# Privacy-Preserving Localization Using Private Linear-Combination Aggregation

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***Abstract*—Distributed state estimation and localization methods have become increasingly popular with the rise of ubiquitous computing, and have led naturally to an increased concern regarding data and estimation privacy. Traditional distributed sensor navigation methods involve the leakage of sensor or navigator information during localization protocols, thus not preserving participants' data privacy. Existing approaches that do provide such guarantees fail to address sensor and navigator privacy in common non-linear measurement model applications and therefore forfeit broad applicability. We define a suitable notion of linear-combination aggregation encryption and provide a cryptographically secure instance that is applied to the Extended Kalman Filter with range-sensor measurements. Our method keeps navigator location, sensors' locations, and sensors' measurements private during navigation, and has been implemented to evaluate its accuracy and performance. The novel computationally plausible and provably secure range-based localization filter has direct applications to environments where participants are not trusted or data is considered sensitive.**

***Index Terms*—State Estimation, Data Privacy, Sensor Fusion, Extended Kalman Filters.**

## I. Introduction

LOCALISATION methods in distributed sensor environments have long been an active topic of research [1], [2]. Ongoing advancements in portable computing power and sensor capabilities have led to the development of various forms of localization methods, from range-free and range-based signal strength measurements [3], [4], acoustic range estimates [5], and magnetic field measurements [6] to the TDOA and TOA pseudo-range multilateration methods for aircraft [2]. Range-based localization methods, including the use of radar measurements, GPS clocks, and signal strength measurements, have found particularly large applications due to their robustness and accuracy. These methods use distance measurements to produce location estimates and typically involve gathering sensor details such as measurement noise, geographical location, and observations centrally. Broadcasting or gathering information in this way, however, implies a trust between participants within a trusted communication network to ensure shared data may not be used for individual malicious gain. With the advancements in cloud and ubiquitous computing, changes in possible use-cases involving distributed sensors have made the requirements of information security and user privacy more apparent than before [7], [8]. For example, an aircraft localization scenario in the presence of privately operated measurement stations may imply measurements and locations of stations should be kept private from navigating vehicles. Or in autonomous vehicle state estimation, benefit from neighboring vehicles' information may be desired while no participant wishes to share information about their hardware. Additionally, the dispatching of any expensive intermediate computations to service-providing cloud operators may require ensuring the privacy of information involved in the computations. All of these scenarios, and the one we will consider, require methods for computing specific operations while ensuring the privacy of some or all of the input data. Our contributions in this work will be presented in two parts. We first propose a novel aggregation-based encryption scheme which allows the private aggregation of weighted sums, before developing a computationally plausible localization filter in the presence of range-only sensors using this scheme, such that sensor and navigator privacy is preserved.

The remainder of the paper is structured as follows. In Section we give an overview of the relevant literature to the topic and in Section we introduce the localization problem, the relevant aggregation operation for its solution and the restrictions on involved parties. Sections and will summarize cryptographic preliminaries and introduce a novel security notion for our required aggregation before defining and proving an encryption scheme which satisfies it. Sections and introduce localization preliminaries and our privacy-preserving localization method using the private aggregation scheme. We show and discuss simulation results in Section and conclude our work in Section .

In this section, we will discuss methods for providing security guarantees in signal processing tasks, and their applications to localization and estimation problems.

Of the methods used to preserve privacy during distributed signal processing, it is common for the information exchanging portion of the algorithm to be reduced to a form of aggregation of involved parties' data. The most common methods for ensuring privacy during aggregation required for signal processing tasks have been described below.

**Differential Privacy** These techniques formalize preserving the privacy of individual datum within an aggregation operation by introducing known distributions of noise to the resulting aggregation. To handle the cases with no trusted aggregator, local differential privacy is achieved by introducing noise to individual inputs [9], [10], [11]. Although differential privacy has strong computational benefits due to its implementational simplicity, it provides relatively weak security guarantees for multi-party networks, with no guarantees in the presence of multiple col-

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luding corrupted parties. Additionally, noisy aggregation results hinder its use in scenarios where high precision is a desired property such as aircraft localization.

**Aggregator Oblivious Encryption** This method of encryption formalizes the privacy of *all* aggregation data from honest parties [12], [13], [14]. Encryptions of aggregation data are proven to be computationally indistinguishable and guarantee that only their aggregation (typically sum) can be learned by an aggregator, whether or not they are trusted. The originally proposed method [12] is restricted to a small input domain which has been expanded and computationally simplified in later works [14], [15]. The aggregation operation in these schemes is, however, limited to the sum operation and restricts its applicability to simple processing tasks such as smart-meter sums or mean calculations [13], [15].

In addition to the private aggregation of data, it may also be the case that private data needs to be modified by an external party before being returned, as with privacy-preserving cloud computing. These methods have been broadly grouped into two domains and described below.

**Fully Homomorphic Encryption** These schemes allow all algebraic computations to be performed on encrypted data without the computing party learning any information about inputs, intermediate values or results [16], [17]. Although theoretically ideal for many data-sensitive tasks, current implementations are still computationally impractical for real-time or large scale tasks [18].

**Partially Homomorphic Encryption** A simplification of fully homomorphic schemes; these schemes provide only a subset of (typically one) algebraic operations on encrypted data [19], [20], [21]. Reduced computational requirements and implementation simplicity have made partially homomorphic encryption schemes commonly used in privacy-preserving signal processing tasks, including private matrix multiplication [22], set intersection computation [23] and control input aggregation [24]. However, as with Aggregator Oblivious Encryption, these works are relatively restricted in application due to the limited operations provided.

Lastly, recent developments in encryption schemes have presented some novel methods also suitable for privacy-preserving computation in distributed environments.

**Multi-Client Functional Encryption** These schemes allow the computing of a specific function given encrypted inputs [25], [26]. They are suitable for data-sensitive distributed processing as encryptions must be combined before a result is obtained, and can be considered a generalization of the aggregation schemes described earlier. While general schemes that allow computing arbitrary functions are computationally expensive and complicate security definitions in distributed settings [27], partial schemes supporting only some functions, such as vector dot products [28], exist with formal security proofs, but have yet to find applications in more complicated signal processing tasks.

In the context of privacy-preserving localization and state estimation, some of the aforementioned schemes have found applications and some specialized encryption methods have been proposed. These are summarized in the next section.

Localization methods exist for a variety of purposes requiring different levels of accuracy and supporting various forms of sensors. We aim to provide an accurate localization filter in the presence of range-only sensors while providing uncertainties associated with estimates.

Relevant estimation methods include both Bayesian estimation methods which make assumptions about target trajectories to form their location estimates, typically based on the Kalman Filter and later derivatives [29], [30], and time-independent methods where location estimates are made independently of each other [31], [32]. The work in [31] uses partially homomorphic encryption to compute time-independent location estimates while preserving the privacy of sensor measurements and locations. The work proposes two approaches, using either polygon intersections suffering from geometric dilution of precision, or alternating projection requiring repeated interactive comparison protocols. Neither method considers measurement uncertainty, and no estimate errors are provided, making them unsuitable for our desired solution. Bayesian approaches can achieve better accuracy due to their exploitation of process dynamic knowledge. In [33], partially homomorphic encryption is used to encrypt measurement information and produce location estimates at a navigator. While producing estimate errors and requiring only uni-directional communication, this method exclusively supports linear measurement models and is therefore not suitable for range-only measurements.

Encrypted model-predictive-control methods [34] also introduce some privacy-preserving methods relevant to estimation. In [24], [35], the novel security notions of private weighted sum aggregation with centralized or hidden weights (pWSAc and pWSAh, respectively) are proposed. The schemes capture a weighted aggregation operation suitable for providing privacy in some control environments and are similar in principle to the aggregation decomposition proposed in this work. The requirements of our aggregation operation differ, however, in that weights are generalized to arbitrary length linear-combinations, which must be consistent across all participants, and can vary over time. We also require a formal security proof capturing these weight requirements as well as the explicit capabilities of participants. We define our alternative security notion in Section .

Although similar to the pWSAc communication protocol in [35], we formalise the assumptions

Along with encryption schemes and their application to localization-relevant algorithms, encryption schemes with homomorphic properties also require suitable encoding when used with real numbers. As is the case with range sensors, real-valued measurements need to be encoded as integers with operations consistent under scheme-provided homomorphic operations. Google's Encrypted BigQuery Client [36] provides an encoding which supports both addition and unencrypted multiplication when used with an additively homomorphic encryption scheme. It, however, leaks the exponent information of encrypted numbers. In [37], a simpler alternative is proposed, which leaks no information about encrypted numbers, but allows only a single unencrypted multiplication between encodings. We use a modified version of [37] due to its simplicity and applicability, and introduce the scheme details in Section .

While some aspects of privacy-preserving localization are solved in the above methods and encryption schemes, they fail to provide a localization solution in our context. In the next section, we will formalize our problem and its accompanying security requirements.

### A. Notation

Throughout this work, we will use the following notation. Lowercase underlined characters are vectors, $\underline{a}$, and random variables are bold, $\boldsymbol{a}$. Uppercase bold characters are matrices, $\mathbf{A}$, with $\mathbf{A}^{-1}$ denoting the inverse and $\mathbf{A}^{\top}$ the transpose. $|a|$ is the absolute value of $a$ and $\|\underline{a}\|$ the vector norm. Subscripts of the form $\boldsymbol{x}_{k|j}$ will denote an estimate of $\boldsymbol{x}_k$ using measurements from times up to and including $j$. $\mathrm{E}\{\cdot\}$ and $\mathrm{Var}\{\cdot\}$ are used for the expected value and variance, respectively, and encryption and decryption with key $k$ are denoted $\mathcal{E}_k(\cdot)$ and $\mathcal{D}_k(\cdot)$. Additionally, key $k$ may be ommited when inferrable from context. $\lfloor\cdot\rceil$ rounds to the nearest integer, $a\|b$ stands for a bitwise concatenation of $a$ and $b$ and sets will be written as $\{\dots\}$ while ordered sequences as $\langle\dots\rangle$.

## II. Problem Statement

In this work, we consider the context of privacy-presering range-sensor navigation, in which we wish for no sensor to learn information about the navigator or other sensors beyond their local measurements, and the navigator to learn no information about individual sensors beyond its location estimate. The problem is two-fold, in that we require explicit cryptographic requirements with a suitable encryption scheme meeting them, as well as an estimation scheme that can use the scheme in the context of range-sensor navigation.

To give a formal cryptographic requirement in a distributed setting, we must first consider the communication requirements of our context, and define the attacker capabilities and the desired security of a suitable encryption scheme. In this section, we will define a communication protocol and the relevant formal definition of security we aim to achieve, followed by the estimation problem to which we will apply it.

