University of Missouri

Aerodynamics of a Soccer Ball

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1 INTRODUCTION

Studying the aerodynamics of a soccer ball is a fascinating and complex problem that incorporates principles of both fluid mechanics and physics into a game that has been enjoyed by billions of people around the globe. The most important aspect of how these concepts affect the soccer ball is how air flow around the ball affects its speed and direction. This effect is heavily influenced by factors such as the ball's shape, surface texture, and angular velocity. The combination of these properties influences fluid mechanic concepts such as drag (otherwise known as air resistance) and the Magnus effect. The Magnus effect is famously responsible for the curved trajectories seen in free kicks, displayed in Figure 1.

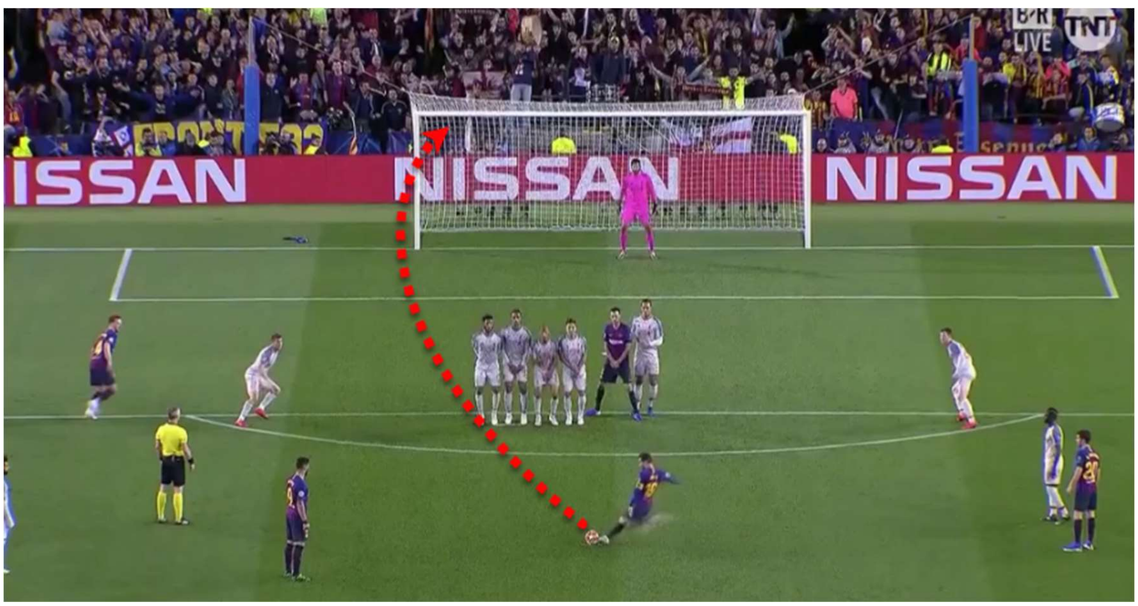


Figure 1. Display of a soccer free kick (Pimental, 2021)

Additionally, modern soccer ball designs, with their unique panel shapes and textures, add another layer of complexity to their aerodynamic properties. While an important factor in the real world for the scope of this paper, all soccer balls will be evaluated as smooth spheres. This paper will look at the effects of air density, spin rate, and the angle of spin on the flight path of a soccer ball through the lens of a simulation.

Players who master the ability to create unique ball flights by leveraging the aerodynamic principles will be able to stand out and move on to higher levels of competition. Understanding all the factors that affect a soccer ball's flight is crucial not only for players trying to improve their skills but also for the engineers and manufacturers creating the soccer balls. Nevertheless, the study of a soccer ball's aerodynamics offers an intriguing example of how fluid mechanic principles impact the world's most popular sport and will be studied through simulation in this paper.

2 METHODS

The aerodynamics of a soccer ball are analyzed in this paper through a simulation created in MATLAB using various fluid mechanic and physic principals. The first step is to determine the axis the ball moves on. For this project the x direction is considered the distance traveled, y is the horizontal variation, and z is the height of the ball. The next step is to realize all the forces that are impacting the ball's trajectory which are displayed in Figure 2. The forces that are evaluated are the initial velocity added to a ball $\vec{\boldsymbol{v}}$, the force from gravity $\vec{\boldsymbol{F}}_{\boldsymbol{g}}$, the drag force $\vec{\boldsymbol{F}}_{\boldsymbol{D}}$, and the lift force $\vec{\boldsymbol{F}}_{\boldsymbol{L}}$.

