

Encrypted Fast Covariance Intersection on the Cloud Without Leaking Weights

Marko Ristic¹ and Benjamin Noack¹

Abstract—State estimate fusion is a common requirement in distributed sensor networks and can be complicated by untrusted participants or network eavesdroppers. We present a method for computing the common Fast Covariance Intersection fusion algorithm on an untrusted cloud without disclosing individual estimates or the fused result. In an existing solution to this problem, fusion weights corresponding to the sensor estimate errors are leaked to the cloud in order to perform the fusion. In this work, we present a method that guarantees no leakage at the cloud by requiring an additional computation step by the party querying the cloud for the fused result. The Paillier encryption scheme is used to homomorphically compute separate parts of the computation that can be combined after decryption. This encrypted Fast Covariance Intersection algorithm can be used in scenarios where the fusing cloud is not trusted and relative sensor performances must remain confidential.

I. INTRODUCTION

Data fusion and distributed state estimation have long been active fields of research and continue to find many applications in modern systems today [1], [2]. Methods relying on the Kalman filter and derivatives [3] have become particularly prevalent in this area due to their recursive structure and suitability to modelling estimate cross-correlations typically required for data fusion [4], [5]. The handling of these cross-correlations is especially important when consistent or optimal fusion is desired [6], [7] and presents a key challenge in data fusion when they are not known in advance. To overcome this, some methods propagate cross-correlations through time at the cost of repeated reconstruction [8] and typically add local computational complexity. Alternative methods approximate error cross-correlations with conservative suboptimal estimates to provide consistent fusion instead [9], [10], [11]. One such popular method is Covariance Intersection [9] and its computationally inexpensive approximation, Fast Covariance Intersection [11]. These methods minimise fusion estimate error given possible conservative estimates and are popular due to their compatibility with the information form of the Kalman filter [4], [12].

While effective and often efficient, these data fusion solutions have traditionally been performed on closed networks with trusted participants and inherently imply a trust between sensors and estimators. In recent years, the ubiquity of distributed public networks has seen the additional challenges of security and privacy become increasingly relevant in distributed sensing environments and has led to

an active field of research in providing security during data processing and fusion tasks [13], [14]. While ensuring transmitted information is kept secret from eavesdroppers on untrusted networks is achievable with common private and public key encryption schemes [15], tasks involving untrusted participants during computations present a greater challenge. Here, partial computations on encrypted data or the leakage of intermediate results are often required for arriving at the final result [16], [17]. One common tool for achieving these requirements is homomorphic encryption [18], [19] which allows operations to be computed on encrypted numbers without decryption. The Paillier encryption scheme [19] allows homomorphic addition and is a frequent choice in fusion application due to its applicability, simplicity and provable security. In [20], the Paillier scheme is used to perform non-Bayesian measurement fusion with range-sensors without disclosing individual sensor locations or measurements, while in [21], it is used in to achieve similar security with a Kalman filter when measurements are linear and sensors form a hierarchical network. When paired with additional schemes that introduce some leakage, tasks in a wider variety of scenarios can often be solved as well. [22] uses private weighted sum aggregation together with the Paillier scheme to compute decentralised local control inputs while leaking only the sum of weighted neighbouring states, while in [16], an untrusted cloud can use order-revealing encryptions to compute and leak relative sensor accuracies allowing the homomorphic computation of the Covariance Intersection fusion algorithm.

In this work, we consider stochastic state estimate fusion on an untrusted cloud similar to [16], but aim to fuse estimates with no leakage at the cloud. Our contribution is a novel method for computing the Fast Covariance Intersection algorithm using only the Paillier encryption scheme and leaking no intermediate information to the untrusted cloud. While in some use-cases, leaked relative sensor accuracies can be useful for prioritising sensor communications, this leakage inherently informs the cloud about sensor performances over time, disclosing the times when sensors are out of range or non-functioning and implicitly leaking information about the fused estimate. Therefore, the presented method finds application when all sensor information is considered sensitive and when the fusing cloud is not trusted.

Section II introduces the formal data fusion and security goals and section III gives relevant preliminaries. Section IV introduces our method, before sections V and VI analyse its security and simulation results, respectively. Concluding remarks and future work are discussed in section VII.

