Privileged Estimate Fusion With Correlated Gaussian Keystreams

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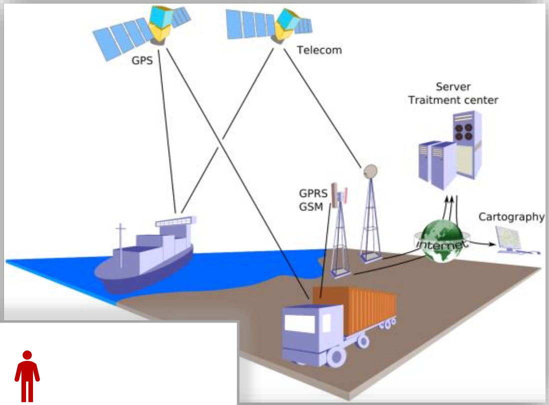
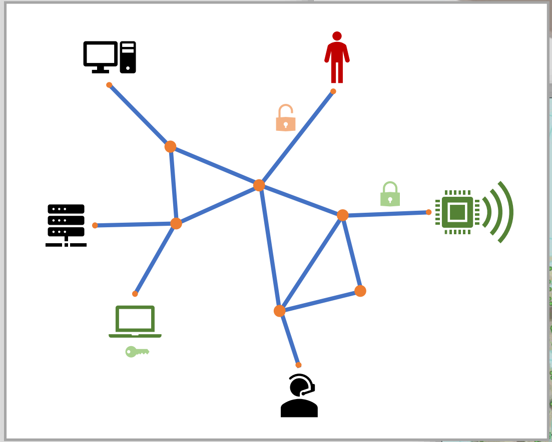
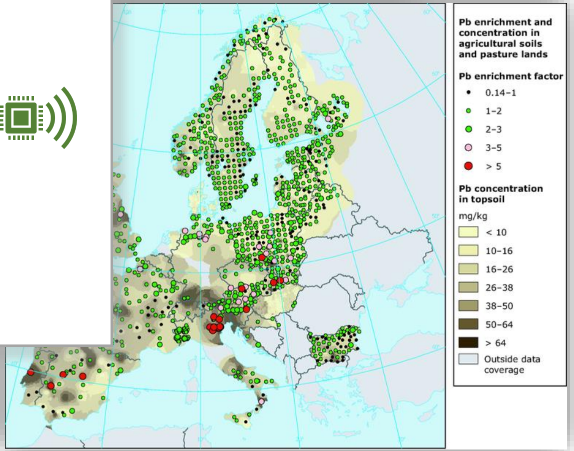
Security in Networks

- Growing number of public networks
- Increasingly used by distributed sensors, IoT devices, cloud computing, etc
- Greater need for security guarantees
- Affected users
  - Private
  - Commercial
  - Government

https://fr.wikipedia.org/wiki/G%C3%A9olocalisation#/media/Fichier:Geolocation.png



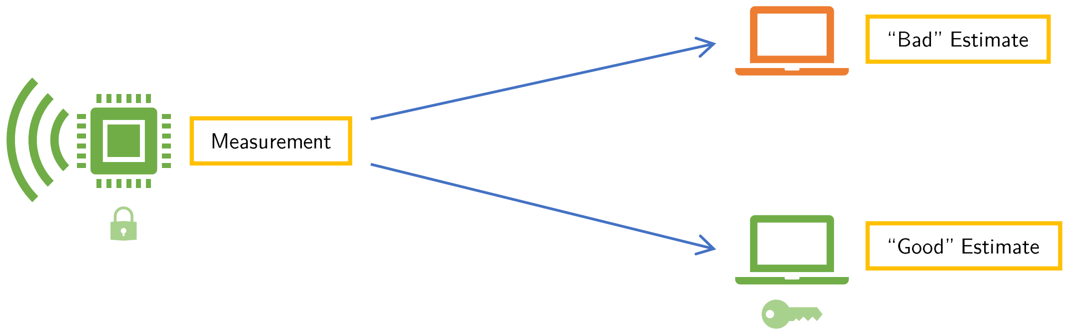




https://www.eea.europa.eu/data-and-maps/figures/soil-contamination-by-heavy-metals

