Cryptographically Privileged State Estimation With Gaussian Keystreams

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***Abstract*—The abstract goes here.**

I. Introduction

State estimation

Wireless and distributed estimation

Security concerns

Traditional methods hide all information, other use-cases exist

Information may be divided into privilege levels authenticating different audiences to different amounts of information [gps, anonymisation]

Our contribution in this work is comprised of a formal definition of privileged state estimation, which allows the quantification of estimation error covariance differences between privileged and unprivileged estimators, before proposing a solution to the problem accompanied by a cryptographic sketch proof and simulation results.

Section summary

A. Notation

Define vectors, matrices, encryption, pseudorandom samples, positive-definitiveness and $\prec$ for matrices, negligible function

II. Problem Statement

The estimation scenario that we consider is for known process and measurement models, where state estimators are either privileged estimators possessing a secret key, or unprivileged estimators without. We aim to develop a scheme for which the difference in their estimation errors is quantifiable and cryptographically guaranteed when process and measurement models are Gaussian, linear and time-invariant.

The process model we consider gives the state $\underline{x}_k \in \mathbb{R}^n$ at a timestep k and is given by

$$\underline{x}_k = \mathbf{F}\underline{x}_{k-1} + \underline{w}\,, \tag{1}$$

with noise term $\underline{w} \sim \mathcal{N}(\underline{0}, \mathbf{Q})$ and a known covariance $\mathbf{Q} \in \mathbb{R}^{n\times n}$. Similarly, the measurement model gives the measurement $\underline{y}_k$ at time k and is given by

$$\underline{y}_k = \mathbf{H}\underline{x}_k + \underline{v}\,, \tag{2}$$

with noise term $\underline{v} \sim \mathcal{N}(\underline{0}, \mathbf{R})$ and a known covariance $\mathbf{R} \in \mathbb{R}^{m\times m}$.

To capture our aim of creating a "better" and "worse" estimator, we need to define how to assess estimator privilege and what algorithms are required to provide a privileged estimation scheme. In the following section, we give relevant formal definitions, which are later referred to when assessing the security of our proposed scheme.

III. Formal cryptographic Problem

While we are interested in Gaussian, linear and time-invariant models, it is of more use to define a broader security notion that can be satisfied given specified conditions on the models. This will allow the use of the same notion in future literature and is more closely in-line with cryptography practice. Later, we will show our proposed scheme meets this broad security notion under the Gaussian, linear and time-invariant model assumptions.

Typical formal cryptographic security notions capture desired privacy properties as well as attacker capabilities [1]. The most commonly desired privacy property, cryptographic indistinguishability, is not suitable for our estimation scenario due to our desire for unprivileged estimators to still gain some information from measurements. Instead, we will require a time series of estimation error covariance differences, given arbitrary known Bayesian models, such that the difference in estimation error between estimators with and without the secret key is lower-bounded at all times by the series.

To formalise this, we introduce the following notations and definitions. We assume the existence of an arbitrary process following a known model exactly, with the state at time k denoted $\underline{x}_k \in \mathbb{R}^n$, as in section II, and model parameters $\mathcal{M}_P$. Similarly, we assume the existence of a means of process measurement following a known measurement model exactly, with the measurement at time k denoted as $\underline{y}_k \in \mathbb{R}^m$, and model parameters $\mathcal{M}_M$. We can now define a *privileged estimation scheme* as a pair of algorithms (Setup, Noise) given by

$\mathsf{Setup}(\mathcal{M}_P, \mathcal{M}_M, \kappa)$ On the input of models $\mathcal{M}_P$, $\mathcal{M}_M$ and the security parameter κ, public parameters pub and a secret key sk are created.

$\mathsf{Noise}(\mathsf{sk}, k, \mathcal{M}_P, \mathcal{M}_M, \underline{y}_1, \ldots, \underline{y}_k)$ On input of secret key sk, time k, models $\mathcal{M}_P$, $\mathcal{M}_M$ and measaurements $y_1, \ldots, y_k$, a noisey measurement $\underline{y}'_k$ (with no required model constraints) is created.

In addition to the scheme description above, we also give the following defintions to help formalise our desired security notion.

Definition III.1. An *estimator* is any algorithm which produces a guess of the state $\underline{x}_k$ for a given time k.

Definition III.2. A *negligible covariance function* is a function

$$\mathsf{neglCov}_m(\kappa) : \mathbb{N} \to \mathbb{R}^{m \times m} \tag{3}$$

that returns a matrix $\mathbf{A}$ such that $\mathbf{A}$ is a valid covariance ($\mathbf{A} \succ 0$ and $\mathbf{A} = \mathbf{A}^\top$) and that for each of its eigenvalues $e \in \mathrm{eig}(\mathbf{A})$, there exists a negligible function η such that $e \leq \eta(\kappa)$.

With the terminology above, we can now introduce the security notion which captures the formal requirements of the estimation problem we want to solve.

