BLG 354E Signals and Systems for CE (Spring 2018) Homework-1

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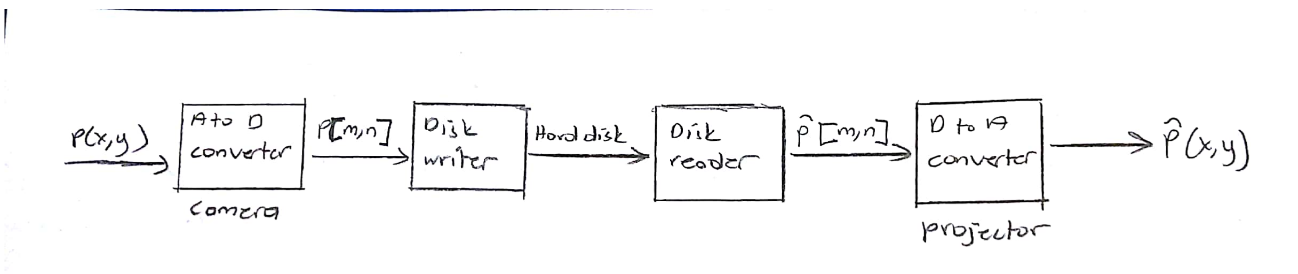
1 Question 1



2 Question 2

According to wikipedia, any value which varies with time or space can be considered a signal that carries information. Therefore, all three figures in the question are signals.

3 Question 3

$z^4 = j$

$j = e^{j\frac{\pi}{2}}$

$z^4 = e^{(j\frac{\pi}{2}+2\pi k)}$

$z = \sqrt[4]{e^{(j\frac{\pi}{2}+2\pi k)}} = e^{j(\frac{\pi}{8}+k\frac{\pi}{2})}$ for $k = 0, 1, 2, 3$

Therefore, the roots are: $e^{j\frac{\pi}{8}}, e^{j\frac{5\pi}{8}}, e^{j\frac{9\pi}{8}}, e^{j\frac{13\pi}{8}}$

4 Question 4

According to Taylor series:

$e^{i\theta} = e^{ia} + e^{ia}i(\theta - a) + \frac{e^{ia}i^2}{2!}(\theta - a)^2 + \frac{e^{ia}i^3}{3!}(\theta - a)^3 + \frac{e^{ia}i^4}{4!}(\theta - a)^4 + ...$

if $a = 0$:

$e^{i\theta} = 1 + i\theta + \frac{i^2}{2!}\theta^2 + \frac{i^3}{3!}\theta^3 + \frac{i^4}{4!}\theta^4 + ...$

$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + ...$

$\cos\theta = \cos a - \sin a(\theta - a) - \frac{1}{2!}\cos a(\theta - a)^2 + \frac{1}{3!}\sin a(\theta - a)^3 + ...$

if $a = 0$: $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + ...$

$\sin\theta = \sin a + \cos a(\theta - a) - \frac{1}{2!}\sin a(\theta - a)^2 - \frac{1}{3!}\cos a(\theta - a)^3 + ...$

if $a = 0$: $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + ...$

$e^{i\theta} = [1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + ...] + i[\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + ...]$

$e^{i\theta} = \cos\theta + i\sin\theta$

5 Question 5

a)

Odd function is a function where $f(-x) = -f(x)$ for all x. This means the function has a symmetry about the origin.

Example: $f(x) = x^3$ is and odd function.

$$f(-2) = -8 = -f(2)$$

b)

Even function is a function where $f(-x) = f(x)$ for all x. This means the function has a symmetry about the y-axis.

Example: $g(x) = x^2$ is and even function.

$$g(-2) = 4 = g(2)$$

c)

$\sin\theta = \cos(\theta - \frac{\pi}{2})$

$\cos(\theta + 2\pi k) = \cos\theta$ when k is an integer

$cos(-\theta) = cos\theta$

$sin(-\theta) = -\sin\theta$

$sin(\pi k) = 0$ when k is an integer

$cos(2\pi k) = 1$ when k is an integer

$cos(2\pi(k + 1/2) = -1$ when k is an integer

d)

i-

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

Therefore:

$e^{j\theta} + e^{-j\theta} = 2\cos\theta$ and $\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$ and $\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$

$sin^2\theta + cos^2\theta = (\frac{e^{j\theta}+e^{-j\theta}}{2})^2 + (\frac{e^{j\theta}-e^{-j\theta}}{2j})^2 = 1$

iv,v-

$e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$e^{i\alpha}e^{i\beta} = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$

$e^{i(\alpha+\beta)} = \cos\alpha\cos\beta + \cos\alpha i\sin\beta + i\sin\alpha\cos\beta - \sin\alpha\sin\beta$

