

Mathematical model:

1. Decision variable:

$$x_{ijk} = \begin{cases} 1, & \text{if element } (i, j) \text{ of the } n \times n \text{ Sudoku matrix contains the integer } k \\ 0, & \text{otherwise.} \end{cases}$$

2. Objective function:

$$\text{Maximize } z = \sum_i \sum_j \sum_k kx_{ijk}$$

(Maximizing or minimizing the objective function doesn't change the result, $z = 405$)

3. Constraints:

- Rule 1: Each cell must contain the digits 1 through 9 exactly once.

$$\sum_k x_{ijk} = 1, \quad \forall i, j$$

$$\text{Ex: } x_{111} + x_{112} + x_{113} + x_{114} + x_{115} + x_{116} + x_{117} + x_{118} + x_{119} = 1$$

- Rule 2: Each row must contain the digits 1 through 9 exactly once.

$$\sum_j x_{ijk} = 1, \quad \forall i, k$$

$$\text{Ex: } x_{113} + x_{123} + x_{133} + x_{143} + x_{153} + x_{163} + x_{173} + x_{183} + x_{193} = 1$$

- Rule 3: Each column must contain the digits 1 through 9 exactly once

$$\sum_i x_{ijk} = 1, \quad \forall j, k$$

$$\text{Ex: } x_{113} + x_{213} + x_{313} + x_{413} + x_{513} + x_{613} + x_{713} + x_{813} + x_{913} = 1$$

- Rule 4: Each 3 x 3 submatrix must contain the digits 1 through 9 once

$$\sum_i \sum_j x_{ijk} = 1, \quad \forall k$$

$$\text{Ex: } x_{113} + x_{123} + x_{133} + x_{213} + x_{223} + x_{233} + x_{313} + x_{323} + x_{333} = 1$$