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1. If $(2 \le r \le n)$, then ${}^nC_r + 2.{}^nC_{r+1} + {}^nC_{r+2}$ is equal to

a. $2.{}^nC_{r+2}$

b. ${}^{n+1}C_{r+1}$

c. ${}^{n+2}C_{r+2}$

d. ${}^{n+1}C_r$

2. The number $(101)^{100}-1$is divisible by

a. 10^4

b. 10^6

c. 10^8

d. 10^{12}

3. If n is even positive integer, then the condition that the greatest term in the expansion of $(i+x)^n$ may also have the greatest coefficient is

a. $\frac{n}{n+2} < x < \frac{n+2}{n}$

b. $\frac{n}{n+1} < x < \frac{n+1}{n}$

c. $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$

d. $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

4. $\text{If} \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A, \text{then} \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix} \text{is}$

a. A^2

b. $A^2 - A + I_3$

c. $A^2 - 3A + I_3$

d. $3A^2 + 5A - 4I_3$

I_3 denotes the det of the identity matrix of order 3

5. If $a_r = (\cos 2r\pi + i\sin 2r\pi)^{1/9}$, then the value of $\begin{vmatrix} a_i & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

a. 1

b. −1

c. 0

d. 2

6. If $Sr = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} S_r$ is independent of

a. x only
b. y only
c. n only
d. x,y,z and n

7. If the following three linear equations have a non –trivial solution, then x + 4ay +az =0, x+ 3by +bz = 0, x + 2cy + cz =0

a. a,b,c are in A.P
b. a,b,c are in G.P.
c. a,b,c are in H.P
d. a + b + c = 0

8. On R, a relation ρ is defined by xρy if and only if x–y is zero or irrational.Then

a. ρ is equivalence relation
b. ρ is reflexive but neither symmetric nor transitive
c. ρ is reflexive & symmetric but not transitive
d. ρ is symmetric & transitive but not reflexive

9. On the set R of real numbers, the relation ρ is defined by xρy,(x,y) $\in$ R.

a. if |x–y| < 2 then ρ is reflexive but neither symmetric nor transitive
b. if x–y < 2 then ρ is reflexive and symmetric but not transitive
c. if |x| ≥ y then ρ is reflexive and transitive but not symmetric
d. if x > |y| then ρ is transitive but neither reflexive nor symmetric

10. If f : R→R be defined by f(x) =e^x and g : R →R be defined by g(x) = x^2. The mapping g of : R →R be defined by (g of)x =g[f(x)] $\forall$ x $\in$ R, then

a. g of is bijective but f is not injective
b. g of is injective and g is injective
c. g of is injective but g is not bijective
d. g of is surjective and g is surjective

11. In order to get a head at least once with probability ≥ 0.9, the minimum number of times a unbiased coin needs to be tossed is

a. 3
b. 4
c. 5
d. 6

12. A student appears for tests I,II and III. The student is successful if he passes in tests I, II or III.The probabilities of the student passing in tests I,II and III are respectively p,q and $\frac{1}{2}$. If the probability of the student to be successful is $\frac{1}{2}$. Then

a. $p(1 + q) = 1$
b. $q(1 + p) = 1$
c. $pq = 1$
d. $\frac{1}{p}+\frac{1}{q}=1$

13. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then general value of θ is
(n is integer)

a. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$
b. $\frac{n\pi}{4}, 4\pi \pm \frac{\pi}{6}$
c. $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$
d. $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

14. If $0 \leq A \leq \frac{\pi}{4}$, then $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ is equal to

a. $\frac{\pi}{4}$
b. π
c. 0
d. $\frac{\pi}{2}$

15. Without changing the direction of the axes, the origin is transferred to the point (2, 3). Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

a. $x^2 + y^2 + 4 = 0$
b. $x^2 + y^2 = 4$
c. $x^2 + y^2 - 8x - 12y + 48 = 0$
d. $x^2 + y^2 = 9$

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16. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is

    a. $x^2 + y^2 + 4x + 6y + 9 = 0$
    b. $x^2 + y^2 - 4x + 6y + 9 = 0$
    c. $x^2 + y^2 - 4x - 6y + 9 = 0$
    d. $x^2 + y^2 + 4x - 6y + 9 = 0$

17. The point Q is the image of the point P(1,5) about the line $y = x$ and R is the image of the point Q about the line $y = -x$. The circumcentre of the ΔPQR is

    a. (5,1)
    b. (−5,1)
    c. (1,−5)
    d. (0,0)

18. The angular points of a triangle are A(−1,−7), B (5,1) and C (1,4).The equation of the dissector of the angle $\angle ABC$ is

    a. $x = 7y + 2$
    b. $7y = x + 2$
    c. $y = 7x + 2$
    d. $7x = y + 2$

19. If one the diameters of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is (2, −3), the radius of S is

    a. $\sqrt{41}$ unit
    b. $3\sqrt{5}$ unit
    c. $5\sqrt{2}$ unit
    d. $2\sqrt{5}$ unit

20. A chord AB is drawn from the point A(0,3) on the circle $x^2 + 4x + (y-3)^2 = 0$, and is extended to M such that AM=2AB. The locus of M is

    a. $x^2 + y^2 - 8x - 6y + 9 = 0$
    b. $x^2 + y^2 + 8x + 6y + 9 = 0$
    c. $x^2 + y^2 + 8x - 6y + 9 = 0$
    d. $x^2 + y^2 - 8x + 6y + 9 = 0$

21. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

    a. 8 : 1
    b. 1 : 8
    c. 9 : 1
    d. 1 : 9

22. Let A,B be two distinct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

a. $-\frac{1}{r}$

b. $\frac{1}{r}$

c. $\frac{2}{r}$

d. $-\frac{2}{r}$

23. Let $P(at^2, 2at)$, $Q(ar^2, 2ar)$ be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are then the value of x areparallel where the co-ordinates of K is (2a, 0), then the value of r is

a. $\frac{t}{1-t^2}$

b. $\frac{1-t^2}{t}$

c. $\frac{t^2+1}{t}$

d. $\frac{t^2-1}{t}$

24. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{x^2}{4}=1$ and the line through P parallel to the y-axis meets the circle $x^2 + y^2 = 9$ at Q, where P, Q are on the same side of the x-axis. If R is a point on PQ such that $\frac{PR}{RQ}=\frac{1}{2}$, then the locus of R is

a. $\frac{x^2}{9}+\frac{9y^2}{49}=1$

b. $\frac{x^2}{49}+\frac{y^2}{9}=1$

c. $\frac{x^2}{9}+\frac{y^2}{49}=1$

d. $\frac{9x^2}{49}+\frac{y^2}{49}=1$

25. A point P lies on a line through Q(1, –2, 3) and is parallel to the line $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$. If P lies on the plane $2x + 3y - 4z + 22 = 0$, then segment PQ equals to

a. $\sqrt{42}$ units

b. $\sqrt{32}$ units

c. 4 unit

d. 5 units

26. The foot of the perpendicular drawn from the point (1, 8, 4) on the line joining the points (0, –11, 4) and (2, – 3, 1) is

a. (4, 5, 2)

b. (–4, 5, 2)

c. (4, –5, 2)

d. (4, 5, –2)

27. The approximate value of sin31º is

a. > 0.5
b. > 0.6
c. < 0.5
d. < 0.4

28. Let $f_1(x)=e^x, f_2(x)=e^{f_1(x)}$, ..., $f_{n+1}(x)= e^{f_n(x)}$ for all $n \geq 1$. The for any fixed n, $\frac{d}{dx}f_n(x)$: is

a. $f_n(x)$
b. $f_n(x)f_{n-1}(x)$
c. $f_n(x)f_{n-1}(x)....f_1(x)$
d. $f_n(x)......... f_1(x)e^x$

29. The domain of definition of $f(x)=\sqrt{\frac{1-|x|}{2-|x|}}$ is

a. $(-\infty,-1) \cup (2, \infty)$
b. $[-1,1] \cup (2, \infty) \cup (-\infty,-2)$
c. $(-\infty,1) \cup (2, \infty)$
d. $[-1,1] \cup (2, \infty)$

30. Let $f : [a, b] \to R$ be differentiable on [a, b] and $k \in R$. Let f(a) = 0 = f'(b). Also let J(x) = f ′(x) + kf(x). Then

a. J(x) > 0 for all $x \in [a, b]$
b. J(x) < 0 for all $x \in [a, b]$
c. J(x) = 0 has atleast one root in (a, b)
d. J(x) = 0 through (a, b)

31. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. Then $\frac{f(1-h)-f(1)}{h^3+3h}$

a. does not exist
b. is $\frac{50}{3}$
c. is $\frac{53}{3}$
d. is $\frac{22}{3}$

32. Let $f : [a, b] \to R$ be such that f is differentiable in (a, b), f is continuous at x = a and x = b and moreover f(a) = 0 = f(b). Then

a. there exists atleast one point c in (a, b) such that f ′(c) = f(c)
b. f ′(x) = f(x) does not hold at any point in (a, b)
c. at every point of (a, b), f ′(x) > f(x)
d. at every point of (a, b), f ′(x) < f(x)

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33. Let f : R →R be a twice continuously differentiable function such that f(0) = f(1) = f ′(0) = 0. Then

a. $f''(0) = 0$

b. $f''(c) = 0$ for some $c \in R$

c. if $c \neq 0$, then $f''(c) \neq 0$

d. $f'(x) > 0$ for all $x \neq 0$

34. If $\int \left[\frac{x\cos^3 x - \sin x}{\cos^2 x}\right] dx = e^{\sin x} f(x) + c$ where c is constant of integration, then f(x) =

a. secx–x

b. x–secx

c. tanx–x

d. x–tanx

35. If $\int f(x)\sin x\cos x\,dx = \frac{1}{2(b^2-a^2)}\log f(x) + c$, where c is the constant of integration, then f(x) =

a. $\frac{2}{(b^2-a^2)\sin 2x}$

b. $\frac{2}{ab\sin 2x}$

c. $\frac{2}{(b^2-a^2)\cos 2x}$

d. $\frac{2}{ab\cos 2x}$

36. If $M = \int_0^{\pi/2} \frac{\cos x}{x+2}dx, N = \int_0^{\pi/4} \frac{\sin x\cos x}{(x+1)^2}dx$, then the value of M – N

a. π

b. $\frac{\pi}{4}$

c. $\frac{2}{\pi - 4}$

d. $\frac{2}{\pi + 4}$

37. The value of the integral $I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx$ is

a. $\frac{\pi}{4}\log 2014$

b. $\frac{\pi}{2}\log 2014$

c. $\pi \log 2014$

d. $\frac{1}{2}\log 2014$

38. Let $I=\int_{\pi/4}^{\pi/3}\frac{\sin x}{x}dx$. Then

a. $\frac{1}{2}\le I\le 1$

b. $4\le I\le 2\sqrt{30}$

c. $\frac{\sqrt{3}}{8}\le I\le\frac{\sqrt{2}}{6}$

d. $1\le I\le\frac{2\sqrt{3}}{\sqrt{2}}$

39. The value of $I=\int_{\pi/2}^{5\pi/2}\frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)}+e^{\tan^{-1}(\sin x)}}dx$, is

a. 1

b. π

c. e

d. $\pi/2$

40. The value of $\lim_{n\to\infty}\frac{1}{n}\left\{\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+..........+\sec^2\frac{n\pi}{4n}\right\}$ is

a. $\mathrm{Log}_c 2$

b. $\frac{\pi}{2}$

c. $\frac{4}{\pi}$

d. e

41. The differential equation representing the family of curves $y^2=2d\left(x+\sqrt{d}\right)$ where d is a parameter, is of

a. order 2

b. degree 2

c. degree 3

d. degree 4

42. Let y(x) be a solution of $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ and $y(0)=-1$. Then y(1) is equal to

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{6}$

d. -1

43. The law of motion of a body moving along a straight line is $x=\frac{1}{2}vt$, x being its distance from a fixed point on the line at time t and v is its velocity there.Then

a. acceleration f varies directly with x
b. acceleration f varies inversely with x
c. acceleration f is constant
d. acceleration f varies directly with t

44. Number of common tangents of $y = x^2$ and $y = -x^2 + 4x - 4$ is

a. 1 b. 2
c. 3 d. 4

45. Given that n numbers of A.Ms are inserted between two sets of numbers a, 2b and 2a, b where a, b $\in$R. Suppose further that the m^{th} means between these sets of numbers are same, then the ratio a : b equals

a. n – m + 1 : m b. n – m + 1 : n
c. n : n – m + 1 d. m : n – m + 1

46. If $x + \log_{10}(1 + 2^x) = x \log_{10}5 + \log_{10}6$ then the value of x is

a. $\frac{1}{2}$ b. $\frac{1}{3}$
c. 1 d. 2

47. If $Z_r = \sin\frac{2\pi r}{11} - \cos\frac{2\pi r}{11}$ then $\sum_{r=0}^{10} Z_r =$

a. –1 b. 0
c. i d. –i

48. If z_1 and z_2 be two non zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then the origin and the points represented by z_1 and z_2

a. lie on a straight line b. form a right angled triangle
c. form an equilateral triangle d. form an isosceles triangle

49. If $b_1b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations.$x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ has

a. real roots
b. purely imaginary roots
c. roots of the form a + ib (a, b $\in$R, ab$\neq$0)
d. rational roots

50. The number of selection of n objects from 2n objects of which n are identical and the rest are different is

a. 2^n
b. 2^{n-1}
c. $2^n - 1$
d. $2^{n-1} + 1$

51. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let B(1, 7) and D(4, – 2) be two points on the circle such that tangents at B and D meet at C. The area of the quadrilateral ABCD is

a. 150 sq. units
b. 50 sq. units
c. 75 sq. units
d. 70 sq. units

52. Let f(x) = $\begin{cases} -2\sin x, & \text{If } x \le -\frac{\pi}{2} \\ A\sin x + B, & \text{If } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{If } x \ge \frac{\pi}{2} \end{cases}$. Then

a. f is discontinuous for all A and B
b. f is continuous for all A = – 1 and B = 1
c. f is continuous for all A = 1 and B = –1
d. f is continuous for all real values of A, B

53. The normal to the curve $y = x^2 - x + 1$, drawn at the points with the abscissa $x_1 = 0, x_2 = -1$ and $x_3 = \frac{5}{2}$

a. are parallel to each other
b. are pair wise perpendicular
c. are concurrent
d. are not concurrent

54. The equation x log x = 3 – x

a. has no root in (1, 3)
b. has exactly one root in (1, 3)
c. x log x – (3 – x) > 0 in [1, 3]
d. x log x – (3 – x) < 0 in [1, 3]

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55. Consider the parabola $y^2 = 4x$. Let P and Q be points on the parabola where P(4, −4) & Q(9, 6). Let R be a point on the arc of the parabola between P & Q. Then the area of ΔPQR is largest when

a. $\angle PQR = 90°$

b. R(4, 4)

c. $R\left(\frac{1}{4},1\right)$

d. $R\left(1,\frac{1}{4}\right)$

56. A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

a. $\frac{8}{3}$

b. $\frac{6}{5}$

c. $\frac{3}{2}$

d. $\frac{17}{4}$

57. For $0 \le p \le 1$ and for any positive a, b; let $I(p) = (a + b)^p$, $J(p) = a^p + b^p$, then

a. $I(p) > J(p)$

b. $I(p) < J(p)$

c. $I(P) < J(P) \text{ in } \left[0,\frac{p}{2}\right] \;\&\; I(p) > J(p) \text{ in } \left[\frac{p}{2},\infty\right)$

d. $I(P) < J(P) \text{ in } \left[\frac{p}{2},\infty\right) \;\&\; J(p) < I(p) \text{ in } \left[0,\frac{p}{2}\right)$

58. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}, \vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{\delta}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by

a. $-\hat{i} - 3\hat{j} - 3\hat{k}$

b. $\hat{i} - 3\hat{j} - 3\hat{k}$

c. $-\hat{i} + 3\hat{j} + 3\hat{k}$

d. $\hat{i} + 3\hat{j} - 3\hat{k}$

59. Let $\vec{\alpha},\vec{\beta},\vec{\gamma}$ be three unit vectors such that $\vec{\alpha}.\vec{\beta}=\vec{\alpha}.\vec{\gamma}$ and the angle between $\vec{\beta}$ and $\vec{\gamma}$ is 30°. Then $\vec{\alpha}$ is

a. $2\left(\vec{\beta}\times\vec{\gamma}\right)$

b. $-2\left(\vec{\beta}\times\vec{\gamma}\right)$

c. $\pm2\left(\vec{\beta}\times\vec{\gamma}\right)$

d. $\left(\vec{\beta}\times\vec{\gamma}\right)$

60. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $Re(z_1) > 0$ and $Im(z_2) < 0$, then $\frac{z_1+z_2}{z_1-z_2}$ is

a. one

b. real and positive

c. real and negative

d. purely imaginary

61. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

a. 284 × 17

b. 285 × 17

c. 284 × 16

d. 285 × 16

62. The least positive integer n such that $\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}^n$ is an identity matrix of order 2 is

a. 4

b. 8

c. 12

d. 16

63. Let ρ be a relation defined on N, the set of natural numbers, as

$\rho = \{(x, y) \in N \times N : 2x + y = 41\}$ Then

a. ρ is an equivalence relation

b. ρ is only reflexive relation

c. ρ is only symmetric relation

d. ρ is not transitive

64. If the polynomial $f(x)=\begin{vmatrix}(1+x)^a & (2+x)^b & 1\\ 1 & (1+x)^a & (2+x)^b\\ (2+x)^b & 1 & (1+x)^a\end{vmatrix}$, then the constant term of f(x) is

a. $2 - 3.2^b + 2^{3b}$

b. $2 + 3.2^b + 2^{3b}$

c. $2 + 3.2^b - 2^{3b}$

d. $2 - 3.2^b - 2^{3b}$

65. A line cuts the x-axis at A(5, 0) and the y-axis at B(0, –3). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P and the y-axis at Q. If AQ and BP meet at R, then the locus of R is

a. $x^2 + y^2 - 5x + 3y = 0$

b. $x^2 + y^2 + 5x + 3y = 0$

c. $x^2 + y^2 + 5x - 3y = 0$

d. $x^2 + y^2 - 5x - 3y = 0$

66. In a third order matrix A, a_{ij} denotes the element in the i-th row and j-th column. If

$a_{ij} = 0$ for i = j

= 1 for i> j

= –1 for i< j

Then the matrix is

a. skew symmetric

b. symmetric

c. not invertible

d. non-singula

67. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k), with the lines y = x and x + y = 2 is h^2. The locus of the point P is

a. x = y – 1

b. x = –(y – 1)

c. x = 1 + y

d. x = –(1 + y)

68. A hyperbola, having the transverse axis of length 2 $\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

a. $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$

b. $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$

c. $(x^2 + y^2) \sin^2\theta = 1 + y^2$

δ. $x^2 \operatorname{cosec}^2\theta = x^2 + y^2 + \sin^2\theta$

69. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$ then assuming k as an integer,

a. f(x) increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

b. f(x) decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

c. f(x) decreases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

d. f(x) increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

70. Consider the function $y = \log_a\left(x + \sqrt{x^2+1}\right), a > 0, a \neq 1$. The inverse of the function

a. does not exist

b. is $x = \log_{1/a}\left(y + \sqrt{y^2+1}\right)$

c. is x = sinh (y ln a)

d. is $x = \cosh\left(-y \ln\frac{1}{a}\right)$

71. Let $I = \int_0^1 \frac{x^3 \cos 3x}{2+x^2} dx$. Then

a. $-\frac{1}{2} < I < \frac{1}{2}$

b. $-\frac{1}{3} < I < \frac{1}{3}$

c. $-1 < I < 1$

d. $-\frac{3}{2} < I < \frac{3}{2}$

72. A particle is in motion along a curve $12y - x^3$. The rate of change of its ordinate exceeds that of abscissa in

a. $-2 < x < 2$

b. $x = \pm 2$

c. $x < -2$

d. $x > 2$

73. The area of the region lying above x-axis, and included between the circle $x^2 + y^2 = 2ax$ & the parabola $y^2 = ax$, $a > 0$ is

a. $8\pi a^2$

b. $a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$

c. $\frac{16\pi a^2}{9}$

d. $\pi\left(\frac{27}{8}+3a^2\right)$

74. If the equation $x - cx + d = 0$ has roots equal to the fourth powers of the roots of $x^2 + ax + b = 0$, where $a^2 > 4b$, then the roots of $x^2 - 4bx + 2b^2 - c = 0$ will be

a. both real

b. both negative

c. both positive

d. one positive and one negative

75. On the occasion of Diwali festival each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards send by students are

a. ${}^{20}C_2$

b. ${}^{20}P_2$

c. $2\times{}^{20}C_2$

d. $2\times{}^{20}P_2$

WBJEE-2018 (Mathematics)