$e^{i(\alpha+\beta)} = (\cos\alpha\cos\beta - \sin\alpha\sin\beta) + i(\cos\alpha\sin\beta + \sin\alpha\cos\beta)$

Therefore:

$\cos(\alpha{\pm}\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha{\pm}\beta) = \cos\alpha\sin\beta + \sin\alpha\cos\beta$

ii-

$\cos 2\theta = \cos\theta + \theta$

$\cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta$ (Proof iv-v)

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

iii-

$\sin 2\theta = \sin\theta + \theta$

$\sin 2\theta = \sin\theta\cos\theta + \sin\theta\cos\theta$ (Proof iv-v)

$\sin 2\theta = 2\sin\theta\cos\theta$

6 Question 6

$\sum_{k=1}^{N} \cos(w_0 t + \phi_k)$

$= \sum_{k=1}^{N} Re(A_k e^{jp(w_0 t + \phi k)})$

$= Re(\sum_{k=1}^{N} A_k e^{j\phi k} e^{jw_0 t})$

$= Re(Ae^{j\phi} e^{jw_0 t})$

$= Re(Ae^{j(w_0 t + \phi)})$

$= A\cos(w_0 t + \phi)$

7 Question 7

a)

$z_1^{(t)} = \cos(wt - \frac{1}{3}\pi) = Re(e^{j(wt + \frac{1}{3}\pi)})$

$z_2^{(t)} = 3\sin(wt - \frac{5}{4}\pi) = 3\cos(wt - \frac{7}{4}\pi) = Re(3e^{j(wt + \frac{7}{4}\pi)})$

$z_3^{(t)} = Re(2e^{j(wt - \frac{3}{2}\pi))})$

$x(t) = Re(e^{j(wt + \frac{1}{3}\pi)} + 3e^{j(wt + \frac{7}{4}\pi)} + 2e^{j(wt - \frac{3}{2}\pi))})$

$x(t) = Re((e^{-j\frac{\pi}{3}} + 3e^{-j\frac{7}{4}\pi} + 2e^{-j\frac{3}{2}\pi})e^{jwt})$

$x(t) = \cos\frac{\pi}{3} - j\sin\frac{\pi}{3} + 3\cos\frac{7\pi}{4} - 3j\sin\frac{7\pi}{4} + 2\cos 3\pi 2 - 2j\sin 3\pi 2$

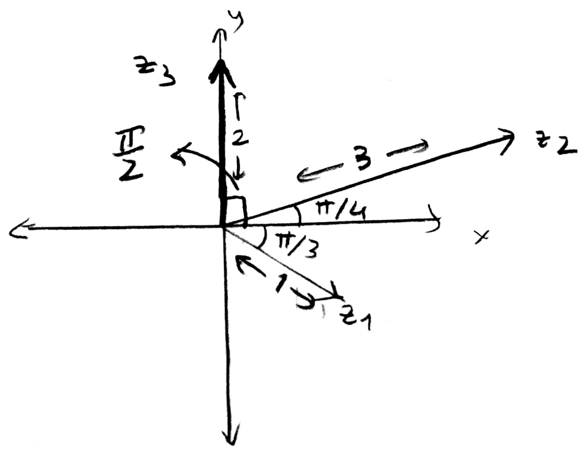
$x(t) = (\frac{1}{2} + \frac{3\sqrt{2}}{2}) + j(-\frac{\sqrt{3}}{2} + \frac{3\sqrt{2}}{2} + 2)$

$x(t) = 2.621 + j3.255 = 4.179e^{j\theta}$

$\theta = \arctan(\frac{3.255}{2.621}) = 51.16 = \frac{3\pi}{10}$

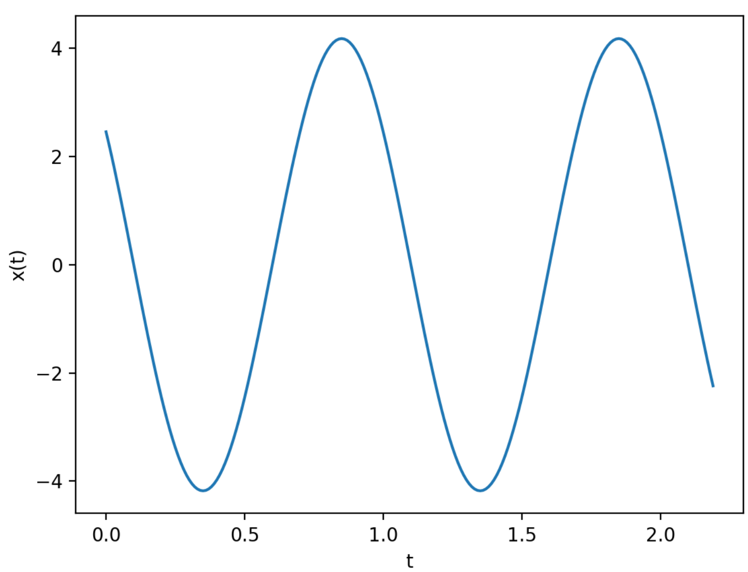
$x(t) = 4.179\cos(wt + \frac{3\pi}{10})$

b)



c)

