CS 551 Final Term Project - Subsurface Scattering

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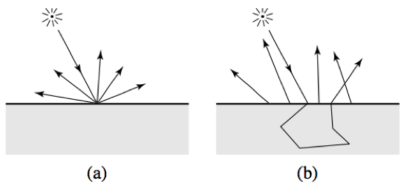

Figure 1: Scattering of light in (a) a BRDF, and (b) a BSSRDF.

Abstract

In this project, I am going to implement a method for rendering the effects of subsurface scattering by using Bidirectional Surface Scattering Reflectance Distribution Function (*BSSRDF*) proposed by Jensen et al. [2001]. Subsurface scattering is a very common material appearance in reality, and happens in all objects with a translucent appearance, such as jade, marble, candles, cola, apples, milk, human skin, and so on. In reality, this semi-transparent material is very common, so how to establish the correct mathematical model for their appearance is an important part of photorealistic rendering. This paper will demonstrate how I implement BSSRDF to simulate translucent materials.

1 Introduction

The model of subsurface scattering is also much more complicated than the general surface reflections because to properly simulate this phenomenon, the light will not only be scattered on the surface of the object but will be refracted into the interior of the object first, and then several scatterings occurs inside the object until it emerges from a point on the surface of the object, such as (b) in Figure 1. Therefore, for materials with subsurface scattering properties, the position where the light exits is different from the position of the incident light, and the brightness of each point depends on the brightness of all other positions on the surface of the object, the shape and thickness of the object, and the like. The surface scattering based on the BSDF model (bidirectional scattering distribution function) can only be used to describe the scattering properties of a point on the surface of an object, so it cannot describe the phenomenon like subsurface scattering.

2 Related Work

2.1 Light Transporting in a Media

To simulate such a phenomenon, the simplest, most accurate, and most computationally intensive and slowest method is to directly solve the rendering equation with the participating Media in the space inside the object.

The existence of Participating Media means that the traditional rendering equation is no longer the same as the integral of all surfaces in the scene as the defined domain, because when the light propagates in the medium space, it does not have to be in contact with the surface, and it is also in space. The medium absorbs and scatters. Usually we use several parameters to describe the nature of the medium in space. Absorption coefficient σ_a, and scattering coefficient s. The former describes the probability of absorption of photons per unit length in space, while the latter describes the probability that photons are scattered per unit length and both propagation directions are changed. The scattering coefficient includes the probability that other directions of light are incident to the natural direction, and also includes the probability that the current direction will exit to other directions. The other parameter is the attenuation coefficient, $\sigma_t = \sigma_a + \sigma_s$, and it is the sum of the absorption coefficient and the scattering coefficient, and represents the total probability that a photon will exit or be absorbed in other directions within a unit distance. The rate of change in radioactivity within a unit distance can be written as:

$$(\vec{w}\cdot\vec{\bigtriangledown})L(x,\vec{w}) = -\sigma_t L(x,\vec{w}) + \sigma_s \int_{4\pi} p(\vec{w},\vec{w'})L(x,\vec{w'})dw' + Q(x,\vec{w}) \quad (1)$$

2.2 Definition of BSSRDF

Another major methods for simulating subsurface scattering is the modeling of materials with BSSRDF (Bidirectional Surface Scattering Reflectance Distribution Function). The difference from BRDF is that BSSRDF can specify different light incidence positions and exit positions. Jensen's paper in 2001 can be said to be the most important paper for subsurface material modeling, deriving many important physical formulas, computational models, parameter conversion during rendering, and many scattering coefficients of common materials in life. Most of the later papers were based on some improvements in the theory in this article. However, there are a lot of mathematics and physics formulas in this paper. It will quickly become confusing for people who don't have a certain mathematics foundation and haven't contacted material modeling and rendering. Fortunately, the significance of most formulas lies in the derivation and proof of BSSRDF calculations, so this article will pick out several important formulas that directly relate to subsurface scattering. If you want to know all the details, please refer to Jensen's paper [2001].