Privilege in Estimation

- Public measurements useable for state estimation
- Trusted or special users may be granted *privilege*
- Privileged users should perform *better* than unprivileged ones



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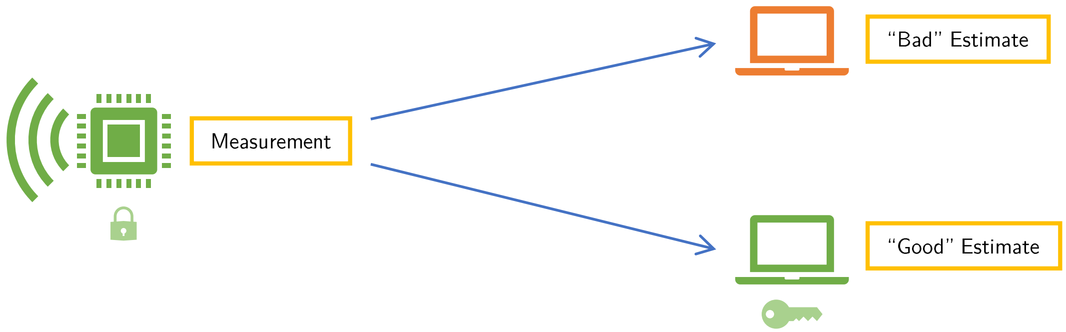

- Security guarantee concerns proving the minimum difference in performance

Single Sensor Privilege

- Add generated Gaussian keystream to measurements
- Anyone holding generation key can remove added noise

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    - System

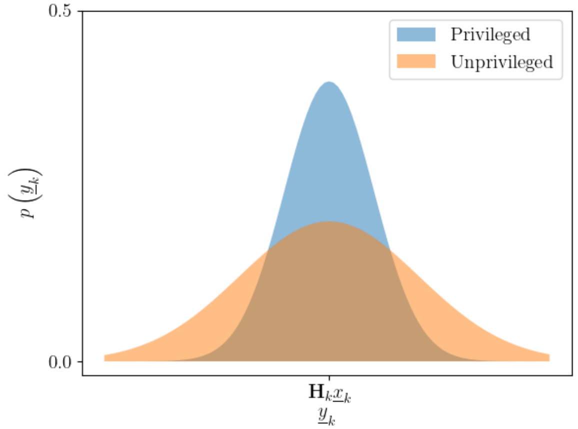
$$\underline{x}_k = \mathbf{F}_k \underline{x}_{k-1} + \underline{w}_k \qquad \underline{w}_k \sim \mathcal{N}(\underline{0}, \mathbf{Q}_k)$$

    - Measurement

$$\underline{y}_k = \mathbf{H}_k \underline{x}_k + \underline{v}_k \qquad \underline{v}_k \sim \mathcal{N}(\underline{0}, \mathbf{R}_k)$$

    - Modified measurement

$$\underline{y}'_k = \underline{y}_k + \underline{g}_k = \mathbf{H}_k \underline{x}_k + \underline{v}_k + \underline{g}_k \qquad \underline{v}_k \sim \mathcal{N}(\underline{0}, \mathbf{R}_k)\,,\ \underline{g}_k \dot{\sim} \mathcal{N}(\underline{0}, \mathbf{Z})$$



Cryptographic Definition for Performance

- Algorithms

$$\mathsf{Setup}\left(\mathcal{M}_S, \mathcal{M}_M, \kappa\right),$$
$$\mathsf{Noise}\left(\mathsf{pub}, \mathsf{sk}, k, \mathcal{M}_S, \mathcal{M}_M, \underline{y}_1, \ldots, \underline{y}_k\right)$$

- Definitions

$$\mathit{estimator},$$
$$\mathsf{neglCov}_m(\kappa) : \mathbb{N} \to \mathbb{R}^{m \times m}$$