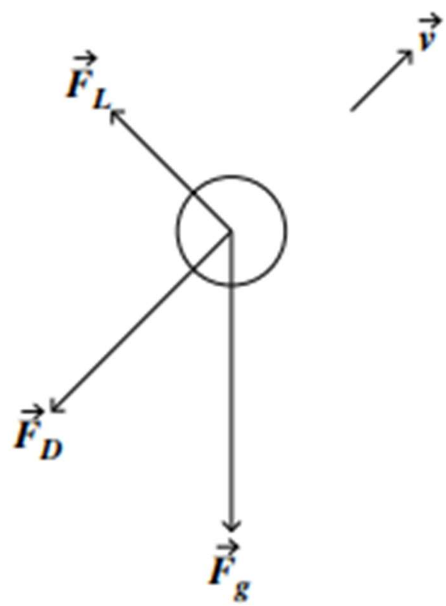


Figure 2. Forces acting on a soccer ball.

2.1 INITIAL VELOCITY

To compute the initial velocity a magnitude and direction are needed. Since this the flight of a soccer ball is a three-dimensional project two angles are needed to describe its path, the X-Y angle, and the Z angle shown in Figure 3. The X-Y angle describes its horizontal variation, and the Z angle describes the vertical direction. Then Equation 1 is used to determine the X, Y, and Z components of the initial velocity from the kick.

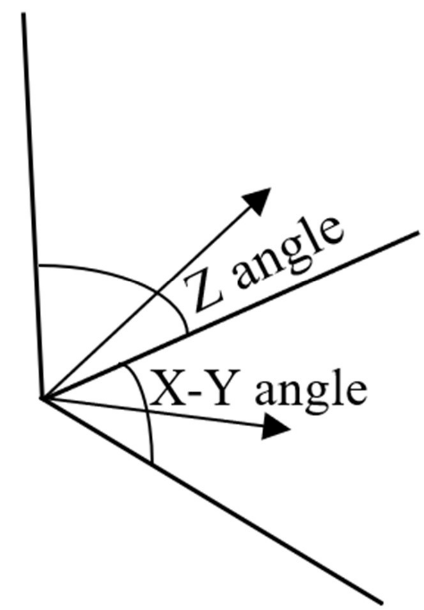


Figure 3. Showing angles of launch.

$$V_X = V_{magnitude} * cos(Z) * cos(X - Y)$$
$$V_y = V_{magnitude} * cos(Z) * sin(X - Y)$$
$$V_z = V_{magniutde} * sin\ (Z)$$

Equation 1. Initial velocity components.

2.2 Force from Gravity

The force from gravity is a constant force acting in the -Z direction. Since the velocity of the ball is needed to compute the position, the force of gravity must be changed into V. This is shown in Equation 2.

$$F_g = acceleration * mass$$

$$Acc_g = \frac{F_g}{mass}$$

$$V_g = \int Acc_g$$

Equation 2. Derivation of velocity due to gravity.

2.3 Drag Force

The drag force is a force relative to the Reynolds number, air density, area of the ball, and the magnitude of the velocity. It always acts in the opposite of the direction that the ball is currently moving in. Therefore, the direction is continuously updated at each time step. The first step in computing the drag force is finding the Reynolds number through Equation 3, where ρ is the air density, V is the magnitude of the ball's initial velocity, D is the diameter of the ball, and μ is the dynamic viscosity. They Reynolds number is then used to determine the drag coefficient, C_d, shown in Figure 4. While the drag coefficient would continuously be updated as the ball slows down, for this project it stays constant and is a result of the initial velocity.

$$Re = \rho V D / \mu$$

Equation 3. Reynolds number

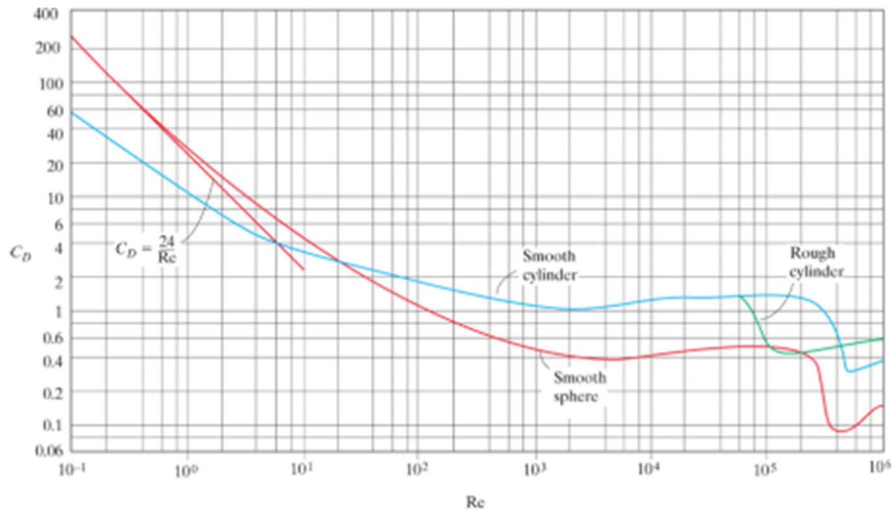


Figure 4. Drag coefficient of a sphere and long cylinder.

