ACTL30007 - Omar Amin - Assignment 1

Omar

2025-06-14

R Markdown

```
library(readxl)
library(tidyr)
library(dplyr)
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##
       filter, lag
## The following objects are masked from 'package:base':
##
       intersect, setdiff, setequal, union
##
df1 = read_excel("1463797.xlsx", sheet=1)
df2 = read_excel("1463797.xlsx", sheet=2)
df3 = read_excel("1463797.xlsx", sheet=3)
# Filter strictly positive claims
sev1 <- df1$claim_size[df1$claim_size > 0]
sev2 <- df2$claim size[df2$claim size > 0]
sev3 <- df3$claim_size[df3$claim_size > 0]
data <- list(df1, df2, df3)</pre>
```

Including Plots

```
You can also embed plots, for example:
```

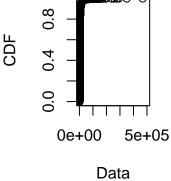
```
## Loading required package: MASS
##
## Attaching package: 'MASS'
```

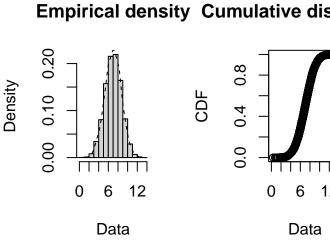
- ## The following object is masked from 'package:dplyr': ##
- ## select
- ## Loading required package: survival
- ## Portfolio 1: 68.641%
- ## Portfolio 2: 90.432%
- ## Portfolio 3: 84.787%

