COMP1210 **Tutorial 5**

1. Let $X = \{a, b, c\}$. Which of the following relations on X are *reflexive*, *symmetric*, *transitive* or an *equivalence relation*.

a. $R_1 = \{(a, a), (a, b), (b, b), (b, a)\}$

b. $R_2 = \{(a, a), (b, b), (c, c)\}$

2. Given that the relation R is defined on the set of natural numbers $\mathbb{N}$ by xRy iff x is a multiple of y or y is a multiple of x.
Determine whether R is an *equivalence relation*.

3. Prove that the relation xRy defined by $x \sim y \iff x - y = 4k$, where $x, y, k \in \mathbb{Z}$, is an equivalence relation on $\mathbb{Z}$.
Find the *equivalence classes* of $x = 2$ and $x = 4$.

4. Prove that the relation xRy defined by $x \equiv y \ mod \ 5$ where $x, y \in \mathbb{Z}$, is an *equivalence relation*. Note that $x \equiv y \ mod \ 5$ means that $5 \,|\, x - y$.
Find the *equivalence classes* of $x = 2$ and $x = 4$.

5. A relation R is defined on the set $S = \mathbb{Z} \times \mathbb{Z}$ by $(a, b)\, R\, (c, d)$ if and only if $a + d = b + c$.
Show that R is an *equivalence relation* on S.
Determine the *equivalence class* of (x, y).

6. A relation R is defined on the set $S = \mathbb{N}$, the set of natural numbers, such that $(a, b) \in R$ iff $\exists\, m \in \mathbb{N}$ such that $b = ma$.
Is R an *equivalence relation*? Please justify your answer.

7. Let R be a relation, defined on $\mathbb{Z}$, the set of integers, by xRy if and only if $x - y$ is a multiple of p, where p is a fixed positive integer.

(i) Show that R is an *equivalence relation*.

(ii) Find the *equivalence class* of $x \in \mathbb{Z}$.

8. Let $\mathbb{Z}$ be the set of integers, and let ∇ be the relation on $\mathbb{Z}$ such that for all $m, n \in \mathbb{Z}$, $(m, n) \in \nabla$ iff $m - n = 2k$, for some $k \in \mathbb{Z}$.

(i) Show that ∇ is an equivalence relation.

(ii) Identify the *equivalence class* of $x \in \mathbb{Z}$.

(iii) Identify the distinct (two of them) *equivalence classes* of ∇.