Next the drag force magnitude is then computed with Equation 4. After the magnitude is calculated the individual components need to be calculated. Since the drag force is in the opposite direction of the currently velocity a negative unit vector of the current velocity can be used as in Equation 5 (Note: |V| specifies the magnitude of V). Once again, the force needs to be converted into a velocity which uses the same concept as gravity in Equation 2.

$$|F_D| = C_D A\left(\frac{\rho * |V|^2}{2}\right)$$

Equation 4. Drag force equation.

$$\overrightarrow{F_d} = -\left(\frac{V}{|V|}\right) * |F_D|$$

Equation 5. Drag force component computation.

2.4 Magnus Force

Lastly the Magnus force must be calculated. This starts with an input of the spin magnitude and the angle in which it is performed on the ball. Using these inputs, the spin rate is decomposed into the X, Y, and Z components by Equation 6. Since the angular velocity decelerates over time Equation 7 is used to calculate the instantaneous angular velocity as a function of initial angular velocity, the diameter, time, and an angular deceleration coefficient (Grider, n.d.). The coefficient has been chosen as 0.2.

$$\omega_x = |\omega| * \sin(\theta_\omega) * \sin(X - Y)$$

$$\omega_y = |\omega| * \sin(\theta_\omega) * \cos(X - Y)$$

$$\omega_z = |\omega| * \cos(\theta_\omega)$$

Equation 6. Angular velocity decomposed.

$$\vec{\omega} = \vec{\omega}_0 * e^{-\frac{C_W}{\frac{D}{2}} * t}$$

Equation 7. Instantaneous angular velocity.

The Magnus force is computed through Equation 8. C_L has been chosen as a constant at 0.25 due to a study by NASA (Hall 2021). Where alpha is represented in Equation 9. This result again must go through the derivation into velocity which uses the same concept as gravity in Equation 2.

$$F_m = \alpha\left(\vec{\omega} \times \vec{V}\right)$$

Equation 8. Magnus force equation.

$$\alpha = \frac{1}{2} * \rho * C_L * A$$

Equation 9. Alpha equation for Magnus force.

2.5 Position Computation from Velocities

Once all the forces have been calculated and derived into velocity for the current time step, they are added to the last time steps instantaneous velocity, Equation 10. Finally, in Equation 11, the change in position is computed through velocity for the current time step and added to the previous time step's position for the current time step's position.

$$\vec{V}(t) = \vec{V}(t-1) + \vec{V_g}(t) + \vec{V_D}(t) + \vec{V_M}(t)$$

Equation 10. Computing instantaneous overall velocity.

$$\vec{X}(t) = \vec{X}(t-1) + \vec{V}(t) * dt$$

Equation 11. Position at current time step computation.

2.6 MATLAB SIMULATION

This computation is done through a MATLAB program and represented in both graphs and a table. The entire simulation window is shown in Figure 5. The graphs and table displayed are shown in Figure 6.

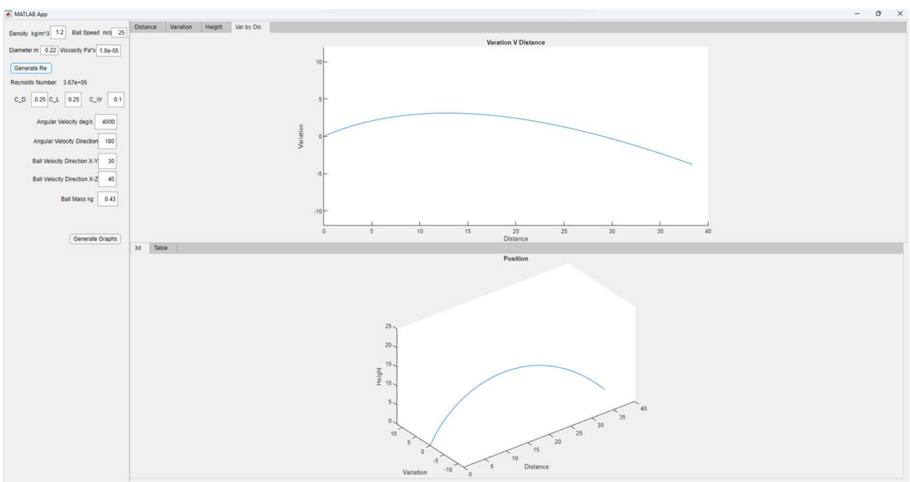
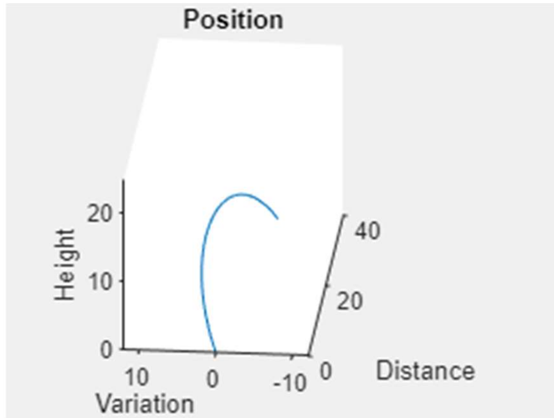


Figure 5. MATLAB simulation window.



