

Why Recursion ??

- Merge Sort / Quick Sort
- Trees → Binary Tree / BST / Tries / Segment Tree
- Dynamic Programming,
- Backtracking
- Graphs

Recursion : function calling itself.

→ Solving a problem using,

Return the sum of 1st
N natural no.

Smaller instances
of the same problem

$$\rightarrow \text{Sum}(N) = 1 + 2 + 3 + \dots + (N-1) + N \quad \text{Subproblem}$$

$$\text{Sum}(N) = \underbrace{\text{Sum}(N-1)}_{\text{Subproblem}} + N$$

Subproblem

$$\text{Sum}(4) = 1 + 2 + 3 + 4$$

Sum(3)

How to Write Recursive Codes

1) Assumption : (i) Decide what your function should do
(ii) Assume it returns a correct answer.

2) Main Logic : Solving assumption using subproblems

3) Base Condition : Inputs for which we want to stop the recursion
↓
At the start of function.

Ex 1 Sum of 1st N natural no.

int sum(N) < Ass: Given N, calculates & returns the correct sum of 1st N natural no's.

```
if (N == 1) <
    return 1;
```

→ Base Case

```
return sum(N-1) + N; // Main Logic
```

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$N \rightarrow N-1 \rightarrow N-2 \rightarrow N-3 \dots \dots \dots$ 1

Ex 2 Calculate $(N!)$

$N > 1$

$$\text{fact}(N) = 1 \times 2 \times 3 \times 4 \dots \dots (N-1) \times N$$

$$\text{fact}(N) = \text{fact}(N-1) \times N$$

Code

int fact(N) & Ass: Given N, calculates &
returns $N!$ correctly

if ($N == 1$) &

return 1;

else

return fact(N-1) \times N;

else

int add(N, M) &

return N+M;

else

int mul(x, y) &

return x*y;

else

int sub(x, y) &

return (x-y);

else

Function Call Tracing

main() &

int $x = 10$;

int $y = 20$;

print (sub (mul (add (x, y), 30), 75));

↓

900

30

$$900 - 75 = 825$$

825

print (sub (mul (add (x, y), 30), 75))

825

900

sub (mul (add (x, y), 30), 75) \Rightarrow 825

900

30

mul (add (x, y), 30) \Rightarrow 900

30

add (x, y) \Rightarrow 30

Data Structure \Rightarrow Stack (last in first out)

Stack

```

add(x, y) (30)
mul(add(x, y), 30) (900)
sub(mul(add(x, y), 30), 75) (825)
point(sub(mul(add(x, y), 30), 75)) (825)

```

Sum Tracing ($N = 4$)

$\uparrow 10$

```

int sum(N = 4) {
    if (N == 1) return 1;
    return sum(N-1) + N;
}

```

\downarrow \uparrow

3 4

sum(1):

sum(2): $\sum(1) + 2$

sum(3): $\sum(2) + 3$

sum(4): $\sum(3) + 4$

\downarrow Stack

10

```

int sum(N = 3) {
    if (N == 1) return 1;
    return sum(N-1) + N;
}

```

\downarrow \uparrow

2 3

\uparrow $3 (+3 = 6)$

```

int sum(N = 2) {
    if (N == 1) return 1;
    return sum(N-1) + N;
}

```

\downarrow \uparrow

1 2

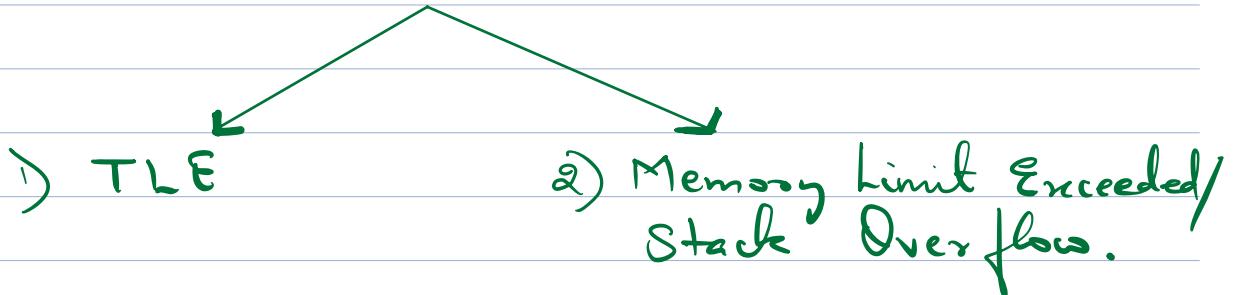
\uparrow $2 (+2 = 3)$

$$T(1+2=3)$$

```
int sum(N = 1) {
    if (N == 1) return 1;
    return sum(N-1) + N;
}
```

Without Base Case \Rightarrow Recursion won't stop

$4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow (-1) \dots \dots \dots -\infty$



How to define Base Case ??

