Warmup Questions

1. Factorise 399, 396, 391, 384. What do you notice? Hence, what is the sum of the prime factors of 6557?

   A. 129; B. 138; C. 146; D. 154; E. 162.

2. Which of the following values is the largest?

   A. $2^{(3^4)}$; B. $2^{(4^3)}$; C. $3^{(2^4)}$; D. $4^{(2^3)}$; E. $4^{(3^2)}$.

3. 2026 is the product of two primes, 2 and 1013. The Winter Olympics are taking place in 2026. Assuming the Winter Olympics continue to occur every 4 years, in which century will another Olympic games occur on a year which is also the product of two primes?

   A. 2000s; B. 2100s; C. 2200s; D. 2300s; E. 2400s.

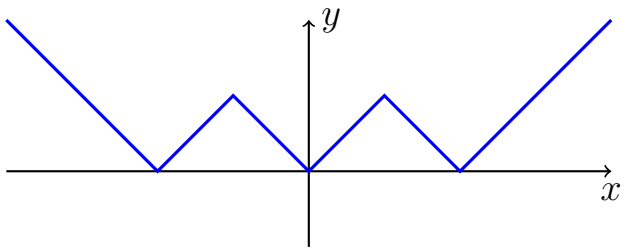
4. Let f and g be functions where $f(1) = 1$, $g(1) = 1$ and for $n \geq 1$,

   $$f(n+1) = 2f(n) \qquad \text{and} \qquad g(n+1) = g(n) + 50.$$

   For how many positive numbers n is it the case that $g(n) > f(n)$?

   A. 0; B. 4; C. 8;
   D. 128; E. Infinitely many!

5. Which of the following equations is drawn below?

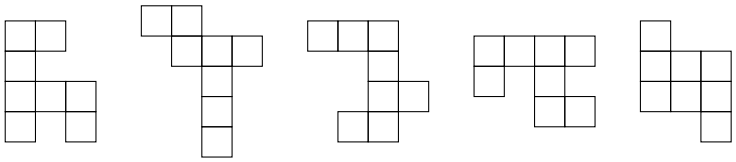


   A. $y = |x|$; B. $y = \big||x| - 1\big|$;

   C. $y = \Big|\big||x| - 1\big| - 1\Big|$; D. $y = \Big|\Big|\big||x| - 1\big| - 1\Big| - 1\Big|$;

   E. $y = \Big|\Big|\Big|\big||x| - 1\big| - 1\Big| - 1\Big| - 1\Big|$.

6. A polyomino is constructed by connecting 1×1 square tiles to each other. Below are examples of polyominoes each with eight tiles.



Two polyominoes are considered the same if one can be transformed into the other with rotations and reflections (so, for example, the first and fourth polyominoes above are the same). How many distinct polyominoes can be constructed with exactly 4 tiles?

A. 3; B. 4; C. 5; D. 6; E. 7.

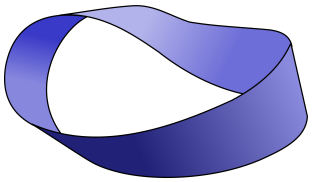
7. Let p be any prime number, and let n be any whole number. Which option can always be uniquely expressed as a difference of two squares?

A. p; B. $4p$; C. $17n$; D. np; E. $(2n+1)^2$.

8. For the polynomial $q(x) = x^4 + ux^2 + v$, $q(\sqrt{2}+\sqrt{3}) = 0$, where u and v are whole numbers. What is the sum of u and v?

A. -9; B. -4; C. 0; D. 4; E. 9.

9. A Möbius strip is constructed by taking a strip of paper, twisting one of the ends by 180° (a 'twist') and gluing the ends:



What do you obtain if you cut a Möbius strip along the middle?

A. One loop with no twists;
B. A Möbius strip;
C. One loop with two twists;
D. Two separate Möbius strips;
E. Two interlocked Möbius strips.

10. Which of the following functions is not equal to the rest?

A. $\sin(x + 720°)$;
B. $\cos(x - 90°)$;
C. $\sin(\arccos(\cos x))$;
D. $\sin^3 x + \sin x \cos^2 x$;
E. $(\tan^2(x)\sin^2(x + 90°) + \cos^2 x)\sin x$.