WB-JEE-2015 (Mathematics)

1. The value of $\lim_{x\to 2}\int_2^x \frac{3t^2}{x-2}$ is
   a. 12
   b. 10
   c. 8
   d. 16

2. If $\cot\frac{2x}{3}+\tan\frac{x}{3}=\operatorname{cosec}\frac{kx}{3}$, then the value of k is
   a. 1
   b. 2
   c. 3
   d. –1

3. If $\theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$, then the value of $\sqrt{4\cos^4\theta+\sin^2 2\theta}+4\cot\theta\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ is
   a. $-2\cot\theta$
   b. $2\cot\theta$
   c. $2\cos\theta$
   d. $2\sin\theta$

4. The number of real solutions of the equation $(\sin x - x)(\cos x - x^2) = 0$ is
   a. 1
   b. 2
   c. 3
   d. 4

5. Which of the following is not always true?
   a. $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other
   b. $|\vec{a}+\lambda\vec{b}|\ge|\vec{a}|$ for all $\lambda\in R$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other
   c. $|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|^2=2(|\vec{a}|+|\vec{b}|^2)$
   d. $|\vec{a}+\lambda\vec{b}|\ge|\vec{a}|$ for all $\lambda\in R$ if $\vec{a}$ is parallel to $\vec{b}$

6. If the four points with position vectors $-2\hat{i}+\hat{j}+k, \hat{i}+\hat{j}+\hat{k}, \hat{j}-\hat{k}$ and $\lambda\hat{j}+k$ are coplanar, then λ =
   a. 1
   b. 2
   c. –1
   d. 0

7. For all real values of a0, a1, a2, a3 satisfying $a_0+\frac{a_1}{2}+\frac{a_2}{3}+\frac{a_3}{4}=0$, the equation $a_0 + a_1x + a_2x^2 + a_3x^3 = 0$ has a real root in the interval
   a. [0, 1]
   b. [–1, 0]
   c. [1, 2]
   d. [–2, –1]

8. Let f R→ R be defined as $f(x)=\begin{cases} 0, x \text{ is irrational} \\ \sin|x|, x \text{ is rational} \end{cases}$ Then which of the following is true?
   a. f is discontinuous for all x
   b. f is continuous for all x
   c. f is discontinuous at x = kπ, where k is an integer
   d. f is continuous at x = kπ, where k is an integer

9. If f: [0,π/2]→ R is defined as $f(\theta)=\begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$. Then the range of *f* is
   a. (2, ∞)
   b. (–, ∞–2)
   c. (2, ∞)
   d. (– ∞, 2]

10. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ equals
   a. 2AB
   b. 2BA
   c. A+B
   d. AB

11. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$ is
 a. -2ω
 b. $-3\omega^2$
 c. -1
 d. 0 (Zero)

12. Let a, b, c, d be any four real numbers. Then $a^n + b^n = c^n + d^n$ holds for any natural number n if
 a. $a + b = c + d$
 b. $a - b = c - d$
 c. $a + b = c + d$, $a^2 + b^2 = c^2 + d^2$
 d. $a - b = c - d$, $a^2 - b^2 = c^2 - d^2$

13. If α, β are the roots of $x^2 - px + 1 = 0$ and γ is root of $x^2 + px + 1 = 0$, then $(\alpha + \gamma)(\beta + \gamma)$ is
 a. 0 (zero)
 b. 1
 c. -1
 d. p

14. Number of irrational terms in the binomial expansion of $(3^{1/2} + 7^{1/3})^{100}$ is
 a. 90
 b. 88
 c. 93
 d. 95

15. The quadratic expression $(2x + 1)^2 - px + q \neq 0$ for any real x if
 a. $p^2 - 16p - 8q < 0$
 b. $p^2 - 8p + 16q < 0$
 c. $p^2 - 8p - 16q < 0$
 d. $p^2 - 16p + 8q < 0$

16. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64} + \left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)$ is
 a. 0 (zero)
 b. -1
 c. 1
 d. i

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17. Find the maximum value of |z| when $\left|z-\frac{3}{2}\right|=2$, z being a complex number.

a. $1+\sqrt{3}$
b. 3
c. $1+\sqrt{2}$
d. 1

18. Given that x is a real number satisfying $\frac{5x^2-26x+5}{3x^2-10x+3}<0$, then

a. $x<\frac{1}{5}$
b. $\frac{1}{5}<x<3$
c. x > 5
d. $\frac{1}{5}<x<\frac{1}{3}$ or 3 < x < 5

19. The least positive value of t so that the lines x = t + α, y + 16 = 0 and y = αx are concurrent is
a. 2
b. 4
c. 16
d. 8

20. If in a triangle ΔABC, $a^2 \cos^2 A - b^2 - C^2 = 0$, then
a. $\frac{\pi}{4}<A<\frac{\pi}{2}$
b. $\frac{\pi}{2}<A<\pi$
c. $A=\frac{\pi}{2}$
d. $A<\frac{\pi}{4}$

21. $\{x \in R : |\cos x| \ge \sin x\} \bigcap \left[0\frac{3\pi}{2}\right] =$

a. $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\frac{3\pi}{2}\right]$
b. $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$
c. $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{5\pi}{4},\frac{3\pi}{2}\right]$
d. $\left[0,\frac{3\pi}{2}\right]$

22. A particle starts moving from rest from a fixed point in a fixed direction. The distance s from the fixed point at a time t is given by x = t^2 + at – b + 17, where a, b are real numbers. If the particle comes to rest after 5 sec at a distance of s = 5 units from the fixed point, then values of a and b are respectively
a. 10, –33
b. –10, –33
c. –8, 33
d. –10, 33

23. $\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+.....+\sqrt{n-1}}{n\sqrt{n}}=0$
a. $\frac{1}{2}$
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. 0 (zero)

24. If $\lim_{x\to 0}\frac{axe^{x}-b\log(1+x)}{x^{2}}=3$ then the values of a, b are respectively
a. 2, 2
b. 1, 2
c. 2, 1
d. 2, 0

25. Let P(x) be a polynomial, which when divided by x – 3 and x – 5 leaves remainders 10 and 6 respectively. If the polynomial is divided by (x – 3) (x – 5) then the remainder is
a. –2x + 16
b. 16
c. 2x – 16
d. 60

26. The integrating factor of the differential equation $\frac{dy}{dx}+\left(3x^{2}\tan^{-1}y-x^{3}\right)\left(1+y^{2}\right)=0$ is
a. e^{x^2}
b. e^{x^3}
c. e^{3x^2}
d. e^{3x^3}

27. If $y = e^{-x}\cos 2x$ then which of the following differential equations is satisfied?
   a. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$
   b. $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+2y=0$
   c. $\frac{d^2y}{dx^2}-5\frac{dy}{dx}-2y=0$
   d. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=0$

28. In a certain town, 60% of the families own a car, 30% own a house and 20% own both a car and a house. If a family is randomly chosen, what is the probability that this family owns a car or a house but not both?
   a. 0.5
   b. 0.7
   c. 0.1
   d. 0.9

29. The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word COCHIN is
   a. 360
   b. 192
   c. 96
   d. 48

30. Let $f: R \to R$ a continuous function which satisfies $f(x)=\int_0^x f(t)dt$. Then the value of $f(\log_e 5)$ is
   a. 0 (zero)
   b. 2
   c. 5
   d. 3

31. The value λ, is such that the following system of equations has no solution, is
   $2x - y - 2z = 2$
   $x - 2y + z = -4$
   $x + y + \lambda z = 4$
   a. 3
   b. 1
   c. 0 (zero)
   d. -3

32. If $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Then f(100) is equal to

a. 0 (zero)
b. 1
c. 100
d. 10

33. If $\sin^{-1}\left(x-\frac{x^2}{4}+\frac{x^3}{4}-\frac{x^4}{8}+.....\right)=\frac{\pi}{6}$ where |x| < 2 then the value of x is

a. $\frac{2}{3}$
b. $\frac{3}{2}$
c. $-\frac{2}{3}$
d. $-\frac{3}{2}$

34. The area of the region bounded by the curve $y = x^3$, its tangent at (1, 1) and x-axis is

a. $\frac{1}{12}$
b. $\frac{1}{6}$
c. $\frac{2}{17}$
d. $\frac{2}{15}$

35. If $\log_{0.2}(x-1) > \log_{0.04}(x+5)$ then

a. $-1 < x < 4$
b. $2 < x < 3$
c. $1 < x < 4$
d. $1 < x < 3$

36. The number of real roots of equation $\log_e x + ex = 0$

a. 0 (zero)
b. 1
c. 2
d. 3

37. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines $x + y = 3$ and $2x + 2y - 1 = 0$ then
    a. $2 < a < 4$
    b. $-\frac{1}{2} < a < 2$
    c. $0 < a < 2$
    d. $-\frac{1}{2} < a < \frac{3}{2}$

38. Number of intersecting points of the conic $4x^2 + 9y^2 = 1$ and $4x^2 + y^2 = 4$ is
    a. 1
    b. 2
    c. 3
    d. 0 (zero)

39. The value of λ for which the straight line $\frac{x-\lambda}{3}=\frac{y-1}{2+\lambda}=\frac{z-3}{-1}$ may lie on the plane $x - 2y = 0$ is
    a. 2
    b. 0
    c. $-\frac{1}{2}$
    d. There is no such λ

40. The value of $2\cot^{-1}\frac{1}{2}-\cot^{-1}\frac{4}{3}$ is
    a. $-\frac{\pi}{8}$
    b. $\frac{3\pi}{2}$
    c. $\frac{\pi}{4}$
    d. $\frac{\pi}{2}$

41. If the point $(2\cos\theta, 2\sin\theta)$, for $\theta \in (0, 2\pi)$ lies in the region between the lines $x + y = 2$ and $x - y = 2$ containing the origin, then θ lies in
    a. $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right)$
    b. $[0, \pi]$
    c. $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$
    d. $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

42. Number of points having distance $\sqrt{5}$ from the straight line x – 2y + 1 = 0 and a distance $\sqrt{13}$ from the lie 2x + 3y – 1 = 0 is
    a. 1
    b. 2
    c. 4
    d. 5

43. Let $f : R \to R$ be defined as $f(x) = \frac{x^2 - x + 4}{x^2 + x + 4}$. Then the range of the function $f(x)$ is
    a. $\left[\frac{3}{5}, \frac{5}{3}\right]$
    b. $\left(\frac{3}{5}, \frac{5}{3}\right)$
    c. $\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{5}{3}, \infty\right)$
    d. $\left[-\frac{5}{3}, -\frac{3}{5}\right]$

44. The least value of $2x^2 + y^2 + 2xy + 2x - 3y + 8$ for real numbers x and y is
    a. 2
    b. 8
    c. 3
    d. 1

45. Let $f : [-2, 2] \to R$ be a continuous function such that $f(x)$ assumes only irrational values. If $f\left(\sqrt{2}\right) = \sqrt{2}$, then
    a. $f(0) = 0$
    b. $f\left(\sqrt{2} - 1\right) = \sqrt{2} - 1$
    c. $f\left(\sqrt{2} - 1\right) = \sqrt{2} + 1$
    d. $f\left(\sqrt{2} - 1\right) = \sqrt{2}$

46. The minimum value of $\cos\theta + \sin\theta + \frac{2}{\sin 2\theta}$ for $\theta \in (0, \pi/2)$ is
    a. $2 + \sqrt{2}$
    b. 2
    c. $1 + \sqrt{2}$
    d. $2\sqrt{2}$

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47. Let $x_n = \left(1-\frac{1}{3}\right)^2 \left(1-\frac{1}{6}\right)^2 \left(1-\frac{1}{10}\right)^2 \ldots\ldots\ldots \left(1-\frac{1}{\frac{n(n+1)}{2}}\right)$, $n \geq 2$.

    Then the value of $\lim\limits_{n\to\infty} x_n$ is

    a. 1/3
    b. 1/9
    c. 1/81
    d. 0 (zero)

48. The variance of first 20 natural numbers is
    a. 133/4
    b. 279/12
    c. 133/2
    d. 399/4

49. A fair coin is tossed a fixed number of times. If the probability of getting exactly 3 heads equals the probability of getting exactly 5 heads, then the probability of getting exactly one head is
    a. 1/64
    b. 1/32
    c. 1/16
    d. 1/8

50. If the letters of the word PROBABILITY are written down at random in a row, the probability that two B-s are together is
    a. 2/11
    b. 10/11
    c. 3/11
    d. 6/11

51. The number of distinct real roots of

    $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is
    a. 0 (zero)
    b. 2
    c. 1
    d. > 2

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52. Let $x_1, x_2, ..., x_{15}$ be 15 distinct numbers chosen from 1, 2, 3, ..., 15. Then the value of $(x_1 - 1)(x_2 - 1), (x_3 - 1)..........(x_{15} - 1)$ is
    a. always ≤ 0
    b. 0 (zero)
    c. always even
    d. always odd

53. Let [x] denote the greatest integer less than or equal to x. Then the value of α for which the function $f(x) = \begin{cases} \dfrac{\sin\left[-x^2\right]}{\left[-x^2\right]}, & x \neq 0 \\ \alpha, x = 0 \end{cases}$ is continuous at x = 0 is
    a. $\alpha = 0$
    b. $\alpha = \sin(-1)$
    c. $\alpha = \sin(1)$
    d. $\alpha = 1$

54. Let $f(x)$ denotes the fractional part of a real number x. Then the value of $\int_0^{\sqrt{3}} f(x)^2 dx$
    a. $2\sqrt{3} - \sqrt{2} - 1$
    b. 0 (zero)
    c. $\sqrt{2} - \sqrt{3} + 1$
    d. $\sqrt{3} - \sqrt{2} + 1$

55. Let S = {(a, b, c) $\in$ N × N × N : a + b + c = 21, a $\leq$ b $\leq$ c } and
    T = {(a, b, c) $\in$ N× N × N : a, b, d are in A.P.}, where N is the set of all natural numbers. Then the number of elements in the set S $\cap$ T is
    a. 6
    b. 7
    c. 13
    d. 14

56. Let $y = e^{x^2}$ and $y = e^{x^2} \sin x$ be two given curves. Then the angle between the tangent to the curves at any point of their intersection is
    a. 0 (zero)
    b. π
    c. $\frac{\pi}{2}$
    d. $\frac{\pi}{4}$

57. Area of the region bounded by $y = |x|$ and $y = -|x| + 2$ is
   a. 4 sq. units
   b. 3 sq. units
   c. 2 sq. units
   d. 1 sq. units

58. Let d(n) denote the number of divisors of n including 1 and itself. Then d(225), d(1125) and d(640) are
   a. in AP
   b. in HP
   c. in GP
   d. consecutive integers

59. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}2a$ has a real solution if
   a. $|a| > \frac{1}{\sqrt{2}}$
   b. $\frac{1}{2\sqrt{2}} < |a| > \frac{1}{\sqrt{2}}$
   c. $|a| > \frac{1}{2\sqrt{2}}$
   d. $|a| \le \frac{1}{2\sqrt{2}}$

60. If $2 + i$ and $\sqrt{5} - 2i$ are the roots of the equation $(x^2 + ax + b)(x^2 + cx + d) = 0$, where a, b, c, d are real constants, ten product of all roots of the equation is
   a. 40
   b. $9\sqrt{5}$
   c. 45
   d. 35

61. In a triangle ABC, $\angle C = 90°$, r and R are the in-radius and circum-radius of the triangle ABC respectively, then $2(r + R)$ is equal to
   a. b + c
   b. c + a
   c. a + b
   d. a + b + c

62. Let α, β be two distinct roots of a $\cos\theta$ + b $\sin\theta$ = c, where a, b and c are three real constants and $q \in [0, 2\pi]$. Then $\alpha + \beta$ is also a root of the same equation if
   a. a + b = c
   b. b + c = a
   c. c + a = b
   d. c = a

63. For a matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$, if U_1, U_2 and U_3 are 3 × 1 column matrices satisfying

$$AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix},\ AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$ and U is 3 × 3 matrix whose columns are U_1, U_2 and U_3

Then sum of the elements of U^{-1} is
   a. 6
   b. 0 (zero)
   c. 1
   d. 2/3

64. Let $f : N \rightarrow R$ be such that f(1) = 1 and
$f(1) + 2f(2) + 3f(3) + \ldots\ldots\ldots + nf(n) = n(n + 1)f(n)$, for all $n \in N$, $n \geq 2$, where is the set of natural numbers and R is the set of real numbers. Then the value of f(500) is
   a. 1000
   b. 500
   c. 1/500
   d. 1/1000

65. If 5 distinct balls are placed at random into 5 cells, then the probability that exactly one cell remains empty is
   a. 48/125
   b. 12/125
   c. 8/125
   d. 1/125

66. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker?
a. 1/140
b. 1/70
c. 3/140
d. 1/10

67. A person goes to office by a car or scooter or bus or train, probability of which are 1/7, 3/7, 2/7 and 1/7 respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is 2/9, 1/9, 4/9 and 1/9 respectively. Given that he reached office in time, the probability that he travelled by a car is
a. 1/7
b. 2/7
c. 3/7
d. 4/7

68. The value of $\int \frac{(x-2)dx}{\left\{(x-2)^2(x+3)^7\right\}^{1/3}}$ is
a. $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3}+c$
b. $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{3/4}+c$
c. $\frac{5}{12}\left(\frac{x-2}{x+3}\right)^{4/3}+c$
d. $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{5/3}+c$

69. Let f : R $\to$ R be differentiable at x = 0. If $f(0) = 0$ and $f'(0) = 2$, then the value of
$\lim_{x\to 0}\frac{1}{x}\left[f(x)+f(2x)f(3x)+.....+f(2015x)\right]$
a. 2015
b. 0 (zero)
c. 2015 × 2016
d. 2015 × 2014

70. If x and y are digits such that 17! = 3556xy428096000, then x + y equals

   a. 15

   b. 6

   c. 12

   d. 13

71. Which of the following is/are always false?

   a. A quadratic equation with rational coefficients has zero or two irrational roots

   b. A quadratic equation with real coefficients has zero or two non-real roots

   c. A quadratic equation with irrational has zero or two rational roots

   d. A quadratic equation with integer coefficients has zero or two irrational roots

72. If the straight line $(a - 1)x - by + 4 = 0$ is normal to the hyperbola $xy = 1$ the which of the followings does not hold?

   a. $a > 1, b > 0$

   b. $a > 1, b < 0$

   c. $a < 1, b < 0$

   d. $a < 1, b > 0$

73. Suppose a machine produces metal parts that contain some defective parts with probability 0.05. How many parts should be produced in order that probability of at least one part defective is ½ or more? (Given $\log_{10}95 = 1.977$ and $\log_{10}2 = 0.3$)
   a. 11
   b. 12
   c. 15
   d. 14

74. Let f: R $\to$ R be such that $f(2x - 1) = f(x)$ for all $x \in R$. If f is continuous at $x = 1$ and $f(1) = 1$, then
   a. $f(2) = 1$
   b. $f(2) = 2$
   c. f is continuous only at $x = 1$
   d. f is continuous at all points

75. If cos x and sin x are solutions of the differential equation $a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0$. Where a_0, a_1, a_2 are real constants then which of the followings is/are always true?

a. A cos x + B sin x is a solution, where A and B are real constants

b. $A\ \cos\left(x+\frac{\pi}{4}\right)$ is solution, where A is real constant

c. A cos x sin x is a solution, where A is real constant

d. $A\cos\left(x+\frac{\pi}{4}\right)+B\sin\left(x-\frac{\pi}{4}\right)$ is a solution, where A and B are real constants

76. Which of the following statements is/are correct for $0<\theta<\frac{\pi}{2}$?

a. $(\cos\theta)^{1/2}\le\cos\frac{\theta}{2}$

b. $(\cos\theta)^{3/2}\ge\cos\frac{3\theta}{4}$

c. $\cos\frac{5\theta}{6}\ge(\cos\theta)^{5/6}$

d. $\cos\frac{7\theta}{8}\le(\cos\theta)^{7/8}$

77. Let $16x^2 - 3y^2 - 32x - 12y = 44$ represent a hyperbola. Then

a. Length of the transverse axis is $2\sqrt{3}$

b. Length of each latus rectum is $32\sqrt{3}$

c. Eccentricity is $\sqrt{19/3}$

d. Equation of a directrix is $x=\frac{\sqrt{19}}{3}$

78. For the function $f(x)\left[\frac{1}{[x]}\right]$, where [x] denotes the greatest integer less than or equal to x, which of the following statements are true?

