MATHS FOR COMPUTING **COMP1210** **Tutorial 6**

Question 1. Let S be the set of rationals which differ from -1. A binary operation $\otimes$ is defined on S by

$$x \otimes y = x + y + xy, \ \forall\, x, y \in S.$$

(i) Determine whether $\otimes$ is commutative.

(ii) Determine whether $\otimes$ is associative.

(iii) Show that S has an identity with respect to $\otimes$.

(iv) Determine which elements of S have inverses with respect to $\otimes$.

(v) Solve the equation $5 \otimes x = 4$.

Question 2. Let $*$ be a binary operation on $\mathbb{R} \times \mathbb{R}$ given by

$$(a, b) * (c, d) = (ad + bc,\ bd)\ \forall\, a, b, c, d \in \mathbb{R}$$

(i) Determine if $*$ is commutative and associative.

(ii) Find the identity of $\mathbb{R} \times \mathbb{R}$ with respect to $*$.

(iii) Determine which elements have inverses and give the inverse of such elements.

Question 3. Let S be the set of ordered pairs (a, b) of real numbers, with the second element of the ordered pair being non-zero. A binary operation $\boxplus$ is defined on S, such that

$$(a, b) \boxplus (c, d) = (a + c,\ bd) \text{ for all ordered pairs in } S.$$

(i) Show that $\boxplus$ is commutative and associative.

(ii) Find the identity element in S for $\boxplus$.

(iii) Find the inverse of the element $(a, b) \in S$ under $\boxplus$.

Question 4. A binary operation $\heartsuit$ is defined on the set of positive reals $\mathbb{R}^+$ by

$$x \heartsuit y = \frac{xy}{x + y + 1} \quad \forall\, x, y\ \in \mathbb{R}^+ .$$

(i) Determine whether $\heartsuit$ is commutative and associative.

(ii) Is there an identity with respect to $\heartsuit$ on $\mathbb{R}^+$? Give reasons for your answer.