¹Marko Ristic and Benjamin Noack are with the Autonomous Multi-sensor Systems Group (AMS), Institute for Intelligent Cooperating Systems (ICS), Otto von Guericke University (OVGU), Magdeburg, Germany {marko.ristic, benjamin.noack}@ovgu.de

A. Notation

II. PROBLEM STATEMENT

In this work, we consider an arbitrary time-varying process defined by its state $\underline{x}_k \in \mathbb{R}^n$ for all timesteps $k \in \mathbb{N}$. This process is estimated by m individual estimators i , $1 \leq i \leq m$, each producing a state estimate and an estimate error covariance,

$$\hat{\underline{x}}_{k,i} \in \mathbb{R}^n \text{ and } \mathbf{P}_{k,i} \in \mathbb{R}^{n \times n}, \quad (1)$$

respectively, at every timestep k . Our aim at every timestep is for all estimates $\hat{\underline{x}}_{k,i}$ and errors $\mathbf{P}_{k,i}$, $1 \leq i \leq m$, to be sent to a cloud server where a fused estimate and error covariance,

$$\hat{\underline{x}}_{k,\text{fus}} \in \mathbb{R}^n \text{ and } \mathbf{P}_{k,\text{fus}} \in \mathbb{R}^{n \times n}, \quad (2)$$

respectively, are computed and are consistent with the estimates in (1).

Simultaneously, we consider the desired security of the fusion process. The fusing cloud and sensors are treated as *honest-but-curious*, that is, they follow protocols correctly but may use learned information for malicious gain, and a trusted third party exists that can query the cloud at any timestep k to obtain the fusion results in (2). To guarantee that no new information about estimates, (1), and fusions, (2), can be learned by eavesdroppers, the cloud and the sensors, we aim for cryptographic *Indistinguishability under the Chosen Plaintext Attack* (IND-CPA) [15] for all transmitted and processed information. This is inline with common security goals in the field and suitable for homomorphic encryption.

III. PRELIMINARIES

When proposing our solution in section IV, we make use of the Fast Covariance Intersection algorithm, the Paillier homomorphic encryption scheme and a common integer encoding for floating-point numbers. These preliminaries are summarised below.

A. Fast Covariance Intersection

The Fast Covariance Intersection (FCI) algorithm [11] provides a consistent suboptimal fusion of estimates in the form (1) when cross-correlations between the measurements are not known, providing a fast, non-iterative and general solution. Fusion is given by

$$\mathbf{P}_{k,\text{fus}} = \left(\sum_{i=1}^m \omega_{k,i} \mathbf{P}_{k,i}^{-1} \right)^{-1} \quad (3)$$

and

$$\hat{\underline{x}}_{k,\text{fus}} = \mathbf{P}_{k,\text{fus}} \sum_{i=1}^m \omega_{k,i} \mathbf{P}_{k,i}^{-1} \hat{\underline{x}}_{k,i}, \quad (4)$$

with positive fusion weights $\omega_{k,i} \in \mathbb{R}^+$, $1 \leq i \leq m$, such that

$$\sum_{i=1}^m \omega_{k,i} = 1 \quad (5)$$

at each timestep k . In FCI, these weights are computed non-iteratively, as

$$\omega_{k,i} = \frac{1/\text{tr}(\mathbf{P}_{k,i})}{\sum_{i=1}^m 1/\text{tr}(\mathbf{P}_{k,i})}. \quad (6)$$

B. Paillier Encryption Scheme

The Paillier encryption scheme [19], [15] is an additively homomorphic encryption scheme meeting the IND-CPA security notion. Key generation of the scheme is performed by choosing two sufficiently large primes of equal bit length, p and q , and computing $N = pq$. The public key is defined by $\text{pk} \triangleq N$ and the private key by $\text{sk} \triangleq (p, q)$.

Encryption of a plaintext message $a \in \mathbb{Z}_N$, resulting in a ciphertext $\mathcal{E}_{\text{pk}}(a) \in \mathbb{Z}_{N^2}^*$, is computed by

$$\mathcal{E}_{\text{pk}}(a) = (N+1)^a r^N \pmod{N^2} \quad (7)$$

with a randomly chosen $r \in \mathbb{Z}_N$. Decryption of the ciphertext is computed by

$$\mathcal{D}_{\text{sk}}(\mathcal{E}_{\text{pk}}(a)) = \frac{L(\mathcal{E}_{\text{pk}}(a)^\lambda \pmod{N^2})}{L((N+1)^\lambda \pmod{N^2})} \pmod{N} \quad (8)$$

where $\lambda = \text{lcm}(p-1, q-1)$ and $L(u) = \frac{u-1}{N}$. In addition to encryption and decryption, the following homomorphic properties are provided by the Paillier encryption scheme. $\forall a_1, a_2 \in \mathbb{Z}_N$,

$$\mathcal{D}_{\text{sk}}(\mathcal{E}_{\text{pk}}(a_1) \mathcal{E}_{\text{pk}}(a_2) \pmod{N^2}) = a_1 + a_2 \pmod{N}, \quad (9)$$

$$\mathcal{D}_{\text{sk}}(\mathcal{E}_{\text{pk}}(a_1)(N+1)^{a_2} \pmod{N^2}) = a_1 + a_2 \pmod{N}, \quad (10)$$

$$\mathcal{D}_{\text{sk}}(\mathcal{E}_{\text{pk}}(a_1)^{a_2} \pmod{N^2}) = a_1 a_2 \pmod{N}. \quad (11)$$

