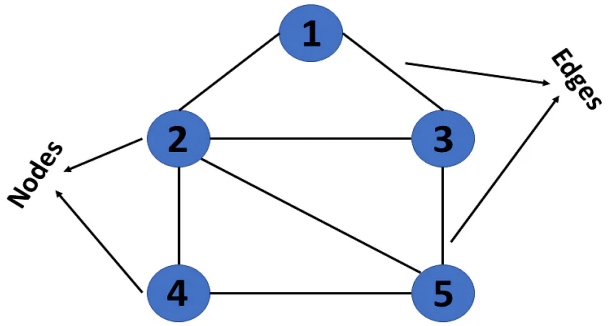
Graph

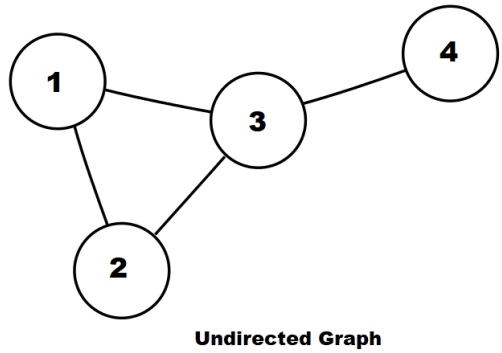
- Graphs in data structures are non-linear data structures made up of a finite number of nodes or vertices and the edges that connect them.
- Graphs in data structures are used to address real-world problems in which it represents the problem area as a network like telephone networks, circuit networks, and social networks.
- For example, it can represent a single user as nodes or vertices in a telephone network, while the link between them via telephone represents edges.
- A graph is a non-linear kind of data structure made up of nodes or vertices and edges. The edges connect any two nodes in the graph, and the nodes are also known as vertices.
- This graph has a set of vertices V= { 1,2,3,4,5} and a set of edges E= { (1,2),(1,3),(2,3),(2,4),(2,5),(3,5),(4,50 }.



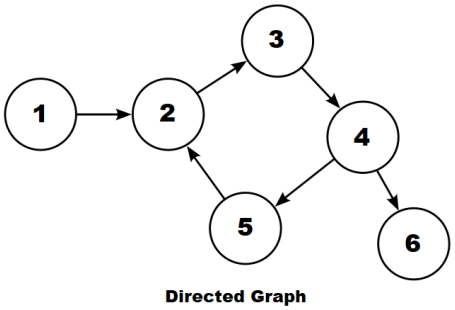
Types of Graphs in Data Structure

The most common types of graphs in the data structure are mentioned below:

1. Undirected: A graph in which all the edges are bi-directional. The edges do not point in a specific direction.



2. Directed: A graph in which all the edges are directional. If an edge between any two nodes is directionally oriented, a graph is referred as directed graph or digraph.

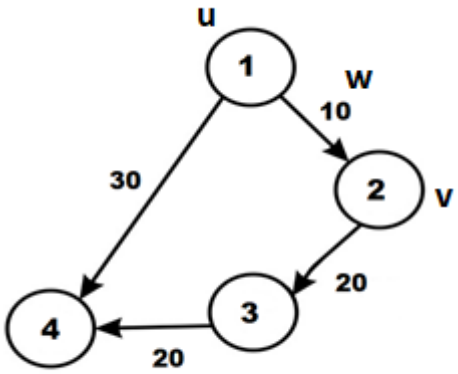


Directed Graph

3. Weighted Graph: A graph that has a value associated with every edge. The values corresponding to the edges are called weights. A value in a weighted graph can represent quantities such as cost, distance, and time, depending on the graph. Weighted graphs are typically used in modeling computer networks.

An edge in a weighted graph is represented as (u, v, w), where:

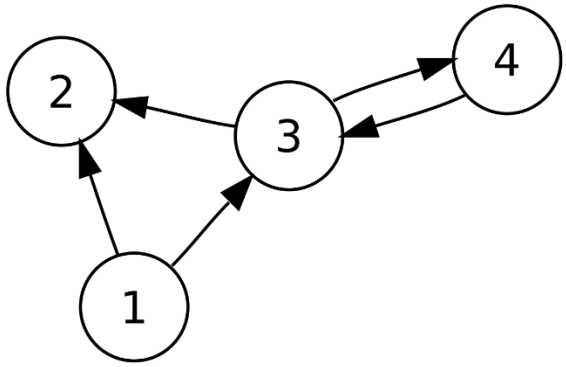
- u is the source vertex
- v is the destination vertex
- w represents the weight associated to go from u to v



Weighted Graph

4. Unweighted Graph: A graph in which there is no value or weight associated with the edge. All the graphs are unweighted by default unless there is a value associated.

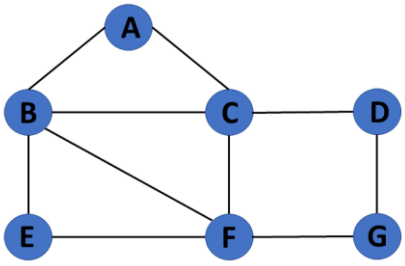
Graph Terminology



Vertex	Every individual data element is called a vertex or a node.
Edge (Arc)	It is a connecting link between two nodes or vertices. Each edge has two ends and is represented as (startingVertex, endingVertex).
Undirected Edge	It is a bidirectional edge.
Directed Edge	It is a unidirectional edge.
Weighted Edge	An edge with value (cost) on it.
Degree	The total number of edges connected to a vertex in a graph.
Indegree	The total number of incoming edges connected to a vertex.
Outdegree	The total number of outgoing edges connected to a vertex.
Self-loop	An edge is called a self-loop if its two endpoints coincide with each other.
Adjacency	Vertices are said to be adjacent to one another if there is an edge connecting them.

