

Unit IV: Introduction to Curves.

(weightage 12 marks)

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5.1: Curve generation: Arc generation using DDA algorithm, Interpolation.

Questions

Explain curve generation using interpolation. (4m)

Explain the process of DDA using arc. (4m)

Define fractal lines. (2m)

Interpolation

* A curve having no simple mathematical definition can be drawn by using interpolation.

* Interpolation is method of constructing new data points within range of discrete set of known data points.

Rules for Interpolation:

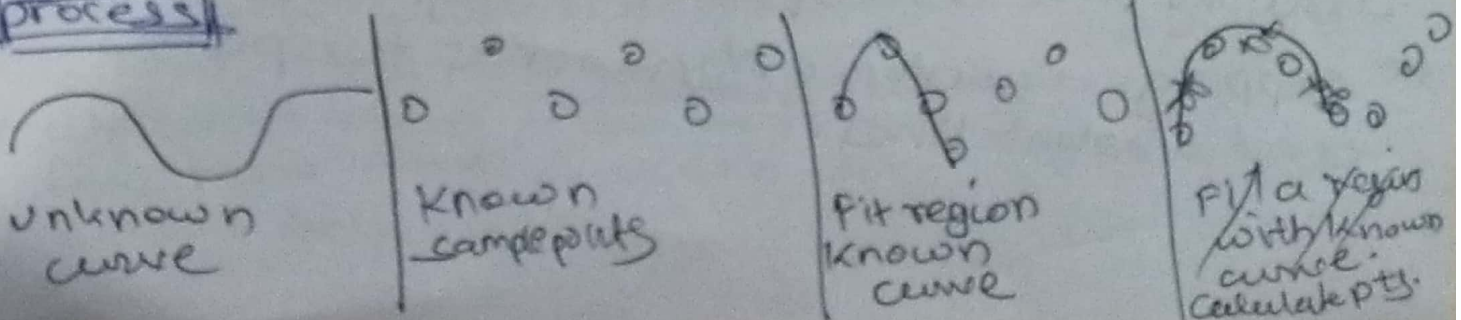
- ① The curve is specified by set of control points.
- ② Positions of the control points controls the shape of curve.
- ③ These control points are fitted with continuous polynomial function.
- ④ If the curve passes through the control points then it is called interpolation.

* Lagrange interpolation techniques is of type of interpolation.

* This technique is used when we have to draw curve by determining intermediate points between known sample point.

* When the given data points are not evenly distributed we can use this interpolation method to find solution.

Process





DDA Arc Generation

* Arc is a part of circle. If we join n numbers of arc then we get a complete circle.

* Digital differential Analyzer Algorithm uses the differential Equation of Curve.

* A curve is an infinitely large set of points.

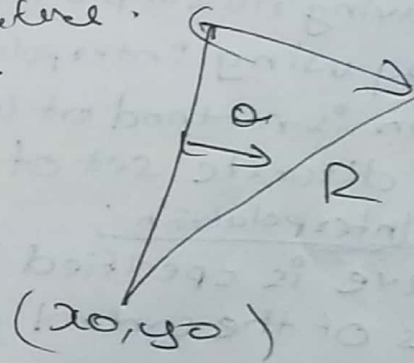
* Each point has two neighbours except endpoints.

X_0, Y_0 - Center of circle / curvature.

R - Radius of curvature.

θ - Angle

X, Y - the starting point of arc.



— Considering input we have to get a arc.

— This Eqⁿ of arc in angle parameter can be given as.

$$X = R \cos A + X_0$$

$$Y = R \sin A + Y_0$$

* It is easy to implement.

* It is easy for generating differential eqⁿs for ellipses and circle.

* Ellipse and circle can be implemented in hardware. So the display device can be capable of drawing arcs as well as line segments.

* Clipping algorithm only works for points and straight lines.

Fractal lines

- * Fractals is a complex picture created using iteration and a single formula.
- * Sometimes, object cannot be drawn with a given eqn or with a given geometry.
- * Example, mountains, clouds.
- * Examples so, their shape cannot be defined so in this case, we use fractals.
- * It is an never ending pattern & it is infinity.