### A. Formal Cryptographic Problem

The communication between the navigator and sensors in our estimation problem will be decomposed into a simple two-step bi-directional protocol that will simplify defining formal security. In section V, we will show how this protocol is sufficient to compute the location estimate at a navigator while meeting our desired privacy goals.

At every *instance* $t$ (used to distinguish from an estimation timestep $k$), the navigator first broadcasts a number of weights $\omega_j^{(t)}, 1 \leq j \leq m$ to all sensors $1 \leq i \leq n$, who compute individual linear-combinations $l_i^{(t)} = \sum_{j=1}^{m} a_{j,i}^{(t)}\omega_i^{(t)}$ based on their measurement data $a_{j,i}$. Linear-combinations are then sent back to the navigator, who computes their sum $\sum_{i=1}^{n} l_i^{(t)}$. This two-step linear-combination aggregation protocol has been visually displayed in figure 1. In addition, we remark that

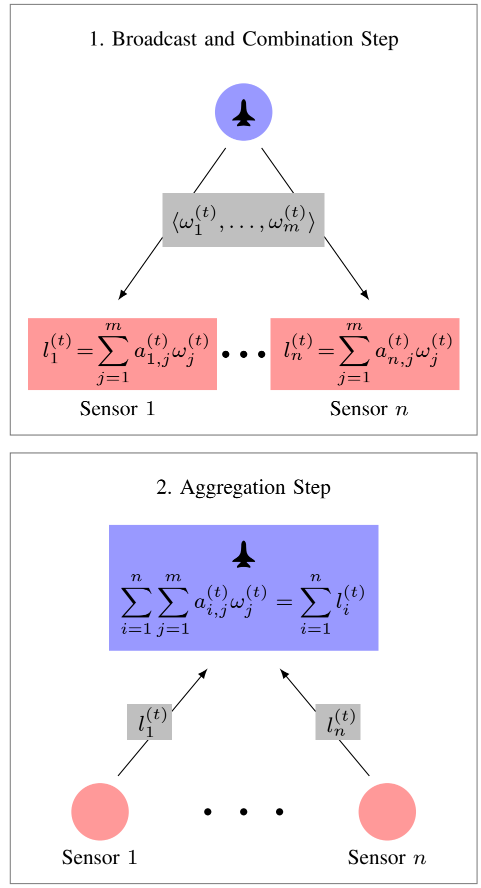


Fig. 1. Required linear-combination aggregation steps at instance $t$.

an alternative approach to the two-step protocol is computing $\sum_{j=1}^{m}(\omega_j^{(t)}\sum_{i=1}^{n} a_{i,j}^{(t)})$ at the navigator, requiring only values $a_{i,j}^{(t)}, 1 \leq j \leq m$ to be sent from each sensor $i$. We justify the use of bi-directional communication by reducing communication costs when the number of weights is larger than the number of sensors, $m > n$, and by sending fewer weights in the presence of repeats, as will be shown to be the case in section V.

Before giving a formal definition for the construction and security of our desired encryption scheme, we make the following assumptions on the capabilities of the navigator and sensors.

**Global Navigator Broadcast** We assume that broadcast information from the navigator is received by *all* sensors involved in the protocol.

**Consistent Navigator Broadcast** We assume that broadcast information from the navigator is received equally by all sensors. This means the navigator may not send different weights to individual sensors during a single instance $t$.

**Honest-but-Curious Sensors** We adopt the honest-but-curious attacker model for all involved sensors, meaning they are assumed to follow the localization procedure correctly but may store or use any gained sensitive information.

We justify the global broadcast assumption by noting that any subset of sensors within the range of the navigator can be considered a complete group and treated as the global set for estimation purposes, generalising the method, while the wide-spread use of cheap non-directional antennas supports the assumption of consistent broadcasts. The final assumption refers to the known problem of misbehaving sensors [32], [38], often requiring additional complicated detection mechanisms, and will not be considered in this work for the sake of simplicity.

We are now ready to define the type of encryption scheme we want for the specified communication protocol and the security guarantees it should provide. We let a linear-combination aggregation scheme be defined as a tuple of the four algorithms $(\mathsf{Setup}, \mathsf{Enc}, \mathsf{CombEnc}, \mathsf{AggDec})$. These will be used by a trusted setup party, the navigator and sensors $i$, $1 \leq i \leq n$. They are defined as follows

$\mathsf{Setup}(\kappa)$ On input of security paramater $\kappa$, generate public parameters pub, the number of weights $m$, the navigator's public and private keys $pk_0$ and $sk_0$ and the sensor private keys $sk_i$, $1 \leq i \leq n$.

$\mathsf{Enc}(pk_0, x)$ The navigator and sensors can encrypt any value $x$ with the navigator's public key $pk_0$ and obtain the encryption $\mathcal{E}_{pk_0}(x)$.

$\mathsf{CombEnc}(t, pk_0, sk_i, \mathcal{E}_{pk_0}(\omega_1^{(t)}),..., \mathcal{E}_{pk_0}(\omega_m^{(t)}), a_{i,1}^{(t)},..., a_{i,m}^{(t)})$ At instance $t$, sensor $i$ computes and obtains the encrypted linear combination $l_i^{(t)} = \mathcal{E}_{pk_0,sk_i}(\sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)})$ using its secret key $sk_i$.

$\mathsf{AggDec}(t, pk_0, sk_0, l_1^{(t)},..., l_n^{(t)})$ At instance $t$, the navigator computes the aggregation of linear-combinations $\sum_{i=1}^{n} l_i^{(t)} = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)}$ using its public and private keys $pk_0$, $sk_0$.

The security notions we want the above algorithms to meet reflect the previously stated estimation privacy goals. The navigator should learn no information from individual sensors while sensors should learning no information from the navigator or any other sensors. In the context of the introduced communication protocol, this can be summarised as the following notions.

**Indistinguishable Weights** No colluding subset of sensors gains any additional knowledge about the navigator weights $\omega_j$, $1 \leq j \leq m$ when receiving only their encryptions from the current and previous instances, and the ability to encrypt plaintexts of their choice.

**Linear-Combination Aggregator Obliviousness** No colluding subset *excluding* the navigator gains additional information about the remaining sensor values to be weighted, $a_{i,j}^{(t)}$, $1 \leq j \leq m$, where sensor $i$ is not colluding, given only encryptions of their linear combinations $l_i$ from the current and previous instances. Any colluding subset *including* the navigator learns only the sum of all linear combinations weighted by weights of their choice, $\sum_{i=1}^{n} l_i^{(t)} = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)}$.

While indistinguishable weights can be achieved by encrypting weights with an encryption scheme meeting the notion of Indistinguishability under the Chosen Ciphertext Attack (IND-CPA) [39], the novel notion of Linear-Combination Aggregator Obliviousness (LCAO) has been formalised as a typical cryptographic game between attacker and challenger in appendix A. Lastly, we conclude the cryptographic problem definition with the following important remark.

*Remark.* A leakage function including weights from the navigator requires extra care to be taken when giving its definition. If an attacker compromises the navigator (which need not follow the estimation protocol), they have control over the weights, and therefore the leakage function. We note that in the leakage function above, $\sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)}$, an individual sum weighted by the same weight may be learned by an attacker, *e.g.*, $\sum_{i=1}^{n} a_{i,1}^{(t)}$ given weights $(1, 0, \ldots, 0)$, but that individual sensor values $a_{i,j}^{(t)}$ remain private due to the assumption of a consistent broadcast.

### B. Estimation problem

The localization scenario we consider in this work is that of Bayesian self-navigation using range-only sensors. We consider localization in the two-dimensional case for simplicity but will derive methods suitable for an extension to the three-dimensional equivalent.

The navigator state captures the position and change in position ($\Delta$), and is of the form

$$\underline{x} = \begin{bmatrix} x & \Delta x & y & \Delta y \end{bmatrix}^\top . \tag{1}$$

A known process model is followed, in which the state at time $k$ is given by

$$\underline{\boldsymbol{x}}_k = \underline{f}(\underline{\boldsymbol{x}}_{k-1}) + \underline{\boldsymbol{w}}_k , \tag{2}$$

with zero-mean Gaussian process noise $\underline{\boldsymbol{w}}_k \sim \mathcal{N}(\underline{0}, \mathbf{Q})$. The measurement model is dependent on sensor $i$ and given by

$$\boldsymbol{z}_{k,i} = h_i(\underline{\boldsymbol{x}}_k) + \boldsymbol{v}_{k,i} , \tag{3}$$

with noise $\boldsymbol{v}_{k,i} \sim \mathcal{N}(0, r_i)$. The measurement function $h_i$, for sensor $i$ at location

$$\underline{s}_i = \begin{bmatrix} s_{x,i} & s_{y,i} \end{bmatrix}^\top , \tag{4}$$

is defined as

$$\begin{aligned} h_i(\underline{x}) &= \left\| \begin{bmatrix} x & y \end{bmatrix}^\top - \underline{s}_i \right\| \\ &= \sqrt{(x - s_{x,i})^2 + (y - s_{y,i})^2} . \end{aligned} \tag{5}$$

We wish to run a state estimation filter with models (2) and (3) such that all involved sensors $1 \leq i \leq n$ do not learn state estimates of $\underline{\boldsymbol{x}}_k$ and the navigator does not learn sensor locations $\underline{s}_i$, $1 \leq i \leq n$, measurement variances $r_i$ or their measurements $\boldsymbol{z}_{k,i}$ at any time $k$. We motivate these goals with the example of aircraft navigation in the presence of privately-operated range-measuring stations. It is reasonable to assume that the current state of an aircraft may not wish to be disclosed to unknown parties and, similarly, that tower locations may wish to be kept private from unidentified navigating aircraft. The additional safety to passengers provided by accurate aircraft localization, however, may be a goal of all those involved, justifying their cooperation.

Leakage of information will be discussed in more detail in Section V-C, however, we stress that in any estimation problem where process and measurement models are known, knowledge of sequential state estimates as is the case at the navigator, or the knowledge of one or more measurements as is the case by one or more malicious sensors, naturally leads to the leakage of some information. For this reason, we accept that average sensor information may be leaked to the navigator and that less accurate estimates of location may be leaked to sensors. We will instead show that individual sensor information remains private to sensors and that the most precise localization estimate, requiring all measurements, remains private to the navigator.

This has been captured in the associated secruity notions i nthe communication protocol

## III. Preliminaries

When defining our aggregation security requirements and encryption scheme we will reference some existing cryptographic security notions, the additively homomorphic Paillier encryption scheme, and the Joye-Libert private aggregation scheme.

The security of a cryptographic scheme is typically defined by a security *game*, which captures both the desired privacy guarantees, as well as the capabilities of attackers [39]. The typical security notion for a homomorphic encryption scheme is Indistinguishability under Chosen Plaintext Attack (IND-CPA) [40], while for private aggregation schemes, the notion is Aggregator Obliviousness (AO) [12].