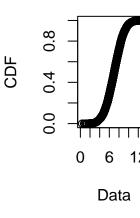
Empirical density Cumulative distribut

Density 0.0e+000e+00 5e+05

Data

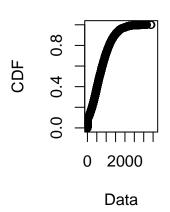


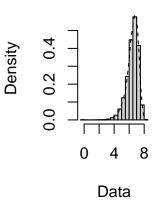


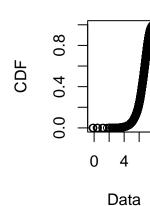


Empirical density Cumulative distribut

6e-04 Density 0 2000 Data





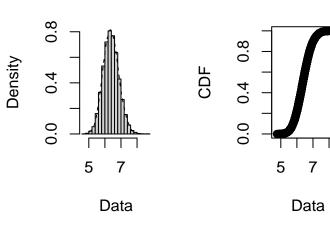


Empirical density Cumulative dis

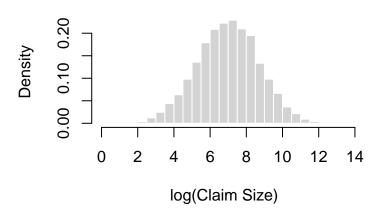
Empirical density Cumulative distribut

Density O 4000 O 4000 O 4000 Data Data

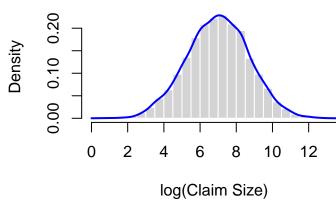
Empirical density Cumulative dis



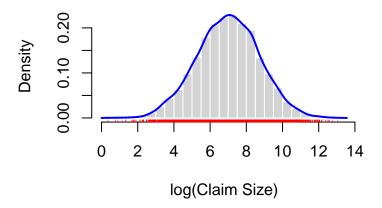
Portfolio 1: Log-Claim Sizes



Portfolio 1: Log-Claim Sizes



Portfolio 1: Log-Claim Sizes



Note that the echo = FALSE parameter was added to the code chunk to prevent printing of the R code that generated the plot.

```
summarize_portfolio <- function(df) {
  count_data <- df %>%
    group_by(id) %>%
    summarise(n_claims = ifelse(length(claim_size) == 1 & all(claim_size == 0), 0L, n()))

zero_prop <- mean(count_data$n_claims == 0)
list(
    zero_prop = zero_prop,
    mean = mean(count_data$n_claims),
    var = var(count_data$n_claims),
    data = count_data
)
}

stats1 <- summarize_portfolio(df1)
stats2 <- summarize_portfolio(df2)
stats3 <- summarize_portfolio(df3)
stats1; stats2; stats3</pre>
```

```
## $zero_prop
## [1] 0.413
##
## $mean
## [1] 0.904
##
## $var
## [1] 0.9426783
##
## $data
## # A tibble: 10,000 x 2
## id n_claims
## <dbl> <int>
```

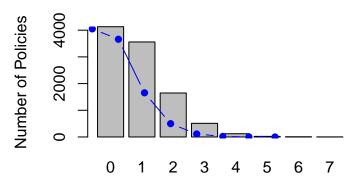
```
## 1
        1
## 2
         2
                  2
## 3
         3
                  2
## 4
         4
                  0
## 5
         5
                  0
## 6
         6
                  1
## 7
         7
                  0
## 8
         8
                  1
## 9
         9
                  0
        10
## 10
                  0
## # i 9,990 more rows
## $zero_prop
## [1] 0.1692
##
## $mean
## [1] 1.5992
##
## $var
## [1] 1.299889
##
## $data
## # A tibble: 10,000 x 2
##
        id n_claims
##
     <dbl>
              <int>
## 1
         1
## 2
         2
                  2
## 3
         3
## 4
         4
                  1
## 5
         5
                  3
## 6
         6
                  3
                  2
## 7
         7
## 8
                  1
         8
## 9
         9
                  2
## 10
        10
                  2
## # i 9,990 more rows
## $zero_prop
## [1] 0.2923
##
## $mean
## [1] 1.6291
##
## $var
## [1] 2.179151
##
## $data
## # A tibble: 10,000 x 2
        id n_claims
##
              <int>
##
      <dbl>
## 1
         1
## 2
                  0
         2
## 3
         3
                  4
## 4
         4
                  3
```

```
## 5
          5
                   0
##
   6
          6
##
   7
          7
                   0
##
  8
          8
                   1
##
   9
          9
                   0
                   0
## 10
         10
## # i 9,990 more rows
cat("Index of Dispersions P1, P2, P3: ", c(stats1$var/stats1$mean, stats2$var/stats2$mean, stats3$var/s
## Index of Dispersions P1, P2, P3: 1.042786 0.8128373 1.337641
# Fit Poisson and (if appropriate) Negative Binomial for claim counts
library(MASS)
df1_counts <- df1 %>% group_by(id) %>% summarise(count = if_else(n()==1 & any(claim_size==0), 0L, as.in
df2_counts <- df2 %>% group_by(id) %>% summarise(count = if_else(n()==1 & any(claim_size==0), OL, as.in