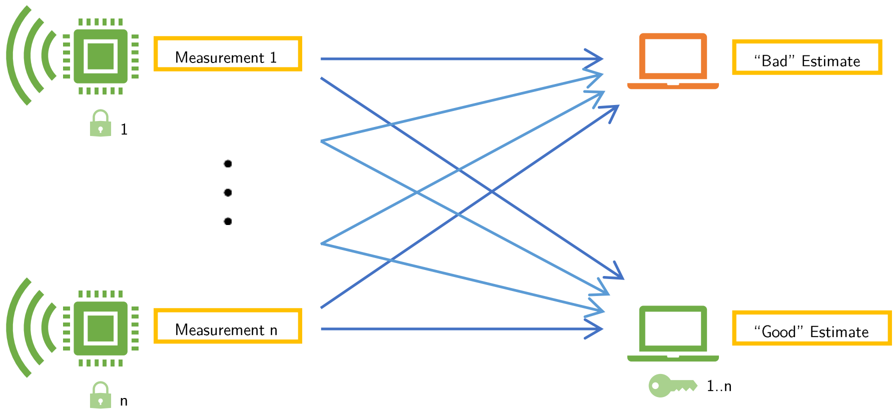
- Covariance Privilege

$(\mathsf{Setup}, \mathsf{Noise})$ meets $\{\mathbf{D}_1, \mathbf{D}_2, \ldots\}$-*Covariance Privilege for Models* $\mathcal{M}_S$ *and* $\mathcal{M}_M$

if for any PPT estimator $\mathcal{A}$, there exists a PPT estimator $\mathcal{A}'$, such that

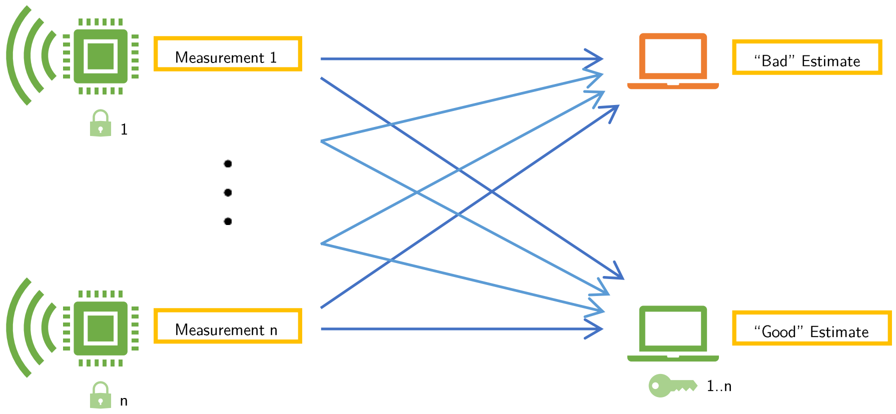
$$\begin{aligned}
&\mathsf{Cov}\left[\mathcal{A}\left(k, \kappa, \mathsf{pub}, \mathcal{M}_S, \mathcal{M}_M, \underline{y}'_1, \ldots, \underline{y}'_k\right) - \underline{x}_k\right] \\
&- \mathsf{Cov}\left[\mathcal{A}'\left(k, \kappa, \mathsf{pub}, \mathcal{M}_S, \mathcal{M}_M, \underline{y}_1, \ldots, \underline{y}_k\right) - \underline{x}_k\right] \\
&\succeq \mathbf{D}_k + \mathsf{neglCov}_m(\kappa)
\end{aligned}$$

- Multiple privileged sensors each adding Gaussian keystream



Complication of Fusion

- Multiple privileged sensors each adding Gaussian keystream



- Two ways of getting better estimates
  - Hold keys to remove added noises (desired)
  - Fuse more measurements (desired only when keys are held as well)

Multisensor Estimation Problem

- Linear models considered
- Kalman Filter optimality in proofs

  - System

$$\underline{x}_k = \mathbf{F}_k \underline{x}_{k-1} + \underline{w}_k \qquad \underline{w}_k \sim \mathcal{N}(\underline{0}, \mathbf{Q}_k)$$

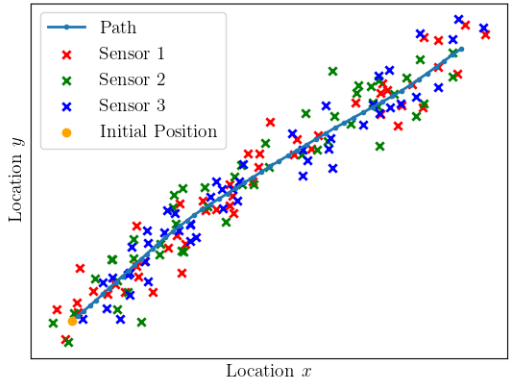
  - Measurements

$$\underline{y}_{k,i} = \mathbf{H}_{k,i} \underline{x}_k + \underline{v}_{k,i} \qquad \underline{v}_{k,i} \sim \mathcal{N}(\underline{0}, \mathbf{R}_{k,i}), \; 1 \leq i \leq n$$

