**COMP1210** **TUTORIAL SET 4**

1. Find the range and the domain of the following functions.

a. $f(x) = 4x - 16$

b. $f(x) = \sqrt{3x - 6}$

c. $f(x) = \sqrt{2 - x}$

d. $f(x) = \dfrac{2x}{(x-1)(3x+9)}$

e. $f(x) = \dfrac{1}{\sqrt{x-3}}$

2. Determine whether the following functions from $\mathbb{R}$ to $\mathbb{R}$ are one to one or onto. [Note: Give a proof in each case]

a. $f(x) = x - 3$

b. $f(x) = 2x + 3$

3. Repeat "Problem 2" this time assuming that the functions are defined from $\mathbb{Z} \to \mathbb{Z}$.

4. a. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$. List all functions from $A$ to $B$. Say which of your functions are injective and which are surjective.

   b. $X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ and let $f = \{(x, x^2) : x \in X\}$. Write $f$ as a set of ordered pairs. Is $f$ injective or surjective?

5. Let $A$ be the set of negative real numbers and $B$ be the set of positive real numbers. Let $f : A \to B$ be given by $f(x) = \frac{1}{2-x}$ and $g : B \to B$ be given by $g(x) = \frac{1}{x+2}$. Show why $g \circ f$ is defined. Find $g \circ f$ and its range.

6. Find the composition, $f \circ g$, of the following function pairs.

a. $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x-2}$. Notice that $f \circ g$ is not defined as a function from the domain of $g$. Find the values of $x$ for which $f \circ g$ is defined. That is, the natural domain of $f \circ g$.

b. $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{x-1}{x+1}$.

7. Show that the following functions are one to one and onto and find their inverses.

a. $f(x) = -x - 4$

b. $f(x) = (x+1)^2,\ x \geq -1$.

8. Let $A = \{x : x \geq 2\}$ and $\{x : x \geq -4\}$. Let $f : A \to B$ be given by $f(x) = x^2 - 4x$. Show that $f(x)$ is one to one and onto. Determine the inverse of $f(x)$.

9. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that

a. If $f$ and $g$ are injective, then $g \circ f$ is injective.

b. If $f$ and $g$ are surjective, then $g \circ f$ is surjective.

10. Let $f : A \to B$ and $C_1, C_2 \subseteq A$ and $D \subseteq B$. Prove that

a. $f(C_1 \cap C_2) \subseteq f(C_1) \cap f(C_2)$.

b. $f[f^{-1}(D)] \subseteq D$.