Oxford Mathematics Team Challenge
Scenes Round Question Booklet

SAMPLE SET

Instructions

1. Do not look at the first set of questions until the invigilator tells you to do so.
2. **Format.** This round contains two sections (the "acts"), which each contain four sets (the "scenes") of 3 questions each. You must submit solutions to your current scene in order to receive the next scene in the act. You cannot update your answers to any set once submitted.
3. **Time limit.** 30 minutes for each section. You may not submit any answers after the allotted time has expired.
4. **No calculators, squared paper or measuring instruments.** Lined paper and blank paper for rough working is allowed. You may use a pen or pencil to preference. Other mediums for working (e.g. digital devices, whiteboards, augury) are strictly forbidden.
5. **Scoring rules.** For the first seven scenes, your team is awarded full points for each correctly answer, and zero points otherwise. Answers must be fully simplified and in exact form; approximations will not be accepted. In the final set, which is the *estimathon*, your score for each question is $9 \times \frac{\min\{A,E\}}{\max\{A,E\}}$, rounded to the nearest integer, where A is the actual answer and E is your estimated answer.
6. Don't expect to solve every question in the time given! You may consider skipping to the estimation set at the end of the round, taking some time to attempt the questions to score partial credit.
7. You are also encouraged to think deeply, rather than to guess.
8. Good luck, and enjoy! ☺

Act	I				II			
Scene	1	2	3	4	1	2	3	4
Points per question	3	4	5	6	3	4	6	10

Act I, Scene 1

Questions in this scene are worth 3 points

1A. How many integers a are there such that $x^2 - ax + 50 = 0$ has two distinct integer roots?

1B. A *lattice point* in the plane is a point (x, y) where x, y are both integers. How many lattice points lie in the region $x^2 + y^2 \leq 12$?

1C. A 3-digit number is called *geometric* if it has 3 distinct digits which, when read from left to right, form a geometric sequence. What is the difference between the largest and smallest geometric numbers?

Act I, Scene 2

Questions in this scene are worth 4 points

2A. What is the third smallest positive integer whose digits sum to 25?

2B. Each morning, Arav has to decide how to get from his home to his maths class, which is 5 miles due south of his home. He can cycle at 10 mph; alternatively, he can jog at 6 mph to the bus stop a mile due north of his home, then take the bus which arrives every 5 minutes and takes a direct route to the maths class at 20 mph.

If Arav leaves his house at a random time, the probability that the bus will get him to class fater than cycling is p/q, where p/q is fully simplified. What is $p + q$?

2C. Hannah is standing on the vertex of a cube. For each of the other vertices, one of her friends is standing on it. The sum of the distances from her to each of her friends is $a + b\sqrt{2} + c\sqrt{3}$. What is $a + b + c$?

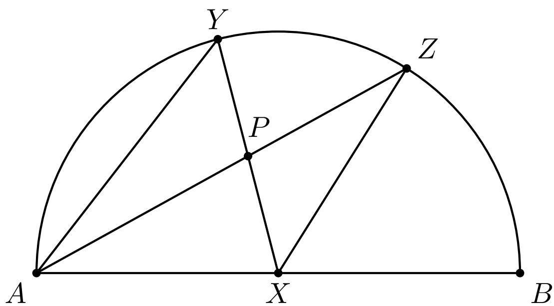
Act I, Scene 3

Questions in this scene are worth 5 points

3A. Tom is writing a list of letters consisting of O, M, T, C. After writing down 7 letters, he discovers that he has spelled out his name in three consecutive letters precisely once. How many possible sequences of letters could Tom have written?

3B. Alex and Brennan each have positive integers. The highest common factor of their numbers is 12, and their lowest common denominator is 720. What's the smallest possible sum of Alex and Brennan's numbers?

3C. Let AB be the diameter of a semicircle, and let X be the midpoint of A and B. Choose points Y and Z on the arc of the semicircle such that $AY < AZ$. Let P be the intersection of AZ and XY, and suppose that $AP = 4$, $PX = 2$, $PY = 2$, and $PZ = 3$. It follows that $AY = \sqrt{a}$. What does a equal?



Act I, Scene 4

Questions in this scene are worth 6 points

4A. Bernie writes down the expression

$$1! + 2! + \cdots + 100!$$

What is the tens digit of Bernie's expression?

4B. Jimbo wants to draw a decagon, but chooses to be a bit chaotic. Each minute, he rolls a standard 6-sided die and draws that many edges until he has drawn all ten edges in which case he stops.

The probability that Jimbo finishes drawing the decagon after exactly three rolls is p/q, when fully simplified. What does $p+q$ equal?

4C. Sam and Owen are stood 200 m apart in a large foggy field. If they ever get within 100 m of each other, they will see each other. Sam and Owen both turn to a random direction and simultaneously begin walking in a straight path. Sam walks at a constant rate of 1 ms^{-1}, and Owen walks at a constant rate of 2 ms^{-1}. The probability that Sam and Owen find each other is $1/p$; what does p equal?