Is subproblem valid ??
 $\text{Sum}(N) = \text{Sum}(N-1) + N;$

↑
transition.

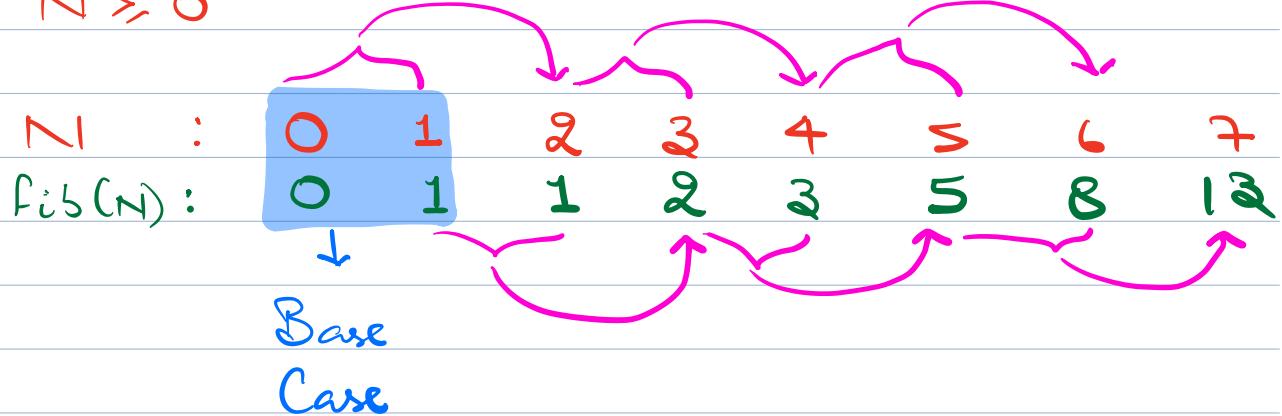
OR
 $\text{Sum}(1) = \frac{\text{Sum}(0)}{\text{invalid}} + 1 \quad (N=1)$

