The Dimension of Superpower Rivalry

A DYNAMIC FACTOR ANALYSIS

JOHN T. WILLIAMS
MICHAEL D. McGINNIS
Indiana University

The security policies of the United States and the Soviet Union can be interpreted as manifestations of a single "rivalry system." If each state's security policies are driven by the same underlying factors, then any effort to separate the contributions of internal and external determinants of the arms race is essentially misleading. We use dynamic factor analysis to evaluate whether an unobservable dimension of rivalry explains the dynamics exhibited by the military expenditures and diplomatic hostility of these two states. A one-factor model explains much of the variance of these data series, although some evidence indicates the possible existence of a second factor. More generally, the results of this analysis question the validity of many structural equation models of dyadic interaction.

For years the mutually reinforcing suspicions of policymakers in the United States and the Soviet Union have provided the foundation for an exceptionally interdependent rivalry system. Neither state can sustain large military budgets without a convincing external threat, and the presence of threat gives proponents of a large military budget a distinct advantage in policy debates. As this syndrome of mutual suspicion abates, political processes in each state should begin to follow their own, internal dynamics. However, even if recent events truly signify the demise of this rivalry system, the recurring patterns

AUTHORS' NOTE: This research was supported by National Science Foundation Grant SES-8810610. An earlier version of this paper was presented at the Annual Meeting of the American Political Science Association, Atlanta, Georgia, August 31-September 3, 1989. We would like to thank Stuart Bremer, Jeff Hart, Gary King, Mark Lichbach, Michael Ward, and members of the Political Methodology Section of the American Political Science Association for their useful comments on earlier versions of this paper, and especially Marc Simon for research assistance and useful comments.

JOURNAL OF CONFLICT RESOLUTION, Vol. 36 No. 1, March 1992 86-118

of dynamic interactions that sustained superpower rivalry for so long merit further study.

Research on international rivalries falls into two categories. In one approach, all cases that satisfy an operational definition of rivalry are identified. For example, Smith (1980) defines arms races as a particular combination of military expenditure increases and hostile statements by both states, and Diehl (1985) uses a series of crises occurring within a specified time period to define cases of "enduring rivalry." Once the relevant universe of cases is identified, conclusions can be drawn about the likelihood of arms races or rivalries escalating to war (see Siverson and Diehl 1989).

A second approach investigates one or a few cases in greater detail. The U.S.-Soviet case has been most extensively studied, because of its substantive prominence and the availability of relatively high quality data series of relatively long length. Continuing controversy over the existence or prevalence of reaction in specific cases conveys the impression that this research approach is designed to prove whether or not an arms race exists. Instead, the basic goal of this research is to better understand dynamic relationships between resource allocation processes in interdependent states. Our research falls within this second tradition.

We take the existence of rivalry between the United States and the Soviet Union as a basic premise of our analysis, and we investigate some implications of that assumption for the statistical analysis of time series data on their military expenditures and hostility levels. Although evidence of reaction in Richardson's sense is based on the magnitude of particular coefficients, we argue that rivalry constitutes a more fundamental linkage that manifests itself in subtle and pervasive relationships among indicators of the foreign policies of these rival superpowers. As a consequence, we test for evidence of a particular type of sophisticated reaction in which the data series move together in a systematic manner across the entire frequency spectrum.

In the first section we compare our conceptualization of rivalry to previous research on arms race models. We then introduce a general formal model of the foreign policies of rival states. This model implies that the common variance in time series data on U.S. and Soviet military expenditures and diplomatic hostility can be attributed to the effects of a common, unobservable factor we call the dimension of rivalry. To test these implications we use dynamic factor analytical techniques developed by Geweke (1975, 1977). These techniques enable us to examine relationships among variables across the entire range of frequencies as reflected in the time series data. After interpreting our specific results, we draw more general implications for the use of structural equation models of international rivalry. Methodological details of the dynamic factor model are summarized in a Methodological Appendix.

RESEARCH STRATEGY

Our point of departure is the body of research building on Richardson's (1960) arms race model, in which each state reacts to the threat posed by the other's military expenditures and to the economic fatigue created by its own expenditures. Statistical work in this research tradition focuses on the relative importance of internal and external factors in supposed cases of arms races. Most analyses suggest that domestic variables explain arms expenditures more effectively than do external variables or indicators of arms race reaction (Moll and Luebbert 1980; Zinnes 1980; Russett 1983; Isard and Anderton 1985).

These puzzling results inspire various efforts to build more elaborate statistical models capable of accounting for more complex domestic and international political processes.[1] Ostrom (1978) develops a model of a specific "reactive linkage" process in which estimates of Soviet defense spending affect different stages of the U.S. budgetary process in different ways. Ostrom reports evidence that U.S. policymakers react to Soviet expenditures, a conclusion supported by more recent work (Ostrom and Marra 1986; Marra 1985; Majeski 1983a, 1983b). Other researchers respond to the puzzle of nonreaction by developing more sophisticated models of the underlying structure of the arms race. For example, in the nonlinear, continuous-time model estimated by Ward (1984) each state's expenditures are shown to react to a measure of the relative balance of weapons stocks.

These analysts find evidence of reaction in the U.S.-Soviet arms race by developing elaborate models that require the imposition of a large number of a priori restrictions about functional forms and exogeneity of variables. We are skeptical that the existing state of our theoretical knowledge about domestic politics and international conflict is sufficient to justify such a large number of specific restrictions.

We offer a novel means of dealing with the complexity of international rivalries. Unlike others, we acknowledge the difficulty of justifying enough restrictions to fully specify models of rivalry. Instead, we adopt a research

1. We believe that direct comparisons between the relative potency of domestic and external "determinants" of an arms race are logically invalid (see McGinnis 1991).

strategy that focuses on the implications of the shared dynamics of rivalry on aggregate indicators of foreign policy.

The flavor of our research strategy is conveyed by a reconsideration of the empirical puzzle of nonreaction (Williams and McGinnis 1988; also McGinnis 1991). There is no doubt that previous research demonstrates that each state's own military expenditures are the best predictors of their own future arms expenditures, but this result hardly constitutes proof of the exclusively domestic basis of the arms race. Although indicating that advocates of high military spending levels frequently win in policy debates, this result can be attributed to the success of policy advocates who take advantage of the persistence of a serious external threat. Without this threat, other political groups would prevail more often in policy debates. In this sense, the two superpowers comprise a single, interdependent rivalry system, and neither state's security policies can be studied in isolation.

We interpret an arms race as one component of the broader relationship between rival states. We define a rivalry as occurring when most influential policy advocates in two states see the other state as the primary threat to their own national security. Our conceptualization of superpower rivalry includes conventional and nuclear arms races, crisis management, diplomacy, alliance commitments, troop deployments, economic and military aid, and interventions in regional conflicts. In short, each superpower tries to counter any gain made by its rival, whether in terms of new weapons, new allies, or new gambits in their ideological struggle.

Rival states need not react to each other in kind, for states have a wide variety of foreign policy instruments from which to choose as they pursue their goals of national security. Thus rivalry necessarily involves foreign policy substitutability (Most and Starr 1984, 1989). Unlike other analysts, however, we do not assume the existence of some single decision entity that substitutes one policy instrument for another. In our conceptualization the security policies of states emerge from complex patterns of political competition among various policy advocates (see McGinnis and Williams 1991). Different aspects of a given state's foreign and security policies are related because they are determined by overlapping sets of policy advocates making deals and compromises on various policy dimensions. Similarly, the foreign policies of rival states are systematically related if policy advocates in each state routinely use evidence about the other state's policies in order to support their own preferred policies. In short, "The actions of one nation affect those of another to the degree that they result in advantages and disadvantages for players in the second nation" (Allison 1971, 178).

This fundamental linkage imparts a common dynamic to the foreign policies of rival states. We posit the existence of an underlying dimension of rivalry, which we define as the intersection of all internal and external factors that influence the security policies of both rival states. We report statistical results concerning the properties of a single-factor explanation of U.S. and Soviet military expenditures and diplomatic hostility.

We examine data on the military budgets of both superpowers and an events data-based measure of diplomatic relations for the years from 1949 to 1978. Both variables play important roles in previous analyses of arms race models, but for our purposes the relevant point is that each variable represents a summary score of important, but different, aspects of their political relationship.

The military budget provides a measure of the aggregate level of effort devoted to military purposes.[2] Unlike many previous arms race modelers, we do not presume that one state reacts directly to the budgetary allocations of the other state. Even though one state cannot observe the other's budget exactly, each state has access to a wide range of intelligence sources that enable it to estimate the activities that would be included in the actual budgetary figures.