5.2 Types of curve : Hilbert's Curve, Koch curve, B-spline, Bezier curve.

Types of curve

- | | |
|------------------|-------------------------------|
| → Bezier Curve | } Spline curve representation |
| → B-spline curve | |
| → Hilbert curve | } Fractals representation. |
| → Koch Curve | |

Questions

- Q1 Explain Koch Curve neat diagram. (1M)
- Q2 Write 'C' program to generate Hilbert's curve. (6M/4M)
- Q3 Draw Bezier curve. (2M)

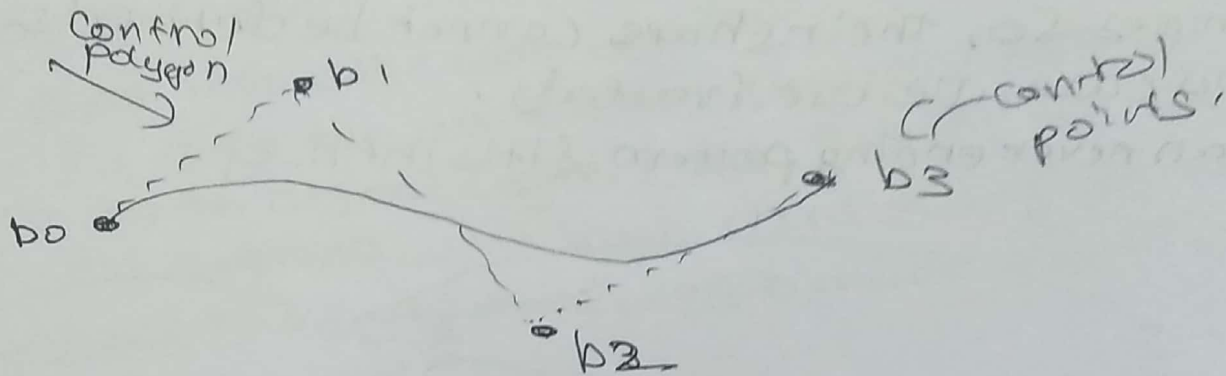
Bezier curve

- * The Bezier Curve is adequate/efficient for most graphic application. This curve requires four points (control).
- * This four control points completely specifies the curve additional points cannot be additional points can added like B-spline curve.
- * We cannot extend Bezier curve, but we can take four more points & we can construct four more second Bezier curve.

Bezier curve is a parametric curve defined by set of control points.

Two points are end of curve.
Others determine the shape of curve.

The following curve is an example,

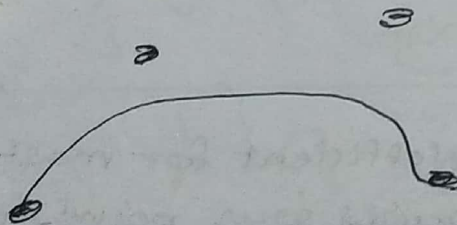


The Bezier curve is defined by a set of control points b_0, b_1, b_2 and b_3 .

b_0 & b_3 are end points. that control point of end.

b_1 & b_2 determines shape.