df3_counts <- df3 %>% group_by(id) %>% summarise(count = if_else(n()==1 & any(claim_size==0), OL, as.in
# Frequency models (intercept-only, since no covariates)
model1_pois <- glm(count ~ 1, data = df1_counts, family = "poisson")</pre>
model2_pois <- glm(count ~ 1, data = df2_counts, family = "poisson")</pre>
model3_pois <- glm(count ~ 1, data = df3_counts, family = "poisson")</pre>
# Negative Binomial fit for portfolios showing overdispersion
           <- glm.nb(count ~ 1, data = df3_counts)</pre>
                                                           # (only portfolio3 needs NB significantly)
coef(model3_nb); model3_nb$theta
                                                           # view NB parameters (intercept log-mean, the
## (Intercept)
     0.4880277
## [1] 3.862073
library(fitdistrplus)
library(actuar)
                 # for Pareto distribution functions
## Attaching package: 'actuar'
## The following objects are masked from 'package:stats':
##
##
       sd, var
## The following object is masked from 'package:grDevices':
##
##
       cm
sev1 <- df1$claim_size[df1$claim_size > 0]
                                              # Portfolio1 claim sizes > 0
sev2 <- df2$claim_size[df2$claim_size > 0]
sev3 <- df3$claim_size[df3$claim_size > 0]
# Fit parametric severity distributions for Portfolio1 (heavy-tailed)
```

```
fit1_gamma <- fitdist(sev1, "gamma", start=list(shape=1, scale=mean(sev1)))</pre>
                                                                                 # Gamma
fit1_lnorm <- fitdist(sev1, "lnorm")</pre>
                                                                                  # Lognormal
fit1_pareto<- fitdist(sev1, "pareto", start=list(shape=2, scale=500))</pre>
                                                                                  # Pareto
gofstat(list(fit1_gamma, fit1_lnorm, fit1_pareto), fitnames = c("Gamma", "Lognormal", "Pareto"))$aic
       Gamma Lognormal
                           Pareto
## 164796.8 161740.5 161931.7
fit1_gamma$estimate; fit1_lnorm$estimate; fit1_pareto$estimate
                                                                                  # view parameter estimates
##
          shape
                        scale
      0.4543333 9873.8802195
##
## meanlog
              sdlog
## 6.989846 1.710560
##
         shape
                      scale
##
      1.096714 1257.609778
# ----- Frequency models for Portfolios 1 and 2 -----
# Prepare claim counts per policy
counts_df <- function(df) {</pre>
 df %>%
    group by(id) %>%
    summarise(n_claims = ifelse(length(claim_size) == 1 & all(claim_size == 0), 0L, n()))
}
counts1 <- counts_df(df1)</pre>
counts2 <- counts_df(df2)</pre>
# Portfolio 1
fit_pois1 <- glm(n_claims ~ 1, family = poisson(), data = counts1)</pre>
lambda1 <- exp(coef(fit_pois1))</pre>
expected1 <- dpois(0:max(counts1$n_claims), lambda1) * nrow(counts1)</pre>
obs1 <- table(counts1$n_claims)</pre>
# Portfolio 2
fit_pois2 <- glm(n_claims ~ 1, family = poisson(), data = counts2)</pre>
lambda2 <- exp(coef(fit_pois2))</pre>
expected2 <- dpois(0:max(counts2$n_claims), lambda2) * nrow(counts2)</pre>
obs2 <- table(counts2$n_claims)</pre>
# Plot for Portfolio 1
barplot(obs1, col = "gray", names.arg = names(obs1),
        main = "Portfolio 1 Claim Count Fit (Poisson)",
        xlab = "Number of Claims", ylab = "Number of Policies")
```