Although controversial, events data provide useful summary measures of the overall nature of relations between the United States and the Soviet Union (see Goldstein and Freeman 1990). We calculate a yearly aggregate score from the Conflict and Peace Data Bank (COPDAB) events data set, with events weighted according to the Azar-Havener (1976) scale.[3] We include

2. Our military expenditure data set for 1949-1984 was calculated from Stockholm International Peace Research Institute (SIPRI) sources, as follows. Figures for Soviet expenditures in the years 1950-1966 are as given in each edition of the SIPRI Yearbook from 1974 through 1979. Those for 1967-1976 were taken from the 1979 edition. The figures for the years 1977-1984 were calculated from data in the 1985 edition, by dividing by a factor of 1.801, the average of the ratio between the 1985 and 1979 figures for the overlapping years of 1975-1976. (As a general rule, we discarded the most recent year of any edition's list, because these values are nearly always revised in subsequent editions.) Finally, the figures for 1948-1949 were calculated from data in the 1969-70 edition, multiplied by a factor of 1.3041, which was the average of the ratios for the overlap years 1950-1952. For consistency, a similar procedure was applied to SIPRI figures on U.S. military expenditures. In this case the base series covered 1952-1972, as published in the 1974-1976 editions. (These figures were in constant 1970 dollars, so this was taken as our base year for both series.) Figures for the years 1973-1984 were calculated from figures in 1980 dollars (the 1984 edition), by dividing by 2.122, a conversion rate calculated to include the overlap for data published in 1973 prices in the 1977-1979 editions. (The ratio for 1970 to 1973 prices was 1.144.) Finally the 1949-1951 data were common to the 1968 through 1972 editions, multiplied by 1.325 (from the overlap years 1952-1955).

3. Our use of budgetary data requires a level of aggregation of 1 year because this is the natural temporal unit. We would like to disaggregate events further, but for this analysis we cannot. One benefit is that random measurement error will be smaller for the aggregated data.

only conflictual events.[4] In our earlier work we demonstrate that this measure of conflictual activity is more closely related to military budgets than a net hostility (conflict minus cooperation) score, because participants in policy debates are more sensitive to surprisingly hostile events and treat cooperative overtures with more cautious skepticism (McGinnis and Williams 1989, 1109-10). In that work we use diplomatic hostility as an important source of information to policymakers debating the size of the military budget, but in this article we treat both expenditures and hostility as coequal policy variables.

Our fundamental hypothesis is that data series on U.S. and Soviet military expenditures and diplomatic hostility are related through a shared dynamic structure of the rivalry system. This dynamic is driven by some unobservable explanatory factor, which we interpret as the single dimension of rivalry.

Our dynamic factor model is consistent with many different structural models that include military expenditures and diplomatic hostility as endogenous variables. In many previous arms race models hostility levels have been included as one of the explanatory variables in an equation for military expenditures (Ward 1984; Rattinger 1975). Our focus is different. We examine whether the military expenditures and diplomatic hostility of these two states exhibit joint movement in response to the same underlying explanatory factor. The existence of extensive feedback among foreign policy variables implies that no variable can be treated as exogenous in a structural equation model (McGinnis and Williams 1989). We exploit the fact that if feedback is so pervasive, then the military expenditures and diplomatic hostility of rival states should follow similar dynamics.

We emphasize that we do not claim that policymakers restrict their attention to these variables, or even that they actually monitor these specific indicators. In fact, policymakers collectively monitor far more pieces of information than could possibly be included in any single analysis. Nonetheless, the relationships among these aggregate indicators of foreign policy convey important insights into the overall nature of superpower relations.

Our theoretical perspective highlights the many complex interactions that make a rivalry system. This perspective requires us to adopt statistical techniques that require as few a priori restrictions as possible. Elsewhere we show that an unrestricted vector autoregression (VAR) model provides evidence of "sophisticated reaction" in U.S.-Soviet military budgets (McGinnis and Williams 1989; Williams and McGinnis 1988). This empirical evidence

4. We use yearly aggregations of all conflictual events (COPDAB categories eight to 15) directed from the United States to the Soviet Union and vice versa (see Azar 1980).

supports our theoretical argument that intense policy competition in both states implies that military budgets closely approximate the optimal allocations of two unitary rational states capable of forming rational expectations of each other's future behavior.[5]

We use dynamic factor analysis to dig deeper into this relationship. If policy variables are indeed closely related through processes of policy competition within each state, then these data series should share common dynamic patterns that can be attributed to a single underlying dimension of explanation. This underlying dimension cannot be directly observed, but its presence can be detected through application of dynamic factor analysis.

Naturally, this research strategy limits the nature of the inferences we can draw. We cannot interpret negative findings as evidence of nonreaction, for rival states might interact in a manner unrelated to sophisticated reaction. On the other hand, positive findings cannot be taken to prove the existence of rivalry, for these dynamics might conceivably be attributed to some other type of relationship. Yet our tests are more powerful than may be immediately apparent, for the dynamic patterns we posit are virtually impossible to generate under any nonrivalry model.

Primarily self-directed processes unique to each state could hardly be expected to produce, purely by chance, systematic similarities in dynamic response over the entire range of the frequency spectrum. Also, it is too facile to attribute such wide-ranging common dynamics to the possible effects of a single confounding factor, such as the behavior of China. Because covariation across frequency ranges indicates a considerably more extensive interrelationship than does correlation in cross-sectional models, the effects of any postulated third factor require a more elaborate justification. Frankly, we find it difficult to even imagine how China's changing relationship with each of the superpowers could explain systematic similarities in the common dynamics exhibited by U.S. and Soviet behavior across the entire frequency spectrum.[6] This is not to say that China is irrelevant to superpower rivalry; far from it. Both superpowers do indeed react to China's behavior, but they also react to each other's reaction to China's behavior, as well as to all other common areas of concern. It is this wide-ranging, mutual reaction that we intend by the term "rivalry."

5. Rational expectations is a very controversial concept that has been interpreted in very different ways. For discussion of our intended meaning, as well as references to the relevant economic literature, see Williams and McGinnis (1988), and McGinnis and Williams (1989).

6. In any event, the absence of a consistent time series on Chinese military expenditures precludes any extension of our analysis to incorporate China. For recent analyses of the strategic triangle based on events data alone, see Goldstein and Freeman (1990, 1991) and Rajmaira and Ward (1990).

Therefore, if our analysis identifies systematic similarities in U.S. and Soviet behavior over a wide range of frequencies, we feel confident in interpreting this evidence as supporting our conceptualization of rivalry.

A GENERAL MODEL OF THE FOREIGN POLICIES OF RIVAL STATES

Let y_i represent some measure of foreign policy behavior and Ω the set of all relevant explanatory variables. The standard practice in the quantitative international relations literature has been to select specific variables ω_k from Ω and to estimate a set of linear equations of the form $y_i = \Sigma_k \beta_k \omega_k$. In the absence of theoretical knowledge that can specify exactly which variables should be included, we instead examine the dynamics of a small set of policy variables.

Let (y_1, y_2) and (x_1, x_2) denote data on the military expenditures and diplomatic hostility of rival states y and x. Let $y = (y_1, y_2, \ldots, y_n)$ and $x = (x_1, x_2, \ldots, x_n)$ denote the vectors of all foreign policy variables of these two states. The substitutability effects identified by Most and Starr (1984, 1989) imply that individual components of y are interrelated, in the sense that each y_i is a function of all the other y_j $(j \neq i)$. Alternatively, one can say that all y_i are determined jointly. In addition, all x_i and y_i are determined jointly if the states are in a rivalry. Rivalry between two states implies some functional interrelationship between x and y.

Not all factors relevant to an explanation of foreign policy are themselves policy variables under the control of a given state or its primary rival. Let $z = (z_1, z_2, \ldots, z_m)$ include all other factors, including policy variables under the control of other states (or other actors) as well as nonpolicy variables such as population or geographic conditions. In very general terms, each foreign policy variable y_i of a given state is a function of all policy variables y, the rival's policies x, all other explanatory factors z and random influences (denoted by ξ). That is,[7]

$$y_i = f(y_j, x, z, \xi) \text{ and } x_i = f(x_j, y, z, \xi), \text{ for } i, j = 1, 2, \ldots, n, \text{ and } j \neq i.$$

If these two states do indeed form a single rivalry system, then one can say that all x_i and y_i are jointly determined, that is, each is a manifestation of the

7. In this formulation each set of policy variables includes values observed at earlier times. That is, each $y_{i,\,t}$ is a function of its own previous values $y_{i,\,t-k}$ as well as those of its other policy variables $y_{j,\,t-k}$ and the other state's policies $x_{j,\,t-k}$ and all other variables $z_{i,\,t-k}$. We suppress the time subscript to minimize notational clutter.

same underlying system. Although our empirical application specifically includes only two components of y and x, the underlying complexity implied by reaction, substitutability, and nonpolicy variables (as well as the importance of actors' expectations and feedback paths) is implicitly encompassed by our analytic techniques.

The generality of this formulation should be apparent. If one state's policies change in a certain direction as a consequence of surprising developments (innovations) in the domestic or international environment, vigilant policymakers in the other state will try to forecast these changes and make corresponding changes in their own state's policies. For example, if Soviet policymakers observe domestic conditions that make it more likely that the U.S. government will embark on an extensive military buildup, Soviet policymakers will most likely implement changes meant to address this increased threat. In any specific instance reaction may be slowed for various reasons, but, in the aggregate, the security policies of rival states should exhibit similar dynamics, if policymakers in both states do indeed pay careful attention to the policies of the other state.

