University of the West Indies

COMP1210: Mathematics for Computing

Tutorial # 3

Using Induction, verify that each equation is true for every positive integer n.

Question 1. $1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$

Question 2. $1(2) + 2(3) + 3(4) + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Question 3. $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

Question 4. $2^n \geq n^2$, $n = 4, 5, \ldots$

Question 5. $2n + 1 \leq 2^n$, $n = 3, 4, \ldots$

Question 6. $5^{2n} - 1$ is a multiple of 8, $\forall\ n \in \mathbf{N}$

Using set notations, obtain the solution to each of the following questions:

Question 7. If E represent the universal set and A and B are subsets of E such that

$E = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$,

$A = \{1, 2, 3, 4, 5, 7\}$ and $B = \{2, 3, 7, 8, 9, 12, 15\}$; Find:

(i) $n(A)$ (ii) $n(A \cup B)$ (iii) $n(A \cap B)^c$ (iv) $(A \cup B)^c$

(v) A^c (vi) $n(B^c)$

(vii) Draw a suitable Venn diagram to represent the information above.

Question 8. In a class of 30 pupils, 18 take French and 17 take German. 3 take neither. How many take both French and German?

Question 9. Of 100 students, 42 take Physics, 35 take Chemistry and 30 take Botany. 20 take none of these subjects. 9 take Botany and Physics, 10 take Botany and Chemistry and 11 take Physics and Chemistry.

i. Draw a suitable Venn diagram to represent the above information.

Find:

ii. the number of students that take all three subjects;

iii. the number that take Physics only;

iv. the number of students that take Botany and Chemistry only.

Question 10. For any sets A, B and C, prove that

i. $(A \cap B)' = A' \cup B'$

ii. $A' - B' = B - A$

iii. $A - (B \cap C) = (A - B) \cup (A - C)$