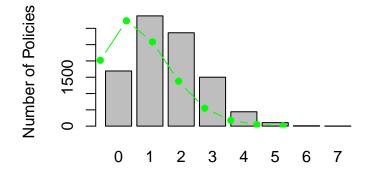
lines(0:max(counts1\$n_claims), expected1, col = "blue", type = "b", pch = 16)

Portfolio 1 Claim Count Fit (Poisson)



Number of Claims

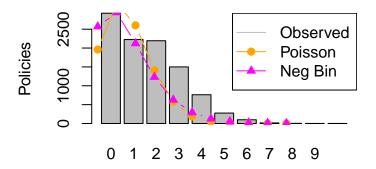
Portfolio 2 Claim Count Fit (Poisson)



Number of Claims

```
# ------- Frequency models for Portfolio 3 ------
# Fit Poisson and Negative Binomial
fit_pois3 <- glm(n_claims ~ 1, family = poisson(), data = stats3$data)
fit_nb3 <- glm.nb(n_claims ~ 1, data = stats3$data)
# Visualize model fit
obs_counts <- table(stats3$data$n_claims)
lambda3 <- exp(coef(fit_pois3))</pre>
```

Portfolio 3 Claim Count

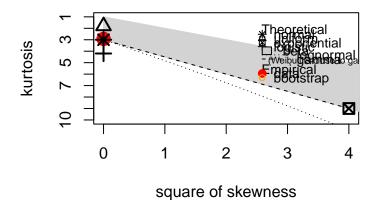


Number of Claims

```
library(fitdistrplus)

# Portfolio 1 - log-transformed claim sizes
sev1 <- df1$claim_size[df1$claim_size > 0]
descdist(log(sev1), boot = 1000)
```

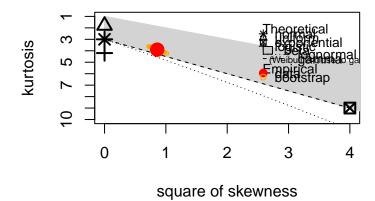
Cullen and Frey graph



```
## summary statistics
## -----
## min: 0.3577616 max: 13.05511
## median: 7.012698
## mean: 6.989846
## estimated sd: 1.710654
## estimated skewness: -0.02848522
## estimated kurtosis: 2.923099
```

sev2 <- df2\$claim_size[df2\$claim_size > 0]
descdist(sev2, boot = 1000)

Cullen and Frey graph

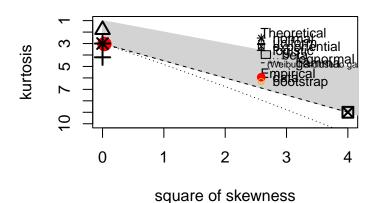


```
## summary statistics
## -----
## min: 0.8236877 max: 3401.825
```

```
## median: 717.4712
## mean: 802.7218
## estimated sd: 506.1836
## estimated skewness: 0.9267448
## estimated kurtosis: 3.901759

sev3 <- df3$claim_size[df3$claim_size > 0]
descdist(log(sev3), boot = 1000)
```

Cullen and Frey graph



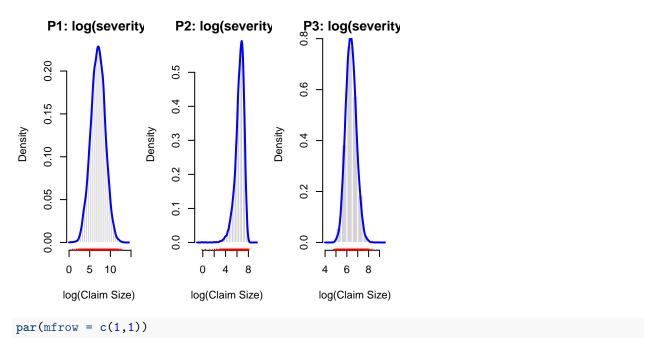
summary statistics

```
## min: 4.75679
                   max: 8.772254
## median: 6.368575
## mean: 6.382347
## estimated sd: 0.4890409
## estimated skewness: 0.173933
## estimated kurtosis: 3.042674
#--- 1. Prepare log-severity vectors
logs1 <- log(df1$claim_size[df1$claim_size > 0])
logs2 <- log(df2$claim_size[df2$claim_size > 0])
logs3 <- log(df3$claim_size[df3$claim_size > 0])
#--- 2. A plotting function
plot_log_severity <- function(logs, title) {</pre>
  x_min <- floor(min(logs))</pre>
  x_max <- ceiling(max(logs)) + 0.5</pre>
  hist(logs,
       breaks = seq(x_min, x_max, by = 0.5),
       freq
            = FALSE,
       xlim
              = c(x_min, x_max),
              = "lightgray",
       col
       border = "white",
```

```
main = title,
    xlab = "log(Claim Size)"
)

dens <- density(logs, from = x_min, to = x_max)
lines(dens, col="blue", lwd=2)
  rug(logs, col="red", ticksize=0.02)
}

#--- 3. Plot side by side
par(mfrow = c(1,3), mar = c(4,4,2,1))
plot_log_severity(logs1, "P1: log(severity)")
plot_log_severity(logs2, "P2: log(severity)")
plot_log_severity(logs3, "P3: log(severity)")</pre>
```