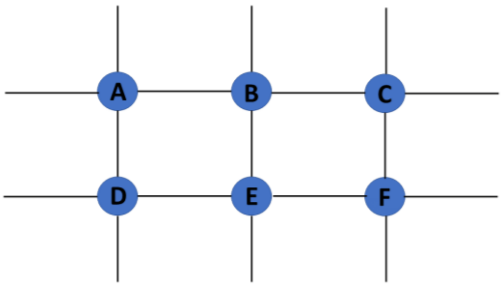
Finite Graph

The graph G=(V, E) is called a finite graph if the number of vertices and edges in the graph is limited in number



Infinite Graph

The graph G=(V, E) is called a finite graph if the number of vertices and edges in the graph is interminable.

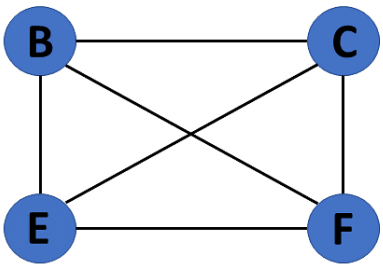


Trivial Graph

A graph G= (V, E) is trivial if it contains only a single vertex and no edges.

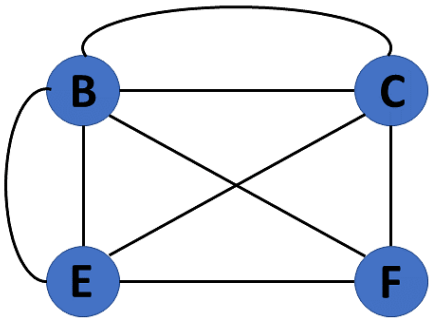
Simple Graph

If each pair of nodes or vertices in a graph G=(V, E) has only one edge, it is a simple graph. As a result, there is just one edge linking two vertices, depicting one-to-one interactions between two elements.



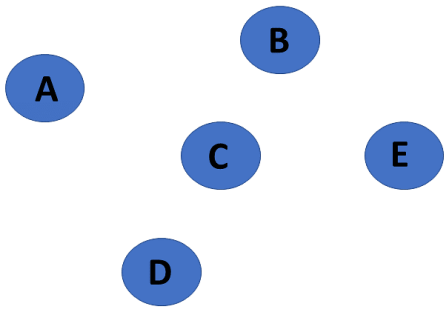
Multi Graph

If there are numerous edges between a pair of vertices in a graph G= (V, E), the graph is referred to as a multigraph. There are no self-loops in a Multigraph.



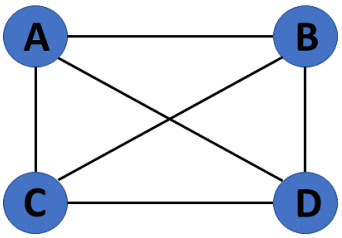
Null Graph

It's a reworked version of a trivial graph. If several vertices but no edges connect them, a graph G= (V, E) is a null graph.



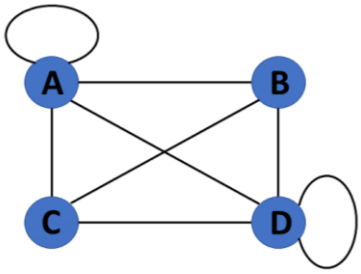
Complete Graph

If a graph G= (V, E) is also a simple graph, it is complete. Using the edges, with n number of vertices must be connected. It's also known as a full graph because each vertex's degree must be n-1.



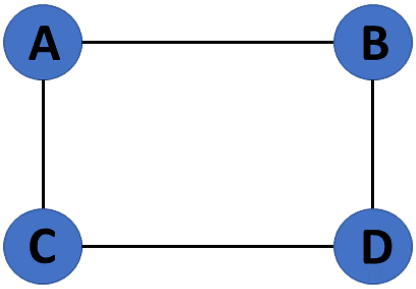
Pseudo Graph

If a graph G= (V, E) contains a self-loop besides other edges, it is a pseudograph.



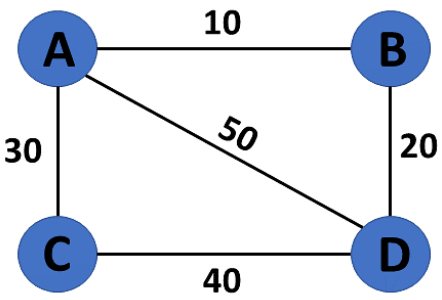
Regular Graph

If a graph G= (V, E) is a simple graph with the same degree at each vertex, it is a regular graph. As a result, every whole graph is a regular graph.



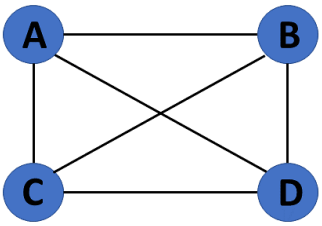
Weighted Graph

A graph G= (V, E) is called a labeled or weighted graph because each edge has a value or weight representing the cost of traversing that edge.



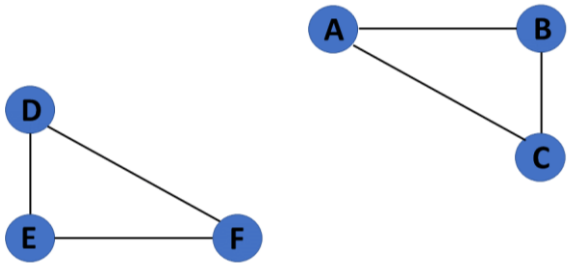
Connected Graph

If there is a path between one vertex of a graph data structure and any other vertex, the graph is connected.



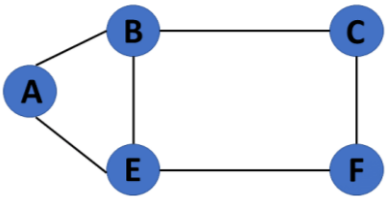
Disconnected Graph

When there is no edge linking the vertices, you refer to the null graph as a disconnected graph.