ANSWER KEYS

1. (c)	2. (a)	3. (a)	4. (a)	5. (c)	6. (d)	7. (c)	8. (c)	9. (d)	10. (c)
11. (b)	12. (a)	13. (a)	14. (b)	15. (b)	16. (d)	17. (d)	18. (b)	19. (a)	20. (c)
21. (a)	22. (c,d)	23. (d)	24. (a)	25. (a)	26. (d)	27. (a)	28. (c)	29. (b)	30. (c)
31. (c)	32. (a)	33. (b)	34. (b)	35. (c)	36. (d)	37. (b)	38. (c)	39. (b)	40. (c)
41. (c)	42. (c)	43. (c)	44. (b)	45. (d)	46. (c)	47. (b)	48. (c)	49. (a)	50. (a)
51. (c)	52. (b)	53. (c)	54. (b)	55. (c)	56. (a)	57. (b)	58. (a,b,c)	59. (c)	60. (d)
61. (a)	62. (b)	63. (d)	64. (a)	65. (a)	66. (a,c)	67. (a,b)	68. (b)	69. (a,c)	70. (c)
71. (a,b,c,d)	72. (c,d)	73. (b)	74. (a,d)	75. (b,c)					

WBJEE-2018 (Mathematics)

Solution

1. (c)

$^nC_r + 2.\,^nC_{r+1} + ^nC_{r+2}$

$\Rightarrow \quad ^nC_r + \,^nC_{r+1} + ^nC_{r+1} + ^nC_{r+2}$

$\Rightarrow \quad ^{n+1}C_{r+1} + \,^{n+1}C_{r+2}$

$\Rightarrow \quad ^{n+2}C_{r+2}$

2. (a)

$(101)^{100}-1$

$\Rightarrow (100+1)^2 - 1$

$\Rightarrow [^{100}C_0 \cdot (100)^{100}\,(1)^0 + \,^{100}C_1\,(100)^{99}(1)^1 + \text{.........} + \,^{100}C_{99}(100)^1(1)^{99}$

$+ \,^{100}C_{100}\,(100)^0(1)^{100}] - 1$

$\Rightarrow [^{100}C_0\,(100)^{100} + \,^{100}C_1\,(100)^{99} + \text{.........} + \,^{100}C_{99}\,(100)^1 + 1 - 1]$

$\Rightarrow ^{100}C_0\,(100)^{100} + \,^{100}C_1\,(100)^{99} + \text{.........} + (100)\,(100)^{100}$

$\Rightarrow 10^4\,[^{100}C_0\,(100)^{98} + \,^{100}C_1\,(100)^{97} + \text{.........} + 1]$

3. (a)

For greatest term we have

$$\frac{n}{2} < \frac{n+1}{1+|x|} \le \frac{n}{2} + 1$$

$$\Rightarrow \frac{n}{2} < \frac{n+1}{1+|x|} \text{ and } \frac{n+1}{|x|+1} \le \frac{n}{2} + 1$$

$$\Rightarrow \frac{1+|x|}{2} < \frac{n+1}{n} \text{ and } \frac{n+1}{n+2} \le \frac{|x|+1}{2}$$

$$\Rightarrow |x| < \frac{2n+2}{n} - 1 \text{ and } \frac{2n+2}{n+2} - 1 \le |x|$$

$$\Rightarrow x < \frac{n+2}{n} \text{ and } \frac{n}{n+2} \le x$$

$$\therefore \frac{n}{n+2} < x < \frac{n+2}{n}$$

4. (a)

Given

$$A = \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$$

$|A| = -1\,(1 + 12) - 7\,(2 + 9) + 0$

$|A| = -13 - 77$

$|A| = -90$

$$\text{Let } B = \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$$

$$B = 5 \times 3 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ -7 & -1 & -1 \end{vmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$B = 15 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ 0 & 0 & -6 \end{vmatrix}$$

$B = 15\,[0 - 0 - 6\,(-90)]$

$B = (-90)\,(-90)$

$B = A.\,A$

$B = A^2$

5. (c)

Given,

$a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9}$

$a_r = e^{\frac{2r\pi i}{9}}$

Now,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} & e^{\frac{6\pi i}{9}} \\ e^{\frac{8\pi i}{9}} & e^{\frac{10\pi i}{9}} & e^{\frac{12\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$$

$$= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \begin{vmatrix} 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$$

$= 0$ { $\because$ two row are same}

6. (d)

$$S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} S_r = \begin{vmatrix} 2\sum_{r=1}^{n} r & x & n(n+1) \\ \sum_{r=1}^{n}(6r^2-1) & y & n^2(2n+3) \\ \sum_{r=1}^{n}(4r^3-2nr) & z & n^3(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} S_r = \begin{vmatrix} 2\left(\frac{n(n+1)}{2}\right) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix}$$

$\sum_{r=1}^{n} S_r = 0$ { ∵ two row are same}

7. (c)

System equations

x + 4ay + az = 0

x + 3by + bz = 0

x + 2cy + cz = 0

For non-trivial solution

Δ = 0

$$\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

⇒ 1(3bc – 2bc) – 1 (4ac – 2ac) + (4ab – 3 ab) = 0

⇒bc – 2ac + ab = 0

⇒bc + ab = 2ac

$b = \frac{2ac}{a+c}$

a, b, c are in H.P.

8. (c)
xRy$\Rightarrow$ x – y is zero or irrational
xRx$\Rightarrow$ 0 $\therefore$ reflexive
ifxRy$\Rightarrow$ x – y is zero or irrational
$\Rightarrow$ y – x is zero or irrational
$\therefore$ yRx symmetric
xRy$\Rightarrow$ x –y is 0 or irrational
yRz$\Rightarrow$ y –z is 0 or irrational
Then (x – y) + (y – z) = x – z may be rational
$\therefore$ It is not transitive

9. (d)
$(x, x) \in R \Rightarrow x > |x|$ {false}
$\therefore$ not reflexive
If $(x, y) \in R \Rightarrow x > |y| \Rightarrow y > |x|$
$\therefore$ not symmetric
If $(x, y) \in R \Rightarrow x > |y|$; $(y, z) \in R \not\Rightarrow y > |z|$
$\Rightarrow x > |z| \Rightarrow (x, z) \in R$
$\therefore$ transitive

10. (c)
Given
$f(x) = e^x$
$g(x) = x^2$
$(g \text{ of})(x) = g[f(x)]$
$= g[e^x]$
$= (e^x)^2$
$= e^{2x} : x \in R$
Clearly g(f(x)) is injective and g(x) is not injective.

11. (b)
$p(H) = \frac{1}{2}$

$p(T) = \frac{1}{2}$

$\Rightarrow P = 1 - \frac{1}{2^n} \geq 0.9$

$\Rightarrow 1 - \frac{9}{10} \geq \frac{1}{2^n}$

$\Rightarrow \frac{1}{10} \geq \frac{1}{2^n}$

$\Rightarrow 10 \leq 2^n$
$n = 4$

12. (a)

Let A, B and C be the events that the student is successful in tests I, II and III respectively. Then p (The student is successful)

$= P\,[(I \cap II \cap III') \cup (I \cap II' \cap III) \cup (I \cap II \cap III)]$

$$\Rightarrow \frac{1}{2} = P(I \cap II \cap III') + P(I \cap II' \cap III) + P(I \cap II \cap III)$$

$$\Rightarrow \frac{1}{2} = P(I)P(II)P(III') + P(I)\,P\,(II')\,P(III) + P(I)P(II)P(III)$$

[$\because$ I, II and III are independent]

$$\Rightarrow \frac{1}{2} = p.q.\left(1 - \frac{1}{2}\right) + p \cdot (1-q) \cdot \frac{1}{2} + p.q.\frac{1}{2}$$

$\Rightarrow 1 = pq + p$

$\Rightarrow p(q + 1) = 1$

13. (a)

$\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow \sin 4\theta + 2\sin \dfrac{8\theta}{2} \cos \dfrac{4\theta}{2} = 0$

$\Rightarrow \sin 4\theta + 2\sin 4\theta \cos 2\theta = 0$

$\Rightarrow \sin 4\theta\,(1 + 2\cos 2\theta) = 0$

$\sin 4\theta = 0$ or $1 + 2\cos 2\theta = 0$

$4\theta = n\pi$ or $\cos 2\theta = \dfrac{-1}{2} = \cos \dfrac{2\pi}{3}$

$\theta = \dfrac{n\pi}{4}$ or $2\theta = 2n\pi \pm \dfrac{2\pi}{3}$

$\theta = \dfrac{n\pi}{4}$ or $\theta = n\pi \pm \dfrac{\pi}{3}$

14. (b)

$$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$$

$$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\frac{2\tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\cot A}{1 - \cot^2 A}\right)$$

$$= \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\tan A}{\tan^2 A - 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) - \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right)$$

$= 0$

15. (b)

By replacing

$X \to x + 2, y \to y + 3$

Given equation of circle is $x^2 + y^2 - 4x + 6y + 9 = 0$

$\therefore (x + 2)^2 + (y + 3)^2 - 4(x + 2) - 6(y + 3) + 9 = 0$

$\Rightarrow x^2 + 4x + 4 + y^2 + 6x + 9 - 4x - 8 - 6y - 18 + 9 = 0$

$\Rightarrow x^2 + y^2 - 4 = 0$

16. (d)

Let the centre be 0, points on circle from where tangents are drawn is A, B and point of intersection of tangent is P.

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$

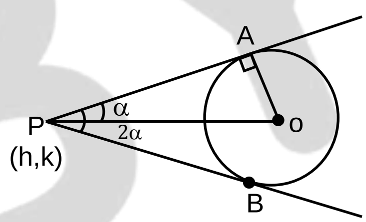
centre of circle O = (–2, 3)

$r = \sqrt{4+9-9\sin^2\alpha-13\cos^2\alpha}$

$r = \sqrt{13-9\sin^2\alpha-13(1-\sin^2\alpha)}$

$r = \sqrt{13\sin^2\alpha-9\sin^2\alpha}$

$r = 2\sin\alpha$



2α is the angle between tangents

$\sin x = \frac{OA}{OP}$

$\Rightarrow \sin x = \frac{2\sin x}{\sqrt{(h+2)^2+(k-3)^2}}$

$\Rightarrow (h+2)^2 + (k-3)^2 = 4$

$\Rightarrow h^2 + k^2 + 4h - 6k + 9 = 0$

Focus of point p is

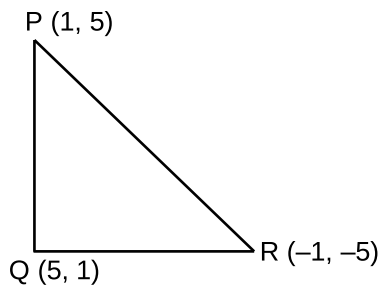
$x^2 + y^2 + 4x - 6y + 9 = 0$

17. (d)

Clearly P (1, 5)

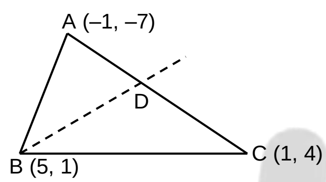
Q = (5, 1) {∵ y = x}

R = (–1, –5) { ∵ y = –x}



∴ Circum center of PQR is $\left(\frac{1-1}{2},\frac{5-5}{2}\right)$=(0, 0)

18. (b)



AB = $\sqrt{(-1-5)^2+(-7-1)^2}=10$

BC = $\sqrt{(1-5)^2+(4-1)^2}=5$

BD divides AC in ratio 2 : 1

D = $\left(\frac{-1+2}{2+1},\frac{-7+8}{2+1}\right)$

D = $\left(\frac{1}{3},\frac{1}{3}\right)$

∴ Equation of BD is

⇒ y – 1 = $\frac{1-\frac{1}{3}}{5-\frac{1}{3}}(x-5)$

⇒ (y – 1) = $\frac{2}{14}(x-5)$

⇒ x – 7 y + 2 = 0

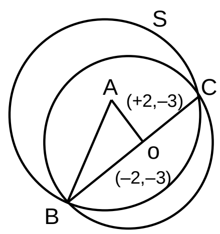
⇒ 7 y = x + 2

19. (a)

Given equation of circle is $x^2 + y^2 + 4x + 6y - 12 = 0$

Whose centre is (2, –3) and radius = $\sqrt{2^2 + (-3)^2 + 12}$ = 5

Now, according to given information, we have the following figure.



$x^2 + y^2 - 4x + 6y - 12 = 0$

Clearly, $AO \perp BC$, as O is mid point of the chord.

Now, in ΔAOB we have

OA = $\sqrt{(2+2)^2 + (-3+3)^2} = \sqrt{16} = 4$

And OB = 5

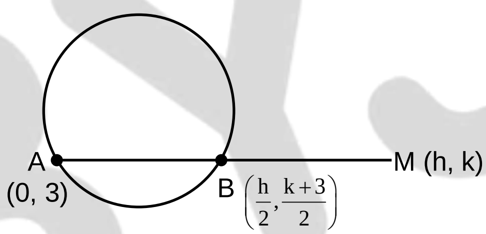
$\therefore$ AB = $\sqrt{OA^2 + OB^2}$ AB = $\sqrt{160+25}$

AB = $\sqrt{41}$

20. (c)

Given equation of circle is $x^2 + 4x + (y - 3)^2 = 0$

AM = 2AB



B is the mid point of AM

$\Rightarrow$ B = $\left(\frac{h}{2}, \frac{k+3}{2}\right)$ lies on the circle

Equation of circle is $x^2 + 4x + (y - 3)^2 = 0$

Let x = $\frac{h}{2}$, y = $\frac{k+3}{2}$

$\therefore$ $\frac{h^2}{4} + 2h + \left(\frac{k+3}{2} - 3\right)^2 = 0$

$\frac{h^2}{4} + 2h + \frac{k^2 - 6K + 9}{4} = 0$

$\therefore$ $k^2 + h^2 + 8h - 6k + 9 = 0$

$\therefore$ Locus of m is $x^2 + y^2 + 8x - 6y + 9 = 0$

WBJEE-2018 (Mathematics)

21. (a)

Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Ellipse: $x^2 + 9y^2 = 9$

$$\frac{x^2}{9}+\frac{y^2}{1}=1$$

Eccentricity of ellipse e = $\sqrt{1-\frac{1}{9}}$ = $\sqrt{\frac{8}{9}}$

$\therefore$ the eccentricity of the hyperbola be reciprocal to the ellipse

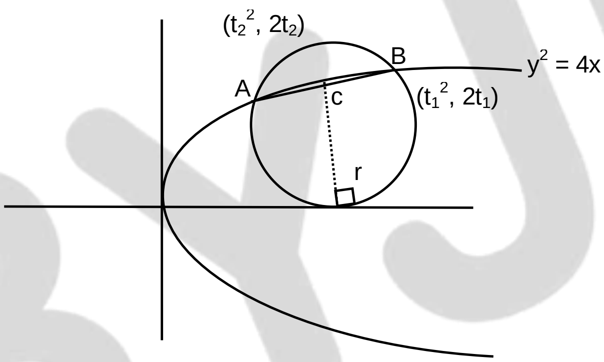
Eccentricity of hyperbola = $\sqrt{\frac{9}{8}}$

$1 + \frac{b^2}{a^2} = \frac{9}{8}$

$\frac{b^2}{a^2}=\frac{1}{8}$

$a^2 : b^2 = 8 : 1$

22. (c, d)



Let, A $\left(t_2^2,\ 2t_2\right)$, B$(t_1^2,\ 2t_1)$ be the two points on the parabola.

AB is the diameter of the circle.

Let, c be the centre of the circle.

$\Rightarrow$ C = $\left(\frac{t_1^2+t_2^2}{2}, t_1+t_2\right)$

This axis of the parabola is x-axis

So, the circle touches the x-axis

Hence, distance of c from x-axis = radius of circle.

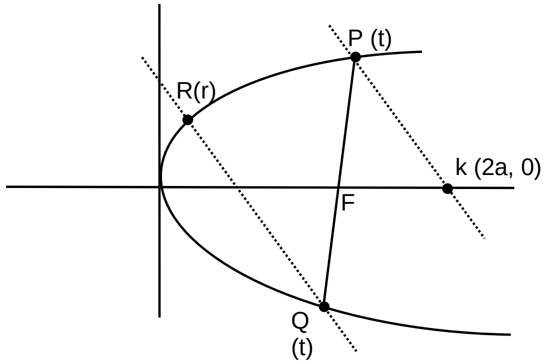
$\Rightarrow |t_1 + t_2| = r$

$\Rightarrow (t_1+t_2) = \pm r$

Slope of AB = $\frac{y_2-y_1}{x_2-x_1}$

$$= \frac{2t_1-2t_2}{t_1^2-t_2^2} = \frac{2}{t_1+t_2} = \pm\frac{2}{r}$$

23. (d)



Slope of line Pk = slope of line QR

$m_{Pk} = m_{QR}$

$$\Rightarrow \frac{2at-0}{at^2-2a} = \frac{2at'-2ar}{a(t')^2-ar^2}$$

$$\Rightarrow \frac{t}{t^2-2} = \frac{t'-r}{(t')^2-r^2}$$

$\Rightarrow -t' - tr^2 = -t - rt^2 - 2t' + 2r$ $\{tt' = -1\}$

$\Rightarrow t' - tr^2 = -t + 2r - rt^2$

$\Rightarrow -tr^2 + r(t^2 - 2) + t' + t = 0$

$$\lambda = \frac{(2-t^2) \pm \sqrt{(t^2-2)^2+4(-1+t^2)}}{-2t}$$

$$= \frac{(2-t^2) \pm \sqrt{t^4}}{-2t} = \frac{2-t^2 \pm t^2}{-2t}$$

$$r = -\frac{1}{t}$$

It is not possible as the R & Q will be one and same.

$$\text{Or } r = \frac{t^2-1}{2}$$

24. (a)

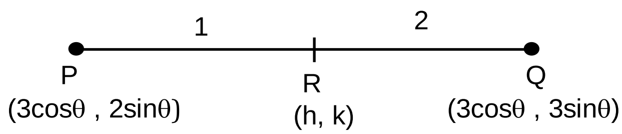
Given

Ellipse: $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

P = (3cosθ, 2sinθ)

Circle: $x^2 + y^2 = 9$

Q = (3cosθ, 3sinθ)



$h = \dfrac{3\cos\theta+6\cos\theta}{3}$; $k = \dfrac{3\sin\theta+4\sin\theta}{3}$

$h = 3\cos\theta$; $k = \dfrac{7}{3}\sin\theta$

$\therefore$ $\sin^2\theta + \cos^2\theta = 1$

$\dfrac{h^2}{9} + \dfrac{9k^2}{49}=1$

Locus is $\dfrac{x^2}{9}+\dfrac{9y^2}{49}=1$

25. (a)

Equation line : $\dfrac{x-1}{1}=\dfrac{y+2}{4}=\dfrac{z-3}{5}=\lambda$ (let)

Point P (λ + 1, 4λ – 2, 5λ + 3) .