The significance of BSSRDF is a fast approximation. For many materials with particularly low absorption coefficients and particularly high scattering coefficients (most turbid translucent objects such as milk, marble, apples, meat, etc.), a beam of light will scatter thousands of times inside the object before being completely absorbed. Simulating directly with the most brute Path Tracing means that each Sample needs to be scattered thousands of times and convergence is very slow.

For the previous BRDF, the calculation of a reflected light is the area of the ball on the normal hemisphere of the light intersection. For the BSSRDF, each reflection has a hemispherical area at each position on the surface of the object, which is an integral. The BRDF is an approximation of the BSSRDF for which it is assumed that light enters and leaves at the same point (i.e., $x_o = x_i$). Given a BSSRDF, the outgoing radiance is computed by integrating the incident radiance over incoming directions and area, A:

$$L_o(x_o,\vec{w}_o) = \int_A \int_{2\pi} S(x_i,\vec{w}_i;x_o,\vec{w}_o)L_i(x_i,\vec{w}_i)(\vec{n}\cdot\vec{w}_i)dw_i dA(x_i) \quad (2)$$

The definition of BSSRDF is:

$$S_d(x_i, \vec{w}_i; x_o, \vec{w}_o) = \frac{1}{\pi} F_t(\eta, \vec{w}_i) R_d(||x_i - x_o||) F_t(\eta, \vec{w}_o) \quad (3)$$

This includes the refractive Fresnel term F_t, a normalized term $1/\pi$, and a diffuse term R_d. Except the diffusion item, the rest of the items are very intuitive. The following focuses on this diffusion item. First of all, R_d only accepts one scalar parameter. The meaning of this parameter is the Manhattan distance between the incident position and the initial position. The intuitive understanding is that BSSRDF attempts to approximate the energy remaining after the light is scattered thousands of times inside the surface of the object as a function of the distance between the point of incidence and the point of exit.

This approximation is based on several assumptions. The first assumption is that the subsurface scattering object is a plane with zero curvature. The second assumption is that the thickness and size of this plane are infinite. The third assumption is that the media parameters inside the plane are uniform, and the final assumption is that the light is always incident on the surface from a vertical direction. Because of these assumptions, it is easy to approximate the intensity of the emergent light and the distance between the exit point and the incident point by a function.

In theory, it needs to calculate the energy of the shot point from the distance of the light incident point as far as possible. First, in the case of the scene and its simplification, for example, a point light source is completely surrounded by a medium having a uniform scattering coefficient, and the relationship between the scattered energy and the distance of the light source can be calculated by the following formula.

$$D \bigtriangledown^2 \phi(x) = \sigma_a \phi(x) - Q_o(x) + 3D \vec{\bigtriangledown} \cdot \vec{Q}_1(x) \quad (4)$$

However, this integral does not have a solution. The solution obtained by the numerical method also diverges. In addition, there is no formula that can accurately calculate the energy of the subsurface scattering at any time in the limited space and at any time in the limited space. It is necessary to seek some kind of approximation. One of the very popular approximation algorithms for these years is the method of solving the Diffusion equation.

$$\phi(x) = \frac{\Phi}{4\pi D} \frac{e^{-\sigma_{tr} r(x)}}{r(x)} \quad (5)$$

Compared to the exact solution above, diffusion approximation is much simpler, just a distance from the natural exponent divided by the distance. This equation is also the core of the Dipole approximation proposed by Eason et al. [1978].

3 Subsurface Scattering

Subsurface scattering is a phenomenon occurring in many objects. It is necessary for rendering realistic translucent materials. Without this feature, a block of marble will look like a cube with pure color and a glass of milk will look like a cylinder with pure color. The light transporting within marble is a really important role to affect its appearance. If there is no subsurface scattering, the marble would look solid and has pure black on sides away from light. The effect of subsurface scattering makes a marble block and a glass of milk have variations of color and the side away from light will be lit. The example is shown on Figure 2 and 3.