**Definition III.1.** An encryption scheme meets IND-CPA security if an attacker who can choose plaintext messages to be encrypted at will, gains no additional information about an unknown plaintext message when they learn only its encryption. The formal security game and definition for IND-CPA has been given in Appendix .

**Definition III.2.** An encryption scheme meets AO security if no colluding subset of participants *excluding* the aggregator gains additional information about the remaining aggregation values given only their encryptions, while any colluding subset *including* the aggregator learns only their sum. The formal security game and definition for AO has been given in Appendix .

The Paillier encryption scheme and the Joye-Libert private aggregation scheme meet the IND-CPA and AO security notions respectively, and are given below.

### A. Paillier Encryption Scheme

The Paillier encryption scheme [20] is an additively homomorphic encryption scheme which bases its security on the decisional composite residuosity assumption (DCRA) and meets the security notion of IND-CPA. Key generation of the Paillier scheme is performed by choosing two sufficiently large primes $p$ and $q$, and computing $N = pq$. A generator $g$ is also required for encryption, which is often set to $g = N+1$ when $p$ and $q$ are of equal bit length [39]. The public key is defined by $(N, g)$ and the secret key by $(p, q)$.

Encryption of a plaintext message $a \in \mathbb{Z}_N$, producing ciphertext $c \in \mathbb{Z}^*_{N^2}$, is computed by

$$c = g^a \rho^N \pmod{N^2} \tag{6}$$

for a randomly chosen $\rho \in \mathbb{Z}_N$. $\rho^N$ can be considered the noise term which hides the value $g^a \pmod{N^2}$, which due to the scheme construction, is an easily computable discrete logarithm. The decryption of a ciphertext is computed by

$$a = \frac{L(c^\lambda \pmod{N^2})}{L(g^\lambda \pmod{N^2})} \pmod{N} \tag{7}$$

where $\lambda = \mathsf{lcm}(p-1, q-1)$ and $L(u) = \frac{u-1}{N}$.

In addition to encryption and decryption, the following homomorphic functions are provided by the Paillier scheme. $\forall a_1, a_2 \in \mathbb{Z}_N$,

$$\mathcal{D}(\mathcal{E}(a_1)\mathcal{E}(a_2) \pmod{N^2}) = a_1 + a_2 \pmod{N}, \tag{8}$$

$$\mathcal{D}(\mathcal{E}(a_1)g^{a_2} \pmod{N^2}) = a_1 + a_2 \pmod{N}, \tag{9}$$

$$\mathcal{D}(\mathcal{E}(a_1)^{a_2} \pmod{N^2}) = a_1 a_2 \pmod{N}. \tag{10}$$

### B. Joye-Libert Private Aggregation Scheme

The Joye-Libert private aggregation scheme [14] is a scheme defined on time-series data, that we refer to as instances for consistency, and meets the security notion of AO. Similarly to the Paillier scheme, it bases its security on the DCRA. A notable difference to a public-key encryption scheme is the need for a trusted party to perform an initial key generation and distribution step.

Key generation is computed by choosing two equal length and sufficiently large primes $p$ and $q$, and computing $N = pq$. Additionally, hash function $H : \mathbb{Z} \to \mathbb{Z}^*_{N^2}$ is defined, and the public key is set to $(N, H)$. $n$ private keys are generated by choosing $sk_i$, $1 \le i \le n$ uniformly from $\mathbb{Z}_{N^2}$ and distributing them to all $n$ users (whose values are to be aggregated), while the last key is set as

$$sk_0 = -\sum_{i=1}^{n} sk_i \pmod{N^2}, \tag{11}$$

and sent to the aggregator.

Encryption of plaintext $a_i^{(t)} \in \mathbb{Z}_N$ to ciphertext $c_i^{(t)} \in \mathbb{Z}_{N^2}$ at instance $t$ is computed by user $i$ as

$$c_i^{(t)} = (N+1)^{a_i^{(t)}} H(t)^{sk_i} \pmod{N^2}, \tag{12}$$

where $H(t)^{sk_i}$ can be considered the noise term which hides the again easily computable discrete logarithm $g^{a_i^{(t)}} \pmod{N^2}$, where $g = N+1$.

When all encryptions $c_i^{(t)}$, $1 \le i \le n$ are sent to the aggregator, summation and decryption are computed by the functions

$$c^{(t)} = H(t)^{sk_0} \prod_{i=1}^{n} c_i^{(t)} \pmod{N^2} \tag{13}$$

and

$$\sum_{i=1}^{n} a_i^{(t)} = \frac{c^{(t)} - 1}{N}. \tag{14}$$

Correctness follows from $\sum_{i=0}^{n} sk_i = 0$, and thus

$$\begin{aligned}
&H(t)^{sk_0}\prod_{i=1}^{n} c_{i,t} \pmod{N^2}\\
\equiv&H(t)^{sk_0}\prod_{i=1}^{n}(N+1)^{a_{i,t}}H(t)^{sk_i} \pmod{N^2}\\
\equiv&H(t)^{\sum_{j=0}^{n} sk_j}\prod_{i=1}^{n} g^{a_{i,t}} \pmod{N^2}\\
\equiv&(N+1)^{\sum_{i=1}^{n} a_{i,t}} \pmod{N^2}
\end{aligned}$$

removing all noise terms.

### C. Integer Encoding for Real Numbers

Our next goal is to apply the LCAO secure encryption scheme introduced to a Bayesian localization filter. The filter we introduce requires the use of real-valued inputs and functions, and relies on a non-linear measurement model. We will make use of a real-number encoding scheme supporting our required homomorphic operations as well as a reformulated Extended Kalman Filter which reduces the filter update step to use only these operations.

In the encryption scheme introduced, weights and values are restricted to integers and all operations are computed $\pmod N$, thus bounding meaningful inputs to $\{a \,|\, a \in \mathbb{Z}_N\}$. For this reason, a quantization and integer mapping method for real numbers is required for their encryption and homomorphic processing. We quantize with a generalized Q number format [41] due to implementation simplicity and applicability.

We define a subset of rational numbers in terms of a range $M \in \mathbb{N}$ and fractional precision $\phi \in \mathbb{N}$. This contrasts with the common definition given in terms of total bits and fractional bits [41], [42], [37], but allows for a direct mapping to integer ranges which are not powers of two. Rational subset $\mathbb{Q}_{M,\phi}$ is given by

$$\mathbb{Q}_{M,\phi} = \left\{ q \,\middle|\, \phi q \in \mathbb{N} \land -\left\lfloor \frac{M}{2} \right\rfloor \le \phi q < \left\lfloor \frac{M}{2} \right\rfloor \right\}, \tag{15}$$

and we quantize any real number $a$ by taking the nearest rational $q \in \mathbb{Q}_{M,\phi}$. That is, $\arg\min_{q\in\mathbb{Q}_{M,\phi}} |a-q|$. In this form, mapping rationals $\mathbb{Q}_{M,\phi}$ to the encryption scheme range $\mathbb{Z}_N$ is achieved by choosing $M = N$, and handling negatives with modulo arithmetic. In addition, we note that the Q number format requires a precision factor $\phi$ to be removed after each encoded multiplication, which is not supported by our encryption scheme. This is captured by a third parameter $d$; the number of multiplication factors to add or remove from encodings.

The function for *combined* quantization and encoding $\mathsf{E}_{M,\phi,d}(a)$, of a given real number $a \in \mathbb{R}$, integer range $\mathbb{Z}_M$, and the desired scaling for $d$ prior encoded multiplications, is given by

$$\mathsf{E}_{M,\phi,d}(a) = \left\lfloor \phi^{d+1} a \right\rceil \pmod M. \tag{16}$$

Decoding of an integer $u \in \mathbb{Z}_M$, is given by

$$\mathsf{E}^{-1}_{M,\phi,d}(u) = \begin{cases} \dfrac{u \pmod M}{\phi^{d+1}}, & u \pmod M \le \left\lfloor \dfrac{M}{2} \right\rfloor \\ -\dfrac{M - u \pmod M}{\phi^{d+1}}, & \text{otherwise} \end{cases}. \tag{17}$$

This encoding scheme provides the following homomorphic operations,

$$\mathsf{E}_{M,\phi,d}(a_1) + \mathsf{E}_{M,\phi,d}(a_2) \pmod M = \mathsf{E}_{M,\phi,d}(a_1 + a_2) \tag{18}$$

and

$$\mathsf{E}_{M,\phi,d}(a_1)\mathsf{E}_{M,\phi,d}(a_2) \pmod M = \mathsf{E}_{M,\phi,d+1}(a_1 a_2), \tag{19}$$

noting that when $M = N$, the modulus corresponds with the encrypted homomorphic operations in (9), (10) and therefore (22).

The choice of a high fractional precision $\phi$ may reduce quantization errors introduced in (16), however, risks an overflow following too many multiplications. Given the largest number of expected multiplications $d_{max}$, and the largest expected decoded value $a_{max}$, the parameter should be chosen such that the following condition holds

$$\left|\phi^{d_{max}+1} a_{max}\right| < \left\lfloor \frac{M}{2} \right\rfloor. \tag{20}$$

In practice, $N$ is typically very large ($N > 2^{1024}$) and this condition can be ignored when $M = N$.

## IV. Private Linear-Combination Aggregation Scheme

From the informal definitions above, it is clear that weights encrypted by an IND-CPA secure encryption scheme are sufficient for the first requirement, while a scheme satisfying AO is not sufficient for the second. To formalize the second requirement, we define a novel encryption type "Linear-Combination Aggregator Oblivious Encryption" and an accompanying security game, which capture the additional weights and modified leakage of AO.

In the next section, we will give a solution to an encryption scheme meeting LCAO security, with IND-CPA secure weight encryption, and give a cryptographic proof for its security.

Our scheme is based on the Paillier and Joye-Libert schemes introduced in Sections III-A and III-B, and similarly bases its security on the DCRA. As with Joye-Libert's private aggregation scheme, a trusted party is required for the initial distribution of user secret keys. Below, we give definitions for the four algorithms comprising the linear-combination aggregation encryption scheme.

$\mathsf{Setup}(\kappa)$ On input parameter $\kappa$, generate two equal length, sufficiently large, primes $p$ and $q$, and compute $N = pq$. Define a hash function $H : \mathbb{Z} \to \mathbb{Z}^*_{N^2}$, choose an $m > 1$ as the number of weights to combine, and set public parameter $\mathsf{pub} = H$, aggregator public key $pk_0 = N$ and aggregator private key $sk_0 = (p, q)$. The remaining user secret keys are generated by choosing $sk_i$, $1 \le i \le n-1$

uniformly from $\mathbb{Z}_{N^2}$ and setting the last key as $sk_n = -\sum_{i=1}^{n-1} sk_i \pmod{N^2}$.