If valid $\text{Sum}(0) = \frac{\text{Sum}(-1)}{\text{invalid}} + 0 \quad (N=0)$

Fibonacci

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$

$n \geq 0$

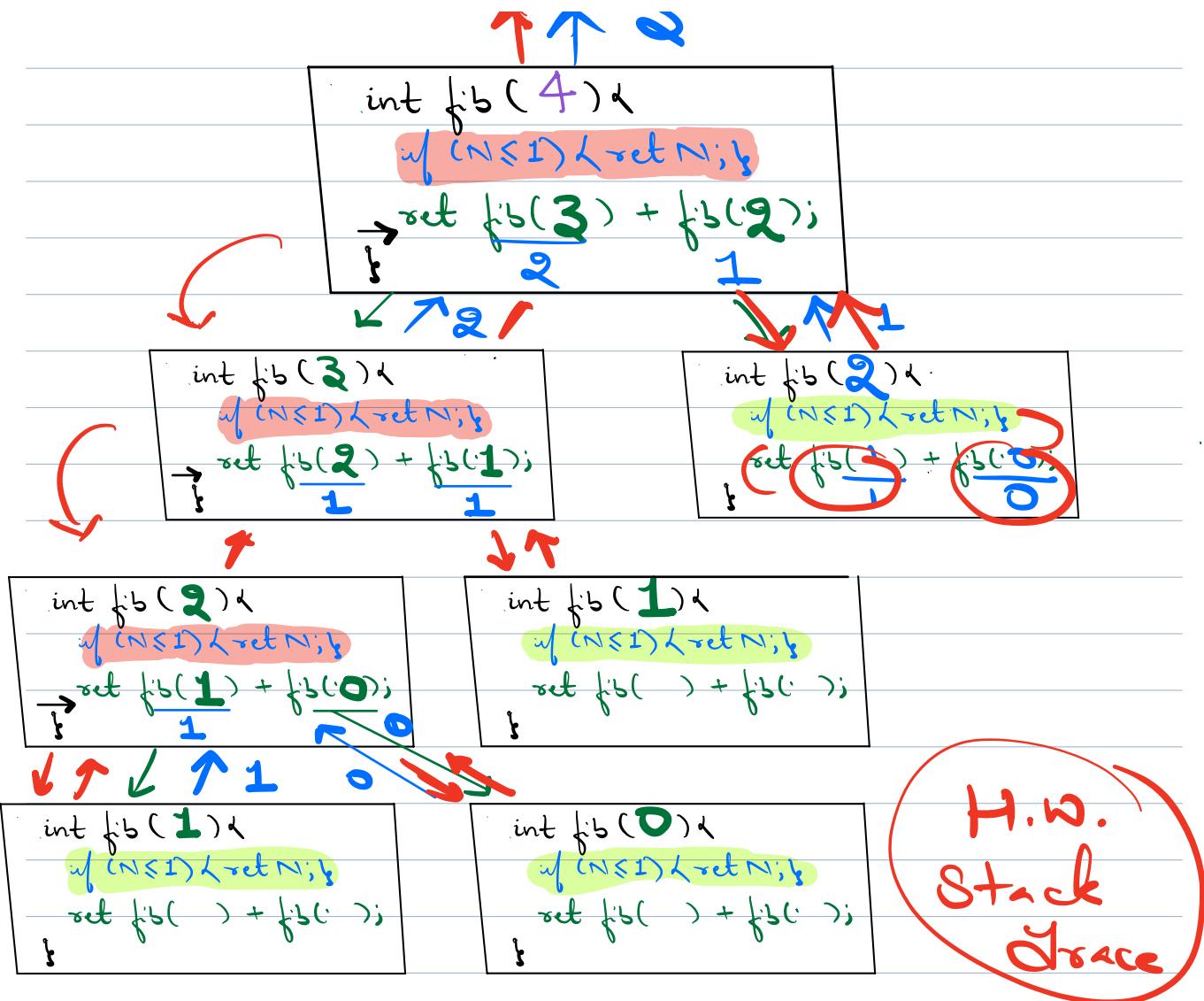


Given N , find the N^{th} fibonacci number.

Base Case $\begin{cases} \text{fib}(0) = \cancel{\text{fib}(0-1)} + \cancel{\text{fib}(0-2)} \\ \text{fib}(1) = \text{fib}(0) + \cancel{\text{fib}(1-1)} \end{cases}$

Code

```
int fib(n) {
    if (n == 0 || n == 1) return n;
    return fib(n-1) + fib(n-2);
```

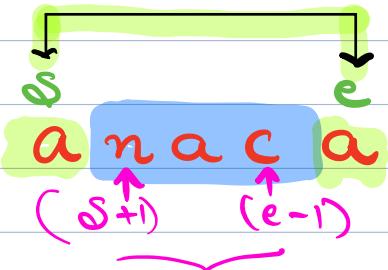
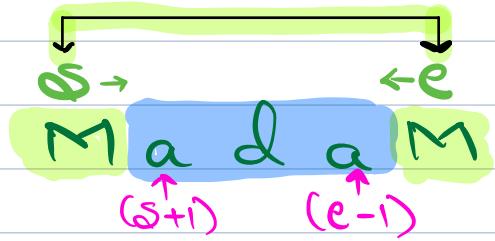


0
==

Given a substring of string (s, e)
Check if it is a palindrome
recursively.

Ex1 s: gooddad (s=4, e=6) ✓

Ex2: s: gooddad (s=2, e=5) X



Not a
palindrome

Code

```
bool isPalindrome (str, s, e) &
```

```
if (s >= e) & return True; }
```