Point p lies on 2x + 3y – 4z + 22 = 0

$\Rightarrow 2(\lambda + 1) + 3 (4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow -6\lambda + 6 = 0$

$\Rightarrow \lambda = 1$

Point p = (2, 2, 8), q = (1, –2, 3)

Distance PQ $= \sqrt{1^2+4^2+5^2}$

$= \sqrt{1+16+25}$

$= \sqrt{42}$

26. (d)

Equation of line joining point (0, –11, 4) and (2, –3, 1)

$\dfrac{x-2}{2} = \dfrac{y+3}{8} = \dfrac{z-1}{-3} = \lambda$ (let)

DR’s of PQ (2λ + 1 , 8λ – 11, –3λ – 3)

Now, $(2\lambda + 1)\,2 + (8\lambda - 11)8 + (-3\lambda - 3)(-3)=0$

$\Rightarrow$ $77\lambda - 77 = 0$

$\Rightarrow$ $\lambda = 1$

Q = (4, 5, –2)

27. (a)

$\because \sin 3\,0 = \frac{1}{2}$

$\therefore \sin 31 > \frac{1}{2}$ { $\because$ sinx is increasing function}

28. (c)

$\frac{d}{dx} f_r(x)$

$= \frac{d}{dx} e^{f_{r-1}(x)}$

$= e^{f_{r-1}(x)} \frac{d}{dx} f_{r-1}(x)$

$= f_r(x) \frac{d}{dx} f_{r-1}(x) \quad \forall r \in N > 1$

$\therefore \frac{d}{dx} f_n(x) = f_n(x) \frac{d}{dx} f_{n-1}(x)$

$= f_n(x)\, f_{n-1}(x) \frac{d}{dx} f_{n-2}(x)$

$= f_n(x)\, f_{n-1}(x)\, f_{n-2}(x) \frac{d}{dx} f_{n-3}(x)$

$\vdots$

$= f_n(x) f_{n-1}(x)\, f_{n-2}(x) \,\ldots\ldots. f_3(x) f_2(x) \frac{d}{dx} f_1(x)$

$= f_n(x) f_{n-1}(x)\, f_{n-2}(x) \,\ldots\ldots. f_3(x) f_2(x) \frac{d}{dx} e^x$

$= f_n(x)\, f_{n-1}(x) f_{n-2}(x) \,\ldots\ldots\ldots. f_3(x) f_2(x) e^x$

$= f_n(x)\, f_{n-1}(x) f_{n-2}(x) \,\ldots\ldots\ldots. f_3(x) f_2(x) f_1(x)$

29. (b)

$f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

$\therefore \frac{1-|x|}{2-|x|} \geq 0 \Rightarrow |x| \leq 1$ or $|x| \geq 2$

$\Rightarrow x \in [-1,1]$ or $x \in (-\infty,-2) \cup (2,\infty)$

$\Rightarrow x \in [-1,1] \cup (-\infty,-2) \cup (2,\infty)$

30. (c)

Let $g(x) = e^{kx}f(x)$

$f(a) = 0 = f(b)$

by Rolle's theorem

$g'(c) = 0 \quad , \qquad c \in (a,b)$

$g'(x) = e^{kx}f'(x) + ke^{kx}f(x)$

$g'(c) = 0$

$\Rightarrow e^{kc}(f'(c) + kf(c)) = 0$

$f'(c) + kf(c) = 0$

foratleast one c in (a, b)

31. (c)

Given

$f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

Now,

$$\lim_{h\to 0} \frac{f(1-h)-f(1)}{h^3+3h}$$

$\Rightarrow \quad \frac{0}{0}$ form, using L' hospital rule

$$\lim_{h\to 0} \frac{-f'(1-h)}{3h^2+3}$$

$$\Rightarrow \quad \frac{-f'(1)}{3}$$

$$\Rightarrow \quad \frac{-[30(1)^9-56(1)^7+30(1)^5-63(1)^2+6(1)]}{3}$$

$$\Rightarrow \quad \frac{-[30-56+30-63+6]}{3}$$

$$\Rightarrow \quad \frac{53}{3}$$

32. (a)

Let, $h(x) = e^{-x} f(x)$

$h(a) = 0, h(b) = 0$

h(x) is continuous and differentiable function

by Rolle's theorem

$h'(c) = 0 \qquad , \qquad c \in (a,b)$

$e^{-x}f'(c) + (-e^{-x}) f(c) = 0$

$e^{-x}f'(c) = e^{-x}f(c)$

$f'(c) = f(c)$

33. (b)

f(x) is continuous and differentiable function

$f(0) = f(1) = 0 \quad \Rightarrow$ by Rolle's theorem

$f'(a) = 0 \quad , \quad a \in (0,1)$

Given $f'(0) = 0$

By Rolle's theorem $f''(0) = 0$ for some c, $c \in (0, a)$

34. (b)

$$I = \int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right)dx$$

$$I = \int e^{\sin x}\left(x\cos x - \tan x\sec x\right)dx$$

$$I = \left(xe^{\sin x} - \int e^{\sin x}\right) - \left[e^{\sin x}\sec x - \int e^{\sin x}dx\right] + c$$

$$I = xe^{\sin x} - e^{\sin x}\sec x + c$$

$$I = e^{\sin x}(x - \sec x) + c$$

35. (c)

$$\int f(x)\sin x\cos x\,dx = \log(f(x))\frac{1}{2(b^2-a^2)} + C$$

Differentiate with respect to x

$$f(x)\ \sin x\cos x = \frac{f'(x)}{f(x)}\frac{1}{2(b^2-a^2)} + C$$

$$\Rightarrow \sin 2x\ (b^2 - a^2) = \frac{f'(x)}{(f(x))^2}$$

On integrating

$$\frac{-1}{f(x)} = \frac{-(b^2-a^2)\cos 2x}{2}$$

$f(x) = \frac{2}{(b^2 - a^2)\cos 2x}$

36. (d)

$N = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{(x+1)^2} dx$

$N = \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{2(x+1)^2} dx$

Let $2x = t \Rightarrow 2dx = dt$ — $\frac{\pi}{2}$ (U.L), 0 (L.L)

$N = \int_0^{\frac{\pi}{2}} \frac{\sin t}{4\left(\frac{t}{2}+1\right)^2} dt$

$N = \int_0^{\frac{\pi}{2}} \frac{\sin t}{(t+2)^2} dt$

Apply by part method

$N = \sin t \left(\frac{-1}{t+2}\right) + \int_0^{\frac{\pi}{2}} \frac{\cos t}{t+2} dt$

$N = \left(-\sin\left(\frac{1}{t+2}\right)\right)_0^{\frac{\pi}{2}} + M$

$M - N = \frac{2}{\pi + 4}$

37. (b)

$I = \int_{\frac{1}{2014}}^{2014} \frac{\tan^{-1} x}{x}\, dx$ ....(1)

Let $x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$

$I = \int_{2014}^{\frac{1}{2014}} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\left(\frac{1}{t}\right)}\left(-\frac{1}{t^2}\right) dt$

$I = \int_{\frac{1}{2014}}^{2014} \frac{\cot^{-1} t}{t} dt$ ......(2)

From eq(1) + eq(2)

$2I = \int_{\frac{1}{2014}}^{2014} \frac{\tan^{-1} t + \cot^{-1} t}{t} dt$

$2I = \int_{\frac{1}{2014}}^{2014} \frac{\pi/2}{t} dt$

$I = \frac{\pi}{4}\left(\ell nt\right)_{\frac{1}{2014}}^{2014}$

$I = \frac{\pi}{4}(\ell n2014 - \ell n\frac{1}{2014})$

$I = \frac{\pi}{4}(2\ell n2014)$

$I = \frac{\pi}{2}\ell n2014$

38. (c)

$I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$

$\frac{\sin x}{x}$ is a decreasing function

So $\frac{\pi}{12} \times \frac{\sin\frac{\pi}{3}}{\frac{\pi}{3}} \le I \le \frac{\pi}{12} \times \frac{\sin\frac{\pi}{4}}{\frac{\pi}{4}}$

$\Rightarrow \frac{1}{4} \times \frac{\sqrt{3}}{2} \le I \le \frac{1}{3} \times \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$$

39. (b)

$$I = \int_{\pi/2}^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$$

$$I = \int_{\pi/2}^{\pi} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \text{ ....(1)}$$

$$I = \int_{\pi/2}^{\pi} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{\frac{5\pi}{2}} \frac{e^{-\tan^{-1}(\sin x)}}{e^{-\tan^{-1}(\sin x)} + e^{-\tan^{-1}(\cos x)}} dx$$

$$\left\{ \text{by} \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx \right\}$$

$$I = \int_{\frac{\pi}{2}}^{\pi} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \text{ ....(2)}$$

From eq (1) + eq(2)

$$2I = \int_{\pi/2}^{\pi} 1 dx + \int_{\pi}^{\frac{5\pi}{2}} 1 dx$$

$$2I = (x)_{\frac{\pi}{2}}^{\frac{5\pi}{2}} = 2\pi$$

$$I = \pi$$

40. (c)

$$L = \lim_{n\to\infty} \frac{1}{n} \left\{ \sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \ldots\ldots + \sec^2 \frac{n\pi}{4n} \right\}$$

$$L = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \sec^2 \left(r \cdot \frac{\pi}{4n} \right)$$

Let $\frac{r}{n} = x \Rightarrow \frac{1}{n} = dx$

$$L = \int_0^1 \sec^2 \left(\frac{\pi}{4} x \right) dx$$

$$L = \left[\tan\left(\frac{\pi}{4} x \right) \right]_0^1 \times \frac{4}{\pi}$$

$$L = \frac{4}{\pi}\left(\tan\frac{\pi}{4} - \tan 0\right)$$

$$L = \frac{4}{\pi}$$

41. (c)

$y^2 = 2d\left(x + \sqrt{d}\right)$ .....(i)

Differentiate with respect to x

$2y\frac{dy}{dx} = 2d \qquad \Rightarrow d = y\frac{dy}{dx}$

Put in equation (i)

$$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$$

$$y^2 = 2y\frac{dy}{dx} + 2y^{3/2}\left(\frac{dy}{dx}\right)^{3/2}$$

$$\left(y^2 - 2xy\frac{dy}{dx}\right)^2 = 4y^3\left(\frac{dy}{dx}\right)^3$$

Degree three.

42. (c)

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{4x^2}{1+x^2}$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\ell n\left|1+x^2\right|} = \left(1+x^2\right)$$

$$\Rightarrow y\left(1+x^2\right) = \int \frac{4x^2}{\left(1+x^2\right)} \times \left(1+x^2\right)dx + c$$

$$\Rightarrow y\left(1+x^2\right) = \int 4x^2 dx + c$$

$$\Rightarrow y\left(1+x^2\right) = \frac{4x^3}{3} + c$$

Put $y(0) = -1 \Rightarrow -1 = c$

$$\therefore y\left(1+x^2\right) = \frac{4x^3}{3} - 1$$

$$y(1) \Rightarrow y(1+1) = \frac{4(1)}{3} - 1$$

$$2y = \frac{1}{3}$$

$$y = \frac{1}{6}$$

43. (c)

$x = \frac{1}{2}vt$

Differentiate with respect to x

$\Rightarrow x = \frac{1}{2}.\frac{dx}{dt}.t$

$\Rightarrow \int \frac{2dt}{t} = \int \frac{dx}{x}$

$\Rightarrow \ell nc + 2\ell nt = \ell nx$

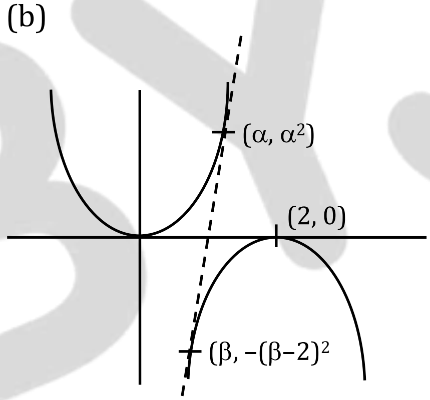
$\Rightarrow x = t^2c$

$\Rightarrow \frac{dx}{dt} = 2tc$

$\Rightarrow \frac{d^2x}{dt^2} = 2c$

Hence, acceleration is constant.

44. (b)



$y = x^2$; $y = -(x-2)^2$

$\frac{\alpha^2 + (\beta-2)^2}{\alpha - \beta} = 2\alpha = -2(\beta - 2)$

$\Rightarrow \alpha = 2 - \beta$ $\Rightarrow \beta = 2 - \alpha$

$\frac{\alpha^2 + \alpha^2}{\alpha - 2 + \alpha} = 2\alpha \Rightarrow \frac{2\alpha^2}{2\alpha - 2} = 2\alpha$

$\Rightarrow \alpha^2 = \alpha(2\alpha - 2)$

$\Rightarrow \alpha^2 = 2\alpha^2 - 2\alpha$

$\Rightarrow \alpha^2 = 2\alpha \quad \Rightarrow \alpha = 0, 2$

$\Rightarrow \alpha = 0 \quad \Rightarrow \beta = 2$

$\Rightarrow \alpha = 2 \quad \Rightarrow \beta = 0$

Hence, two common tangent.

45. (d)

a..........n A.Ms...........2b

(Difference)d = $\frac{2b-a}{n+1}$

$A_m = a + m\left(\frac{2b-a}{n+1}\right)$ .....(i)

2a..........n A.Ms...........b

$d = \frac{b-2a}{n+1}$

$A_m = 2a + m\left(\frac{b-2a}{n+1}\right)$ .....(ii)

Equating equation (i) & (ii)

$a = \frac{m}{n+1}(b+a)$

$\Rightarrow \frac{a}{b} = \frac{m}{n-m+1}$

46. (c)

$x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$

$\Rightarrow x\log_{10}10 + \log_{10}(1 + 2^x) = \log_{10}5^x + \log_{10}6$

$\Rightarrow \log_{10}(1 + 2^x) = \log_{10}5^x + \log_{10}6 - \log_{10}10^x$

$\Rightarrow \log_{10}(1 + 2^x) = \log_{10}\left(\frac{5^x.6}{10^x}\right)$

$\Rightarrow 1 + 2^x = \frac{6\,.\,5^x}{2^x\,.\,5^x}$

$\Rightarrow 1 + 2^x = \frac{6}{2^x}$

let $2^x = t$

$\Rightarrow 1 + t = \frac{6}{t}$

$\Rightarrow t^2 + t - 6 = 0$

$\Rightarrow \quad (t+3)(t-2) = 0$

$t = -3$	$t = 2$
$2^x = -3$	$2^x = 2$
(not possible)	$x = 1$

47. (b)

Given

$$Z_r = \sin\frac{2\pi r}{11} - i\cos\frac{2\pi r}{11}$$

$$Z_r = -i\left(\cos\frac{2\pi r}{11} + i\sin\frac{2r\pi}{11}\right)$$

$$Z_r = -ie^{\frac{i2r\pi}{11}}$$

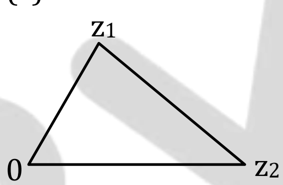
Now

$$\sum_{r=0}^{10} Z_r = -i\sum_{r=0}^{10}\left(e^{\frac{i2r\pi}{11}}\right)$$

$= -i(0)$

$= 0$

48. (c)



$$\Rightarrow z_1 = z_2 e^{\frac{i\pi}{3}}$$

$$\Rightarrow 2z_1 = z_2\left(1 + i\sqrt{3}\right)$$

$$\Rightarrow 2z_1 = z_2 + i\sqrt{3}z_2$$

$$\Rightarrow 2z_1 - z_2 = i\sqrt{3}z_2$$

Squaring both side

$$\Rightarrow 4z_1^2 + z_2^2 - 4z_1z_2 = -3z_2^2$$

$$\Rightarrow 4z_1^2 + z_2^2 = 4z_1z_2$$

Hence from equilateral triangle.

49. (a)

Suppose the equations $x^2 + b_1x + c_1 = 0$ & $x^2 + b_2x + c_2 = 0$ have real roots.

then $b_1^2 \geq 4c_1$ .....(i)

$b_2^2 \geq 4c_2$ .....(ii)

Given that $b_1b_2 = 2(c_1 + c_2)$

On squaring $b_1^2 b_2^2 = 4\left(c_1^2 + c_2^2 + 2c_1c_2\right) = 4\left[\left(c_1 - c_2\right)^2 + 4c_1c_2\right]$

$\Rightarrow b_1^2 b_2^2 - 16c_1c_2 = 4\left(c_1 - c_2\right)^2 \geq 0$

Multiplying (i) & (ii), we get

$b_1^2 b_2^2 \geq 16c_1c_2$

Therefore, at least one equation have real roots.

50. (a)

Total no. of ways = ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots + {}^nC_n$

$= 2^n$

51. (c)

s : $x^2 + y^2 - 2x - 4y - 20 = 0$

Centre of circle A = (1, 2)

Equation of tangent at B(1, 7)

$\Rightarrow x + 7y - (x + 1) - 2(y + 7) - 20 = 0$

$\Rightarrow 5y = 35$

$\Rightarrow y = 7$

Equation of tangent at D(4, –2)

$\Rightarrow 4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$

$\Rightarrow 3x - 4y = 20$

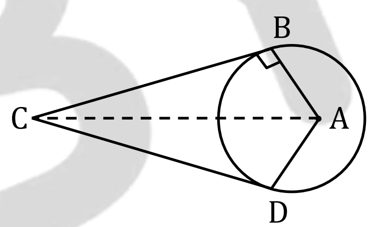
$\therefore$ coordinates of C are (16, 7)

Length of AB = $\sqrt{\left(1-1\right)^2 + \left(7-2\right)^2} = 5$

Length of BC = $\sqrt{\left(16-1\right)^2 + \left(7-7\right)^2} = 15$

$\therefore$ The area of quadrilateral ABCD = $2 \times \frac{1}{2} \times 5 \times 15$

= 75 sq. units.



52. (B)

Given,

$$f(x) = \begin{cases} -2\sin x & \text{if } x \leq -\frac{\pi}{2} \\ A\sin x + b & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

From above conditions function f(x) is continuous throughout the real line, when function f(x) is continuous at $x=-\frac{\pi}{2}$ and $\frac{\pi}{2}$ for continuity at $x=-\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x)=f\left(-\frac{\pi}{2}\right) \qquad .....(ii)$$

$$\lim_{x\to\frac{\pi^-}{2}} f(x)=2$$

$\therefore$ from equation (ii) we get

–A + B = 2 .....(iii)

For continuity at $x=\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x)=f\left(\frac{\pi}{2}\right) \qquad .....(iv)$$

Here $\lim_{x\to\frac{\pi^-}{2}} f(x)=A+B$

$\Rightarrow \lim_{x\to\frac{\pi^+}{2}} f(x)=0$

and $f\left(\frac{\pi}{2}\right)=0$

$\therefore$ from equation (iv)

A + B = 0 .....(v)

From equation (iii) & (iv)

A = –1, B = 1

53. (c)

$y = x^2 - x + 1$

$$\frac{dy}{dx}=2x-1=m_T$$

$$\because m_N=\frac{-1}{m_T}\Rightarrow m_N=\frac{1}{1-2x}$$

For tangent

$m_{x_1}=1$ Point (0, 1)

(y – 1) = 1(x – 0) $\Rightarrow$ x – y + 1 = 0 .....(i)

For tangent

$m_{x_2}=\frac{1}{3}$ point (–1, 3)

(y – 3) = $\frac{1}{3}$(x + 1) $\Rightarrow$ 3y – 9 = x + 1

$\Rightarrow$ x – 3y + 10 = 0 .....(ii)

For normal

$m_{x_3}=-\frac{1}{4}$ point $\left(\frac{5}{2},\frac{19}{4}\right)$

$$\left(y-\frac{19}{4}\right)=-\frac{1}{4}\left(x-\frac{5}{2}\right) \quad \Rightarrow \quad x+4y=\frac{43}{2} \quad .....(iii)$$

To find intersection point tangent (1) & tangent (2)

$\Rightarrow y - 1 - 3y + 10 = 0$

$\Rightarrow -2y = -9 \Rightarrow y = \dfrac{9}{2}$

Intersection point is $\left(\dfrac{7}{2}, \dfrac{9}{2}\right)$ passes (3)

Hence, normal are concurrent.

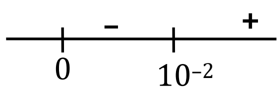
54. (b)

f(x) = x log x – 3 + x

Differentiate with respect to x

$$f'(x)=x.\frac{1}{x}+\log.x+1$$

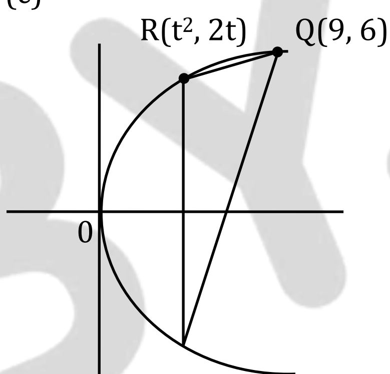
f'(x) = 2 + log x



f(1)f(3) = –2(3log3) = –ve

Hence, one root is (1, 3).