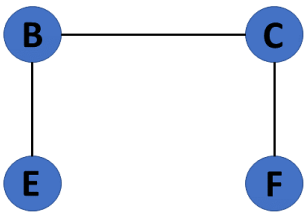
Cyclic Graph

If a graph contains at least one graph cycle, it is considered to be cyclic.



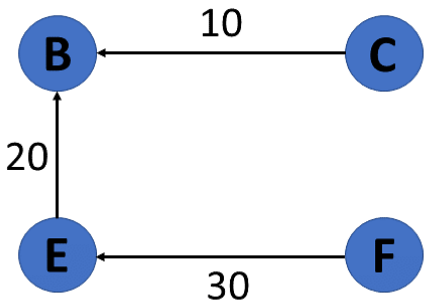
Acyclic Graph

When there are no cycles in a graph, it is called an acyclic graph.



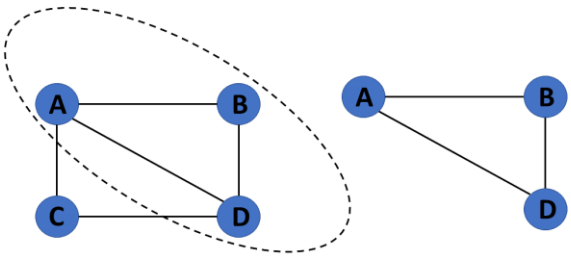
Directed Acyclic Graph

It's also known as a directed acyclic graph (DAG), and it's a graph with directed edges but no cycle. It represents the edges using an ordered pair of vertices since it directs the vertices and stores some data.



Subgraph

The vertices and edges of a graph that are subsets of another graph are known as a subgraph.



Representation of Graph

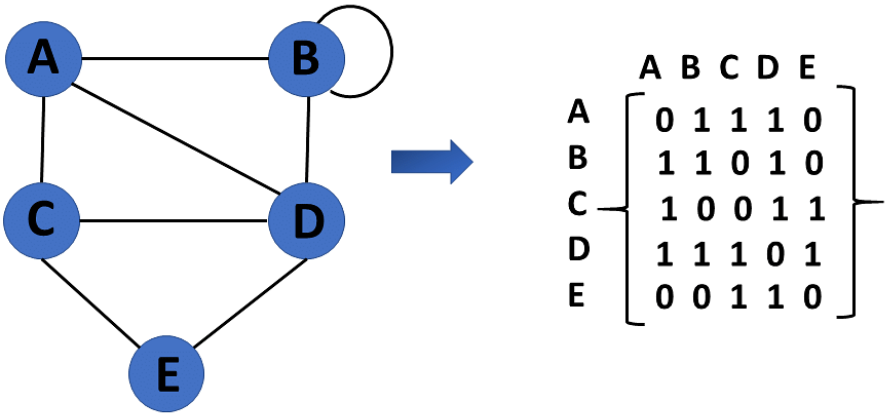
The most frequent graph representations are the two that follow:

- using array as a Adjacency matrix (sequential representation)
- using linked list as a Adjacency list (linked list representation)

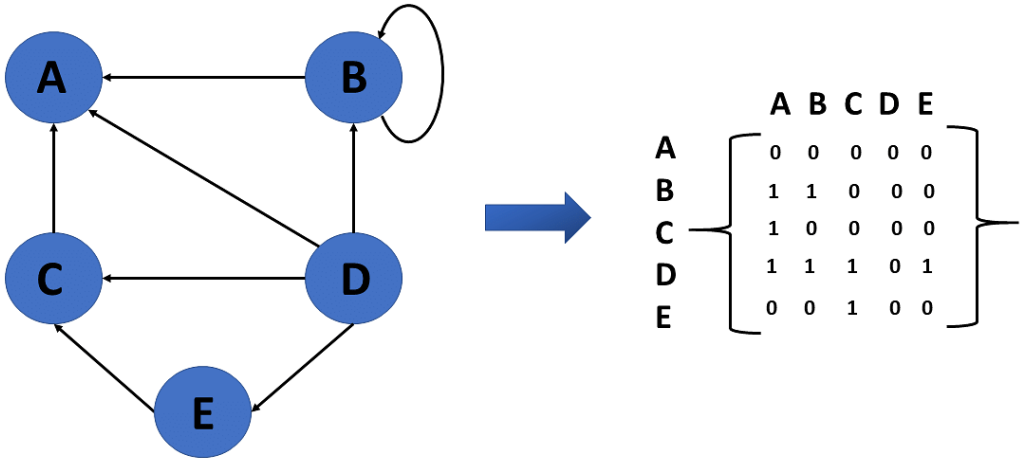
Adjacency Matrix

- A sequential representation is an adjacency matrix.
- It's used to show which nodes are next to one another. I.e., is there any connection between nodes in a graph?
- You create an MXM matrix G for this representation. If an edge exists between vertex a and vertex b, the corresponding element of G, gi,j = 1, otherwise gi,j = 0.
- If there is a weighted graph, you can record the edge's weight instead of 1s and 0s.