Another Example



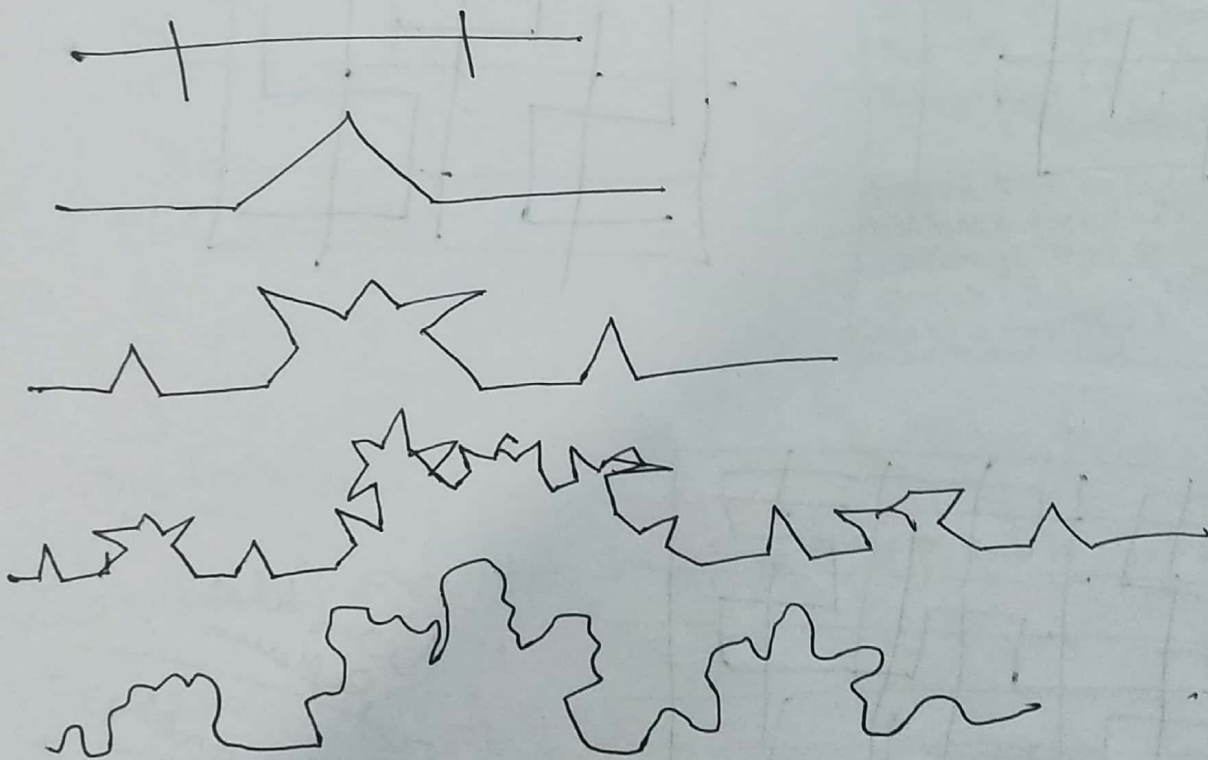
Properties

- * They generally follow the shape of control polygon, which consists of segments join the control points.
- * They always pass through first and last control pts.
- * A Bezier curve generally follows the shape of the defining polygon.
- * No straight line intersects a Bezier curve more times than it intersects control polygon.

Kotch Curve / Koch curve

- * In Koch curve, begin at line segment.
- * Divide line segment into three equal part.
- * Replace center part by same measure as equilateral triangle without third line.
- * The again follow the same iteration
- * This will give the curve which starts and ends at same place as the original segment but is built of unequal lengths segments, with each $1/3$ rd of original length.
- * The curve has gained more wiggles and its length is $16/9$ times original.

$$D = \log_3 4 / \log 3 = 1.261$$



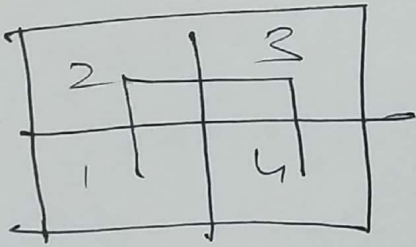
It is an fractal curve

It follows iterations

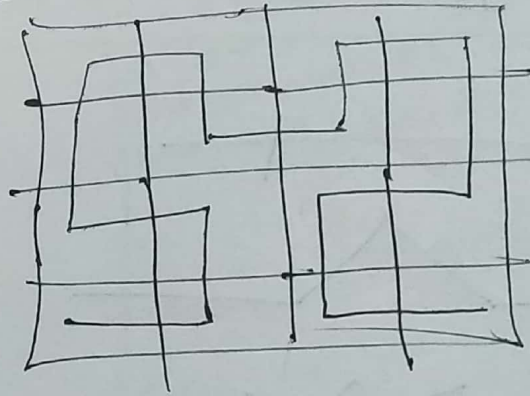
Hilbert curve

- The Hilbert curve is a space filling curve that visits every point in square grid with size of 2×2 , 4×4 , 8×8 , 16×16 or any power of 2.
- It was described by David Hilbert in 1892.
- Application - used in image processing especially compression.
- This Hilbert curve can be built by following successive approximation.
- It fractal representation.

