# Portfolio Optimisation and Risk Management: A Case Study

## Introduction:

In this analysis, we delve into the formation of an optimal investment portfolio using data on four firms traded on the LSE – Shell, Sainsbury, National Grid, and Easy Jet – along with the FTSE100 Index. The period under consideration spans from 01-09-2019 to 01-11-2023, with a constant risk-free rate of return assumed at 0.04% per month. The essay aims to provide a step-by-step exploration of portfolio optimisation, addressing tasks such as expected returns, risk assessment, and the impact of different risk attitudes. Key assumptions guiding this exploration include negligible transaction costs, a linear relationship between assets and reliable historical data.

## Portfolio Formation:

To commence, logged monthly returns for each stock were calculated. The correlation matrix (Table 1) was then generated to identify pairs with the best diversification opportunities. Shell and Sainsbury emerged as the optimal combination as they have the lowest positive correlation (0.1751), demonstrating their potential for effective diversification.

| | SHEL | SBRY | NG | EZJ | FTSE100 |
|---|---|---|---|---|---|
| **SHEL** | 1 | 0.1751107 | -0.0394717 | 0.5147852 | 0.6475576 |
| **SBRY** | 0.17511073 | 1 | 0.49190371 | 0.3132149 | 0.3536933 |
| **NG** | -0.0394717 | 0.4919037 | 1 | 0.212033 | 0.3270044 |
| **EZJ** | 0.5147852 | 0.3132149 | 0.212033 | 1 | 0.7706061 |
| **FTSE100** | 0.6475576 | 0.3536933 | 0.32700444 | 0.7706061 | 1 |

*Table 1: Correlation Matrix*

| | SHEL | | | SBRY |
|---|---|---|---|---|
| Expected Return | 0.51% | | | 0.77% |
| Standard Deviation | 8.23% | | | 7.63% |
| Correlation Coefficient | | 0.18 | | |
| Covariance | | | 0.11% | |

*Table 2: Descriptive Statistics for the Assets*

## Global Minimum-Variance Portfolio:

Moving forward, Excel's Solver tool was used to determine the weights that minimise the portfolio variance, adhering to the optimisation problem outlined in Equation 1. The resulting weights were calculated as 45.46% for Shell and 54.54% for Sainsbury.

$$Min\ Portfolio\ Variance\ (\sigma^2)$$
$$= \omega_{SHEL}^2 \cdot \sigma_{SHEL}^2 + \omega_{SBRY}^2 \cdot \sigma_{SBRY}^2 + 2 \cdot \omega_{SHEL} \cdot \omega_{SBRY} \cdot Cov_{SHEL,SBRY}$$
$$Subject\ to\ \omega_{SHEL} + \omega_{SBRY} = 1\,.$$

*Equation 1: Global Minimum Variance Optimisation Problem*

Equipped with these weights, we then computed the expected return and standard deviation of the Global Minimum-Variance Portfolio (GMVP) using Equations 2 and 3.

$$E(R) = \omega_{SHEL} \cdot E(R_{SHEL}) + \omega_{SBRY} \cdot E(R_{SBRY})$$

*Equation 2: Portfolio Expected Return*

$$\sigma = \sqrt{\sigma^2},$$

*Equation 3: Portfolio Risk (Standard Deviation)*

The calculated values for the GMVP were a standard deviation, $\sigma_{GMVP}$, of 6.06% and an expected return, $E(R_{GMVP})$, of 0.65%. These metrics provide a foundational understanding of the risk-return profile of the chosen portfolio.

## Investment Opportunity Set and Capital Allocation Line (CAL):

Next, we explored the Investment Opportunity Set by considering the different investment proportions for the two assets, in increments of 10%, ensuring the sum of the weights was 1. For each combination of investment opportunities expected return and standard deviation are calculated using Equations 2 and 3, respectively.