But rivalry is unlikely to be the sole determinant of any foreign policy instrument. U.S. military spending is affected by perceptions of the prevailing military balance and the state of U.S.-Soviet relations, but it is also affected by more clearly domestic factors such as elections, efforts to manage the economy, and the practice of spreading governmental projects around to the constituencies of important congressmen. The Soviet system may not lend itself to such blatant pork barrel politics, but Soviet defense allocations are also affected by bureaucratic and political battles (see Meyer 1984). U.S. foreign policy is often affected by international events that have very little to do with U. S.-Soviet rivalry per se, such as the trade balance with Japan. And although Sino-Soviet relations occasionally influence U.S.-Soviet rivalry, we do not see this additional linkage as a fundamental part of superpower rivalry. Rather, we interpret China's role in U.S.-Soviet rivalry as that of a sporadic interloper and not a systematic participant.[8]

In short, foreign policy data will contain both rivalry and nonrivalry components. For example, to the extent that Soviet policymakers discount U.S. diplomatic statements as being primarily directed at domestic audiences, these statements would not be related to the rivalry factor. That part of U.S. diplomatic hostility toward the Soviet Union that is explained by the common factor can be thought of as being part of the rivalry system; that part

8. The Korean and Vietnam Wars are also part of rivalry, at least in part. Although some of the war spending may be discounted by the Soviets, we argue that at least some of the actions by the U.S. are reciprocated in some way by the Soviets.

that is not explained by the single factor may be due to exogenous international events or to the systematic influence of domestic politics, such as attempts by the president to increase his public support through inflammatory rhetoric.

For ease of presentation we assume that there exists one common rivalry factor and one separate factor for each of the two states. Let r denote the underlying factor of rivalry and s_k the other systematic influence on the security policy of state k (for k = y, x). Thus, each policy variable y_i and x_i is a function of r, s_y or s_x, and ξ (denoting, as before, random influences). That is, $y_i = f(r, s_y, \xi_i)$ and $x_i = f(r, s_x, \xi_i)$ for i = 1, 2, . . . , n.[9]

The general mathematical forms of these functions may be intricate, but we follow the standard practice of using a linear model in the absence of knowledge about alternative functional forms. In this case, linearity is not overly restrictive. A linear factor structure is capable of accounting for very complex dynamics, as we will estimate a different factor structure for different time periods or frequency ranges. Thus, in principle, we can estimate a model of the following form:

$$\begin{aligned} y_i &= \lambda_{yi} r + \alpha_{yi} s_{yi} + \varepsilon_{yi}, \\ x_i &= \lambda_{xi} r + \alpha_{xi} s_{xi} + \varepsilon_{xi} \qquad \text{for } i = 1, 2, \ldots, n. \end{aligned}$$

However, a great deal of information is needed to estimate a three-factor model. Fortunately, important insights into the dynamics of rivalry can be gained by estimating the following one-factor model:

$$\begin{aligned} y_i &= \lambda_{yi} r + \varepsilon_{yi}, \\ x_i &= \lambda_{xi} r + \varepsilon_{xi} \qquad \text{for } i = 1, 2, \ldots, n. \end{aligned}$$

This rivalry factor model focuses attention on the underlying dynamics of interaction between the policy variables of the two rival states. In short, rivalry is unobserved but relates to each policy variable in a way described by each λ_{ki}, and nonrivalry components are included in the ε_{ki} terms.[10]

More specifically, our analysis will concentrate on comparisons among the proportion of variance explained by the rivalry factor. In general, time

9. More than one nonrivalry factor may be involved but we subsume all other systematic influences into a residual category, because we only estimate a single-factor model. A further complication is that rivalry itself may be multidimensional, as would occur if, for example, the security and ideological aspects of superpower rivalry are mostly separable dimensions of interaction. For simplicity we assume that all aspects of their competition are interrelated and thus that rivalry is unidimensional.

10. See King (1989b) for a related but very different approach to the decomposition of variance of data on U.S.-Soviet relations.

series data on any foreign policy variable y_i exhibits some finite variance $var(y_i)$. This variance can be decomposed (at least in principle) into the proportion of variance explained by the underlying rivalry factor, random influences, and the systematic influences of other factors. These factors will vary in their relative importance for explaining different policy variables, but in each case the variance can be partitioned into the variance $\phi_{ki}[r]$ of variable i for state k that is explained by the rivalry factor r and the variance $\tau_{ki}[s_k]$ explained by any remaining systematic factors s_k. Letting υ_{ki} denote the remaining unexplained variance, we can write

$$\begin{aligned} var(y_i) &= \phi_{yi}[r] + \tau_{yi}[s_y] + \upsilon_{yi}, \\ var(x_i) &= \phi_{xi}[r] + \tau_{xi}[s_x] + \upsilon_{xi} \qquad \text{for } i = 1, 2, \ldots, n. \end{aligned}$$

In summary, our argument for the preeminent role of rivalry in the determination of U.S. and Soviet military expenditures and diplomatic hostility implies that, for these data series, the variance explained by rivalry ($\phi_{ki}[r]$) will be much larger than the sum of the variance explained by other systematic influences ($\tau_{ki}[s_k]$) and the remaining unexplained random errors (υ_{ki}). Additional implications are discussed subsequently.

We test for the existence of a single-factor model, and we interpret the unobserved dimension as that part of these foreign policy instruments that is due to interaction with the rival. That is, this rivalry factor includes all shared sources of the security policies of the two superpowers. Any remaining variance may result either from random influences or systematic influences on one state's behavior that do not directly affect the other state's behavior. Because we test only a one-factor model we will not be able to distinguish between these competing explanations for the remaining unexplained variance. However, the overall proportion of variance explained by the common factor indicates the relative importance of rivalry in determining the values of each state's policy instruments.

The nature of this inference requires qualifications. If we find common dynamics in the data in the sense that a common factor is identified by the statistical procedure, then we interpret this common factor as rivalry. Strictly speaking, this unobserved factor can only be said to determine jointly the dynamics of the policy variables included in our model. Even if a single factor structure fits the data, this procedure does not constitute a direct test of our earlier definition of "rivalry" as involving a reciprocal pattern of perceptions among prominent policymakers as to the primacy of the security threat from the other state. That is, a common dynamic could exist for some other reason, unrelated to rivalry as we define it. However, this common dynamic would necessarily represent some common source of variance in military expenditures and diplomatic hostility, and rivalry is a reasonable name for that common source, regardless of its exact nature. Thus, if the one-factor model fits the data, this constitutes evidence in favor of our conceptualization of rivalry.

One competing explanation for a one-factor model is that the U.S. and Soviet Union react to exogenous events and shocks for some reason other than their rivalry. Our interpretation indicates that if both rivals react to changing situations, these reactions, both in direction and intensity, are largely a reflection of rivalry. An alternative interpretation, one that disallows our interpretation of the one-factor model, would focus on independent reactions of each superpower to each political event.[11]

We are confident that most actions during the period under investigation are rivalry related. This does not preclude the alternative explanation, as we realize that our method is subject to omitted variables problems.[12] Nevertheless, we feel confident that our interpretation is easily the most plausible. Our fundamental assumption is that rivalry pervades the behavior of the superpowers, which implies a one-factor model should fit this set of data.

RIVALRY AND THE FREQUENCY SIGNATURES OF TIME SERIES DATA

Our conceptualization of rivalry implies that a common but unobservable explanatory dimension underlies observed data on the military expenditures and diplomatic hostility of the United States and the Soviet Union. Factor analysis is the standard means to discover whether some unobserved dimensions explain a given set of data. Its application to cross-sectional data is widely familiar to analysts of foreign policy from the earlier work of Rummel (1972). Our application is similar, in that we are trying to reduce the dimensionality of a set of data, but the nature of time series data requires a different estimation strategy.

In cross-sectional analysis, we describe possible relationships using correlation. For time series data, correlation between variables imply that the

11. These reactions to exogenous variables would require a congruence (they must be consistently in the same or opposite direction) that is unlikely in the absence of rivalry. Otherwise, the reaction to the exogenous shocks will appear randomly distributed over time.

12. China's relationship with the Soviet Union represents the best counterexample because their interaction is much too involved simply to be a function of U.S.-Soviet rivalry (see Goldstein and Freeman 1990). However, it is not clear exactly how a diplomatic clash between China and the Soviet Union would affect U.S. behavior toward the Soviets. In view of this uncertainty, it would not be appropriate to simply include Chinese-Soviet relations as an exogenous variable because this relationship may also be affected by superpower relations.

variables have similar dynamics. That is, A and B may be related if both variables are described by the same dynamic. In short, two variables must move together over time in a systematic manner in order for them to be related to one another.[13] For purposes of estimating the dynamic factor model, we use the frequency domain representation of the variables, and the following discussion introduces the analog of correlation in the frequency domain.

Any set of data over time can be represented as a "Fourier transform," a mathematical construct in which the data is represented as a weighted sum of sine waves at all possible frequencies. Intuitively, a Fourier transform is a function that changes data from its original metric to another metric more suitable for analysis. A logarithmic function fulfills a similar purpose, but in the case of Fourier transforms the change is more drastic, because an observed series of numbers in the time domain is transformed to sine waves in the frequency domain. These waves are represented as complex numbers, which indicate both an amplitude (or magnitude) and a phase (or direction in the space of complex numbers). Complex numbers are useful for representing oscillatory movements in variables across time, especially because they are necessary for solving differential and difference equations.[14] Although it may be difficult to visualize a series of defense expenditure figures as a set of frequency waves, it is important to remember that the original data can be recovered by using an inverse Fourier transform, just as logged data can be recovered using an antilog function.

Figure 1 illustrates the description of a time series as a sum of sine waves. In this simplified example, a time series is defined as the sum of three regular cycles at low, medium, and high frequencies. (A long cycle time corresponds to low frequencies.) The resulting composite series is more complicated than any of its component parts, but it exhibits a clear regularity over time. Fourier analysis builds on the famous result that any time series, no matter how irregular, can be described as a summation of a large number of cycles.