55. (c)



$$\text{Area}=\frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[t^2(10)-2t(5)+1(-60)\right]$$

$$=\frac{10}{2}\left(t^2-t-6\right)$$

f(t)= $5t^2$ – 5t – 30

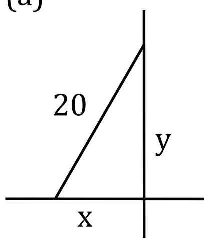
f'(t) = 10t – 5 = 0

$t = \frac{1}{2}$

Point $R = \left(\left(\frac{1}{2}\right)^2, 2\left(\frac{1}{2}\right)\right)$

$= \left(\frac{1}{4}, 1\right)$

56. (a)



Using right angle triangle concept

$x^2 + y^2 = 400$

Differentiate with respect to t

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Given

$\frac{dy}{dt} = 2ft/sec$

$x = 12 \Rightarrow 16$

$\Rightarrow 2(12)\frac{dx}{dt} + 2(16)\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{-8}{3}$

57. (b)

a = 9

b = 16

I(P) = 5 and J(P) = 7

J(P) > I(P)

Now,

$a = \frac{1}{9}$ and $b = \frac{1}{16}$

$I(P) = \frac{5}{12}$ & $J(P) = \frac{7}{12}$

J(P) > I(P)

58. (a,b,c)

Given that

$\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$

$\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$

$\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k} \qquad \Rightarrow |\vec{\gamma}| = \sqrt{3}$

$\vec{\delta} = \vec{\alpha} + n\vec{\beta} = \left(\hat{i} + \hat{j} + \hat{k}\right) + n\left(\hat{i} - \hat{j} - \hat{k}\right)$

$\vec{\delta} = (1+n)\hat{i} + (1-n)\hat{j} + (1-n)\hat{k}$

Now,

Projection on $\vec{\delta} = \frac{1}{\sqrt{3}} = \frac{\vec{\delta}.\vec{\gamma}}{|\vec{\gamma}|}$

$\Rightarrow \frac{\left[(1+n)\hat{i} + (1-n)\hat{j} + (1-n)\hat{k}\right].\left(-\hat{i} + \hat{j} - \hat{k}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{|-1-n+1-n-1+n|}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow |n + 1| = 1$

$\Rightarrow n = 0$ or $n = -2$

$\vec{\delta} = -\hat{i} + 3\hat{j} + 3\hat{k}$

59. (c)

$\vec{\alpha}.\vec{\beta} = \vec{\alpha}.\vec{\gamma}$

Thus, α is perpendicular to b and c.

A unit vector perpendicular to b and c

$= \pm \frac{\left(\vec{\beta} \times \vec{\gamma}\right)}{\left|\vec{\beta} \times \vec{\gamma}\right|}$

$= \pm \frac{\left(\vec{\beta} \times \vec{\gamma}\right)}{\left|\vec{\beta}\right| \left|\vec{\gamma}\right| \sin \frac{\pi}{6}}$

$= \pm \frac{\left(\vec{\beta} \times \vec{\gamma}\right)}{1/2}$

$= \pm 2\left(\vec{\beta} \times \vec{\gamma}\right)$

60. (d)

$z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$R_e(z_1) > 0 \Rightarrow x_1 > 0$ and $I_m(z_2) < 0 \Rightarrow y_2 < 0$

$|z_1| = |z_2| \Rightarrow |z_1|^2 = |z_2|^2 \Rightarrow \ z_1\bar{z}_1 = z_2\bar{z}_2$

Now,

$\left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \overline{\left(\frac{z_1 + z_2}{z_1 - z_2}\right)}$

$= \left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2}\right)$

$$=\frac{z_1\overline{z}+z_2\overline{z}_1-z_1\overline{z}_2-z_2\overline{z}_2+z_1\overline{z}_1+z_1\overline{z}_2-z_2\overline{z}_1-z_2\overline{z}_1}{\left(z_1-z_2\right)\left(\overline{z}_1-\overline{z}_2\right)}$$

$$=\frac{2\left(\left|z_1\right|^2-\left|z_2\right|^2\right)}{\left(z_1-z_2\right)\left(\overline{z}_1-\overline{z}_2\right)}=0 \qquad \{\because |z_1|^2=|z_2|^2\}$$

$\Rightarrow \dfrac{z_1+z_2}{z_1-z_2}$ is purely imaginary

61. (a)

1, 2, 3, 4, 5, ..., 19, 20

There are 17 ways for four consecutive number

Number ways = $^{20}C_4 - 17$

$$= \frac{20\times19\times18\times17}{1\times2\times3\times4} - 17$$

$= 285 \times 17 - 17$

$= 284 \times 17$

62. (b)

$$\text{Let } A = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}^n$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^n$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

63. (d)

$P = \{(x, y) \leftarrow N \times N : 2x + y = 41\}$

For reflexive relation

$\Rightarrow xRx \Rightarrow 2x + x = 41 \Rightarrow x = \frac{41}{3} \notin N$ (not reflexive)

For symmetric

$\Rightarrow xRy \Rightarrow 2x + y = 41 \neq yRx$ (not symmetric)

For transitive

$\Rightarrow xRy \Rightarrow 2x + y = 41$ and $yRz \Rightarrow 2y + z = 41$ $x \not R z$ (not transitive)

64. (a)

$$f(x)=\begin{vmatrix}(1+x)^a & (2+x)^b & 1\\ 1 & (1+x)^a & (2+x)^b\\ (2+x)^b & 1 & (1+x)^a\end{vmatrix}$$

For constant term [put x = 0]

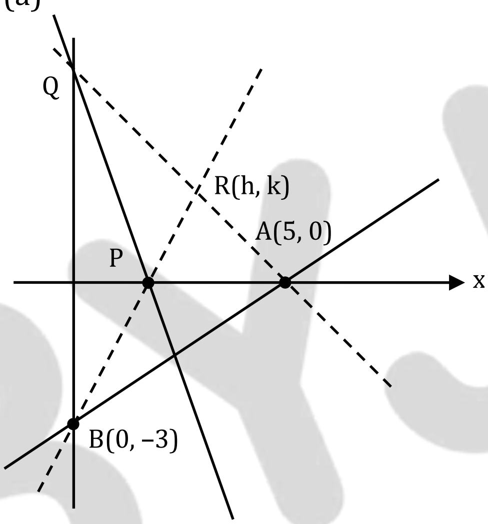
$$f(x)=\begin{vmatrix}1 & 2^b & 1\\ 1 & 1 & 2^b\\ 2^b & 1 & 1\end{vmatrix}$$

$f(x) = 1(1 - 2^b) - 2^b(1 - 2^{2b}) + 1(1 - 2^b)$

$f(x) = 1 - 2^b - 2^b + 2^{3b} + 1 - 2^b$

$f(x) = 2 - 3.2^b + 2^{3b}$

65. (a)



Line AB is $\frac{x}{5}+\frac{y}{-3}=1 \quad \Rightarrow 3x - 5y = 15$

Any perpendicular line to AB

$5x + 3y = \lambda \qquad$ So $P\left(\frac{\lambda}{5},0\right)$, $Q\left(0,\frac{\lambda}{3}\right)$

AQ is $\frac{x}{5}+\frac{y}{\lambda/3}=1$

$\Rightarrow \frac{3y}{\lambda}=1-\frac{x}{5}$

$\Rightarrow \frac{1}{\lambda}=\frac{1}{3y}\left(1-\frac{x}{5}\right) \qquad$ .....(i)

And BP is $\frac{x}{\lambda/5} - \frac{y}{3} = 1$

$\Rightarrow \frac{5x}{\lambda} = 1 + \frac{y}{3}$

$\Rightarrow \frac{1}{\lambda} = \frac{1}{5x}\left(1 + \frac{y}{3}\right)$ .....(ii)

Equation (i) = equation (ii)

$\Rightarrow \frac{1}{3y}\left(1 - \frac{x}{5}\right) = \frac{1}{5x}\left(1 + \frac{y}{3}\right)$

$\Rightarrow 5x\left(1 - \frac{x}{5}\right) = 3y\left(1 + \frac{y}{3}\right)$

$\Rightarrow 5x - x^2 = 3y + y^2$

$\Rightarrow x^2 + y^2 - 5x + 3y = 0$

66. (a,c)

Given

$$a_{ij} = \begin{cases} 0, & \text{for } i = j \\ 1, & \text{for } i > j \\ -1, & \text{for } i < j \end{cases}$$

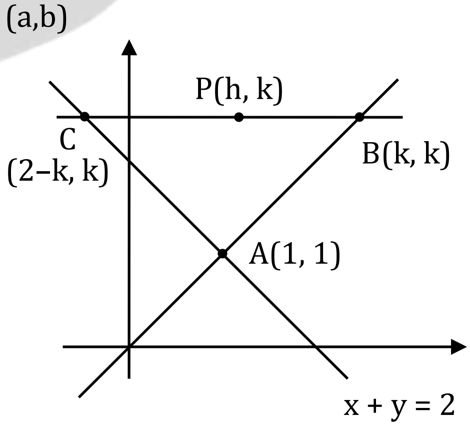
$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow \text{skew symmetric matrix}$$

$$|A| = \begin{vmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$|A| = 0 + 1 - 1$

$|A| = 0 \Rightarrow$ non invertible.

67. (a,b)



Area of triangle = h^2

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & k & k \\ 1 & 2-k & k \end{vmatrix} = \pm h^2$$

$\Rightarrow 1(k^2 - (2 - k)) - 1(k - k) + 1(2 - k - k) = \pm 2h^2$

$\Rightarrow k^2 - 2k + k^2 + 2 - 2k = \pm 2h^2$

$\Rightarrow 2k^2 - 4k + 2 = \pm 2h^2$

$\Rightarrow k^2 - 2k + 1 = \pm h^2$

Locus is $(k - 1)^2 = h^2 \Rightarrow y - 1 = \pm x$

$x - y + 1 = 0$ or $x + y = 1$

$x = y - 1$ or $x = -(y - 1)$

68. (b)

The length of transverse axis = $2\sin\theta = 2a$

$\therefore\ a = \sin\theta$

Also for ellipse, $3x^2 + 4y^2 = 12$

i.e. $\left(\frac{x^2}{4}\right) + \left(\frac{y^2}{3}\right) = 1$

$\therefore\ a^2 = 4$ & $b^2 = 3$

Now,

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{3}{4}} = \pm\frac{1}{2}$$

$\therefore$ focus of ellipse is = $(ae, 0)$ & $(-ae, 0)$

$\therefore$ focus = $(1, 0)$ and $(-1, 0)$

As the hyperbola if confocal

$\Rightarrow$ focus is same

And length of transverse axis = $2\sin\theta$

$\therefore$ length of semi transverse axis = $\sin\theta$

i.e. $A = \sin\theta$

And $C = 1$ where A, B, C are parameters in hyperbola similar to ellipse

$\therefore\ C^2 = A^2 + B^2$

$\therefore\ B^2 = 1 - \sin^2\theta = \cos^2\theta$

$\therefore$ Equation of hyperbola is $\left(\frac{x^2}{A^2}\right) - \left(\frac{y^2}{B^2}\right) = 1$

$$\Rightarrow \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

$$x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$$

69. (a,c)

$f(x) = \cos\left(\frac{\pi}{x}\right)$

Differentiate with respect to x

$f'(x) = -\sin\left(\frac{\pi}{x}\right).-\left(\frac{\pi}{x}\right)$

$f'(x) = \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right) > 0$

For increasing function f'(x) > 0

$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow (2k\pi) < \frac{\pi}{x} < (2k+1)\pi \qquad \Rightarrow \frac{1}{2k} > x > \frac{1}{2k+1}$

For decreasing function f'(x) < 0

$\sin\left(\frac{\pi}{x}\right) < 0$

$\Rightarrow \frac{\pi}{x} \in ((2k+1)\pi, (2k+2)\pi)$

$\Rightarrow \frac{\pi}{x} \in \left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

70. (c)

Given

$y = \log_a\left(x + \sqrt{x^2+1}\right)$

$a^y = x + \sqrt{x^2+1} \qquad$ .....(i)

Now,

$a^{-y} = \frac{1}{x+\sqrt{x^2+1}} \times \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x}$

$a^{-y} = \frac{\sqrt{x^2+1}-x}{x^2+1-x^2}$

$a^{-y} = \sqrt{x^2+1} - x$

Equation (i) – equation (ii)

$\Rightarrow a^y - a^{-y} = 2x$

$\Rightarrow x = \frac{a^y - a^{-y}}{2} \qquad \Rightarrow f^{-1}(y) = x = \frac{a^y - a^{-y}}{2}$

$f^{-1}(y) = \frac{e^{y\ell na} - e^{-y\ell na}}{2} \qquad \left\{\because \sin h x = \frac{e^x - e^{-x}}{2}\right\}$

$f^{-1}(y) = \sin h\,(y\ell na)$

71. (a,b,c,d)

We know that

$-1 < \cos 3x < 1$

$-x^3 < x^3 \cos 3x < x^3$

$$\frac{-x^3}{2+x^2} < \frac{x^3 \cos 3x}{2+x^2} < \frac{x^3}{2+x^2}$$

Taking integration from 0 to 1

$$\Rightarrow \int_0^1 -x^2 dx < I < \int_0^1 x^2 dx$$

$$\Rightarrow \left(\frac{-x^3}{3}\right)_0^1 < I < \left(\frac{x^3}{3}\right)_0^1$$

$$\Rightarrow -\frac{1}{3} < I < \frac{1}{3}$$

72. (c,d)

Given

$$\frac{dy}{dt} > \frac{dx}{dt} \qquad \text{.....(i)}$$

$12y = x^3$

Differentiate with respect to

$$\Rightarrow 12\frac{dy}{dt} = 3x^2 \frac{dx}{dt} \qquad \text{.....(ii)}$$

From equation (i)

$$3x^2 \frac{dx}{dt} > 12\frac{dx}{dt}$$

$\Rightarrow \quad x^2 - 4 > 0$

$\Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$

73. (b)

Given

C_1: $x^2 + y^2 = 2ax$

C_2: $y^2 = ax$

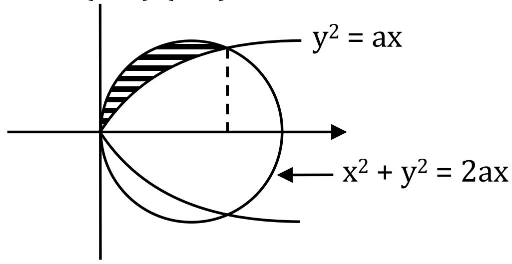
To find intersection points

$x^2 + (ax) = 2ax$

$x^2 = ax$

$x(x - a) = 0 \quad \Rightarrow \quad x = 0, a$

$\therefore \quad (0, 0)\ (a, a)$



$$\text{Area} = \frac{1}{4}(\text{area of circle}) - \int_0^a \sqrt{ax}\, dx$$

$$= \frac{1}{4}(\pi a^2) - \sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a$$

$$= \frac{\pi a^2}{4} - \frac{2a^2}{3}$$

$$= a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$$

74. (a,d)

Let $x^2 + ax + b = 0$ has roots α and β

$x^2 - cx + d = 0$ roots are α^4 and β^4

$\alpha + \beta = -a$ .....(i)

$\alpha\beta = b$ .....(ii)

$\alpha^4 + \beta^4 = c$ .....(iii)

$(\alpha\beta)^4 = d$ .....(iv)

From equation (ii) & (iv)

$b^4 = d$

And $\alpha^4 + \beta^4 = c$

$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = c$

$((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2 = c$

$\Rightarrow$ $(a^2 - 2b)^2 - 2b^2 = c \Rightarrow 2b^2 + c = (a^2 - 2b)^2$

$\Rightarrow$ $2b^2 - c = 4a^2b - a^2$

$\Rightarrow$ $2b^2 - c = a^2(4b - a^2)$

Now for equation

$x^2 - 4bx + 2b^2 - c = 0$

$D = (4b)^2 - 4(1)(2b^2 - c)$

$D = 16b^2 - 8b^2 + 4c$

$D = 4(2b^2 + c)$

$D = 4(a^2 - 2b)^2 > 0$ $\Rightarrow$ real roots

Now,

$f(0) = 2b^2 - c$

$f(0) = a^2(4b - a^2) < 0$ {since $a^2 > 4b$}

Roots are opposite in sign

75. (b,c)

Total students = 20

Number of ways = ${}^{20}C_2 \times 21$

$$= \frac{20 \times 19}{2} \times 2$$

$= 20 \times 19$

$= {}^{20}P_2$

WBJEE 2018 (Chemistry)

1. Ferric ion forms a Prussian blue precipitate due to the formation of
   a. $K_4[Fe(CN)_6]$
   b. $K_3[Fe(CN)_6]$
   c. $Fe(CNS)_3$
   d. $Fe_4\left[Fe(CN_6)\right]_3$

2. The nucleus $^{64}_{29}$Cu accepts an orbital electron to yield.
   a. $^{65}_{28}Ni$
   b. $^{64}_{30}Zn$
   c. $^{64}_{28}Ni$
   d. $^{65}_{30}Zn$

3. How many moles of electrons will weigh one kilogram?
   a. 6.023×10^{23}
   b. $\frac{1}{9.108}\times10^{31}$
   c. $\frac{6.023}{9.108}\times10^{54}$
   d. $\frac{1}{9.108\times6.023}\times10^{8}$

4. Equal weights of ethane and hydrogen are mixed in an empty container at 25°C. The fraction of total pressure exerted by hydrogen is
   a. 1: 2
   b. 1 : 1
   c. 1 : 16
   d. 15 : 16

5. The heat of neutralization of a strong base and a strong acid is 13.7 kcal. The heat released when 0.6 mole HCl solution is added to 0.25 mole of NaOH is
   a. 3.425 kcal
   b. 8.22 kcal
   c. 11.645 kcal
   d. 13.7 kcal

6. A compound formed by elements X and Y crystallizes in the cubic structure, where X atoms are at the corners of a cube and Y atoms are at the centres of the body. The formula of the compound is :

a. XY
b. XY_2
c. X_2Y_3
d. XY_3

7. What amount of electricity can deposit 1 mole of Al metal at cathode when passed through molten $AlCl_3$?

a. 0.3 F
b. 1 F
c. 3F
d. 1/3F

8. Given the standard half-cell potentials (E°) of the following as

$Zn = Zn^{2+} + 2e^-$ $\quad E^0 = +0.76V$

$Fe = Fe^{2+} + 2e^- \ E^0 = 0.41V$

Then the standard e.m.f. of the cell with the reaction $Fe^{2+} + Zn \rightarrow Zn^{2+} + Fe$ is

a. –0.35V
b. + 0.35 V
c. +1.17 V
d. –1.17 V

9. The following equilibrium constants are given :

$N_2 + 3H_2 \leftrightharpoons 2NH_3 ; K_1$

$N_2 + O_2 \leftrightharpoons 2NO ; K_2$

$H_2 + \frac{1}{2}O_2 \leftrightharpoons H_2O ; K_3$

The equilibrium constant for the oxidation of 2 mol of NH_3 to give NO is

a. $K_1 . \frac{K_2}{K_3}$
b. $K_2 . \frac{K_3^3}{K_1}$ c
c. $K_2 . \frac{K_3^2}{K_1}$
d. $K_2^2 . \frac{K_3}{K_1}$

10. Which one of the following is a condensation polymer?
 a. PVC
 b. Teflon
 c. Dacron
 d. Polystyrene

11. Which of the following is present in maximum amount in 'acid rain'?
 a. HNO_3
 b. H_2SO_4
 c. HCl
 d. H_2CO_3

12. Which of the set of oxides are arranged in the proper order of basic, amphoteric, acidic?
 a. SO_2, P_2O_5, CO
 b. BaO, Al_2O_3, SO_2
 c. CaO, SiO_2, Al_2O_3
 d. CO_2, Al2O_3, CO

13. Out of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one?
 a. $(n-1)d^8 ns^2$
 b. $(n-1)d^5 ns^2$
 c. $(n-1)d^3 ns^2$
 d. $(n-1)d^5 ns^1$

14. At room temperature, the reaction between water and fluorine produces
 a. HF and H_2O_2
 b. HF, O_2 and F_2O_2
 c. F^-, O_2 and H^+
 d. HOF and HF

15. Which of the following is least thermally stable?
 a. $MgCO_3$
 b. $CaCO_3$
 c. $SrCO_3$

d. $BeCO_3$

16. Cl_2O_7 is the anhydride of

a. HOCl

b. $HClO_2$

c. $HClO_3$

d. $HClO_4$

17. The main reason that SiCl4 is easily hydrolyzed as compared to CCl_4 is that

a. Si-Cl bond is weaker than C-Cl bond.

b. $SiCl_4$ can form hydrogen bonds.

c. $SiCl_4$ is covalent.

d. Si can extend its coordination number beyond four.