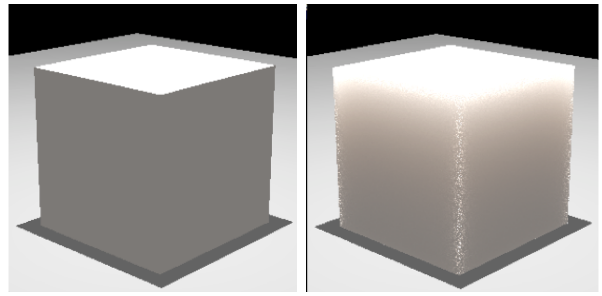

Figure 2: The left figure is a cube without subsurface scattering. The right figure is simulating a marble block with BSSRDF model.

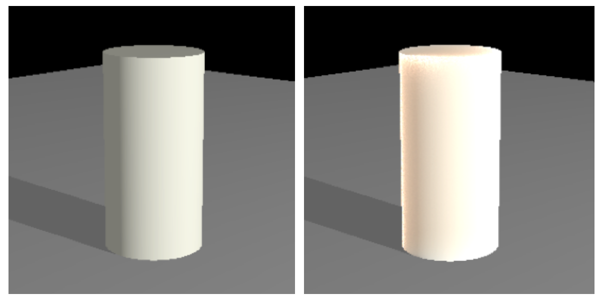

Figure 3: The left figure is a cylinder without subsurface scattering. The right figure is simulating a glass of milk with BSSRDF model.

The previous method for rendering subsurface scattering is BRDF. In order to simulating light transporting by using the BRDF, the leaving point and the entering point must be the same point. Jensen et al. [2001] proposed a method for rendering the effects of subsurface scattering by using the BSSRDF. In this method, light can transport between any two points, so it is much more accurate to the reality and able to simulate real translucent materials. That is reason using BSSRDF to render the effects of subsurface scattering. In order to use the approximation, making assumptions about the properties of medium is necessary. There are two coefficients which are σ_s and σ_a to represent the properties of materials in term of subsurface scattering. σ_a is absorption coefficient describing how likely the material is to absorb light and σ_s is scattering coefficient describing how likely the material is to scatter light. A material can have different value for different wavelengths of light such as red, green and blue. Also, the approximation assumes that $\sigma_s >> \sigma_a$ and the distribution of each scattering is evenly distributed over the hemisphere. For those materials satisfying those assumption, the approximation can provide good result while for the other types of materials, it would not provide good result.

The BSSRDF model is a sum of the diffusion approximation and the single scattering term:

$$S(x_i, \vec{w}_i; x_o, \vec{w}_o) = S_d(x_i, \vec{w}_i; x_o, \vec{w}_o) + S^{(1)}(x_i, \vec{w}_i; x_o, \vec{w}_o) \quad (6)$$

Single scattering occurs only when the refracted incoming and outgoing rays intersect, and is computed as an integral over path length s along the refracted outgoing. The BRDF can be extended to a BSSRDF in order to account for local variations in lighting over the surface.

3.1 Single Scattering

The single scattering significantly affects the overall appearance of the object since the energy of single scattering would not scatter or absorb too much. Using BSSRDF to compute the contribution of single scattering means that the all path of single scattering must be traced, and the amount of radiance must be computed for each segment. The overall outgoing radiance is computed by integrating the incident radiance along the refracted outgoing ray:

$$\begin{aligned} L_o^{(1)}(x_o,\vec{w}_o) &= \sigma_s(x_o)\int_{2\pi} F\,p(\vec{w}_i{}'\cdot\vec{w}_o{}')\int_0^\infty e^{-\sigma_{tc}s}L_i(x_i,\vec{w}_i)dsd\vec{w}_i \\ &= \int_A\int_{2\pi} S^{(1)}(x_i,\vec{w}_i;x_o,\vec{w}_o)L_i(x_i,\vec{w}_i)(\vec{n},\vec{w}_i)dw_idA(x_i) \end{aligned} \tag{7}$$

where F is the product of the Fresnel transmission terms for the incoming and outgoing directions, σ_{tc} is the combined extinction coefficient term. It combines σ_t at x_o and σ_t at x_i, but σ_t at x_i needs to multiply a geometry factor G.