Time	X	Y	Z
0.1500	2.5164	1.2244	2.2331
0.1600	2.6852	1.2928	2.3705
0.1700	2.8541	1.3596	2.5065
0.1800	3.0229	1.4248	2.6411
0.1900	3.1918	1.4885	2.7743
0.2000	3.3607	1.5508	2.9062
0.2100	3.5295	1.6115	3.0367
0.2200	3.6983	1.6707	3.1658
0.2300	3.8671	1.7285	3.2935
0.2400	4.0358	1.7849	3.4199
0.2500	4.2045	1.8399	3.5449
0.2600	4.3730	1.8934	3.6687
0.2700	4.5415	1.9456	3.7910
0.2800	4.7099	1.9964	3.9121
0.2900	4.8781	2.0458	4.0318
0.3000	5.0463	2.0939	4.1502
0.3100	5.2143	2.1407	4.2673
0.3200	5.3822	2.1862	4.3831
0.3300	5.5499	2.2304	4.4975
0.3400	5.7174	2.2733	4.6107

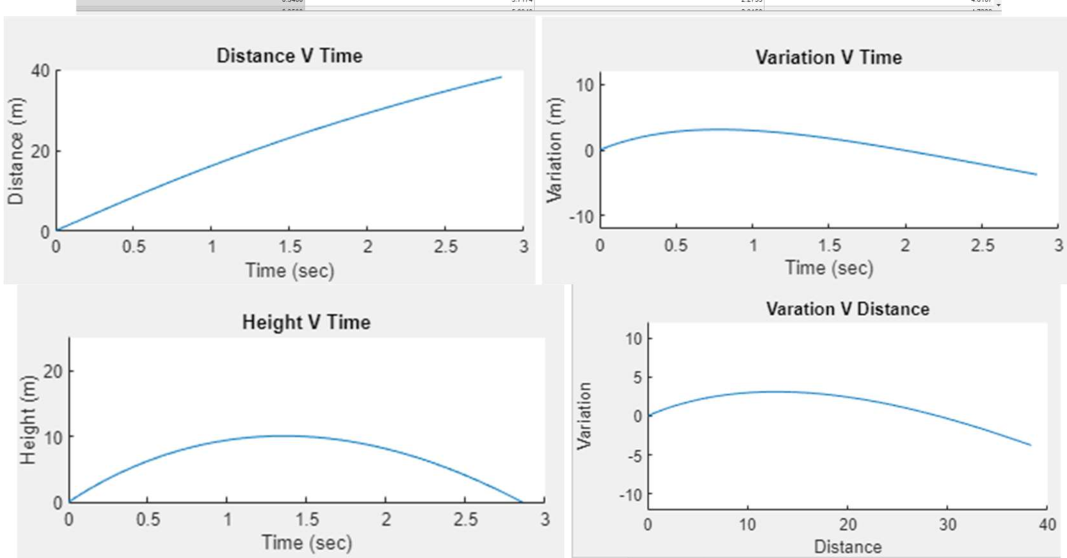


Figure 6. Display of graphs and tables shown in MATLAB simulation.

3 DATA, RESULTS, AND DISCUSSION

In this section, this paper will explore the differences in ball path and speed due to changes in elevation (air pressure), ball size, spin rate, spin direction, and ball speed. It will also investigate sources of errors.

3.1 CHANGE IN ELEVATION

The first experiment done was what happens when there is a change in elevation. The main driving point of this experiment is that the elevation increases so does the air density. Air density affects both the Reynolds number and the drag force that is created on the ball. For this experiment only the air density will change. The ball is propelled at a velocity of 20 m/s. The viscosity of air is 1.8E-5 Pa*s. The diameter of the ball is 0.22 m. The Angular Velocity will be 1000 deg/s with a direction of 90 degrees. As there is very little angular velocity, there should not be much variation in the balls path. The ball is kicked in the X-Y direction of 0 degrees and 40 degrees in the Z direction. Finally, the temperature is 59 degrees Fahrenheit, relative humidity % is 40%, and the Barometer is 29.92 in. All air density calculations were performed by Engineers Edge's "Air Density and Specific Weight Table, Equations and Calculator" (Engineers Edge. n.d.)[1]. The drag coefficient is determined by the Reynolds number shown in Figure 4. In Table 1, 3 different elevations are tested to see their effects in the distance the ball travels, the maximum height the ball reaches, and the amount of time the ball stays in the air (hang time). Figure 7 shows the flight path of a ball in the height and distance axis at these elevations with the prior specified parameters.

