**	(B)	**118.**	(A)	**119.**	(B)	**120.**	(A)
121.	(C)	**122.**	(C)	**123.**	(C)	**124.**	(D)	**125.**	(B)
126.	(B)	**127.**	(B)	**128.**	(B)	**129.**	(B)	**130.**	(C)
131.	(D)	**132.**	(D)	**133.**	(A)	**134.**	(A)	**135.**	(C)
136.	(B)	**137.**	(B,D)	**138.**	(A)	**139.**	(A)	**140.**	(C)
141.	(B)	**142.**	(B,C)	**143.**	(D)	**144.**	(B)	**145.**	(C)
146.	(B)	**147.**	(B)	**148.**	(D)	**149.**	(C)	**150.**	(B)
151.	(A)	**152.**	(B)	**153.**	(A)	**154.**	(C)	**155.**	(B)
156.	(A,B)	**157.**	(D)	**158.**	(A)	**159.**	(B)	**160.**	(ABC)
161.	(D)	**162.**	(D)	**163.**	(B)	**164.**	(*)	**165.**	(D)
166.	(B)	**167.**	(D)	**168.**	(B)	**169.**	(B)	**170.**	(A)
171.	(A)	**172.**	(D)	**173.**	(B)	**174.**	(C)	**175.**	(D)
176.	(A)	**177.**	(C)	**178.**	(A)	**179.**	(CD)	**180.**	(D)
181.	(C)	**182.**	(A)	**183.**	(A)	**184.**	(D)		