Solution to Tutorial Set 7

Comp 1210

Question 1

The procedure of assigning offices to these two employees consists of assigning an office to Sanchez, which can be done in 12 ways, then assigning an office to Patel different from the office assigned to Sanchez, which can be done in 11 ways.
By the product rule, there are $12 \times 11 = 132$ ways to assign offices to these two employees.

Alternatively, using the permutation concept, we have

$$^{12}P_2 = \frac{12!}{(12-2)!} = \frac{12 \times 11 \times \cancel{10!}}{\cancel{10!}}$$

$$= 12 \times 11 = 132.$$

Note: The product rule states that suppose that a procedure can be broken down into a sequence of two tasks.

If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 \times n_2$ ways to do the procedure.

Question 2

There are 37 ways to choose a member of the Mathematics faculty and there are 83 ways to choose a student who is a Mathematics major. Choosing a member of the Mathematics faculty is never the same as choosing a student who is a Mathematics major because no one is both a faculty member and a student. By the sum rule it follows that there are $37+83=120$ possible ways to pick this representative.

Alternatively, using the sum of the permutations, we have

$$^{37}P_1 + {}^{83}P_1 = \frac{37!}{(37-1)!} + \frac{83!}{(83-1)!} =$$

$$\frac{37 \times \cancel{36!}}{\cancel{36!}} + \frac{83 \times \cancel{82!}}{\cancel{82!}} = 37 + 83 = 120.$$

Note: The sum rule states that if a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

Question 3

Because some of the letters of SUCCESS are the same, the answer is not given by the number of permutations of seven letters. This word contains three Ss, two Cs, one U, and one E. To determine the number of different strings that can be made by reordering the letters, first note that the three Ss can be placed among the seven positions in ${}^{7}C_{3}$ different ways, leaving four positions free. Then the two Cs can be placed in ${}^{4}C_{2}$ ways, leaving two free positions.

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The U can be placed in ${}^{2}C_{1}$ ways, leaving just one position free.

Hence, E can be placed in ${}^{1}C_{1}$ way.

Consequently, from the product rule, the number of different strings that can be made is

$$ {}^{7}C_{3} \times {}^{4}C_{2} \times {}^{2}C_{1} \times {}^{1}C_{1} = $$

$$ \frac{7!}{(7-3)!\,3!} \times \frac{4!}{(4-2)!\,2!} \times \frac{2!}{(2-1)!\,1!} \times \frac{1!}{(1-1)!\,1!} $$

$$ = \frac{7!}{4!\,3!} \times \frac{4!}{2!\,2!} \times \frac{2!}{1!\,1!} \times \frac{1!}{0!\,1!} = $$

$$ \frac{7 \times \cancel{6} \times 5}{\cancel{3} \times \cancel{2} \times 1} \times \frac{4 \times 3}{2 \times 1} \times \frac{2}{1} \times \frac{1}{1} = $$

$$ 35 \times 6 \times 2 = 420. $$

Note:

For the combination, we have that, for instance, ${}^{7}C_{3} = \frac{7!}{(7-3)!\,3!}$

$$ = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!} \times 3!} = \frac{7 \times 6 \times 5}{3!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1}. $$

Question 4

We will use the product rule to solve this problem. To begin, note that the first player can be dealt 5 cards in ${}^{52}C_5$ ways. The second player can be dealt 5 cards in ${}^{47}C_5$ ways, because only 47 cards are left. The third player can be dealt 5 cards in ${}^{42}C_5$ ways. Finally, the fourth player can be dealt 5 cards in ${}^{37}C_5$ ways. Hence, the total number of ways to deal four players 5 cards each is

$$ {}^{52}C_5 \times {}^{47}C_5 \times {}^{42}C_5 \times {}^{37}C_5 = $$

$$ \frac{52!}{47!\,5!} \times \frac{47!}{42!\,5!} \times \frac{42!}{37!\,5!} \times \frac{37!}{32!\,5!} = $$

$$ \frac{52!}{5!\,5!\,5!\,5!\,32!}. $$

Remark: The result will be a very huge number that would have to be expressed in standard form (scientific notation).

Note: The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ..., and n_k indistinguishable objects of type k, is

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}.$$

Remark: The solution to Question 4 equals the number of permutations of 52 objects, with 5 indistinguishable objects of each of four different types, and 32 objects of a fifth type. This equality can be seen by defining a one-to-one correspondence between permutations of this type and distributions of cards to the players.