a. The domains is $(-\infty, \infty)$

b. The range is $\{0\}\cup\{-1\}\cup\{1\}$

c. The domain is $\{-\infty, 0\}\cup[1, \infty)$

d. The range is $\{0\}\cup\{1\}$

79. Let f be any continuously differentiable function on [a, b] and twice differentiable on (a, b) such that $f(a) = f'(a) = 0$ and $f(b) = 0$. Then
    a. $f''(a) = 0$
    b. $f'(x) = 0$ for some $x \in (a, b)$
    c. $f''(x) = 0$ for some $x \in (a, b)$
    d. $f'''(x) = 0$ for some $x \in (a, b)$

80. A relation ρ on the set of real number R is defined as follows:
    a. ρ is reflexive and symmetric
    b. ρ is symmetric but not reflexive
    c. ρ is symmetric and transitive
    d. ρ is an equivalence relation

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ANSWER KEY

1. (b)	2. (b)	3. (b)	4. (c)	5. (d)	6. (a)	7. (a)	8. (d)	9. (c)	10. (c)
11. (b)	12. (d)	13. (a)	14. (G)	15. (c)	16. (b)	17. (b)	18. (d)	19. (d)	20. (b)
21. (a)	22. (b)	23. (c)	24. (a)	25. (a)	26. (b)	27. (a)	28. (a)	29. (c)	30. (a)
31. (d)	32. (a)	33. (a)	34. (a)	35. (c)	36. (b)	37. (b)	38. (d)	39. (c)	40. (d)
41. (c)	42. (c)	43. (a)	44. (G)	45. (d)	46. (a)	47. (b)	48. (a)	49. (b)	50. (a)
51. (c)	52. (b)	53. (c)	54. (c)	55. (b)	56. (a)	57. (c)	58. (c)	59. (d)	60. (c)
61. (c)	62. (d)	63. (b)	64. (d)	65. (a)	66. (c)	67. (a)	68. (a)	69. (c)	70. (a)
71. (c)	72. (a,c)	73. (c,d)	74. (a,d)	75. (a,d)	76. (a,c)	77. (a,b,c)	78. (b,c)	79. (b,c)	80. (b,c)

WB-JEE-2015 (Mathematics)

SOLUTIONS

1. (a)

$$\lim_{x\to 2}\int_2^x \frac{3t^2}{x-2}dt$$

$$= \lim_{x\to 2}\frac{\int_2^x 3t^2 dt}{x-2}\left(\frac{0}{0}\ form\right)$$

Applying L' hospital rule

$$= \lim_{x\to 2}\frac{\frac{d}{dx}\int_2^x 3t^2 dt}{\frac{d}{dx}x-2}$$ {In numerator we will use Newton Leibnitz's theorem}

$$= \lim_{x\to 2}\frac{3x^2(1)-3(2)^2.0}{1}$$

$= 3.(2)^2 = 12$

2. (b)

$\frac{2x}{3}+\tan\frac{x}{3}$ = cosec $\frac{kx}{3}$, k = ?

Let $\frac{x}{3} = \theta$

$\therefore$ $\cot 2\theta + \tan\theta = \operatorname{cosec} k\theta$

Now L.H.S. = $\frac{\cos 2\theta}{\sin 2\theta}+\frac{\sin\theta}{\cos\theta}$

$$= \frac{cos2\theta.cos\theta+\sin\theta\sin 2\theta}{sin2\theta.cos\theta}$$

$$= \frac{cos(2\theta-\theta)}{sin2\theta.cos\theta}$$

$$= \frac{cos\theta}{sin2\theta.\cos\theta}$$

$= \frac{1}{sin2\theta}$ = cosec 2θ $\therefore$ k = 2

3. (b)

$$\sqrt{4\cos^4\theta+\sin^2 2\theta} + 4\cot\theta.\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$= \sqrt{4\cos^4\theta+(2.\sin\theta\cos\theta)^2} + 2\cot\theta.\ 2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$= \sqrt{4.\cos^4\theta+4\sin^2\theta\cos^2\theta} + 2.\cot\theta\left(1+\cos 2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$$

$$= \sqrt{4.\cos^2\theta(\cos^2\theta+\sin^2\theta)} + 2.\cot\theta\left(1+\cos\left(\frac{\pi}{2}-\theta\right)\right)$$

$$= \sqrt{4\cos^2\theta} + 2\cot\theta\,(1+\sin\theta)$$

$$= |2\cos\theta| + 2\cot\theta + 2\cot\theta.\sin\theta$$

$$= -2\cos\theta + 2\cot\theta + 2\,\frac{\cos\theta}{\sin\theta}\times\sin\theta \quad \left\{\left(Given\ \theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\right)\right.$$

i.e. in IInd & IIIrd Quadrant where $\cos\theta$ is negative. }

$\therefore$ $|2\cos\theta| = -2\cos\theta$ }

$= -2\cos\theta + 2\cot\theta + 2\cos\theta$

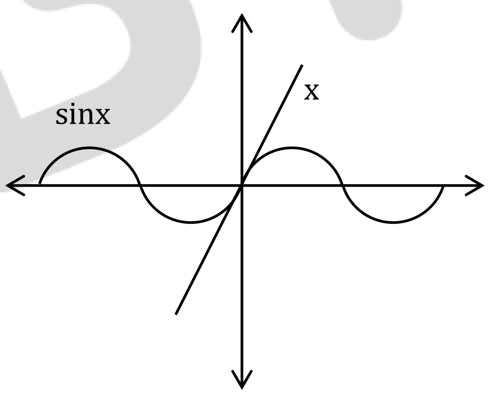
$= 2\cot\theta$

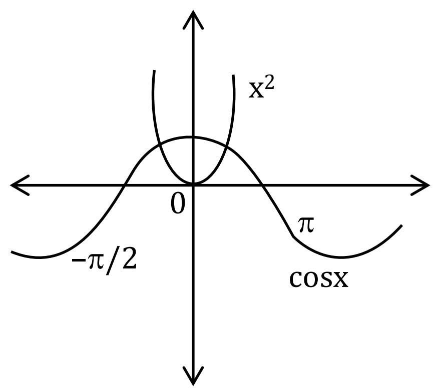
4. (c)

$(\sin x - x)(\cos x - x^2) = 0$

i.e. $\sin x = x$ OR $\cos x = x^2$

Now let's use graphs to get their solutions





x and sinx meets at x = 0 only — x^2 and cos x meets at two points

$\therefore$ one solution — $\therefore$ two solutions

Therefore, we have total 3 solutions

5. (d)

(A) $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\vec{a}.\vec{b}$

$= \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta \quad (\theta = \vec{a} \wedge \vec{b})$

Now, if $\vec{a} \perp \vec{b}$ i.e. $\theta = 90°$

$= \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|^2 . 0 \quad (\cos 90 = 0)$

$= \left|\vec{a}\right|^2+\left|\vec{b}\right|^2$

True

(B) $|\vec{a}+\lambda\vec{b}|^2 = |\vec{a}|^2+|\lambda\vec{b}|^2+2\vec{a}\lambda\vec{b}$

$= |\vec{a}|^2+\lambda^2|\vec{b}|^2+2\lambda\vec{a}.\vec{b}$

Now, $\vec{a} \perp \vec{b} \therefore \vec{a} . \vec{b} = 0$

$|\vec{a}+\lambda\vec{b}|^2 = |\vec{a}|^2+\lambda^2|\vec{b}|^2 \geq |\vec{a}|^2 \quad \{\lambda^2|\vec{b}|^2 \geq 0)\}$

$\therefore |\vec{a}+\lambda\vec{b}|^2 \geq |\vec{a}|^2$

OR $|\vec{a}+\lambda^2\vec{b}| \geq |\vec{a}|$ (equality holds for $\lambda = 0$)

True

(C) $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2$

$= (|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}^2)+(|\vec{a}|^2+|\vec{b}|^2-2\vec{a}.\vec{b})$

$= 2(|\vec{a}|^2+|\vec{b}|^2)$

(D) $|\vec{a}+\lambda\vec{b}|^2=|\vec{a}|+\lambda^2|\vec{b}|^2+2\lambda. \vec{a}.\vec{b}$

Now $\vec{a} \,||\, \vec{b}$ i. e. $\theta = \vec{a} \wedge \vec{b} = 0$

$= |\vec{a}|^2+\lambda^2|\vec{b}|^2+2\lambda|\vec{a}||\vec{b}|$

$= (|\vec{a}|+\lambda|\vec{b}|)^2$

Now, If $\lambda > 0$, $(|\vec{a}|+\lambda|\vec{b}|)^2 > |\vec{a}|^2$

$|\vec{a}+\lambda\vec{b}|^2 = (|\vec{a}+\lambda\vec{b}|)^2 > |\vec{a}|^2$

$|\vec{a}+\lambda\vec{b}| > |\vec{a}|$

If $\lambda < 0$, $(|\vec{a}+\lambda\vec{b}|)^2 < |\vec{a}|^2$

$|\vec{a}+\lambda\vec{b}| < |\vec{a}|$

If $\lambda = 0$, $|\vec{a}+\lambda\vec{b}| = |\vec{a}|$

$\therefore$ Option D is not true for all $\lambda \in R$

WB-JEE-2015 (Mathematics)

6. (a)

If four points A(−2, 1, 1), B(1,1,1), C(0, 1, −1) & D(0, λ, 1) are coplanar or lies in the same plane then vectors determined by them will also be coplanar.

Consider $\overrightarrow{AB} = 3\hat{i} + 0\hat{j} + 0\hat{k}$

$\overrightarrow{AC} = 2\hat{i} + 0\hat{j} - 2\hat{k}$

$\overrightarrow{AD} = 2\hat{i} + (\lambda - 1).\hat{j} + 0\hat{k}$

Now applying the condition of co-planarity of three vectors

i.e. their scalar triple product equals zero.

$\therefore \begin{vmatrix} 3 & 0 & 0 \\ 2 & 0 & -2 \\ 2 & \lambda - 1 & 0 \end{vmatrix} = 0$ {expanding along R_1}

$3(0 + 2(\lambda - 1)) + 0 + 0 = 0$

$\Rightarrow 6(\lambda - 1) = 0$

$\lambda = 1$

7. (a)

Here we are talking about roots of an equation function in some interval so think about Rolle's theorem.

Now, let $f(x) = a_0x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} + a_3 \frac{x^4}{4} + c$

(i) $f(0) = c$

$f(1) = a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} + c$

$= c$ ($\because \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$, given)

$f(0) = f(1)$

(ii) Also f(x) is a polynomial so continuous and differential everywhere.

$\therefore$ By Rolle's theorem f'(x) i.e. $a_0 + a_1x + a_2x^2 + a_3x^2$

Will have at least one real root in [0,1]

8. (d)

$f: R \to R$; $f(x) = \begin{cases} 0, x \text{ is irrational} \\ \sin|x|, x \text{ is rational} \end{cases}$

You know on the real line 'x' takes rational and irrational values in very close neighborhood.

Therefore, the values of given function f(x) will oscillate to '0' or sin|x|, corresponding to irrational and rational values of 'x' and so will be discontinuous at infinite points.

The f(x) will be continuous at points where y = 0 & y = sin|x| meets (or equal)

i.e. $\sin|x| = 0$

$\Rightarrow x = k\pi, k \in I$

9. (c)

$$f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$$

Expanding along R_1

$= 1(1+\tan^2\theta) - \tan\theta(-\tan\theta + \tan\theta) + 1(\tan^2\theta + 1)$

$= 2\sec^2\theta$

Now, for $\theta \in [0, \pi/2]$

$\Rightarrow \sec^2\theta \in [1,\infty)$

$\Rightarrow 2\sec^2\theta \in [2,\infty)$

i.e. Range of $f(\theta) \to [2, \infty)$

10. (c)

$A^2 + B^2 = A.A + B.B$

$= A.(BA) + B(AB)$ {Given $AB = B$ & $BA = A$}

$= (AB)A + (BA)B$

$= B.A + AB$

$= A + B$

11. (b)

$$\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$$

$C_1 \to C_1 + C_2$, $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 1+\omega+\omega^2 & \omega+\omega^2 & -\omega \\ 1+\omega+\omega^2 & \omega+\omega^2 & -\omega^2 \\ \omega+\omega+\omega^2 & \omega+\omega^2 & -\omega^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -1 & -\omega \\ 0 & -1 & -\omega^2 \\ \omega-1 & -1 & -\omega^2 \end{vmatrix} \quad \{\because 1 + \omega + \omega^2 = 0, \omega + \omega^2 = -1\}$$

Expanding along R_3

$= 0 + 0 + (\omega-1)(\omega^2-\omega)$

$= \omega^3 - \omega^2 - \omega^2 + \omega$

$= 1-2\omega^2 + \omega$

$= (1+\omega) - 2\omega^2$ $\{\because 1 + \omega + w^2 = 0, 1 + \omega = -\omega^2\}$

$= -3\omega^2$

12. (d)

$a, b, c, d \in R$

$a^n + b^n = c^n + d^n \quad \forall\, n \in N$

For $n = 1$, $a + b = c + d$ .....(i)

$n = 2$, $a^2 + b^2 = c^2 + d^2$ .....(ii)

$n = 3$, $a^3 + b^3 = c^3 + d^3$ .....(iii)

$\Rightarrow (a + b)(a^2 + b^2 - ab) = (c + d)(c^2 + d^2 - cd)$

$\Rightarrow a^2 + b^2 - ab = c^2 + d^2 - cd$ (using (i) & (ii))

$\Rightarrow ab = cd$ .....(iv)

Now consider a quadratic equation with roots a^3 & b^3

$x^2 - (a^3 + b^3)x + a^2b^3 = 0$ .....(v)

Similarly, equation with roots c^3 & d^3 will be

$x^2 - (c^3 + d^3)x + c^3d^3 = 0$ .....(vi)

both equation (v) & (vi) are same with four roots a^3, b^3, c^3, d^3 which is not possible since a quadratic equation cannot have more than two roots.

Therefore, it is possible only when $a = c$ & $b = d$ OR $a = d$ & $b = c$.

13. (a)

α, β all roots of $x^2 - px + 1 = 0$ ....(i)

If we replace x by $-x$

(i) becomes $x^2 + px + 1 = 0$ .....(ii)

$\Rightarrow$ (ii) will have roots $-\alpha$ & $-\beta$ (transformation of equations)

But we are given root of (ii) γ

$\therefore$ Either $\gamma = -\alpha$ or $-\beta$

$\Rightarrow (\alpha + \gamma)(\beta + \gamma) = 0$

14. Bonus (G)

$(3^{1/5} + 7^{1/3})^{100}$

Writing general term of given binomial

i.e. $T_{r+1} = {}^{100}C_r\, (3^{1/5})^{100-r}\, (7^{1/3})^r$

$= {}^{100}C_r \,.\, 3^{\frac{100-r}{5}} \,.\, 7^{\frac{r}{3}}$

Now the term will be rational when exponents of 3 and 7 both become a whole number

i.e. $\dfrac{100-r}{5}$ & $\dfrac{r}{3}$

$r = 0, 15, 30, 45, 60, 75$ & 90 (7 rational terms)

Since total no. of terms are 101.

$\therefore$ No. of irrational terms $= 101 - 7 = 94$.

15. (c)

$(2x + 1)^2 - px + q \neq 0$ or $4x^2 + (4-p)\,x + 1 + q \neq 0$ for any real 'x'

$\Rightarrow$ Equation, $4x^2 + (4-p)\,x + 1 + q = 0$ has no real roots

$\therefore$ $D < 0$

$(4 - p)^2 - 4.\,4\,(1 + q) < 0$

$\Rightarrow 16 + p^2 - 8p - 16 - 19q < 0$

$\Rightarrow p^2 - 8p - 16q < 0$

16. (b)

$1 + \sqrt{3}\,i = -2\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right) = -2\,\omega^2$

$1 - \sqrt{3}\,i = -2\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right) = -\,2\omega$

Substituting these values in given equation

$\omega^{64} + \frac{1}{\omega^{64}} = \omega^{3\times 21 + 1} + \frac{1}{\omega^{3\times 21+1}}$

$= \omega + \frac{1}{\omega}$ $\{\omega^3 = 1, \omega^3 = \frac{1}{\omega}, 1 + \omega + \omega^2 = 0, \omega + \omega^2 = -\,1\}$

$= \omega + \omega^2 = -1$

17. (b)

Given, $\left|z - \frac{3}{2}\right| = 2$

From triangle inequality

$\left||z_1| - |z_2|\right| \le |z_1 + z_2| \le |z_1| + |z_1|$

$z_1 = z$, $z_2 = -\,\frac{3}{z}$

$\left||z| - \left|\frac{3}{z}\right|\right| \le \left|z - \frac{3}{z}\right| = 2$

$\therefore \left||z| - \frac{3}{|z|}\right| \le 2$

$-2 \le |z| - \frac{3}{|z|} \le 2$

From right inequality

$|z| - \frac{3}{|z|} \le 2$

$\Rightarrow |z|^2 - 2|z| - 3 \le 0$

$\Rightarrow (|z| - 3)(|z| + 1) \le 0$

$\Rightarrow -1 \le |z| \le 3$

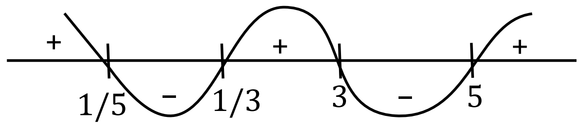
$\Rightarrow |z|_{max} = 3$

18. (d)

$\frac{5x^2-26x+5}{3x^2-10x+3} < 0$

$\Rightarrow \frac{(5x-1)(x-5)}{(3x-1)(x-3)} < 0$

Now using wavy curve



$\therefore$ x $\in$ (1/5 ,1/3) U (3,5)

19. (d)

$y = -16$......(i) $\quad$ $y = \alpha x$......(ii) $\quad$ $x = t + \alpha$ .... (iii)

From (i) and (ii)

$x = \frac{-16}{\alpha}$ now put in (iii)

$\Rightarrow \frac{-16}{\alpha} = t + \alpha$

$\Rightarrow t = -\alpha - \frac{-16}{\alpha}$

$\Rightarrow t = -\left(\alpha+\frac{16}{\alpha}\right)$

Now, $\alpha+\frac{16}{\alpha} \geq 8$ for $\alpha > 0$ and $\alpha+\frac{16}{\alpha} \leq -8$ for $\alpha < 0$ {using A.M. $\geq$ G.M.}

$\therefore t = -\left(\alpha+\frac{16}{\alpha}\right) \leq -8$; $\alpha > 0$

$t = -\left(\alpha+\frac{16}{\alpha}\right) \geq 8$; $\alpha < 0$

Therefore, least positive value of 't' is 8

Note: You can also use concept of maxima & minima.