$\mathsf{Enc}(pk_0, x)$ Encryption of values is computed as a Paillier encryption with implicit generator $g = N + 1$. This is given by

$$\mathcal{E}_{pk_0}(x) = (N+1)^x \rho^N \pmod{N^2}, \tag{21}$$

for a randomly chosen $\rho \in \mathbb{Z}_N$.

$\mathsf{CombEnc}(t, pk_0, sk_i, \mathcal{E}_{pk_0}(\omega_1^{(t)}),..., \mathcal{E}_{pk_0}(\omega_m^{(t)}), a_{i,1}^{(t)},..., a_{i,m}^{(t)})$ The linear combination encryption step at instance $t$ is computed as

$$l_i^{(t)} = H(t)^{sk_i} \prod_{j=1}^{m} \mathcal{E}_{pk_0}(\omega_j^{(t)})^{a_{i,j}^{(t)}} \pmod{N^2}, \tag{22}$$

and makes use of the homomorphic property (10). Correctness follows from

$$\begin{aligned} l_i^{(t)} &= H(t)^{sk_i} \prod_{j=1}^{m} \mathcal{E}_{pk_0}(\omega_j^{(t)})^{a_{i,j}^{(t)}} \pmod{N^2} \\ &= H(t)^{sk_i} \prod_{j=1}^{m} \mathcal{E}_{pk_0}(a_{i,j}^{(t)}\omega_j^{(t)}) \pmod{N^2} \\ &= H(t)^{sk_i} \prod_{j=1}^{m} (N+1)^{a_{i,j}^{(t)}\omega_j^{(t)}} \rho_j^N \pmod{N^2} \\ &= H(t)^{sk_i} (N+1)^{\sum_{j=1}^{m} a_{i,j}^{(t)}\omega_j^{(t)}} \rho_i^N \pmod{N^2}, \end{aligned}$$

for some values $\rho_i, \rho_j \in \mathbb{Z}_N, 1 \le j \le m$. Here, $\rho_i^N$ and $H(t)^{sk_i}$ can be considered the noise terms corresponding to the two levels of encryption from $pk_0$ and $sk_i$, respectively.

$\mathsf{AggDec}(t, pk_0, sk_0, l_1^{(t)},..., l_n^{(t)})$ Aggregation is computed as $l^{(t)} = \prod_{i=1}^{n} l_i^{(t)} \pmod{N^2}$, removing aggregation noise terms, and is followed by Paillier decryption

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{j=1}^{m} a_{i,j}^{(t)}\omega_j^{(t)} = \\ &\qquad \frac{L((l^{(t)})^{\lambda} \pmod{N^2})}{L((N+1)^{\lambda} \pmod{N^2})} \pmod{N}, \end{aligned} \tag{23}$$

with $\lambda = \mathsf{lcm}(p-1, q-1)$ and $L(u) = \frac{u-1}{N}$. The correctness of aggregation can be seen from

$$\begin{aligned} l^{(t)} &= \prod_{i=1}^{n} H(t)^{sk_i} (N+1)^{\sum_{j=1}^{m} a_{i,j}\omega_j} \rho_i^N \pmod{N^2} \\ &= H(t)^{\sum_{i=1}^{n} sk_i} \cdot \\ &\qquad \prod_{i=1}^{n} (N+1)^{\sum_{j=1}^{m} a_{i,j}^{(t)}\omega_j^{(t)}} \rho_i^N \pmod{N^2} \\ &= (N+1)^{\sum_{i=1}^{n}\sum_{j=1}^{m} a_{i,j}^{(t)}\omega_j^{(t)}} \rho'^N \pmod{N^2}, \end{aligned}$$

for some values $\rho_i, \rho' \in \mathbb{Z}_N$, $1 \le i \le n$.

Additionally, we note that in the above construction, all weights $\omega_j^{(t)}$ and values $a_{i,j}^{(t)}$ are integers, and that resulting linear combinations and summations are computed $\pmod{N}$.

*Remark.* The construction of this scheme additionally supports the linear combination of the implicit weight, $\omega_j^{(t)} = 1$, by replacing

$$\mathcal{E}_{pk_0}(1)^{a_{i,j}^{(t)}} = (N+1)^1 \rho_j^N \pmod{N^2} \tag{24}$$

in (22) by

$$(N+1)^{a_{i,j}^{(t)}} \pmod{N^2}, \tag{25}$$

due to the removal of $\rho_j^N$ terms during decryption. This can be used to reduce the broadcast communication cost by one weight.

To prove the security of our introduced scheme, we recall the desired security properties of an LCAO secure scheme with IND-CPA secure encrypted weights. From the definition above, weights encrypted with public key $pk_0$ are identical to encryptions of the Paillier scheme and therefore meet security notion IND-CPA. We omit this proof here and refer readers to the security proof of the Paillier encryption scheme [20] instead.

To show our scheme meets the security notion of LCAO, we prove by contrapositive that for an adversary $\mathcal{A}$ playing against a challenger using *our scheme*, we can create an adversary $\mathcal{A}'$ playing against a challenger $\mathcal{C}$ using the *Joye-Libert scheme*, such that

$$\mathsf{Adv}^{LCAO}(\mathcal{A}) > \eta_1(\kappa) \implies \mathsf{Adv}^{AO}(\mathcal{A}') > \eta_2(\kappa),$$

for some negligible functions $\eta_1$, $\eta_2$ and security parameter $\kappa$. (*i.e.*, if we assume our scheme is not LCAO secure, then the Joye-Libert scheme is not AO secure.) Given the Joye-Libert AO proof in [14], we know our scheme must be LCAO secure and will thus conclude our proof. The proof overview has been given in Figure 2. Additionally, the function $H$ used by our scheme is treated as a *random oracle* in the Joye-Libert AO proof and will, therefore, prove our scheme secure in the random oracle model as well.

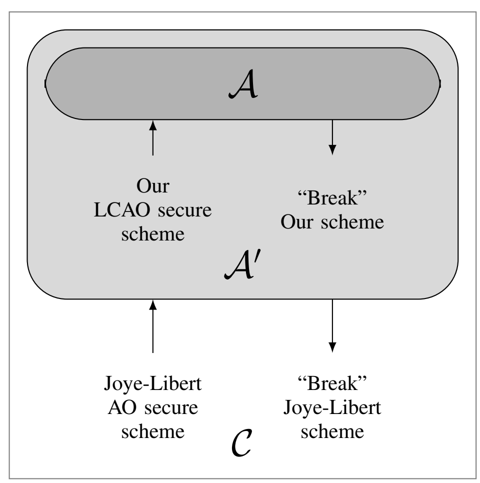


Fig. 2. High-level overview of our scheme's LCAO security proof.

## V. Privacy-Preserving Localization

With the relevant preliminaries, we now apply the LCAO secure scheme to our original localization problem. Recall we are considering a navigator in the presence of $n$ range sensors. The navigator runs a local filter, computing its update step with equations and , where received information from each sensor $i$ consists of the measurement vector

$$\underline{\boldsymbol{i}}_{k,i} = \mathbf{H}_{k,i}^{\top} r_i^{-1} (\boldsymbol{z}_{k,i} - h_i(\underline{\boldsymbol{x}}_{k|k-1}) + \mathbf{H}_{k,i}\underline{\boldsymbol{x}}_{k|k-1}) \tag{26}$$

and the measurement matrix

$$\mathbf{I}_{k,i} = \mathbf{H}_{k,i}^{\top} r_i^{-1} \mathbf{H}_{k,i}\,, \tag{27}$$

at each time-step $k$. In this form, all sensitive sensor information is exclusively found in $\underline{\boldsymbol{i}}_{k,i}$ and $\mathbf{I}_{k,i}$. Namely, their measurements $\boldsymbol{z}_{k,i}$, measurement variances $r_i$ and locations $\underline{s}_i$; captured in measurement functions $h_i$ and Jacobians $\mathbf{H}_{k,i}$.

The reason for using a linear-combination aggregation scheme in this scenario is due to wanting to compute $\underline{\boldsymbol{i}}_{k,i}$ and $\mathbf{I}_{k,i}$ at the sensors, which would require the navigator to disclose some information in $\underline{\boldsymbol{x}}_{k|k-1}$ for the computing of Jacobians $\mathbf{H}_{k,i}$. We want to reformulate $\underline{\boldsymbol{i}}_{k,i}$ and $\mathbf{I}_{k,i}$ such that the sensors can use the LCAO secure scheme, with location information in $\underline{\boldsymbol{x}}_{k|k-1}$ as the weights from the navigator. As was shown in Section , this will guarantee that sensors cannot learn navigator location information $\underline{\boldsymbol{x}}_{k|k-1}$, and that the navigator cannot learn individual sensor values required for computing $\underline{\boldsymbol{i}}_{k,i}$ and $\mathbf{I}_{k,i}$. From (26) and (27) we see that such a linear reformulation will set requirements on the possible measurement functions $h_i$.

### *A. Range Measurement Modification*

Recalling the state definition (1) and two-dimensional measurement model (3), we can see that $h_i$ *cannot* be rearranged to a linear combination of elements private to the navigator. In addition, the Jacobian of $h_i$ at $\underline{\boldsymbol{x}}_{k|k-1}$,

$$\mathbf{H}_{k,i} = \begin{bmatrix} \frac{\boldsymbol{x}_{k|k-1} - s_{x,i}}{\sqrt{(\boldsymbol{x}_{k|k-1} - s_{x,i})^2 + (\boldsymbol{y}_{k|k-1} - s_{y,i})^2}} \\ \frac{\boldsymbol{y}_{k|k-1} - s_{y,i}}{\sqrt{(\boldsymbol{x}_{k|k-1} - s_{x,i})^2 + (\boldsymbol{y}_{k|k-1} - s_{y,i})^2}} \end{bmatrix}, \tag{28}$$

cannot be either. Therefore, we consider the modified measurement functions

$$h_i'(\underline{x}) = h_i(\underline{x})^2\,. \tag{29}$$

A measurement function in this form allows rearrangement to a linear combination of navigator information, and thus of the corresponding modified measurement vector $\underline{\boldsymbol{i}}'_{k,i}$ and measurement matrix $\mathbf{I}'_{k,i}$ as well. The modified measurement functions $h_i'$ can be written as

$$\begin{aligned} h_i'(\underline{x}) &= \left\| \begin{bmatrix} x & y \end{bmatrix}^{\top} - \underline{s}_i \right\|^2 \\ &= (x - s_{x,i})^2 + (y - s_{y,i})^2 \\ &= x^2 + y^2 - 2s_{x,i}x - 2s_{y,i}y + s_{x,i}^2 + s_{y,i}^2 \end{aligned}\,, \tag{30}$$

and the corresponding Jacobians $\mathbf{H}'_{k,i}$ as

$$\mathbf{H}'_{k,i} = \begin{bmatrix} 2\boldsymbol{x}_{k|k-1} - 2s_{x,i} \\ 2\boldsymbol{y}_{k|k-1} - 2s_{y,i} \end{bmatrix}. \tag{31}$$

From the above we see that $h_i'(\underline{\boldsymbol{x}}_{k|k-1})$ and $\mathbf{H}'_{k,i}$ can be rearranged as linear combinations of the weights $\boldsymbol{x}_{k|k-1}^2$, $\boldsymbol{y}_{k|k-1}^2$, $\boldsymbol{x}_{k|k-1}$ and $\boldsymbol{y}_{k|k-1}$, which capture all the required information that is private to the navigator.

