(Computer graphics - 22318) Pranjal Save (CO-10)

Unit III: Overview of transformations

(weightage - 12 marks)

Introduction

* In Computer graphics we can use to draw variety of pictures on screen. But in many applications there is need for altering or manipulating pictures on displays.
* One of the applications of CGR is animation in that one needs to move object along the path.
* Such changes in orientation, size and shape of an object changes or alters the co-ordinates of the object and this is done in geometric transformation.

Basic Geometric Transformations	Portion
1 - Translation, 2 - Scaling, 3. Rotation } 2D+3D	3.1 2D Transformations
Other extra transformations: 4. Reflection, 5 - Shear } 2D	3.4 3D Only Basic

Questions

Q1 Define i) Scaling ii) Reflection } 2 marks.

Q2 List out Basic transformations techniques. Explain with respect to 2D. (4 marks)

~~Q3// Transformation of of shear (2M)~~

① Translation

* It is the process of repositioning an object along a straight line path from one co-ordinate location to new - co-ordinate locates.
* Translate is rigid motion of points to new locations and moves object to new place without deformation.
* To translate an 2D object from one point to another point we need to add translation distance tx and ty along x-axis and y-axis respectively to originally coordinates positions such as.

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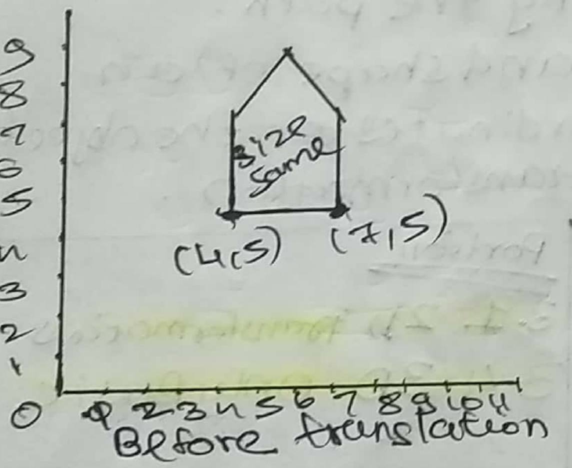
$x' = x + tx$
$y' = y + ty$

where (x, y) = original co-ordinate point
(x', y') = New co-ordinate point.
(tx, ty) = translation distance

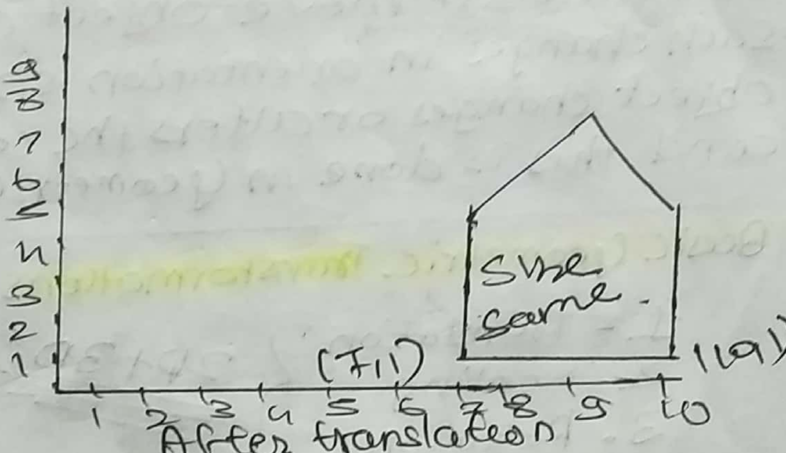
matrix Representation

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad P = \begin{bmatrix} x \\ y \end{bmatrix} \quad T = \begin{bmatrix} tx \\ ty \end{bmatrix}$$

$$\boxed{P' = P + T}$$



Before translation



After translation

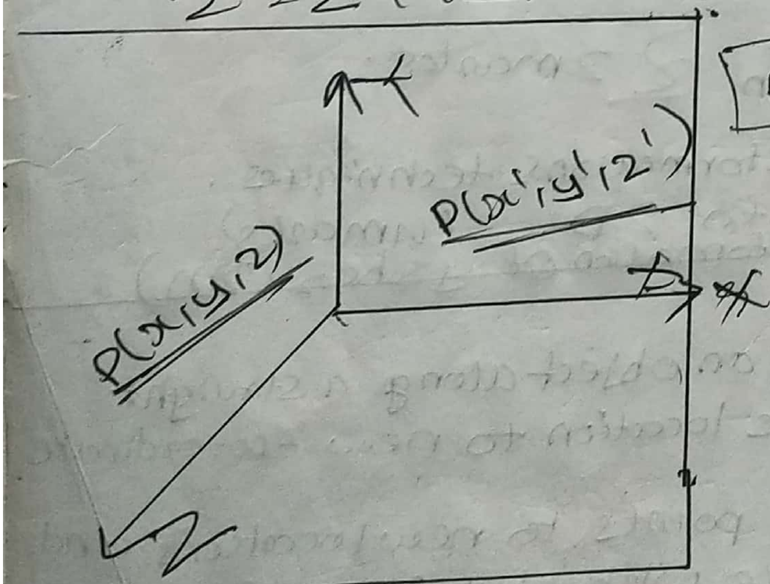
3D representation -

$x' = x + tx$
$y' = y + ty$
$z' = z + tz$

matrix

$$P' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad T = \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix}$$

$$\boxed{P' = P + T}$$



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⑤ Scaling

* Scaling changes the size of object along x and y or both axis respectively.
* In scaling one needs to increase the size of the object or reduce the size of the object in x or y axis both axis.
* New transformationed co-ordinates of a point (x, y) can be calculated by multiplying it with scaling factor (S_x) & (S_y) in x and y directions respectively.
* In short scaling is stretching of points along axes.