18. Silver chloride dissolves in excess of ammonium hydroxide solution. The cation present in the resulting solution is

a. $\left[Ag\left(NH_3\right)_6\right]^+$

b. $\left[Ag\left(NH_3\right)_4\right]^+$

c. Ag^+

d. $\left[Ag\left(NH_3\right)_2\right]^+$

19. The ease of hydrolysis in the compounds CH_3COCl(I), CH_3–CO–O–$COCH_3$(II), $CH_3COOC_2H_5$(III) and CH_3CONH_2(IV) is of the order

a. I > II > III > IV

b. IV > III > II > I

c. I > II > IV > III

d. II > I > IV > III

20. $CH_3-C\equiv C\,MgBr$ can be prepared by the reaction of

a. $CH_3-C\equiv C-Br$ with $MgBr_2$

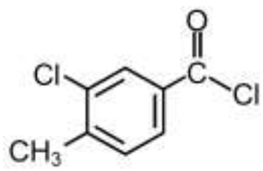
b. $CH_3-C\equiv CH$ with $MgBr_2$

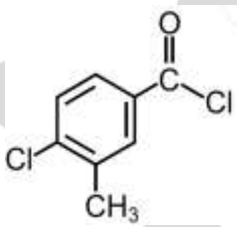
c. $CH_3-C\equiv CH$ with KBr and Mg metal

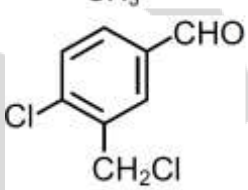
d. $CH_3-C\equiv CH$ with CH_3 MgBr

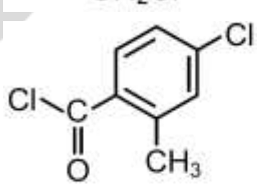
21. The number of alkene(s) which can produce-2-butanol by the successive treatment of (i) B_2H_6 in tetrahydrofuran solvent and (ii) alkaline H_2O_2 solution is
    - a. 1
    - b. 2
    - c. 3
    - d. 4

22. Identify **'M'** in the following sequence of reactions :

a. 

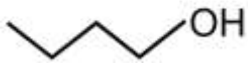
b. 

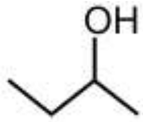
c. 

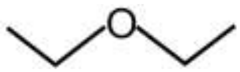
d. 

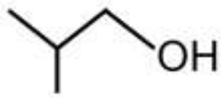
23. Methoxybenzene on treatment with HI produces :
    - a. Iodobenzene and methanol
    - b. Phenol and methyl iodide
    - c. Iodobenzene and methyl iodide
    - d. Phenol and methanol

24. $$\underset{N}{C_4H_{10}O} \xrightarrow[H_2SO_4]{K_2Cr_2O_7} C_4H_8O \xrightarrow[\text{warm}]{I_2/NaOH} CHI_3$$

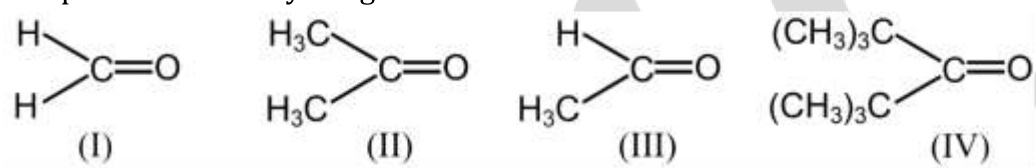
a. 

b. 

c. 

d. 

25. The correct order of reactivity for the addition reaction of the following carbonyl compounds with Ethyl Magnesium Iodide is :



a. I >III >II >IV
b. IV >III >II >I
c. I >II >IV >III
d. III >II >I >IV

26. If aniline is treated with conc. H2SO4 and heated at 200 °C, the product is

a. Anilinium sulphate
b. Benzenesulphonic acid
c. m-Aminobenzenesulphonic acid
d. Sulphanilic acid

27. Which of the following electronic configuration is not possible?

a. n = 3, l = 0, m = 0
b. n = 3, l = 1, m = – 1
c. n = 2, l = 0, m = –1
d. n = 2, l = 1, m = 0

28. The number of unpaired electrons in Ni (atomic number = 28) are
   a. 0
   b. 2
   c. 4
   d. 8

29. Which of the following has the strongest H-bond?
   a. O – H … S
   b. S – H … O
   c. F – H … F
   d. F – H … O

30. The half-life of C_{14} is 5760 years. For a "200" mg sample of C_{14}, the time taken to change to 25 mg is
   a. 11520 years
   b. 23040 years
   c. 5760 years
   d. 17280 years

31. During a reversible adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio $\frac{C_P}{C_v}$ for the gas is
   a. $\frac{3}{2}$
   b. $\frac{7}{2}$
   c. $\frac{5}{3}$
   d. $\frac{9}{7}$

32. $[X] + \text{dil.}H_2SO_4 \longrightarrow [Y]$: Colourless, suffocating gas
   $[Y] + K_2Cr_2O_7 + H_2SO_4 \longrightarrow$ Green colouration of solution
   Then, [X] and [Y] are
   a. SO_3^{2-}, SO_2
   b. Cl^-, HCl
   c. S^{2-}, H_2S
   d. CO_3^{2-}, CO_2

33. $[P]\xrightarrow{Br_2}C_2H_4Br_2\xrightarrow[NH_3]{NaNH_2}Q$

$[Q]\xrightarrow[H_g^{2+},\Delta]{20\%H_2SO_4}[R]\xrightarrow{Zn-Hg/HCl}[S]$

The species P, Q, R and S respectively are

a. ethene, ethyne, ethanal, ethane
b. ethane, ethyne, ethanal, ethene
c. ethene, ethyne, ethanal, ethanol
d. ethyne, ethane, ethene, ethanal

34. The number of possible organobromine compounds which can be obtained in the allylic bromination of 1–butene with N–bromosuccinimide is

a. 1
b. 2
c. 3
d. 4

35. A metal M (specific heat 0.16) forms a metal chloride with ≈ 65% chlorine present in it. The formula of the metal chloride will be

a. MCl
b. MCl_2
c. MCl_3
d. MCl4

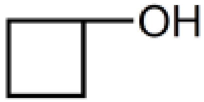
36. Among the following, the extensive variables are

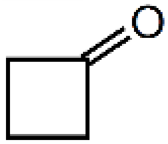
a. H (Enthalpy)
b. P (Pressure)
c. E (Internal energy)
d. V (Volume)

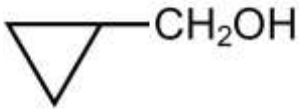
37. White phosphorus P_4 has the following characteristics :

a. 6 P – P single bonds
b. 4 P – P single bonds
c. 4 lone pair of electrons
d. P – P – P angle of 60°

38. The possible product (s) to be obtained from the reaction of cyclobutyl amine with HNO_2 is/are

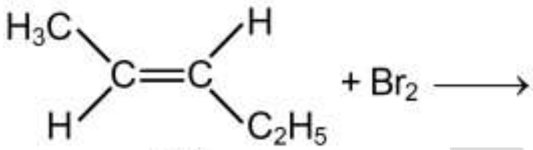
a. 

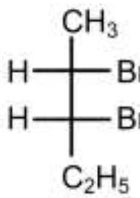
b. 

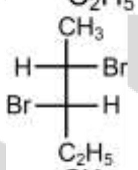
c. 

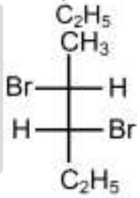
d. $H_2C = CH_2$.

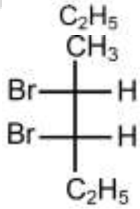
39. The major product(s) obtained in the following reaction is/are



a. 

b. 

c. 

d. 

40. Which statements are correct for the peroxide ion?

a. It has five completely filled anti-bonding molecular orbitals.
b. It is diamagnetic
c. It has bond order one.
d. It is isoelectronic with neon

WBJEE 2018 (Chemistry)

ANSWER KEYS

1. (d)	2. (c)	3. (d)	4. (d)	5. (a)	6. (a)	7. (c)	8. (b)	9. (b)	10. (c)
11. (b)	12. (b)	13.(b)	14.(c)	15. (d)	16. (d)	17. (d)	18.(d)	19. (a)	20. (d)
21. (a)	22. (b)	23.(b)	24.(b)	25. (a)	26. (d)	27. (c)	28.(b)	29. (c)	30. (d)
31. (a)	32. (a)	33.(a)	34.(d)	35. (b)	36. (a,c,d)	37. (a,c,d)	38.(a,c)	39. (a,d)	40. (b,c)

WBJEE 2018 (Chemistry)

Solution

1. (d)

Ferric ion forms Prussian blue precipitate due to formation of $Fe_4[Fe(CN)_6]_3$.

$Fe^{3+} + [Fe(CN)_6]^{-4} \longrightarrow \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3}$

2. (c)

$_{29}Cu^{64} + {-1}^{e^{\circ}} \longrightarrow {}_{28}Ni^{64}$

(k electron capture)

During k- capture mass number remain same but atomic number decrease by 1 unit for each capture.

3. (d)

Mass of one electron = 9.108×10^{-31} kg

So, mass of one mole of electron = $9.108 \times 10^{-31} \times 6.023 \times 10^{23}$ kg

Then no. of mole of e^- of in 1 kg

$$= \frac{1}{9.108 \times 6.023 \times 10^{-31} \times 10^{23}} = \frac{1}{9.108 \times 6.023} \times 10^{8} \text{ mole of } e^-$$

4. (d)

Given – weight of ethane = weight of hydrogen gas

	C_2H_6	H_2	
Initial weight =	W gm	W gm	
Mole =	$\frac{W}{30}$	$\frac{W}{2}$	$\left\{\text{Mole} = \frac{\text{mass}}{\text{M.wt}}\right\}$

We know that,

$P_{H_2} = X_{H_2}.P_{total}$

The fraction of total pressure exerted by hydrogen = $\frac{P_{H_2}}{P_{total}} = X_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{C_2H_6}}$

$$\frac{P_{H_2}}{P_{total}} = \frac{\frac{W}{2}}{\frac{W}{2} + \frac{W}{30}} = \frac{\frac{W}{2}}{\frac{15W + W}{30}} = \frac{W}{2} \times \frac{30}{16W}$$

$$\frac{P_{H_2}}{P_{total}} = \frac{15}{16}$$

5. (a)

$HCl + NaOH \longrightarrow NaCl + H_2O$; $\Delta H = -13.7$ kcal

1 mole 1 mole

Now, according to question,

$HCl + NaOH \longrightarrow NaCl + H_2O$

0.6 mole 0.25 mole

So, NaOH is limiting reagent,

0.6 – 0.25 0 0.25 mole

Then, release energy = 13.7 × 0.25 = 3.425 kcal

6. (a)

There are 8 corners in a cube. So, as the X atoms are at corners, so each of the X atom will contribute $\frac{1}{8}$ part towards a particular unit cell.

The no. of X atoms per unit cell = $8 \times \frac{1}{8} = 1$

Further, the Y atoms are at the centres of the body, so Y atom will contribute 1 part towards a particular unit cell.

The no. of Y atoms per unit cell = 1 × 1 = 1

So, the formula of the compound will be "XY".

7. (c)

$AlCl_3 \longrightarrow Al^{3+} + 3Cl^-$

When 1 mole of $AlCl_3$ dissociate, it gives 1 mole of Al^{3+} and 3 mole of Cl^- ions. The positively charged aluminium ions moves towards negative electrode or cathode and negatively charged chloride ions will move towards positively charged electrode or anode.

Reduction (on cathode):- $Al^{3+} + 3e^- \longrightarrow Al$

1 mole of aluminium ions will gain 3 mole of electrons to produce 1 mole of aluminium.

1 electron carries charge = 1.6×10^{-19} C

1 mole of electron carries charge = $1.6 \times 10^{-19} \times 6.023 \times 10^{23}$ C = 96500 C

and 95600 C = 1 faraday

So, as 1 mole of electrons contain 1 faraday. So, 3 mole of electrons carries 3 faraday of electricity.

8. (b)

Given $E^{o}_{Zn/Zn^{2+}} = +0.76\,V$

$E^{o}_{Fe/Fe^{2+}} = +0.41\,V$

The cell reaction –

$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$

At anode (oxidation) – $Zn \longrightarrow Zn^{2+} + 2e^{-}$; $E° = +0.76$ V

At cathode (reduction) – $Fe^{2+} + 2e^{-} \longrightarrow Fe$; $E° = -0.41$ V

E^{o}_{cell} = SOP of anode + SRP of cathode

= 0.76 + (–0.41) = 0.35 V

9. (b)

According to question,

$$2NH_3 + \frac{5}{2}O_2 \rightleftharpoons 2NO + 3H_2O$$

$$K = \frac{[NO]^2[H_2O]^3}{[NH_3]^2[O_2]^{5/2}}$$

According to question for given reaction,

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

$$K_1 = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$N_2 + O_2 \rightleftharpoons 2NO$$

$$K_2 = \frac{[NO]^2}{[N_2][O_2]}$$

$$H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O$$

$$K_3 = \frac{[H_2O]}{[H_2][O_2]^{1/2}}$$

So, on comparing values of K_1, K_2, K_3 with K

We will get

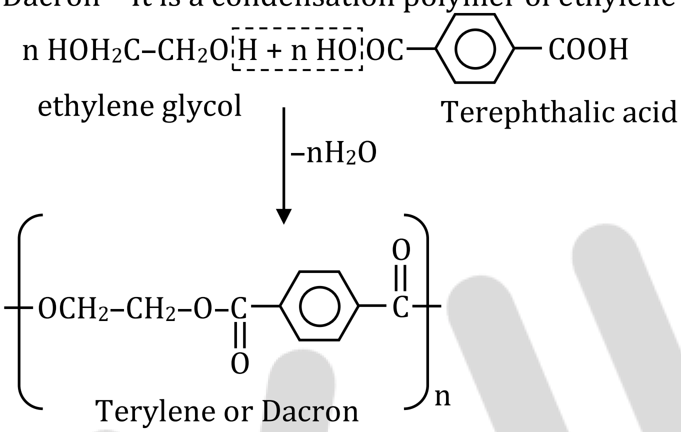
$$K = K_2 . \frac{K_3^3}{K_1} = \frac{[NO]^2}{[N_2][O_2]} \times \frac{[H_2O]^3}{[H_2]^3[O_2]^{3/2}} \times \frac{[N_2][H_2]^3}{[NH_3]^2}$$

$$K = K_2 . \frac{K_3^3}{K_1} = \frac{[NO]^2 [H_2O]^3}{[NH_3]^2 [O_2]^{5/2}}$$

10. (c)

Condensation polymers are any kind of polymers formed through a condensation reaction, where molecules join together and losing small molecules as by products such as water or methanol.

Dacron – It is a condensation polymer of ethylene glycol and terephthalic acid.



11. (b)

Sulphuric acid is the main constituent of acid rain because of following reaction.

$2SO_2(g) + O_2(g) + 2H_2O(\ell) \longrightarrow 2H_2SO_4(aq.)$

12. (b)

Basic	Amphoteric	Acidic
BaO	Al_2O_3	SO_2

13. (b)

All outer electronic configuration of element are below to d-block.

So, $(n - 1)d^5ns^2 \longrightarrow [Mn]$

Mn show highest oxidation state = +7

14. (c)

$$F_2 + H_2O \xrightarrow{\text{room temperature}} HF + O_2$$

$$\downarrow$$

$$H^+ + F^-$$

15. (d)

According to Fajan's rule,

Polarizing power $\propto$ covalent character $\propto \frac{1}{\text{size of cation}}$

And size of cation $\propto$ ionic character $\propto$ thermal stability

$$\xrightarrow{\quad BeCO_3 \quad MgCO_3 \quad CaCO_3 \quad SrCO_3 \quad}$$

size of cation $\uparrow$ ionic character $\uparrow$ thermal stability $\uparrow$

So, least thermal stable is $BeCO_3$.

16. (d)

$Cl_2O_7 + H_2O \longrightarrow 2HClO_4$ (perchloric acid)

Thus, Cl_2O_7 is an anhydride of perchloric acid.

17. (d)

In $SiCl_4$ vacant d-orbitals are present but in CCl_4, C does not have vacant d-orbital. So by hydrolysed SI form co-ordination bond and Si expand its co-ordination number beyond four.

18. (d)

$AgCl + NH_4OH \longrightarrow [Ag(NH_3)_2]^+ Cl^- + H_2O$

Silver chloride dissolves in excess of ammonium hydroxide solution to $[Ag(NH_3)_2]^+$ and Cl^-.

So the cation is $[Ag(NH_3)_2]^+$.

19. (a)

Reactivity towards hydrolysis of acid derivation is

Acid halide > Anhydride > Ester > Amide

So, $\underset{(I)}{CH_3COCl} > \underset{(II)}{CH_3{-}CO{-}O{-}COCH_3} > \underset{(III)}{CH_3COOC_2H_5} > \underset{(IV)}{CH_3CONH_2}$

20. (d)

$\underset{\text{(grignard reagent)}}{CH_3{-}C{\equiv}C{-}H} + CH_3MgBr \longrightarrow CH_3{-}C{\equiv}MgBr + CH_4$

21. (a)

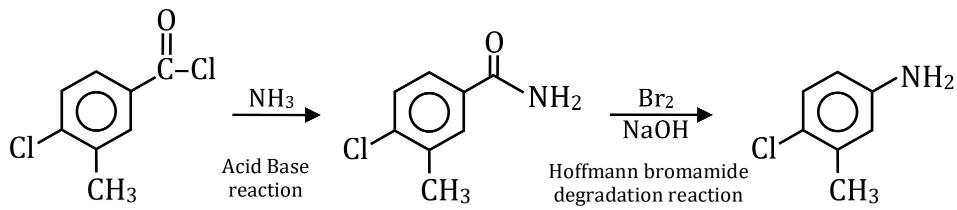
$$\underset{\text{(alkene)}}{CH_3{-}\overset{H}{\overset{|}{C}}{=}\overset{H}{\overset{|}{C}}{-}CH_3} \xrightarrow[\text{(ii) } H_2O_2/OH^-]{\text{(i) } B_2H_6} CH_3{-}\underset{H}{\underset{|}{CH}}{-}CH_2{-}CH_3$$

butan-2-ol

or

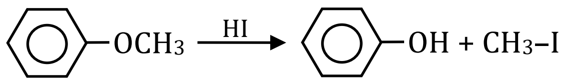
2-butanol

22. (b)



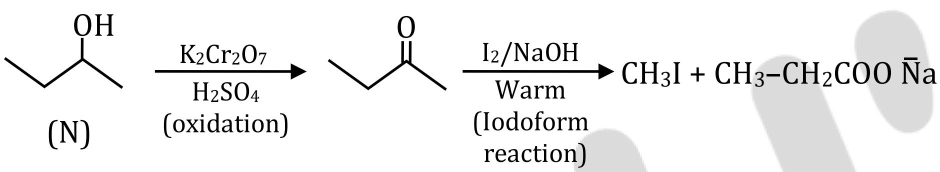
(M)

23. (b)



Methoxy benzene

24. (b)



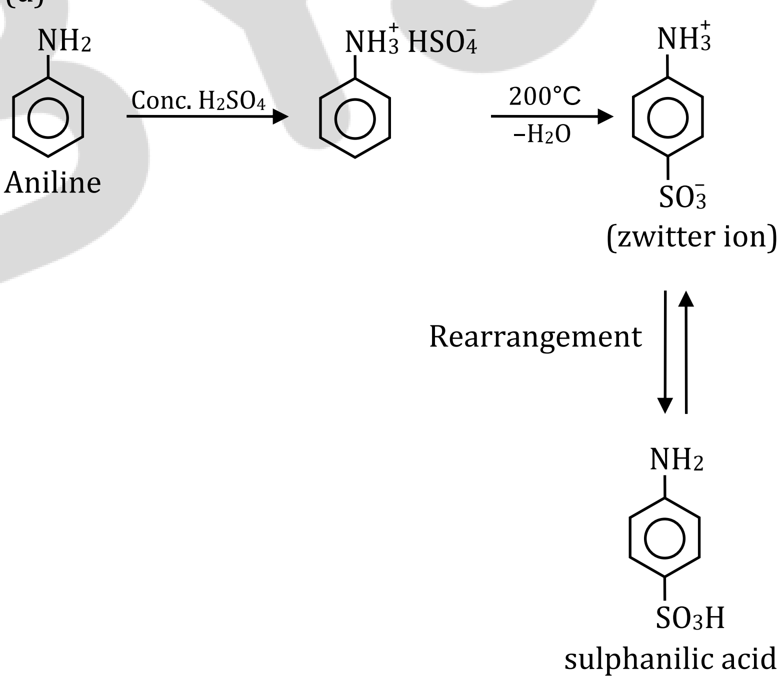
25. (a)

$$\text{Reactivity} \propto \frac{1}{\text{steric crowding}}$$

So, order of nucleophilic addition reaction is,

$$\begin{matrix} H \\ H \end{matrix}\!\!>\!C=O \;>\; \begin{matrix} CH_3 \\ H \end{matrix}\!\!>\!C=O \;>\; \begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>\!C=O \;>\; \begin{matrix} (CH_3)_3C \\ (CH_3)_3C \end{matrix}\!\!>\!C=O$$

26. (d)



27. (c)

We know that,

if n = a then ℓ = 0 to a – 1 and m = $-\ell$ to $+\ell$

(A) n = 3, ℓ = 0, m = 0, 3S

(B) n = 3, ℓ = 1, m = 1, 3P

(C) n = 2, ℓ = 0, m = –1, (not possible)

(D) n = 2, ℓ = 1, m = 0, 3P

28. (b)

$_{28}Ni \longrightarrow [Ar]3d^8 4s^2$

↿⇂	↿⇂	↿⇂	↿	↿

3d

↿⇂

4s

No. of unpaired electrons = 2

29. (c)

Due to highest electronegativity of F, it is more polar, so it is formed strongest H–bonding.