  - Modified measurements and keys

$$\underline{y}'_{k,i}, \; \mathsf{sk}_i \qquad 1 \leq i \leq n$$

Example: Linear Constant Velocity Model ($n = 3$)



Sequential Assumption

- Estimators can access q measurements $(1 \leq q \leq n)$
- Estimators have privilege p (the number of keys they hold) $(0 \leq p \leq n)$

$$\mathsf{e}^{[\text{privilege, access}]}$$

Sequential Assumption

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$\mathsf{e}^{[0,q]}$	Access to:	$\underline{y}'_{k,i}\,,\ 1 \leq i \leq q$
$\mathsf{e}^{[p,p]}$	Access to:	$\underline{y}'_{k,i}\,,\ \mathsf{sk}_i\,,\ 1 \leq i \leq p$
$\mathsf{e}^{[p,q]}$	Access to:	$\underline{y}'_{k,i}\,,\ 1 \leq i \leq q \qquad \mathsf{sk}_j\,,\ 1 \leq j \leq p$

Bounds on Performance Loss and Gain

- Capture desired performance differences in multisensor environment

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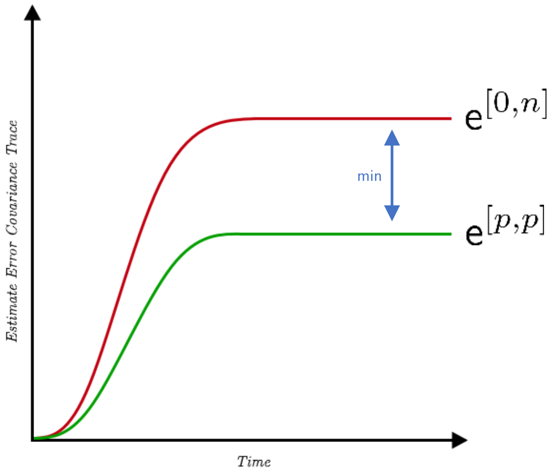
Performance Loss Lower Bound



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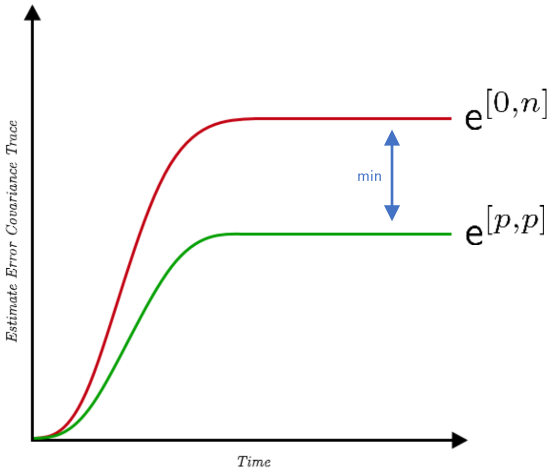
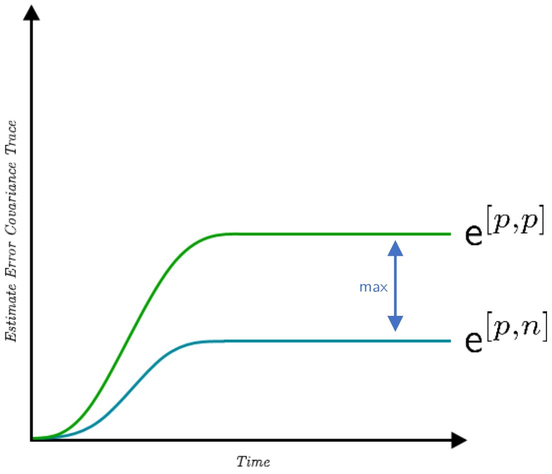