C. Integer Encoding for Homomorphic Encryption

IV. ENCRYPTED FAST COVARIANCE INTERSECTION

V. SECURITY ANALYSIS

VI. SIMULATION AND RESULTS

VII. CONCLUSION

REFERENCES

- [1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Dover Publications, 1979.
- [2] D. Simon, *Optimal State Estimation: Kalman, H Infinity and Nonlinear Approaches*. Wiley-Interscience, 2006.
- [3] A. J. Haug, *Bayesian Estimation and Tracking: A Practical Guide*. John Wiley & Sons, 2012.
- [4] A. G. O. Mutambara, *Decentralized Estimation and Control for Multisensor Systems*. CRC press, 1998.
- [5] M. Liggins, C. Y. Chong, D. Hall, and J. Llinas, *Distributed Data Fusion for Network-Centric Operations*. CRC Press, 2012.
- [6] Y. Bar-Shalom, "On The Track-to-track Correlation Problem," *IEEE Transactions on Automatic Control*, vol. 26, no. 2, pp. 571–572, 1981.
- [7] S. L. Sun and Z. L. Deng, "Multi-sensor Optimal Information Fusion Kalman Filter," *Automatica*, vol. 40, no. 6, pp. 1017–1023, 2004.
- [8] J. Steinbring, B. Noack, M. Reinhardt, and U. D. Hanebeck, "Optimal Sample-based Fusion for Distributed State Estimation," in *19th Intl. Conf. on Information Fusion (Fusion 2016)*, 2016, pp. 1600–1607.
- [9] S. J. Julier and J. K. Uhlmann, "A Non-divergent Estimation Algorithm in the Presence of Unknown Correlations," in *American Control Conf. (ACC 1997)*, vol. 4, 1997, pp. 2369–2373.

- [10] B. Noack, J. Sijs, M. Reinhardt, and U. D. Hanebeck, "Decentralized data fusion with inverse covariance intersection," *Automatica*, vol. 79, pp. 35–41, 2017.
- [11] W. Niehsen, "Information Fusion Based On Fast Covariance Intersection Filtering," in *5th Intl. Conf. on Information Fusion (Fusion 2002)*, vol. 2, 2002, pp. 901–904.
- [12] F. Pfaff, B. Noack, U. D. Hanebeck, F. Govaers, and W. Koch, "Information Form Distributed Kalman Filtering (IDKF) with Explicit Inputs," in *20th Intl. Conf. on Information Fusion (Fusion 2017)*, 2017, pp. 1–8.
- [13] K. Ren, C. Wang, and Q. Wang, "Security Challenges for the Public Cloud," *IEEE Internet Computing*, vol. 16, no. 1, pp. 69–73, 2012.
- [14] M. Brenner, J. Wiebelitz, G. von Voigt, and M. Smith, "Secret Program Execution in the Cloud Applying Homomorphic Encryption," in *5th IEEE International Conference on Digital Ecosystems and Technologies (DEST)*, 2011, pp. 114–119.
- [15] J. Katz and Y. Lindell, *Introduction to Modern Cryptography: Principles and Protocols*. Chapman & Hall, 2008.
- [16] M. Ristic, B. Noack, and U. D. Hanebeck, "Secure Fast Covariance Intersection Using Partially Homomorphic and Order Revealing Encryption Schemes," *IEEE Control Systems Letters*, vol. 5, no. 1, pp. 217–222, 2021.
- [17] E. Shi, T.-H. H. Chan, and E. Rieffel, "Privacy-Preserving Aggregation of Time-Series Data," *Annual Network & Distributed System Security Symposium (NDSS)*, p. 17, 2011.
- [18] C. Gentry, "Fully Homomorphic Encryption Using Ideal Lattices," in *41st ACM Symposium on Theory of Computing (STOC)*, 2009, pp. 169–178.
- [19] P. Paillier, "Public-Key Cryptosystems Based on Composite Degree Residuosity Classes," in *Annual Intl. Conf. on the Theory and Applications of Cryptographic Techniques (EUROCRYPT)*. Springer, 1999, pp. 223–238.
- [20] A. Alanwar, Y. Shoukry, S. Chakraborty, P. Martin, P. Tabuada, and M. Srivastava, "ProLoc: Resilient Localization with Private Observers Using Partial Homomorphic Encryption," in *16th ACM/IEEE International Conference on Information Processing in Sensor Networks (IPSN)*, 2017, pp. 41–52.
- [21] M. Aristov, B. Noack, U. D. Hanebeck, and J. Müller-Quade, "Encrypted Multisensor Information Filtering," in *21st International Conference on Information Fusion (Fusion 2018)*, 2018, pp. 1631–1637.
- [22] A. B. Alexandru, M. S. Darup, and G. J. Pappas, "Encrypted Cooperative Control Revisited," in *58th IEEE Conf. on Decision and Control (CDC 2019)*, vol. 58, 2019.