2D

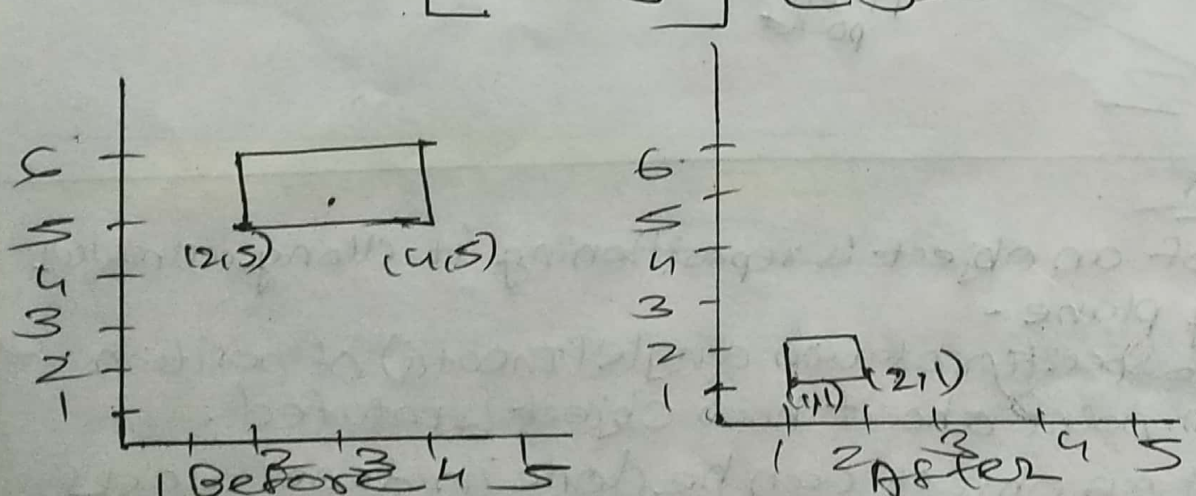
$$x' = S_x \cdot x$$
$$y' = S_y \cdot y$$

where
(x, y) = orginal co-ordinates
(x', y') = New co-ordinate
S_x, S_y = Scaling factor

matrix

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad S = \cancel{\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$P' = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Case 1: If $S_x = S_y = 1$
then scaling do not change size of object.

Case 2: If S_x and $S_y > 1$
increases size

Case 3: If S_x and $S_y < 1$
Reduces size

~~(2) Rotation~~ Case 4: $Sx = Sy$

then it has two possible condition either increases parallely in upward side or decreases in downward side uniformally.

Case 5: $Sx \neq Sy$

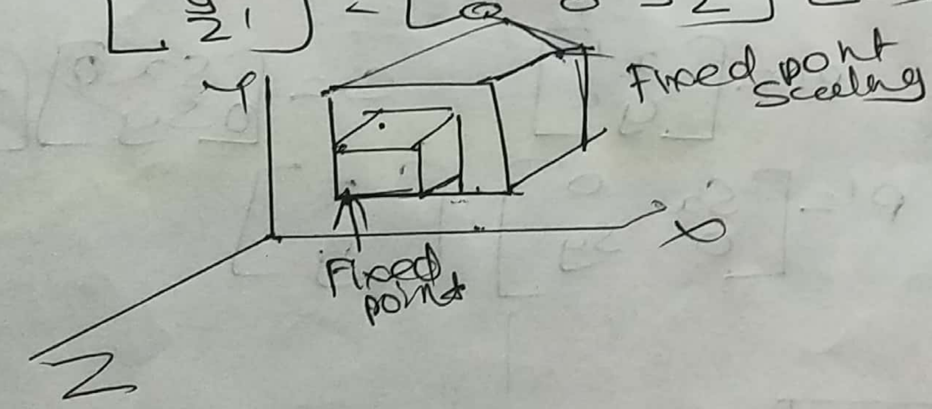
It also has two conditions.
one increases in negative & other +ve.
or else, one in +ve and other -ve.

3D.

$x' = x \cdot Sx$
$y' = y \cdot Sy$
$z' = z \cdot Sz$

matrix

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & Sz \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



(3) Rotation

* Rotation of an object is repositioning it along a circular path in xy plane -
* we need to specify rotation angle (theata) of position of rotation point about how object is rotated.
* Rotation of an object can be done in two ways
    - Clock wise
    - Counter Clock wise

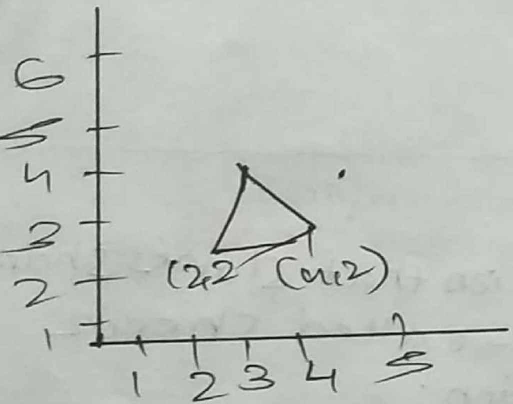
If positive angle = Counter clock wise.
negative angle = clock wise.

fn: $x' = x \cdot \cos\theta - y \cdot \sin\theta$

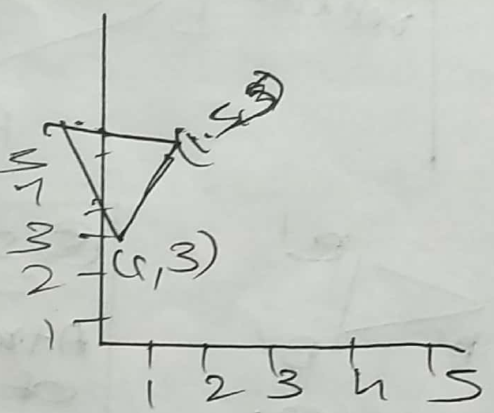
$y' = x \cdot \sin\theta + y \cdot \cos\theta$

matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

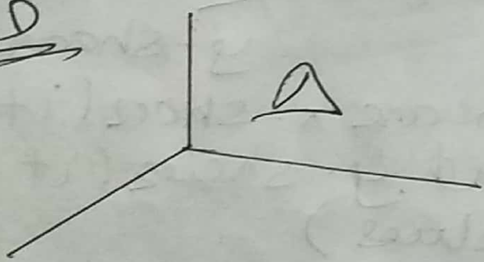


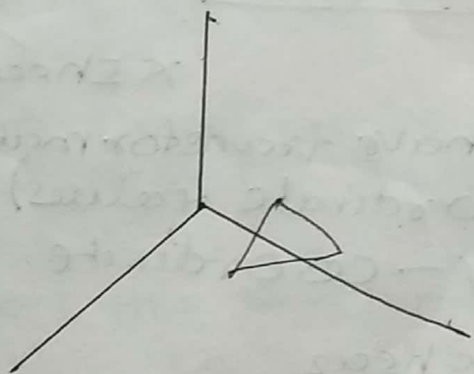
Before



After.