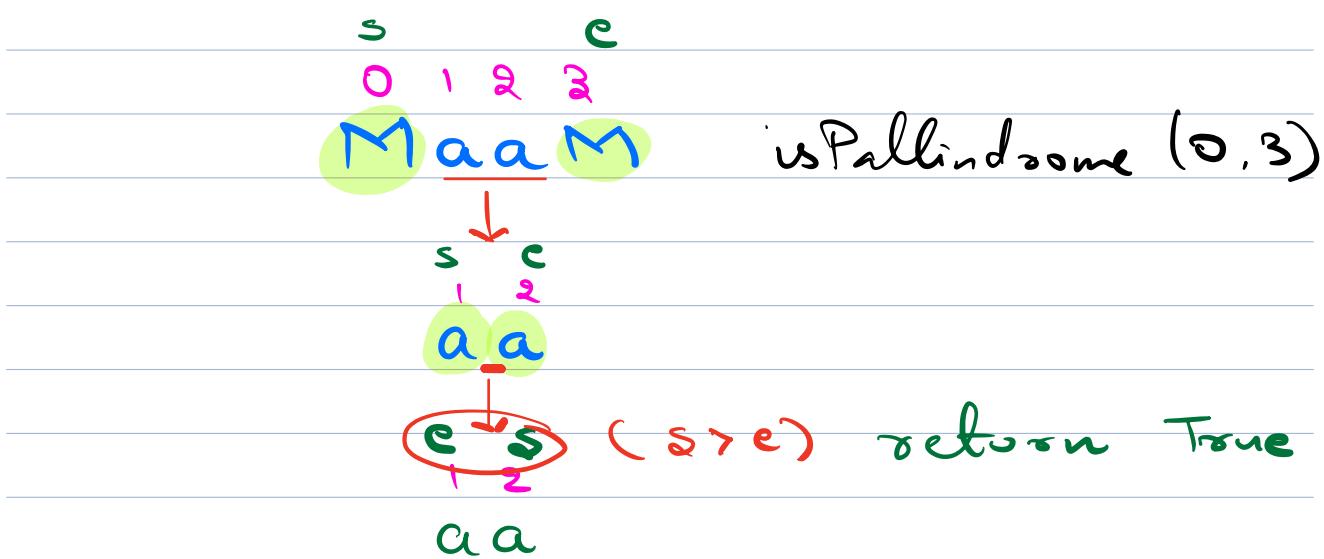
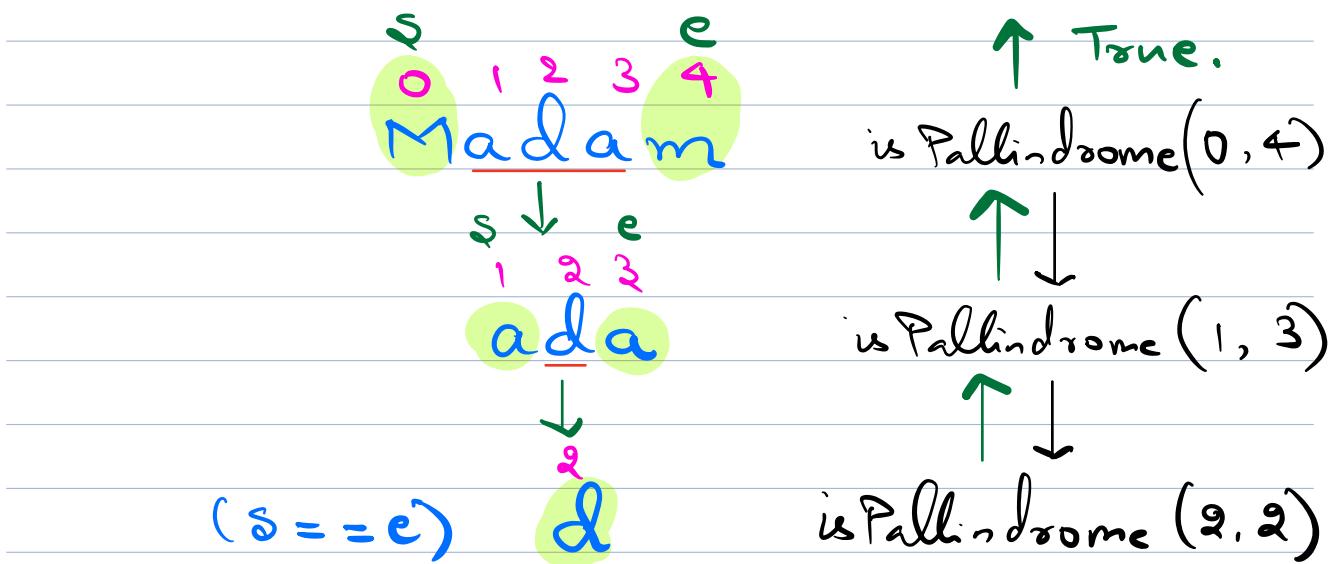
```
if (str[s] == str[e]) &
```

```
return isPalindrome(str, s++, e--);
```

```
else
```

```
return false;
```

```
;
```



isPalindrome (7, 5)

NOTE : Don't use postfix increment
operators in Recursion
Use prefix