Definition III.3. A privileged estimation scheme meets $\{\mathbf{D}_1, \mathbf{D}_2, \ldots\}$*-Covariance Privilege for Models* $\mathcal{M}_P$ *and* $\mathcal{M}_M$ if for any probabilistic polynomial-time (PPT) estimator $\mathcal{A}$, there exists a PPT estimator $\mathcal{A}'$, such that

$$\begin{aligned} &\mathrm{Cov}\left[\mathcal{A}\left(\mathrm{k}, \kappa, \mathsf{pub}, \mathcal{M}_\mathrm{P}, \mathcal{M}_\mathrm{M}, \underline{\mathrm{y}}'_1, \ldots, \underline{\mathrm{y}}'_\mathrm{k}\right) - \underline{\mathrm{x}}_\mathrm{k}\right] \\ &\quad - \mathrm{Cov}\left[\mathcal{A}'\left(\mathrm{k}, \kappa, \mathsf{pub}, \mathcal{M}_\mathrm{P}, \mathcal{M}_\mathrm{M}, \underline{\mathrm{y}}_1, \ldots, \underline{\mathrm{y}}_\mathrm{k}\right) - \underline{\mathrm{x}}_\mathrm{k}\right] \\ &\qquad \succeq \mathbf{D}_k + \mathsf{neglCov}_m(\kappa) \end{aligned} \tag{4}$$

for valid covariances $\mathbf{D}_1, \ldots, \mathbf{D}_k$ and some negligible covariance for all $k > 0$. Here, estimators $\mathcal{A}$ and $\mathcal{A}'$ are running in polynomial-time with respect to the security parameter κ, and all probabilities are taken over models $\mathcal{M}_P$ and $\mathcal{M}_M$, estimators $\mathcal{A}$ and $\mathcal{A}'$, and algorithms Setup and Noise.

Informally, the above definition states that no estimator with access to only noisey measurements $\underline{y}'_1, \ldots, \underline{y}'_k$ can estimate a state $\underline{x}_k$ at time k with an RMSE covariance less than an equivalent estimator with normal measurements $\underline{y}_1, \ldots, \underline{y}_k$, by a margin of at least $\mathbf{D}_k$. Next, we will propose a scheme meeting the aforementioned notion for a derivable series of covariances given Gaussian, linear, and time-invariant models $\mathcal{M}_P$ and $\mathcal{M}$.

IV. Privileged Estimation

General idea

(picture ?)

Use a cryptographically secure key stream to generate pseudorandom Gaussian samples

Samples are used to increase the uncertainty of estimation and are known and removable only by those with the key used to generate them

A. Gaussian Keystream

To generate pseudorandom Gaussian samples, we rely on first generating a traditional pseudorandom bitstream given a secret key.

Using well-studied methods for the generation of pseudorandomness guarantees robustness and an easy means of updating only the relevant component when the methods used are no longer considered safe.

Any implementation of a cryptographic stream cipher can be used for our purpose and will produce a stream of bits typically combined with plaintexts to provide secure encryption.

Rather than encrypting plaintext, we interpret the bitstream as sequential pseudorandom integers and use these to generate pseudorandom uniform real numbers in the range (0,1). u

While the uniform real samples are only approximated by floating-point numbers in the conversion from integers, we argue this is sufficiently random and discuss this further in the Security section.

Finally, independent standard Gaussian samples can then be generated from the uniform real numbers using the Box-Muller transform, and are ready to be used by our sensor and privileged filter. z

B. Additional Gaussian Noise

To use the pseudorandom Gaussian samples at the sensor and privileged estimator, they need to be converted to multivariate Gaussian samples suitable for use in the measurement model and need a means of controlling how much uncertainty is added to the unprivileged estimators.

We define the additional noise term $Z > 0$ and can transform the Gaussian samples z into pseudorandom samples o of a multivariate zero-mean Gaussian distribution with covariance Z.

Before estimation, we assume that a secret key is shared between the sensor and the privileged estimator.

During estimation, the sensor modifies its measurements at each timestep.

There are now two estimation problems present for the privileged and unprivileged estimators respectively.

For the privileged estimator who holds the shared secret key, values z, and therefore o, can be computed at any time k and received measurements modified to their original form. This in turn results in exactly the measurement model from the problem formulation.

In the case where pseudorandomness is indistinguishable from randomness, as is the case at an unprivileged estimator when using cryptographically secure Gaussian keystreams and the secret key is not known, the measurement model noise covariance can now be written as $R + Z$.

Intuitively, we can already see that the two estimators will have an estimation error covariance differing by some value dependent on Z at each time k. In the security section we will show that the best possible error covarances achievable by the privileged and unprivileged estimators can be computed exactly by computing the Cramér–Rao lower-bound for both measurement models, and that the difference between them will give an exact lower-bound on the difference between the two estimator error covariances.

C. Multiple Privileges

In the above scenario, we have considered a single level of estimation privileged with one private key, dividing estimation error covariance into two groups; privileged and unprivileged estimators.

As a direct extension, it may be desirable to define multiple levels of privilege, such that the best estimation performance would depend on the privilege level of the estimator.