For the plot, I chose $w = 2\pi$. My sampling space goes from 0 to 2.2 in steps which are 0.01



8 Question 8

a)

$$4\cos(500\pi t + \tfrac{5\pi}{4}) = 2e^{j(500\pi t + \frac{5\pi}{4})} + 2e^{-j(500\pi t + \frac{5\pi}{4})}$$

$4\cos(500\pi t + \frac{5\pi}{4}) = 2e^{j\frac{5\pi}{4}}e^{j500\pi} + 2e^{-j\frac{5\pi}{4}}e^{-j500\pi}$

Phasors: $2e^{j\frac{5\pi}{4}}$, $2e^{-j\frac{5\pi}{4}}$

Frequencies: 250Hz, -250Hz

$-3\sin(60\pi t) = -3\cos(60\pi t + \frac{3\pi}{2})$

$-3\cos(60\pi t + \frac{3\pi}{2}) = -\frac{3}{2}e^{j(60\pi t - \frac{\pi}{2})} - \frac{3}{2}e^{-j(60\pi t + \frac{\pi}{2})}$

$-3\cos(60\pi t + \frac{3\pi}{2}) = (\frac{3}{2}e^{j(60\pi t - \frac{\pi}{2})} + \frac{3}{2}e^{-j(60\pi t + \frac{\pi}{2})})e^{j\pi}$

$-3\cos(60\pi t + \frac{3\pi}{2}) = \frac{3}{2}e^{j(60\pi t + \frac{\pi}{2})} + \frac{3}{2}e^{-j(60\pi t + \frac{\pi}{2})}$

$-3\cos(60\pi t + \frac{3\pi}{2}) = \frac{3}{2}e^{j\frac{\pi}{2}}e^{j60\pi t} + \frac{3}{2}e^{-j\frac{\pi}{2}}e^{-j60\pi t}$

Phasors: $\frac{3}{2}e^{j\frac{\pi}{2}}$, $\frac{3}{2}e^{-j\frac{\pi}{2}}$

Frequencies: 30Hz, -30Hz

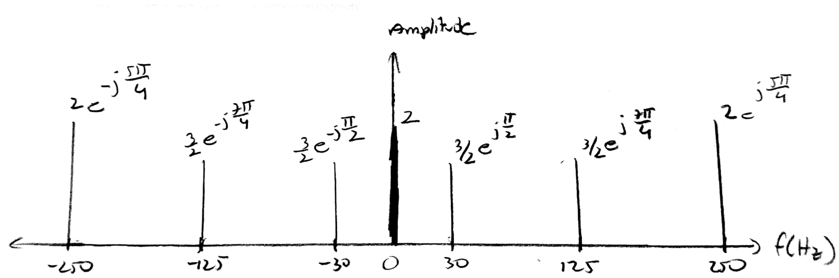
$-3\cos(250\pi(t - 10^{-3})) = -3\cos(250\pi t - \frac{\pi}{4}) = -3\cos(250\pi t + \frac{7\pi}{4})$

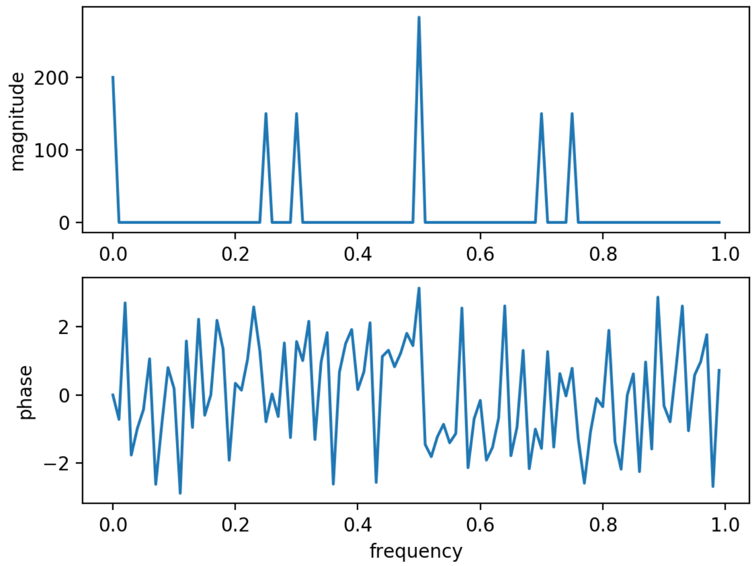
$-3\cos(250\pi t + \frac{7\pi}{4}) = \frac{3}{2}e^{j(250\pi t + \frac{7\pi}{4})} + \frac{3}{2}e^{-j(250\pi t + \frac{7\pi}{4})}$