To show how $\underline{\boldsymbol{i}}'_{k,i}$ and $\mathbf{I}'_{k,i}$ can be formulated in a similar manner using , we require the existance of a modified measurement model of the form

$$\boldsymbol{z}'_{k,i} = h_i'(\underline{\boldsymbol{x}}_k) + \boldsymbol{v}'_{k,i}\,, \tag{32}$$

where $\boldsymbol{z}'_{k,i}$ is the modified measurement, and noise $\boldsymbol{v}'_{k,i}$ is zero-mean with known variance. The approximation of $\boldsymbol{z}'_{k,i}$ and its noise variance $\boldsymbol{r}_{k,i}$ from original measurements and variances $\boldsymbol{z}_{k,i}$ and $r_i$ will be shown in Section V-A.

In Section , we assumed the existance of a measurement model (32) where modified sensor measurements $\boldsymbol{z}'_{k,i}$ had zero-mean noises with variances $\boldsymbol{r}_{k,i}$. In practice, conversion of range measurements $\boldsymbol{z}_{k,i}$ to $\boldsymbol{z}'_{k,i}$ is complicated by the noise term $\boldsymbol{v}_{k,i} \sim \mathcal{N}(0, r_i)$ in (3). Squaring the range measurement produces

$$\begin{aligned} \boldsymbol{z}_{k,i}^2 &= (h_i(\underline{\boldsymbol{x}}_k) + \boldsymbol{v}_{k,i})^2 \\ &= h_i(\underline{\boldsymbol{x}}_k)^2 + 2h_i(\underline{\boldsymbol{x}}_k)\boldsymbol{v}_{k,i} + \boldsymbol{v}_{k,i}^2 \\ &= h_i'(\underline{\boldsymbol{x}}_k) + 2h_i(\underline{\boldsymbol{x}}_k)\boldsymbol{v}_{k,i} + \boldsymbol{v}_{k,i}^2\,, \end{aligned} \tag{33}$$

with new noise term, $2h_i(\underline{\boldsymbol{x}}_k)\boldsymbol{v}_{k,i} + \boldsymbol{v}_{k,i}^2$, now dependent on the measurement function $h_i$ and no longer zero-mean or Gaussian. We can compute the mean of the new noise term (a function of the Gaussian term $\boldsymbol{v}_{k,i}$) as

$$\mathrm{E}\left\{2h_i(\underline{\boldsymbol{x}}_k)\boldsymbol{v}_{k,i} + \boldsymbol{v}_{k,i}^2\right\} = r_i \tag{34}$$

and define modified measurements as

$$\begin{aligned} \boldsymbol{z}'_{k,i} &= \boldsymbol{z}_{k,i}^2 - r_i \\ &= h_i(\underline{\boldsymbol{x}}_k)^2 + 2h_i(\underline{\boldsymbol{x}}_k)\boldsymbol{v}_{k,i} + \boldsymbol{v}_{k,i}^2 - r_i \\ &= h_i'(\underline{\boldsymbol{x}}_k) + \boldsymbol{v}'_{k,i}\,, \end{aligned} \tag{35}$$

with now zero-mean noise $\boldsymbol{v}'_{k,i} = 2h_i(\underline{\boldsymbol{x}}_k)\boldsymbol{v}_{k,i} + \boldsymbol{v}_{k,i}^2 - r_i$. The noise in this case (again a function of white Gaussian term $\boldsymbol{v}_{k,i}$) has variance

$$\mathrm{Var}\left\{\boldsymbol{v}'_{k,i}\right\} = 4h_i(\underline{\boldsymbol{x}}_k)^2 r_i + 2r_i^2 \tag{36}$$

and is dependent on $h_i(\underline{\boldsymbol{x}}_k)$. To use the modified measurement function (35) with an EIF, we require an estimate of its noise variance using only the values available to the sensor, $\boldsymbol{z}_{k,i}$ and $r_i$. A conservative estimate of the variance (*i.e.*, a larger estimate resulting in less confidence in measurements) is desirable to avoid increasing the likeliness of filter divergence. While replacing $h_i(\underline{\boldsymbol{x}}_k)$ with $\boldsymbol{z}_{k,i}$ only provides a conservative estimate when

$$\begin{aligned} &\boldsymbol{z}_{k,i} > h_i(\underline{\boldsymbol{x}}_k) \\ \implies &\boldsymbol{z}_{k,i} - h_i(\underline{\boldsymbol{x}}_k) > 0 \\ \implies &\boldsymbol{v}_{k,i} > 0\,, \end{aligned} \tag{37}$$

and cannot be guaranteed, we can instead provide a conservative estimate with 95% confidence, by shifting measurement $\boldsymbol{z}_{k,i}$ by two standard deviations $\sqrt{r_i}$. Thus, the modified measurement variance for sensor $i$ at time $k$ is conservatively approximated by

$$\begin{aligned} \boldsymbol{r}_{k,i} &= 4(\boldsymbol{z}_{k,i} + 2\sqrt{r_i})^2 r_i + 2r_i^2 \\ &\gtrapprox \mathrm{Var}\left\{\boldsymbol{v}'_{k,i}\right\} \end{aligned}. \tag{38}$$

Together, the model and variance, (35) and (38), provide a suitable substitute for the required measurement model (32) and can be used to achieve the desired linear properties using only the available range sensors.

### B. Localization

With the existence of (32), measurement vector and measurement matrix linear combinations can be given in the same form as , by

$$\begin{aligned}\underline{\boldsymbol{i}}'_{k,i} &= \mathbf{H}'^{\top}_{k,i}\boldsymbol{r}^{-1}_{k,i}(\boldsymbol{z}'_{k,i} - h'_i(\underline{\boldsymbol{x}}_{k|k-1}) + \mathbf{H}'_{k,i}\underline{\boldsymbol{x}}_{k|k-1}) \\ &= \begin{bmatrix} \begin{aligned}&(2\boldsymbol{r}^{-1}_{k,i})\boldsymbol{x}^3_{k|k-1} + (2\boldsymbol{r}^{-1}_{k,i})\boldsymbol{x}_{k|k-1}\boldsymbol{y}^2_{k|k-1} \\ &\quad + (-\boldsymbol{r}^{-1}_{k,i}s_{x,i})\boldsymbol{x}^2_{k|k-1} + (-2\boldsymbol{r}^{-1}_{k,i}s_{x,i})\boldsymbol{y}^2_{k|k-1} \\ &\quad + (2\boldsymbol{r}^{-1}_{k,i}\boldsymbol{z}'_{k,i})\boldsymbol{x}_{k|k-1} + (-2\boldsymbol{r}^{-1}_{k,i}s^2_{x,i})\boldsymbol{x}_{k|k-1} \\ &\quad + (-2\boldsymbol{r}^{-1}_{k,i}s^2_{y,i})\boldsymbol{x}_{k|k-1} + (2\boldsymbol{r}^{-1}_{k,i}s^3_{x,i}) \\ &\quad + (2\boldsymbol{r}^{-1}_{k,i}s_{x,i}s^2_{y,i}) + (-2\boldsymbol{r}^{-1}_{k,i}s_{x,i}\boldsymbol{z}'_{k,i})\end{aligned} \\ \begin{aligned}&(2\boldsymbol{r}^{-1}_{k,i})\boldsymbol{y}^3_{k|k-1} + (2\boldsymbol{r}^{-1}_{k,i})\boldsymbol{x}^2_{k|k-1}\boldsymbol{y}_{k|k-1} \\ &\quad + (-2\boldsymbol{r}^{-1}_{k,i}s_{y,i})\boldsymbol{x}^2_{k|k-1} + (-2\boldsymbol{r}^{-1}_{k,i}s_{y,i})\boldsymbol{y}^2_{k|k-1} \\ &\quad + (2\boldsymbol{r}^{-1}_{k,i}\boldsymbol{z}'_{k,i})\boldsymbol{y}_{k|k-1} + (-2\boldsymbol{r}^{-1}_{k,i}s^2_{x,i})\boldsymbol{y}_{k|k-1} \\ &\quad + (-2\boldsymbol{r}^{-1}_{k,i}s^2_{y,i})\boldsymbol{y}_{k|k-1} + (2\boldsymbol{r}^{-1}_{k,i}s_{y,i}s^2_{x,i}) \\ &\quad + (2\boldsymbol{r}^{-1}_{k,i}s^3_{y,i}) + (-2\boldsymbol{r}^{-1}_{k,i}s_{y,i}\boldsymbol{z}'_{k,i})\end{aligned} \end{bmatrix}\end{aligned} \tag{39}$$

and

$$\begin{aligned}\mathbf{I}'_{k,i} &= \mathbf{H}'^{\top}_{k,i}\boldsymbol{r}^{-1}_{k,i}\mathbf{H}'_{k,i} \\ &= \begin{bmatrix}\beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22}\end{bmatrix},\end{aligned} \tag{40}$$

with

$$\begin{aligned}\beta_{11} &= (4\boldsymbol{r}^{-1}_{k,i})\boldsymbol{x}^2_{k|k-1} + (-8\boldsymbol{r}^{-1}_{k,i}s_{x,i})\boldsymbol{x}_{k|k-1} + (4\boldsymbol{r}^{-1}_{k,i}s^2_{x,i}), \\ \beta_{12} &= (4\boldsymbol{r}^{-1}_{k,i})\boldsymbol{x}_{k|k-1}\boldsymbol{y}_{k|k-1} + (-4\boldsymbol{r}^{-1}_{k,i}s_{y,i})\boldsymbol{x}_{k|k-1} \\ &\qquad + (-4\boldsymbol{r}^{-1}_{k,i}s_{x,i})\boldsymbol{y}_{k|k-1} + (4\boldsymbol{r}^{-1}_{k,i}s_{x,i}s_{y,i}), \\ \beta_{21} &= \beta_{12}, \text{ and} \\ \beta_{22} &= (4\boldsymbol{r}^{-1}_{k,i})\boldsymbol{y}^2_{k|k-1} + (-8\boldsymbol{r}^{-1}_{k,i}s_{y,i})\boldsymbol{y}_{k|k-1} + (4\boldsymbol{r}^{-1}_{k,i}s^2_{y,i}).\end{aligned}$$

The above rearrangements result in $\underline{\boldsymbol{i}}'_{k,i}$ and $\mathbf{I}'_{k,i}$ as linear comabinations of the weights

$$\begin{aligned}&\langle \boldsymbol{x}^3_{k|k-1},\ \boldsymbol{y}^3_{k|k-1},\ \boldsymbol{x}^2_{k|k-1}\boldsymbol{y}_{k|k-1},\ \boldsymbol{x}_{k|k-1}\boldsymbol{y}^2_{k|k-1}, \\ &\quad \boldsymbol{x}^2_{k|k-1},\ \boldsymbol{y}^2_{k|k-1},\ \boldsymbol{x}_{k|k-1}\boldsymbol{y}_{k|k-1},\ \boldsymbol{x}_{k|k-1},\ \boldsymbol{y}_{k|k-1}\rangle,\end{aligned} \tag{41}$$

which again capture all the required information private to the navigator.