Question 1: Summarize the recommended modelling parameters for each portfolio of risk.

```
# Load libraries & data
library(readxl); library(dplyr); library(fitdistrplus); library(MASS)

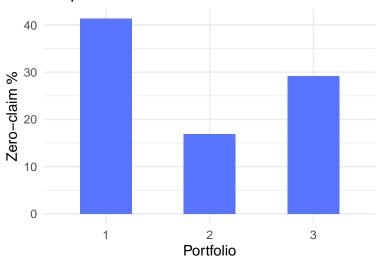
df_list <- lapply(1:3, function(i) read_excel("1463797.xlsx", sheet = i))

# Summarize frequency & choose model
freq_summary <- lapply(df_list, function(df) {
   counts <- df %>% group_by(id) %>% summarise(
        zero = all(claim_size==0),
        claims = ifelse(zero,OL,n())
   )
   m <- mean(counts$claims); v <- var(counts$claims)
   zero_pct <- 100*mean(counts$claims==0)</pre>
```

```
if (v/m > 1.05) {
    size <- m^2/(v-m); model <- "Negative Binomial"; params <- c(mu=m, size=size)</pre>
    model <- "Poisson"; params <- c(lambda=m)</pre>
 list(zero_pct=round(zero_pct,1), model=model, params=round(params,3))
})
# Summarize severity & choose model
sev_summary <- lapply(df_list, function(df) {</pre>
  sev <- df$claim_size[df$claim_size > 0]
  # fit both
  gfit <- fitdist(sev, "gamma", start=list(shape=1, scale=mean(sev)))</pre>
  lfit <- fitdist(sev, "lnorm")</pre>
  # pull out AIC
  aic_g <- gofstat(gfit)$aic</pre>
  aic_l <- gofstat(lfit)$aic</pre>
  if (aic_l < aic_g) {</pre>
    model <- "Lognormal"</pre>
    params <- lfit$estimate[c("meanlog","sdlog")]</pre>
  } else {
    model <- "Gamma"
    params <- gfit$estimate[c("shape", "scale")]</pre>
 list(model = model, params = round(params,3))
})
# Build and print summary table
summary_tbl <- tibble::tibble(</pre>
 Portfolio
            = 1:3,
  ZeroClaimPct = sapply(freq_summary, `[[`, "zero_pct"),
 FreqModel = sapply(freq_summary, `[[`, "model"),
  FreqParams = sapply(freq_summary, function(x) paste(names(x*params),"=",x*params,collapse=", ")),
 SevModel
             = sapply(sev_summary, `[[`, "model"),
  SevParams = sapply(sev_summary, function(x) paste(names(x*params),"=",x*params,collapse=", "))
print(summary_tbl)
## # A tibble: 3 x 6
   Portfolio ZeroClaimPct FreqModel
                                                FreqParams
                                                                    SevModel SevParams
         <int>
                      <dbl> <chr>
                                                                             <chr>
##
                                                <chr>
                                                                    <chr>
## 1
             1
                       41.3 Poisson
                                                lambda = 0.904
                                                                    Lognorm~ meanlog ~
## 2
             2
                       16.9 Poisson
                                                lambda = 1.599
                                                                    Gamma
                                                                             shape = ~
## 3
             3
                       29.2 Negative Binomial mu = 1.629, size ~ Lognorm~ meanlog ~
# Barplot of zero-claim %
library(ggplot2)
ggplot(summary_tbl, aes(x = factor(Portfolio), y = ZeroClaimPct)) +
```

```
geom_col(width = 0.5, fill = "#5874fc") +
labs(
    x = "Portfolio",
    y = "Zero-claim %",
    title = "Proportion of Policies with No Claims"
) +
theme_minimal()
```