3D

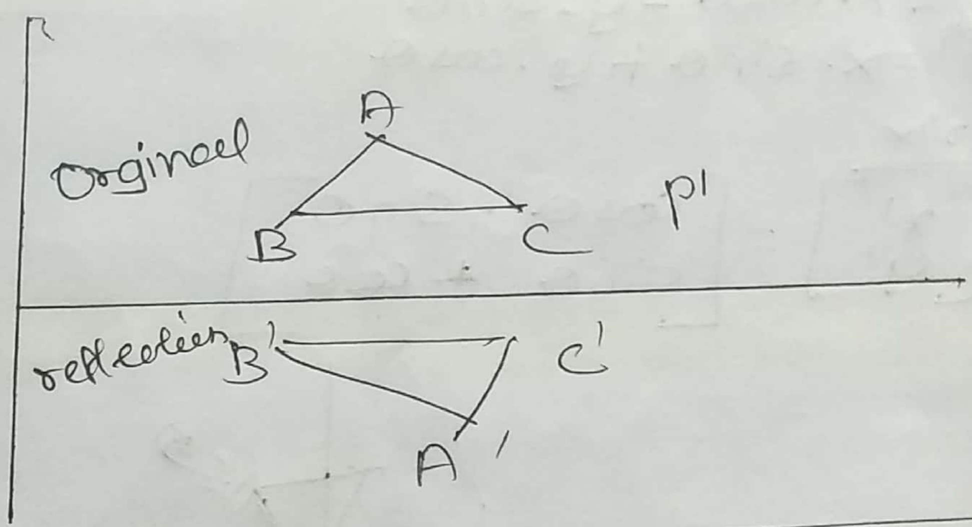


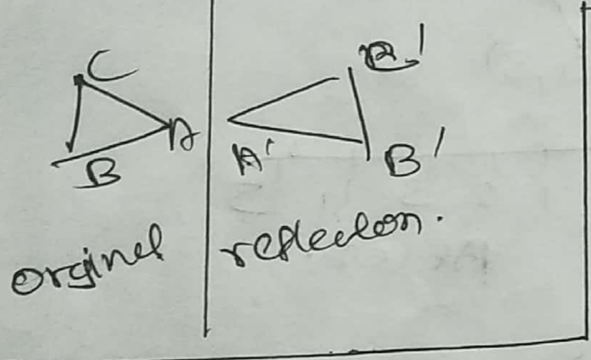


Other transformations − 2D (3D not in syllabus)

④ Reflection

* It is transformation that produces a mirror image of an object relative to a axis of reflection.
* We can choose an axes of reflection in xy plane or perpendicular to xy plane.





(5) Shearing

A transformation that slants shape of object is called sheer transformation.

types
- x sheer
- y-shear.

* Types of hair transformation are x-sheer (it shifts x co-ordinate values) and y-sheer (it shifts y-coordinate values)

* I x-sheer

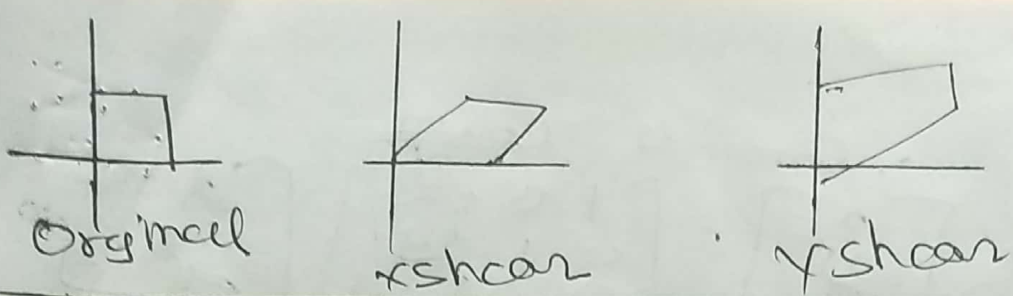
It shifts x co-ordinates and preserves y co-ordinate. Therfore object get tilted toward either right or left.

matrix $\begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Sh_x = x sheer factor

2 Y-sheer

It shifts y co-ordinate and preserves x coordinates Therfore objet get tilted towards either up or down.

matrix $\begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ sh_y = y sheer factor



3.2 matrix representation and homogeneous coordinates (Translation, Scaling, Rotation, Reflection, Shearing)

Questions
- List any four properties of homogeneous coordinate system. (2m)
- Explain the need of homogeneous co-ordinate System/matrix (2m)
- What is homogeneous co-ordinate? why it required? (2m)
- Advantages of homogeneous co-ordinate? (2m)

Homogeneous Cordinates & Matrix representation.

* In 2-D we represent a point by two coordinate values as x and y.
* where in Homogeneous Co-ordinates system a point is represented by a triple as (x, y, w)

$\boxed{w=1}$

* For 2-D transformations this Homogeneous parameter w is equal to 1.
* Therefore each two dimensional point can be represented with Homogeneous co-ordinates $(x, y, 1)$
* So, Homogeneous Co-ordinate system represents 2*2 matrix forms of transformations 3*3 matrix introducing additionally dummy co-ordinate w of unit value.
* It is used to get most accurate values.

Advantages

Very important advantage of this system is it allow us to express all transformation eqn as matrix multiplications which is useful in composite transformers

Translation

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

∴ Cordinate colum

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

∴ row

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ colum

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

row

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

row

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

colum

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} \cos & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

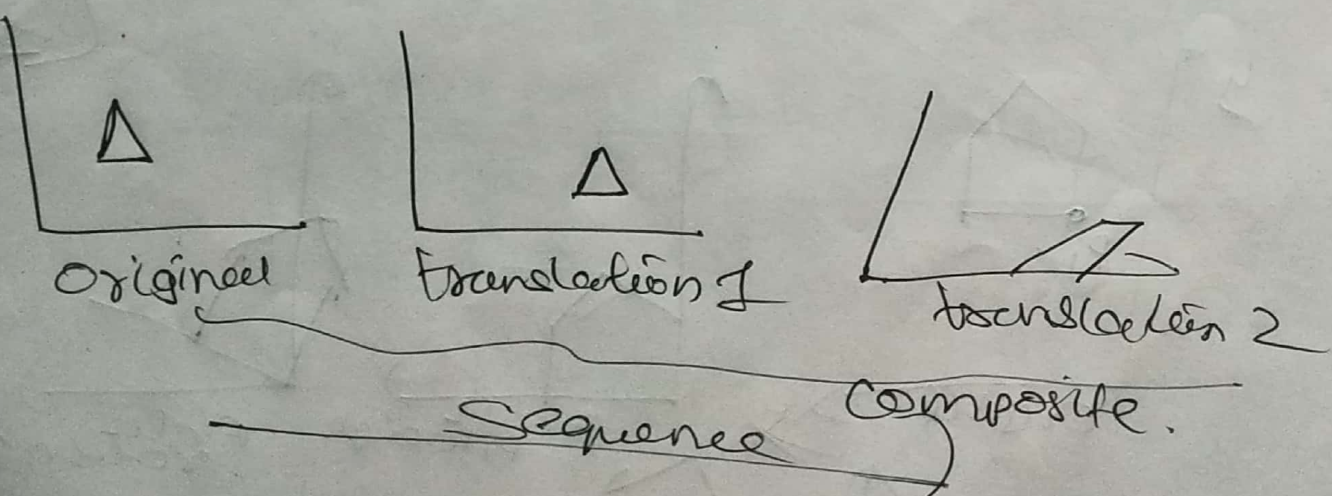
3.3 Composite transformations - rotation about an arbitary point.