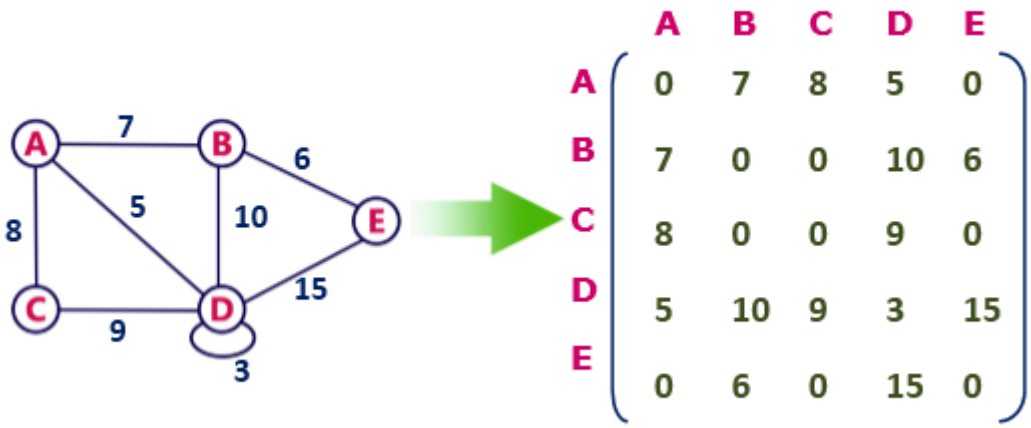
Undirected Graph Representation



Directed Graph Representation



Undirected weighted graph represenation



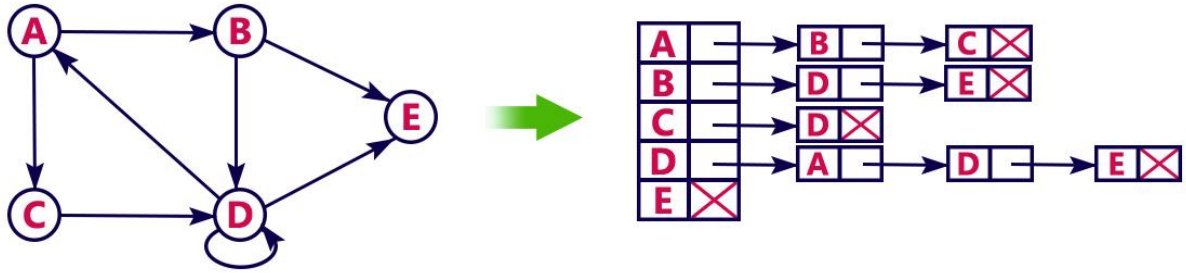
Advantages : Representation is easier to implement and follow.

Disadvantages: It takes a lot of space and time to visit all the neighbors of a vertex, we have to traverse all the vertices in the graph, which takes quite some time.

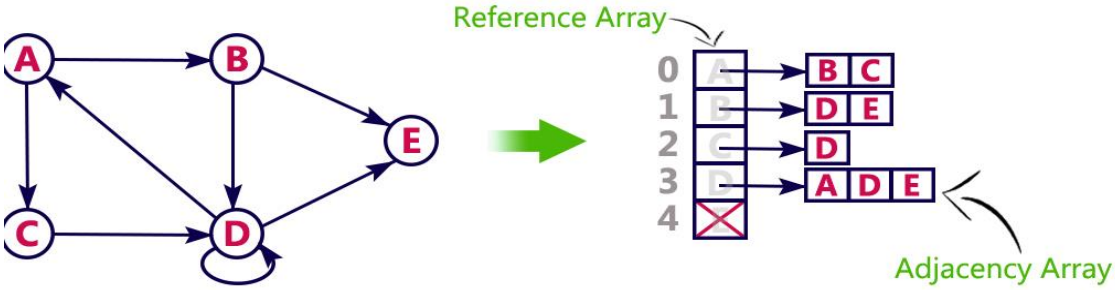
Adjacency list

- Adjacency list is a linked representation.
- In this representation, for each vertex in the graph, we maintain the list of its neighbors. It means, every vertex of the graph contains list of its adjacent vertices.
- We have an array of vertices which is indexed by the vertex number and for each vertex v, the corresponding array element points to a **singly linked list** of neighbors of v.

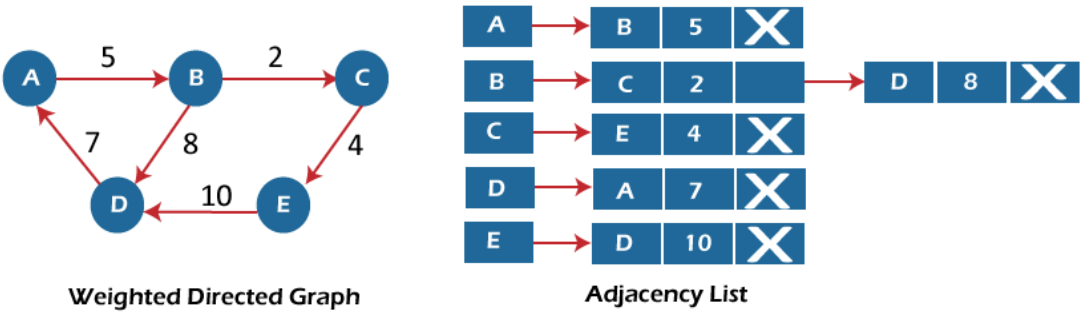
- In this representation, every vertex of a graph contains list of its adjacent vertices.
- For example, consider the following directed graph representation implemented using linked list...



This representation can also be implemented using an array as follows.



Weighted directed graph



Advantages :

- Adjacency list saves lot of space.
- We can easily insert or delete as we use linked list.
- Such kind of representation is easy to follow and clearly shows the adjacent nodes of node.

Disadvantages :

- The adjacency list allows testing whether two vertices are adjacent to each other but it is slower to support this operation.

Graph traversal

Graph traversal is a technique used for searching a vertex in a graph. The graph traversal is also used to decide the order of vertices is visited in the search process. A graph traversal finds the edges to be used in the search process without creating loops. That means using graph traversal we visit all the vertices of the graph without getting into looping path.

There are two graph traversal techniques and they are as follows...