Table 1. Changes in Elevation (Air Density)

Elevation Above Sea Level (m)	Air Density $\left(\frac{kg}{m^3}\right)$	Reynolds Number	Drag Coefficient	Distance Traveled (m)	Max Height (m)	Hang Time (seconds)
0	1.2256	3.00E+05	0.39	23.46	5.23	1.94
805	1.1426	2.79E+05	0.40	23.93	5.36	1.97
1609	1.0653	2.6E+05	0.42	24.28	5.45	2.00

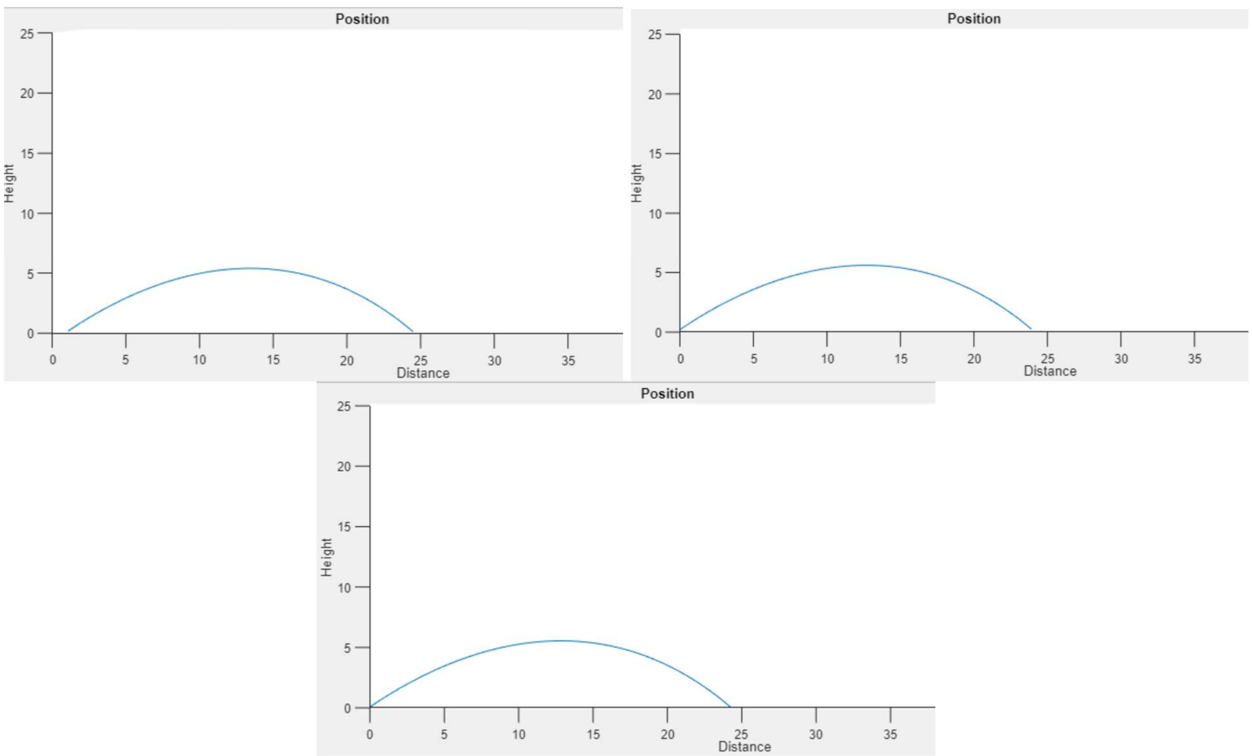


Figure 7. Ball flight at elevations of 0, 805, and 1609 m above sea level. From left to right, top to bottom.

These results show us that at higher elevations, which have lower air density, the ball will travel further and higher from the same initial force. For example, the ball traveled more than 0.8 m further, stayed in the air for 0.06 seconds, and reach 0.22 m higher at 1609 m compared to 0 m. This is due to the fact the drag force lessens on the ball with lower air densities.

3.2 CHANGE IN SPIN RATE

The next experiment tests the change of the spin rate. The change in spin rate affects the magnitude of the magnus force. For this experiment only the spin rate will change. The ball is propelled at a velocity of 20 m/s. The air density is 1.22 kg/m^3. The viscosity of air is 1.8E-5 Pa*s. The diameter of the ball is 0.22 m. The ball is kicked in the X-Y direction of 0 degrees and 40 degrees in the Z direction. The drag coefficient is determined by the Reynolds number shown in Figure 4 and is computed to be 0.39. The spin direction that will be tested in Table 2 is 180 degrees. In other words, the ball will only have a Magnus force pushing it right, as it is spinning clockwise around the Z axis.