$.....\overset{\delta-}{F}—\overset{\delta+}{H}.....\overset{\delta-}{F}—\overset{\delta+}{H}.....$

30. (d)

Time taken = 17280 years

200 mg of C

$\downarrow t_{1/2}$ = 5760 years

100 mg of C

$\downarrow t_{1/2}$ = 5760 years

50 mg of C

$\downarrow t_{1/2}$ = 5760 years

25 mg of C

It takes 3 half–life to reduce 200 mg of C^{14} to 25 mg i.e. = 3 × 5760 = 17280 years.

31. (a)

$PT^{\frac{\gamma}{1-\gamma}}$ = constant, for reversible adiabatic process.

Given –

$P \propto T^3$

PT^{-3} = constant

On comparing,

$$\frac{\gamma}{1-\gamma} = -3$$

$\gamma = -3 + 3\gamma \Rightarrow 2\gamma = 3$

So, $\gamma = \dfrac{3}{2}$

And we know that $\gamma = \dfrac{C_P}{C_V}$

So, $\dfrac{C_P}{C_V} = \dfrac{3}{2}$

32. (a)

$SO_3^{2-} + \text{dil. } H_2SO_4 \longrightarrow SO_2(g)$: colourless, suffocating gas

[X] [Y]

$3SO_2(g) + K_2Cr_2O_7 + H_2SO_4 \longrightarrow Cr_2(SO_4)_3 + K_2SO_4 + H_2O$

[Y] (green colouration of solution)

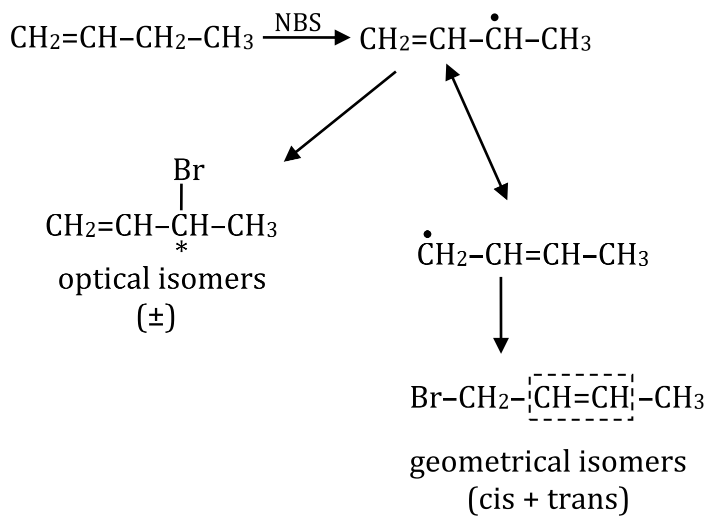
33. (a)

$$\underset{\text{(ethene) [P]}}{CH_2{=}CH_2} \xrightarrow{Br_2} Br{-}CH_2{-}CH_2{-}Br \xrightarrow[NH_3]{NaNH_2} \overset{\text{(ethyne)}}{HC{\equiv}CH} \text{ [Q]}$$

$$\downarrow Hg^{2+}, \Delta \;\; 20\% \; H_2SO_4$$

$$\text{[S] } \underset{\text{(ethane)}}{CH_3{-}CH_3} \xleftarrow[\text{(clemmensen reduction)}]{Zn{-}Hg/HCl} \underset{\text{(ethanal)}}{CH_3{-}\overset{\overset{O}{\|}}{C}{-}H} \text{ [R]}$$

34. (d)



So, total number of possible organobromine compound = 4

35. (b)

Let, the formula of metal chloride is MCl_x then,

We know that,

$$\text{Atomic weight of metal} = \frac{6.4}{\text{specific heat}}$$

$$\text{Atomic weight of metal} = \frac{6.4}{0.16} = 40$$

According to question –

% age of Cl = 65% in metal chloride

$$\text{\% age of Cl} = \frac{\text{mass of Cl in compound}}{\text{Total mass of compound}} \times 100$$

$$65 = \frac{35.5 \times x}{40 + 35.5x} \times 100 \qquad \Rightarrow x = 2.09 \approx 2$$

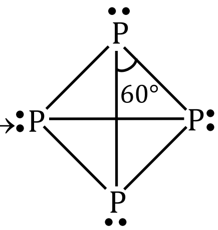
x = 2 approx.

So, metal chloride $MCl_x = MCl_2$

36. (a,c,d)

Extensive variable $\longrightarrow$ H(enthalpy), E(Internal energy) and V(volume), Since variable depends on the amount of substance or volume or size of the system.

37. (a,c,d)

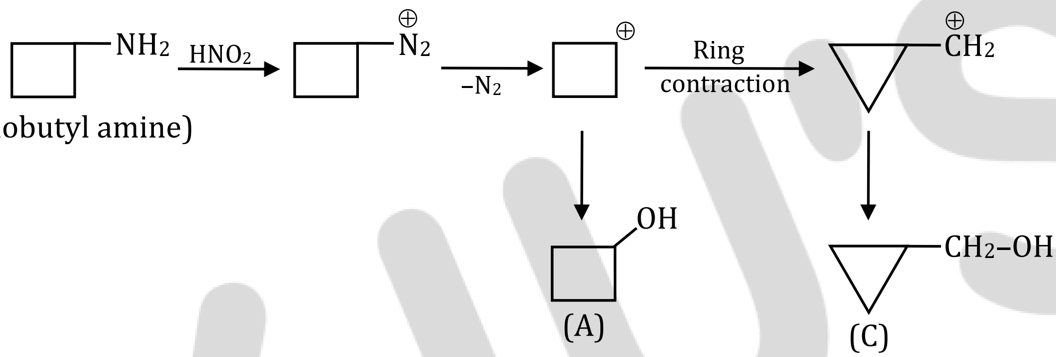
White phosphorus (P_4) ⟶ 

(A) 6 P–P single bond

(B) 4 lone pair of electrons

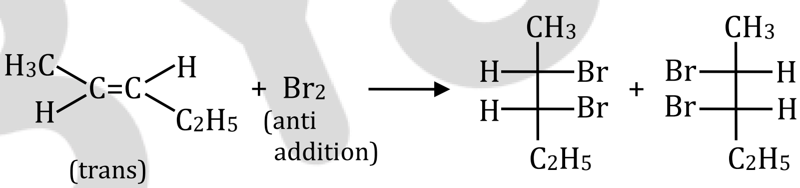
(D) Bond angle < P–P–P = 60°

38. (a,c)



(Cyclobutyl amine)

39. (a,d)



(pair of enantiomers)

40. (b,c)

$O_2^{2-} \longrightarrow$ (Peroxide ion) = $18e^-$

By MOT

$\sigma_{1s}^2, \sigma^{*2}_{1s}, \sigma_{2s}^2, \sigma^{*2}_{2s}, \sigma_{2P_z}^2 \left[\pi_{2P_x}^2 = \pi_{2P_y}^2\right], \left[\pi^{*2}_{2P_x} = \pi^{*2}_{2P_y}\right], \sigma^{*0}_{2P_z}$

B.O. = $\frac{N_b - N_a}{2}$,N_b = No. of bonding e^- and N_a = No. of antibonding e^-

B.O. = $\frac{10-8}{2} = 1$

No. of unpaired electron = 0 (diamagnetic)

WBJEE-2018 (Physics)

Category - I (Q.1 to Q.30)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

1. The velocity (v) of a particle (under a force F) depends on its distance (x) from the origin (with x > 0) $v \propto \frac{1}{\sqrt{x}}$. Find how the magnitude of the force (F) on the particle depends on x.

   a. $F \propto \frac{1}{x^{3/2}}$

   b. $F \propto \frac{1}{x}$

   c. $F \propto \frac{1}{x^2}$

   d. $F \propto x$

2. The ratio of accelerations due to gravity $g_1 : g_2$ on the surfaces of two planets is 5 : 2 and the ratio of their respective average densities $\rho_1 : \rho_2$ is 2 : 1. What is the ratio of respective escape velocities $v_1 : v_2$ from the surface of the plants ?

   a. 5 : 2

   b. $\sqrt{5} : \sqrt{2}$

   c. $5 : 2\sqrt{2}$

   d. 25 : 4

3. A spherical liquid drop is placed on a horizontal plane. A small disturbance causes the volume of the drop to oscillate. The time period of oscillation (T) of the liquid drop depends on radius (r) of the drop, density (ρ) and surface tension (s) of the liquid. Which among the following will be a possible expression for T (where k is a dimensionless constant)?

   a. $k\sqrt{\frac{\rho r}{s}}$

   b. $k\sqrt{\frac{\rho^2 r}{s}}$

   c. $k\sqrt{\frac{\rho r^3}{s}}$

   d. $\sqrt{\frac{\rho r^3}{s^2}}$

4. The stress along the length of a rod (with rectangular cross section) is 1% of the Young's modulus of its material. What is the approximate percentage of change of its volume? (Poisson's ratio of the material of the rod is 0.3)
   a. 3%
   b. 1%
   c. 0.7%
   d. 0.4%

5. What will be the approximate terminal velocity of a rain drop of diameter 1.8×10^{-3} m, when density of rain water ≈103 kgm^{-3} and the co-efficient of viscosity of air $\approx 1.8 \times 10^{-5}$ Nsm^{-2}? (Neglect buoyancy of air).
   a. 48 ms^{-1}
   b. 98 ms^{-1}
   c. 392 ms^{-1}
   d. 980 ms^{-1}

6. The water equivalent of a calorimeter is 10 g and it contains 50 g of water at 15°C. Some amount of ice, initially at −10°C is dropped in it and half of the ice melts till equilibrium is reached. What was the initial amount of ice that was dropped (when specific heat of ice = 0.5 cal $gm^{-1}°C^{-1}$, specific heat of water = 1.0 cal $gm^{-1}°C^{-1}$ and latent heat of melting of ice = 80 cal gm^{-1})?
   a. 10 g
   b. 18 g
   c. 20 g
   d. 30 g

7. One mole of a mono-atomic ideal gas undergoes a quasi-static process, which is depicted by a straight line joining points (V_0, T_0) and ($2V_0$, $3T_0$) in a V-T diagram. What is the value of the heat capacity of the gas at the point (V_0, T_0)?
   a. R
   b. $\frac{3}{2}$R
   c. 2R
   d. 0

8. For an ideal gas with initial pressure and volume P_i and V_i, respectively, a reversible isothermal expansion happens, when its volume becomes V_0. Then it is compressed to its original volume V_i by a reversible adiabatic process. If the final pressure is P_f then which of the following statements is true?

   a. $P_f = P_i$

   b. $P_f > P_i$

   c. $P_f < P_i$

   d. $\frac{P_f}{V_o} = \frac{P_i}{V_i}$

9. A point charge – q is carried from a point A to another point B on the axis of a charged ring of radius 'r' carrying a charge +q. If the point A is at a distance $\frac{4}{3}$ r from the centre of the ring and the point B is $\frac{3}{4}$ r from the centre but on the opposite side, what is the net work that needs to be done for this?

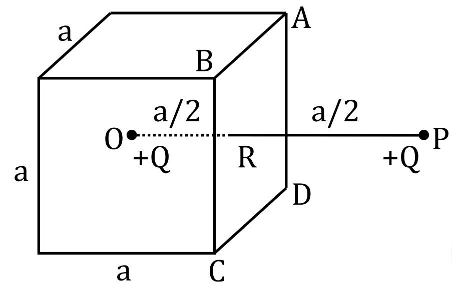
   a. $-\frac{7}{5}\frac{q^2}{4\pi\varepsilon_o r}$

   b. $-\frac{1}{5}\frac{q^2}{4\pi\varepsilon_o r}$

   c. $\frac{7}{5}\frac{q^2}{4\pi\varepsilon_o r}$

   d. $\frac{1}{5}\frac{q^2}{4\pi\varepsilon_o r}$

10. Consider a region in free space bounded by the surfaces of an imaginary cube having sides of length 'a' as shown in the diagram. A charge +Q is placed at the centre 'O' of the cube. P is such a point outside the cube that the line OP perpendicularly intersects the surface ABCD at R and also OR = RP = a/2. A charge +Q is placed at point P also. What is the total electric flux through the five faces of the cube other than ABCD?



a. $\frac{Q}{\varepsilon_o}$

b. $\frac{5Q}{6\varepsilon_o}$

c. $\frac{10Q}{6\varepsilon_o}$

d. Zero

11. Four equal charges of value +Q are placed at any four vertices of a regular hexagon of side 'a'. By suitable choosing the vertices, what can be the maximum possible magnitude of electric field at the centre of the hexagon?

a. $\frac{Q}{4\pi\varepsilon_o a^2}$

b. $\frac{\sqrt{2}Q}{4\pi\varepsilon_o a^2}$

c. $\frac{\sqrt{3}Q}{4\pi\varepsilon_o a^2}$

d. $\frac{2Q}{4\pi\varepsilon_o a^2}$

12. A proton of mass 'm' moving with a speed v (<< c, velocity of light in vacuum) completes a circular orbit in time 'T' in a uniform magnetic field. If the speed of the proton is increased to 2 v, what will be time needed to complete the circular orbit?

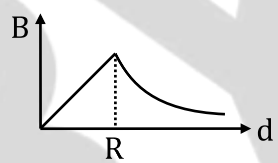
a. $\sqrt{2}\,T$

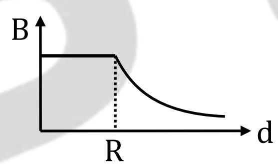
b. T

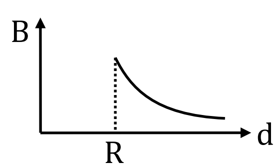
c. $\frac{T}{\sqrt{2}}$

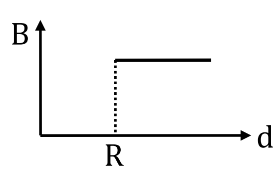
d. $\frac{T}{2}$

13. A uniform current is flowing along the length of an infinite, straight, thin, hollow cylinder of radius 'R'. The magnetic field 'B' produced at a perpendicular distance'd' from the axis of the cylinder is plotted in a graph. Which of the following figures looks like the plot?

a. 

b. 

c. 

d. 

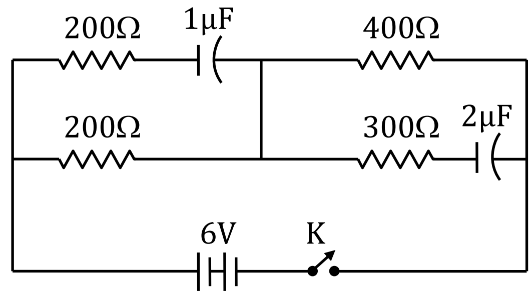
14. A circular loop of radius 'r' of conducting wire connected with a voltage source of zero internal resistance produces a magnetic field 'B' at its centre. If instead, a circular loop of radius '2r' made of same material, having the same cross section is connected to the same voltage source, what will be the magnetic field at its centre?
    a. $\frac{B}{2}$
    b. $\frac{B}{4}$
    c. 2B
    d. B

15. An alternating current is flowing through a series LCR circuit. It is found that the current reaches a value of 1 mA at both 200 Hz and 800 Hz frequency. What is the Resonance frequency of the circuit?
    a. 600 Hz
    b. 300 Hz
    c. 500 Hz
    d. 400 Hz

16. An electric bulb, a capacitor, battery and a switch are all in series in a circuit. How does the intensity of light vary when the switch is turned on?
    a. Continues to increase gradually.
    b. Gradually increases for some time and then becomes steady.
    c. Sharply rises initially and then gradually decreases.
    d. Gradually increases for some time and then gradually decreases.

17. Four resistors, 100Ω, 200 Ω, 300 Ω, and 400 Ω, are connected to form four sides of a square. The resistors can be connected in any order. What is the maximum possible equivalent resistance across the diagonal of the square?
    a. 210 Ω
    b. 240 Ω
    c. 300 Ω
    d. 250 Ω

18. What will be current through the 210 Ωresistor in the given circuit a long time after the switch 'K' is made on?

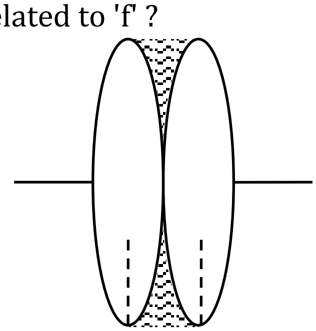


a. Zero
b. 100 mA
c. 10 mA
d. 1 mA

19. A point source is placed at co-ordinates (0, 1) in X-Y plane. A ray of light from the source is reflected on a plane along the X-axis and perpendicular to the X-Y plane. The reflected ray passes through the point (3, 3). What is the path length of the ray from (0, 1) to (3, 3)?

a. 5
b. $\sqrt{13}$
c. $2\sqrt{13}$
d. $1+2\sqrt{13}$

20. Two identical equi-convex lenses, each of focal length 'f' are placed side by side in contact with each other with a layer of water in between them as shown in the figure. If refractive index of the material of the lenses is greater then that of water, how the combined focal length 'F' is related to 'f' ?



a. F > f
b. $\frac{f}{2} < F < f$
c. $F < \frac{f}{2}$
d. F = f

21. There is a small air bubble at the centre of a solid glass sphere of radius 'r' and refractive index 'μ'. What will be the apparent distance of the bubble from the centre of the sphere, when viewed from outside?

a. r

b. $\dfrac{r}{\mu}$

c. $r\left(1-\dfrac{1}{\mu}\right)$

d. Zero

22. If Young's double slit experiment is done with white light, which of the following statements will be true?

a. All the bright fringes will be coloured.

b. All the bright fringes will be white.

c. The central fringe will be white.

d. No stable interference pattern will be white.