1. **DFS (Depth First Search)**
2. **BFS (Breadth First Search)**

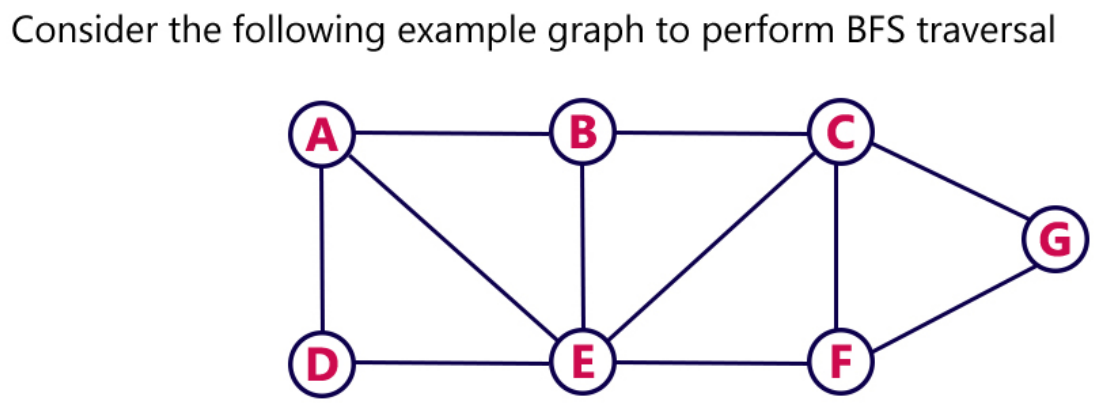
BFS (Breadth First Search)

BFS traversal of a graph produces a **spanning tree** as final result. **Spanning Tree** is a graph without loops. We use **Queue data structure** with maximum size of total number of vertices in the graph to implement BFS traversal.

We use the following steps to implement BFS traversal...

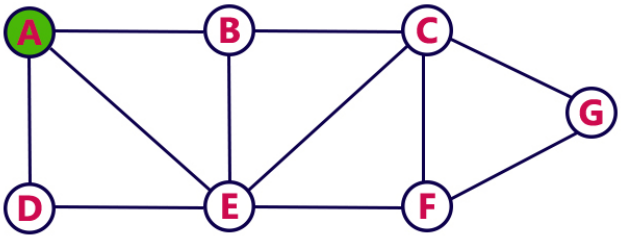
- **Step 1 -** Define a Queue of size total number of vertices in the graph.
- **Step 2 -** Select any vertex as **starting point** for traversal. Visit that vertex and insert it into the Queue.
- **Step 3 -** Visit all the non-visited **adjacent** vertices of the vertex which is at front of the Queue and insert them into the Queue.
- **Step 4 -** When there is no new vertex to be visited from the vertex which is at front of the Queue then delete that vertex.
- **Step 5 -** Repeat steps 3 and 4 until queue becomes empty.
- **Step 6 -** When queue becomes empty, then produce final spanning tree by removing unused edges from the graph

Example:



Step 1:

- Select the vertex **A** as starting point (visit **A**).
- Insert **A** into the Queue.

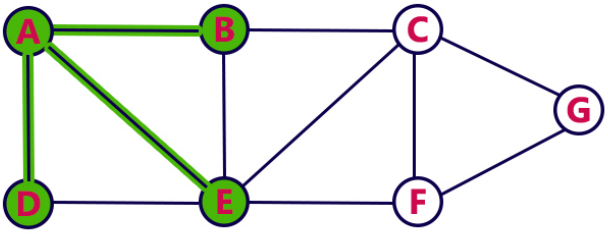


Queue

A						

Step 2:

- Visit all adjacent vertices of **A** which are not visited (**D**, **E**, **B**).
- Insert newly visited vertices into the Queue and delete A from the Queue..

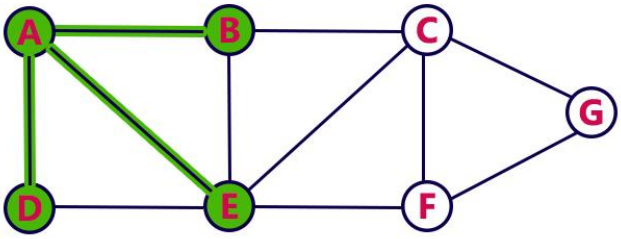


Queue

	D	E	B			

Step 3:

- Visit all adjacent vertices of **D** which are not visited (there is no vertex).
- Delete D from the Queue.

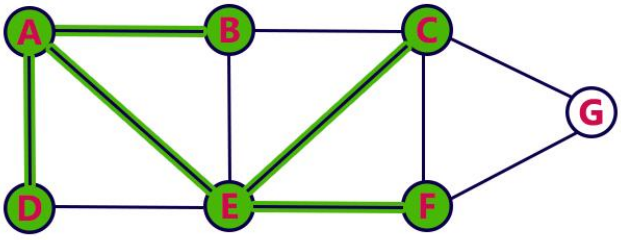


Queue

		E	B			

Step 4:

- Visit all adjacent vertices of **E** which are not visited (**C**, **F**).
- Insert newly visited vertices into the Queue and delete E from the Queue.

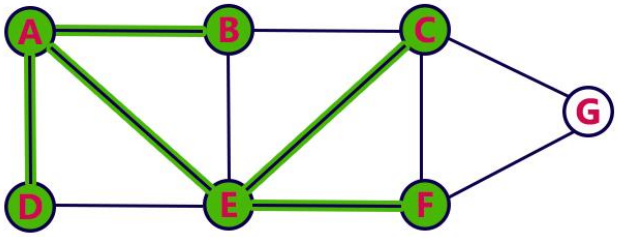


Queue

			B	C	F	

Step 5:

- Visit all adjacent vertices of **B** which are not visited (**there is no vertex**).
- Delete **B** from the Queue.