Questions → Q// Explain composite transformation over arbitary point. (4 marks)

Q// Obtain a transformation matrix for rotation an object about a specified pivot point. (4 marks)

Composite transformations

* In many situation of transformation more than one transformation are needed to be executed in a sequence to give final output.
* Thus if more than one transformations are performed in a sequence then resultant transformation is called composite transformation.
* This composite transformation is achieved by concatenating transformations in sequence.
* For an Example 1, Translate an object from old location to new location and the scale the object on new location.
* Translate an object from old location to new location after that again translate to new location



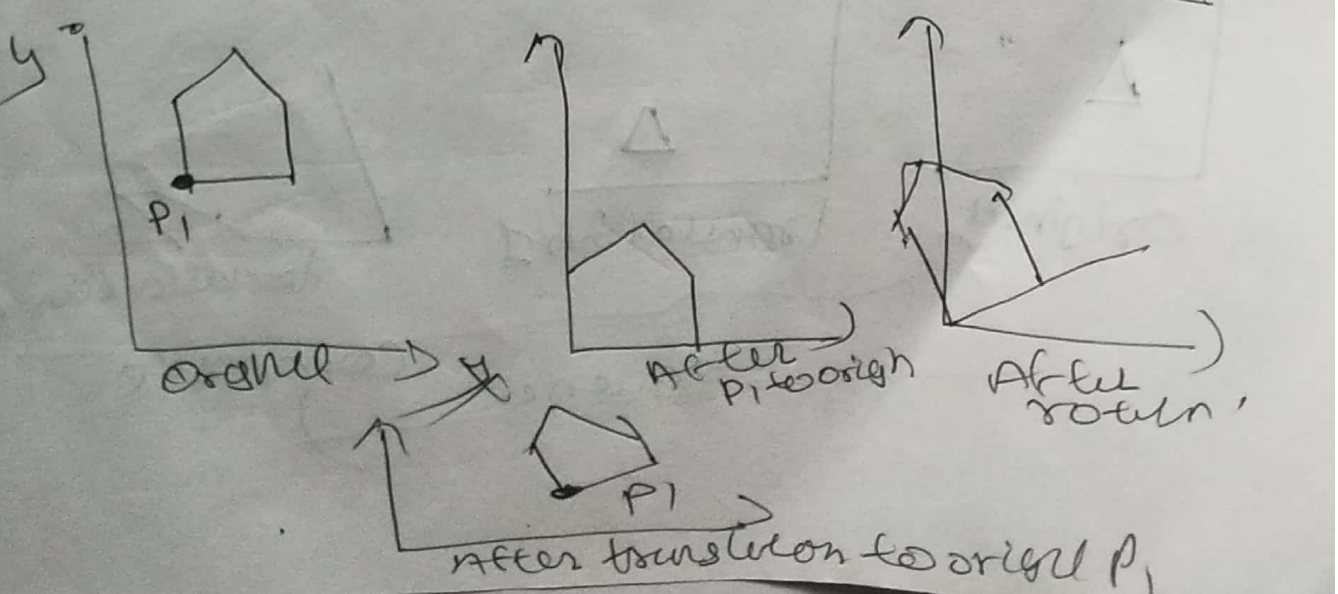
Rotation About Arbitrary point (Pivot point) :

* In previously Section, we have already seen rotation of an object. But it was strictly about origin
* To do rotation of an object about any selected arbitrary point $P_1(x_1, y_1)$, following sequence Should be followed/performed.
  - Translate - Arbitary point P_1 is moved to co-ordinate origin.
  - Rotate : Rotate object about origin
  - Translate : translate object so that abitrary point P_1 is moved back to original position.

- Rotate about a point $P_1(x_1, y_1)$
  1. Translate P_1 to origin.
  2. Rotate
  3. Translate back to P_1.

$$\boxed{Eqn^n = T(x_1, y_1) \cdot R(\theta) \cdot T(-x_1, -y_1)}$$

$$P' = \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



3.5 Types of Projections : Perspectives and Parallel Projections

Questions → Q/ Define projection. Explain
i) Perspective projection.
ii) Parallel projection
} 6 marks.

Q// Explain perspective projection with its any one type. (4 marks)

Q// Explain types of Projection. (4 marks)

Q// Difference between parallel & perspective.

⇒ **Projection**

* Transforming 3D points into 2D points is called projection.
* We can say that Projection transforms 3D objects on to a 2D plane like screen, paper etc.
* Projection is process of representing 3D object onto a screen that is 2D.
* It is basically mapping of any point P(x, y, z) to the image (x', y', z') onto a plane called projection plane

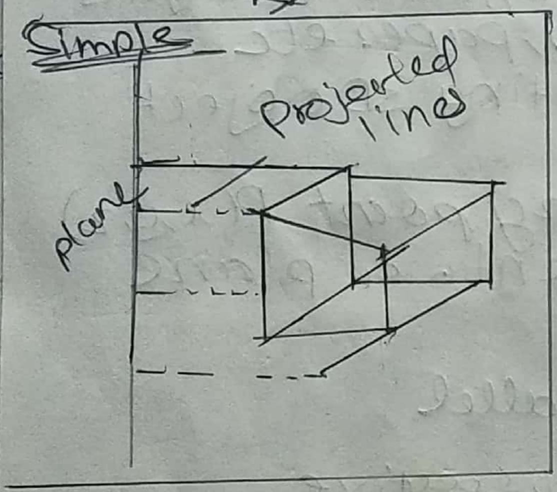
Types of projections → Parallel, Perspective

Parallel projections → Orthographic, Oblique

Perspective projections
- → One point
- → Two point
- → Three point.