23. How the linear velocity 'v' of an electron in the Bohr orbit is related to its quantum number 'n'?

a. $v \propto \dfrac{1}{n}$

b. $v \propto \dfrac{1}{n^2}$

c. $v \propto \dfrac{1}{\sqrt{n}}$

d. $v \propto n$

24. If the half life of a radioactive nucleus is 3 days, nearly what fraction of the initial number of nuclei will decay on the 3rd day? (Given that $\sqrt[3]{0.25} \approx 0.63$)

a. 0.63

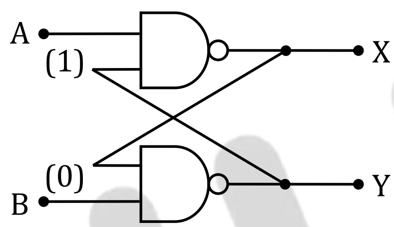
b. 0.5

c. 0.37

d. 0.13

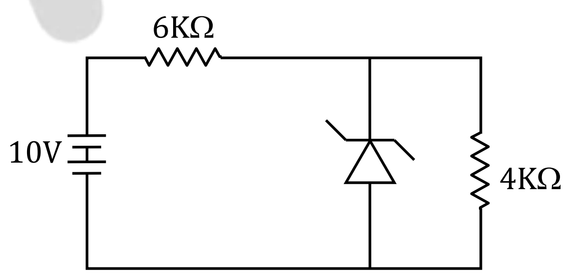
25. An electron accelerated through a potential of 10,000 V from rest has a de-Broglie wave length 'λ'. What should be the accelerating potential so that the wave length is doubled?
    a. 20,000 V
    b. 40,000 V
    c. 5,000 V
    d. 2,500 V

26. In the circuit shown, inputs A and B are in states ‘1’ and ‘0’ respectively. What is the only possible stable state of the outputs ‘X’ and ‘Y’?



    a. X = ‘1’, ‘Y’ = ‘1’
    b. X = ‘1’, ‘Y’ = ‘0’
    c. X = ‘0’, ‘Y’ = ‘1’
    d. X = ‘0’, ‘Y’ = ‘0’

27. What will be the current flowing through the 6KΩ resistor in the circuit shown, where the breakdown voltage of the Zener is 6 V?



    a. $\frac{2}{3}$ mA
    b. 1 mA
    c. 10 mA
    d. $\frac{3}{2}$ mA

28. In case of a simple harmonic motion, if the velocity is plotted along the X-axis and the displacement (from the equilibrium position) (is plotted along the Y-axis, the resultant curve happens to be an ellipse with the ratio:

$$\frac{\text{major axis (along X)}}{\text{minor axis (along Y)}} = 20\pi$$

What is the frequency of the simple harmonic motion?

a. 100 Hz
b. 20 Hz
c. 10 Hz
d. $\frac{1}{10}$ Hz

29. A block of mass m_2 is placed on a horizontal table and another block of mass m_1 is placed on top of it. An increasing horizontal force $F = \alpha t$ is exerted on the upper block but the lower block never moves as a result. If the co-efficient of friction between the blocks is μ_1 and that between the lower block and the table is μ_2, then what is the maximum possible value of μ_1/μ_2?

a. $\frac{m_2}{m_1}$
b. $1+\frac{m_2}{m_1}$
c. $\frac{m_1}{m_2}$
d. $1+\frac{m_1}{m_2}$

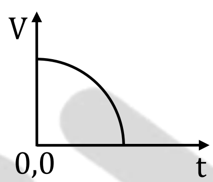
30. In a triangle ABC, the sides AB and AC are represented by the vectors $3\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ respectively. Calculate the angle $\angle ABC$.

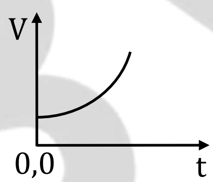
a. $\cos^{-1}\sqrt{\frac{5}{11}}$
b. $\cos^{-1}\sqrt{\frac{6}{11}}$
c. $\left(90^\circ - \cos^{-1}\sqrt{\frac{5}{11}}\right)$
d. $\left(180^\circ - \cos^{-1}\sqrt{\frac{5}{11}}\right)$

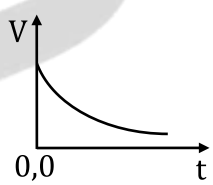
Category - II (Q.31 to Q.35)

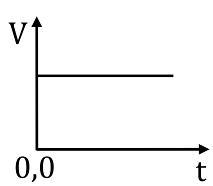
Carry 2 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

31. The insulated plates of a charged parallel plate capacitor (with small separation between the plates) are approaching each other due to electrostatic attraction. Assuming no other force to be operative and no radiation taking place, which of the following graphs approximately shows the variation with time (t) of the potential difference (V) between the plates?

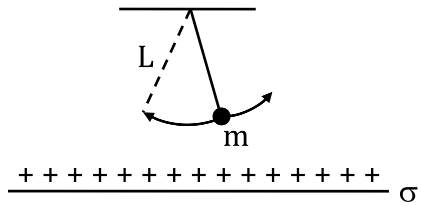
a. 

b. 

c. 

d. 

32. The bob of a pendulum of mass 'm', suspended by an inextensible string of length 'L' as shown in the figure carries a small charge 'a'. An infinite horizontal plane conductor with uniform surface charge density 'σ' is placed below it. What will be the time period of the pendulum for small amplitude oscillations



a. $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{mq}{\varepsilon_o\sigma}\right)}}$

b. $\sqrt{\dfrac{L}{\left(g-\dfrac{mq\sigma}{\varepsilon_o}\right)}}$

c. $\dfrac{1}{2\pi}\sqrt{\dfrac{L}{\left(g-\dfrac{q\sigma}{\varepsilon_o m}\right)}}$

d. $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{q\sigma}{\varepsilon_o m}\right)}}$

33. A light charged particle is revolving in a circle of radius 'r' in electrostatic attraction of a static heavy particle with opposite charge. How does the magnetic field 'B' at the centre of the circle due to the moving charge depend on 'r'?

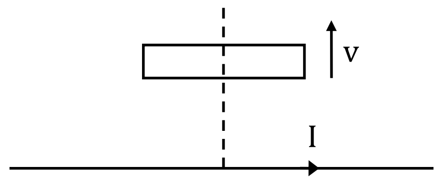
a. $B \propto \dfrac{1}{r}$

b. $B \propto \dfrac{1}{r^2}$

c. $B \propto \dfrac{1}{r^{3/2}}$

d. $B \propto \dfrac{1}{r^{5/2}}$

34. As shown in the figure, a rectangular loop of a conducting wire is moving away with a constant velocity 'v' in a perpendicular direction from a very long straight conductor carrying a steady current 'I'. When the breadth of the rectangular loop is very small compared to its distance from the straight conductor, how does the e.m.f. 'E' induced in the loop vary with time't'?



a. $E \propto \frac{1}{t^2}$

b. $E \propto \frac{1}{t}$

c. $E \propto \ln(t)$

d. $E \propto \frac{1}{t^3}$

35. A solid spherical ball and a hollow spherical ball of two different materials of densities ρ_1 and ρ_2 respectively have same outer radii and same mass. What will be the ratio of the moment of inertia (about an axis passing through the centre) of the hollow sphere to that of the solid sphere?

a. $\frac{\rho_2}{\rho_1}\left(1-\frac{\rho_2}{\rho_1}\right)^{5/3}$

b. $\frac{\rho_2}{\rho_1}\left[1-\left(1-\frac{\rho_2}{\rho_1}\right)^{5/3}\right]$

c. $\frac{\rho_2}{\rho_1}\left(1-\frac{\rho_1}{\rho_2}\right)^{5/3}$

d. $\frac{\rho_2}{\rho_1}\left[1-\left(1-\frac{\rho_1}{\rho_2}\right)^{5/3}\right]$

WBJEE-2018 (Physics)

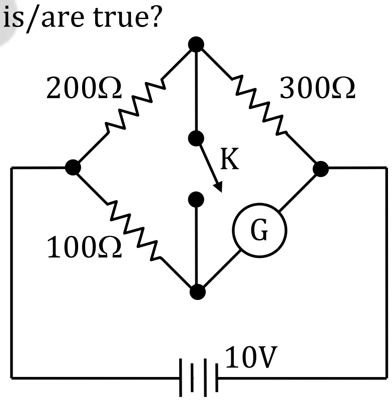
Category - III (Q.36 to Q.40)

Carry 2 mark each and one or more option(s) is/are correct. If all correct answers are not marked and also noincorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correctanswers. If any wrong option is marked or if any combination including a wrong option is marked, the answerwill considered wrong but there is no negative marking for the same and zero marks will be awarded.

36. Which of the following statement(s) is/are true?
"Internal energy of an ideal gas ……………."
   a. Decreases in an isobaric process.
   b. Remains constant in an isothermal process.
   c. Increases in an isobaric process.
   d. Decreases in an isobaric expansion.

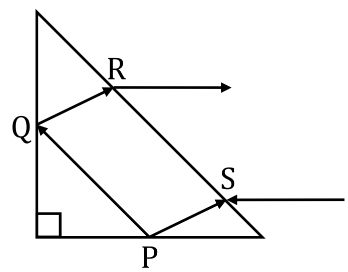
37. Two positive charges Q and 4Q are placed at points A and B respectively, where B is at a distance 'd' units to the right of A. The total electric potential due to these charges is minimum at P on the line through A and B, What is (are) the distance(s) of P from A?
   a. $\frac{d}{3}$ units to the right of A
   b. $\frac{d}{3}$ units to the left of A
   c. $\frac{d}{3}$ units to the right of A
   d. d units to the left of A

38. A non–zero current passes through the galvanometer G shown in the circuit when the key 'K' is closed and its value does not change when the key is opened. Then which of the following statement(s) is/are true?



   a. The galvanometer resistance is infinite.
   b. The current through the galvanometer is 40 mA.
   c. After the key is closed, the current through the 200Ω resistor is same as the current through the 300Ω resistor.
   d. The galvanometer resistance is 150Ω.

39. A ray of light is incident on a right angled isosceles prism parallel to its base as shown in the figure. Refractive index of the material of the prism is $\sqrt{2}$. Then which of the following statement(s) is/are true?



a. The reflection at P is total internal.

b. The reflection at Q is total internal.

c. The ray emerging at R is parallel to the ray incident at S.

d. Total deviation of the ray is 150°.

40. The intensity of a sound appears to an observer to be periodic. Which of the following can be the cause of it?

a. The intensity of the source is periodic.

b. The source is moving towards the observer.

c. The observer is moving away from the source.

d. The source is producing a sound composed of two nearby frequencies.

WBJEE-2018 (Physics)

ANSWER KEY

1. (c)	2. (c)	3. (c)	4. (d)	5. (b)	6. (c)	7. (c)	8. (b)	9. (b)	10. (a)
11. (c)	12. (b)	13. (c)	14. (b)	15. (d)	16. (c)	17. (d)	18. (c)	19. (a)	20. (b)
21. (d)	22. (c)	23. (a)	24. (d)	25. (c)	26. (c)	27. (a)	28. (c)	29. (b)	30. (a)
31. (a)	32. (d)	33. (d)	34. (a)	35. (d)	36. (b)	37. (a)	38. (bcd)	39. (ac)	40. (ad)

WBJEE-2018 (Physics)

SOLUTIONS

1. (c)

Given that, $v \propto \frac{1}{\sqrt{x}}$

On differentiating with respect to time 't'.

$$\Rightarrow \frac{dv}{dt} = \frac{1}{2x^{3/2}} \cdot \frac{dx}{dt} - \frac{dv}{dt} \propto \frac{1}{x^{3/2}} \times \frac{1}{x^{1/2}}$$

$$\Rightarrow \frac{dv}{dt} \times \frac{1}{x^2} \Rightarrow F \propto \frac{1}{x^2}$$

2. (c)

Given that, $\frac{g_1}{g_2} = \frac{5}{2}$

The ratio of their respective average densities $\frac{\rho_1}{\rho_2} = \frac{2}{1}$

Now, we have to calculate the ratio of respective escape velocities $v_1:v_2$ from the surface of planets.

We know that

$$-\frac{Gmm}{R} + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^2}R} = \sqrt{2gR}$$

$$\frac{\dfrac{G\rho_1 \times \frac{4}{3}\pi(R_1)^3}{(R_1)^2}}{\dfrac{G\rho_2 \times \frac{4}{3}\pi(R_2)^3}{(R_2)^2}} = \frac{5}{2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{5}{2} \times \frac{\rho_2}{\rho_1} = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1R_1}{g_2R_2}} = \sqrt{\frac{5}{2}\times\frac{5}{4}} = \frac{5}{2\sqrt{2}}$$

3. (c)

Now, according to the question, we can write the equation.

$T = Kr^x \rho^y s^z$

$T \to T$, $r \to L$, $\rho \to ML^{-3}$, $s \to MT^{-2}$

$S = \frac{F}{\ell} = \frac{ma}{\ell} = \frac{m}{s^2}$

$T^1 = KL^x (ML^{-2})^y (MT^{-2})^z$

$\Rightarrow T^1 = KL^{(x-3y)} M^{(y+z)} T^{(-2z)}$

$-2z = 1 \Rightarrow z = -\frac{1}{2}$, $y + z = 0$, $y = -z = +\frac{1}{2}$

$x - 3y = 0 \Rightarrow x = 3y = \frac{3}{2}$

$T = K6^{3/2}\rho^{1/2}s^{-1/2} = K\sqrt{\frac{\rho r^3}{S}}$

4. (d)

We know that,

$v = xyz$

$$\frac{dv}{v} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

$$\Rightarrow \frac{d\ell}{\ell} - \mu\frac{d\ell}{\ell} - \mu\frac{d\ell}{\ell} = (1 - 2\mu)\frac{d\ell}{\ell}$$

Now, we know that

Stress = y × strain

$0.01y = y\frac{d\ell}{\ell} \Rightarrow \frac{d\ell}{\ell} = 0.01$

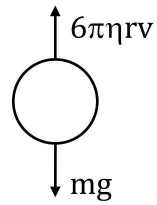
So, $\frac{dv}{v} = 0.01 \times (1 - 0.6) = 0.01 \times 0.4$

Percentage volume change is = 0.4%

5. (b)

We know that,

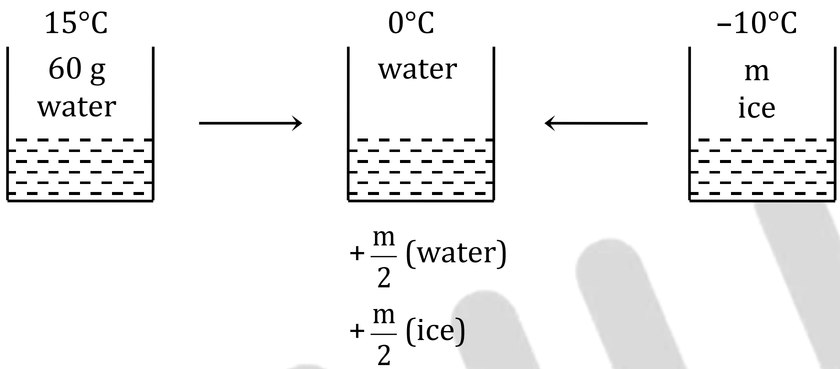
$6\pi\eta rv = \frac{4}{3}\pi r^3\rho g$



$$v = \frac{2}{9\eta r^2\rho g} = \frac{2\times0.9\times0.9\times10^{-6}\times10^{3}\times9.8}{9\times1.8\times10^{-5}}$$

$\Rightarrow \quad v = 9.8 \times 10^{-6} \times 10^{7} = 98$ m/sec

6. (c)

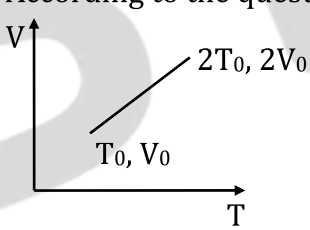


$m \times \frac{1}{2} \times 10 + \frac{m}{2} \times 80 = 60 \times 1 \times 15$

$m = \frac{60\times15}{45} = 20$ gm

7. (c)

According to the question, we can draw its (V–T) graph.



Now, we can write

$Pdv + nC_VdT = ncdT$

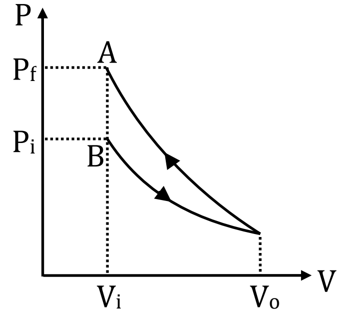
$\Rightarrow \quad C = C_V + \frac{p}{n}\frac{dv}{dT}$

At, V_o, T_o

$\frac{dv}{dT} = \frac{V_o}{2T_o}$, $P_oV_o = nRT_o$

$$C = \frac{3}{2}R + \frac{RT+0}{V_o}\times\frac{V_o}{2T_o} = \frac{3}{2}R + \frac{R}{2} = 2R$$

8. (b)
Now, according to the question, we can draw (P-V) graph.



There is reversible adiabatic process in the curve AC, and isothermal process in the curve BC. Now, from the graph we can say that $p_f > p_i$

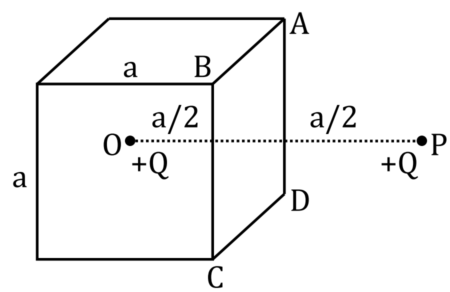
9. (b)
We know that,
$W = U_B = U_A$

$$\Rightarrow \quad W = -\frac{Kqq}{\sqrt{r^2 + \frac{9}{16}r^2}} + \frac{Kq^2}{\sqrt{r^2 + \frac{16}{9}r^2}}$$

$$W = -\frac{4Kq^2}{5r} + \frac{3Kq^2}{5r} = -\frac{Kq^2}{5r}$$

$$W = -\frac{1}{5}\frac{q^2}{4\pi\varepsilon_o r}$$

10. (a)
Flux due to charge at 0



Now, for five faces of cube other than ABCD,

$$\Rightarrow \quad \phi_1 = 5 \times \frac{\theta}{6\varepsilon_o}$$

Flux due to charge at P, through face ABCD will be equal to through all other five faces.

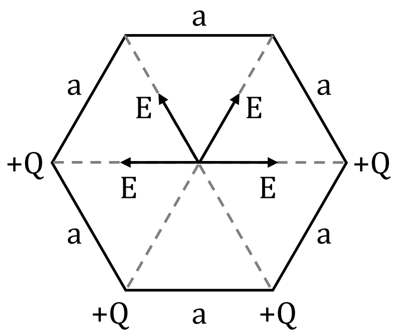
$$\Rightarrow \quad \phi_2 = \frac{\theta}{6\varepsilon_o}$$

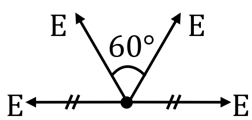
$$\Rightarrow \quad \phi_{net} = \phi_1 + \phi_2 = \frac{\theta}{\varepsilon_o}$$

11. (c)

Now, according to the questions, we will draw a regular hexagon of side 'a'.

According to the question, on distributing four charges at any four vertices of a regular hexagon of side 'a'.





$$|\vec{E}| = \frac{Kq}{a^2}$$

$$|\vec{E}_{net}| = \sqrt{(E)^2 + (E)^2 + 2E^2\cos 60^\circ}$$

$$\Rightarrow \quad |\vec{E}_{net}| = \sqrt{2E^2 + 2E^2\left(\tfrac{1}{2}\right)}$$

$$\Rightarrow \quad |\vec{E}_{net}| = \sqrt{2E^2 + E^2}$$

$$\Rightarrow \quad |\vec{E}_{net}| = \sqrt{3E^2}$$

$$\Rightarrow \quad |\vec{E}_{net}| = \sqrt{3}\,E$$

On substituting the value of E

$$\Rightarrow \quad |\vec{E}_{net}| = \frac{\sqrt{3}q}{4\pi\varepsilon_o a^2} \qquad \left(\because K = \frac{1}{4\pi\varepsilon_o}\right)$$

12. (b)

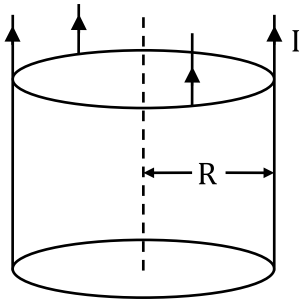
Since time period (T) of a photon will be given by-

$$T = \frac{2\pi m}{qB}$$

We can see that, time period is independent to the velocity/speed.