Question 5

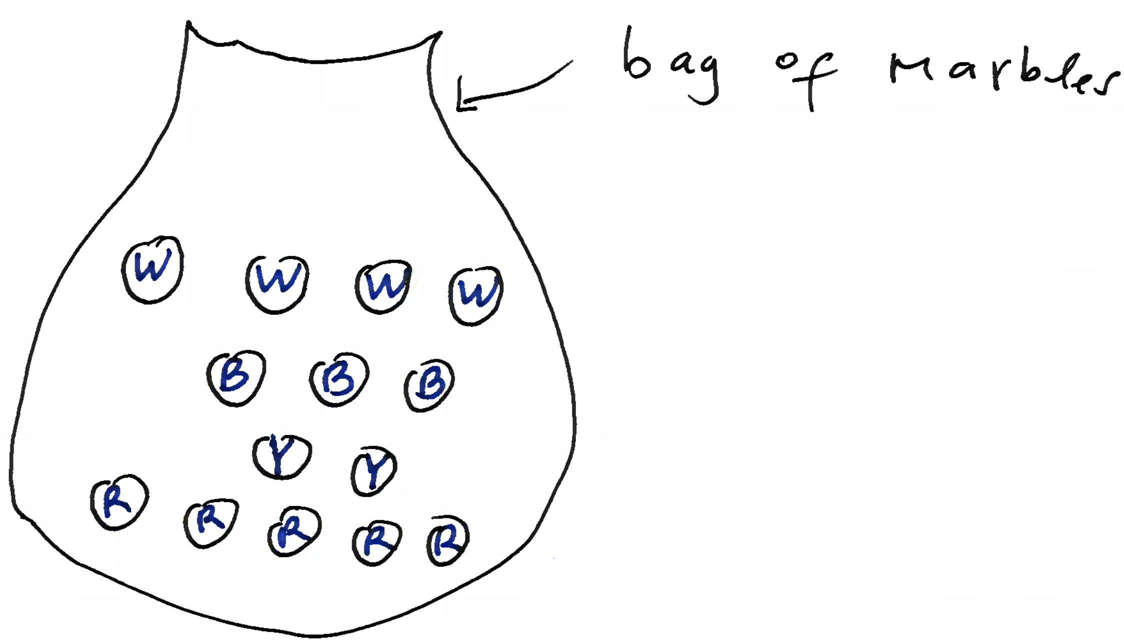
Now, from the 52 cards, there is only 1 jack of spade. So,

$$p(\text{jack of spade}) = \frac{1}{52}.$$

Note that the sample size is 52.

Question 6

A diagram can be shown. See diagram below:



Key: B represent blue, R represent red, W represent white and Y represent yellow.

When the first marble was drawn, the sample space, which is 14, was reduced to 13.

Seeing that it was a blue marble then there is only 2 blue marbles left. Therefore,

$$p(\text{drawing another blue marble}) = \frac{2}{13}.$$

Question 7

Now, since there are 100 discs, then the sample size is 100. Now, of the 100 discs, there are 24 discs which are divisible by 4. Therefore,

$$p(\text{disc divisible by } 4) = \frac{24}{100} = \frac{6}{25}.$$

Question 8

Now, given that $p(A) = 0.3$, $p(B) = 0.1$ and $p(A \cap B) = 0.02$.

(i) Events A and B are mutually exclusive if

$$p(A \cup B) = p(A) + p(B).$$

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So, $P(A \cup B) = 0.3 + 0.1 = 0.4$.

Therefore, the events A and B are mutually exclusive.

(ii) Events A and B are independent if

$$P(A \cap B) = P(A) \times P(B).$$

Now, $P(A \cap B) = 0.02$ (given).

We have that the

$$P(A \cap B) = P(A) \times P(B)$$
$$= 0.3 \times 0.1 = 0.03.$$

Since $P(A \cap B) = 0.02 \neq 0.03$, which is $P(A) \times P(B)$, then

$$P(A \cap B) \neq P(A) \times P(B),$$

and so the events A and B are not independent.

(iii) Now, based on the total probability law, $P(A \cap B)' = 1 - P(A \cap B) = 1 - 0.02 = 0.98$.

Question 9

Given that $p(A) = 0.4$, $p(B) = 0.7$ and $p(A \cap B) = 0.3$.

(i) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$= 0.4 + 0.7 - 0.3 = 0.8$.

This is based on Set theory.

(ii) $p(A \cup B)' = 1 - p(A \cup B)$

$= 1 - 0.8 = 0.2$.

(iii) $p(A \cap B') = p(A) - p(A \cap B)$

$= 0.4 - 0.3 = 0.1$.

(iv) $p(B \cap A') = p(B) - p(A \cap B)$

$= 0.7 - 0.3 = 0.4$.

Note: For any events A and B,

$P(A) = P(A \cap B) + p(A \cap B')$.

Also, $P(A \cup B) = p(B) + p(A \cap B')$.

Finally, $p(A \cap B') = p(A) - p(A \cap B)$ and $p(B \cap A') = p(B) - p(A \cap B)$.

Question 10

Now since there are 52 playing cards, then the Sample space is 52.
Now, the probability that it is a king is $\frac{4}{52} = \frac{1}{13}$. The probability that it is a heart is $\frac{13}{52} = \frac{1}{4}$, and the probability that it is both a King and a heart is $\frac{1}{52}$.
Therefore, the conditional probability that the card is a king given that it is a heart is

$$p(\text{King} \mid \text{heart}) = \frac{p(\text{King and heart})}{p(\text{heart})}$$

$$= \frac{p(\text{King} \cap \text{heart})}{p(\text{heart})} = \frac{\frac{1}{52}}{\frac{1}{4}}$$

$$= \frac{1}{52} \times \frac{4}{1} = \frac{1}{3}.$$