Table 2. Angular Velocity by Max Variation

Angular Velocity (deg/s)	Max Variation (m)
1000	2.39
4000	9.27
7000	15.20

From this data as the angular velocity increases so does the deviation from the path. This is in line with the idea of the Magnus force. A higher spin rate causes a higher-pressure difference, and therein produces a high force towards the low-pressure zone. In Figure 8, it displays examples of a ball with the same parameters as above, but the ball is kicked at 40 degrees in the X-Y direction and at spin rates of 1000, 4000, and 7000 degrees per second. From this figure, it shows how the increase in spin rate, and therefore increase in the Magnus force, can generate the unique ball flights often seen in soccer free kicks, such as Figure 1.

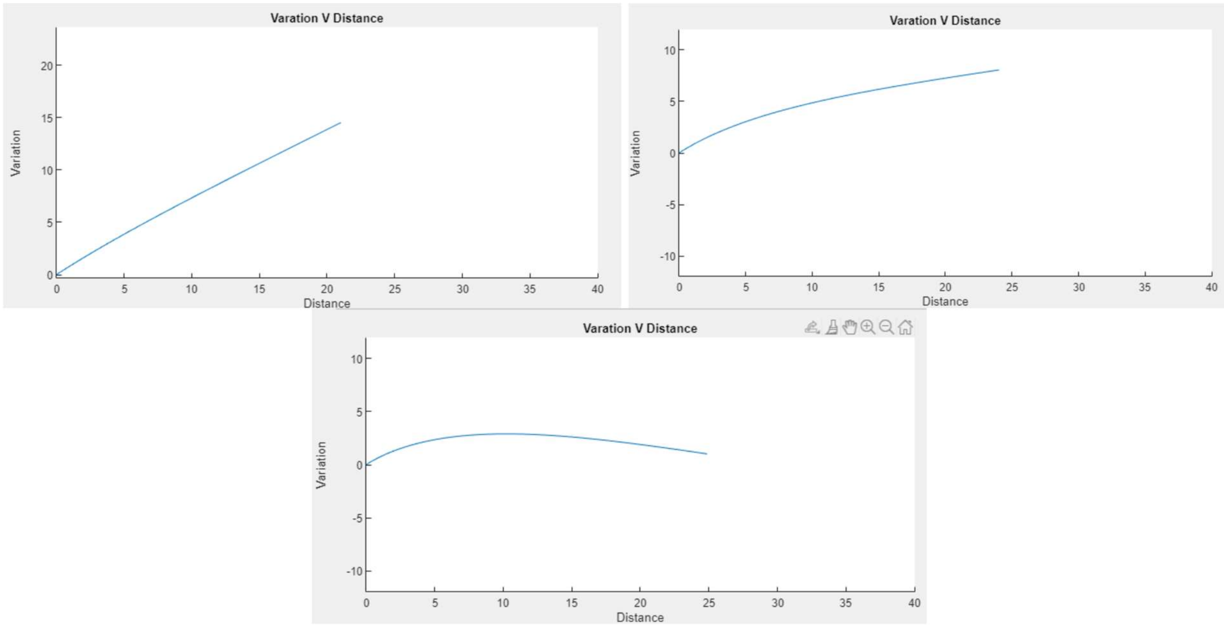


Figure 8. Ball flight at spin rates of 1000, 4000, and 7000 deg/s at a 40-degree X-Y angle. From left to right, top to bottom.

3.3 CHANGE IN SPIN DIRECTION

The final experiment will involve changing the spin direction to see the change in the balls' flight. For this experiment only the spin direction will change. The ball is propelled at a velocity of 20 m/s. The air density is 1.22 kg/m^3. The viscosity of air is 1.8E-5 Pa*s. The diameter of the ball is 0.22 m. The ball is kicked in the X-Y direction of 0 degrees and 40 degrees in the Z direction. The drag coefficient is determined by the Reynolds number shown in Figure 4 and is computed to be 0.39. The spin rate will stay constant at 7000 degrees per second. In Figure 9, the spin rate direction is 0 degrees, or it is spinning counterclockwise around the Z axis. This spin direction results in a Magnus Force pushing the ball left. In Figure 10, the spin rate direction is 180 degrees, or it is spinning clockwise around the Z axis. This spin direction results in a Magnus Force pushing the ball right.