Proportion of Policies with No Claims

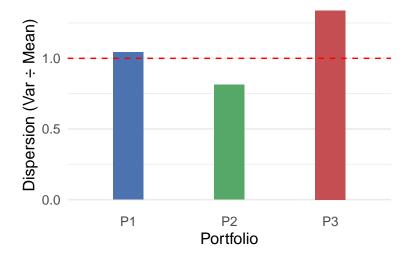


Question 2

Part 1: Dispersion of the policies

```
library(dplyr)
library(ggplot2)
get_disp <- function(df, name) {</pre>
  counts <- df %>% group_by(id) %>%
    summarise(claims = ifelse(all(claim_size==0), OL, n())) %>% ungroup()
  m <- mean(counts$claims); v <- var(counts$claims)</pre>
  data.frame(Portfolio = name, Dispersion = v/m)
}
disp1 <- get_disp(df1, "P1")</pre>
disp2 <- get_disp(df2, "P2")</pre>
disp3 <- get_disp(df3, "P3")</pre>
disp_df <- bind_rows(disp1, disp2, disp3)</pre>
ggplot(disp_df, aes(x = Portfolio, y = Dispersion, fill = Portfolio)) +
  geom_col(width = 0.3, show.legend = FALSE) +
  scale_x_discrete(
    expand = c(.3,0)
  ) +
```

Claim Count Dispersion by Portfolio



Part 2: Demonstrating heterogeneity in portfolio 3

```
library(dplyr)
library(ggplot2)
library(MASS)

# 1. Prepare the data
cts3 <- df3 %>%
    group_by(id) %>%
    summarise(claims = ifelse(all(claim_size == 0), OL, n())) %>%
    ungroup()

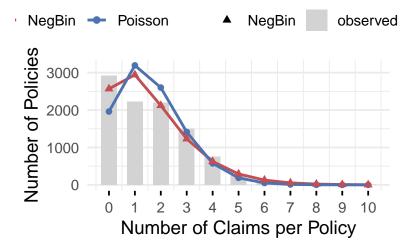
obs_df <- cts3 %>% count(claims) %>% rename(observed = n)

pois3 <- glm(claims ~ 1, family = poisson(), data = cts3)
nb3 <- glm.nb(claims ~ 1, data = cts3)</pre>
```

```
lam3 <- exp(coef(pois3))</pre>
    <- exp(coef(nb3)); th3 <- nb3$theta</pre>
maxc <- max(cts3$claims)</pre>
model_df <- tibble(</pre>
 claims = 0:maxc,
 observed = obs_df$observed,
 Poisson = dpois(claims, lam3) * nrow(cts3),
 NegBin = dnbinom(claims, size = th3, mu = mu3) * nrow(cts3)
# 2. Pivot longer for ggplot
plot df <- model df %>%
 pivot_longer(cols = c("observed", "Poisson", "NegBin"),
              names_to = "Model", values_to = "Count")
# 3. Plot
ggplot(plot_df, aes(x = claims, y = Count, fill = Model, color = Model, shape = Model)) +
 # observed as bars
 geom_col(data = filter(plot_df, Model == "observed"),
          aes(x = claims, y = Count, fill = Model),
          color = NA, width = 0.6) +
 # Poisson & NegBin as lines + points
 geom_line(data = filter(plot_df, Model != "observed"),
           aes(x = claims, y = Count, color = Model), size = 1) +
 geom_point(data = filter(plot_df, Model != "observed"),
            aes(x = claims, y = Count, color = Model, shape = Model), size = 2) +
 # manual scales
 scale_fill_manual(
  name = "",
   values = c("observed" = "lightgray", "Poisson" = NA, "NegBin" = NA)
 scale_color_manual(
  name = "",
   values = c("observed" = NA, "Poisson" = "#4C72B0", "NegBin" = "#C44E52")
 ) +
 scale_shape_manual(
  name = "",
   scale_x_continuous(breaks = 0:maxc) +
 labs(
         = "Number of Claims per Policy",
         = "Number of Policies",
   title = "Portfolio 3: Observed vs Poisson & NegBin Fits"
 theme_minimal(base_size = 14) +
 theme(
                 = element_text(face = "bold"),
   plot.title
   panel.grid.major.x = element_blank(),
   axis.ticks.x = element_line(),
                    = element_text(margin = margin(t = 5)),
   axis.text.x
   legend.position = "top"
```