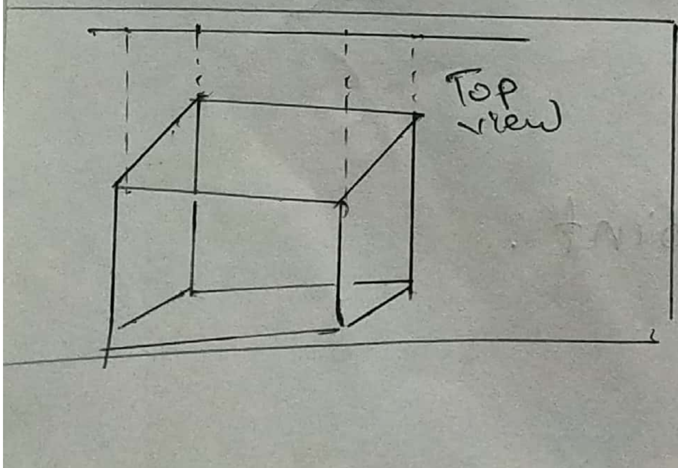
parallel projection

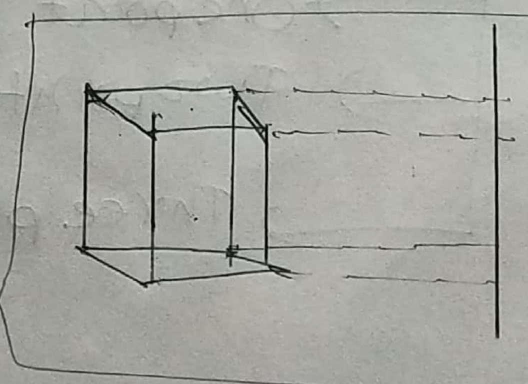
* Parallel projection is a kind of projection where there projecting lines emerge parallely from the polygon surface and then incident parallely on plane.
* In parallel projection, the view of object obtained at the plane is less-realistic as there is no for shortcoming.
* In this, the centre of the projection lies at infinity.
* We connect the projected vertices by line segments which corresponding to connected on original object.
* This type of projection are of two type
  - Orthographic
  - oblique.



Orthographic

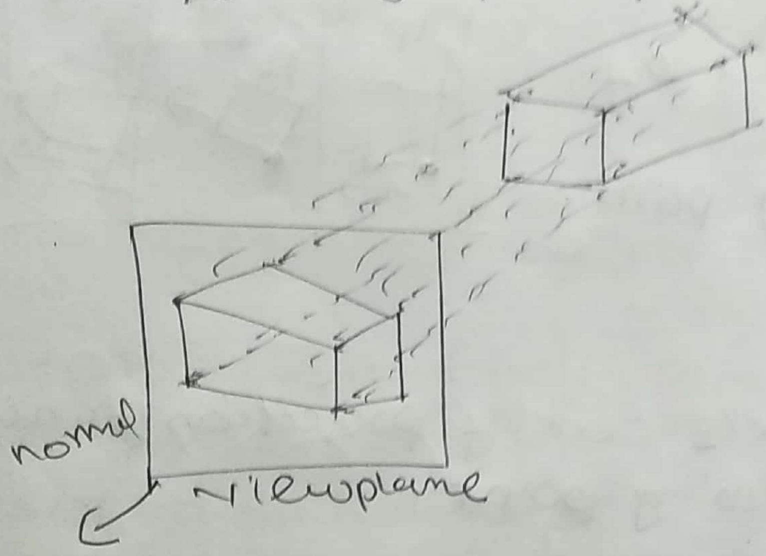
* It is a kind of parallel projection where projecting line emerge parallely from the object surface and incident perpendicular at plane
* this utilizes perpendicular projects from object to plane of projection to generate system of drawing views.
* Describe the design and feature of object. widely used.
* **Types**: Front, Top, Side Projections





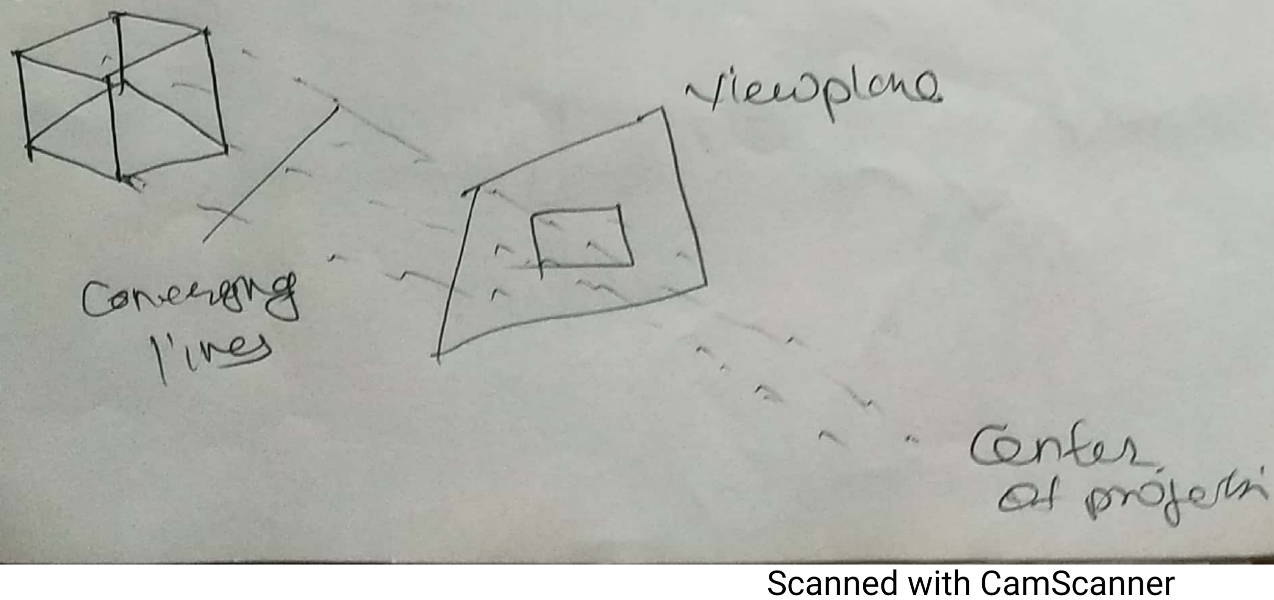
Oblique Projection

* It is a kind of parallel projection where projection rays emerges parallely from the surface of polygon and incident at angle other than 90° on plane.
* In simple words, the direction of projection is not perpendicular to projection of plane.
* In this, we can view the object better than orthographic projection.