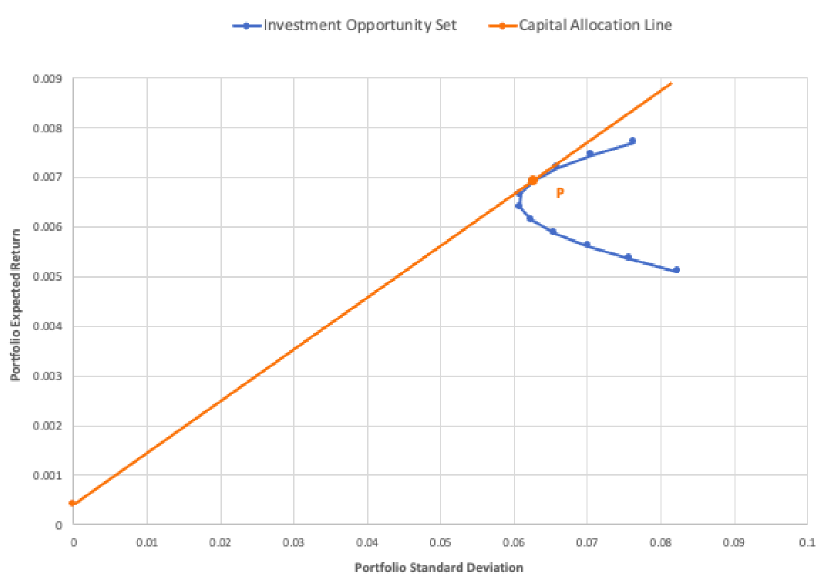


*Figure 2: Investment Opportunity Set*

Assuming a risk-free rate of 0.04%, the Capital Allocation Line (CAL) is introduced. The CAL is tangential to the Investment Opportunity Set at a point denoted as P. Point P represents the optimal portfolio on the efficient frontier, achieving the maximum Sharpe ratio, indicating the best risk-return trade-off.

## Sharpe Ratio and Optimal Portfolio Weights:

The weights of assets SHEL and SBRY in the portfolio at point P represent the optimal allocation for maximising the investor's utility.

$$Max\ Sharpe\ Ratio\ (S) = \frac{\omega_{SHEL} \cdot E(R_{SHEL}) + \omega_{SBRY} \cdot E(R_{SBRY}) - R_f}{\sqrt{\omega_{SHEL}^2 \cdot \sigma_{SHEL}^2 + \omega_{SBRY}^2 \cdot \sigma_{SBRY}^2 + 2 \cdot \omega_{SHEL} \cdot \omega_{SBRY} \cdot Cov_{SHEL,SBRY}}}$$

$$Subject\ to\ \omega_{SHEL} + \omega_{SBRY} = 1$$

*Equation 4: Maximised Sharpe Ratio*

Excel's Solver tool was used to find the weights that maximise S (Equation 4), resulting in 30.51% for Shell and 69.49% for Sainsbury. These weights define the composition of the portfolio at point P on the CAL.

Using Equations 2 and 3 the expected return and standard deviation at point P can be calculated, returning $\sigma_p = 6.25\%$ and $E(R_p) = 0.69\%$.

## Complete Portfolio (C):

The complete portfolio (C) is a combination of the optimal risky portfolio, at point P, and the risk-free asset. The composition of the complete portfolio (C) can be found by considering the investor's risk aversion parameter (A) and is given by Equation 5.

$$\omega_C = \frac{E(R_p) - R_f}{A \cdot \sigma_p^2}$$

*Equation 5: Complete Portfolio Composition*

$$E(R_C) = \omega_C \cdot E(R_P) + (1 - \omega_C) \cdot R_f$$

*Equation 6: Complete Portfolio Expected Return*

$$\sigma_C = \omega_C \cdot \sigma_p$$

*Equation 7: Complete Portfolio Standard Deviation*

Assuming a risk aversion parameter of A = 3, the proportion allocated to the risky portfolio, $\omega_C$, was found to be 55.69%. This reflects both the investor's risk aversion and the trade-off between risk and return. The expected return and standard deviation of C can be calculated using Equations 6 and 7, respectively, returning $E(R_C) = 0.41\%$ and $\sigma_C = 3.48\%$.

While the complete portfolio (C) expects slightly lower returns compared to investing solely in Shell or Sainsbury, it significantly reduces the overall risk associated with individual stocks.

## Investor Risk Attitudes:

An investor's risk attitude plays a pivotal role in shaping portfolio composition, influencing both expected return and standard deviation. A risk loving investor, might increase their allocation to the risky portfolio in C, leading to a higher expected return. However, the standard deviation of the complete portfolio will also increase. Similarly, a risk neutral investor is indifferent to risk. A risk neutral investor may allocate based on maximising expected returns. By considering only expected returns, the expected return and standard deviation of C will increase. Alternatively, an investor more risk-averse than the assumed A = 3, might decrease their allocation to the risky portfolio. This shift toward the risk-free asset leads to a lower expected return and standard deviation.

### FTSE 100 vs. Two-Asset Portfolio:

Considering the provided data on the FTSE 100, which indicates a negative expected return of -0.01%, investing in the FTSE 100 seems less appealing compared to the two-asset portfolio. The negative return signals a potential loss, whereas the two-asset portfolio, even with its slightly lower expected returns compared to individual stocks, offers a more favourable risk-return trade-off.