Oxford Mathematics Team Challenge
Maps Round Question Booklet

SAMPLE SET

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.

2. FORMAT. This round contains 46 numerical questions. All answers are non-negative integers; in your 9×9 grid, write down your answers as **non-negative integers** in the cells corresponding to the question.

3. TIME LIMIT. 75 minutes. You may not write anything into your Answer Booklet, including your team name, after the allotted time has expired.

4. NO CALCULATORS, SQUARED PAPER OR MEASURING INSTRUMENTS. Lined paper and blank paper for rough working is allowed. You may use a pen or pencil to preference, however you may want to use a pencil for the Answer Sheet in order to edit your submission. Other mediums for working (e.g., digital devices, whiteboards, enchanted scrolls) are strictly forbidden.

5. SCORING RULES. Your team is awarded full points for any correctly-answered questions. Any correctly-answered question which is connected to the centre by a series of horizontal and vertical paths of correct answers scores **twice as many points**.

   The points for each question are in the bottom right of the cells in the Answer Sheet, as well as at the end of the questions in the Question Booklet, and are marked in [square brackets]. Each zone is out of 20 points; there are six zones.
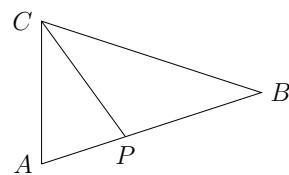
6. Don't expect to complete the whole paper in the time! The questions further from the centre are generally worth more marks, but are also generally harder.

7. Keep in mind that some questions further in a zone may use the ideas or results from previous questions in that zone.

8. You are also encouraged to think deeply, rather than to guess.

9. Good luck, and enjoy! ☺

Red Zone (R)

The Red Zone is out of **20 points**.

R1. In the diagram, $AB = BC$, $AC = BP$, $BP = CP$ and $\angle BPC \neq \angle BAC$. What is the angle $\angle ABC$ in degrees?
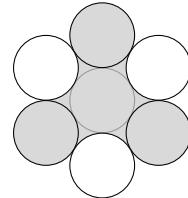

[2]

R2. A circle drawn in the Cartesian plane has diameter with endpoints at $P = (2, 8)$ and $Q = (8, 16)$. What is the shortest distance from the origin to a point on the circle? [1]

R3. In the diagram shown, each of the seven circles have radius 1 cm. The area of the shaded region is $a\sqrt{3} + b\pi$. What is $a + b$?

[4]

R4. Petar has a straight strip of paper 1 cm wide and lays another straight strip 2 cm wide overlapping it. The resulting overlapping region of the two strips of paper has area 4 cm². What is the acute angle between the two strips of paper? [3]

R5. Luke is going on a camping trip with a tent in the shape of a square pyramid that has height 4 m and base length 6 m. He needs a tarp to cover the sides of the tent to protect himself from the rain. In m², what is the smallest area of tarp that Luke can buy? [3]

R6. The triangle ABC has lengths $AB = 20$ and $AC = 25$. The midpoint of AB is labelled M, and the midpoint of AC is labelled N. If the circle with diameter BM is tangential to the circle with diameter CN, what is the length of BC? [2]

R7. How many integer side-length triangles are there where two of the sides are length 20 and 25? [2]

R8. Three circles X, Y, Z having centres A, B, C respectively are externally tangential to each other. Let D on AB, E on BC, F on CA be the intersection points of each pair of circles. Let T_D, T_E, T_F be the circles' tangent lines at points D, E, F respectively.

Suppose the lines T_D, T_E form an angle 90°, the lines T_E, T_F form an angle 120°, the lines T_F, T_D form an angle 150°, and $AB = 5$. Then $BC = \frac{a\sqrt{3}}{b}$ for some positive integers a, b. What is $10a + b$? [3]

Yellow Zone (Y)

The Yellow Zone is out of **20 points**.

Y1. The median of the dataset $\{4, 6, 7, 7, 9, x\}$ equals the mean of the dataset $\{4, 6, 7, 7, 9, x, y\}$ where x and y are both positive integers. What is $x + y$? [1]

Y2. What is the sum of the coefficients of

$$(1 + x)(1 + x^2)(1 + x^4)$$

including the constant term? [1]

Y3. What is the sum of the coefficients of

$$(1 + x)(5 - x^2)(7 + x^4)(17 - x^8)$$

including the constant term? [4]

Y4. What is the sum of all distinct solutions to $(x^2 - 6x + 9)^{(x^2 + 2x)} = 1$? [2]

Y5. 2025 is a square number. How many years is it (from the year 2025) until the next year which is a square number? [1]