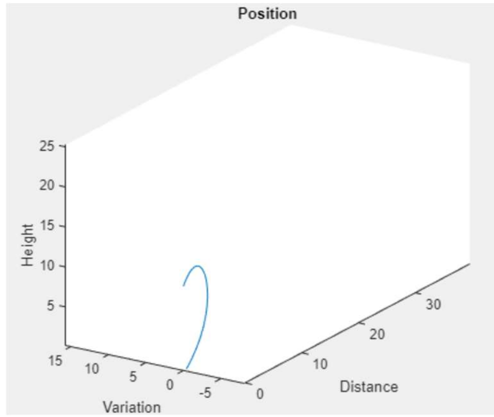


Figure 9. *Spin rate of 7000 deg/s applied at 0 degrees.*

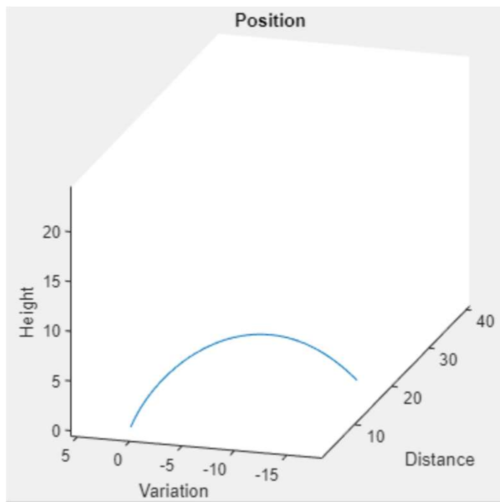


Figure 10. *Spin rate of 7000 deg/s applied at 180 degrees.*

Figure 11 represents a normal ball flight. There is very little spin on the ball resulting in a small Magnus force barely changing the ball flight. The normal, low spin ball flight has 2.26 seconds in the air. Figure 12 shows a ball with a spin rate of 7000 deg/s at 90 degrees, or it is spinning clockwise around the y axis. This results in a large Magnus force pushing the ball downwards, resulting in a short time in the air (0.94 seconds). Figure 13 displays a ball with a spin rate of 7000 deg/s at 270 degrees, or it is spinning counterclockwise around the y axis. This results in a large Magnus force pushing the ball upward, resulting in a long time in the air (3.44 seconds).