So, we can say that there will be no change in the time period of photon.

13. (c)



From the above figure, we can see that-

For $d < R$,

$B \times 2\pi d = \mu_o I_{in} = 0$

$\Rightarrow B = 0$

For $d > R$,

$B \times 2\pi d = \mu_o I = 0$

$\Rightarrow B = \frac{\mu_o I}{2\pi d}$

14. (b)

When we will double its radius, the resistance in the circuit will also be doubled.

When resistance in circuit will double, then current in the circuit will be decreased by halved.

$\frac{\mu_o I}{2r} = B$

$\frac{\mu_o I'}{2r'} = B'$ $[\because I' = I/2 \text{ and } r' = 2r]$

$\therefore B' = \frac{\mu_o I}{8r}$

$\Rightarrow B' = \frac{B}{4}$

15. (d)

Since according to the questions, we can write.

$\omega L = \frac{1}{\omega C}$ $\quad$ $[\because \omega = 2\pi f \text{ and } f = \frac{1}{2\pi\sqrt{LC}}]$

$\Rightarrow$ $\omega^2 = \frac{1}{LC}$

$\Rightarrow$ Squaring both sides, we get

$\omega = \sqrt{\frac{1}{LC}}$

$\Rightarrow$ X_L and X_C will get interchanged.

$\Rightarrow$ $200L = \frac{1}{800\ C}$

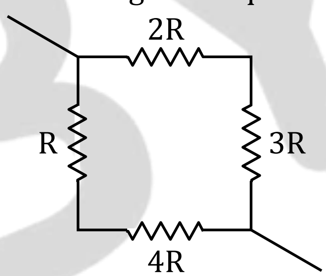
$\Rightarrow$ $\frac{1}{\sqrt{LC}} = \sqrt{200 \times 800} = 400$ Hz

16. (c)

Initially there will be no voltage drop across capacitor so intensity of bulb will rise sharply and gradually voltage drop across capacitor will increase as a result voltage drop across bulb decreases so intensity of bulb will decrease.

17. (d)

According to the question, circuit diagram will be given by-



Now, we can write the maximum possible equivalent resistance across the diagonal of the square.

$R_{eff} = \frac{5R \times 5R}{5R + 5R}$ $\quad$ [Where R = 100]

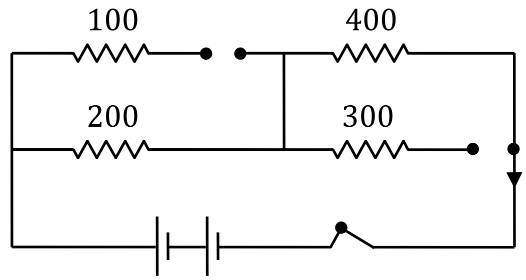
$\Rightarrow$ $R_{eff} = \frac{25R^2}{10R}$

$\Rightarrow$ $R_{eff} = \frac{25R}{10}$

$\Rightarrow$ $R_{eff} = \frac{25 \times 100}{10}$

$\Rightarrow$ $R_{eff} = 250\ \Omega$

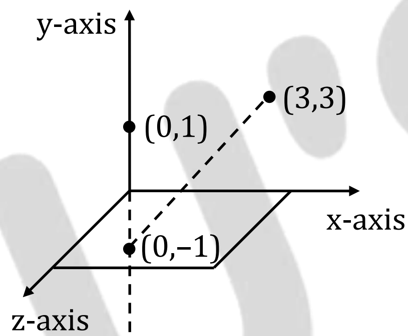
18. (c)



After long time, when switch will be on, then it will become steady state and in steady state the capacitor will behave as infinite resistance. The steady current will be given by-

$I_{steady} = \frac{V}{R} = \frac{6}{600} = 10\text{ mA}$

19. (a)



Magnitude of 'a' will be given by

$$|\vec{a}| = \sqrt{(3)^2 + (4)^2} = 5$$

$\Rightarrow \quad |\vec{a}| = 5$

20. (b)

To combine the eqi-convex lenses, such that (F) is related to (f)-

Now, according to formula

$$\frac{1}{f_w} = (\mu_w - 1)\left(-\frac{2}{R}\right) = \frac{\mu_w - 1}{\mu_\ell - 1}\left(\frac{1}{f_\ell}\right)$$

Now, $\frac{1}{f_\ell} = (\mu_\ell - 1)\frac{2}{R}$

$\because \quad \mu_\ell > \mu_w$

Now, $\frac{1}{f_{eq}} = \frac{2}{f_\ell} - \frac{1}{f_\ell}\left[\frac{\mu_w - 1}{\mu_\ell - 1}\right]$

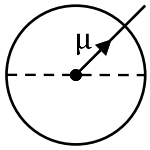
$\Rightarrow \quad \mu_\ell - 1 > \mu_w - 1$

$\Rightarrow \quad \frac{\mu_w - 1}{\mu_\ell - 1} < 1$

$\Rightarrow \quad \frac{1}{f} < \frac{1}{f_{eq}} < \frac{2}{f}$

$\Rightarrow \quad \frac{2}{f} < f_{eq} < f$

21. (d)



As the object is at centre, all rays will fall normally on surface, hence they will not deviate

$\therefore$ Apparent depth = Real depth

22. (c)

If Young's double slit experiment is done with white light, then the centre fringe will be white because $\Delta x = 0$ at centre for all the wavelengths.

23. (a)

The linear velocity 'v' of on electron in the Bohr orbit is related to its quantum number (n) will be given by-

$v = ze^2/2\epsilon_0 nh$

24. (d)

Fractional decay on the third day will be given by-

$$= \frac{[N_o e^{-2/\tau} - N_o e^{-3/\tau}]}{N_o}$$

$$\because \quad \tau = \frac{t_{1/2}}{\ln 2} = \frac{3}{\ln 2}$$

$$= e^{-\frac{2\ln 2}{3}} - e^{-\ln 2}$$

$$= 2^{-2/3} - 2^{-1}$$

$$= 0.63 - 0.5 = 0.13$$

25. (c)

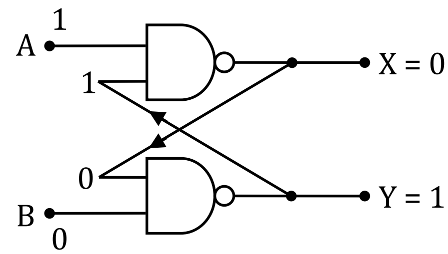
Since we know that-

$$Ve = \frac{hc}{\lambda}$$

$$(10000)e = \frac{hc}{\lambda}$$

$$\therefore \quad \frac{hc}{2\lambda} = (5000)e$$

26. (c)



Firstly checking this from output side-

When X = 0 and Y = 1

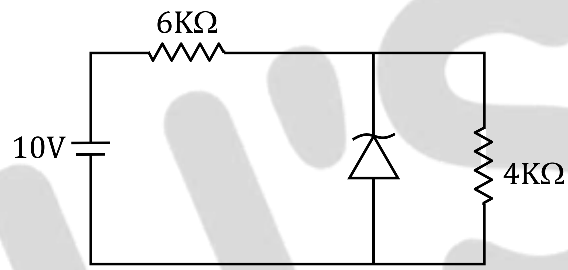
And A = 1 and B = 0 is given so

By A.B = AND GATE

0.0 = 0

1.1 = 1

27. (a)



∵ Zener break done = 6V

So potential across 4KΩ= 6V

And potential across 6KΩ= (10 – 6) = 4V

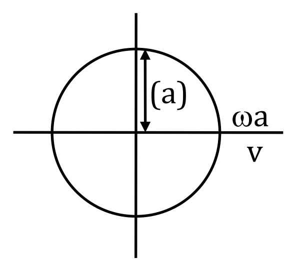
Current through the 6KΩ= $\frac{4}{6000}$ A

$\Rightarrow \quad \frac{2}{3000}$ A = $\frac{2}{3}$ mA

28. (c)

The relation between velocity (v) and displacement (x) in SHM is H.

$\frac{v^2}{w^2A^2} - \frac{x^2}{A^2}$ = 1 [It is equation of Ellipse]



Elliptical path having radius equal to (a)

Major axis will be = 2wA

And minor axis will be = 2a

Given: $\frac{2\omega a}{2a}$

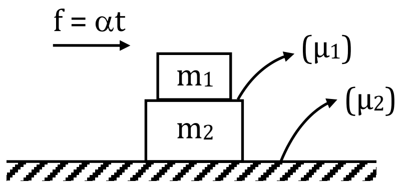
$\omega = 20\pi$

$2\pi f = 20\pi$

f = 10 Hz

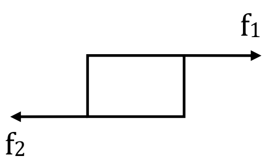
29. (b)

Now, according to the question we can draw the figure.



(μ_1) is co-efficient of friction between the block (m_1) and (m_2).And (μ_2) is co-efficient of friction between the block (m_2) and the surface.

Now, F.B.D. of the lower block (m_2)



(f_2) is the friction between lower block and the table and (f_1) is the friction between lower block and upper block.

$\because$ m_2 is not moving

So, $f_1 \le f_2$

So, friction acting between the block is always less than or equal to friction acting between lower block and table.

So, $\mu_1 m_1 g \le \mu_2(m_1 + m_2)g$

$$\frac{\mu_1}{\mu_2} \le \frac{m_1 + m_2}{m_1}$$

$$\frac{\mu_1}{\mu_2} \le 1 + \frac{m_2}{m_1}$$

So, maximum value of $\frac{\mu_1}{\mu_2} = 1 + \frac{m_2}{m_1}$

30. (a)

According to the question-

$\overrightarrow{AB} = 3\hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{AC} = \hat{i}+2\hat{j}+\hat{k}$

So, $\overrightarrow{CB}$ will be given by

$$\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$$

$$\Rightarrow \quad \overrightarrow{CB} = (3\hat{i}+\hat{j}+\hat{k}) - (\hat{i}+2\hat{j}+\hat{k})$$

$$\Rightarrow \quad \overrightarrow{CB} = 2\hat{i}-\hat{j}$$

Now, $\angle B$ is between $\overrightarrow{AB}$ and $\overrightarrow{CB}$.

So, it will be given by dot product of two vectors.

$$\overrightarrow{AB}\cdot\overrightarrow{CB} = |\overrightarrow{AB}|\,|\overrightarrow{CB}|\cos\theta$$

$$\because \quad |\overrightarrow{AB}| = \sqrt{(3)^2+(1)^2+(1)^2} = \sqrt{11}$$

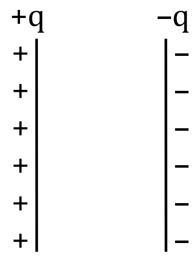
And $|\overrightarrow{CB}| = \sqrt{(2)^2+(-1)^2} = \sqrt{5}$

$$\Rightarrow \quad \overrightarrow{AB}\cdot\overrightarrow{CB} = |\sqrt{11}|\,|\sqrt{5}|\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{5}}{\sqrt{11}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{\sqrt{5}}{\sqrt{11}}\right)$$

31. (a)



As the separation between the plates are decreasing as they are approaching towards each other.

Also, V = E.d

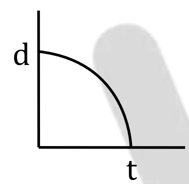
Now, electric field will remains constant between the plates.

So, $V \propto d$

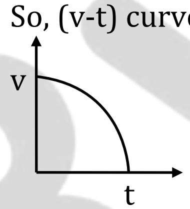
Now, force on each plate will be = $\frac{q^2}{2A\varepsilon_o}$

And acceleration = $\frac{q^2}{2A\varepsilon_o(m)}$

$\because$ a = constant so



So, (v-t) curve will be given by



32. (d)

Force on the bob of pendulum of mass (m) will be given by

$F = qE$

$\because$ $F = ma$

$ma = qE$

$\Rightarrow$ $a = \frac{qE}{m}$

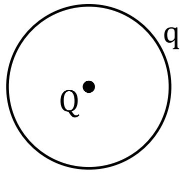
Now, time period (T) of a simple pendulum will be given by-

$T = 2\pi\sqrt{\frac{L}{g'}}$

$g' = g - a$

$\Rightarrow$ $T = 2\pi\sqrt{\frac{L}{\left(g - \frac{qE}{m}\right)}} \Rightarrow T = 2\pi\sqrt{\frac{L}{g - \frac{q\sigma}{\varepsilon_o(m)}}}$

33. (d)



The electrostatic force of attraction will be given by

$F = \dfrac{KqQ}{r^2}$

Since, the charge is in circular motion, it will experience centripetal force

$\because \quad F = \dfrac{mv^2}{r}$

$\Rightarrow \quad \dfrac{mv^2}{r} = \dfrac{KqQ}{r^2}$

$\Rightarrow \quad mv^2 = \dfrac{KqQ}{r}$

$\Rightarrow \quad v = \sqrt{\dfrac{KqQ}{rm}}$

$\Rightarrow \quad v \propto \dfrac{1}{\sqrt{r}}$

Time period $= \dfrac{2\pi r}{v}$ $\qquad \because$ Time period $\propto \dfrac{r}{v}$

$\therefore$ Time period $\propto r^{3/2}$

$I \propto \dfrac{Q}{\text{Time period}}$ $\qquad \therefore (I \propto r^{-3/2})$

$B = \dfrac{\mu_o I}{2r}$

$B \propto I$ or $B \propto \dfrac{I}{r}$

$B \propto \dfrac{I}{r}$ or $B \propto \dfrac{r^{-3/2}}{r}$

$\Rightarrow \quad B \propto r^{-5/2}$

34. (a)

Since we know that

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \varepsilon = -\frac{d(B.A)}{dt}$$

$$\Rightarrow \quad \varepsilon = -A\frac{dB}{dt}$$

$$\Rightarrow \quad \varepsilon = -A\frac{d}{dt}\frac{\mu_o i}{2\pi(vt)} \qquad [\because B = \frac{\mu_o i}{2\pi(vt)}]$$

$$\Rightarrow \quad -A\frac{\mu_o i}{2\pi v}\frac{d}{dt}(t^{-1})$$

After differentiating,

$$\Rightarrow \quad A\frac{\mu_o i}{2\pi v}t^{-2}$$

$$\Rightarrow \quad \varepsilon \propto \frac{1}{T^2}$$

35. (d)

According to the question

$\rho_1 \rightarrow$ Solid spherical ball

$\rho_2 \rightarrow$ Hollow spherical ball

Now,

$$\mu = \frac{4}{3}\pi\rho_1 R^3 = \frac{4}{3}\pi\rho_2[R^3 - R^3_{inner}]$$

$$\Rightarrow \quad R^3 - R^3_{inner} = \frac{\rho_1 R^3}{\rho_2}$$

$$\Rightarrow \quad R^3_{inner} = R^3\left[1 - \frac{\rho_1}{\rho_2}\right]$$

$$\Rightarrow \quad R_{inner} = R\left[1 - \frac{\rho_1}{\rho_2}\right]^{1/3}$$

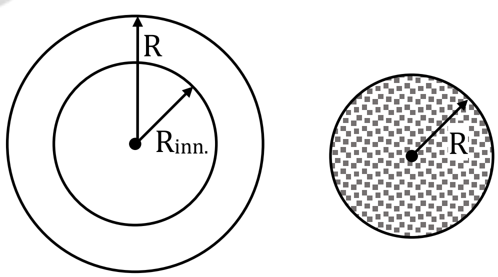
Ratio of the moment of inertia will be:

$$\frac{I_{Hollow}}{I_{Solid}} = \frac{\frac{4}{3}\pi R^3 \rho_2 \times \frac{2}{5}R^2 - \frac{4}{3}\pi R^3_{inner}\rho_2 \times \frac{2}{5}R^2_{inner}}{\frac{2}{5}\times\frac{4}{3}\pi R^3 \rho_1}$$

$$= \frac{\rho_2}{\rho_1}\left[\frac{R^5 - R^5_{inner}}{R^5}\right]$$

$$= \frac{\rho_2}{\rho_1}\left[1 - \left(\frac{R_{inner}}{R}\right)^5\right]$$

$$= \frac{\rho_2}{\rho_1}\left[1 - \left[1 - \left(\frac{\rho_1}{\rho_2}\right)\right]^{5/3}\right]$$

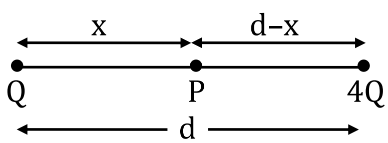


36. (b)

As we know, that in isothermal process the change in internal energy is always zero and in isobaric expansion, volume is directly proportional to temperature. So, change in internal energy will increase.

37. (a)

As we know that, electric potential will be minimum, where electric field will be zero. Now, let us assume that the charge Q is at (x) distance from point P as well as charge 4Q is at (d–x) distance from point P.



The electric potential due to charge (Q) at point P will be

$$E_P = \frac{KQ}{x^2}$$

Similarly, the electric potential due to charge (4Q) at point P will be:

$$E_P = \frac{K4Q}{(d-x)^2}$$

On equating

$$\frac{KQ}{x^2} = \frac{K4Q}{(d-x)^2}$$

$$\Rightarrow \quad \frac{1}{(x)^2} = \frac{4}{(d-x)^2}$$

$$\Rightarrow \quad \left(\frac{1}{x}\right)^2 = \left(\frac{2}{d-x}\right)^2$$

$$\Rightarrow \quad \frac{1}{x} = \frac{2}{d-x}$$

$$\Rightarrow \quad x = \frac{d}{3}$$

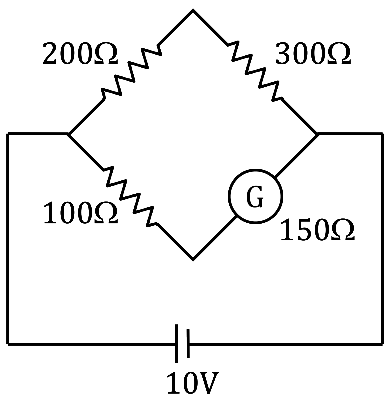
Electric potential will be minimum at $\left(\frac{d}{3}\right)$ units, at right of point A.

38. (b), (c), (d)

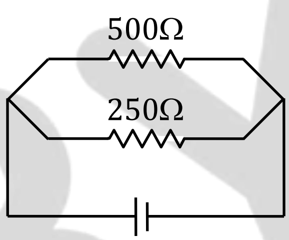
When switch will be opened then it will be a case of balanced wheat-stone bridge

$$\Rightarrow \quad \frac{200}{300} = \frac{100}{a}$$

$$\Rightarrow \quad G = 150\ \Omega$$



Now, we will draw the circuit diagram



$$R_{eq.} = \frac{500 \times 250}{750} = \frac{500}{3}\Omega$$

Now, current through the galvanometer

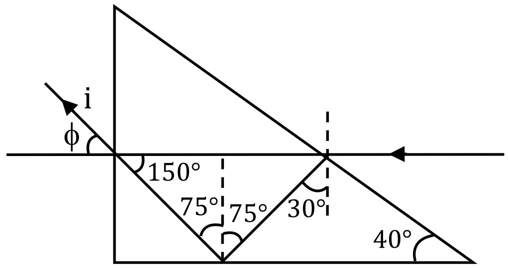
$$\frac{10}{250} = \frac{1}{25} = 0.04\text{ A}$$

200 Ω and 300 Ω are in series.

100 Ω, 150 Ω and (G) are in series.

39. (a) (c)

Applying geometry to the given figure



On applying Snell's law in the given figure

$1 \sin(i) = \sqrt{2}\sin(15°)$

$1 \sin(i) = \sqrt{2}\left[\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right]$

$\sin(i) = \dfrac{\sqrt{3}-1}{2}$

This ray will go under 'total internal reflection' at point P only. So, option (a) is correct.

If the ray emerge from R (partially) it is possible, then it will become parallel to the incident ray. So, option (c) is correct.

40. (a) (d)

Due to difference of frequencies, phenomenon of beat occurs.