20. (b)

Given $a^2\cos^2A - b^2 - c^2 = 0$

$\Rightarrow \cos^2A = \frac{b^2+c^2}{a^2} < 1$ $\quad$ {cos A $\leq$ 1 but A $\neq$ 0 $\therefore$ cos A < 1}

Now from cosine rule

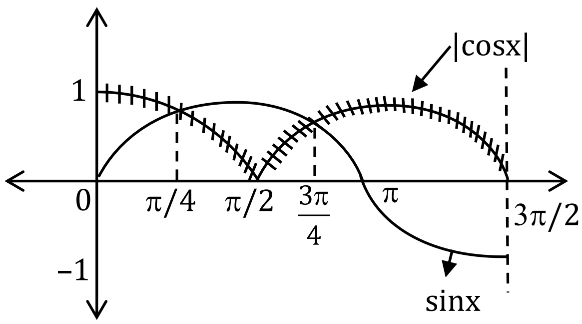
$\cos A = \frac{b^2+c^2-a^2}{2bc} < 0$ {$\because b^2 + c^2 < a^2$}

$\Rightarrow \cos A < 0$

$\therefore A \in (\pi/2, \pi)$

21. (a)

For $x \in \left[0, \frac{3\pi}{2}\right]$



From above graph

$|\cos x| \geq \sin x$

For $x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right]$

22. (b)

$s = t^2 + at - b + 17$

$v = \left.\frac{ds}{dt}\right|_{t=5} = 2t + a|_{t=5} = 0$

$\therefore\ 10 + a = 0$

$\Rightarrow a = -10$

Also at $t = 5$, $s = 25$

$\therefore\ 25 = 5^2 + 5(-10) - b + 17 \Rightarrow b = -33$

23. (c)

$$\lim_{x\to\infty} \frac{\sqrt{1} + \sqrt{2} + \ldots\ldots\sqrt{n-1}}{n\sqrt{n}}$$

$$= \lim_{x\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \ldots\ldots\sqrt{\frac{n-1}{n}}\right)$$

$$= \lim_{x\to\infty} \frac{1}{n}\sum_{r=1}^{n-1}\left(\sqrt{\frac{r}{n}}\right) \text{ \{Using definite integral as limit of sum\}}$$

$$= \int_0^1 \sqrt{x}\, dx = 2/3$$

24. (a)

$\lim\limits_{x\to 0}\dfrac{a\times e^{x}-b\,\ell n(1+x)}{x^{2}}$ = 3

Using expansions

= $\lim\limits_{x\to 0}\dfrac{ax\left(1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\ldots\right)-b\left(x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\ldots\right)}{x^2}$

= $\lim\limits_{x\to 0}\dfrac{(a-b)x+\left(a+\dfrac{b}{z}\right)x^2+kx^3+\ell x^4+\ldots}{x^2}$ = 3

For above limit to be defined coefficient of x i.e. a – b must be zero

$\Rightarrow$ a = b

Again, the value of limit will be coefficient of x^2

i.e. a + $\dfrac{b}{2}$ = 3

$\Rightarrow$ a + $\dfrac{a}{2}$ = 3 $\Rightarrow$ a = 2 = b

25. (a)

P(x) = (x–3) Q_1(x) + 10 $\Rightarrow$ P(3) = 10

P(x) = (x–5) Q_2(x) + 6 $\Rightarrow$ P(5) = 6

P(x) = (x–3) (x–5) Q_3(x) + ax + b

Put x = 3 & 5

P(3) = 0 + 3a + b = 10

P(5) = 0 + 5a + b = 6

Solving above equations

a = –2 & b = 16

$\therefore$ Remainder = –2x + 16

26. (b)

$\dfrac{dy}{dx}$ + ($3x^2 \tan^{-1}y - x^3$) ($1 + y^2$) = 0

$\Rightarrow \dfrac{1}{1+y^2}\dfrac{dy}{dx}$ + $\tan^{-1}y$. $3x^2 = x^3$

Now, put $\tan^{-1}y$ = z

Differentiate with respect to x

$\dfrac{1}{1+y^2}\dfrac{dy}{dx}=\dfrac{dz}{dx}$

$\therefore \dfrac{dz}{dx}$ + z. $3x^2 = x^3$

P(x) = $3x^2$; Q(x) = x^3

I. F. = $e^{\int P(x)dx}=e^{\int 3x^2dx}=e^{x^3}$

27. (a)

$y = e^{-x}.\cos 2x$

$\Rightarrow y_1 = e^{-x}(-1)\cos 2x + e^{-x}(-\sin 2x)(2)$

$\Rightarrow y_1 = -y - 2e^{-x}\sin 2x$

$\Rightarrow y_2 = -y_1 - 2(-e^{-x}\sin 2x + e^{-x}\cos 2x.\ 2)$

$y_2 = -y_1 + 2e^{-x}\sin 2x - 4e^{-x}\cos 2x$

$y_2 = -y_1 + (-y-y_1) - 4y$

$y_2 + 2y_1 + 5y = 0$

28. (a)

$A \rightarrow$ Car, $B \rightarrow$ House

$n(A) = 60\%$, $n(B) = 30\%$, $n(A \cap B) = 20\%$

No. of families which owns car or house but not both

$= n(A \cup B) - n(A \cap B)$

$= n(A) + n(B) - n(A \cap B) - n(A \cup B)$

$= 70 + 30 - 2 \times 20 = 50\%$

$\therefore$ Probability $= \dfrac{50}{100} = 0.5$

29. (c)

COCHIN

CCHINO – Alphabetical order

(i) $\boxed{C}$ (C) HINO (No. of words starting with CC)

No. of arrangements = 4!

(ii) $\boxed{C}$ (H) CINO $\rightarrow$ 4!

(iii) $\boxed{C}$ (I) CHNO $\rightarrow$ 4!

(iv) $\boxed{C}$ (N) CIHO $\rightarrow$ 4!

(v) $\boxed{C}$ (O) CHIN $\rightarrow$ 4!

Rank of COCHIN = 4! + 4! + 4! + 4! + 1

= 4 × 4! + 1 = 97

No. of words before COCHIN = 97–1 = 96

30. (a)

$f(x) = \int_0^x f(t)dt$

Differentiate {using Newton Leibnitz theorem on R.H.S.}

$\Rightarrow f'(x) = f(x)$

$\Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrate w.r.t. x

$\Rightarrow \int \frac{f'(x)}{f(x)} dx = \int 1.dx$

$\Rightarrow \ell n\, |f(x)| = x + c$

$\Rightarrow f(x) = e^{x+c} = e^c \, . \, e^x$

Now, $f(0) = \int_0^0 f(t)dt = 0$

$\therefore$ k must be 0

$\Rightarrow f(x) = 0$ (const.)

$\Rightarrow f(\log_e 5) = 0$

31. (d)

For number solution

$\Delta = 0$ and at least one of Δx, Δy, Δz must be non-zero (Cramer's rule)

$\therefore \Delta = \begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 2(-2\lambda-1) + 1\,(\lambda-1) - 2\,(1+2) = 0$

$\Rightarrow -3\lambda - 9 = 0$

$\Rightarrow \lambda = -3$

32. (a)

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$$

$C_2 \rightarrow \frac{C_2}{x}$, $C_3 \rightarrow \frac{C_3}{x+1}$, $R_3 \rightarrow \frac{R_3}{x-1}$

$$= x(x-1)(x+1) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix}$$

$C_1 \rightarrow C_1 - C_3$, $C_2 \rightarrow C_2 - C_3$

$$= x(x^2-1) \begin{vmatrix} 0 & 0 & 1 \\ x & -1 & x \\ 2x & -2 & x \end{vmatrix}$$

$= x(x^2-1)\{1.(-2x + 2x)\}$

$= 0$

$\therefore f(100) = 0$

33. (a)

$$\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \ldots\infty\right) = \frac{\pi}{6}$$

$$\Rightarrow x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \ldots\infty = \frac{1}{2}$$

Infinite G.P.; Common ratio = $\frac{-x}{2}$

$$\Rightarrow \frac{x}{1-\left(-\frac{x}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \frac{2x}{2+x} = \frac{1}{2}$$

$x = \frac{2}{3}$

34. (a)

$y = x^3$

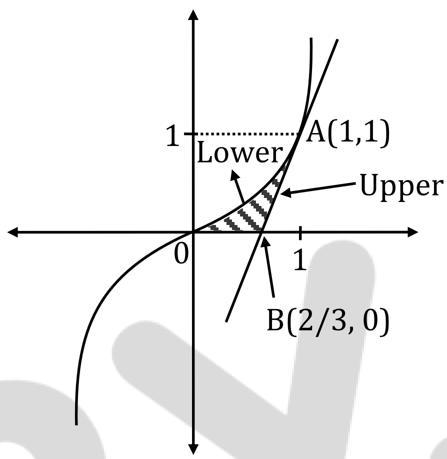
$\frac{dy}{dx} = 3x^2 \quad \left.\frac{dy}{dx}\right|_{A(1,1)} = 3(1)^2 = 3$

Equation of tangent at A (1, 1)

$y - 1 = 3(x - 1)$

$\Rightarrow$ $y = 3x - 2$

Now,



Area of shaded region = Along Y-axis

$= \int_0^1 (\text{Upper} - \text{Lower})dy$

$= \int_0^1 \left(\frac{y+2}{3} - y^{1/3}\right)dy$

$= \frac{1}{3}\left(\frac{y^2}{2} + 2y\right) - \left.\frac{y^{4/3}}{4/3}\right|_0^1$

$= \frac{1}{3}\left(\frac{1}{2} + 2\right) - \frac{3}{4} - 0$

$= \frac{1}{12}$

35. (c)

(I) For $\log_{0.2}(x-1)$ and $\log_{0.04}(x+5)$ to be defined

$x-1>0$ and $x+5>0$

$\Rightarrow$ $x > 1$ ....(i)

(II) Now, $\log_{0.2}(x-1) > \log_{0.04}(x+5)$

$\Rightarrow$ $\log_{0.2}(x-1) > \log_{0.2^2}(x+5)$

$\Rightarrow$ $\log_{0.2}(x-1) > \frac{1}{2}\log_{0.2}(x+5)$

$\Rightarrow$ $2\log_{0.2}(x-1) > \log_{0.2}(x+5)$

$\Rightarrow$ $\log_{0.2}(x-1)^2 > \log_{0.2}(x+5)$

Taking antilog {note the inequality will change because base < 1}

$\Rightarrow$ $(x-1)^2 < x+5$

$\Rightarrow$ $x^2 - 3x - 4 < 0$

$\Rightarrow$ $(x-4)(x+2) < 0$

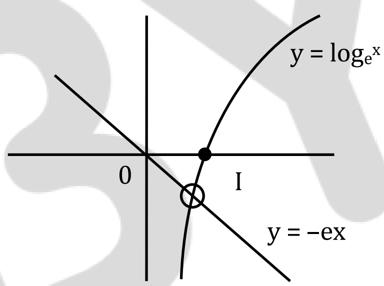
$\Rightarrow$ $-1 < x < 4$ {but $x > 1$, (i)}

$\therefore$ $1 < x < 4$

36. (b)

$\log_e x + ex = 0$

$\Rightarrow \log_e x = -ex$



The two curves meet at one point

$\therefore$ One solution

37. (b)

$y^2 - 4y = 4x - 4a$

$\Rightarrow$ $(y-2)^2 = 4(x - a + 1)$ {Parabola}

$\Rightarrow$ Vertex $(a-1, 2)$

Now, $(a-1, 2)$ always lies between lines $x + y - 3 = 0$ & $2x + 2y - 1 = 0$

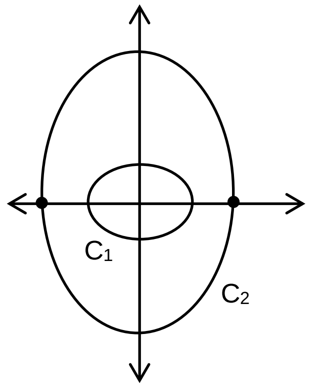
$\Rightarrow (a-1+2-3)(2(a-1)+2(2)-1) < 0$ {Location of point with respect to line}

$\Rightarrow (a-2)(2a+1) < 0$

$\Rightarrow -\frac{1}{2} < a < 2$

38. (d)

C1: $\frac{x^2}{1/4}+\frac{y^2}{1/9}=1, C_2:\frac{x^2}{1}+\frac{y^2}{4}=1$



Clearly no point of intersection

39. (c)

If given line lies on the plane, the normal of plane must also be normal (perpendicular) to line.

Direction ratios of line $(3, 2+\lambda, -1)$

Direction ratios Normal $(1, -2, 0)$

$\therefore\ (3)(1) + (2+\lambda)(-2) + (-1)(0) = 0$

$\Rightarrow -1 - 2\lambda = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

40. (d)

$2\cot^{-1}\frac{1}{2} - \cot^{-1}\frac{4}{3}$

$= 2\tan^{-1}2 - \cot^{-1}\frac{4}{3}$

$= \pi + \tan^{-1}\left(\frac{2(2)}{1-2^2}\right) - \cot^{-1}\frac{4}{3}$ $\quad \left\{2\tan^{-1}x = \pi + \tan^{-1}\frac{2x}{1-x^2};\ x>1\right\}$

$= \pi + \tan^{-1}\left(-\frac{4}{3}\right) - \cot^{-1}\frac{4}{3}$ $\quad \{\tan^{-1}(-x) = -\tan^{-1}x\}$

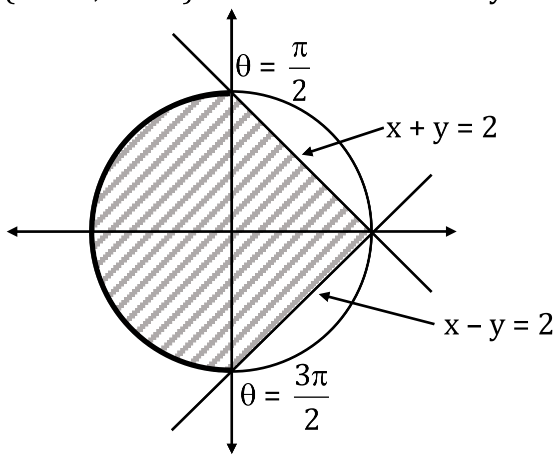
$= \pi - \left(\tan^{-1}\frac{4}{3} + \cot^{-1}\frac{4}{3}\right)$

$= \pi - \frac{\pi}{2}$

$= \frac{\pi}{2}$

41. (c)

$(2\cos\theta, 2\sin\theta)$ lies on the circle $x^2 + y^2 = 4$



The given point will lie in shaded region for $\theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

42. (c)

Let the point be (a, b)

Now, $\frac{|a-2b+1|}{\sqrt{5}} = \sqrt{5} \Rightarrow a - 2b + 1 = \pm 5$ ...(1)

Also $\frac{|2a+3b-1|}{\sqrt{13}} = \sqrt{13} \Rightarrow 2a + 3b - 1 = \pm 13$ ...(2)

Anyone of equations (1) will give a point of intersection with anyone of equations (2).

$\therefore$ total 4 points.

43. (a)

$f(x) = y = \frac{x^2 - x + 4}{x^2 + x + 4}$; $x \in R$

$\Rightarrow y(x^2 + x + 4) = x^2 - x + 4$

$\Rightarrow (y - 1)x^2 + (y + 1)x + 4y - 4 = 0$

For x to be real, $D \geq 0$

$\Rightarrow (y + 1)^2 - 4\cdot 4\cdot (y - 1)^2 \geq 0$

$\Rightarrow (5y - 3)(-3y + 5) \geq 0$

$\Rightarrow (5y - 3)(3y - 5) \leq 0$

$\Rightarrow \frac{3}{5} \leq y \leq \frac{5}{3}$

Also, check for y = 1; x = 0, which is valid.

44. Bonus (G)

$z = 2x^2 + y^2 + 2xy + 2x - 3y + 8$

$= \frac{1}{2}\{4x^2 + 2y^2 + 4xy + 4x - 6y + 16\}$

$= \frac{1}{2}\{(2x + y + 1)^2 + (y - 4)^2 - 1\}$

$= \frac{1}{2}\{(2x + y - 1)^2 + (y - 4)^2\} - \frac{1}{2} \geq -\frac{1}{2}$

So the least value is $-\frac{1}{2}$ at $x = -\frac{5}{2}$ & $y = 4$

45. (d)

A continuous function assuming only irrational (or rational) value must be a constant function.

$\therefore f(x) = \sqrt{2}, \forall x \in R$

46. (a)

$y = \sin\theta + \cos\theta + \frac{2}{\sin 2\theta}; \theta \in \left(0, \frac{\pi}{2}\right)$

$= \sin\theta + \cos\theta + \frac{1}{\sin\theta\cos\theta}$

As the expression remains unchanged by interchanging $\sin\theta$ and $\cos\theta$ so minimum is achieved for $\sin\theta = \cos\theta$

i.e. for $\theta = \frac{\pi}{4}$ in $\left(0, \frac{\pi}{2}\right)$

So, minimum value = $2 + \sqrt{2}$

47. (b)

$$\frac{X_n}{X_{n-1}} = \left(1 - \frac{1}{\frac{n(n+1)}{2}}\right)^2 = \left(\frac{n(n+1)-2}{n(n+1)}\right)^2 = \left(\frac{(n+2)(n-1)}{n(n+1)}\right)^2 = \frac{\left(\frac{n+2}{n}\right)^2}{\left(\frac{n+1}{n-1}\right)^2}$$

By comparing, $X_n = K\left(\frac{n+2}{n}\right)^2$

$X_2 = \frac{4}{9} \Rightarrow K = \frac{1}{9}$

Now, $\lim_{n\to\infty} X_n = \lim_{n\to\infty} \frac{1}{9}\left(1 + \frac{2}{n}\right)^2 = \frac{1}{9}$

48. (a)

Variance of first n natural numbers is $\frac{n^2-1}{12}$

$$\sigma^2 = \frac{\sum X_i^2}{n} - \left(\frac{\sum Xi}{n}\right)^2$$

$$= \frac{\frac{n(n+1)(2n+1)}{6}}{n} - \left(\frac{\frac{n(n+1)}{2}}{n}\right)^2$$

$$= \frac{n+1}{2}\left\{\frac{2n+1}{3} - \frac{n+1}{2}\right\}$$

$$= \frac{(n+1)}{2}\cdot\left(\frac{n-1}{6}\right)$$

$$= \frac{n^2-1}{12}$$

Now, for n = 20

$$\sigma^2 = \frac{20^2-1}{12} = \frac{133}{4}$$

49. (b)

Let, coin be tossed n times,

Probability of getting exactly 3 heads = $^nC_3 \underbrace{\left(\frac{1}{2}\right)^3}_{H} \underbrace{\left(\frac{1}{2}\right)^{n-3}}_{T}$

Probability for exactly 5 heads = $^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5}$

(Using Binomial probability distribution)

Now, $^nC_3\left(\frac{1}{2}\right)^n = {}^nC_5\left(\frac{1}{2}\right)^n$

$\Rightarrow$ n = 8

Thus, required probability = $^8C_1\left(\frac{1}{2}\right)^8 = \frac{1}{32}$

50. (a)

Required probability alignments $= \dfrac{\overbrace{\left(\frac{10!}{2!}\right)}^{\text{Two B's together}}}{\underbrace{\left(\frac{11!}{2!2!}\right)}_{\text{Total number of alignments}}} = \frac{2}{11}$

51. (c)

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} \sin x - \cos x & 0 & \cos x \\ 0 & \sin x - \cos x & \cos x \\ \cos x - \sin x & \cos x - \sin x & \sin x \end{vmatrix} = 0$$

Taking common (sinx – cosx) from C_1 & C_2

$$\Rightarrow (\sin x - \cos x)^2 \begin{vmatrix} 1 & 0 & \cos x \\ 0 & 1 & \cos x \\ -1 & -1 & \sin x \end{vmatrix} = 0$$

$\Rightarrow (\sin x - \cos x)^2\{1\cdot(\sin x + \cos x) + 0 + \cos x(1)\} = 0$

$\Rightarrow (\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\Rightarrow \tan x = 1 \to x = \dfrac{\pi}{4}$

or $\tan x = -2 \to$ No value of x in $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

52. (b)

Among $x_1, x_2, \ldots, x_{15}$; one of them, say x_k is 1.

$\Rightarrow x_k - 1 = 0$

$\Rightarrow$ the given product is '0'.