The theoretical BSSRDF model can only apply to semi-infinite homogeneous media. The derivation is really hard to compute in the presence of arbitrary geometry and texture variation. However, using some intuitive method based on the theory allows BSSRDF be applied for computer graphics. However, we can use some of the intuition behind the theory to extend it to a practical model for computer graphics. In raytracing, the contribution of single scattering can be computed using discrete version. The progresses are sampling the distance along the outgoing direction to the scattering points, casting a ray from the scatter points towards a light source and find the intersection with the object, and finally casting a ray from this point to the light source. The distance along the refracted outgoing is sampled based on s_o' = log(ξ) / $\sigma_t(x_o)$ where ξ is a random value between 0 and 1. The result of single scattering is shown on Figure 4 and 5. For this sample location we can compute the outscattered radiance as:

$$L_o^{(1)}(x_o,\vec{w}_o) = \frac{\sigma_s(x_o)F\,p(\vec{w}_i\cdot\vec{w}_o)}{\sigma_{tc}}e^{-s_i'\sigma_t(x_i)}e^{-s_o'\sigma_t(x_o)}L_i(x_i,\vec{w}_i) \tag{8}$$

where s_i' is the distance between x_i and the scatter point. The contribution of single scattering is average the number of samples. It is really difficult to compute for arbitrary geometry since it requires finding the point at the surface where the shadow ray is refracted. However, there is a good approximation assuming light source is really far away, then use Snells law to estimate the distance through the medium of the incoming ray.

$$s_i' = s_i\frac{|\vec{w}_i\cdot\vec{n}_i|}{\sqrt{1-(\frac{1}{\eta})^2(1-|\vec{w}_i\cdot\vec{n}(x_i)|^2)}} \tag{9}$$

Here s_i' is the observed distance and s_i is the refracted distance.

3.2 Diffusion approximation

Diffusion approximation can be solved by using BSSRDF. In this method, the points should be picked randomly on the surface and average those points since it is an efficient way to compute this value. The contributions of diffusion approximation are based on the distance between scattering points and x_o. The dipole source is used for computing the diffusion approximation. It transforms an incoming ray into a dipole source. The dipole method places two source points near the surface in such a way as to satisfy the required boundary condition. In the ideal case that the geometry is flat, using a dipole source would get a good diffusion approximation for flat materials. In this case the positive light source would be placed at a depth of $1/_t$' below x_i and the negative light source is place above x_o, so it satisfies the boundary condition. This approach works well for the BSSRDF. After setting up the dipole source, rays are casted from the random point to the light sources and the radiance is computed. In order to get a good diffusion approximation, a large number of sample points should be picked. The diffusion approximation with dipole source can be computed as below:

$$\begin{aligned} R_d(r) &= -D\frac{(\vec{n}\cdot\vec{\bigtriangledown}\phi(x_s))}{d\Phi_i} \\ &= \frac{\alpha'}{4\pi}[(\sigma_{tr}d_r+1)\frac{e^{-\sigma_{tr}d_r}}{\sigma_t' d_r^3}+z_v(\sigma_{tr}d_v+1)\frac{e^{-\sigma_{tr}d_v}}{\sigma_t' d_v^3}] \end{aligned} \tag{10}$$

However, this approach has a trouble for arbitrary geometries. For an instance of a cube, if x_o and x_i are on the opposite sites of an edge of the cube, the positive light source will be very close to the x_i. This makes the edge extremely white. To solve this problem, the minimum distance between x_i and the light sources should be set.

Another problem is to choose a random sample based on the distance between the sample point and the light source. However, choosing a sample using this method would cause a computational problem since finding the intersection between the primitive and a sphere need to compute the distance between the light source and any point on the screen. This approach is really computational consuming. In order to solve this computational problem, I use another approach choosing a random point on the surface of the object.