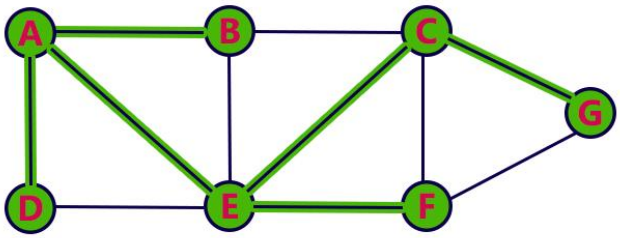


Queue

				C	F	

Step 6:

- Visit all adjacent vertices of **C** which are not visited (**G**).
- Insert newly visited vertex into the Queue and delete **C** from the Queue.

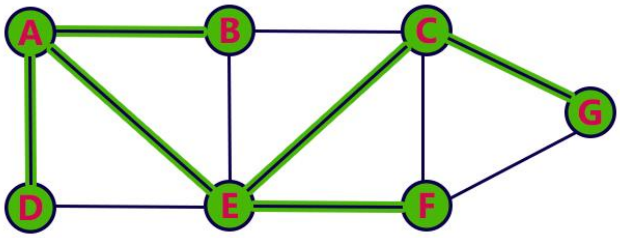


Queue

					F	G

Step 7:

- Visit all adjacent vertices of **F** which are not visited (**there is no vertex**).
- Delete **F** from the Queue.

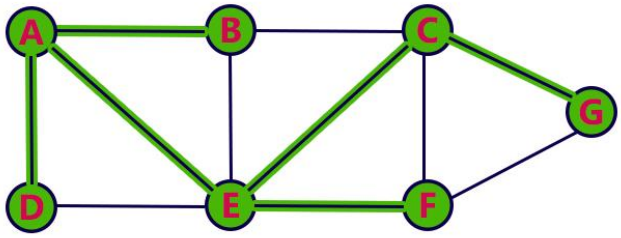


Queue

						G

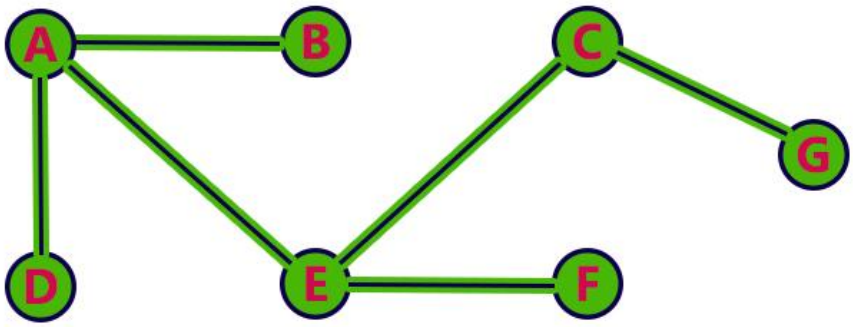
Step 8:

- Visit all adjacent vertices of **G** which are not visited (**there is no vertex**).
- Delete **G** from the Queue.



Queue

- Queue became Empty. So, stop the BFS process.
- Final result of BFS is a Spanning Tree as shown below...



DFS (Depth First Search)

DFS traversal of a graph produces a **spanning tree** as final result. **Spanning Tree** is a graph without loops. We use **Stack data structure** with maximum size of total number of vertices in the graph to implement DFS traversal.

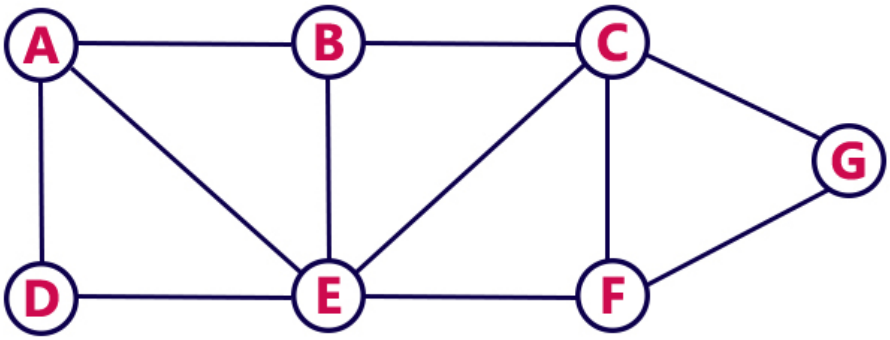
We use the following steps to implement DFS traversal...

- **Step 1 -** Define a Stack of size total number of vertices in the graph.
- **Step 2 -** Select any vertex as **starting point** for traversal. Visit that vertex and push it on to the Stack.
- **Step 3 -** Visit any one of the non-visited **adjacent** vertices of a vertex which is at the top of stack and push it on to the stack.
- **Step 4 -** Repeat step 3 until there is no new vertex to be visited from the vertex which is at the top of the stack.
- **Step 5 -** When there is no new vertex to visit then use **back tracking** and pop one vertex from the stack.
- **Step 6 -** Repeat steps 3, 4 and 5 until stack becomes Empty.
- **Step 7 -** When stack becomes Empty, then produce final spanning tree by removing unused edges from the graph

Back tracking is coming back to the vertex from which we reached the current vertex.