Here we will discuss the case of multiple privilege levels where a single secret key corresponds to each level, and where noise is added in the same manner as above, for each keyindividually.

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N noise terms are added to the original measurement equation, with variances Z_i

From the equation, we can see that obtaining any single key sk_i would lead to a measurement model with added non-removable pseudorandom Gaussian noise of variance Z_j.

The above restricts possible estimation error bounds of each privilege level due to the dependence of measurement noise at an estimator with key i on the remaining noise terms $Z_j, j \neq i$.

If we write the desired measurement model noise variances at each privileged estimator i as E_i, we can cature this dependance as $E_i = \sum_{j=0,j\neq i}^{N} Z_j$ where both $E_i > 0$ and $Z_i > 0$.

Since choosing values E_i directly controls the estimation error bound computed using the CRLB, we are interested in the numerical restrictions on $E_i > 0$ which will produce valid covariances $Z_j > 0$, that can be used when adding noise at the sensor.

The dependencies between the covariances can be captured by the block matrix equation.

...equation and also block matrix inequality (might need some defining as it uses $\prec$)

From the equation, we can see that the only restriction on arbitrary choices of additional noise variances E_i at each privilege level, can be chosen as long as the condition is met.

We have chosen the case with a single shared key per privilege level due to its simplicity and the ability to change privilege estimation error bounds without the need for key redistribution.

Alternative methods involving multiple or overlapping keys among privilege levels may allow choices of E_i to be less restricted than in the equation above and have been left as future work the topic.

V. Scheme Security

The security of the proposed scheme will be primarily considered in the single privileged and unprivileged estimator case.

A proof sketch will be provided to show the cryptographic guarantees of the scheme.

The extension to multiple privilege levels as described in the section above will be informally discussed.

A. Single Privileged Case

Given the models (1) and (2), the optimal estimator with respect to mean squared error is given by the linear Kalman filter []. Estimates are computed recursively, following the combined state prediction and update equations

$$\hat{\underline{x}}_k = \dots \tag{5}$$

Optimality in terms of estimation error covariance has been proved by the Cramér–Rao lower bound (CRLB) []

(CRLB details - unbiased/biased/when is it the best/what assumptions)

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With the security notion we aim for defined formally above, we can now provide a proof sketch, with conditions on $\mathcal{M}_P$ and $\mathcal{M}_M$, and a covariance series $\mathbf{D}_1, \dots, \mathbf{D}_k$, for which our privileged estimation scheme meets the defined security notion.

Our proposed method in the sections previously can be seen as an implementation of a privileged estimation scheme as defined above, where parameter κ is the security parameter for the required stream cipher and the secret key sk is the same as the stream cipher key, while $/mathsfpub$ contains the covariance of the added pseudorandom Gaussian noise and an initial estimate and covariance. The noise algorithm corresponds to ().

Proof sketch: We consider the process model () and measurement model () exactly, that is, any linear models with known zero-mean Gaussian additive noises.

The idea behind the proof relies on the fact that the CRLB gives the smallest RMSE covariance achievable for *any* estimator, when all measurements $\underline{y}_1, \dots, \underline{y}_k$ are observed and can be computed exactly when process and measurement models are linear and Gaussian.

The CRLB at time k can, denoted $\mathbf{J}_k^{-1}$ satisfies

$$\mathbf{J}_k^{-1} \preceq \mathrm{Cov}\left[\mathcal{A}\left(\mathrm{k}, \mathcal{M}_\mathrm{P}, \mathcal{M}_\mathrm{M}, \underline{\mathrm{y}}_1, \dots, \underline{\mathrm{y}}_\mathrm{k}\right) - \underline{\mathrm{x}}_\mathrm{k}\right] \tag{6}$$

for any estimator $\mathcal{A}$

As we use a cryptographically pseudorandom keystream, noisy measurements $\underline{y}'_k$ are indistinguishable from measurements following the modified measurement model () exactly.

Similarly, the generation of uniform ...

We can compute the CRLB recursively for both the true measurement model () and the modified model (), and take their differences to get the infinite covariance series $\mathbf{D}_1, \mathbf{D}_2, \dots$.

As we know from the CRLB proof, any estimators with the two models () and () will at least differ by $\mathbf{D}_k$ at time k.

A reduction proof can be easily constructed where an unprivileged estimator in our scheme, which produces estimates such that (neglcov eq) does not hold, can be used to construct an estimator with an error covariance lower than that of the CRLB given the modified model, known to be impossible.

In addition to the security definitions and proof above, we stress caution when assuming such guarantees in the presence of a measured physical process. The following implicit assumptions are made when applying models $\mathcal{M}_P$ and $\mathcal{M}_M$ to an observable phenomenon.

the Bayesian interpretation of probability

the assumption is that model is exactly correct

an assumption that uniform floating points are uniform enough (here or in negligible difference discussion above?)

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B. Multiple Additional Noises

VI. Simulation and Results

VII. Conclusion

The conclusion goes here.

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References

[1] J. Katz and Y. Lindell, *Introduction to Modern Cryptography: Principles and Protocols*. Chapman & Hall, 2008.