Performance Gain Upper Bound



Bounds on Performance Loss and Gain

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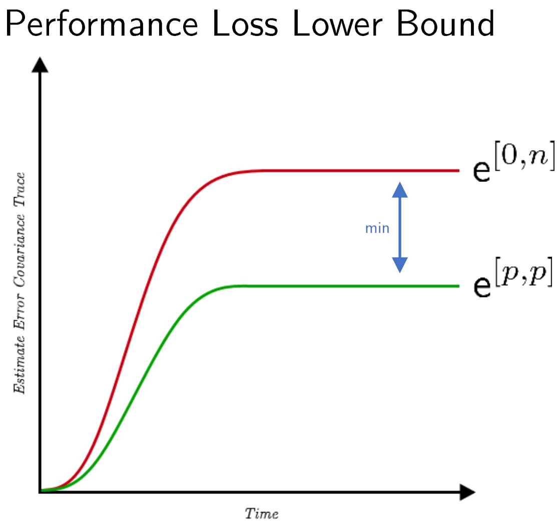

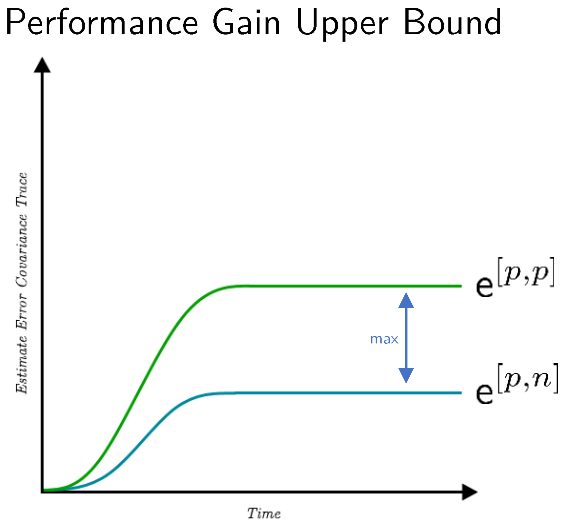

- Both bounds desired for each privilege p

Correlated Noise Generation

- Generate uniform keystreams from sk_i for $1 \leq i \leq n$
- Correlated uniform keystreams with Box-Muller transform

$$\begin{bmatrix} \underline{g}_{k,1} \\ \vdots \\ \underline{g}_{k,n} \end{bmatrix} \sim \mathcal{N}\left(\underline{0}, \mathbf{S}^{(n)}\right) \qquad \mathbf{S}^{(n)} = \underbrace{\begin{bmatrix} \mathbf{Z} & \cdots & \mathbf{Z} \\ \vdots & \ddots & \vdots \\ \mathbf{Z} & \cdots & \mathbf{Z} \end{bmatrix}}_{\text{Correlated component}} + \underbrace{\begin{bmatrix} \mathbf{Y} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y} \end{bmatrix}}_{\text{Uncorrelated component}}$$

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- Add generated keystreams to measurements

$$\underline{y}'_{k,i} = \underline{y}_{k,i} + \underline{g}_{k,i} = \mathbf{H}_{k,i}\underline{x}_k + \underline{v}_{k,i} + \underline{g}_{k,i} \qquad \underline{v}_k \sim \mathcal{N}(\underline{0}, \mathbf{R}_{k,i})$$

Correlated Noise Reconstruction

- Each added noise depends on multiple keys!
- Need to correctly reconstruct partial noises when $p < n$
- Lower-triangular decomposition (e.g. Cholesky) in Box-Muller transform ensures

$$\underline{g}_{k,i} \quad \text{depends on} \quad \mathsf{sk}_j \,, \; 1 \leq j \leq i$$

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$$\underline{g}_{k,i} \quad \text{depends on} \quad \mathsf{sk}_j\,,\ 1 \leq j \leq i$$

- Can reconstruct first p noises with $\mathsf{sk}_i\,,\ 1 \leq i \leq p$ exactly (recall sequential assumption)

$$\begin{bmatrix} \underline{g}_{k,1} \\ \vdots \\ \underline{g}_{k,p} \end{bmatrix} \sim \mathcal{N}\left(\underline{0}, \mathbf{S}^{(p)}\right) \qquad \mathbf{S}^{(p)} = \begin{bmatrix} \mathbf{Z} & \cdots & \mathbf{Z} \\ \vdots & \ddots & \vdots \\ \mathbf{Z} & \cdots & \mathbf{Z} \end{bmatrix} + \begin{bmatrix} \mathbf{Y} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y} \end{bmatrix}$$