53. (c)

$$\lim_{x\to 0}\frac{\sin[-x^2]}{[-x^2]} = \lim_{x\to 0}\frac{\sin(-1)}{(-1)} = \sin(1)$$

for f(x) to be continuous,

$\lim\limits_{x\to 0} f(x) = f(0)$

$\Rightarrow \alpha = \sin(1)$

54. (c)

$$\int_0^{\sqrt{3}} f(x^2)dx = \int_0^{\sqrt{3}} \{x^2\}dx$$

$$= \int_0^{\sqrt{3}} \{x^2 - [x]\}dx$$

$$= \int_0^{\sqrt{3}} x^2\,dx - \left(\int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx\right)$$

$$= \sqrt{3} - \left(0 + \int_1^{\sqrt{2}} 1\cdot dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\cdot dx\right)$$

$$= \sqrt{3} - \left(\sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2}\right)$$

$$= 1 + \sqrt{2} - \sqrt{3}$$

55. (b)

S : a + b + c = 21 and T: a + c = 2b

$\Rightarrow$ a + c = 14 & b = 7

$\downarrow$

(1, 13), (2, 12), …….(6, 8) or a = b = c = 7

$\therefore$ 7 triplets

56. (a)

$C_1 : y = e^{x^2}, C_2 : y = e^{x^2}\sin x$

Point of intersection

$e^{x^2} = e^{x^2}\sin x$

$\Rightarrow$ sin x = 1

$\Rightarrow$ $x = (4n+1)\dfrac{\pi}{2}$; $n \in I$

Now,

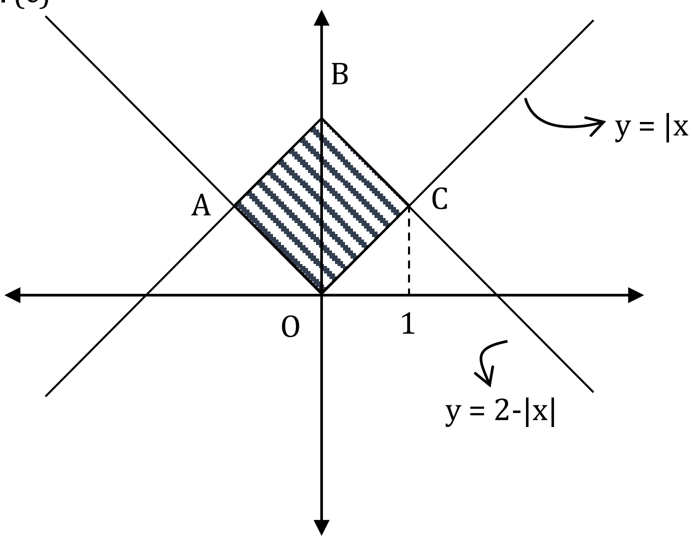
$C_1 : \dfrac{dy}{dx} = 2xe^{x^2}$, $C_2 : \dfrac{dy}{dx} = 2xe^{x^2}\sin x + e^{x^2}.\cos x$

$= 2xe^{x^2}$ ($\because$ when sin x = 1, cos x = 0)

$\Rightarrow$ Slopes of tangents are equal.

$\Rightarrow$ Angle of intersection zero.

57. (c)



In figure; A(-1,1), B(0,2), C(1,1), 0(0,0)
OABC is a square with side length $\sqrt{2}$,
$\therefore$ Area = $\left(\sqrt{2}\right)^2 = 2$ sq. units.

58. (c)

(i) $225 = 3^2 \times 5^2 \Rightarrow d(225) = (2+1)(2+1) = 9$
(ii) $1125 = 3^2 \times 5^3 \Rightarrow d(1125) = 3 \times 4 = 12$
(iii) $640 = 2^7 \times 5 \Rightarrow d(640) = 8 \times 2 = 16$
9, 12, 16 are in GP

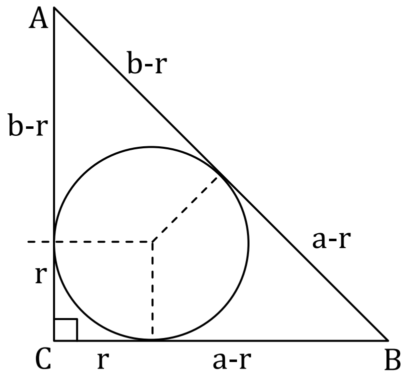
59. (d)

$$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$$
$$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1} 2a \le \frac{\pi}{2}$$
$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1} 2a \le \frac{\pi}{4}$$
$$\Rightarrow \frac{-1}{\sqrt{2}} \le 2a \le \frac{1}{\sqrt{2}}$$
$$\Rightarrow \frac{-1}{2\sqrt{2}} \le a \le \frac{1}{2\sqrt{2}}$$
$$\Rightarrow |a| \le \frac{1}{2\sqrt{2}}$$

60. (c)

If 2 – i and $\sqrt{5}+2i$ are two roots then other roots must be $2+i$ & $\sqrt{5}-2i$
So, Product of roots = $(2-i)(2+i)(\sqrt{5}+2i)(\sqrt{5}-2i) = 5 \times 9 = 45$

61. (c)



Now, AB = 2R = b – r + a – r

$\Rightarrow$ 2 (R + r) = b + a

62. (d)

$a\cos\theta + b\sin\theta = C$

$$\Rightarrow \quad a\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)+b\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)=c$$

$$\Rightarrow \quad (c+a)\tan^2\frac{\theta}{2}-2b\tan\frac{\theta}{2}+c-a=0 \quad ......(1)$$

α, β are roots of equation (1)

$$\therefore \tan\frac{\alpha}{2}+\tan\frac{\beta}{2}=\frac{+2b}{c+a}, \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{c-a}{c+a}$$

If $\alpha + \beta$ is also roots of given equation

$$\text{Then, } \tan\frac{\alpha+\beta}{2}=\frac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}.\tan\frac{\beta}{2}}=\frac{\frac{2b}{c+a}}{1-\frac{c-a}{c+a}}=\frac{b}{a} \text{ will satisfy (1)}$$

$$\text{i.e. } (c+a)\ \frac{b^2}{a^2}-2b.\frac{b}{a}+c-a=0$$

$$\Rightarrow \quad (c+a)b^2-2b^2a+(c-a)a^2=0$$

$$\Rightarrow \quad b^2(c-a)+a^2(c-a)=0$$

$$\Rightarrow \ (c-a)(b^2+a^2)=0$$

$\Rightarrow$ c – a = 0

$\Rightarrow$ c = a

63. (b)

Let $U_i = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix}; i = 1,2,3$

$$AU_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a_1 \\ 2a_1 + b_1 \\ 3a_1 + 2b_1 + c_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$a_1 = 1$, $b_1 = -2$, $c_1 = 1$

$$AU_2 = \begin{pmatrix} a_2 \\ 2a_2 + b_2 \\ 3a_2 + 2b_2 + c_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$a_2 = 2$, $b_2 = -1$, $c_2 = -4$

$$AU_3 = \begin{pmatrix} a_3 \\ 2a_3 + b_3 \\ 3a_3 + 2b_3 + c_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$a_3 = 2$, $b_3 = 1$, $c_3 = -3$

$$\therefore U = \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -4 \end{pmatrix}$$

$|U| = 1 (3 - 4) - 2 (c + 1) + 2 (8 + 1) = 3$

$$U^{-1} = \frac{1}{|U|}\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}^T$$

$$= \frac{1}{3}\begin{pmatrix} -1 & -7 & 9 \\ -2 & -5 & +6 \\ 0 & -3 & +3 \end{pmatrix}^T$$

Sum of elements of $U^{-1} = 0$

64. (d)

$f(1) = 1$

$n = 2;\ f(1) + 2f(2) = 2.3\ f(2)$

$$f(2) = \frac{1}{4}$$

$n = 3;\ f(1) + 2f(2) + 3f(3) = 12f(3)$

$$f(3) = \frac{1}{6}$$

$$\Rightarrow \quad f(n) = \frac{1}{2h}$$

$$\therefore f(500) = \frac{1}{1000}$$

65. (a)

Total no. of ways to put 5 balls in 5 cells = 5^5

No. of ways so that exactly one cell remains empty

(i) Select one cell to be left empty = ${}^5C_1$

(ii) Distribute 5 balls to 4 cells so that no cell will be left empty.

Distribution will be of 1, 1, 1 & 2

$$= {}^5C_1 \underbrace{\frac{5!}{(1!)^3 \times 2! \times 3!}}_{\text{Division}} \times \underbrace{4!}_{\text{Arrangement}}$$

$$\text{Now, required probability} = \frac{{}^5C_1 \times \frac{5!}{2! \times 3!} \times 4!}{5^5}$$

$$= \frac{48}{125}$$

66. (c)

E $\rightarrow$ death due to lung cancer

$E_1 \rightarrow$ Person is a smoker

$E_2 \rightarrow$ Person is a non-smoker

$$P(E_1) = \frac{1}{5}, P(E_2) = \frac{4}{5}$$

$$P(E) = 0.006, \quad P\left(\frac{E}{E_1}\right) = 10P\left(\frac{E}{E_2}\right)$$

Now,

$$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$

$$\Rightarrow \quad 0.006 = \frac{1}{5} \cdot P\left(\frac{E}{E_1}\right) + \frac{4}{5} \times \frac{1}{10} \cdot P\left(\frac{E}{E_1}\right) \Rightarrow \quad P\left(\frac{E}{E_1}\right) = \frac{3}{140}$$

67. (a)

Using Baye's Theorem

$$P\left(\frac{E}{E_1}\right)=\frac{P(E_1)\cdot P\left(\frac{E}{E_1}\right)}{\Sigma P(E_i)\cdot P\left(\frac{E}{E_i}\right)}$$

$E_i \rightarrow$ Probability of travelling by car, scooter, bus, train

$P\left(\frac{E}{E_1}\right) \rightarrow$ Probability of reaching on time

$$=\frac{\frac{1}{7}\times\frac{7}{8}}{\frac{1}{7}\times\frac{7}{9}+\frac{3}{7}\times\frac{8}{9}+\frac{2}{7}\times\frac{5}{9}+\frac{1}{7}\times\frac{8}{9}}$$

$$=\frac{1}{7}$$

68. (a)

$$I=\int\frac{dx}{(x-2)^{-\frac{1}{3}}.(x+3)^{\frac{7}{3}}},\left\{\frac{1}{3}-\frac{7}{3}=-2(-\text{ve even integer})\right\}$$

$$=\int\frac{dx}{(x-2)^2\left(\frac{x+3}{x-2}\right)^{\frac{7}{3}}}$$

Put $\frac{x+3}{x-2}=t$

$$\frac{-5}{(x-2)^2}dx=dt$$

$$I=\frac{-1}{5}\int\frac{dt}{t^{7/3}}=\frac{3}{20t^{4/3}}+C=\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3}+C$$

69. (c)

Applying L' hospital rule

$$\lim_{x\to 0}\frac{f'(x)+2f'(2x)+3f'(3x)+.........2015f'(2015x)}{1}$$

$$=\lim_{x\to 0} f'(0)(1+2+3+........2015)$$

$$=\cancel{2}\times\frac{2015\times 2016}{\cancel{2}} = 2015\times 2016$$

70. (a)

Since 17!is divisible by 9,

$\therefore$ Sum of digits must be divisible by 9

$\Rightarrow$ 48 + x + y is divisible by 9

$\Rightarrow$ x + y = 6 or 15 ; x, y $\in \{0,1,2,.......a\}$

Again,17! is divisible by 11,

|Sum of digits at odd places – sum of digits at even places|

Must be divisible by 11

i.e. |10 + x - y| is divisible by 11

$\Rightarrow$ |x - y| = 1 (only possible value)

$\therefore$ x + y = 15, (x, y) = (8,7) or (7,8)

71. (c)

A $\rightarrow$ A quadratic equation with rational coefficient can have any type of roots.

Ex:

(i) $x^2 = 0 \rightarrow$ zero roots

(ii) $x^2 - x - 1 = 0 \rightarrow$ Irrational roots

so can be true

B $\rightarrow$ Again a Quadratic Equation with real coefficients can have any type of roots.

So, can be true

C $\rightarrow$ A Quadratic Equation with irrational distinct coefficients can never have zero or rational roots.

So, always false

D $\rightarrow$ It can have any roots

So, can be true.

72. (a,c)

x.y = 1

Slope of normal $= \frac{-dx}{dy} = x^2$ or $\frac{1}{y^2} > 0$

$\therefore$ given line is normal if its slope,

i.e. $\frac{a-1}{b} > 0$

A and C are possible

73. (c,d)

Probability of at least one defective part

=1- non defective part

$= 1 - (0.95)^n \geq \frac{1}{2}$

$\Rightarrow \quad (0.95)^n \leq \frac{1}{2}$

Taking log

$\Rightarrow \quad n\log_{10}(0.95) \leq \log_{10}\left(\frac{1}{2}\right)$

$\Rightarrow \quad n\{\log_{10}95 - \log_{10}100\} \leq \log_{10}2^{-1}$

$\Rightarrow \quad n\,\{1.977 - 2\} \leq -1\cdot 0.3$

$\Rightarrow \quad n\,(-0.023) \leq -0.3$

$\Rightarrow \quad n \geq \frac{300}{23}$

$\therefore n = 14, 15$

74. (a,d)

$f(2x-1) = f(x)\ \ \forall\ x \in R$

Replacing $x \to \frac{x+1}{2}$

$\Rightarrow\ f(x) = f\left(\frac{x+1}{2}\right)$

$= f\left(\frac{\frac{x+1}{2}+1}{2}\right)$ $\quad\{$Again replacing $x \to \frac{x+1}{2}\}$

$= f\left(\frac{x+1+2}{22}\right)$

$= f\left(\frac{x+1+2+2^2+.......2^{n-1}}{2^n}\right)$

Now taking $\lim n \to \infty$

We have $f(x) = \lim\limits_{n\to\infty} f\left(\frac{x+\frac{2^n-1}{1}}{2^n}\right)$

$= \lim\limits_{n\to\infty} f\left(\frac{x-1}{2^n}+1\right)$

$= f(1)$ $\{\because f(x)$ is cont. at $x = 1\}$

$= 1$ $\qquad \therefore f(x)$ is constant function, continuous everywhere.

75. (a,d)

$y = a \sin x + b \cos x$

$\frac{dy}{dx} = a \cos x - b \sin x$

$\Rightarrow \frac{d^2y}{dx^2} = -a \sin x - b \cos x = -y$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$

The given difference = n is general solution of $y = a \sin x + b \cos x$

$\therefore$ a and d are correct options.

76. (a,c)

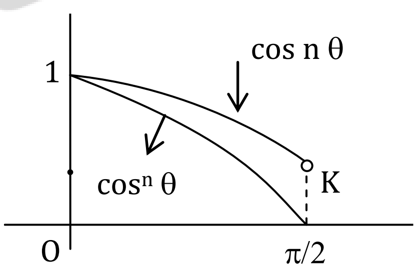
$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}\ ;\ 0 < n\theta \le \frac{n\pi}{2} \quad \{0 < n < 1 \therefore \frac{n\pi}{2} < \frac{\pi}{2}\}$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \cos^n\theta < 1$ ……(1)

$K < \cos n\theta < 1$

Where, $0 < K < 1$



$\therefore \cos^n\theta \le \cos n\theta$

77. (a,b,c)

$16x^2 - 3y^2 - 32x - 12y = 44$

$\Rightarrow \quad 16(x^2-2x) - 3(y^2+4y) = 44$

$\Rightarrow \quad 16(x^2-2x+1) - 3(y^2+4y+4) = 44+4$

$\Rightarrow \quad 16(x-1)^2 - 3(y+2)^2 = 48$

$$\Rightarrow \quad \frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$$

Length of transverse axis = 2a = $2\sqrt{3}$

Length of Latus Rectum = $\frac{2b^2}{a} = \frac{32}{\sqrt{3}}$

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{\frac{19}{3}}$$

Directrix; $X = \pm\frac{a}{e}$

$$x-1 = \pm\frac{\sqrt{3}}{\sqrt{19/3}}$$

$$x = 1 \pm \frac{3}{\sqrt{19}}$$

78. (b,c)

$$f(x) = \left[\frac{1}{[x]}\right]$$

For domain: $[x] \neq 0$

$x \neq [0,1)$

$\therefore D_f = R - [0, 1)$

Range: -

$$\frac{1}{[x]} \to 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \quad -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4} ....$$

$$\left[\frac{1}{[x]}\right] \to 1, 0, -1$$

$\therefore \quad R_f = \{-1, 0, 1\}$

79. (b,c)

Applying Rolle's Theorem on f(x) in [a, b]

$f(a) = f(b) = 0$

f(x) is constant and differential function

$f'(c) = 0$ for $c \in (a, b)$

Now, again applying Rolle's Theorem on f'(x)

$f'(a) = f'(c) = 0$

$\therefore$ In (a, c), f''(x) will be zero at some point.

80. (b,c)

$x \rho y \Rightarrow x.y > 0$

Reflexive: $x \rho x \Rightarrow x.x > 0$

$x^2 > 0$

Not true for $x = 0$

$\therefore$ not reflexive

Symmetric: $x \rho y \Rightarrow x.y > 0$

$\Rightarrow y.x > 0$

$\Rightarrow y \rho x$

$\therefore$ Symmetric

Transitive: $x \rho y \Rightarrow x.y > 0$ .....(i)

$y \rho z \Rightarrow y.z > 0$ ..... (ii)

From (i) and (ii)

$x.z.y^2 > 0$

$\Rightarrow x.z > 0$

$\Rightarrow x \rho z$

$\therefore$ Transitive

WBJEE-2015 (Chemistry)

1. Match the flame colours of the alkaline earth metal salts in the Bunsen burner.

(A) Calcium	(p) brick red
(B) Strontium	(q) apple green
(C) Barium	(r) crimson

a. a-p, b-r, c-q b. a-r, b-p, c-q

c. a-q, b-r, c-p d. a-p, b-q, c-r

2. Extraction of gold (Au) involves the formation of complex ions 'X' and 'Y'.

$$\text{Gold ore} \xrightarrow[CN^-,\,H_2O,\,O_2]{\text{Roasting}} HO^- + \text{'X'} \xrightarrow{Zn} \text{'Y'} + Au$$

'X' and 'Y' are respectively

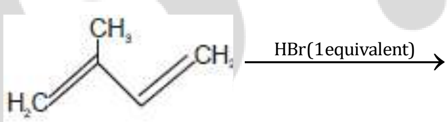
a. $Au(CN)_2^-$ and $Zn(CN)_4^{2-}$ b. $Au(CN)_4^{3-}$ and $Zn(CN)_4^{2-}$

c. $Au(CN)_3^-$ and $Zn(CN)_6^{4-}$ d. $Au(CN)_4^-$ and $Zn(CN)_3^-$

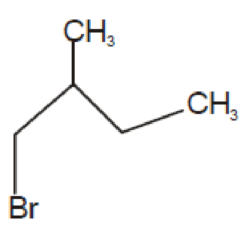
3. The atomic number of cerium (Ce) is 58. The correct electronic configuration of Ce3+ ion is

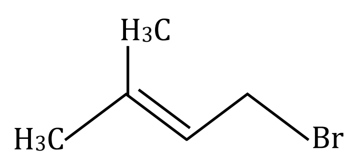
a. $[Xe]4f^1$ b. $[Kr]4f^1$

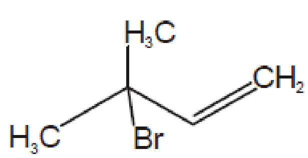
c. $[Xe]4f^{13}$ d. $[Kr]4d^1$

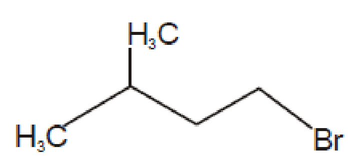
4. 

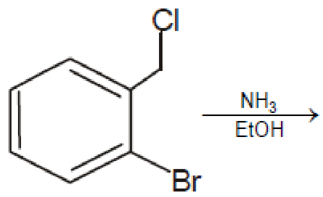
The major product of the above reaction is

a. 

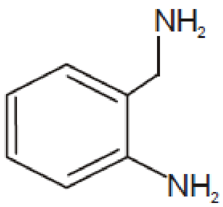
b. 

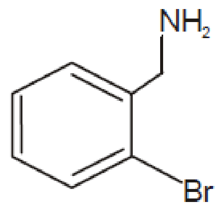
c. 

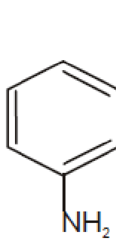
d. 

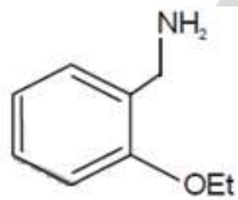
5. 

The product of the above reaction is

a. 

c. 

b. 

d. 