Each cycle has a certain amplitude, and cycles of higher amplitude are more directly reflected in the composite series. The low and medium frequencies have the most influence on the overall series presented in Figure 1, but by varying the relative amplitude of the cycles the resulting composite time series can be made to exhibit more pronounced swings at high, low, or middle frequencies.

In general, a series is said to have the most "power" at those frequencies associated with cycles of higher relative magnitude. Spectral analysis is a

13. The requirement that variables must move together and have similar cyclical properties in order to be dynamically related does not necessarily mean that these variables are causal in the Granger sense, just as correlation does not mean causation for cross-sectional analysis.

14. See Allen (1959, chap. 4) for an introduciton to complex numbers and variables.

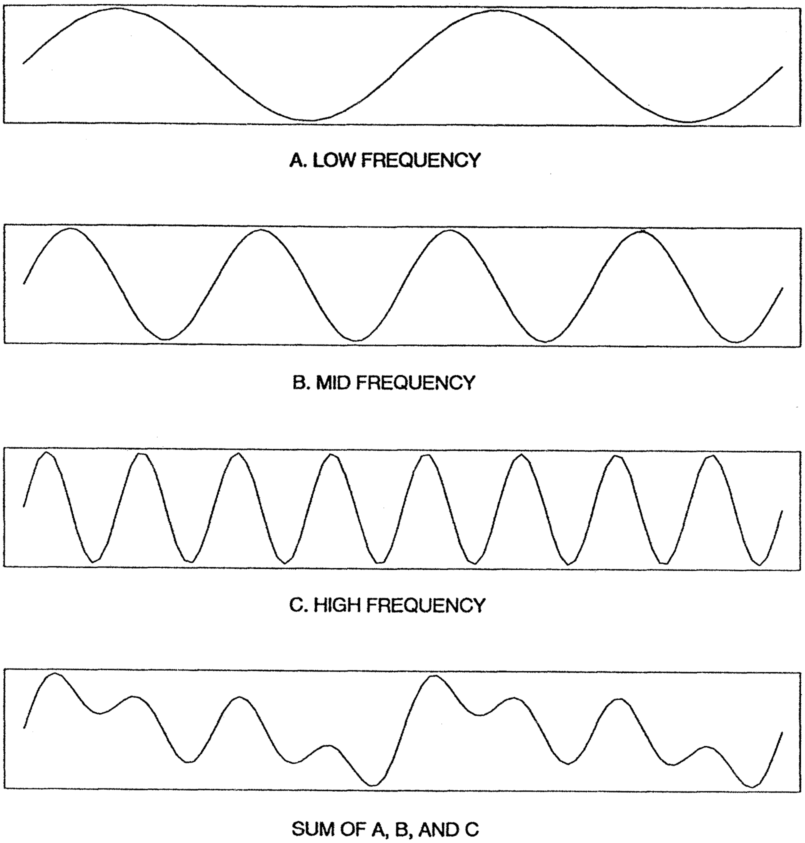

Figure 1: Example of the Summation of Cycle Frequencies

technique capable of estimating the importance or power of each frequency needed to describe a given time series. The relative power of each frequency is reflected in the coefficient estimate at each frequency, and the overall distribution of frequency weights constitutes the frequency spectrum (or "signature") of that data. Some signatures can be very simple, as would be the case if a variable was solely determined by, say, the 4-year presidential election cycle. However, most policy variables have complex frequency signatures because they are affected by many different influences that also exhibit complex dynamics.

In the time series context, if two variables are related then they will have similar frequency signatures. Two unrelated variables may exhibit similarly large power at specific frequencies purely by coincidence, but similarity of frequency signatures indicates that variables exhibit similar dynamics across a wide range of frequencies.

The crux of our argument is that the underlying dimension of rivalry imparts similar dynamics to a wide range of foreign policy instruments of rival states. Of course, each foreign policy instrument is also affected by random influences and other factors unrelated to rivalry. We focus our attention most closely on the decomposition of the variance of data series into that part that can be explained by an underlying factor of rivalry versus that part that remains unexplained.

IMPLICATIONS OF THE RIVALRY DIMENSION

By combining our general model with knowledge about the nature of the foreign policy variables under analysis, we can derive testable hypotheses within the context of the dynamic factor model. We make the following two assumptions about time series data on U.S. and Soviet military expenditures and diplomatic hostility.

Assumption 1: For either foreign policy area, data series on U.S. policies should exhibit more random variance than Soviet behavior in the same policy area.

Assumption 2: Hostility measures based on events data should have more powerful high frequency dynamics than military budgets.

Our first assumption is based on the observation that more groups with more widely divergent interests directly participate in the formation of policy in the United States than in the more closed political system of the Soviet Union. Elsewhere we develop a formal model of foreign policy as the outcomes of social choice functions defined over the actions of rational individuals motivated by policy concerns (McGinnis and Williams 1991).[15] It is impossible for participants or outside observers to predict policy outcomes exactly, but under conditions of rivalry each state's policies reflect relatively efficient forecasts of the rival's future behavior (Williams and McGinnis 1988; McGinnis and Williams 1989). The inherent uncertainties of preference aggregation processes preclude representation in simple struc-

15. The process of preference aggregation need not satisfy desirable properties, as demonstrated by Arrow (1963) and subsequent work in the field of social choice (see Riker 1982; Ordeshook 1986).

tural models. But the characteristics of social choice functions vary in a systematic manner in different foreign policy areas in the same state or in the same policy area in different states. Thus, in general and ceteris paribus, we expect Assumption 1 to hold.[16]

Our second assumption asserts that budgets, in general, change more slowly than diplomatic relations. Military expenditures are much more difficult to change in the short run, whereas diplomacy by its very nature often involves sudden change. Budgets tend to change incrementally, and these changes are shaped by the expectations of relevant actors (Williams and McGinnis 1988). But a single Presidential speech (especially one including surprising phrases such as "evil empire") can result in a noticeable change in perceived hostility levels. Thus, in general, we expect hostility data to cycle more strongly in the higher frequencies (even when aggregated yearly) than data on military budgets.[17]

Our next assumption more directly concerns the common rivalry dimension. Recent dramatic changes demonstrate that for some 40 years the U.S.-Soviet rivalry system has been essentially stable, in the sense that superpower relations have remained within a relatively narrow range of conflict and cooperation despite many important political and technological changes (Gaddis 1986). This connection between rivalry and stability might seem to suggest that the rivalry factor should be most effective in explaining the low frequency dynamics of any particular variables. However, short cycles in diplomatic hostility may also be important sources of rivalry dynamics.

A more reasonable presumption is that the intensity of policy debates insures that the rivalry dimension comes to explain the most important parts of any relevant policy variable. If a variable has more power in the higher frequency range, then the rivalry dimension should do best in explaining that range of frequencies, whereas in other, less important, frequency bands the effects of the rivalry dimension might be washed out by random noise or other factors. This argument justifies our third and final assumption.

16. Systematic measurement error provides an alternative explanation for observation of more variance in data series on U.S. policies. That is, analysts may tend to provide more smoothed estimates of Soviet behavior, because they are required to extrapolate from limited data. Thus it remains theoretically possible that if we had access to more complete information about Soviet behavior then their data series might exhibit comparable levels of variance, but we find it more plausible that there is indeed a fundamental divergence in the variability of these two state's policy outputs.

17. This difference between events data hostility measures and military expenditures is reflected in the time periods used to aggregate events data in previous research. Some insist that events data should be aggregated at the monthly (Freeman 1990) or daily (Jones 1988) levels. However, analysis of budgetary data is meaningful only at the yearly level.

Assumption 3: For any data series, the rivalry factor should explain proportionally more of that variable's variance in the frequency ranges with the most power.

These three assumptions provide the foundation for the specific hypotheses we test in this article:

Hypothesis 1: For all variables, the rivalry factor should explain proportionally more of the variance in the Soviet series than the U.S. series.
Hypothesis 2: For both states, the rivalry factor should explain proportionally more of the variance for the military expenditure series than for the hostility series at lower frequencies, and proportionally more of the hostility series than for the expenditure series at higher frequencies.

Hypothesis 1 derives from Assumption 1. Because U.S. data series exhibit more random variance than comparable Soviet series, the rivalry dimension should explain proportionally more of the variance of the Soviet series.[18] Hypothesis 2 derives from Assumptions 2 and 3. If higher frequencies are relatively more important (powerful) for the diplomatic hostility series and the lower frequencies are most important for budgetary data, then the rivalry dimension should be most effective (i.e., it should explain proportionally more of the variance) for high frequencies in hostility data and for lower frequencies in the budgetary data.

Our theoretical perspective precludes the imposition of the large number of a priori restrictions needed to identify a specific structural model. As a consequence, individual parameter estimates will be virtually impossible to interpret. These hypotheses are examples of the type of analysis that remains possible despite our inability to identify a specific structural model of political competition.