Warning: Using 'size' aesthetic for lines was deprecated in ggplot2 3.4.0.
i Please use 'linewidth' instead.
This warning is displayed once every 8 hours.
Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
generated.

Portfolio 3: Observed vs Pois

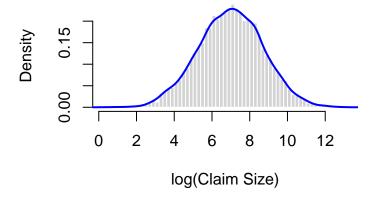


Question 3

```
library(readxl)
library(fitdistrplus)
library(actuar)
                   # for Pareto
library(extRemes) # for mrlplot
## Loading required package: Lmoments
## Loading required package: distillery
## Attaching package: 'extRemes'
## The following objects are masked from 'package:stats':
##
       qqnorm, qqplot
##
# 1. Data
      <- read_excel("1463797.xlsx", sheet="Sheet1")</pre>
sev1 <- df1$claim_size[df1$claim_size > 0]
n <- length(sev1)</pre>
```

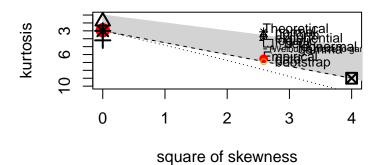
```
logs1 <- log(sev1)</pre>
# 2. Fit candidates (for Q-Q)
fit_gamma <- fitdist(sev1, "gamma", start=list(shape=1,scale=mean(sev1)))</pre>
fit_lnorm <- fitdist(sev1, "lnorm")</pre>
fit_pareto <- fitdist(sev1, "pareto", start=list(shape=2,scale=min(sev1)))</pre>
# Emprical density (logged data)
par(mar = c(5,4,4,2) + 0.1) # extra top margin for title
hist(logs1,
     breaks = 50,
     freq = FALSE,
           = "lightgray",
     col
     border = "white",
     xlab = "log(Claim Size)",
     ylab = "Density",
     main = "P1: Log Scale Histogram & Density"
)
lines(density(logs1), col = "blue", lwd = 2)
```

P1: Log Scale Histogram & Density

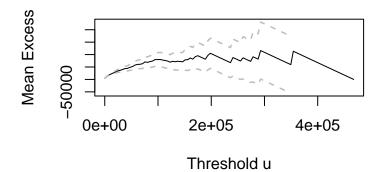


```
# 3a) Cullen-Frey on log-data
par(mar = c(5,4,6,2))  # bottom, left, top, right margins
descdist(logs1, boot=1000, discrete=FALSE)
```

Cullen and Frey graph

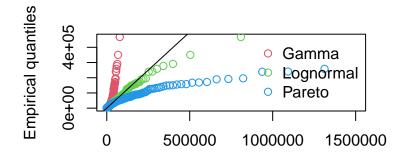


P1: Mean-Excess Plot



```
# 3c) Q-Q comparison
par(mar = c(5,4,6,2))
qqcomp(
  list(fit_gamma, fit_lnorm, fit_pareto),
  legendtext = c("Gamma", "Lognormal", "Pareto"),
  main = "P1: Q-Q Plot - Severity Models",
  xlab = "Theoretical quantiles",
  ylab = "Empirical quantiles",
  xlim = c(0, 1500000)
)
```