Observable Measurement Models

- Gaussian keystream indistinguishable from random without key
- Leads to three observable measurement models

Observable Measurement Models

Notation

$$\mathbf{R}_k^{(1:q)} = \begin{bmatrix} \mathbf{R}_{k,1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{R}_{k,q} \end{bmatrix}$$

$$\bar{\mathbf{Z}} = \begin{bmatrix} \mathbf{Z} & \cdots & \mathbf{Z} \\ \vdots & \ddots & \vdots \\ \mathbf{Z} & \cdots & \mathbf{Z} \end{bmatrix}$$

- Gaussian keystream indistinguishable from random without key
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$$\mathsf{e}^{[0,q]} \qquad \underline{y}_k = \mathbf{H}_k^{(1:q)} \underline{x}_k + \underline{v}'_k \qquad \underline{v}'_k \sim \mathcal{N}\left(\underline{0}, \mathbf{R}_k^{(1:q)} + \mathbf{S}^{(q)}\right)$$

$$\mathsf{e}^{[p,p]} \qquad \underline{y}_k = \mathbf{H}_k^{(1:p)} \underline{x}_k + \underline{v}'_k \qquad \underline{v}'_k \sim \mathcal{N}\left(\underline{0}, \mathbf{R}_k^{(1:p)}\right)$$

$$\mathsf{e}^{[p,q]} \qquad \underline{y}_k = \mathbf{H}_k^{(1:q)} \underline{x}_k + \underline{v}'_k \qquad \underline{v}'_k \sim \mathcal{N}\left(\underline{0}, \begin{bmatrix} \mathbf{R}_k^{(1:p)} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}^{(q-p)} - \bar{\mathbf{Z}}\left(\mathbf{S}^{(p)}\right)^{-1} \bar{\mathbf{Z}} + \mathbf{R}_k^{(p+1:q)} \end{bmatrix}\right)$$

- Exact linear models mean optimal estimates

Proving Performance Loss and Gain Bounds

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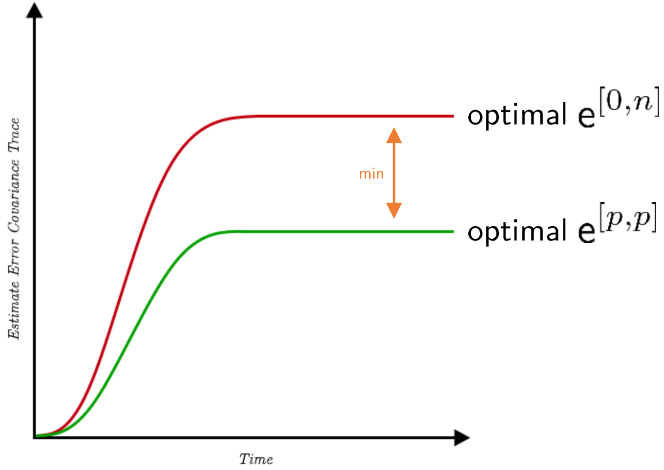
Performance Loss Lower Bound



$$\mathbf{P}_0 = \mathbf{0} \implies \text{Lowest possible } \operatorname{tr}\left(\mathbf{P}_k\right)$$