THE DYNAMIC FACTOR MODEL

Factor analysis has been used extensively in cross-sectional research, especially when a large number of observed indicators are thought to be correlated with a few unobserved factors. In a dynamic factor analysis representation of rivalry the observed variables are indicators of the security

18. If, instead, Sino-Soviet rivalry constitutes the second most important dimension underlying the security policies of the United States and the Soviet Union, then this hypothesis may no longer hold. However, a domestic U.S. politics factor is considerably more likely to have a significant impact on our specific data series than is a Sino-Soviet rivalry factor.

and foreign policies of rival states, and the unobserved dimension is rivalry. A dynamic single-factor model takes the form:

$$y_{it} = \lambda_i r_t + \varepsilon_{it}, \text{ for } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T,$$

where y_i represents observed policy variables for both states,[19] r the factor, T the number of time periods and I the number of variables. This model will fit only those time series that are highly correlated with each other (and thus implicitly with the unobserved factor) at different frequencies or time lags. In other words, different time series data must be highly correlated to be represented as a function of the same underlying factor. If the one-factor model is an adequate representation of the y_i's, then the λ_i's describe the relationship between the underlying factor and the variables, and the ε_i's account for that part of each variable's power that is not accounted for by the factor. If the remaining unexplained variance of ε_{it} remains relatively small and random, then the one-factor model fits the data relatively well. Errors include measurement error and the effects of systematic and random factors.[20]

Because there is almost always serial correlation in different variables and the factor across t, the conventional factor model is usually inappropriate for time series applications (Anderson 1963). For example, attempts to specify the required lag structure and serial correlation of the errors prove unwieldy in the maximum likelihood factor model of Jöreskog (1967).[21] Indeed, factor analysis in the time domain has all but been ignored because of the difficulty of specifying reasonable disturbance structures. We overcome these difficulties by factor analyzing the spectral density of the variables, as the assumption of independence of errors across time is much more tenable for the frequency domain.[22]

The frequency domain factor model we use was developed by Geweke (1975; 1977) and involves performing maximum likelihood factor analysis on a spectral density matrix. Briefly, a spectral density matrix is essentially the analogue of the covariance matrix in the frequency domain; it includes

19. We use y_i to represent policies of both states to simplify the presentation.

20. This model requires that variables have common cycles for all frequencies. In contrast, the common trends model of cointegrated variables implies only a common component for long-term movements between variables (see Harvey 1989, 449-56).

21. Watson and Engle (1983) and Beck (1989) show that the dynamic factor model can be estimated using the Kalman filter. However, the restrictions necessary for using the Kalman filter model are not defendable or even desirable for our application.

22. Data series are observed at regular intervals in the time domain, but the same information can be conveyed by organizing the data in terms of frequency of cycles per time unit in the frequency domain. The Fourier transform is used to move from the time to the frequency domain.

spectral densities on the diagonal and cross-spectral densities on the off-diagonal. For multiple time series, a spectral density matrix exists for each frequency, just as a covariance matrix exists for each time lag pairing (t, t; t, t – 1; t, t – 2, etc.). The off-diagonals of the spectral densities are cross-spectra, and these complex valued elements represent the relationships across variables for each frequency.

As already noted, we expect that for similar frequencies the spectrum for each series should be relatively similar if a one-factor model fits the data. Also, the various cross-spectra should themselves be relatively similar, with any differences between the implied spectral density matrix and the actual one being random. Thus, factor analysis in the frequency domain implies that the dimensionality of the spectral density matrix (in our case a 4×4 complex valued matrix) can be reduced (in our case to one dimension).

By factor analyzing the spectral and cross-spectral densities we can get around the problem that, in the time domain, the assumption that the error and factors are uncorrelated with each other at all time points is unlikely to hold. In the frequency domain, this assumption holds, at least in large samples, because spectral density estimates are independent across frequencies.

The factor structure of the dynamic factor model can be written:

$$S_y = \alpha(\omega)\alpha(\omega)' + \beta(\omega)\beta(\omega)', \tag{1}$$

where $\alpha(\omega)$ represents a 4×1 vector of factor loadings and $\beta(\omega)\beta(\omega)'$ is a 4×4 real diagonal matrix of common factors for each variable. Thus, $\alpha(\omega)\alpha(\omega)'$ represents the spectral density matrix implied by the one-factor model, and $\beta(\omega)\beta(\omega)'$ unique sources of variation.

A familiar correspondence between equation 1 and the conventional factor model is (see Appendix):

$$\Sigma = \Lambda\Lambda' + \Psi, \tag{2}$$

where Λ represents $\alpha(\omega)$ and Ψ equals $\beta(\omega)\beta(\omega)'$. The primary difference between the static and dynamic models is that for each ω a different factor analysis is performed. For technical reasons, we average the spectral densities over several frequencies (ω represents some frequency subinterval), and perform factor analysis over a number of subintervals. For our analysis, we choose four different subintervals for estimating the model. The appendix offers a more detailed description of the theoretical foundations of the analysis and the steps taken for estimation.

The assumptions required to use maximum likelihood factor analysis in the frequency domain are analogous to those for the cross-sectional model.[23] They are:

1. $\hat{y} \sim CN(0, S_y[\omega])$, the variables for analysis are distributed multivariate complex normal, with zero mean and an unknown variance.
2. $E(r_t, \varepsilon_{ij}) = 0$ for all t and i and j. The factors and the errors are uncorrelated.
3. $E(\varepsilon_t, \varepsilon_{it-j}) = 0$ for all t and j. The errors are serially uncorrelated and each is uncorrelated with other errors.

The first assumption is the most problematic, but not more than the multivariate normality assumption in the conventional factor model. Violations of this assumption make the distributional assumptions for hypothesis testing questionable, but maximum likelihood methods are, in general, subject to this objection. The second assumption is standard for applications of factor analysis. Under some alternative conceptualization of rivalry the factor and errors may be correlated, but we conceive rivalry as accounting for all shared variation. The third assumption is questionable given the small number of cases. However, the alternative factor model estimated in the time domain clearly violates this assumption. The robustness of this model to violations of these assumptions is not yet established, and further work will be needed to address these important practical considerations.

On estimation, interpretation of $\alpha(\omega)\alpha(\omega)'$ and $\beta(\omega)\beta(\omega)'$ (or equivalently $\Lambda\Lambda'$ and Ψ) is generally made in terms of the proportion of variance explained by the factor. However, the diagonals of the spectral densities implied by the one-factor model ($\alpha(\omega)\alpha(\omega)'$) are reported in Table 1, along with the real nonzero elements of $\beta(\omega)\beta(\omega)'$ and their standard errors.[24] The standard errors are large as expected given the small number of cases.

Because the precision of individual parameter estimates is low, we focus our analysis on the behavior of the likelihood function for different models. We also consider the ability of the one-factor model to account for variance in the observed variables. The relatively large stochastic components of these series inflates standard errors just as do the small number of cases. By

23. See King (1989a) for an analogous treatment of the assumptions behind the general maximum likelihood factor model.

24. Standard errors are obtained from the approximation of the Hessian matrix built during estimation and are calculated in the usual way. We require that $\alpha' (\beta(\omega)\beta(\omega)')\alpha$ be diagonal to identify the α elements for estimation, and our search is only over $\beta(\omega)\beta(\omega)'$, for once these elements are determined, the α elements have an analytical solution. Thus, as with the conventional maximum likelihood factor model, we provide standard errors for the $\beta(\omega)\beta(\omega)'$ elements

focusing on the likelihood, we can ask whether a one-factor model fits well, even if the stochastic component is large. Thus we are concerned just as much with the fit of the model as we are the ability of the factor model to precisely describe dynamics in each variable.

The parameters are best interpreted in terms of the implied relative power for each frequency band for the one-factor model.[25] For example, the power implied for U.S. spending by the one-factor model for the lowest frequency band is .40, about the same as Soviet hostility but much less than Soviet spending. The power in U.S. spending not accounted for by the model is .36. Thus, the $\alpha(\omega)\alpha(\omega)'$ simply gives the power spectrum implied by the one-factor model. We only report the square of the diagonal elements (the power) because the off-diagonals are complex and are typically converted by spectral analysts to coherences, which themselves merely represent explained variances (Granger and Newbold 1977, 57). Thus, for the rest of this article, we focus on the behavior of the likelihood and the amount of explained variance of the one-factor model.

Before discussing our results, it is worth noting that a dynamic one-factor model is closely tied to a vector autoregression (VAR) model like the models we test elsewhere.[26] Specifically, the one-factor model can be viewed as a restriction on an unrestricted VAR. A VAR model can be written as a moving average representation:

$$y_t = \sum_{j=1}^{\infty} A(L)Y_{t-j} + u_t, \tag{3}$$

where Y denotes the vector of I observed variables of both states, A is an I × I matrix of coefficients, u an I-vector of error terms, and L is the lag operator. The one-factor model can be interpreted as estimating equation 3 where the rank of A is assumed to be one. The factor model restriction is more consistent with our concept of rivalry than are conventional zero-order restrictions, and the factor representation of equation 3 can be quite valuable in reducing the many dynamic interactions of variables to a more easily interpretable single

only. Those standard errors not given in Table 1 represent corner solutions for which the standard errors are undefined (see Jöreskog 1967).

25. Because the spectral density matrices were normalized to have ones on the diagonals before estimation, the entries are comparable across variables.

26. See Sims (1980) or Freeman, Williams, and Lin (1989) for an introduction to vector autoregression models. We use VAR models in Williams and McGinnis (1988); McGinnis and Williams (1989).

dynamic.[27] That is, under the assumption that rivalry provides a single underlying dimension of explanation, the dynamics of the variables should be well represented by our estimated one-factor model.