The final step required for the application of our LCAO secure encryption scheme to the linear combinations (39) and (40) is the handling of the scheme's instance variable $t$ in (22). As six linear-combination aggregations occur at each time-step $k$ (four elements in $\mathbf{I}'_{k,i}$ and two in $\underline{\boldsymbol{i}}'_{k,i}$) and it is required for the security of the scheme that $t$ be unique for each aggregation at each time, this is handled by setting $t$ to the concatenation

$$t = k \parallel v \parallel w \parallel \tau \tag{42}$$

for aggregation in row $v$ and column $w$, with $\tau = 0$ for aggregations in $\underline{\boldsymbol{i}}'_{k,i}$ and $\tau = 1$ otherwise.

*Remark.* The solutions (39) and (40) have been derived for two-dimensional localization, but can be similarly extended to the three-dimensional case. We note, however, the additional cost with increasing dimension. The number of weights is increased in the rearranged functions $h'_i$, and therefore combinatorially increased in $\underline{\boldsymbol{i}}'_{k,i}$ and $\mathbf{I}'_{k,i}$ by . This results in a combinatorial increase in weights, and therefore communication, with respect to the number of state parameters required for computing functions $h_i$.

### C. Leakage

With the algorithm defined, we can analyze the localization leakage when the strictly defined LCAO leakage is taken into account. We recall the assumptions on participant capabilities made in Section . Global and consistent broadcasting is in line with the LCAO definition and required to guarantee the scheme's leakage. Honest-but-curious sensors are not required for scheme privacy, but avoid the possibility of malicious sensors gaining knowledge about other sensors' measurements when navigator behavior is dependent on its estimate. This may be achievable by detecting changes in measurements after reporting false ones. As misbehaving sensors are also a known problem which complicates homomorphic operations, active malicious sensors are not considered in the scope of this work.

The aggregation operation of our encryption scheme leaks the sums of sensor measurement vectors and matrices $\sum_{i=1}^{n} \underline{\boldsymbol{i}}'_{k,i}$ and $\sum_{i=1}^{n} \mathbf{I}'_{k,i}$, however, as stated in Remark II-A, weights chosen by a corrupted navigator mean that individual sums of coefficients can be leaked as well. That is, sums of coefficients in (39) and (40) (*i.e.*, $\sum_{i=1}^{n} 2\boldsymbol{r}^{-1}_{k,i}$, $\sum_{i=1}^{n} -\boldsymbol{r}^{-1}_{k,i}s_{x,i}$, $\sum_{i=1}^{n} -2\boldsymbol{r}^{-1}_{k,i}s_{x,i}$, $\sum_{i=1}^{n} 2\boldsymbol{r}^{-1}_{k,i}\boldsymbol{z}'_{k,i}$, $\dots$) are leaked to a corrupted navigator.

From the leaked sums of coefficients, we see that all information private to the sensors and broadcast to the navigator, namely their modified measurements $\boldsymbol{z}'_{k,i}$, variance estimates $\boldsymbol{r}_{k,i}$ and locations $\underline{s}_i$, are present only in the sums

$$\sum_{i=1}^{n} \boldsymbol{z}'_{k,i},\ \sum_{i=1}^{n} \boldsymbol{r}_{k,i},\ \sum_{i=1}^{n} s_{x,i} \text{ and } \sum_{i=1}^{n} s_{y,i}. \tag{43}$$

From these sums and the definitions (35) and (38), we conclude that the navigator can *at most* learn the sums of sensor private data, which in practice corresponds to the average sensor modified measurements, estimated variances and locations.

**Theorem V.1.** In the context of our localization scheme, leakage of the incorporated LCAO secure encryption scheme corresponds to the leakage of average sensor private information, given by the sums in (43).

### D. Pseudocode

In this section, we piece together our LCAO secure encryption scheme in Section with the modified measurement model in Sections and and define our privacy-preserving localization algorithm.

The privacy-preserving localization filter can be described by the following steps.

**Setup** The Setup algorithm from Section is executed by a trusted third party, $N$ and $H$ are made known to the navigator and all sensors, and the navigator and sensor secret keys, $sk_0 = \lambda = \mathsf{lcm}(p-1, q-1)$ and $sk_i$, $1 \leq i \leq n$ respectively, are distributed accordingly. Additionally, we assume that fractional precision $\phi$ from Section III-C is also a public parameter and made known to the navigator and sensors. We will thus simplify the encoding notation $\mathsf{E}_{N,\phi,d}(\cdot)$ to $\mathsf{E}_d(\cdot)$.

**Prediction** The navigator computes the typical EKF prediction equations before encrypting state variables and broadcasting to sensors. This is given by Algorithm 1.

**Measurement** Sensors modify their measurements before homomorphically computing measurement vectors and matrices $\underline{\boldsymbol{i}}'_{k,i}$ and $\mathbf{I}'_{k,i}$, encrypting them for aggregation, and sending them back to the navigator. Care must be taken when encoding during homomorphic operations to ensure fractional factor $d$ satisfies (18), in particular when using the property from Remark IV. This is described by Algorithm 2.

**Update** The navigator aggregates measurement vectors and matrices before decrypting them. The typical EIF update equations can then be computed. This is shown in Algorithm 3.

**Algorithm 1** Navigator Prediction

**procedure** PREDICTION($\underline{\boldsymbol{x}}_{k-1|k-1}$, $\mathbf{P}_{k-1|k-1}$, $\underline{f}$, $\mathbf{Q}$, $N$)
 Compute $\mathbf{F}_k$ by
 $\underline{\boldsymbol{x}}_{k|k-1} \leftarrow \underline{f}(\underline{\boldsymbol{x}}_{k-1|k-1})$
 $\mathbf{P}_{k|k-1} \leftarrow \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^\top + \mathbf{Q}$
 Compute $\mathsf{E}_0(\boldsymbol{x}^3_{k|k-1})$ by (16)
 Compute $\mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}^3_{k|k-1}))$ by (21)
 Broadcast $\mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}^3_{k|k-1}))$ to sensors
 **for** Remaining weights in (41) **do**
  Broadcast weight in the form above
 **end for**
 **return** $\underline{\boldsymbol{x}}_{k|k-1}, \mathbf{P}_{k|k-1}$
**end procedure**

Algorithms 1, 2 and 3 have been summarized graphically in Figure 3, where encryptions and encodings of vectors and matrices have been denoted with $\mathcal{E}_{pk_0,sk_i}(\cdot)$ and $\mathsf{E}_d(\cdot)$ for brevity, and represent element-wise operations with the same parameters.

**Algorithm 2** Measurement at Sensor $i$

**procedure** MEASUREMENT($i$, $s_{x,i}$, $s_{y,i}$, $r_i$, $N$, $H$)
 Measure $\boldsymbol{z}_{k,i}$
 Compute $\boldsymbol{z}'_{k,i}$ by (35)
 Compute $\boldsymbol{r}_{k,i}$ by (38)
 Recieve $\mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}^3_{k|k-1}))$
 **for** Remaining weights in (41) **do**
  Recieve weight in the form above
 **end for**
 Let $\alpha_v^{(i)}$ represent an encryption of element $v$ in $\underline{\boldsymbol{i}}'_{k,i}$ from (39)

$$\begin{aligned}\alpha_1^{(i)} \leftarrow\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}^3_{k|k-1}))^{\mathsf{E}_0(2\boldsymbol{r}^{-1}_{k,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}_{k|k-1}\boldsymbol{y}^2_{k|k-1}))^{\mathsf{E}_0(2\boldsymbol{r}^{-1}_{k,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}^2_{k|k-1}))^{\mathsf{E}_0(-\boldsymbol{r}^{-1}_{k,i}s_{x,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{y}^2_{k|k-1}))^{\mathsf{E}_0(-2\boldsymbol{r}^{-1}_{k,i}s_{x,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}_{k|k-1}))^{\mathsf{E}_0(2\boldsymbol{r}^{-1}_{k,i}\boldsymbol{z}'_{k,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}_{k|k-1}))^{\mathsf{E}_0(-2\boldsymbol{r}^{-1}_{k,i}s^2_{x,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}_{k|k-1}))^{\mathsf{E}_0(-2\boldsymbol{r}^{-1}_{k,i}s^2_{y,i})} \cdot \\ & (N+1)^{\mathsf{E}_1(2\boldsymbol{r}^{-1}_{k,i}s^3_{x,i})}(N+1)^{\mathsf{E}_1(2\boldsymbol{r}^{-1}_{k,i}s_{x,i}s^2_{y,i})} \cdot \\ & (N+1)^{\mathsf{E}_1(-2\boldsymbol{r}^{-1}_{k,i}s_{x,i}\boldsymbol{z}'_{k,i})} H(k \,\|\, 1 \,\|\, 1 \,\|\, 0) \pmod{N^2}\end{aligned}$$

 Compute $\alpha_2^{(i)}$ using (39), (22) and Remark IV in the form above
 **for** $v \leftarrow 1$ to $2$ **do**
  Send $\alpha_v^{(i)}$ to the navigator
 **end for**
 Let $\beta_{vw}^{(i)}$ represent an encryption of element $(v, w)$ in $\mathbf{I}'_{k,i}$ from (40)

$$\begin{aligned}\beta_{11}^{(i)} \leftarrow\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}^2_{k|k-1}))^{\mathsf{E}_0(4\boldsymbol{r}^{-1}_{k,i})} \cdot \\ & \mathcal{E}_{pk_0}(\mathsf{E}_0(\boldsymbol{x}_{k|k-1}))^{\mathsf{E}_0(-8\boldsymbol{r}^{-1}_{k,i}s_{x,i})} \cdot \\ & (N+1)^{\mathsf{E}_1(4\boldsymbol{r}^{-1}_{k,i}s^2_{x,i})} \cdot \\ & H(k \,\|\, 1 \,\|\, 1 \,\|\, 1) \pmod{N^2}\end{aligned}$$

 Compute remaining $\beta_{vw}^{(i)}$ using (40), (22) and Remark IV in the form above
 **for** $v \leftarrow 1$ to $2$ **do**
  **for** $w \leftarrow 1$ to $2$ **do**
   Send $\beta_{vw}^{(i)}$ to the navigator
  **end for**
 **end for**
**end procedure**

## VI. SIMULATION AND RESULTS

To demonstrate our proposed approach, we have implemented the algorithm in Section and simulated the measurement and navigation of an object following a linear, time-invariant, process model in two-dimensional space.