Proving Performance Loss and Gain Bounds

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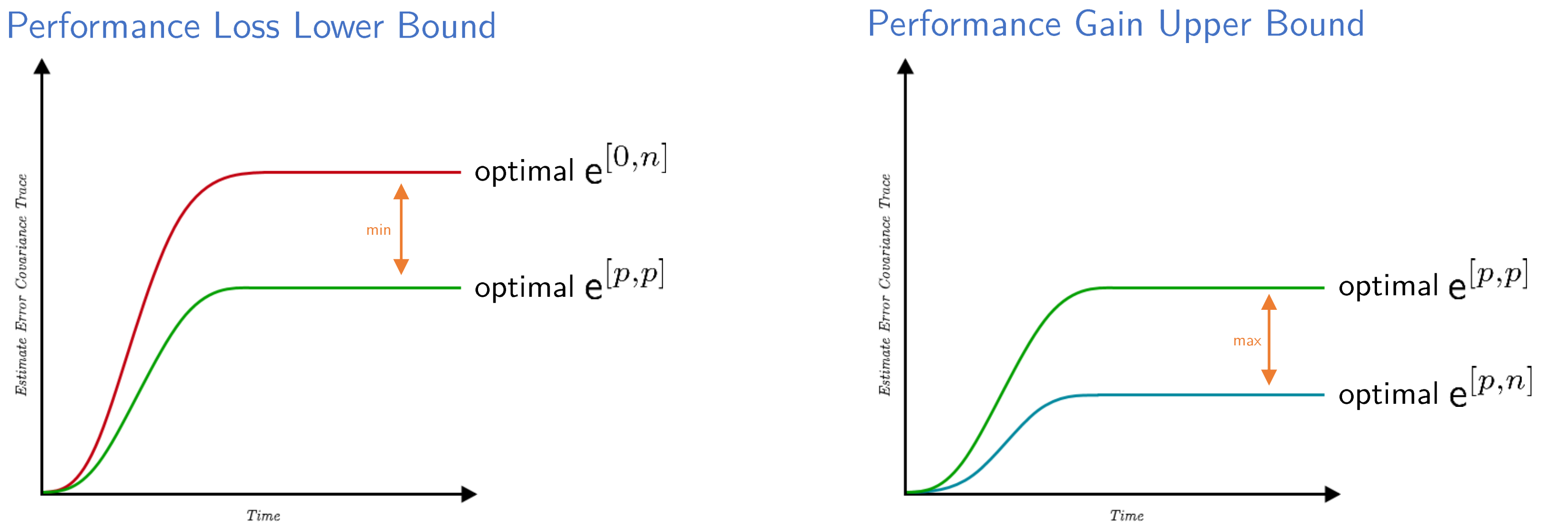

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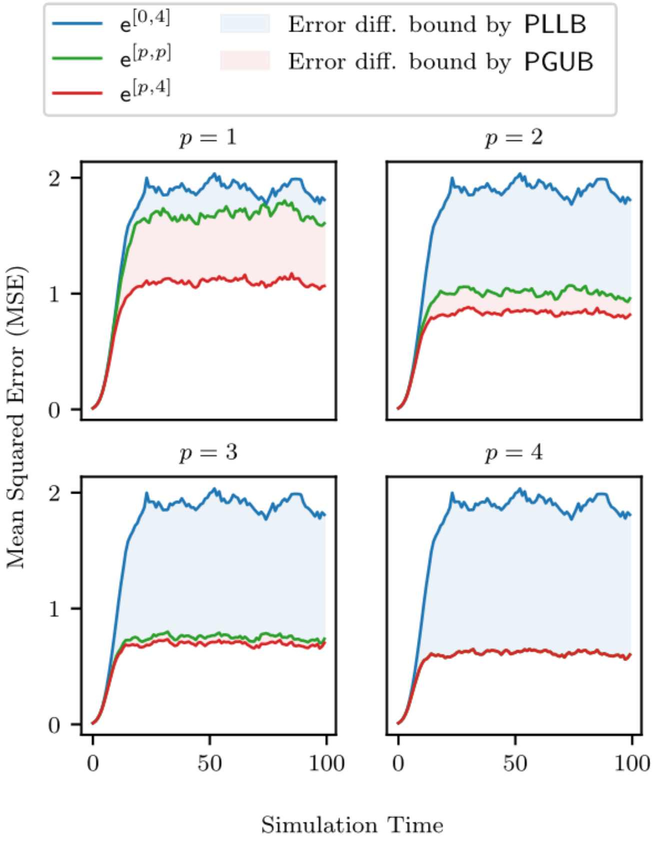
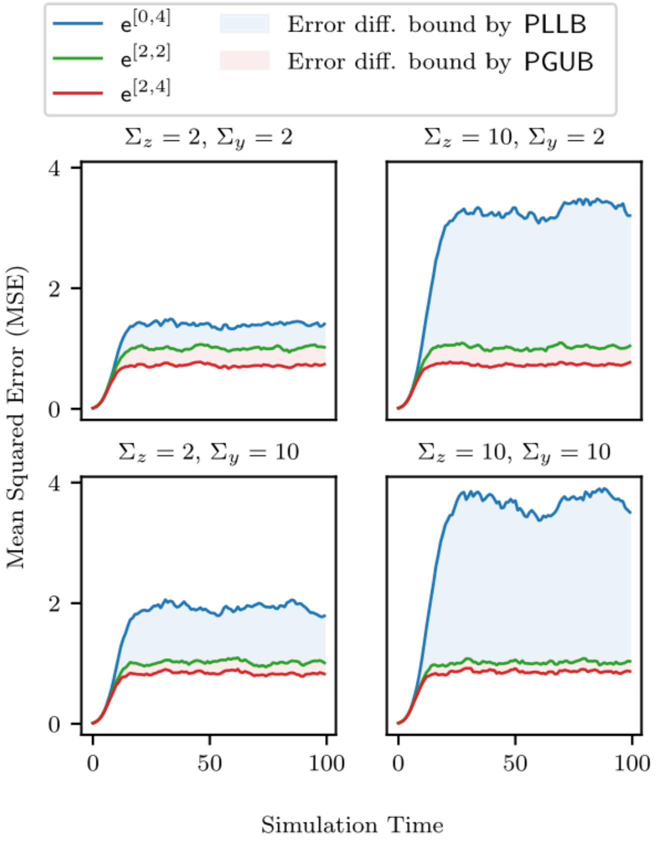
Simulation Results