A multiple factor model is especially interesting as a restriction of the VAR. If a model with k unobserved factors is estimated, equation system 3 can be recovered by applying the inverse Fourier transform on $\alpha(\omega)\alpha(\omega)'$ in order to move back to the time domain for more direct analysis of the dynamics of the variables (see Sargent and Sims 1977). This approach reduces the parameterization of equation 3 by restricting the rank of A to be k, and as long as k is small relative to the number of variables, the factor model reduces the parameter space considerably. However, this form of analysis does not provide any new and interesting information for the case when $k = 1$, and so we will not apply this specific technique in this article.[28]

RESULTS

For an initial validity check we examine the spectral densities of our four variables of U.S. and Soviet military expenditures and diplomatic hostility. Figure 2 presents the log spectrum (the logarithm of the real diagonal elements of the spectral density matrix) for each variable, with higher values denoting more power at specific frequencies.[29]

If the underlying dimension of rivalry is as important as we claim, then the spectral densities of security policy variables should have relatively similar power at the same frequencies. All of the series, with the exception of U.S. hostility, have most power at low frequencies. U.S. hostility has a strong cycle of around 6 years that is missing in the other variables. In general, the spectral density of many typical time series has strongest density at low frequencies, and sharing this spectral pattern does not necessarily mean that variables are causally related (Granger and Newbold 1977).

27. Geweke (1977) interprets the factor in his dynamic factor model as the common disturbance for all of the variables. This interpretation is consistent with the moving average representation.

28. As with the conventional factor model, identification of the multiple factor model is problematic. One option would be to make restrictions as in the confirmatory factor model. In this application these restrictions could be used to define the domestic politics factors.

29.The log spectrum is plotted in order to accentuate differences across and within series. In social science data, low-frequency power is often substantially greater than high-frequency power, and the convention of using a log transformation allows for more detailed plots of the high-frequency, low-power spectrum to be placed in the same figure as the higher-power, low-frequency spectrum.

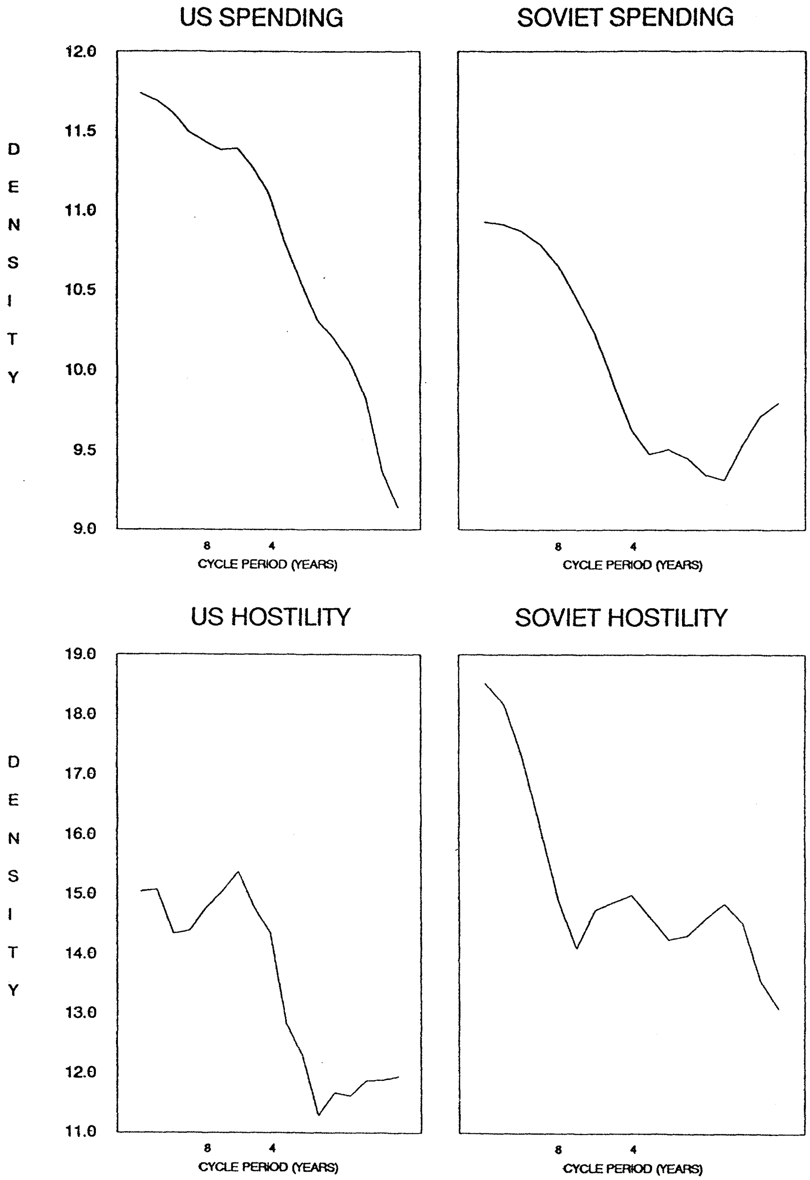

Figure 2: Log Spectrum of U.S. and Soviet Military Expenditures and Hostility

TABLE 1
Parameter Estimates for the Dynamic One-Factor Model

Frequency		U.S. Spending	Soviet Spending	U.S. Hostility	Soviet Hostility
			Estimates of $\|\alpha_i\|^2$		
(1)	.125π	.40	1.0	.03	.44
(2)	.375π	.62	.78	.44	.64
(3)	.625π	1.0	.44	.15	.16
(4)	.875π	.31	.34	.58	.71
			Estimates of β_{ii}		
(1)	.125π	.60	0	.97	.56
		(.34)	–	(.49)	(.34)
(2)	.375π	.37	.20	.56	.35
		(.42)	(.39)	(.45)	(.41)
(3)	.625π	0	.56	.84	.85
		–	(.31)	(.47)	(.46)
(4)	.875π	.69	.66	.42	.28
		(.47)	(.44)	(.44)	(.44)

NOTE: Each reported frequency is the midpoint for the frequency band for which the dynamic factor model was estimated. Each entry represents the variance or power for each variable that is explained by the one factor model divided by the total power or variance of that variable. The periods of the frequency bands are: (1) greater than 8 years; (2) 4 to 8 years; (3) 2.67 to 4 years; and (4) less than 2.67 years. Standard errors are in parentheses.

A likelihood ratio test statistic can be applied in the usual way, testing the hypothesis that Σ and S are the same.[30] This test provides a convenient method of describing the behavior of the likelihood. As Geweke (1975) warns, this test is based on large sample theory, and in many cases, including ours, the number of parameters estimated in proportion to the number of data points makes large sample assumptions dubious. Thus the likelihood ratio statistics presented in Table 2 should be used for heuristic purposes only. However, it is well known that in small samples the chi-square distribution has fat tails, thereby biasing tests against acceptance of the null (Bollen 1989, 266-68). In this light, the results do suggest that the one-factor model fits the sample data well, as in no case would we reject the null at a probability level less

30. The statistic is $2(F' - F)$, where F' is the value of the log-likelihood function for the unrestricted case, and F is the value of the function for the one factor restriction. Bollen (1989) presents a discussion of the merits of the likelihood ratio test in comparison with the Langrangian multiplier and Wald tests, all of which are asymptotically equivalent. Usually the latter two tests are used when the null is unknown, which is not the case here.

TABLE 2
Likelihood Ratio Statistics for the One-Factor Model

Frequency		*Test Statistic*	*Probability*
(1)	$.125\pi$	$\chi^2(5) = 8.29$	.141
(2)	$.375\pi$	$\chi^2(5) = 2.04$	.843
(3)	$.625\pi$	$\chi^2(5) = 7.51$	.185
(4)	$.875\pi$	$\chi^2(5) = 7.81$	.167
Total		$\chi^2(20) = 25.65$	.178

NOTE: Each reported frequency is the midpoint for the frequency band for which the dynamic factor model was estimated.

than or equal to .1. Thus, these statistics suggest that the one-factor model is not an unreasonable restriction of the spectral density matrices.

The evidence in favor of the one-factor model does not preclude the existence of additional factors because of the problems with the likelihood-ratio tests indicated above. If the second factor merely accounts for variance otherwise left to the residual, then no problem of interpretation results, as we are merely expanding the disturbance terms in choosing not to estimate a second factor. Indeed, our argument hinges on the notion that rivalry accounts for all the common variation in foreign policies of the superpowers. However, this interpretation would not be appropriate if the second factor is correlated with the first. In this analysis we do not have enough data to test for this possibility. Even with more variables and longer time series, a multiple factor structure is not without problems. If the additional factors are seen as more than residual factors, then either some rotation method or zero restrictions must be made on the factor matrix. The former method is dubious, and the latter method, analogous to confirmatory factor analysis in the conventional factor model, is a direction for future research. We continue with interpretations under the assumption that any additional factors are indeed residual.

Table 3 presents the proportion of variance explained by the one-factor model. Overall, the one-factor model performs well. However, for frequencies corresponding to periods of about 2.5 to 4 years, the factor falls short of explaining 50% of the variance in all of the series. The most remarkable divergence from the one-factor model concerns U.S. hostility in the lowest frequency band, where the factor accounts for only 3% of the variance. But the other variables are explained very well at these frequencies. This failure of the model to account for U.S. hostility is remarkable because for the other

TABLE 3
Proportion of Variance of Each
Variable Explained by the One-Factor Model

Frequency		U.S. Spending	Soviet Spending	U.S. Hostility	Soviet Hostility	All Variables
(1)	$.125\pi$	.53	1.0	.03	.59	.54
(2)	$.375\pi$	.82	.95	.59	.84	.80
(3)	$.625\pi$	1.0	.59	.17	.18	.49
(4)	$.875\pi$	.39	.44	.76	.90	.62

NOTE: Each reported frequency is the midpoint for the frequency band for which the dynamic factor model was estimated.

frequency bands the two hostility variables behave very similarly. In addition, the behavior of military expenditures across the two states and across all the frequencies is similar.