Code was written in the C programming language, using the MPI library [43] to support simultaneous computations by sensors and navigator as different processes. The OpenSSL library's, [44], mask generation function MGF1 and hash function SHA256 were used to implement the required hash function $H$, while the Libpaillier library [45] was used for the implementation of the Paillier encryption scheme. Additionally, GNU libraries GSL [46] and GMP [47] were used for algebraic operations and the handling of multiple-precision integers, respectively. Simulation execution was performed on a 3.00GHz Intel i7-9700 CPU, and run on the Windows Subsystem for Linux (WSL), where no real-time kernel was ensured but sufficiently consistent runtimes were observed for the purposes of our simulation.

Recalling the dependence of the modified measurement noise on the true measurement in (35), the noise distribution of the model changes with varying sizes of measurements. We considered four layouts with varying sensor distances when running our simulations to observe differences in estimation error when measurements are of different sizes. The considered layouts have been plotted and named in Figure 4. To evaluate the accuracy of our method, we have run 100

**Algorithm 3** Navigator Update

**procedure** UPDATE($\underline{\boldsymbol{x}}_{k|k-1}$, $\mathbf{P}_{k|k-1}$, $N$, $\lambda$)
  **for** $v \leftarrow 1$ to $2$ **do**
    Receive $\alpha_v^{(i)}$ from each sensor $1 \leq i \leq n$
  **end for**
  **for** $v \leftarrow 1$ to $2$ **do**
    **for** $w \leftarrow 1$ to $2$ **do**
      Receive $\beta_{vw}^{(i)}$ from each sensor $1 \leq i \leq n$
    **end for**
  **end for**
  Let $\alpha_v$ represent an encryption of element $v$ in $\sum_{i=1}^{n} \underline{\boldsymbol{i}}'_{k,i}$
  **for** $v \leftarrow 1$ to $2$ **do**
    $\alpha_v \leftarrow \prod_{i=1}^{n} \alpha_v^{(i)}$
    Compute $\mathcal{D}_{sk_0}(\alpha_v)$ with $\lambda$ by (23)
    Compute $\mathsf{E}_1^{-1}(\mathcal{D}_{sk_0}(\alpha_v))$ by (17)
  **end for**
  Construct $\sum_{i=1}^{n} \underline{\boldsymbol{i}}'_{k,i}$ from decoded decryptions above
  Let $\beta_{vw}$ represent an encryption of element $(v, w)$ in $\sum_{i=1}^{n} \mathbf{I}'_{k,i}$
  **for** $v \leftarrow 1$ to $2$ **do**
    **for** $w \leftarrow 1$ to $2$ **do**
      $\beta_{vw} \leftarrow \prod_{i=1}^{n} \beta_{vw}^{(i)}$
      Compute $\mathcal{D}_{sk_0}(\beta_{vw})$ with $\lambda$ by (23)
      Compute $\mathsf{E}_1^{-1}(\mathcal{D}_{sk_0}(\beta_{vw}))$ by (17)
    **end for**
  **end for**
  Construct $\sum_{i=1}^{n} \mathbf{I}'_{k,i}$ from decoded decryptions above
  $\underline{\boldsymbol{y}}_{k|k} \leftarrow \mathbf{P}_{k|k-1}^{-1} \underline{\boldsymbol{x}}_{k|k-1} + \sum_{i=1}^{n} \underline{\boldsymbol{i}}'_{k,i}$
  $\mathbf{Y}_{k|k} \leftarrow \mathbf{P}_{k|k-1}^{-1} + \sum_{i=1}^{n} \mathbf{I}'_{k,i}$
  $\underline{\boldsymbol{x}}_{k|k} \leftarrow \mathbf{Y}_{k|k}^{-1} \underline{\boldsymbol{y}}_{k|k}$
  $\mathbf{P}_{k|k} \leftarrow \mathbf{Y}_{k|k}^{-1}$
  **return** $\underline{\boldsymbol{x}}_{k|k}, \mathbf{P}_{k|k}$
**end procedure**

simulations, all with 50 filter iterations, for each of the sensor layouts with sensor measurement variances set to $r_i = 5$ simulation units. Due to the large encoding modulus $N$, no notable advantage is gained when choosing a low fractional precision factor $\phi$, thus, $\phi = 2^{32}$ was chosen to ensure minimal loss of precision when compared to the floating-point computations in the unencrypted EIF. The plotted average RMSE of our proposed method as well as that of the unencrypted EIF filter can be seen in Figure 5. From the results, it can be seen that sensor distance has little effect on the performance of the filter and that the measurement modification from Section V-A has a negligible effect when compared to a normal EIF filter. The robustness of the privacy-preserving method can be attributed to the conservativeness of variance estimates in (38) and the sufficiently large fractional precision parameter $\phi$.

In addition to accuracy, computational requirements are an important factor in the adoption of novel cryptographic methods. Figure 6 shows the average simulation runtimes when encryption key bit lengths (*i.e.*, bit lengths of $N$) and the number of sensors are varied. Every plotted time is again the average of 100 simulations, each running for 50 filter iterations. As sensor and navigator computations run in parallel, increases in runtime from additional sensors are primarily due to the longer aggregation operation at the navigator. The predominant computational cost is dependent on the encryption scheme key size, which must be chosen such that sufficient security is provided by the scheme. The current recommendation for a secure implementation relying on prime factorisation (difficulty of factorising $N$) is to use 2048 bit-long keys [48].

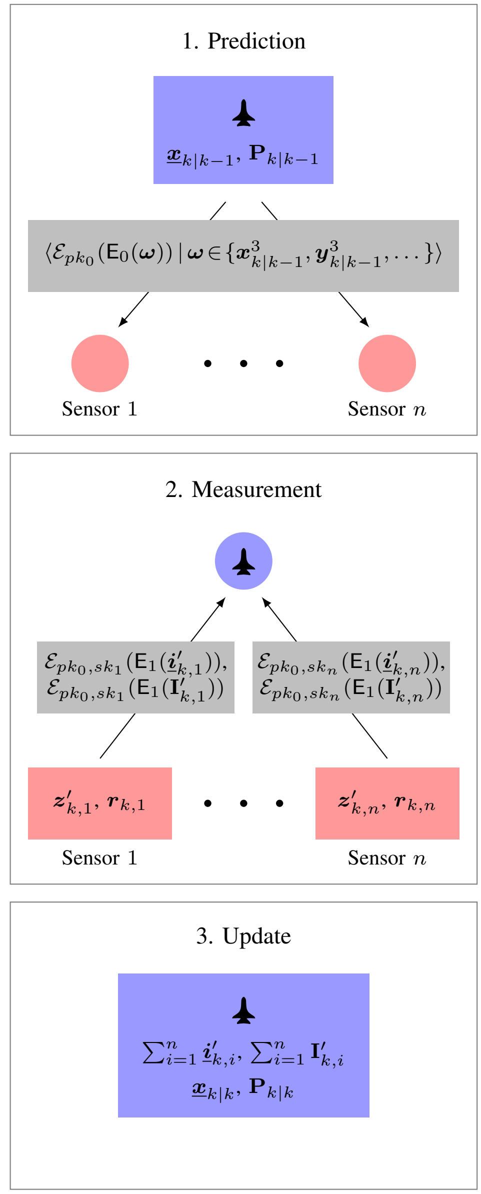


Fig. 3. The summary of steps involved in our proposed privacy-preserving EIF.

## VII. Conclusion

To develop a Bayesian localization algorithm which preserves navigator and sensor privacy, we first defined the novel notion of an LCAO secure encryption scheme and gave a provably secure implementation based on the Paillier and Joye-Libert encryption and aggregation schemes, respectively. This

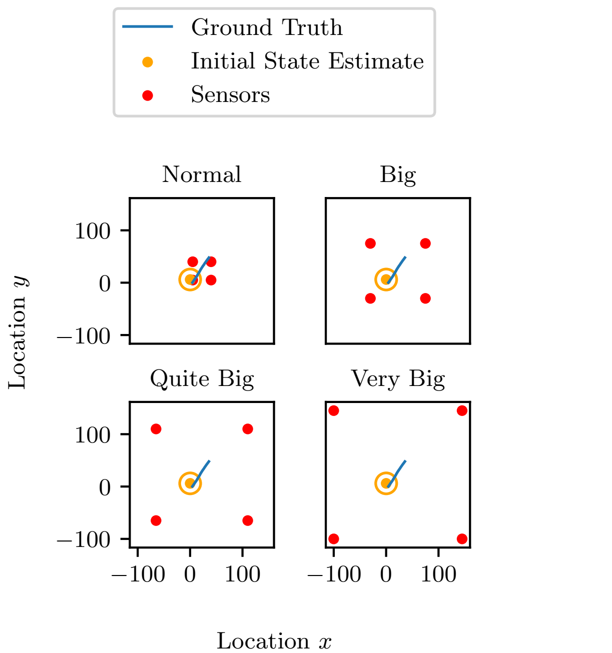


Fig. 4. The considered layouts, with varying average distances between navigator and sensors.