Perspective projection

* In perspective projection the distance from the center of projection to project plane is finite and size of object varies inversely with distance which looks more realistic.
* The distance and angles are not preserved and parallel lines do not remain parallel instead, they all converge at a single point called center of projection.



There are 3 types of perspective projections

One point

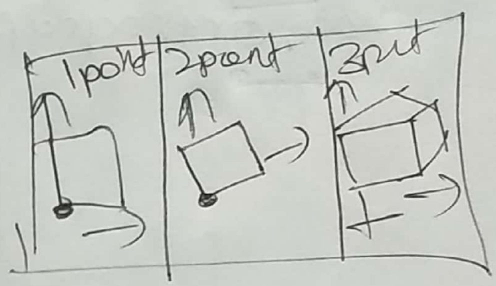
Two point

Three point.

Onepoint

One principle axis cut by projection plane.

One axis vanishing point.



Two-point

Two principle axis cut by projection plane

Two axis vanishing point.

Three point

Three principle axes cut by projection plane.

Three axis vanishing points.

Difference Between

Parallel Projection	Perspective Projection
(1) In Parallel Projection, the centre of projection is at infinity	(1) In perspective projection the centre of projection is at finite distance.
(2) Here, all projectors are parallel to each other.	(2) Here, all projectors are not parallel.
(3) It is a less realistic view.	(3) It is more realistic view.
(4) These are linear transforms implemented with matrix.	(4) These are non-linear transforms.
(5) It is used applications where exact measurement is required.	(5) It is used in camera as well eye human.
(6) Examples, Use of drawing schematic diagram.	(6) Example use in architectural rendering realistic view.

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Numericals

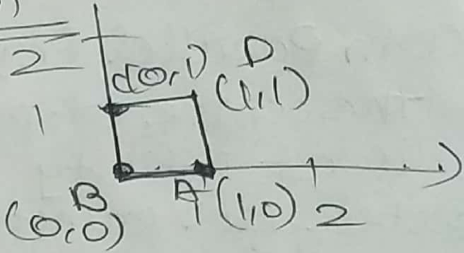
Q/ Consider a square A(1, 0), B(0,0), C(0,1), D(1, 1)
Rotate the square by 45° anticlockwise direction followed by reflection about x-axis.

⇒ Given

$\theta = 45°$

A(1,0)
B(0,0)
C(0,1)
D(1,1)

Graph



Formula

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (i)$$

matrix Reflection about x-axis,

$$X_{ref} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (ii)$$

note

$$Y_{ref} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Firstly, we rotate square by 45° anticlockwise direction and followed by reflection about x axis.

$$\therefore R_{ref} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & -2/\sqrt{2} & 1 \end{bmatrix}$$

$A' = (-1/\sqrt{2}, -1/\sqrt{2})$

$B' = (0, 0)$

$C' = (-1/\sqrt{2}, -1/\sqrt{2})$

$D' = (0, -2/\sqrt{2})$

Q2 Consider the square A(1,0), B(0,0), C(0,1), D(1,1). Rotate the square ABCD by 45° anti-clockwise about point A(1,0).

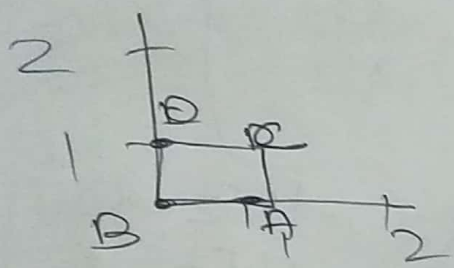
⇒ given

~~A(1,0)~~ A(1,0)
B(0,0)
C(0,1)
D(1,1)

$\theta = 45°$

~~xp, yp~~

$\boxed{(x, y) = (1, 0)}$



Sol^n

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -X\cos\theta + Y\sin\theta + X & -X\sin\theta - Y\cos\theta + Y & 1 \end{bmatrix}$$

... Formula

$$\begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ -1\cos 45 + 0\sin 45 + 1 & -1\sin 45 - 0\cos 45 + 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -\frac{1}{\sqrt{2}} + 1 & -\frac{1}{\sqrt{2}} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -\frac{1}{\sqrt{2}}+1 & -\frac{1}{\sqrt{2}} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{\sqrt{2}}+1 & -1/\sqrt{2} & 1 \\ 1-\sqrt{2} & 0 & 1 \\ 1-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} A' &= (1, 0) \\ B' &= (-1/\sqrt{2}+1, -1/\sqrt{2}) \\ C' &= (1-\sqrt{2}, 0) \\ D' & \\ & (1-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}) \end{aligned}}$$

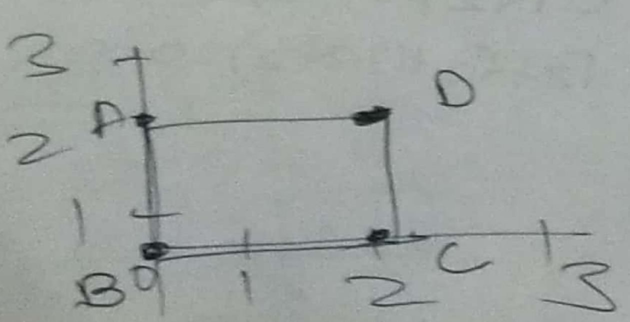
Q3 Consider the square A(2,0), B(0,0), C(0,2), D(2,2). Rotate the square ABCD by 45° anticlockwise about point D(2,2).