The factor model explains U.S. hostility reasonably well for the frequency band with the greatest power, namely for periods of 4 to 8 years. Further, in all frequency bands but the third, where neither hostility series is well explained, Soviet hostility is explained much better than U.S. hostility. This indicates that, in general, U.S. hostility is less important as an indicator of rivalry. This would indeed be the case if the Soviets discounted many changes in U.S. rhetoric as being driven by domestic political concerns and, thus, as not actually constituting changes in the level of American threat.[31]

Our results support both of our formal hypotheses. Hypothesis 2 implies that for the lower frequencies, the military spending variable of one state will be explained better by the factor than the hostility variable of that same state, whereas the reverse would hold for higher frequencies. For the three lowest frequency bands the proportion of variance explained in military spending is substantially greater than that explained in the hostility variables. However, as expected, for the highest frequencies the factor is more closely related to hostility.

Hypothesis 1 asserts that the single factor will explain Soviet variables better than the corresponding U.S. variables. In only one of the eight cases—the third frequency band for military spending—is a U.S. variable

31. In McGinnis and Williams (1989) we discuss reasons why the Soviets should discount much of the United States' hostility toward them. We have also found that the Soviets discount defense agency requests and focus more on presidential requests in predicting future U.S. expenditures (Williams and McGinnis forthcoming).

explained better by the rivalry factor than a Soviet variable. This indicates that more dynamics in U.S. than Soviet policies can be attributed to domestic politics, for any other explanation of this pattern does not suggest itself.

An interesting finding is that U.S. military spending defines the common factor for that third frequency band just as Soviet military spending defines the lowest frequency band. This means that the implied dynamics of the underlying dimension of rivalry is virtually identical to these variables for these specified frequency ranges. We previously discussed why we expect lower power in high frequencies for data on Soviet policy variables, and it does indeed appear that the dynamics of the lowest frequency range are defined by Soviet spending.

These findings also reinforce our earlier analysis of the dynamic response of estimated VAR models of these same variables, in which we find that U.S. military spending and hostility both exhibit a higher frequency response to exogenous shocks than the comparable Soviet variables (McGinnis and Williams 1989). There exists a relatively short cycle in U.S. military spending (in this third frequency band) that is partially mirrored by Soviet spending but is not reflected in either state's hostility. This tendency is relatively unimportant overall, given that at this frequency the power in the U.S. spending series is relatively low. In fact, because the power of all series are relatively small in the third frequency band, any patterns in this frequency band will be less important, but these patterns should not be ignored.

Finally, it is worth noting that, as expected, the rivalry dimension does not always explain lower frequencies better than higher frequencies. Overall, the factor model performs best for the second frequency band and worst for the first and the third. The underlying dimension of rivalry has significant influences on the dynamics of foreign policy at various frequency levels, not just in the low frequencies. Thus stability in superpower rivalry is not merely a manifestation of low-frequency dynamics but is instead a more subtle and pervasive phenomenon.

IMPLICATIONS

The single rivalry dimension uncovered here summarizes the dynamics of U.S. and Soviet military spending and diplomatic hostility. The combination of poor fit to the low-frequency dynamics of U.S. hostility and the importance of an approximately 6-year cycle in both hostility terms suggests the existence of a second factor. However, it is not necessarily the case that all these as yet unexplained dynamics can be attributed to a domestic U.S.

politics factor, or, for that matter, to any other single factor. There may be many sources of remaining influences, the overall effect of which must be treated as essentially random.

Estimation of a two-factor model could settle some of these issues, but, unfortunately, because of degrees of freedom restrictions we are unable to estimate a two-factor model with the limited data available on U.S. and Soviet military expenditures and diplomatic hostility. The next logical step in this research program would be to estimate a model including several factors that would require a large body of additional data. Data on U.S. domestic politics, such as public opinion on foreign policy, approval ratings for the president's handling of foreign affairs, and election results would help resolve the issue of a second U.S. domestic politics factor. None of these data are available in long yearly data series, and analysis would require some creative combination of partial data series. It would also be very useful if some of these series could be measured on a quarterly or monthly basis to increase the degrees of freedom available for estimation. However, budgetary data are inherently measured at the yearly level.

This dynamic factor analysis of the superpower rivalry system demonstrates that the behavior of these two states is closely related. Therefore, the continued separation of domestic and international factors is no longer an appropriate research strategy for the analysis of foreign policy. This conclusion presents modelers with a daunting challenge, for any formal model that explicitly integrates the domestic and foreign policies of different states is likely to be very complicated and case specific. Dynamic factor analysis provides one means of dealing with this complexity.

One final implication of our analysis has broader significance for the quantitative international relations literature in general. We argue elsewhere that superpower rivalry connects the policies of these two states in such an intimate way that conventional econometric structural models of the arms race fail to show interaction (Williams and McGinnis 1988), and our finding that the one-factor model fits the data relatively well is consistent with this conviction. More generally, Geweke (1977) shows that, for almost any case, if a dynamic one-factor model describes the disturbances of a set of variables, then there exists no conventional linear structural representation for the data, because no exogenous variables will be available (among this set) to identify the set of equations. Frankly, this result casts doubts on the validity of efforts to develop and estimate structural equation models of the arms race.[32] In this

32. In McGinnis and Williams (1989) and McGinnis (1991) we discuss other reasons to doubt the usefulness of structural or institutional models of the arms race.

article we illustrate methods based on frequency domain factor models that should prove very useful whenever the dynamics exhibited by the data fail to meet the requirements for building a structural model.

METHODOLOGICAL APPENDIX

This appendix describes how the spectral density matrix is prepared for maximum likelihood factor analysis and how analysis in the frequency domain compares to factor analysis in the conventional cross-sectional model.

Estimates of the spectral densities for the four series are obtained by first taking the finite Fourier transform of the detrended variables,[33]

$$\hat{y}(\omega) = \sum_{t=1}^{T} y(t)e^{-i2\pi jt/T}, \quad j = 0, 1 \ldots, T/2. \tag{A1}$$

The next step is calculation of the periodogram:

$$I_y(\omega) = \hat{y}(\omega)\hat{y}(\omega)', \quad j = 0, 1, \ldots, T/2. \tag{A2}$$

Here, $\omega = 2\pi j/T$ is the frequency, T is the padded number of data points, and $i = \sqrt{-1}$. The periodogram provides inconsistent estimates of the spectrum and cross-spectrum, and the spectral density ($S_y(\omega)$), a smoothed version of the periodogram, provides consistent estimates. This smoothed estimate is obtained by prewhitening the initial series and averaging the periodogram ordinates over a series of frequencies.[34]

The spectral density can also be written as a function of the autocovariance of the series (C):

$$S_y(\omega) = \sum_{t=1}^{T} C_t e^{-i2\pi jt/T}, \quad j = 0, 1 \ldots, T/2. \tag{A3}$$

We can perform analysis in the frequency domain because the Fourier transform provides a representation in the frequency domain that is merely a transformation from autocovariances in the time domain. Thus, the spectral density matrix can be partitioned using factor analysis in the same way as a covariance matrix can be factor analyzed, and the maximum likelihood factor model can be used to estimate the parameters.[35]

The spectral density matrix must be carefully prepared for factor analysis, because the distribution theory of the method assumes that the spectral estimates are un-

33. In the presentation of the spectrum given (Figure 2), variables are filtered through a regression with a trend term to insure that trend is removed and that their means will be zero. For performing factor analysis, we use more elaborate prewhitening procedures.

34. See Gottman (1981, 210 and 216-22) for discussion of inconsistency of the periodogram and a description of methods for estimating a consistent spectral density.

35. The following application is based closely on Geweke's (1977) interpretation of the frequency domain factor model.

correlated across all intervals.[36] The data are prewhitened, and the analysis uses the resulting white residual from the autoregressive filter. The spectral-density matrices for a small number of ordinates are averaged over several frequency bands, and prewhitening insures that those averaged spectral densities will be approximately constant across these bands. This thorough data preparation is required because the distribution theory is asymptotic, and averaging several frequency bands makes sense given the number of parameters that must eventually be estimated.

The number of subintervals must be small due to the small number of cases, and the frequency bands should be reasonably wide and must have equal width. These criteria led us to select four distinct subintervals for estimation, with approximate boundaries of 2.5 years, 4 years, and 8 years in cycle periods.[37] The value of 4 years is the most problematic because the cycle corresponding to presidential elections divides the two middle subintervals. However, as discussed below, 6-year cycles appear more important in the U.S. data, and thus we are less concerned with this boundary than we would be if a 4-year cycle exhibited stronger power in these variables.

Under the condition that both of the following components are uncorrelated, the autocovariance matrix can be partitioned in the following way:

$$C_y(s) = \sum_{s=0}^{\infty} a(s)a(s-t)' + \sum_{s=0}^{\infty} b(s)b(s-t)', \tag{A4}$$

where a and b are coefficient vectors. In this way the autocovariance matrix at each time lag can be written in terms of systematic and unsystematic components, with the first summation interpreted as systematic. Similarly, the spectral density matrix can be partitioned into systematic (factor) and error components, each based on the Fourier transform of the corresponding component of equation (A4):

$$S_y = \alpha(\omega)\alpha(\omega)' + \beta(\omega)\beta(\omega)', \quad |\omega| \leq \pi, \tag{A5}$$

where

$$\alpha(\omega)\alpha(\omega)' = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} a(s)a(s-r)'e^{-ir\omega}, \tag{A6}$$

and

$$\beta(\omega)\beta(\omega)' = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} b(s)b(s-r)'e^{-ir\omega}. \tag{A7}$$

36. The fact that we have a small number of data points makes the preparation of the spectral density matrices most important. Indeed, given the small number of time points, we will not take the distribution theory very seriously and will instead focus on the variance explained by the factor as the primary indicator of goodness-of-fit.