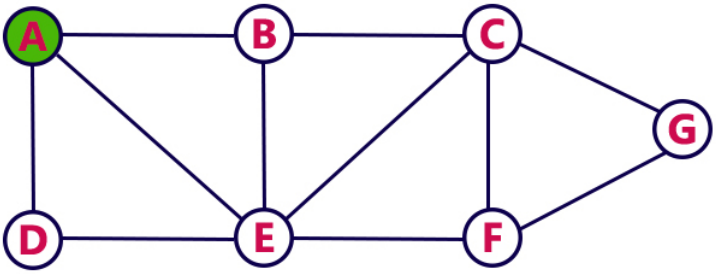
Example

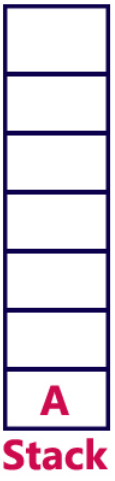
Consider the following example graph to perform DFS traversal



Step 1:

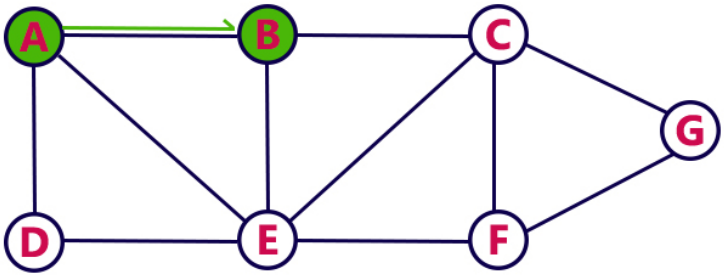
- Select the vertex **A** as starting point (visit **A**).
- Push **A** on to the Stack.





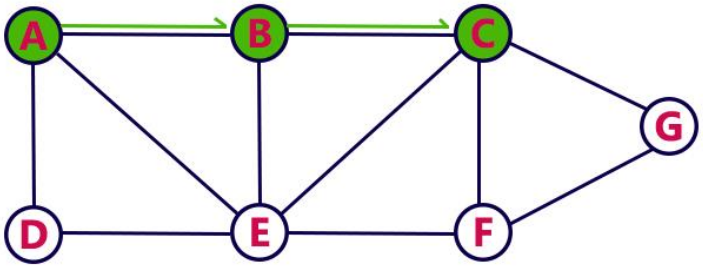
Step 2:

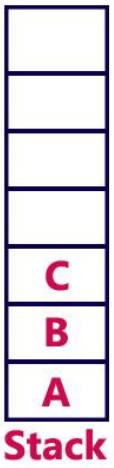
- Visit any adjacent vertex of **A** which is not visited (**B**).
- Push newly visited vertex B on to the Stack.



Step 3:

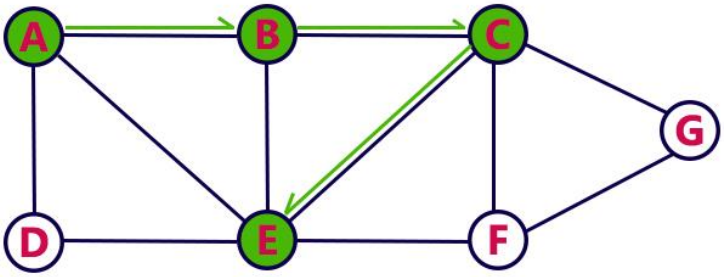
- Visit any adjacent vertext of **B** which is not visited (**C**).
- Push C on to the Stack.

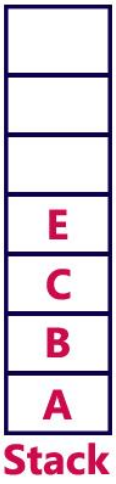




Step 4:

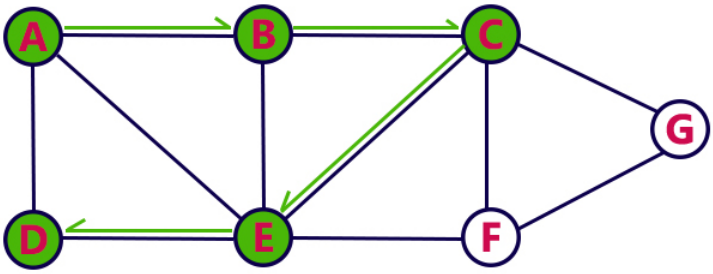
- Visit any adjacent vertext of **C** which is not visited (**E**).
- Push E on to the Stack





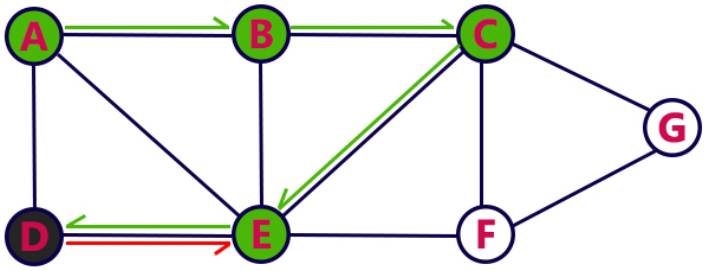
Step 5:

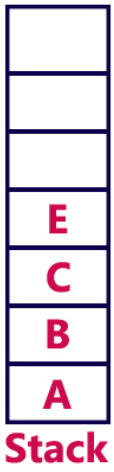
- Visit any adjacent vertext of **E** which is not visited (**D**).
- Push D on to the Stack



Step 6:

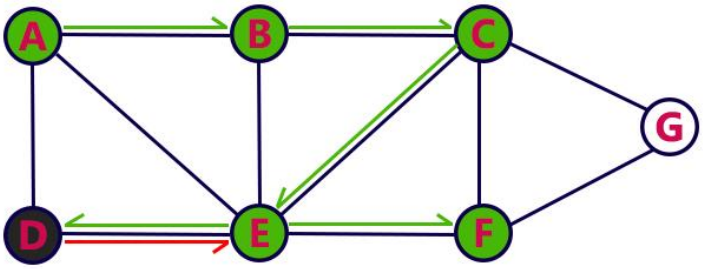
- There is no new vertiex to be visited from D. So use back track.
- Pop D from the Stack.