6. Sulphuryl chloride (SO_2Cl_2) reacts with white phosphorus (P_4) to give

a. PCl_5, SO_2

b. $OPCl_3$, $SOCl_2$

c. PCl_5, SO_2, S_2Cl_2

d. $OPCl_3$, SO_2, S_2Cl_2

7. The number of lone pair of electrons on the central atoms of H_2O, $SnCl_2$, PCl_3 and XeF_2 respectively, are

a. 2,1,1,3

b. 2,2,1,3

c. 3,1,1,2

d. 2,1,2,3

8. Consider the following salts: NaCl, $HgCl_2$, Hg_2Cl_2, $CuCl_2$, CuCl and AgCl. Identify the correct set of insoluble salts in water

a. Hg_2Cl_2, CuCl, AgCl

b. $HgCl_2$, CuCl, AgCl

c. Hg_2Cl_2, $CuCl_2$, AgCl

d. Hg_2Cl_2, CuCl, NaCl

9. In the following compound, the number of 'sp' hybridized carbon is

$$CH_2 = C = CH - \underset{\displaystyle |\atop \displaystyle CN}{CH} - C \equiv CH$$

a. 2
b. 3
c. 4
d. 5

10. For the reaction A + 2B → C, the reaction rate is doubled if the concentration of A is doubled. The rate is increased by four times when concentrations of both A and B are increased by four times. The order of the reaction is

a. 3
b. 0
c. 1
d. 2

11. At a certain temperature, the value of the slope of the plot of osmotic pressure (π) against concentration (C in mol L^{-1}) of a certain polymer solution is 291R. The temperature at which osmotic pressure is measured is (R is gas constant)

a. 271°C
c. 564 K
b. 18°C
d. 18 K

12. The rms velocity of CO gas molecules at 27°C is approximately 1000 m/s. For N2 molecules at 600 K the rms velocity is approximately

a. 2000 m/s
b. 1414 m/s
c. 1000 m/s
d. 1500 m/s

13. A gas can be liquefied at temperature T and pressure P provided

a. $T = T_c$ and $P < P_c$
b. $T < T_c$ and $P > P_c$
c. $T > T_c$ and $P > P_c$
d. $T > T_c$ and $P < P_c$

14. The dispersed phase and dispersion medium of fog respectively are

a. solid, liquid
b. liquid, liquid
c. liquid, gas
d. gas, liquid

15. The decreasing order of basic character of K_2O, BaO, CaO and MgO is

a. $K_2O > BaO > CaO > MgO$
b. $K_2O > CaO > BaO > MgO$
c. $MgO > BaO > CaO > K_2O$
d. $MgO > CaO > BaO > K_2O$

16. In aqueous alkaline solution, two electron reduction of HO_2^- gives

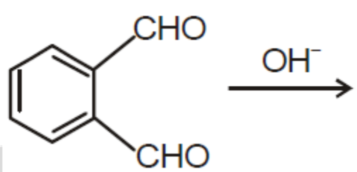
a. HO^-
b. H_2O
c. O_2
d. O_2^-

17. Cold ferrous sulphate solution on absorption of NO develops brown colour due to the formation of
    a. paramagnetic $[Fe(H_2O)_5(NO)]SO_4$
    b. diamagnetic $[Fe(H_2O)_5(N_3)]SO_4$
    c. paramagnetic $[Fe(H_2O)_5(NO_3)](SO_4)_2$
    d. diamagnetic $[Fe(H_2O)_4(SO_4)]NO_3$

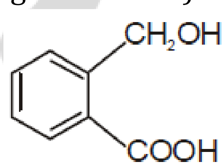
18. Amongst Be, B, Mg and Al the second ionization potential is maximum for
    a. B
    b. Be
    c. Mg
    d. Al

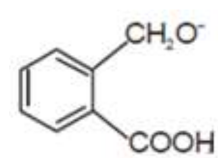
19. In a mixture, two enantiomers are found to be present in 85% and 15% respectively. The enantiomeric excess (e, e)
    a. 85%
    b. 15%
    c. 70%
    d. 60%

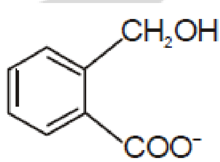
20. 1,4-dimethylbenzene on heating with anhydrous $AlCl_3$ and HCl produces
    a. 1,2-dimethylbenzene
    b. 1,3-dimethylbenzene
    c. 1,2,3-trimethylbenzene
    d. Ethylbenzene

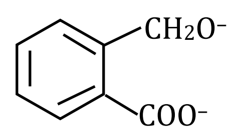
21. 

The product of the above reaction is (Unique set of options is provided for both English and Bengali versions)

a. 

b. 

c. 

d. 

22. Suppose the mass of a single Ag atom is 'm'. Ag metal crystallizes in fcc lattice with unit cell of length 'a'. The density of Ag metal in terms of 'a' and 'm' is
    a. $\frac{4m}{a^3}$
    b. $\frac{2m}{a^3}$
    c. $\frac{m}{a^3}$
    d. $\frac{m}{4a^3}$

23. For the reaction $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ at 300K, the value of ΔG^0 is – 690.9R. The equilibrium constant value for the reaction at that temperature is (R is gas constant)
    a. 10 atm^{-1}
    b. 10 atm
    c. 10
    d. 1

24. At a particular temperature the ratio of equivalent conductance to specific conductance of a 0.01 (N) NaCl solution is
    a. 10^5 cm^3
    c. 10 cm^3
    b. 10^3 cm^3
    d. 10^5 cm^2

25. The units of surface tension and viscosity of liquids are respectively
    a. kg $m^{-1}s^{-1}$, N m^{-1}
    b. kg s^{-2}, kg m^{-1} s^{-1}
    c. N m^{-1}, kg $m^{-1}s^{-2}$
    d. kg s^{-1}, kg m^{-2} s^{-1}

26. The ratio of volumes of CH_3COOH 0.1 (N) to CH_3COONa 0.1 (N) required to prepare a buffer solution of pH 5.74 is (given : pKa of CH_3COOH is 4.74)
    a. 10 : 1
    b. 5 : 1
    c. 1 : 5
    d. 1 : 10

27. The reaction of methyltrichloroacetate (Cl_3CCO_2Me) with sodium methoxide (NaOMe) generates
    a. Carbocation
    b. Carbene
    c. Carbanion
    d. Carbon radical

28. Best reagent for nuclear iodination of aromatic compounds is
    a. KI/CH_3COCH_3
    b. I_2/CH_3CN
    c. KI/CH_3COOH
    d. I_2/HNO_3

29. In the Lassaigne's test for the detection of nitrogen in an organic compound, the appearance of blue coloured compound is due to
    a. ferric ferricyanide
    b. ferrous ferricyanide
    c. ferric ferrocyanide
    d. ferrous ferrocyanide

30. In the following reaction

$$RMgBr + HC(OEt)_3 \xrightarrow{\text{ether}} \xrightarrow{H_3O^+} P$$

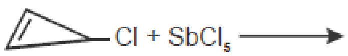
The product 'P' is
    a. RCHO
    b. R_2CHOEt
    c. R_3CH
    d. $RCH(OEt)_2$

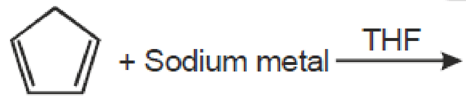
31. Addition of sodium thiosulphate solution to a solution of silver nitrate gives 'X' as white precipitate, insoluble in water but soluble in excess thiosulphate solution to give 'Y'. On boiling in water, 'Y' gives 'Z'. 'X', 'Y' and 'Z' respectively, are
    a. $Ag_2S_2O_3$, $Na_3[Ag(S_2O_3)_2]$, Ag_2S
    b. Ag_2SO_4, $Na[Ag(S_2O_3)_2]$, Ag_2S_2
    c. $Ag_2S_2O_3$, $Na_5[Ag(S_2O_3)_3]$, AgS
    d. Ag_2SO_3, $Na_3[Ag(S_2O_3)_2]$, Ag_2O

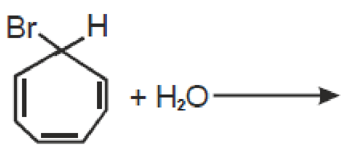
32. At temperature of 298 K the emf of the following electrochemical cell Ag(s) | Ag^+ (0.1 M) || Zn^{2+}(0.1M) | Zn(s) will be (given E^ocell= – 1.562 V)
   a. – 1.532 V
   b. – 1.503 V
   c. 1.532 V
   d. – 3.06 V

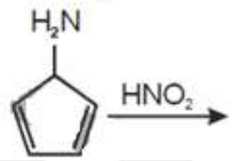
33. For the reaction $X_2Y_4(l) \rightarrow 2\ XY_2(g)$ at 300 K the values of ΔU and ΔS are 2 kCal and 20 Cal K^{-1} respectively. The value of ΔG for the reaction is
   a. – 3400 Cal
   b. 3400 Cal
   c. – 2800 Cal
   d. 2000 Cal

34. The total number of aromatic species generated in the following reactions is

(I) 

(II) 

(III) 

(IV) 

   a. Zero
   b. 2
   c. 3
   d. 4

35. Roasted copper pyrite on smelting with sand produces
   a. $FeSiO_3$ as fusible slag and Cu_2S matte'
   b. $CaSiO_3$ as infusible slag and Cu_2O matte'
   c. $Ca_3(PO_4)_2$ as fusible slag and Cu_2S matte'
   d. $Fe_3(PO_4)_2$ as infusible slag and Cu_2S matte'

36. Ionization potential values of noble gases decrease down the group with increase in atomic size. Xenon forms binary fluorides by the direct reaction of elements. Identify the correct statement(s) from below
   a. Only the heavier noble gases form such compounds
   b. It happens because the noble gases have higher ionization energies.
   c. It happens because the compounds are formed with electronegative ligands.
   d. Octet of electrons provides the stable arrangements.

37. Optical isomerism is exhibited by (ox = oxalate anion; en = ethylenediamine)
   a. *cis*-$[CrCl_2(ox)_2]^{3-}$
   b. $[Co(en)_3]^{3+}$
   c. *trans*-$[CrCl_2(ox)_2]^{3-}$
   d. $[Co(ox)(en)_2]^+$

38. The increase in rate constant of a chemical reaction with increasing temperature is (are) due to the fact(s) that

a. The number of collisions among the reactant molecules increases with increasing temperature.

b. The activation energy of the reaction decreases with increasing temperature.

c. The concentration of the reactant molecules increases with increasing temperature.

d. The number of reactant molecules acquiring the activation energy increases with increasing temperature.

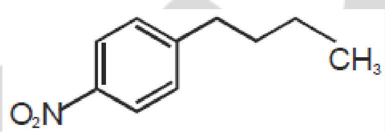
39. Within the list shown below, the correct pair of structures of alanine in pH ranges 2-4 and 9-11 is

(I) $H_3N^+ - CH(CH_3)CO_2H$ (II) $H_2N - CH(CH_3)CO_2^-$

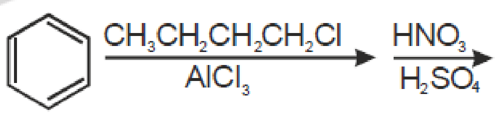
(II) $H_3N^+ - CH(CH_3)CO_2^-$ (IV) $H_2N^+ - CH(CH_3)CO_2H$

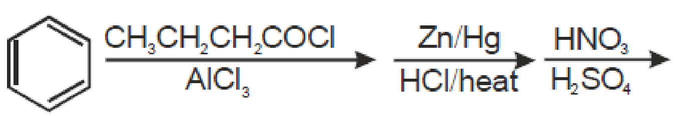
a. I, II b. I, III

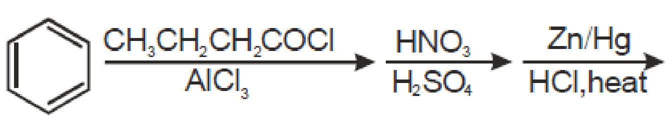
c. II, III d. III, IV

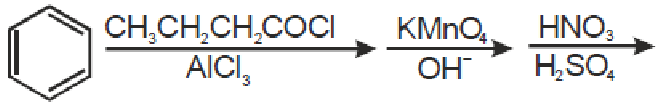
40. 

Identify the correct method for the synthesis of the compound shown above from the following alternatives

a. 

b. 

c. 

d. 

ANSWER KEYS

1. (a)	2. (a)	3. (a)	4. (b)	5. (c)	6. (a)	7. (a)	8. (a)	9. (c)	10. (c)
11. (b)	12. (b)	13. (b)	14. (c)	15. (a)	16. (a)	17. (a)	18. (a)	19. (c)	20. (b)
21. (c)	22. (a)	23. (a)	24. (a)	25. (b)	26. (d)	27. (b)	28. (d)	29. (c)	30. (a)
31. (a)	32. (a)	33. (c)	34. (c)	35. (a)	36. (a,c)	37. (a,b,d)	38. (a,d)	39. (a)	40.(b)

WBJEE-2015 (Chemistry)

Solution

1. (a)

The flame colours of the alkaline earth metal salts in Bunsen burner are as shown:-

- Calcium gives brick red colour $(a \rightarrow p)$
- Strontium gives crimson colour $(b \rightarrow r)$
- Barium gives apple green colour $(c \rightarrow q)$

The electron can easily be excited to higher energy levels. This results in characteristic colour during de excitation of electrons.

2. (a)

Extraction of gold (Au) involves formation of complex ions x and y. x and y are $Au(CN)_2^-$ and $Zn(CN)_4^{2-}$ respectively.

$$\text{Gold ore} \xrightarrow[CN^-, H_2O, O_2]{\text{Roasting}} HO^- + \underset{(X)}{Au(CN)_2^-} \xrightarrow{Zn} \underset{(Y)}{Zn(CN)_4^{2-}} + Au\downarrow$$

Note

- Complex X is water soluble.
- More electropositive zinc displaces gold from complex.

3. (a)

Z = 54 is Xe

Z = 56 is Ba

Electronic configuration of Ba = [Xe] $6s^2$

Z = 57 is La

Electronic configuration of La = [Xe] $5d^1 6s^2$

(Electron in 5d in place of expected 4f)

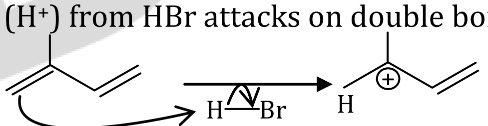
But in Z = 58 (Ce) electron enters 4f and 5d' also shifts to 4f. Hence electronic configuration of Ce (58) = [Xe] $4f^2\, 5d^0\, 6s^2$

$\therefore Ce^{3+} = [Xe]\, 4f^1$

4. (b)

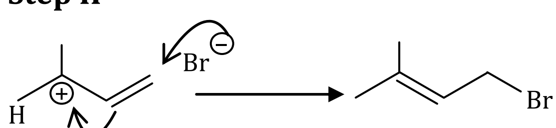
Step I

(H^+) from HBr attacks on double bond to make stable carbocation



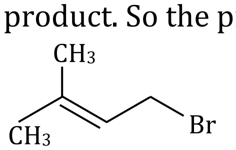
More stable carbocation)

Step II

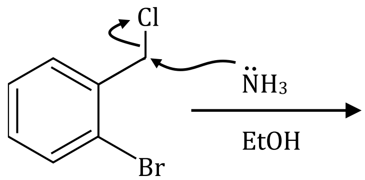


(1,4 addition product)

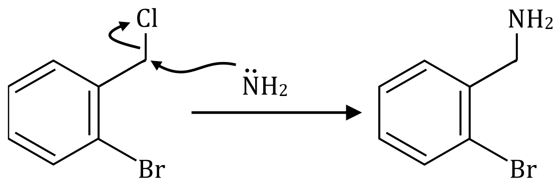
Note: For 1, 3 butadiene at room temperature there is formation of 1,4 addition product. So the product (major) is



5. (c)



Substitution will take place. We see Cl is a better leaving group than Br. Also, substitution at Cl position is easy so,



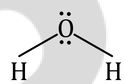
6. (a)

Sulphuryl chloride (SO_2Cl_2) reacts with white phosphorous (P_4) to give phosphorous pentachloride and sulphur dioxide.

$10SO_2Cl_2 + P_4 \longrightarrow 4PCl_5 + 10SO_2$

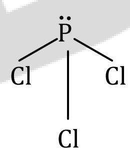
7. (a)

The number of lone pairs of electrons on central atoms of H_2O, $SnCl_2$, PCl_3 and XeF_2 are

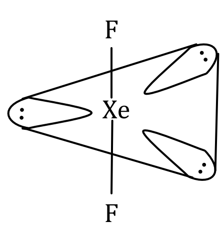


Number of lone pair = 2

Number of lone pairs = 1



Number of lone pair = 1



Number of lone pairs = 3

8. (a)

Hg_2Cl_2, CuCl, AgCl are insoluble in water due to high lattice enthalpy and low hydration enthalpy.

NaCl, $HgCl_2$, $CuCl_2$ are soluble in water due to low lattice enthalpy and high hydration enthalpy.

9. (c)

$$\overset{sp^2}{H_2C} = \overset{sp}{C} = \overset{sp^2}{CH} - \overset{sp^3}{\underset{\underset{\underset{sp}{C}\equiv N}{|}}{CH}} - \overset{sp}{C} \equiv \overset{sp}{CH}$$

Number of sp-hybridised carbon = 4

10. (c)

For the reaction A+2B $\rightarrow$C, the reaction rate is doubled. if the concentration of A is doubled. This indicates that the reaction is of first order with respect to A.

The rate is increased by four times when concentration of both A and B are increased by four times.

This indicates that the reaction is zero order with respect to B.

(Note – When the concentration of A is increased four times the rate is increased by four times.

Hence when concentration of B is increased by four times, rate is not increased.

The order of reaction = 1 + 0 = 1

11. (b)

The relationship between the osmotic pressure and the concentration is $\pi = CRT$

For the plot of osmotic pressure (π) against concentration (C in mol/L)

Slope $\Rightarrow$ RT = 291R.

T = 291 K = 18°C.

12. (b)

$$U_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{U_{rms,N_2}}{U_{rms,CO}} = \sqrt{\frac{T_{N_2} \times M_{CO}}{T_{CO} M_{N_2}}}$$

$$\frac{U_{rms,N_2}}{1000} = \sqrt{\frac{600 \times 28}{300 \times 28}}$$

$$U_{rms,N_2} = 1414\,m/s$$

13. (b)
When the temperature of the gas is equal to the critical temperature, the liquefaction is possible only when the pressure is equal to the critical pressure.
Thus when the temperature is the critical temperature and the pressure is less than the critical pressure, the liquefaction of gas is not possible.
So, for a gas to liquefy
$T < T_C$ and $P > P_C$

14. (c)
The dispersed phase and dispersion medium of log respectively are liquid and gas. It is an example of liquid aerosol. Other examples of this type of aerosol include mist and clouds.

15. (a)
Alkali metal oxides are more basic than their corresponding alkaline earth metal oxides. Thus K_2O is the most basic. Further, basic character of alkaline earth metal oxides increases down the group as the electropositive character of metal increases.
$BaO > CaO > MgO$
Combining the two trends, basic character increases in order
$K_2O > BaO > CaO > MgO$

16. (a)
In aqueous alkaline solution, two electrons reduction of HO_2^- gives HO^-

$$HO_2^- + H_2O + 2e^- \longrightarrow 3OH^-$$

17. (a)
Cold ferrous sulphate solution + NO $\longrightarrow$ Brown ring
For this we need to understand the brown ring concept.
When $FeSO_4$ reacts with H_2O in presence of NO
Following reaction is obtained

$$FeSO_4 + 5H_2O + NO \longrightarrow \underset{\text{Brown ring complex}}{[Fe(H_2O)_5NO]SO_4}$$

It has magnetic moment of 3.89 BM
i.e., μ = 3.89 BM
That is 3 unpaired electrons.
Therefore formation of Brown ring complex is due to the paramagnetic nature of complex.

18. (a)

Among B, Be, Mg and Al Boron has second highest ionization potential. Both beryllium and magnesium belong to group II A and boron and aluminium belongs to III A electronic configuration of Boron is $2s^2\ 2p^1$

Whereas, electronic configuration of B^+ is $2s^2$

Hence, it is difficult to remove second electron from $2s^2$ shell as half-filled and fully filled orbitals are more stable than others.