⇒ Given

A(2,0)
B(0,0)
C(0,2)
D(2,2)

$\theta = 45°$

$(2,2) = (x, y)$



Sol^n

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x\cos\theta + y\sin\theta + x & -x\sin\theta - y\cos\theta + y & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ -2\cos 45 + 2\sin 45 + 2 & -2\sin 45 - 2\cos 45 + 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 2 & 2-2\sqrt{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 2 & 2-2\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}+0+2 & \sqrt{2}+0+(2-2\sqrt{2}) & 0+0+1 \\ 0+0+2 & 0+0+(2-2\sqrt{2}) & 0+0+1 \\ 0+(-\sqrt{2})+2 & 0+\sqrt{2}+(2-2\sqrt{2}) & 0+0+1 \\ \sqrt{2}+(-\sqrt{2})+2 & \sqrt{2}+\sqrt{2}+(2-2\sqrt{2}) & 0+0+1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3.4142 & 0.5857 \\ 2 & -0.8284 \\ 0.5857 & 0.5857 \\ 2 & 2 \end{bmatrix}$$

$$\boxed{\begin{aligned} A' &= (3.4142, 0.5857) \\ B' &= (2, -0.8284) \\ C' &= (0.5857, 0.5857) \\ D' &= (2, 2) \end{aligned}}$$

Shearing

Appy the shearing transformation to square with co-ordinates A(0,0); B(1,0), C(1,1) and D(0,1) as given below.

i) shear parameter value of 0.5 relative to line $Y_{ref} = -1$.

ii) shear parameter value of 0.5 relative to line $X_{ref} = -1$.

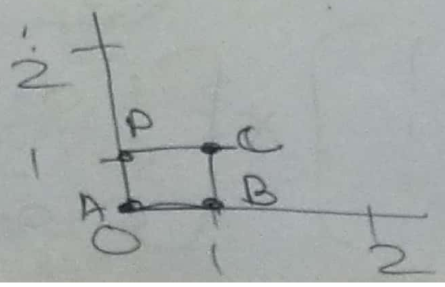
— (6 marks)

⇒ Given

A(0,0)
B(1,0)   of Square.
C(1,1)
D(0,1)

To find : shear parameter value of 0.5 relative line Y_{ref} & $X_{ref} = -1$.

Graph :



i) Here $SH_x = 0.5$ and $y_{ref} = -1$

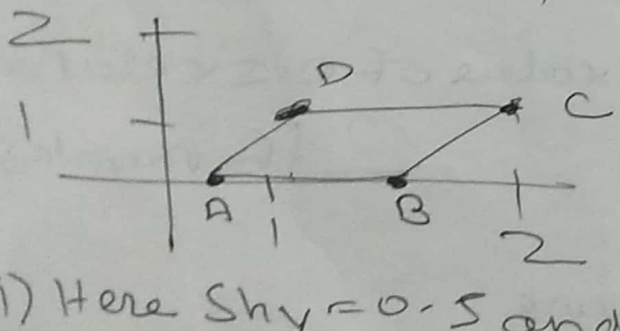
$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ -Sh_x y_{ref} & 0 & 1 \end{bmatrix}$$

... Formula.

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 1 \\ 1.5 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$A' = (0.5, 0)$, $B(1.5, 0)$, $C(2,1)$, $D(1,1)$

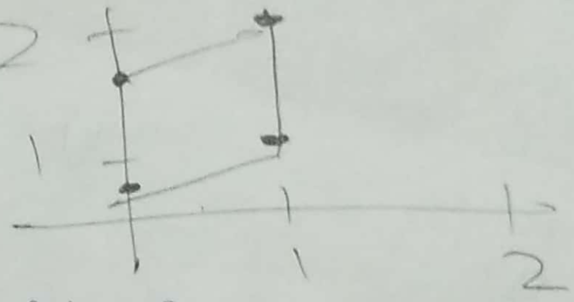


ii) Here $Sh_y = 0.5$ and $x_{ref} = -1$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \times \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & \cancel{0}\ -Sh_y \cdot x_{ref} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.5 & 1 \\ 1 & 2 & 1 \\ 0 & 1.5 & 1 \end{bmatrix}$$



Q) Apply shearing transformation to rectangle, with A(0,0), B(2,0), C(2,2) & D(0,2) as.

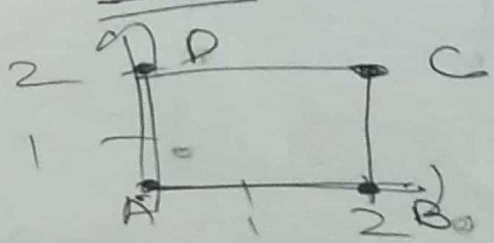
i) Shear parameter value of 1.5 relative to line $y_{ref} = -1$.

ii) Shear parameter value of 1.5 relative to line $x_{ref} = -1$.

⟹ Given

A(0,0)
B(2,0)
C(2,2)
D(0,2)

Graph



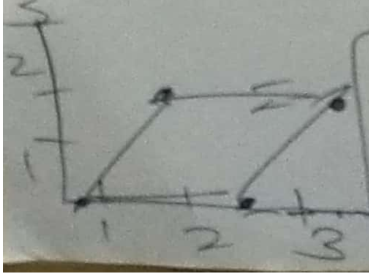
To find: Shear parameter value of 1.5 relative to y_{ref} & $x_{ref} = -1$

Solution:

i) Here $y_{ref} = -1$, $Sh_x = 0.5$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ -Sh_x \times Y_{ref} & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}$$

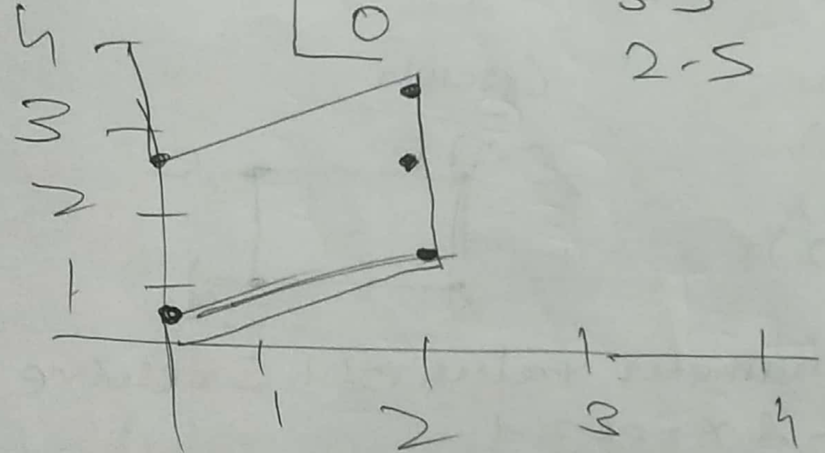


$$\begin{bmatrix} 0.5 & 0 & 1 \\ 2.5 & 0 & 1 \\ 3.5 & 2 & 1 \\ 1.5 & 2 & 1 \end{bmatrix}$$

ii) Xref = -1   Shy = 0.5

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.5 & 1 \\ 2 & 1.5 & 1 \\ 2 & 3.5 & 1 \\ 0 & 2.5 & 1 \end{bmatrix}$$