Y6. How many distinct values of x satisfy the equation $(1+x)(1+x^2)(1+x^4) = 8$? [3]

Y7. Let $x_0 = 1$. We define the sequence x_n iteratively by $x_{n+1} = \dfrac{x_n + 6}{3x_n + 1}$. As n gets very large, x_n approaches $\sqrt{k}$. What is k? [3]

Y8. Rebekah chooses 5 random distinct non-zero digits A, B, C, D, E and computes the product $(A^A - 1)(B^B - 1)(C^C - 1)(D^D - 1)(E^E - 1)$. Let p be the probability that the last digit of the product is 0. What is $1/p$? [5]
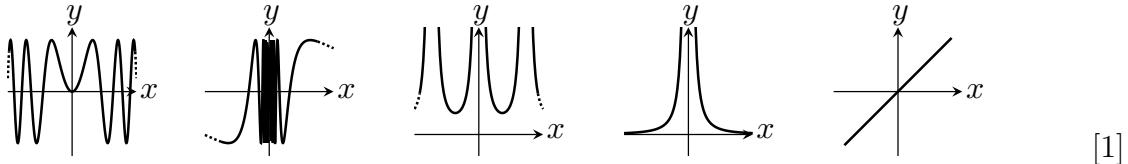
Amber Zone (A)

The Amber Zone is out of **20 points**.

A1. Let $g(x) = \cos(x - 270°) + \cos(90° - x)$. Let K be the maximum value of g, and k the minimum value. What is $K - k$? [1]

A2. Let $p(x) = \sin x$, $q(x) = 1/x$ and $r(x) = x^2$. Cora chooses one of p, q, r, then Derek applies one of p, q, r (possibly the same) to Cora's function, then sketches the function. How many functions depicted below could be Derek's sketch?

[1]

A3. Let $f(x) = \sin x - \cos^2 x + \sin^3 x - \cos^4 x + \cdots + \sin^{99} x - \cos^{100} x$. Let M be the maximum value of f, and m the minimum value. What is $M - m$? [1]

A4. A goat is tied at the corner of a 3×4 shed by a rope of length ℓ, where ℓ is a rational number. How long does the rope need to be for the goat to be able to graze in an area 28π? Submit your answer as $a + b$, where $\ell = a/b$ is the simplest fraction for ℓ. [3]

A5. For $0° \leq x < 90°$, the minimum value of $\tan^2 x - 4 \tan x + 5$ is achieved when the sine of x equals $a/\sqrt{b}$ where a and b are integers that share no common factors. What is the value of $a + b$? [3]

A6. Consider a cylinder with an army of 6 ants living on its curved surface. The ants each have a territory, which is the region of points which are less than 1 cm away when distance is measured along the surface. Territories aren't allowed to cross the edges of the cylinder, and – because they don't get along – no two ants' territories can overlap.

Let P, H be its base perimeter and height of the cylinder respectively. The quantity $C = 2(P + H)$ has a minimum value of $a + b\sqrt{3}$ cm, where a, b are integers. What is the value of $a + 10b$? *[Hint: what does C represent geometrically?]* [5]

A7. Consider an equilateral triangle ABC of side length 1. Let P be a point in the interior, and consider the region of points within ABC that is closer to P than to any of A, B, C. For different points, P, this region has different areas. The maximum area is M and the minimum area is m. What is the ratio M/m equal to? [2]

A8. Longname's quadrilateral $ABCD$ satisfies $AB = BC = CD = 5$ and $\angle ABC = \angle ADB = 90°$. What is the area of $ABCD$? [4]

Green Zone (G)

The Green Zone is out of **20 points**.

G1. For what c does the equation

$$c\left(y - x\right)^2 - 4x - cy = 0$$

have reflective symmetry about the line $y = x$? [2]

G2. For $a \neq 0$, how many segments does the curve

$$\frac{1}{x - a} + \frac{x}{x - 2a} + \frac{x^2}{x - 3a}$$

split the plane into? [2]

G3. Samantha is thinking of a function f that takes positive integer inputs and outputs positive integers. She tells you that f has the following properties:

- If p is a prime number, then $f(p)$ is also a prime number.

- For all integers $n > 1$, $f(n^2 - 1) = (f(n))^2 + 1$.