$-3\cos(250\pi t + \frac{7\pi}{4}) = \frac{3}{2}e^{j\frac{7\pi}{4}}e^{j250\pi t} + \frac{3}{2}e^{-j\frac{7\pi}{4}}e^{-j250\pi t}$

Phasors: $\frac{3}{2}e^{j\frac{7\pi}{4}}$, $\frac{3}{2}e^{-j\frac{7\pi}{4}}$

Frequencies: 125Hz, -125Hz





b)

$x(t)$ is periodic. The period is equal to $\frac{1}{5}s$

c)

Fundamental frequency = 5Hz (Greatest common divisor of 250, 125, 30)

$\frac{250}{5} = 50$, $\frac{125}{5} = 25$, $\frac{30}{5} = 6$

50th, 25th and 6th harmonics exist in this signal.

9 Question 9

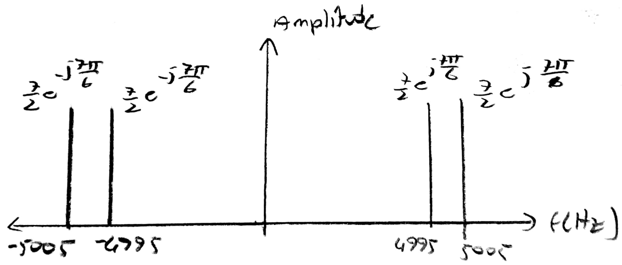
a)

$x(t) = 2\cos(10\pi t)7\cos(10000\pi t + \frac{7\pi}{6}))$

$x(t) = 14(\frac{1}{2}e^{j10\pi t} + \frac{1}{2}e^{-j10\pi t})(\frac{1}{2}e^{j10000\pi t + \frac{7\pi}{6}} + \frac{1}{2}e^{-j10000\pi t + \frac{7\pi}{6}})$

$x(t) = \frac{7}{2}e^{j(10010\pi t + \frac{7\pi}{6})} + \frac{7}{2}e^{-j(9990\pi t + \frac{7\pi}{6})} + \frac{7}{2}e^{j(9990\pi t + \frac{7\pi}{6})} + \frac{7}{2}e^{-j(10010\pi t + \frac{7\pi}{6})}$

b)



c)

$x(t) = 7\cos(10010\pi t + \frac{7\pi}{6}) + 7\cos(9990\pi t + \frac{7\pi}{6})$

10 Question 10

If two functions are orthagonal, integration of the multiplication of these functions over a given interval is zero.

- $\sin(2\pi nft)$ and $\sin(2\pi mft)$

  $\int_{-L}^{L} \sin(2\pi nft)\sin(2\pi mft)dt$

  $= \int_{-L}^{L} \frac{1}{2}(\cos(2\pi nft - 2\pi mft) - \cos(2\pi nft + 2\pi mft))dt$

  $= \frac{1}{2}(\frac{\sin(2\pi ft(n-m))}{2\pi f(n-m)} - \frac{\sin(2\pi ft(n+m))}{2\pi f(n+m)})\Big|_{-L}^{L} = 0$ (These functions are orthagonal)

- $\cos(2\pi nft)$ and $\cos(2\pi mft)$

  $\int_{-L}^{L} \cos(2\pi nft)\cos(2\pi mft)dt$

  $= \int_{-L}^{L} \frac{1}{2}(\cos(2\pi nft - 2\pi mft) + \cos(2\pi nft + 2\pi mft))dt$

  $= \frac{1}{2}(\frac{\sin(2\pi ft(n-m))}{2\pi f(n-m)} + \frac{\sin(2\pi ft(n+m))}{2\pi f(n+m)})\Big|_{-L}^{L} = 0$ (These functions are orthagonal)

- $\sin(2\pi nft)$ and $\cos(2\pi mft)$

  $\int_{-L}^{L} \sin(2\pi nft)\cos(2\pi mft)dt$

  $= \int_{-L}^{L} \frac{1}{2}(\cos(2\pi nft + 2\pi mft) + \sin(2\pi nft - 2\pi mft))dt$

  $= -\frac{1}{2}(\frac{\cos(2\pi ft(n+m))}{2\pi f(n+m)} + \frac{\cos(2\pi ft(n-m))}{2\pi f(n-m)})\Big|_{-L}^{L} = 0$ (These functions are orthagonal)

11 Question 11

Gibbs phenomenon id s case where Fourier estimation overshoots when the estimated function which has a jump discontinuity jumps. This overshoot happens because of trying to approximate a discontinuous function by a finite series of sine and cosine waves.