19. (c)

Two enantiomers are found to be present in 85% and 15% respectively.

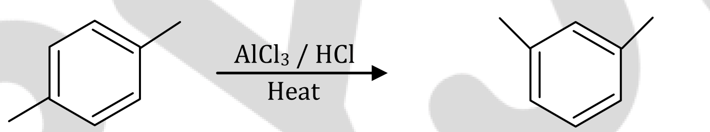
Now, 15% will form racemic mixture with other 15%.

Now enantiomeric excess = (85-15)% =70%

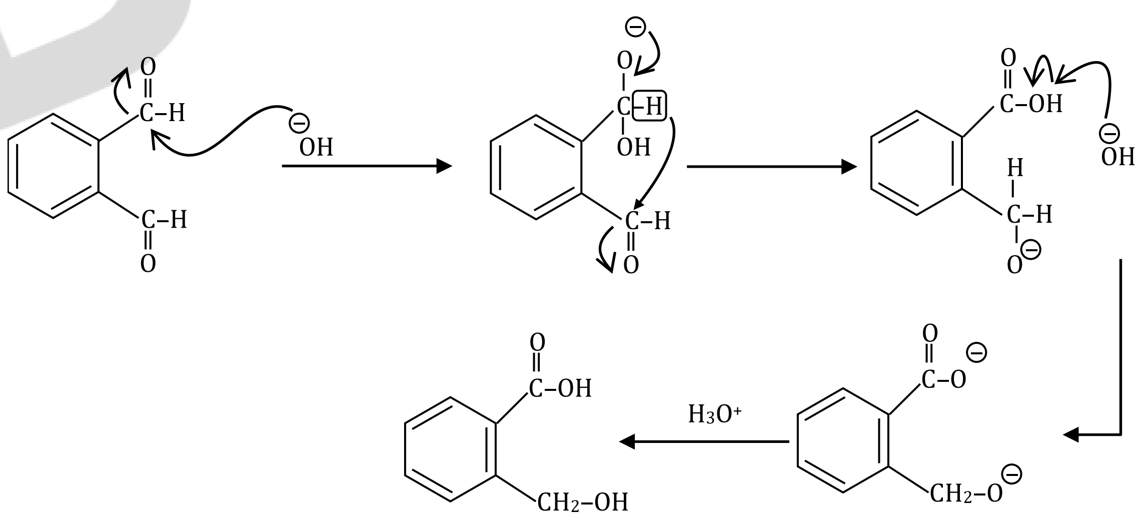
20. (b)

1,4-dimethyl benzene ring on heating with anhydrous $AlCl_3$ and HCl produces 1,3-dimethyl benzene.

The net reaction is isomerisation of di-substituted benzene.



21. (c)



Here intermolecular cannizaro reaction takes place.

22. (a)

Edge length of the unit cell = a

Volume of unit cell = a^3

One FCC unit cell contains 4 Ag atoms as [as 6 atoms present at face centre and 8 at corner]

Contribution at face $= \frac{1}{2}$.

Contribution at corner $= \frac{1}{8}$.

Rank (z) = $\frac{1}{2} \times 6 + \frac{1}{8} \times 8$

= 3 + 1

= 4

Mass of one Ag atom = m.

Mass of 4 Ag atoms = 4 m.

Density = $\frac{\text{mass}}{\text{volume}}$

$d = \frac{4m}{a^3}$

23. (a)

Given reaction

$2SO_2(g) + O_2(g) \rightleftharpoons 3SO_3(g)$

$\Delta G^o = -690.9R$ …… (1)

at T = 300 K

Gibbs free energy in terms of equilibrium constant is given as

$\Delta G^o = -RT \,\ell n$ keq.

Using equation (1)

$-690.9R = -RT\ell n$ Keq.

$\cancel{-}690.9\cancel{R} = \cancel{-}\cancel{R}(300)\ell n\text{Keq.}$

$\ell n\text{Keq.} = \frac{690.9}{300} = 2.303$

Using $\ell nk_{eq.} = 2.303 \log_{10} K_{eq.}$

$2.303 \log_{10} K_{eq} = 2.303$

$\log_{10} K_{eq} = 1$

$K = 10^1 = 10$

Unit of K

K = $(atm)^{\Delta n}$ ......(3)

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$\Delta n = 2 - 3 = -1$ ...... (2)

Using (2) in (3)

K = $(atm)^{-1}$

Therefore K = 10 atm^{-1}

24. (a)

Relation between equivalent conductance and specific conductance is given as

$$\lambda = \frac{K \times 1000}{C}$$

Here C = 0.01 N (given)

$$\frac{\lambda}{K} = \frac{1000}{C} = \frac{1000}{0.01} = 10^5 cm^3 eq^{-1}$$

$$\therefore \quad \lambda = \frac{\Omega^{-1} cm^2 eq^{-1}}{\Omega^{-1} cm^{-1}} = cm^3 eq^{-1}$$

25. (b)

Surface tension (γ) is given as work done by Area.

$$r = \frac{\Delta\omega}{\Delta A} = \frac{J}{m^2} = \frac{Kgm^2s^{-2}}{m^2} = Kgs^{-2}$$

Work done in Joules

Area in M^2

Also 1J = 1 Kg m^2 s^{-2}

Viscous drag (F) given as

$$F \propto A$$

$$F \propto \frac{dv}{dx}$$

$$F = \eta A \frac{dv}{dx}$$

(Viscosity)

$$\eta = \frac{F}{A \cdot \frac{dv}{dx}} = \frac{N}{m^2 \cdot \frac{ms^{-1}}{m}} = Nm^{-2}s$$

$$\eta = \frac{N}{m^2}s = \frac{Kgms^{-2} \cdot s}{m^2} \qquad [1N = Kg\ ms^{-2}]$$

= Kg $m^{-1}s^{-1}$

26. (d)

pH= 5.74 pK_a = 4.74

Let volume of acid solution = XL

Volume of salt solution = YL

pH = buffer is given as

$$pH = pK_a + \log \frac{[Salt]}{[Acid]}$$

$$CH_3COOH + NaOH \rightleftharpoons CH_3COONa + H_2O$$

$$\log_{10} \frac{[CH_3COONa]}{[CH_3COOH]} = 5.74 - 4.74$$

$$\frac{[CH_3COONa]}{[CH_3COOH]} = \frac{1}{10} \qquad \text{...... (1)}$$

Using Molarity (M) = $\frac{\text{Moles}}{\text{Volume}} \frac{(n)}{v}$

Moles (n) = Mv

Total volume = x + y

Number of moles of CH_3COONa = 0.1 × x

Number of moles of CH_3COOH = 0.1 y

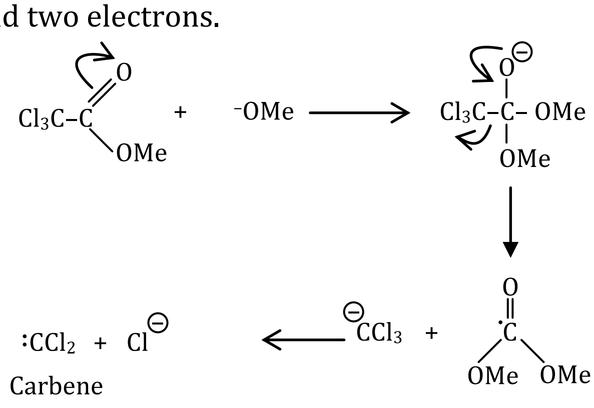
Putting these values in (1)

$$\frac{\frac{0.1x}{x+y}}{\frac{0.1y}{x+y}} = \frac{1}{10}$$

$$\frac{x}{y} = \frac{1}{10}$$

27. (b)

The reaction of methyl trichloroacetate (Cl_3CCO_2Me) with sodium methoxide (NaOMe) generates carbene. Carbene are neutral species having a carbon atom with two bonds and two electrons.



28. (d)

Best reagent for nuclear iodination of aromatic compounds is I_2/HNO_3. During iodination of aromatic compounds, HI is produced which is a strong reducing agent and can reduce iodobenzene back to benzene.

To prevent this, oxidising agent such as iodic acid, nitric acid or mercuric oxide is used.

29. (c)

Carbon and nitrogen present in the organic compound on fusion with sodium metal gives sodium cyanide (NaCN) soluble in water.

This is converted into sodium ferrocyanide by the addition of sufficient quantity of ferrous sulphate.

Ferric ions generated during the process react with ferrocyanide to form Prussian blue precipitate of ferric ferrocyanide.

$Na + C + N \rightarrow NaCN$

$$6NaCN + FeSO_4 \longrightarrow \underset{\text{Sodium ferrocyanide}}{Na_4\left[Fe(CN)_6\right]} + Na_2SO_4$$

$$Na_4\left[Fe(CN)_6\right] + Fe^{3+} \longrightarrow \underset{\text{Ferric Ferro cyanide}}{Fe_4\left[Fe(CN)_6\right]_3}$$

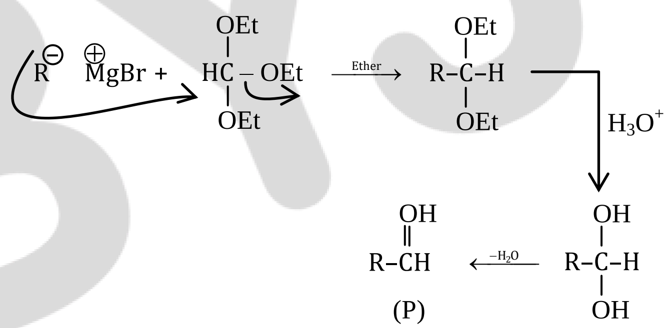
30. (a)

In the reaction

$$RMgBr + HC(OEt)_3 \xrightarrow[H_3O]{Ether} P$$

The product P is RCHO

$$RMgBr + HC(OEt)_3 \xrightarrow{Ether}$$



31. (a)

The addition of sodium thiosulphate solution to a solution of silver nitrate gives 'X' as a white precipitate, insoluble in water but soluble in excess thiosulphate solution to give 'y'

On boiling in water y gives z

x, y and z respectively are:

$Ag_2S_2O_3, Na_3[Ag(S_2O_3)_2]$ and Ag_2S respectively

$$Na_2S_2O_3 \xrightarrow{2AgNO_3} \underset{(x)}{Ag_2S_2O_3}$$

$$\underset{(x)}{Ag_2S_2O_3} \xrightarrow{Na_2S_2O_3(excess)} \underset{(y)}{Na_3[Ag(S_2O_3)_2]}$$

$$\underset{(y)}{Na_3[Ag(S_2O_3)_2]} \xrightarrow[Boil]{Water} \underset{(z)}{Ag_2S}$$

32. (a)

From the given cell, the cell reaction is

$$2Ag_{(s)} + Zn^{2+}(0.1M) \longrightarrow 2Ag^{+}(0.1) + Zn(s)$$

The Nernst equation is

$$E_{Cell} = E^{o}_{cell} - \frac{0.0591}{n} \log \frac{\left[Ag^{+}\right]^{2}}{\left[Zn^{2+}\right]}$$

Or $E_{cell} = (-1.562) - \frac{0.0591}{2} \log \frac{(0.1)^{2}}{0.1}$

$$\left[E^{o}_{cell} - 1.562V\right]$$

$$E_{cell} = -1.562 - \frac{0.0591}{2} \log 10^{-1}$$

$$= -1.562 + \frac{0.0591}{2}$$

$$= -1.562 + 0.02955$$

$$= -1.532 \text{ V}$$

33. (c)

Reaction $X_2Y_4(\ell) \rightarrow 2 \times Y_2(g)$

$$\Delta n_g = n_{g(p)} - n_{g(R)}$$

$$= 2 - 0 = 2$$

We know $\Delta H = \Delta U + \Delta n_g RT$

$$\Delta H = 2 + \frac{2 \times 2 \times 300}{1000} \quad \begin{bmatrix} R = 2 \text{cal } K^{-1} mol^{-1} \\ T = 300K \end{bmatrix}$$

$$\Delta H = 3.2 Kcal$$

We know

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G = 3.2 \times 10^{3} - 300 \times 20 \qquad [\Delta S = 20 \text{cal} K^{-1}]$$

$$= 3.2 \times 10^{3} - 6 \times 10^{3}$$

$$= -2800 \text{ Cal}$$

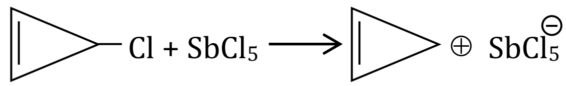
34. (c)

For a species to be aromatic

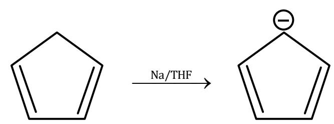
(1) It should be planar

(2) Conjugation present

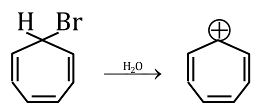
(3) Should have (4n + 2) π electrons.

(i) 

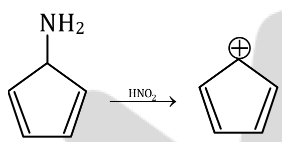
2π electron System (Aromatic)

(ii) 

6π electron system (aromatic)

(iii)  Tropylium cation

6π electron system (Aromatic)

(iv) 

4π electron $\rightarrow$ Not aromatic

35. (a)

Roasted copper pyrite on smelting with sand produces $FeSiO_3$ as fusible slag and Cu_2S as matte

$$2CuFeS_2 + O_2 \longrightarrow Cu_2S + 2FeS + SO_2$$

$$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$$

$$FeO + SiO_2 \longrightarrow FeSiO_3 \longrightarrow FeSiO_3(\text{slag})$$

$FeSiO_3$ is fusible slag

Cu_2S as matte.

36. (a,c)

Option a and c are correct

(a) Only the heavier noble gases form such compounds as they have relatively lower values of ionisation enthalpies.

(c) It happens because the compounds are formed with electronegative ligands.

Option b and d are incorrect.

(b) Since the noble gases have higher ionisation energies the tendency for compound formation will be lower.

(d) Octet of electrons provides the stable arrangements. During compound formation, the octet is broken and stability is lost.

37. (a,b,d)

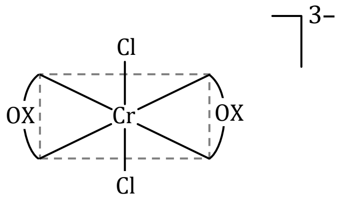
(a) cis – $[CrCl_2\,(ox)_2]^{3+}$

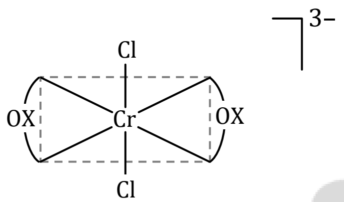
(b) $[Co(en)_3]^{3+}$

(c) $[Co(ox)(en)_2]^{+}$

trans – $[CrCl_2(ox)_2]^{3-}$ isomer optically inactive (super imposable mirror images and plane of symmetry)

cis $[CrCl_2\,(ox)_2]^{3-}$, $[Co(en)_3]^{3+}$ and $[Co(ox)(en)_2]^{+}$ exhibited optical isomerism.





38. (a,d)

The increase in rate constant of a chemical reaction with increasing temperature is

(a) The number of collisions among the reactant molecules increases with increasing temperature.

(d) The number of reactant molecules acquiring the activation energy increases with increasing temperature.

39. (a)

As the isoelectric point of Alanine is 6.1

At pH below pI it has $NH_3^{\oplus}$

At pH above pI it has COO^{-}

for pH $\longrightarrow$ 2 – 4

It forms

$$\overset{\oplus}{N}H_3 - \underset{}{\overset{\overset{\displaystyle CH_3}{|}}{C}H} - COOH$$

For pH $\longrightarrow$ 9 – 11

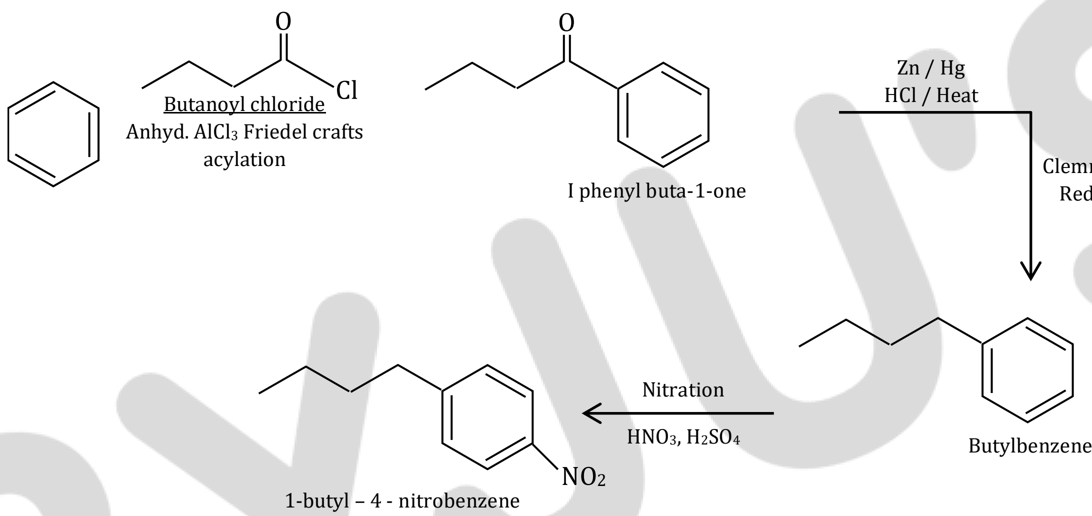
It forms $$NH_2 - \overset{\overset{\displaystyle CH_3}{|}}{C}H - COO^{\ominus}$$

40. (b)

The synthesis is as shown

Friedel crafts acylation of benzene with butanoyl chloride in presence of anhydrous aluminium chloride gives 1-phenylbutan-1-one

Clemmensen's Reduction with Zn/Hg in presence of HCl gives butyl benzene. Nitration with nitrating mixture gives 1-butyl-4-nitrobenzene



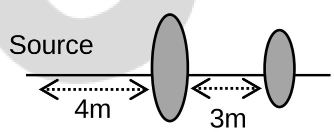
WB-JEE-2015 (Physics)

1. Two particles of mass m_1 and m_2 approach each other due to their mutual gravitational attraction only. Then
   a. accelerations of both the particles are equal
   b. acceleration of the particle of mass m_1 is proportional to m_1
   c. acceleration of the particle of mass m_1 is proportional to m_2
   d. acceleration of the particle of mass m_1 is inversely proportional to m_1

2. Three bodies of the same material and having masses m, m and 3m are at temperatures 40°C, 50°C and 60°C respectively. If the bodies are brought in thermal contact, the final temperature will be
   a. 45°C b. 54°C
   c. 52°C d. 48°C

3. A satellite has kinetic energy K, potential energy V and total energy E. Which of the following statements is true?
   a. K = –V/2 b. K = V/2
   c. E = K/2 d. E = –K/2

4. An object is located 4 m from the first of two thin converging lenses of focal lengths 2m and 1m respectively. The lenses are separated by 3 m. The final image formed by the second lens is located from the source at a distance of



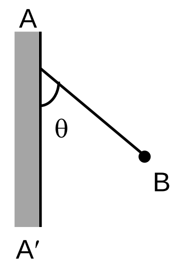
   a. 8.0 m b. 7.5 m
   c. 6.0 m d. 6.5 m

5. A simple pendulum of length L swings in a vertical plane. The tension of the string when it makes an angle θ with the vertical and the bob of mass m moves with a speed v is (g is the gravitational acceleration
   a. mv^2/L b. $mg\cos\theta + mv^2/L$
   c. $mg\cos\theta - mv2/L$ d. $mg\cos\theta$

6. The length of a metal wire is L_1 when the tension is T_1 and L_2 when the tension is T_2. The unstretched length of the wire is equilibrium, the final temperature becomes 19°C. What is the specific heat of the metal in C.G.S. units?
   a. $\dfrac{L_2 + L_2}{2}$ b. $\sqrt{L_1 L_2}$
   c. $\dfrac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$ d. $\dfrac{T_2 L_1 + T_1 L_2}{T_2 + T_1}$