Given these properties, there exists a positive integer N such that $f(N)$ must be a 3-digit number. What is N? [2]

G4. The curve

$$\frac{1}{x - a} + \frac{x}{x - b} + \frac{x^2}{x - c}$$

is graphed, alongside a parabola $Ax^2 + B$. Let M and m be the maximum and minimum possible number of intersections of the parabola and the curve respectively. What is $M - m$? [4]

G5. Gunther foolishly claims that he has proven $\log(x + y) = \log(x) + \log(y)$, as he has found positive integers $a \leq b \leq c$ such that $\log(a + b + c) = \log(a) + \log(b) + \log(c)$. How many different (a, b, c) could Gunther have found? [2]

G6. $2x - \ln(x^2 - y^2) = c$ is graphed for positive x and y. What is the minimum value of c for which the graph has an x-intercept? *[Hint: the tangent to $\ln(x)$ has gradient equal to $1/x$ for each x.]* [4]

G7. Let $f(x)$ be a function with the property $f(f(x)) = x$. If $f(x) = 0$ whenever x is an integer, what is the greatest integer that is in the range of f? [2]

G8. Brad has a function that satisfies $f(xy) = f(x) + f(y)$, and also knows the values of f at n distinct points. What is the least value of n such that Brad can know the values of $f(1), f(2), \ldots, f(50)$? [4]

Cyan Zone (C)

The Cyan Zone is out of **20 points**.

C1. We say a function f is *shrike* if it satisfies the following properties: (i) the domain and range of f is the set $\{1, 2, 3, 4, 5\}$; (ii) for all x in the domain, $f(f(x)) = x$.

How many shrike functions are there such that $f(4) = 4$ and $f(5) = 5$? [2]

C2. Rosie, Angie and Ella are standing in a line, with Rosie being at the front and Ella at the back. Each of them are wearing a jersey with a distinct natural number picked from the set $\{1, 2, 5, 10, 19\}$ on their back, which they cannot see themselves. Each person can see the numbers of people ahead of them.

First, Ella says that she doesn't know if her number is even or odd. Then Angie says she doesn't know if her number is even or odd. Then Rosie then says she knows whether her number is even or odd.

What are the sum of the numbers that Rosie could be wearing? [2]

C3. How many shrike functions are there such that $f(1) = 5$? *[See C1. for the definition of a shrike function.]* [2]

C4. Yoshi starts with a capital of £150. A game consists of a sequence of up to fifty **A**'s and **B**'s, where **A** and **B** are the following actions:

A: Yoshi loses £1.

B: If Yoshi's capital is even, he wins £3. Otherwise, he loses £5.

Yoshi breaks even after a sequence of games if his capital is £150. What is the sum of all game-lengths where Yoshi can break even? [2]

C5. How many shrike functions are there in total? *[See C1. for the definition of a shrike function.]* [4]

C6. Bobby has five distinguishable socks. How many ways can he make two pairs of socks? [2]

C7. A friendly enemy writes the numbers $1, 2, 3, 4, 5$ onto a dark blue wall. Repeatedly, you erase two numbers – call them a and b, with $a > b$ – and write either $2a + b$ or $a + 7b$ onto the wall. This process repeats until there is only one number left. What is the largest number you can create? [6]

Blue Zone (B)
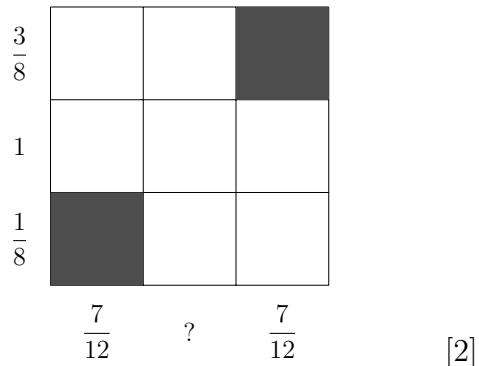
The Blue Zone is out of **20 points**.

B1. With the unit fractions

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}, \frac{1}{24},$$

Angus arranges them in the white squares in the grid below, and then writes the sums of the rows and columns as shown in the diagram. What's the reciprocal of the sum of the middle column?

[2]

B2. For a positive integer N, let d be the sum of its digits. We say N is well-fed if $2d < N < 4d$. What is the largest well-fed number? [3]

B3. The unit fractions $1/a$, $1/b$ and $1/c$ sum to 1 where a, b, c are positive integers such that $a \le b \le c$. How many distinct solutions of (a, b, c) are there? [4]

B4. What, from left to right, are the last 4 digits of 11^{2025}? [3]

B5. A cuboid has side-lengths that are all integers in cm; its surface area is N cm^2, and its volume is N cm^3. What is the maximum possible value of N? [4]

B6. There is only one integer n between 1 and 100 such that the sum of the digits of n is half of the sum of the digits of $3n$. What is n? [2]

B7. When 25! is multiplied by 5^n, it has the highest possible number of zero digits at the end of the number. What is the least value of n? [2]