Oxford Mathematics Team Challenge
Scenes Round Question Booklet

SAMPLE SET

Instructions

1. Do not look at the first set of questions until the invigilator tells you to do so.

2. **Format.** This round contains two sections (the "acts"), which each contain four sets (the "scenes") of 3 questions each. You must submit solutions to your current scene in order to receive the next scene in the act. You cannot update your answers to any set once submitted.

3. **Time limit.** 30 minutes for each section. You may not submit any answers after the allotted time has expired.

4. **No calculators, squared paper or measuring instruments.** Lined paper and blank paper for rough working is allowed. You may use a pen or pencil to preference. Other mediums for working (e.g. digital devices, whiteboards, augury) are strictly forbidden.

5. **Scoring rules.** For the first seven scenes, your team is awarded full points for each correctly answer, and zero points otherwise. Answers must be fully simplified and in exact form; approximations will not be accepted. In the final scene, the *estimathon*, your score for each question is $9 \times \frac{\min\{A,E\}}{\max\{A,E\}}$, rounded to the nearest integer, where A is the actual answer and E is your estimated answer.

6. Don't expect to solve every question in the time given! You may consider skipping to the estimation set at the end of the round, taking some time to attempt the questions to score partial credit.

7. You are also encouraged to think deeply, rather than to guess.

8. Good luck, and enjoy! ☺

Set	1	2	3	4	5	6	7	8
Points per question	3	4	5	6	3	4	6	10

Act II, Scene 1

Questions in this scene are worth 3 points

1A. What's the sum of the solutions of

$$\log(x-2)+2\log(x-3)=\log(10x-38)?$$

1B. The rectangle $ABCD$ has side lengths $AB=9$, $BC=6$. Point P is chosen inside the rectangle such that the areas of $ABPD$, BCP and CDP are all equal. The length AP equals $\sqrt{k}$; what does k equal?

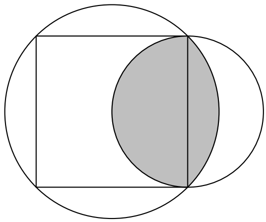
1C. An n-sided regular polygon has an internal angle just less than 145°. What is the largest value of n?

Act II, Scene 2

Questions in this scene are worth 4 points

2A. Abigail the Albatross starts on the top-left square of a 50×25 grid. Every move, she can either walk to the square directly below, walk to the square directly to the right, or fly to any square on the board. What is the fewest number of times Abigail needs to fly to visit all the squares on the board?

2B. Jimbo is drawing a coffee bean. He first draws a square of side length 2, draws a circle whose diameter is a side of the square, and then draws another circle circumscribing the square. The intersecting region of the two circles is his coffee bean. The area of his coffee bean is $a\pi + b$. What does $5a + b$ equal?



2C. The sequence a_n has initial values $a_0 = 1$ and $a_1 = 3$. For $n \geq 2$,

$$a_n = 6a_{n-1} - 9a_{n-2}$$

If $a_{20} = 2^\ell 3^m 5^n 7^o$, what does $\ell + m + n + o$ equal?

Act II, Scene 3

Questions in this scene are worth 6 points

3A. Let S be the set containing the terms in the arithmetic sequence $1, 8, 15, \ldots,$ that are less than 2000. Mr Healy takes a subset of S with n elements. No matter what subset of size n Mr Healy chose, n is just large enough such that Mr Dobby can always find two elements in Mr Healy's subset which sum to 2025.

What does n equal?

3B. Zach spends £1 to play a game. Each round, a coin is flipped. If it comes up as heads, he doubles his money; if it comes up as tails, he loses all of his money. He can withdraw at any point and claim all of the money he has won.

Zach maximises the probability that he wins at least £1 back; this probability is p/q, where p, q have highest common factor 1. What is $p + q$?

3C. The graph $\ln x$ is stretched vertically by a factor b and results in the function $f(x)$. Given that $8^{f(x)} = x^2$ for all x, what is the value of $e^{2/b}$?

Act II, Scene 4: Estimathon

Questions in this scene are worth up to 9 points: where A is the actual answer and E is your estimate for a question, your score is $9 \times \frac{\min(A,E)}{\max(A,E)}$ points, rounded to the nearest integer

4A. How many prime factors does 1000! have? *[For example,* $12 = 2^2 \times 3$ *has three prime factors.]*

4B. *Recamán's alternative sequence* is defined as follows. Initially, $u_0 = 1$. For all subsequent terms,

$$u_{n+1} = \begin{cases} u_n/n & \text{if } n \text{ is a factor of } u_n \\ u_n \times n & \text{otherwise} \end{cases}$$

Which is the first term to exceed 100 digits in length?

4C. Ivy has a perfectly circular pizza and an atomically precise pizza cutter. Every second, she chooses two random points on the circumference of the pizza and then makes a perfectly straight cut connecting those two points. After she makes 2025 cuts, what is the expected number of pieces of pizza she makes? That is to say, if Ivy were to repeat this procedure sufficiently many times and take the mean of all her results, what number would this mean approach?