WB-JEE-2015 (Physics)

7. The line AA′ is on a charged infinite conducting plane which is perpendicular to the plane of the paper. The plane has a surface density of charge σ and B is a ball of mass m with a like charge of magnitude q. B is connected by a string from a point on the line AA′. The tangent of the angle (θ) formed between the line AA′ and the string is



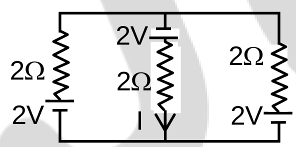
a. $\frac{q\sigma}{2\epsilon_0 mg}$

b. $\frac{q\sigma}{4\pi\epsilon_0 mg}$

c. $\frac{q\sigma}{3\pi\epsilon_0 mg}$

d. $\frac{q\sigma}{\epsilon_0 mg}$

8. The current I in the circuit shown is



a. 1.33A

b. zero

c. 2.00 A

d. 1.00 A

9. A hollow sphere of external radius R and thickness t (<< R) is made of a metal of density ρ, sphere will float in water if

a. $t \le \frac{R}{\rho}$

b. $t \le \frac{R}{3\rho}$

c. $t \le \frac{R}{2\rho}$

d. $t \ge \frac{R}{2\rho}$

10. A metal wire of circular cross-section has a resistance R_1. The wire is now stretched without breaking so that its length is doubled and the density is assumed to remain the same. If the resistance of the wire now becomes R_2 then $R_2 : R_1$ is

a. 1 : 1

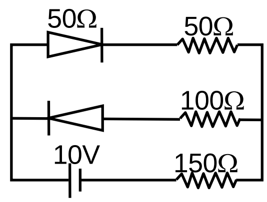
b. 1 : 2

c. 4 : 1

d. 1 : 4

WB-JEE-2015 (Physics)

11. Assume that each diode shown in the figure has a forward bias resistance of 50Ω and an infinite reverse bias resistance. The current through the resistance 150Ω is



a. 0.66A
b. 0.05A
c. zero
d. 0.04A

12. The r.m.s speed of oxygen is v at a particular temperature. If the temperature is doubled and oxygen molecules dissociate into oxygen atoms, the r.m.s speed becomes

a. V
b. $\sqrt{2}v$
c. 2v
d. 4v

13. Two particles, A and B, having equal charges, after being accelerated through the same potential difference enter a region of uniform magnetic field and the particles describe circular paths of radii R_1 and R_2 respectively. The ratio of the masses of A and B is

a. $\sqrt{R_1 / R_2}$
b. R_1 / R_2
c. $(R_1 / R_2)^2$
d. $(R_2 / R_1)^2$

14. A large number of particles are placed around the origin, each at a distance R from the origin. The distance of the center of mass of the system from the origin is

a. = R
b. ≤ R
c. > R
d. ≥ R

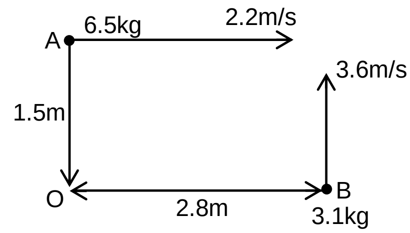
15. A straight conductor 0.1m long moves in a uniform magnetic field 0.1 T. The velocity of the conductor is 15 m/s and is directed perpendicular to the field. The e.m.f. induced between the two ends of the conductor is

a. 0.10 V
b. 0.15V
c. 1.50 V
d. 15.00V

16. A ray of light is incident at an angle i on a glass slab of refractive index μ. The angle between reflected and refracted light is 90°. Then the relationship between i and μ is

a. $i = \tan^{-1}\left(\frac{1}{\mu}\right)$
b. $\tan i = \mu$
c. $\sin i = \mu$
d. $\cos i = \mu$

17. Two particles A and B are moving as shown in the figure. Their total angular momentum about the point O is



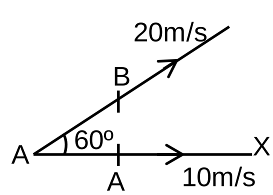
a. 9.8 kg m^2/s
b. Zero.
c. 52.7 kg m^2/s.
d. 37.9 kg m2/s

18. A 20 cm long capillary tube is dipped vertically in water and the liquid rises upto 10 cm. If the entire system is kept in a freely falling platform, the length of water column in the tube will be
a. 5 cm
b. 10 cm
c. 15 cm
d. 20cm

19. A train is moving with a uniform speed of 33 m/s and an observer is approaching the train with the same speed. If the train blows a whistle of frequency 1000 Hz and the velocity of sound is 333 m/s then the apparent frequency of the sound that the observer hears is
a. 1220 Hz
b. 1099 Hz
c. 1110 Hz
d. 1200 Hz

20. A photon of wavelength 300 nm interacts with a stationary hydrogen atom in ground state. During the interaction, whole energy of the photon is transferred to the electron of the atom. State which possibility is correct? (Consider, Planck's constant = 4×10^{-15} eVs, velocity of light = 3×10^{8} m/s, ionization energy of hydrogen = 13.6eV)
a. Electron will be knocked out of the atom
b. Electron will go to any excited state of the atom
c. Electron will go only to first excited state of the atom
d. Electron will keep orbiting in the ground state of atom

21. Particle A moves along X-axis with a uniform velocity of magnitude 10 m/s. Particle B moves with uniform velocity 20 m/s along a direction making an angle of 60° with the positive direction of X-axis as shown in the figure. The relative velocity of B with respect to that of A is



a. 10 m/s along X-axis
b. 10 $\sqrt{3}$ m/s along Y-axis (perpendicular to X-axis)
c. 10 $\sqrt{5}$ along the bisection of the velocities of A and B
d. 30 m/s along negative X-axis

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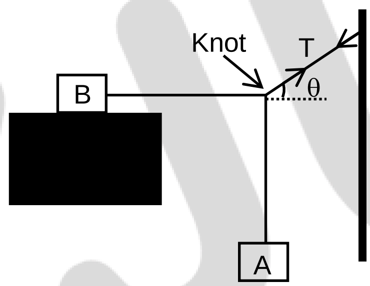
22. When light is refracted from a surface, which of its following physical parameters does not change?
    a. velocity
    b. amplitude
    c. frequency
    d. wavelength

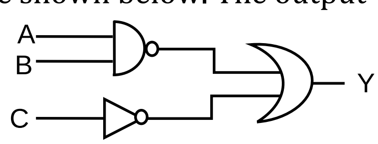
23. A solid maintained at t_1° C is kept in an evacuated chamber at temperature t_2°C ($t_2 >> t_1$). The rate of heat absorbed by the body is proportional to
    a. $t_2^4 - t_1^4$
    b. $(t_2^4 + 273) - (t_1^4 + 273)$
    c. $t_2 - t_1$
    d. $t_2^2 - t_1^2$

24. Block B lying on a table weighs W. The coefficient of static friction between the block and the table is μ. Assume that the cord between B and the knot is horizontal. The maximum weight of the block A for which the system will be stationary is



    a. $\frac{w \tan\theta}{\mu}$
    b. $\mu w \tan\theta$
    c. $\mu w\sqrt{1+\tan^2\theta}$
    d. $\mu w \sin\theta$

25. The inputs to the digital circuit are shown below. The output Y is



    a. $A+B+\overline{C}$
    b. $(A+B)\overline{C}$
    c. $\overline{A}+\overline{B}+\overline{C}$
    d. $\overline{A}+\overline{B}+C$

26. Two particles A and B having different masses are projected from a tower with same speed. A is projected vertically upward and B vertically downward. On reaching the ground
    a. velocity of A is greater than that of B
    b. velocity of B is greater than that of A
    c. both A and B attain the same velocity
    d. the particle with the larger mass attains higher velocity

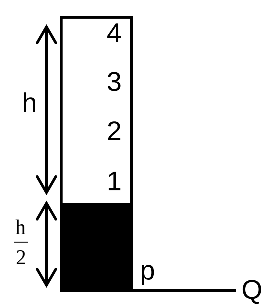
27. The work function of metals is in the range of 2 eV to 5eV. Find which of the following wavelength of light cannot be used for photoelectric effect. (Consider, Planck constant = 4×10^{-15}eVs, velocity of light = 3×10^{8}m/s)
   a. 510 nm
   b. 650 nm
   c. 400 nm
   d. 570nm

28. A thin plastic sheet of refractive index 1.6 is used to cover one of slits of a double slit arrangement. The central point on the screen is now occupied by what would have been the 7^{th} bright fringe before the plastic was used. If the wavelength of light is 600 nm, what is the thickness (in μm) of the plastic?
   a. 7
   b. 4
   c. 8
   d. 6

29. The length of an open organ pipe is twice the length of another closed organ pipe. The fundamental frequency of the open pipe is 100 Hz. The frequency of the third harmonic of the closed pipe is
   a. 100 Hz
   b. 200 Hz
   c. 300 Hz
   d. 150 Hz

30. A 5 μF capacitor is connected in series with a 10 μF capacitor. When a 300 Volt potential difference is applied across this combination, the energy stored in the capacitors is
   a. 15 J
   b. 1.5 J
   c. 0.15 J
   d. 0.10 J

31. A cylinder of height h is filled with water and is kept on a block of height h/2. The level of water in the cylinder is kept constant. Four holes numbered 1, 2, 3 and 4 are at the side of the cylinder and at heights 0, h/4 and 3h/4 respectively. When all four holes are opened together, the hole from which water will reach farthest distance on the plane PQ is the hole no.



   a. 1
   b. 2
   c. 3
   d. 4

32. The pressure p, volume v and temperature T for a certain gas are related by p= $\frac{AT - BT^2}{v}$, where A and B are constants. The work done by the gas when the temperature changes from T_1 to T_2 while the pressure remains constant, is given by

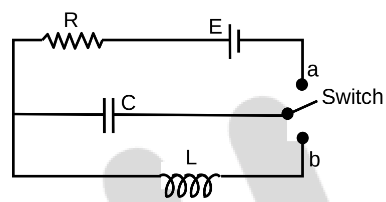
a. $A(T_2 - T_1) + B(T_2^2 - T_1^2)$

b. $\frac{A(T_2 - T_1)}{v_2 - v_1} - \frac{B(T_2^2 - T_1^2)}{v_2 - v_1}$

c. $A(T_2 - T_1) - B(T)$

d. $\frac{A(T_2 - T_2^2)}{v_2 - v_1}$

33. In the circuit shown below, the switch is kept in position 'a' for a long time and is then thrown to position 'b'. The amplitude of the resulting oscillating current is given by



a. $F\sqrt{L / C}$

b. F / R

c. Infinity

d. $F\sqrt{C / L}$

34. A charge q is placed at one corner of a cube. The electric flux through any of the three faces adjacent to the charge is zero. The flux through any one of the other three faces is

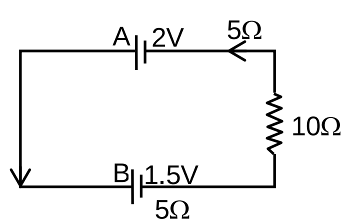
a. $q / 3\in_0$

b. $q / 6\in_0$

c. $q / 12 \in_0$

d. $q / 24 \in_0$

35. Two cells A and B of e.m.f 2V and 1.5V respectively, are connected as shown in figure through an external resistance 10Ω. The internal resistance of each cell is 5Ω. The potential difference E_A and E_B across the terminals of the cells A and B respectively are



a. $E_A = 2.0V, E_B = 1.5V$

b. $E_A = 2.12V, E_B = 1.375V$

c. $E_A = 1.875V, E_B = 1.625V$

d. $E_A = 1.875V, E_B = 1.375V$

36. Two charges +q and −q are placed at a distance 'a' in a uniform electric field. The dipole moment of the combination is $\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right)$, where θ is the angle between the direction of the field and the line joining the two charges. Which of the following statement(s) is/are correct?
    a. The torque exerted by the field on the dipole vanishes
    b. The net force on the dipole vanishes
    c. The torque is independent of the choice of coordinates
    d. The net force is independent of 'd'.

37. Find the right condition(s) for Fraunhoffer diffraction due to a single slit.
    a. Source is at infinite distance and the incident beam has converged at the slit.
    b. Source is near to the slit and the incident beam is parallel.
    c. Source is at infinity and the incident beam is parallel.
    d. Source is near to the slit and the incident beam has converged at the slit.

38. A conducting loop in the form of a circle is placed in a uniform magnetic field with its plane perpendicular to the direction of the field. An e.m.f. will be induced in the loop if
    a. It is translated parallel to itself.
    b. It is rotated about one of its diameters.
    c. It is rotated about its own axis which is parallel to the field.
    d. The loop is deformed from the original shape.

39. A circular disc rolls on a horizontal floor without slipping and the centre of the disc moves with a uniform velocity v. Which of the following values the velocity at a point on the rim of the disc can have?
    a. v
    b. −v
    c. 2 v
    d. zero

40. Consider two particles of different masses. In which of the following situations the heavier of the two particles will have smaller de Broglie wavelength?
    a. Both have a free fall through the same height
    b. Both move with the same kinetic energy.
    c. Both move with the same linear momentum
    d. Both move with the same speed

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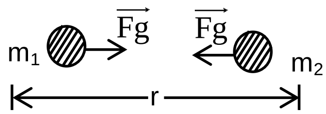
ANSWER KEYS

1. (c)	2. (b)	3. (a)	4. (b)	5. (b)	6. (c)	7. (d)	8. (a)	9. (b)	10. (c)
11. (d)	12. (c)	13. (c)	14. (b)	15. (b)	16. (b)	17. (a)	18. (d)	19. (a)	20. (d)
21. (b)	22. (c)	23. (G)	24. (b)	25. (c)	26. (c)	27. (b)	28. (a)	29. (c)	30. (c)
31. (b)	32. (c)	33. (d)	34. (d)	35. (c)	36. (b,c,d)	37. (b,c)	38. (b,d)	39. (c,d)	40. (a,b,d)

Solution

1. (c)

According to question



Two masses are approaching each other at that time distance b/w them is r.

Now according to Newton's law of gravitation,

$\vec{F}_g = \frac{Gm_1m_2}{r^2}$ (force of attraction)

(Equal and opposite)

Newton's 2nd law of motion -: $F = ma \Rightarrow a = \frac{f}{m}$

for $\vec{a}$ of mass $m_1 \Rightarrow \vec{a}_1 = \frac{\vec{F}_g}{m_1} = \frac{\frac{Gm_1m_2}{r^2}}{m_1} = \frac{Gm_2}{r^2}$

For $\vec{a}$ of mass $m_2 \Rightarrow \vec{a}_2 = \frac{\vec{F}_g}{m_2} = \frac{Gm_1}{r^2}$

$\therefore$ $G = 6.67 \times 10^{-11} \frac{N-m^2}{kg^2}$ and $r \rightarrow$ const. (at any time)

So, $\vec{a}_1 \propto m_2$

And $\vec{a}_2 \propto m_1$

2. (b)

According to law of conservation of energy,

Energy lost = Energy gained

Let the final temperature be T.

Let the specific heat capacity of the material be C.

Then,

$mcT_1 + mcT_2 + 3mcT_3 = (mc + mc + 3mc)\ T$

$\Rightarrow mT_1 + mT_2 + 3\ mT_3 = (m + m + 3m)\ T$

$\Rightarrow mT_1 + mT_2 + 3mT_3 = 5mT$

$\Rightarrow T_1 + T_2 + 3T_3 = 5T$

$\Rightarrow T = \frac{T_1 + T_2 + 3T_3}{5} \Rightarrow \frac{40 + 50 + 3 \times 60}{5}$

$\Rightarrow T = \frac{270}{5} = 54^{o}C$

3. (a)

The kinetic energy of satellite is given as:

$$K = \frac{GMm}{2r} \ldots\ldots\ldots\ldots(i)$$

Similarly, the potential energy is given as;

$$V = \frac{-GMm}{2r} \ldots\ldots\ldots\ldots(ii)$$

Total energy of satellite is given as:

$$E = K + V$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r} \ldots\ldots\ldots\ldots(iii)$$

From equation, (i), (ii) and (iii)

$$K = -\frac{V}{2}$$

4. (b)

The image obtained by first lens will act as object for the second lens.

Here, for the first lens.

$u = -4$ m

$f_1 = 2$ m

According to lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{v_1} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{4}$$

$\Rightarrow$ $v_1 = 4$m

For the second lens,

$u_2 = 1$ m

And $f_2 = 1$m

Again apply lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

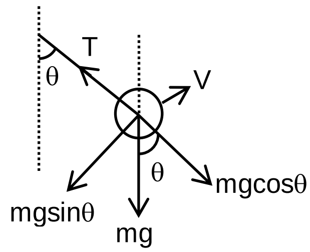
$$\Rightarrow \frac{1}{v_2} - \frac{1}{1} = \frac{1}{1}$$

$$\Rightarrow \frac{1}{v_2} = 2$$

$$\Rightarrow v_2 = \frac{1}{2} = 0.5$$

Therefore distance from the object = 4 + 3 + 0.5 = 7.5 m

5. (b)



From the diagram, we can see that

mg cos θ is the component of mg.

Also a centripetal force act on the pendulum which is $\frac{mv^2}{L}$

So, the tension in the string is mg cos θ + $\frac{mv^2}{L}$

6. (c)

Let the initial length of the metal wire is L

The strain at tension T_1 is $\Delta L_1 = L_1 - L$

The strain at tension T_2 is $\Delta L_2 = L_2 - L$

Suppose, the young's modulus of the wire is Y,

$$\frac{\frac{T_1}{A}}{\frac{\Delta L_1}{L}} = \frac{\frac{T_2}{A}}{\frac{\Delta L_2}{L}}$$

$\because$ young's modulus (Y) = $\frac{\text{stress}(\sigma)}{\text{strain}(\varepsilon)} = \frac{F/A}{\Delta L/L}$

Where, A is area of cross section of the wire assume to be same at all the situations.

$\Rightarrow \frac{T_1}{A} \times \frac{L}{\Delta L_1} = \frac{T_2}{A} \times \frac{L}{\Delta L_2}$

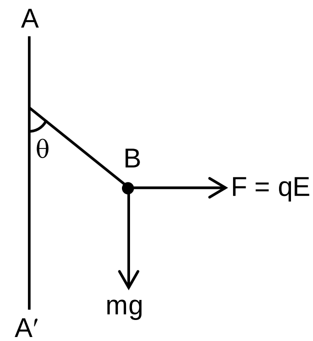
$\Rightarrow \frac{T_1}{(L_1 - L)} = \frac{T_2}{(L_2 - L)}$

$\Rightarrow T_1 (L_2 - L) = T_2 (L_1 - L)$

$\Rightarrow L = \frac{T_2L_1 - T_1L_2}{T_2 - T_1}$

7. (d)

The diagram is as follows



The electric field due to charged infinite conducting sheet is E = $\frac{\sigma}{\varepsilon_0}$

Now, force (electric force) on the charged ball is F = qE = $\frac{q\sigma}{\varepsilon_0}$

The resultant of electric force and mg balance the tension produced in the string.

So, $\tan\theta = \frac{F_e}{mg} = \frac{q\sigma/\varepsilon_0}{mg} = \frac{q\sigma}{\varepsilon_0 mg}$

8. (a)

We first try to find the equivalent emf and internal resistance of the two side branches.

$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$= \frac{\frac{2}{2} + \frac{2}{2}}{\frac{1}{2} + \frac{1}{2}}$$

= 2V

$\because r_{eq} = \frac{r_1 r_2}{r_1 + r_2} = 1\,\Omega$

This equivalent cell is in series with the central branch.

Hence current = $\frac{E_{eq} + E_3}{r_{eq} + r_3}$

$= \frac{2v + 2v}{1\Omega + 2\Omega} = \frac{4}{3}$ A = 1.33 A

9. (b)

The density of material is ρ.

The hollow sphere will float if its weight is less than the weight of the water displaced by the volume of the sphere. This implies mass of the sphere is less than that for the same volume of water. Now, mass of spherical shell,

$m_1 = 4\pi R^2 \times t \times \rho$

While the mass of water having same volume

$$m_2 = \frac{4}{3}\pi R^3 \times \rho_g = \frac{4}{3}\pi R^3$$

Where, ρ_g = density of water = 1 gm/cm^3

For the floatation of sphere,

$m_1 \le m_2$

$$4\pi R^2 \times t \times \rho \le \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad t.\rho \le \frac{R}{3}$$

$$\Rightarrow \quad t \le \frac{R}{3\rho}$$

10. (c)

The wire is stretched to double its length.