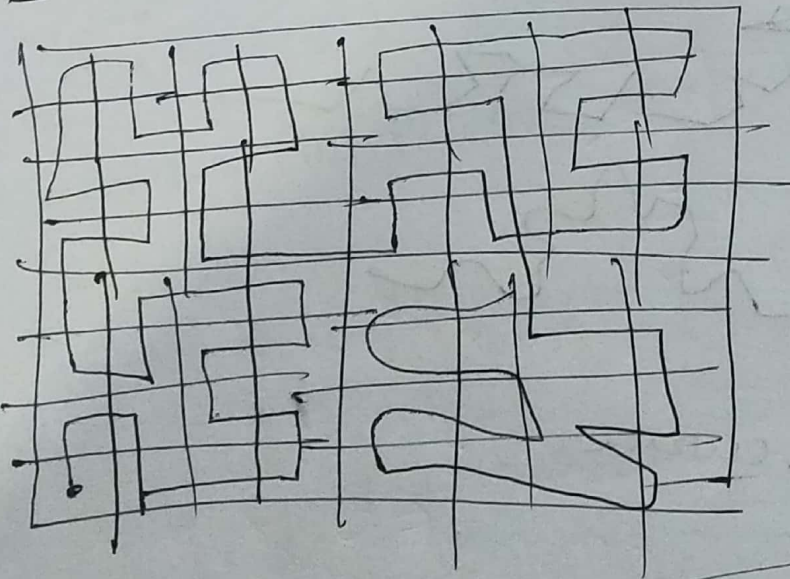
1st Approximation



Second



Third



works
repeatedly
continuously

It follows
iteration

'C' Program for Hilbert Curve

```
#include <stdio.h>
#include <math.h>
#include <graphics.h>
```

```
void move(int j, int h, int &x, int &y)
```

```
{
    if (j == 1)
    {
        y -= h;
    }
    if (j == 2)
    {
        x += h;
    }
    if (j == 3)
    {
        y += h;
    }
    if (j == 4)
    {
        x -= h;
    }
}
```

```
line to(x,y);
```

```
}
void hilbert(int o, int d, int l, int u, int i, int h, int &x, int &y)
```

```
{
    if (i > 0)
    {
        hilbert(d, r, u, l, i, h, x, y);
        move(r, h, x, y);
        delay(100);
        hilbert(r, d, l, u, i, h, x, y);
        move(d, h, x, y);
        hilbert delay(100);
        hilbert(r, d, l, u, i, h, x, y);
        move(l, h, x, y);
        delay(100);
        hilbert(u, d, r, l, i, h, x, y);
    }
}
```

```
int main() {
    int n, x1, y1;
    int x0 = 50, y0 = 150, x, y, h = 10;
    o = 2, d = 3, l = 4, u = 1;
    clrscr();
    printf("give value of n ");
    scanf("%d", &n);
    x = x0;
    y = y0;
    int gm, gd = DETECT;
    initgraph(&gd, &gm, "C:\\Turbo3\\bgi");
    move to(x, y);
    hilbert(o, d, l, u, n, h, x, y);
    delay(100);
    getch();
    closegraph();
    return 0;
}
```

Hilbert curve

```
void move( int direction, int step, int x, int y )
```

```
{  
    if ( direction == 0 )
```

```
    {  
        y -= step;  
    }
```

```
    else if ( direction == 1 )
```

```
    {  
        x += step;  
    }
```

```
    else if ( direction == 2 )
```

```
    {  
        y += step;  
    }
```

```
    else if ( direction == 3 )
```

```
    {  
        x -= step;  
    }
```

```
    line(x, y);
```

```
void hilbert( int right, int down, int left, int up,  
              int iterations, int step, int x, int y )
```

```
{  
    if ( iterations > 0 )
```

```
    {  
        iterations --;
```

```
        hilbert( d, r, u, l, iterations, step, x, y );
```

```
        move( right, step, x, y );
```

```
        hilbert( r,
```

d, r, u, l ⇒ right
r, d, l, u ⇒ down.

d, r, d, l, u ⇒ left.
u, l, d, r ⇒