P1: Q-Q Plot - Severity Models



AIC

Lognormal 161740.5 161754.8

164796.8 164811.0

161931.7 161945.9

Gamma

Pareto

BIC

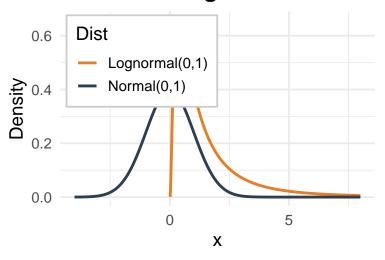
Theoretical quantiles

```
# 3d) AIC and BIC comparison
fits <- list(</pre>
  Gamma = fit_gamma,
  Lognormal = fit_lnorm,
  Pareto = fit_pareto
)
ic_tab <- t(sapply(fits, function(fit) {</pre>
  k <- length(fit$estimate)</pre>
  11
         <- fit$loglik
         <- 2*k - 2*11
  AIC
  BIC
         <- log(n)*k - 2*ll
  c(AIC = round(AIC,1), BIC = round(BIC,1))
print(ic_tab)
```

```
# Estimating paramters of log normal distribution
fit_lnorm <- fitdist(sev1, "lnorm")</pre>
meanlog <- fit lnorm$estimate["meanlog"]</pre>
sdlog <- fit_lnorm$estimate["sdlog"]</pre>
meanlog; sdlog
## meanlog
## 6.989846
##
   sdlog
## 1.71056
library(ggplot2)
# Parameters
mu <- 0
sigma <- 1
# Data frames
norm_df <- data.frame(</pre>
 x = seq(-4, 8, length.out = 1000),
 y = dnorm(seq(-4, 8, length.out = 1000), mean = mu, sd = sigma),
 Dist = "Normal(0,1)"
)
log_df <- data.frame(</pre>
 x = seq(0.01, 8, length.out = 1000),
 y = dlnorm(seq(0.01, 8, length.out = 1000), meanlog = mu, sdlog = sigma),
 Dist = "Lognormal(0,1)"
# Combine
plot_df <- rbind(norm_df, log_df)</pre>
ggplot(plot_df, aes(x = x, y = y, color = Dist)) +
  geom_line(size = 1) +
  scale_color_manual(
   values = c("Normal(0,1)" = "#2C3E50", "Lognormal(0,1)" = "#E67E22")
  ) +
  labs(
   title = "Normal vs Lognormal Densities",
   x = "x",
         = "Density"
  theme_minimal(base_size = 14) +
                 = element_text(face = "bold"),
    plot.title
   legend.position = c(0.02, 0.98), # near top-left
   legend.justification = c(0, 1),
                                            # left/top corner of legend at that point
   legend.background = element_rect(fill = "white", color = "gray80")
```

```
## Warning: A numeric 'legend.position' argument in 'theme()' was deprecated in ggplot2
## 3.5.0.
## i Please use the 'legend.position.inside' argument of 'theme()' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
## generated.
```

Normal vs Lognormal Densitie



Question 4

[1] 0.8308

```
tibble(
  Model = c("Binomial", "Poisson"),
  Mean = c(meanN, meanN),
  Variance = c(varN, meanN),
  Dispersion = c(varN/meanN, 1)
)
```

```
## # A tibble: 2 x 4
## Model Mean Variance Dispersion
## <chr> <dbl> <dbl> <dbl>
## 1 Binomial 8308 1406.
                              0.169
## 2 Poisson 8308 8308
# Binomial Dispersion = 0.41 vs Poisson Dispersion = 1.0
library(fitdistrplus)
cts3 <- df3 %>%
 group_by(id) %>%
 summarise(
   # count how many non-zero claim_size entries each policy has
   claims = sum(claim_size > 0)
 ) %>%
 ungroup()
fit_nb3b <- fitdist(cts3$claims, "nbinom")</pre>
fit_nb3b$estimate
      size
## 3.860821 1.629247
```