$\Rightarrow \ell' = 2\ell$

The volume in the process remains the same. (Since mass is unchanged, and density remains same)

$\Rightarrow \ell'A' = \ell A$

$$\Rightarrow A' = \frac{\ell A}{\ell'} \Rightarrow \frac{A\ell}{2\ell}$$

$$\Rightarrow A' = \frac{A}{2}$$

If the initial resistance was, $R = \dfrac{\rho\ell}{A}$

Hence, $R' = \dfrac{\rho\ell'}{A'}$

$$\Rightarrow R' = \frac{\rho.(2\ell)}{(A/2)} = \frac{4\rho\ell}{A}$$

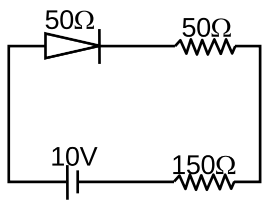
$\Rightarrow \therefore R' = 4R$

$$\Rightarrow \frac{R'}{R} = \frac{4}{1} \text{ or } 4:1$$

11. (d)

Since the positive terminal of battery is attached to positive terminal of the upper diode, it is forward bias, and hence conducts current offering 50Ω resistance.

Since the positive terminal of battery is attached to negative of the lower diode, it is reverse bias, and hence does not conduct current at all. Hence that branch can be ignored in the circuit.



Thus the current flowing in rest of the

Circuit = $\dfrac{10\text{v}}{150\Omega + 50\Omega + 50\Omega}$ = 0.04 A

12. (c)

The root mean square speed of a gas is given by $\sqrt{\dfrac{3RT}{M}}$

Where T is the temperature of gas

M is the molecular weight of gas.

When oxygen molecules dissociate into oxygen atoms, the molecular weight of gas halves.

Hence, $T' = 2T$

$M' = \dfrac{M}{2}$

$$\frac{V'}{V} = \sqrt{\frac{T'/M'}{T/M}} = \sqrt{\frac{2T/M/2}{\frac{T}{M}}}$$

$$\Rightarrow \quad \frac{V'}{V} = \sqrt{\frac{4}{1}} \Rightarrow 2$$

$$\Rightarrow \quad V' = 2V$$

13. (c)

Since charge on both particles is same and they are accelerated through the same voltage, work done by electric field will be same for both the charges so the increase in kinetic energy will be same. Assuming that charges started from rest, we have :

$$\frac{1}{2}m_1v_1^2 = qv = \frac{1}{2}m_2v_2^2 \Rightarrow \frac{v_1^2}{v_2^2} = \frac{m_2}{m_1}$$

Where V is the potential difference through which the charges are accelerated.

The magnetic force only provides the necessary centripetal force for circular motion :

$$\frac{m_1v_1^2}{R_1} = Bqv_1$$

$$\frac{m_2v_2^2}{R_2} = Bqv_2$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{m_1v_1}{m_2v_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}} \Rightarrow \frac{m_1}{m_2} = \frac{R_1^2}{R_2^2}$$

14. (b)

As large number of particles are situated at a distance R from the origin. If particles are uniformly distributed and make a circular boundary around the origin, then centre of mass will be at the origin.

While, if the particles are not uniformly distributed then center of mass will lie between particle and origin. This implies that the distance between centre of mass and origin is always less than or equal to R.

So, Answer will be $\leq$ R.

15. (b)

The EMF induced across a conductor of length ℓ, moving perpendicular to a magnetic field B with velocity V is equal to $BV\ell$

EMF induced in given problem = $BV\ell$

= 0.1 T × 15 m/s × 0.1 m

= 0.15 V

16. (b)

Let the angle of reflection and refraction be r_1, r_2 respectively.

Since angle between reflected and refracted ray is 90°,

$$r_1 + r_2 = 90°$$

From law of reflection, $r_1 = i$

From snell's law,

sin i = μ sinr_2

= μ sin (90° – r_1)

sin i = μ cosr_1 = μ cos i

$\Rightarrow$ tan i = μ

17. (a)

Total angular momentum about 0 is given as,

$L = L_1 + L_2$

$= m_1v_1r_1 + m_2v_2r_2$

$= -6.5 \times 2.2 \times 1.5 + 3.1 \times 3.6 \times 2.8$

(Considering anticlockwise direction as negative angular momentum)

$= -21.45 + 31.24$

$L = 9.8 \text{ kg} - \frac{m^2}{sec}$

18. (d)

The height raised by liquid in capillary tube,

$$H = \frac{2\ell \cos\theta}{\rho g h}$$

Where, H is rise in the capillary tube.

As in freely falling platform, a body experience weightlessness.

$g_{eff} = 0$

So, the liquid will rise upto length of the capillary. i.e. height raised by the liquid will be 20 cm.

19. (a)

Doppler shifted frequency of sound radiation from a source is given as

$f = f_0 \left(\frac{c - v_0}{c - v_s}\right)$

$c \rightarrow$ velocity of sound

$v_0 \rightarrow$ velocity of observer

$v_s \rightarrow$ velocity of source

Following the sign convention of velocity of observer and source

$f = 1000 \left(\frac{333 + 33}{333 - 33}\right)$ Hz

$f = 1220$ Hz

20. (d)

$\because$ Energy of photon, $E_{ph} = h\nu$

$$E_{ph} = \frac{hc}{\lambda}$$

$$\because [c = \nu\lambda]$$

Now put the given values,

$$E_{ph} = \frac{4\times10^{-15}\times3\times10^{8}}{300\times10^{-9}}$$

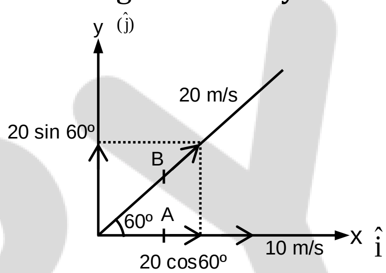
$$E_{ph} = 4ev$$

$\because$ Ionisation energy of hydrogen = 13.6 eV

$$E_{Ionisation} = 13.6 \text{ eV}$$

The ionization energy is 13.6 eV which is greater than energy of photon
13.6 eV > 4 eV, Hence no excitation takes place and electron will keep orbiting in ground state of atom.

21. (b)
Resolving the velocity in horizontal and vertical components



The relative velocity of B with respect to that of A is given as $\vec{V}_{BA}$,

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$= (20\cos 60^\circ\ \hat{i} + 20\sin 60^\circ\ \hat{j}) - 10\,\hat{i}$

$= 20 \times \frac{1}{2}\hat{i} + 20\ \frac{\sqrt{3}}{2}\hat{j} - 10\hat{i}$

$= 10\ \hat{i} + 10\sqrt{3}\ \hat{j} - 10\hat{i}$

$= \vec{v}_{BA} = 10\sqrt{3}\ \hat{j}$

So, the relative velocity of B with respect to that of A is $10\sqrt{3}\,\text{m/s}$ along Y – axis and it is perpendicular to x – axis.

22. (c)
Frequency of the light depends upon the source so it doesn't change in case of reflection or refraction or polarization.

23. (G) Bonus
According to Stefan-Boltzman law

$\mu = \sigma A T^4$

Where, $\sigma \rightarrow$ Stefan – Boltzman constant

$\because t_{solid} < t_{surrounding}$ (given)

Now, temperature of solid increase and, solid and surrounding will be at equilibrium.

So, the rate of emission in case of solid will be proportional at,

$R_{solid} \propto T_1^4$ $\quad$ R→ rate of emission

$\because$ $T_1 = (t_1 + 273)^4$

$\Rightarrow$ $R_{solid} \propto (t_1 + 273)^4$

Similarly, $R_{surrounding}$ & T

$R_{surrounding} \propto T_2^4$

$\because$ $T_2 = (t_2 + 273)^4$

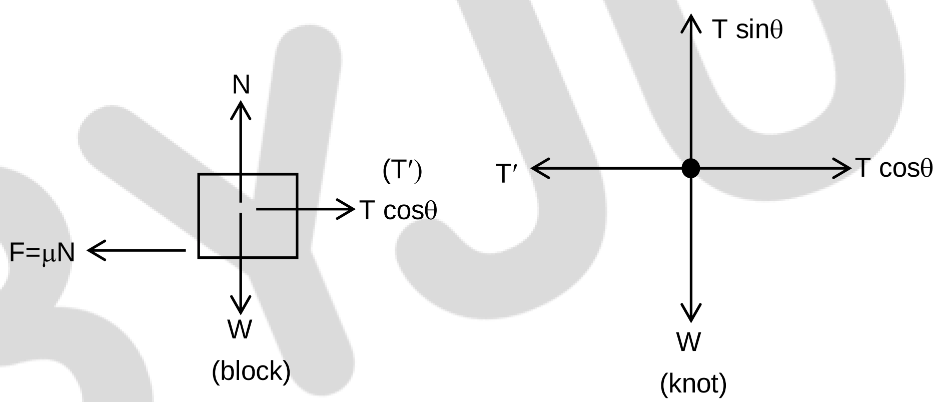
$\Rightarrow$ $R_{surrounding} \propto (t_2 + 273)^4$

$\therefore$ Net Rate $\propto T_2^4 - T_1^4$

$\Rightarrow$ Net Rate $\propto (t_2 + 273)^4 - (t_1 + 273)^4$

So, the correct option is this

24. (b)



The block B is under equilibrium by action of tension force on it leftwards (say T′), and force of static friction on it left wards (f).

Hence, $f = \mu N = \mu W = T'$

Consider the force acting on the knot

Balancing the forces on knot horizontally,

$T' = T\cos\theta$

Also balancing the forces on knot vertically,

$T \sin\theta = w$

$\Rightarrow$ $w = \frac{T'}{\cos\theta}.\sin\theta$

$w = T' \tan\theta$

$w = \mu w \tan\theta$

25. (c)

Here, A, B and C are inputs and Y is output

A and B are inputs to the NAND gate. The output of that gate is

$\overline{(A.B)} = \overline{A} + \overline{B}$

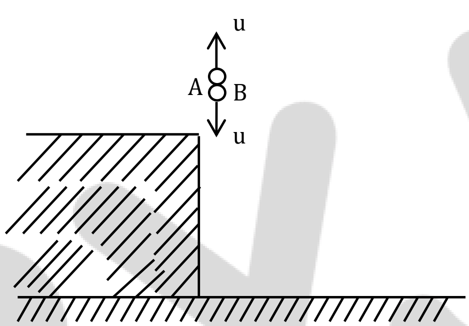
The output of c being input to the NOT gate is $\overline{C}$.

These two $\left[\left(\overline{A}+\overline{B}\right),\overline{C}\right]$ are inputs of OR gate.

Hence, $Y = \left(\overline{A}+\overline{B}\right)+\overline{C}$

$Y = \overline{A}+\overline{B}+\overline{C}$

26. (c)



Let v_A and v_B be the final velocities of A and B respectively since initial velocity of both particles is same.

So, using third equation of motion i.e,

$v^2 = u^2 + 2as$

$\therefore \quad v_A^2 = u^2 - 2g(-h) = u^2 + 2gh$ ....(1)

and $\quad v_B^2 = u^2 + 2gh$ ....(2)

$\therefore$ from (1) and (ii)

$v_A = v_B$

27. (b)

The work function of metal is given as:

$E = \frac{hc}{\lambda}$

So, $\lambda = \frac{hc}{E}$

Now,

$$\lambda_{max} = \frac{hc}{E_{min}}$$

And, $\lambda_{min} = \frac{hc}{E_{max}}$

$$\lambda_{min} = \frac{4\times10^{-15}\times3\times10^{8}}{5} = \frac{12\times10^{-7}}{5} = 2.4\times10^{-7}\,m$$

$\Rightarrow$ $\lambda_{min} = 240$ nm

$$\lambda_{max} = \frac{4\times10^{-15}\times3\times10^{8}}{2} = \frac{12\times10^{-7}}{2} = 6\times10^{-7}\,m$$

$\Rightarrow$ $\lambda_{max} = 600$ nm

So, the wavelength of light that cannot be used for photo electric effect is 650 nm.

28. (a)

If the central fringe shifts to 'm'th bright fringe,

$(\mu - 1)\,t = m\lambda$

$\because$ $\mu = 1.6$, $m = 7$

$\lambda = 600$ nm

t = thickness

$\therefore$ $(1.6 - 1)\,t = 7 \times 600$

$\Rightarrow$ $t = \frac{7\times600}{0.6}$

$\Rightarrow$ $t = 7000$ nm

$\because$ 1000 nm = 1μm

$\Rightarrow$ $t = 7\ \mu$m

29. (c)

The fundamental frequency of the organ open pipe is given as:-

$f_0 = \frac{v}{2\ell_0} = 100\text{Hz}$

Similarly, the third harmonic of the closed pipe is given as:

$f_c = \frac{3}{4}\frac{v}{\ell_c}$

It is given that, $\ell_0 = 2\ell_c$

$f_c = \frac{3}{4}\left(\frac{v}{\frac{\ell_0}{2}}\right) \Rightarrow \frac{3}{2}\left(\frac{v}{\ell_0}\right)$

$f_0 = 3\left(\frac{v}{2\ell_0}\right)$

$= 3 \times 100$

$f_0 = 300$ Hz

30. (c)

Let us assume that,

$C_1 = 5\mu F$

$C_2 = 10\mu F$

Equivalent capacitances are given as:

$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$

$C_{eq} = \frac{5\times10}{5+10} = \frac{50}{15} = \frac{10}{3}\mu F$

Hence,

Energy stored in the capacitor is given as:

$U = \frac{1}{2} C_{eq} V^2$

$= \frac{1}{2}\left(\frac{10}{3}\times10^{-6}\right)(300)^2$

$U = 0.15$ J

31. (b)

We know from torricelli's theorem, that the range of the liquid falling from a certain height is given by,

$R = 2\times\sqrt{h(H-h)}$

Where H is the total height of the container and h is the height where the hole is,

For R = R_{max};

$$\frac{dR}{dh} = 0$$

$$\frac{dR}{dh} = 2\times\left(\frac{1}{2\sqrt{h}}\sqrt{H-h} + \sqrt{h}\frac{-1}{2\sqrt{H-h}}\right)$$

$$\frac{dR}{dh} = \left(\frac{\sqrt{H-h}}{\sqrt{h}} + \frac{-\sqrt{h}}{\sqrt{H-h}}\right)$$

$$\frac{dR}{dh} = \frac{H-h-h}{\sqrt{h(H-h)}} = 0$$

$\Rightarrow$ H–h–h = 0

$\Rightarrow$ H = 2h

For R = R_{max}

$h = \frac{H}{2}$

Taking PQ as the reference,

$H = h + \frac{h}{2} = \frac{3h}{2}$

So hole must be at height,

$$\frac{H}{2} = \frac{\left(\frac{3h}{2}\right)}{2} = \frac{3h}{4}$$

For hole 1,

$h_1 = \frac{h}{2} + 0 = \frac{h}{2}$

For hole 2,

$h_2 = \frac{h}{2} + \frac{h}{4} = \frac{3h}{4}$

For hole 3,

$h_3 = \frac{h}{2} + \frac{h}{2} = h$

For hole 4,

$h_4 = \frac{h}{2} + \frac{3h}{4} = \frac{5h}{4}$

Since, hole 2 is at the $\frac{H}{2}$ height required for longest range.

$\therefore$ Hole 2 is from which water will reach farthest distance on the plane PQ.

32. (c)

We have, $P = \dfrac{AT - BT^2}{V}$

$\Rightarrow \quad V = \dfrac{AT - BT^2}{p}$

Work done is given as:-

$W = p\Delta V = p\,[V_2 - V_1]$

Now,

$V_2 = \dfrac{AT_2 - BT_2^2}{P}$

$V_1 = \dfrac{AT_1 - BT_1^2}{p}$

So,

$$W = P\left[\frac{AT_2 - BT_2^2}{p} - \left(\frac{AT_1 - BT_1^2}{p}\right)\right]$$

$$= P\left[\frac{AT_2 - BT_2^2 - AT_1 + BT_1^2}{p}\right]$$

$$w = \left[A(T_2 - T_1) - B\left(T_2^2 - T_1^2\right)\right]$$

33. (d)

The charge through the capacitor is given as :

$q_0 = CE$

Where, C is capacitance and E is potential difference.

Now by conservation of energy,

Energy stored by capacitor = Energy stored by inductor

$$\frac{q_0^2}{2c} = \frac{1}{2} L I_0^2$$

$$\Rightarrow \frac{C^2E^2}{2C} = \frac{1}{2} L I_0^2$$

$$\Rightarrow I_0 = E\sqrt{\frac{C}{L}}$$

34. (d)

Consider a cube as this charge is at centre so, this cube for which the charge is at the corner becomes $\frac{1}{8}$. As we know from Gauss Law, the total flux produced by a charge is,

$$\phi_0 = \frac{Q_{encl}}{\varepsilon_0}$$

In this amount $\frac{1}{8^{th}}$ part goes from the cube we are considering. So, flux from the given cube is

$$\phi = \frac{Q_{enclosed}}{8\varepsilon_0} = \frac{q}{8\varepsilon_0}$$

Therefore, flux passing from the three surfaces in contact with the charges is zero and the flux passes only from the remaining faces. Because of the symmetry, the flux from the remaining three faces will be equal. Therefore, flux passing from each of these faces is

$$\phi = \frac{q}{24\varepsilon_0}$$

35. (c)

Let i be the current flowing through the circuit on applying KVL,

$$i = \frac{2-1.5}{20} = \frac{1}{40}\,A$$

The potential difference through battery A is given as,

$$E_A = 2 - ir = 2 - \frac{1}{40}\times 5 = 1.875\ V$$

Similarly potential difference through B is given as,

$$E_B = 1.5 + ir = 1.625\ v$$

36. (b, c, d)

The net force on a dipole in a uniform electric field is zero because opposite and equal forces on both charges of dipole cancels each other but the torque is given as,

$T = PE \sin\theta$

Where, θ is angle between electric field and dipole.

Now, torque is only zero when dipole is kept along electric field.

37. (b, c)

In fraunhoffer diffraction, the source has to be kept at infinite distance effectively from the slit because then only the incident rays are parallel. This can be achieved if

(i) Point source is kept at the focus of a converging lens

(ii) Point source is at infinite distance

38. (b, d)

Emf will be induced in the loop only when flux changes. Now, if the loop is translated parallel to itself then there is no flux change.

Hence option A is not correct.

If the loop is rotated about one of its diameters then the flux passing through loop changes so, B is correct option.

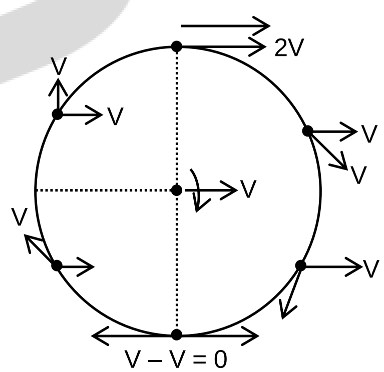
If the axis is parallel to field then again there will be no change in flux and hence no emf will be induced in the loop.

If the loop is deformed then obviously its area will change and hence the flux flowing through it will also change.

So, the correct option is (B) and (D)

39. (c, d)

At each point on the rim, the velocity is tangent due to circular motion and horizontal to the ground due to linear motion of the centre of mass of the disc. So, the resultant velocity will never be v or –v.



It is clear from the figure that at topmost position on the rim, the velocity is 2v and at lower most position, the velocity is 0.

So, the correct options are C and D.

40. (a, b, d)

De Broglie wave length (λ), it can be given as;

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

If both particles fall freely from the same height, then

$$v = \sqrt{2gh}$$

So, De Broglie wavelength

$$\lambda = \frac{h}{m\sqrt{2gh}}$$

So, $\lambda \propto \frac{1}{m}$ for same v

From the formula of De Broglie wave length, it can be seen that option B and option D are correct.