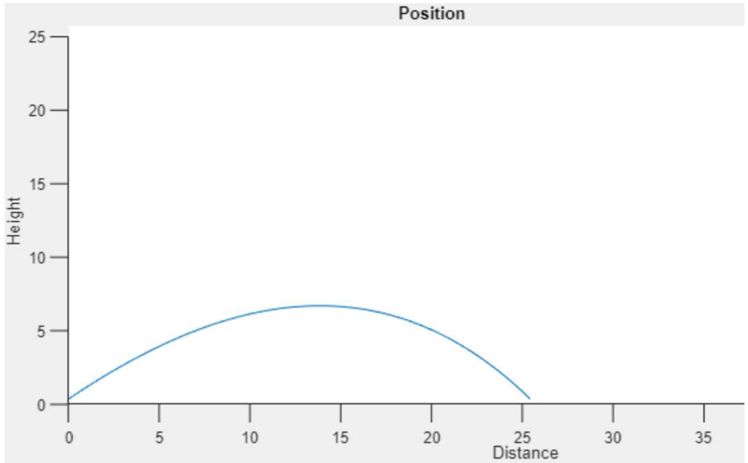


Figure 11. *Normal ball flight with low angular velocity*

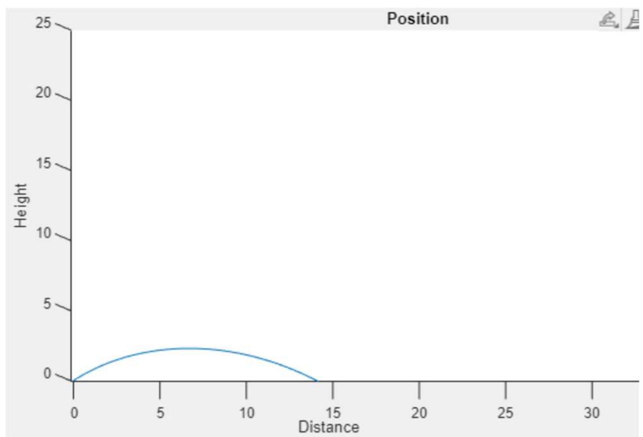


Figure 12. *Spin rate of 7000 deg/s applied at 90 degrees.*

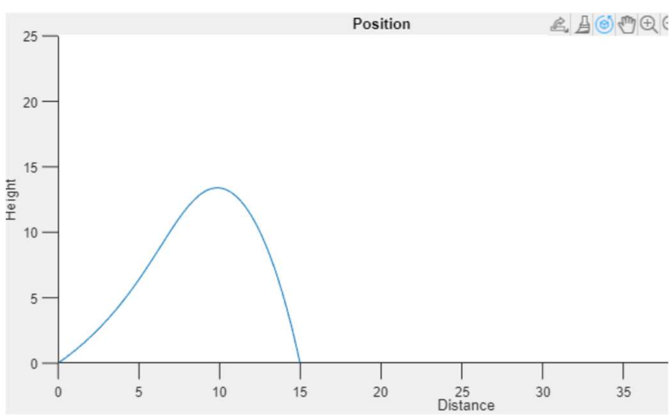


Figure 13. *Spin rate of 7000 deg/s applied at 270 degrees.*

The spin rate direction can be in any of the 360 degrees. To compute the ball flight in these cases the spin rate around the Y axis and the spin rate around the Z axis must be considered to find the Magnus force components in the Z and Y directions respectively. Figure 14 shows a ball path where spin direction is 210 degrees. This results in a hang time of 3.04 seconds and a variation of 14.3 m. This creates an increase in the hang time of the low spin ball while also creating variation.

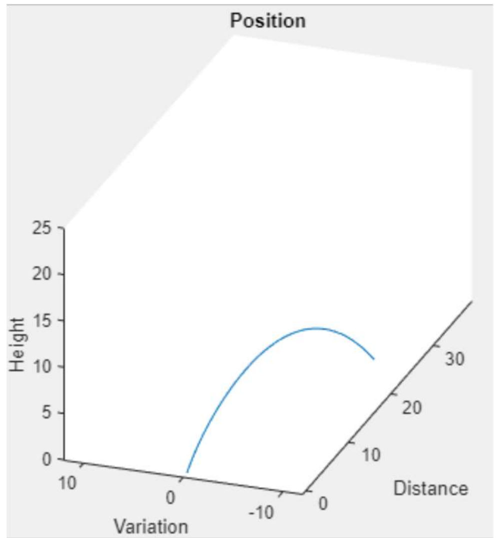


Figure 14. Spin rate of 7000 deg/s applied at 210 degrees.

3.4 DISCUSSION OF ERRORS

While the simulation does encompass the general ball flight of a soccer ball from the given parameters, there are certain components that can introduce error. For one, the way to determine the drag coefficient in Figure 4 is challenging as the Reynolds number hovers around 3.0E+05 which is a large dip. Secondly the lift coefficient and the drag coefficients remains constant under all circumstances, while in practice they would change. Thirdly, the simulation takes a soccer ball as a smooth sphere, but it has textures that change the ball's flight. Another item that introduces error is the deceleration of the angular velocity. There was nothing fund online about deceleration constants, so the number was chosen based on how the simulation ran. Lastly, no wind is considered in the simulation.

4 CONCLUSION

This report sets out to explore the effects of aerodynamics on a soccer ball. It explored areas of how various factors such as air density, spin rates, and the angle of spin influence the ball’s trajectory and speed. The approach implemented a MATLAB simulation to model these variables and their impacts. The results from the simulation provided valuable insights into the understanding of how the initial velocity, force of gravity, drag force, and Magnus force interact to create unique ball flights.

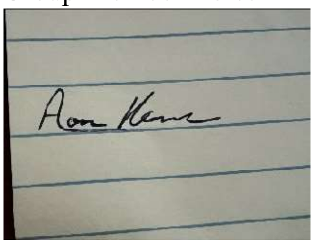
First, it examined how changes in elevation (resulting in changes in air density) affect the ball's flight path. It was discovered that higher elevations, with lower air densities, allow the ball to travel further and reach higher altitudes. Additionally, the effects of varying spin rates and directions on the ball's trajectory were explored. The findings aligned with the principles of the Magnus effect, showing that increased spin rates resulted in higher deviations from a straight line. These findings lay the foundation for the unique paths seen in soccer free kicks.

In terms of the initial objectives, the significant role that aerodynamics plays in soccer was explored and showed that mastering these principles can give players a competitive edge. While the simulation provided valuable insights, it's important to note the limitations we encountered. The simulation considered the soccer ball as a smooth sphere, not accounting for real-world textures that might affect flight. Furthermore, the unchanging from the initial drag and lift coefficients, the estimation of angular velocity deceleration, and the exclusion of environmental factors like wind are items that could be improved in future work.

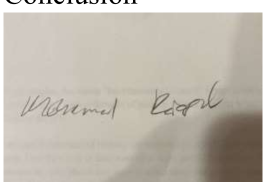
Nevertheless, this report has successfully explored the role of aerodynamics in soccer. It showed the value it can add to a player’s skill set, as well as the importance for engineers and manufacturers to understand the role aerodynamics has in soccer.

5 GROUP MEMBER ROLES

Aaron Harrison

- Creation of MATLAB simulation
- Methods
- Data, Results, and Discussions –team effort
- Group Member Roles
- 

Mohammed Raifet

- Introduction
- Data, Results, and Discussions – team effort
- Conclusion
- 

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