37. The midpoints of the frequency bands are .125π, .735π, .625π, and .875π. The cycle periods corresponding to the four frequency bands are as follows: Band 1, 8 or more years; Band 2, 4 to 8 years; Band 3, 2.67 to 4 years; Band 4, less than 2.67 years.

All variation in Y(t) thus can be decomposed into variation across frequencies.

Finally, the connection between the frequency domain factor model and the cross-sectional factor model can be summarized by writing $S_y(\omega) = \Sigma$, $\alpha(\omega) = \Lambda$, and $\beta(\omega)\beta(\omega)' = \Psi$. Then we can represent our model in the same form as Jöreskog's (1967) maximum likelihood factor model, that is,

$$\Sigma = \Lambda\Lambda' + \Psi. \tag{A8}$$

For each distinct subinterval, estimates of Λ give the spectral densities of the variable implied by the one factor model, and estimates of the diagonal elements of Ψ give that portion of the implied spectral densities that are not explained by the one factor model.

Programs that estimate the conventional maximum likelihood factor model cannot be used directly because Σ and Λ are complex. However, Geweke (1975) derives the relevant first order conditions for the complex case, and Jöreskog's (1967) estimation method can be used with some modifications.[38]

REFERENCES

Allen, R.G.D. 1959. *Mathematical economics*, 2d ed. London: Macmillan.

Allison, Graham T. 1971. *Essence of decision: Explaining the Cuban missile crisis*. Boston: Little, Brown.

Anderson, T. W. 1963. The use of factor analysis in the statistical analysis of multiple time series. *Psychometrika* 28:1-25.

Arrow, Kenneth J. 1963. *Social choice and individual values*, 2d ed. New Haven, CT: Yale University Press.

Azar, Edward. 1980. The conflict and peace data bank (COPDAB) project. *Journal of Conflict Resolution* 24:143-52.

Azar, Edward, and Thomas Havener. 1976. Discontinuities in the symbolic environment: A problem in scaling events. *International Interactions* 2:231-46.

Beck, Nathaniel. 1989. Estimating dynamic models using Kalman filtering. In *Political analysis*, edited by James Stimson, vol. 1. Ann Arbor: University of Michigan Press.

Bollen, Kenneth A. 1989. *Structural equations with latent variables*. New York: Wiley.

Diehl, Paul F. 1985. Contiguity and military escalation in major power rivalries, 1816-1980. *Journal of Politics* 47:1203-11.

Freeman, John R. 1990. Systematic sampling, temporal aggregation, and the study of political relationships. In *Political analysis*, edited by James Stimson, vol. 1. Ann Arbor: University of Michigan Press.

Freeman, John R., John T. Williams, and Tse-min Lin. 1989. Vector autoregression and the study of politics. *American Journal of Political Science* 33:842-77.

Gaddis, John Lewis. 1986. The long peace: Elements of stability in the postwar international system. *International Security* 10:99-142.

Geweke, John. 1975. Employment turnover and wage dynamics in U.S. manufacturing. Ph.D. diss., University of Minnesota.

38. The resulting modification is straightforward. Technical details about the estimation method can be obtained from the first listed author.

———. 1977. The dynamic factor analysis of economic time-series models. In *Latent variables in socio-economic models*, edited by D. J. Aigner and Arthur S. Goldberger. Amsterdam: North-Holland.

Goldstein, Joshua S., and John R. Freeman. 1990. *Three-way street: Strategic reciprocity in world politics*. Chicago: University of Chicago Press.

———. 1991. U.S.-Soviet-Chinese relations: Routine, reciprocity, or rational expectations?. *American Political Science Review*, 85:17-35.

Gottman, John M. 1981. *Time series analysis*. New York: Cambridge University Press.

Granger C.W.J., and Paul Newbold. 1977. *Forecasting economic time series*. New York: Academic Press.

Harvey, Andrew C. 1989. *Forecasting, structural time series modeles and the Kalman Filter*. Cambridge: Cambridge University Press.

Isard, Walter, and Charles H. Anderton. 1985. Arms race models: A survey and synthesis. *Conflict Management and Peace Science* 8:27-122.

Jones, David L. 1988. *The dynamics of contemporary international conflict*. Ph.D. diss., Indiana University, Bloomington.

Jöreskog, Karl G. 1967. Some contributions to factor analysis. *Psychometrika* 32:443-82.

King, Gary. 1989a. *Unifying political methodology: The likelihood theory of statistical inference*. Cambridge: Cambridge University Press.

———. 1989b. Event count models for international relations: Generalizations and applications. *Internaional Studies Quarterly* 33:123-47.

Majeski, Stephen J. 1983a. Dynamic properties of the U.S. military expenditure decision-making process. *Conflict Management and Peace Science* 7:65-86.

———. 1983b. Mathematical models of the U.S. military expenditure decision-making process. *American Journal of Political Science* 27:485-514.

Marra, Robin F. 1985. A cybernetic model of the U.S. defense expenditure policy-making process. *International Studies Quarterly* 29:357-84.

McGinnis, Michael D. 1991. Richardson, rationality, and restrictive models of arms races. *Journal of Conflict Resolution* 35:443-73.

McGinnis, Michael D., and John T. Williams. 1989. Change and stability in superpower rivalry. *American Political Science Review* 83:1101-23.

———. 1991. Correlated equilibria in domestic and international policy games, unpublished paper, Indiana University.

Meyer, Stephen M. 1984. Soviet national security decisionmaking: What do we know and what do we understand? In *Soviet decisionmaking for national security*, edited by Jiri Valenta and William Potter, 255-97. London: Allen & Unwin.

Moll, Kendall D., and Gregory M. Luebbert. 1980. Arms race and military expenditure models: A review. *Journal of Conflict Resolution* 24:153-85.

Most, Benjamin A., and Harvey Starr. 1984. International relations theory, foreign policy substitutability, and "nice laws". *World Politics* 36:383-406.

———. 1989. *Inquiry, logic and international politics*. Columbia: University of South Carolina Press.

Ordeshook, Peter C. 1986. *Game theory and political theory*. Cambridge: Cambridge University Press.

Ostrom, Charles W. Jr. 1978. A reactive linkage model of the U.S. defense expenditure policymaking process. *American Political Science Review* 72:941-57.

Ostrom, Charles W., Jr., and Robin F. Marra. 1986. U.S. defense spending and the Soviet estimate. *American Political Science Review* 80:819-42.

Rajmaira, Sheen, and Michael D. Ward. 1990. Evolving foreign policy norms: Reciprocity in the superpower triad. *International Studies Quarterly* 34:457-75.

Rattinger, Hans. 1975. Armaments, detente, and bureaucracy: The arms race in Europe. *Journal of Conflict Resolution* 19:571-95.

Richardson, Lewis Fry. 1960. *Arms and insecurity: A mathematical study of the causes and origins of war*. Pittsburgh: Boxwood Press.

Riker, William H. 1982. *Liberalism against populism*. San Francisco: Freeman.

Rummel, Rudolph J. 1972. *Dimensions of nations*. Beverly Hills: Sage.

Russett, Bruce M. 1983. International interactions and processes: The internal vs. external debate revisited. In *Political science: The state of the discipline*, edited by Ada W. Finifter, 541-68. Washington, DC: American Political Science Association.

Sargent, Thomas, and Christopher Sims. 1977. Business cycle modeling without pretending to have too much a priori economic theory. In *New methods of business cycle research: Proceedings from a conference*, edited by C. Sims, 45-109. Minneapolis: Federal Reserve Bank of Minneapolis.

Sims, Christopher A. 1980. Macroeconomics and reality. *Econometrica* 48:1-48.

Siverson, R. M., and P. F. Diehl. 1989. Arms races, the conflict spiral, and the onset of war. In *Handbook of war studies*, edited by Manus I. Midlarsky, 195-218. Winchester, MA: Unwin Hyman.

Smith, Theresa C. 1980. Arms race instability and war. *Journal of Conflict Resolution* 24:253-84.

Stockholm International Peace Research Institute (SIPRI). Various years. *World armaments and disarmament*. London: Taylor and Francis, Oxford University Press.

Ward, Michael D. 1984. Differential paths to parity: A study of the contemporary arms race. *American Political Science Review* 78:297-317.

Watson, Mark W., and Robert F. Engle. 1983. Alternative algorithms for the estimation of dynamic factor, mimic and varying coefficient regression models. *Journal of Econometrics* 23:385-400.

Williams, John T., and Michael D. McGinnis. 1988. Sophisticated reaction in the U.S.-Soviet arms race: Evidence of rational expectations. *American Journal of Political Science* 32: 968-95.

———. Forthcoming. Expectations and the dynamics of U.S. defense budgets. In *The political economy of military spending in the U.S.*, edited by Alex Mintz. Harper Collins.

Zinnes, Dina A. 1980. Three puzzles in search of a researcher. *International Studies Quarterly* 24:315-42.