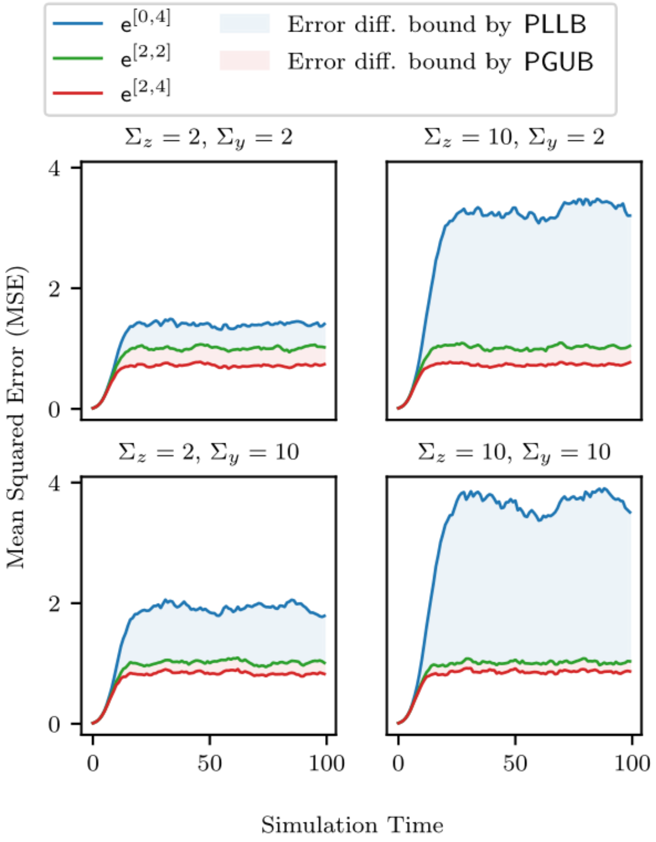


- $n = 4$
- Fixed fully correlated component $\mathbf{Z}$ and uncorrelated component $\mathbf{Y}$

Simulation Results



- $n = 4$
- Varied $\mathbf{Z} = \Sigma_z \times \mathbf{I}$ and $\mathbf{Y} = \Sigma_y \times \mathbf{I}$



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- Varied $\mathbf{Z} = \Sigma_z \times \mathbf{I}$ and $\mathbf{Y} = \Sigma_y \times \mathbf{I}$

- Effect of $\mathbf{Z}$ and $\mathbf{Y}$ on bounds?

Future Work

- Search for correlation matrix parameters that affect bounds independently
- Relaxations of sequential assumption

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Thank you!