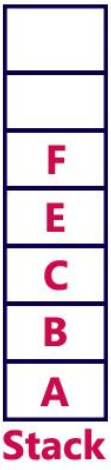




Step 7:

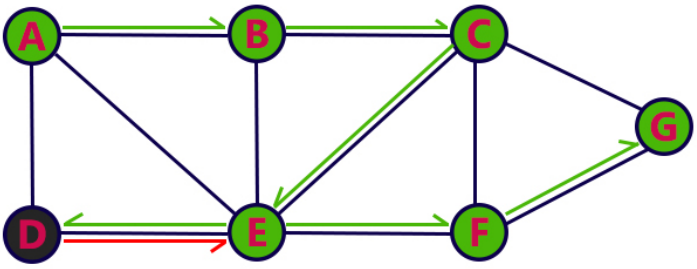
- Visit any adjacent vertex of **E** which is not visited (**F**).
- Push **F** on to the Stack.





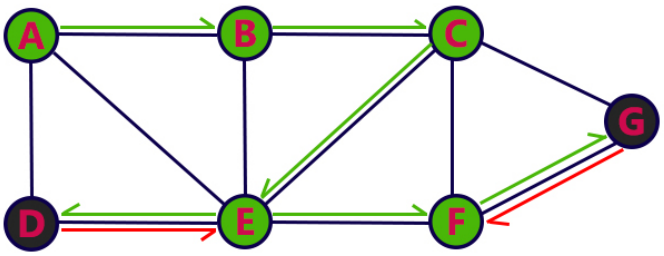
Step 8:

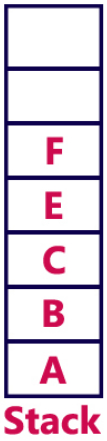
- Visit any adjacent vertex of **F** which is not visited (**G**).
- Push **G** on to the Stack.



Step 9:

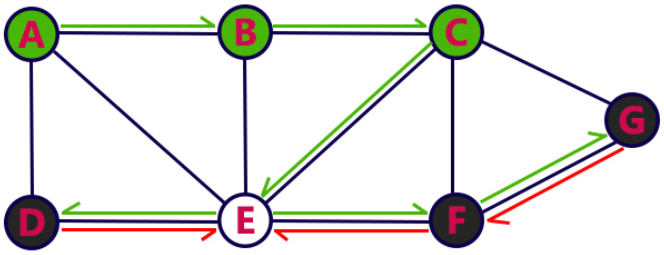
- There is no new vertiex to be visited from G. So use back track.
- Pop G from the Stack.

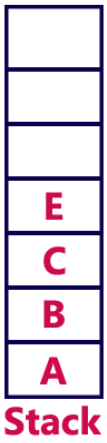




Step 10:

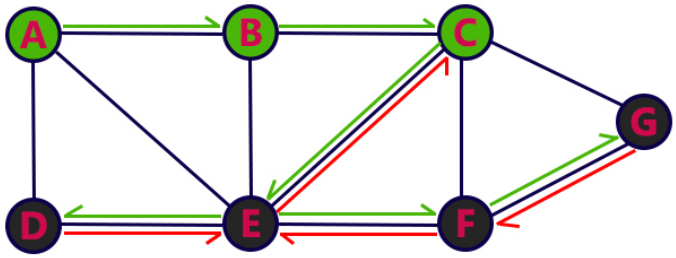
- There is no new vertiex to be visited from F. So use back track.
- Pop F from the Stack.

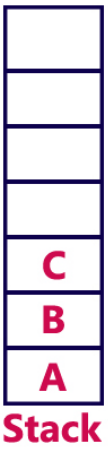




Step 11:

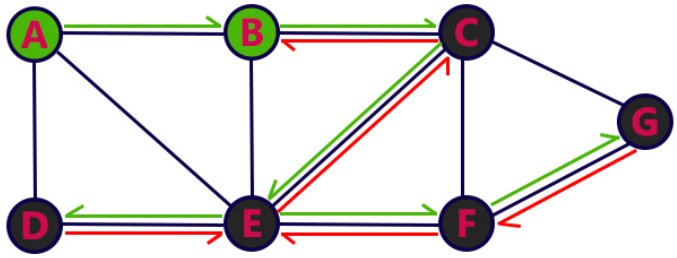
- There is no new vertiex to be visited from E. So use back track.
- Pop E from the Stack.





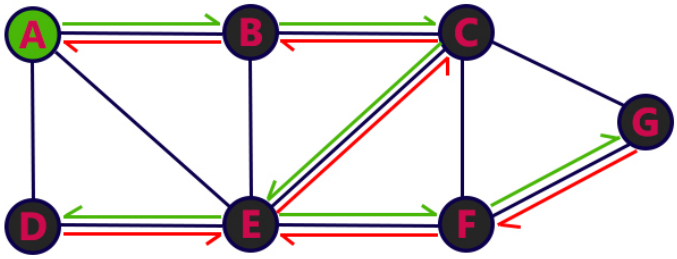
Step 12:

- There is no new vertiex to be visited from C. So use back track.
- Pop C from the Stack.



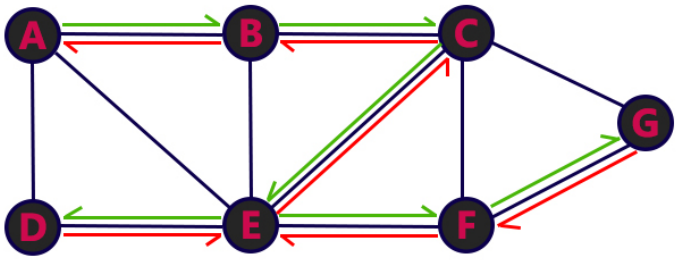
Step 13:

- There is no new vertiex to be visited from B. So use back track.
- Pop B from the Stack.



Step 14:

- There is no new vertiex to be visited from A. So use back track.
- Pop A from the Stack.



- Stack became Empty. So stop DFS Treversal.
- Final result of DFS traversal is following spanning tree.