Q// Translate polygon with co-ordinates A(3,6), B(8,11) & C(11,3) by 2 units in x direction and 3 units in y direction

$\Rightarrow$ A (3, 6)

$\Rightarrow$ $3+2=5$
$6+3=9$

B (8, 11)

$\Rightarrow$ $8+2=10$
$11+3=14$

C(11,3)

$\rightarrow$ $11+2=13$
$3+3=9$

A' = (5, 9)
B' = (10, 14)
C' = (11, 3)

Triangle

* Rotation a triangle defined by A(0,0), B(6,0), & C (3,3) by 90° about origin in anticlock wise direction

⇒ Given : for triangle,

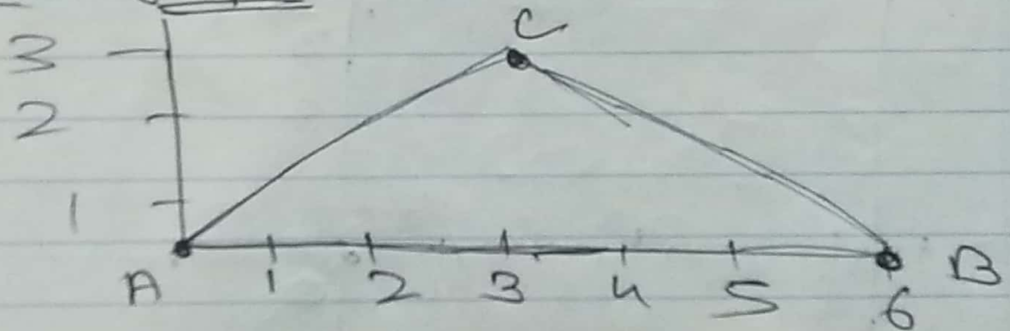
A(0,0) $\theta = 90°$

B(6,0)

C(3,3)

To find : values of A, B & C anticlockwise about origin & , 90°.

Solution graph



$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

Anticlock wise (matrix)T

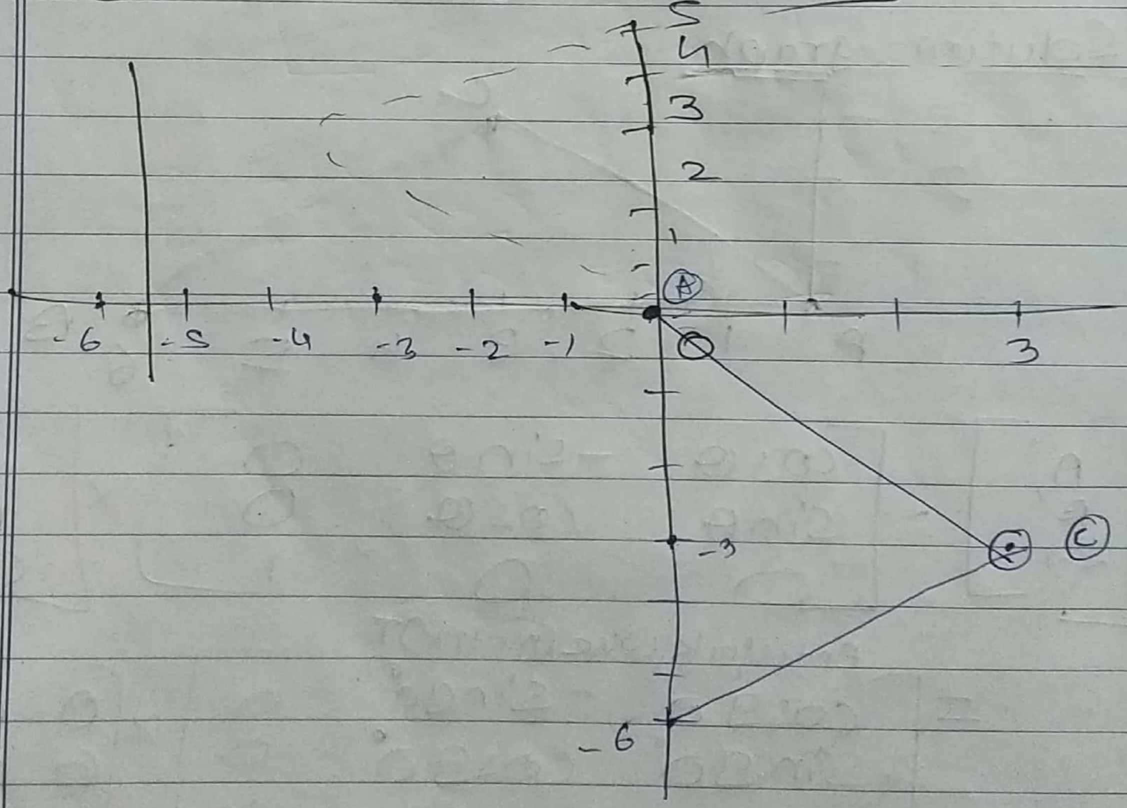
$$= \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & \cancel{0}+0+0 & 0+0+1 \\ 0+0+0 & -6+0+0 & 0+0+1 \\ 0+3+0 & -3+0+0 & 0+0+1 \end{bmatrix}$$

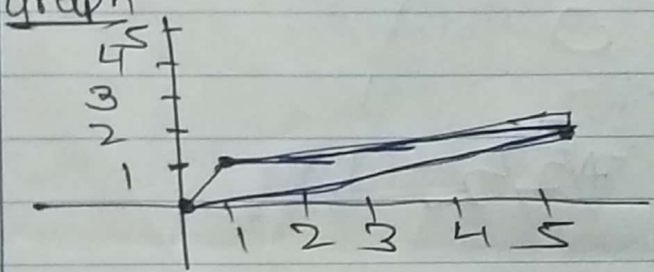
$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -6 & 1 \\ 3 & -3 & 1 \end{bmatrix}$$



Q) Perform a 45° rotation of triangle A(0,0), B(1,1) C(5,2)
i) About the origin.
ii) About P(-1,-1)

→ given
A(0,0)
B(1,1) $\theta = 45°$ for triangle Δ
C(5,2)

graph



Soln. A.B.C = $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$ — (1)

Rotation = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Anticlockwise

$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin 45 & 0 \\ -\sin\theta & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 3\sqrt{2}/2 & 7\sqrt{2}/2 & 1 \end{bmatrix}.$$

$A' = (0,0)$
$B' = (0, \sqrt{2})$
$C' = (3\sqrt{2}/2, 7\sqrt{2}/2)$