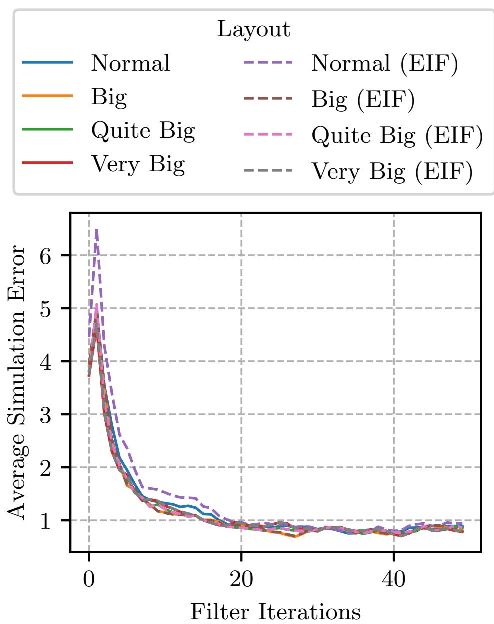


Fig. 5. Filter error for different layouts, where solid lines show the results from our privacy-preserving filter and dashed lines from an unmodified EIF.

scheme was then used in our proposed privacy-preserving EIF, where sensors compute measurement information homomorphically such that it can be privately used by a navigator. Our privacy-preserving estimation method may find uses in a variety of untrusted distributed localization environments including airspaces and autonomous vehicle networks as well as those where alternative measurement models satisfying our defined linearity requirements can be applied. Possible future work in this topic includes the expanding of the LCAO security notion to *ensure* that the same weights are broadcast to all

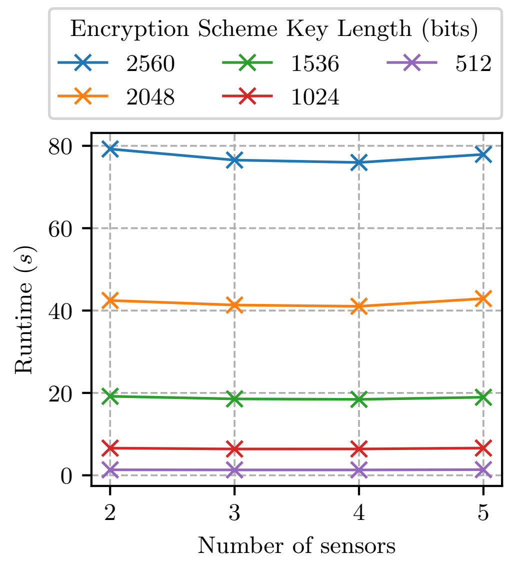


Fig. 6. Runtimes for varying key sizes and numbers of sensors.

sensors, the application of LCAO secure schemes to alternative measurement models and the exploring of implications behind an active sensor attacker model.

## Appendix A
## Linear-Combination Aggregator Oblivousness (LCAO)

**Setup** The challenger chooses security parameter $\kappa$, runs the $\mathsf{Setup}(\kappa)$ algorithm and gives pub, $m$ and $pk_0$ to the attacker

**Queries** The attacker can now perform encryptions or submit queries that are answered by the challenger. The types of actions are:

1) *Encryption:* The attacker chooses a value $x$ and computes an encryption of $x$ under the aggregator's public key $pk_0$, obtaining $\mathcal{E}_{pk_0}(x)$.
2) *Weight Queries:* The attacker chooses an instance $t$ and receives the weights for that instance encrypted with the aggregator's public key, $\mathcal{E}_{pk_0}(\omega_j^{(t)})$, $1 \leq j \leq m$.
3) *Combine Queries:* The attacker chooses a tuple $(i, t, a_{i,1}^{(t)}, \dots, a_{i,m}^{(t)})$ such that for any two chosen combine query tuples $(i, t, a_{i,1}^{(t)}, \dots, a_{i,m}^{(t)})$ and $(i', t', a_{i',1}^{\prime(t')}, \dots, a_{i',m}^{\prime(t')})$, the following condition holds:

$$i = i' \wedge t = t' \implies a_{i,j}^{(t)} = a_{i',j}^{\prime(t')},\ 1 \leq j \leq m\,.$$

The attacker is then given back the encryption of the linear combination $\mathcal{E}_{pk_0, sk_i}(\sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)})$ encrypted under both the aggregator public key $pk_0$ and the secret key $sk_i$.
4) *Compromise queries:* The attacker chooses $i$ and receives the secret key $sk_i$. The aggregator's secret key may also be compromised (when choosing $i = 0$).

**Challenge** Next, the attacker chooses an instance $t^*$, and a subset of users $S \subseteq U$ where $U$ is the complete set of users for which no combine queries, for the instance $t^*$, and no compromise queries, are made for the duration of the game. The attacker then chooses two series of tuples

$$\left\langle \left(i, t^*, a_{i,1}^{(t^*)(0)}, \ldots, a_{i,m}^{(t^*)(0)}\right) \middle| i \in S \right\rangle$$

and

$$\left\langle \left(i, t^*, a_{i,1}^{(t^*)(1)}, \ldots, a_{i,m}^{(t^*)(1)}\right) \middle| i \in S \right\rangle ,$$

and gives them to the challenger. In the case that $0 \in S$ (*i.e.*, the aggregator is compromised) and $S = U$, it is additionally required that

$$\sum_{i \in S} \sum_{j=1}^{m} a_{i,j}^{(t^*)(0)} \omega_j^{(t^*)} = \sum_{i \in S} \sum_{j=1}^{m} a_{i,j}^{(t^*)(1)} \omega_j^{(t^*)} ,$$

for weights $\omega_j^{(t^*)}$, $1 \leq j \leq m$ returned by a *Weight Query* with chosen instance $t^*$. The challenger then chooses a random bit $b \in \{1, 0\}$ and returns encryptions

$$\left\langle \mathcal{E}_{pk_0, sk_i} \left( \sum_{j=1}^{m} a_{i,j}^{(t^*)(b)} \omega_j^{(t^*)} \right) \middle| i \in S \right\rangle .$$

**More Queries** The attacker can now perform more encryptions and submit queries, so long as the queries do not break the requirements in the Challenge stage. That is, $S \subseteq U$.

**Guess** At the end, the attacker outputs a bit $b'$ and wins the game if and only if $b' = b$. The advantage of an attacker $\mathcal{A}$ is defined as

$$\mathsf{Adv}^{LCAO}(\mathcal{A}) := \left| \mathrm{P}[b' = b] - \frac{1}{2} \right| .$$

**Definition A.1.** An encryption scheme meets LCAO security if no adversary, running in probabilistic-time with respect to security parameter $\kappa$, has more than a negligible advantage in winning the above security game. Probabilities are taken over randomness introduced by $\mathcal{A}$, and in $\mathsf{Setup}$, $\mathsf{Enc}$ and $\mathsf{CombEnc}$.

## APPENDIX B
## PROOF OF OUR SCHEME SECURITY

*Proof.* Consider adversary $\mathcal{A}$ playing the LCAO game defined in Section . The following is a construction of an adversary $\mathcal{A}'$ playing the AO game in Appendix against a challenger $\mathcal{C}$ using the Joye-Libert aggregation scheme from Section III-B.

**Setup** When receiving $N$ and $H$ as public parameters from $\mathcal{C}$, choose an $m > 1$ and give public parameter $H$, number of weights $m$, and $pk_0 = N$ to $\mathcal{A}$.

**Queries** Handle queries from $\mathcal{A}$:

***Weight Query*** When $\mathcal{A}$ submits a weight query $t$, choose weights $\omega_j^{(t)}$, $1 \leq j \leq m$ and random values $\rho_j \in \mathbb{Z}_N$, $1 \leq j \leq m$, and return encryptions

$$(N+1)^{\omega_j^{(t)}} \rho_j^N \pmod{N^2}, \, 1 \leq j \leq m$$

to $\mathcal{A}$.

***Combine Query*** When $\mathcal{A}$ submits combine query $(i, t, a_{i,1}^{(t)}, \ldots, a_{i,m}^{(t)})$, choose weights $\omega_j^{(t)}, 1 \leq j \leq m$ if not already chosen for the instance $t$, and make an AO encryption query $(i, t, \sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)})$ to $\mathcal{C}$. The received response will be of the form $(N+1)^{\sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)}} H(t)^{sk_i}$; multiply it by $\rho^N$ for a random $\rho \in \mathbb{Z}_N$ and return

$$(N+1)^{\sum_{j=1}^{m} a_{i,j}^{(t)} \omega_j^{(t)}} \rho^N H(t)^{sk_i} \pmod{N^2}$$

to $\mathcal{A}$.

***Compromise Query*** When $\mathcal{A}$ submits compromise query $i$, make the same compromise query $i$ to $\mathcal{C}$, and return the recieved secret key $sk_i$ to $\mathcal{A}$.

**Challenge** When $\mathcal{A}$ submits challenge series

$$\left\langle \left(i, t^*, a_{i,1}^{(t^*)(0)}, \ldots, a_{i,m}^{(t^*)(0)}\right) \middle| i \in S \right\rangle$$

and

$$\left\langle \left(i, t^*, a_{i,1}^{(t^*)(1)}, \ldots, a_{i,m}^{(t^*)(1)}\right) \middle| i \in S \right\rangle ,$$

choose weights $\omega_j^{(t^*)}, 1 \leq j \leq m$ for instance $t^*$ and submit AO challenge series

$$\left\langle \left(i, t^*, \sum_{j=1}^{m} a_{i,j}^{(t^*)(0)} \omega_j^{(t^*)}\right) \middle| i \in S \right\rangle$$

and

$$\left\langle \left(i, t^*, \sum_{j=1}^{m} a_{i,j}^{(t^*)(1)} \omega_j^{(t^*)}\right) \middle| i \in S \right\rangle ,$$

to $\mathcal{C}$. The received response will be of the form

$$\left\langle (N+1)^{\sum_{j=1}^{m} a_{i,j}^{(t^*)(b)} \omega_j^{(t^*)}} H(t^*)^{sk_i} \middle| i \in U \right\rangle ,$$

for an unknown $b \in \{0, 1\}$. Multiply series elements by $\rho_i^N$, $1 \leq i \leq n$ for randomly chosen $\rho_i \in \mathbb{Z}_N$ and return

$$\left\langle (N+1)^{\sum_{j=1}^{m} a_{i,j}^{(t^*)(b)} \omega_j^{(t^*)}} \rho_i^N H(t^*)^{sk_i} \middle| i \in U \right\rangle$$

to $\mathcal{A}$.

**Guess** When $\mathcal{A}$ makes guess $b'$, make the same guess $b'$ to $\mathcal{C}$.

In the above construction, $\mathcal{C}$ follows the Joye-Libert scheme from Section III-B exactly, and to $\mathcal{A}$, $\mathcal{A}'$ follows our scheme in Section exactly. Since $\mathcal{A}'$ runs in polynomial-time to security parameter when $\mathcal{A}$ does, and no non-neglibile advantage adversary to $\mathcal{C}$ exists [14], we conclude that no non-negligible advantage adversary $\mathcal{A}$ exists. That is, there exists a negligible function $\eta$, such that

$$\mathsf{Adv}^{LCAO}(\mathcal{A}) \leq \eta(\kappa)$$

for security parameter $\kappa$. □

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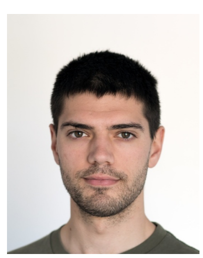


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