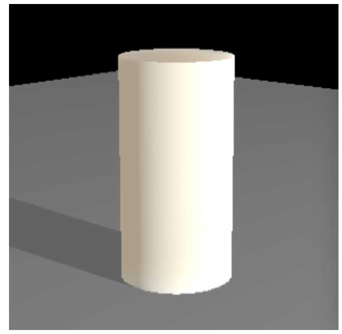

Figure 4: A cylinder of milk with single scattering.

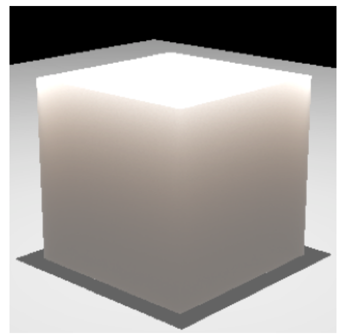

Figure 5: A cube of marble with single scattering.

Figure 6: A cylinder of milk with diffusion approximation.

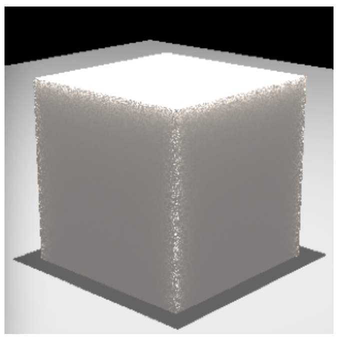

Figure 7: A cube of marble with diffusion approximation.

First, I need to compute the surface area of the object. Then there is a routine to pick points on the surface randomly. This approach will pick the points over the entire object. This approach is not ideal but does not cost too much computational resource. For the diffusion approximation, it is really important because there are a hug number of sample points needed to be picked in order to simulate the real translucent materials. The result of diffusion approximation is shown on Figure 6 and 7.

4 Result

The result is able to approximate the effect of subsurface scattering and render translucent materials. Jensen [2001] also provide the measured parameters for several materials, σ_a and σ_s, for computing the contribution of single scattering and diffusion approximation. Those parameters are measured based on the properties of those materials. The objects generated by the model of the subsurface scattering look realistic. After applying the single scattering, the diffusion approximation and those measured parameters, the light is able to transport within the object. It also includes transport the different colors of any light. The figures, a cylinder of milk and a cube of marble, are generated by the model shown on Figure 8 and 9. The block of marble is lit by the light on the top. The subsurface scattering model allows the light to go through it and scatter in different points on the surface. The speckling points around the marble is caused by the insufficient sample points. This noise can be reduced, and the surface can be smoother by adding more sample points, but it would be very computationally expensive. However, the model works correctly and well, and is able to render reasonable effects of subsurface scattering.

Figure 8: A cylinder of milk with BSSRDF model.

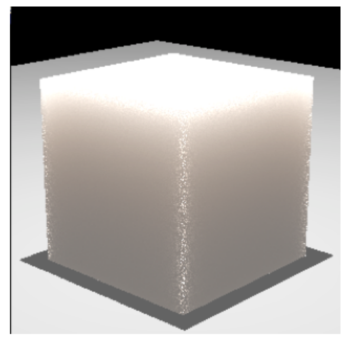

Figure 9: A cube of marble with BSSRDF model.

5 Conclusion and Future Work

The subsurface scattering is really important to render the effect of subsurface scattering using the BSSRDF model for computer graphics. This is because when light go through a translucent object, the light would transport many times within the object and leave from arbitrary surface. The model combines the single scattering and the diffusion approximation. Also, there are some results of simulating translucent materials using this model. Those results of BSSRDF model are look realistic.

In the future, I would like to make the model to be able to render multiple layers in an object since in the real world there are many different materials overlapping together. For instance, in the Figure 8 it is trying to render a glass of milk. Although this image is able to render the properties of milk, it does not like a glass of milk since the model cannot render a material like glass. Also, the model cannot render an object overlapping multiple different kind of materials. It can only render one layer of material instead of multiple layers. This affects the overall quality of appearance a lot. Therefore, in the future work I plan to extend the model to be able to render multiple layers in an object. It makes the result more real and accurate.

References

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