BLG 354E Signals and Systems for CE (Spring 2018)
Homework-3

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May 12, 2018

1 Question 1

a) This system is not causal because it depends on a future value. It is stable because if x(t) is bounded, then y(t) will also be bounded.

b) This system is causal because it depends on past and present values. The result of the integration is $x(t) - x(\inf)$. Since t is unbounded, y(t) is also unbounded. Therefore this system is not stable.

c) Since $u(t-5) = 0$ when t is bigger than 5, $h(t) = e^{-(t-5)}$ for $t > 5$. This system is causal because it only depends on past values. Since $e^{-(t-5)}$ is between 0 and 1 when $t > 5$, this system is bounded and stable.

d) Since the system only depends on the present value, it is causal. When $t < 0$, $h(t) = 0$. When $t \geq 0$, $u(t) = 1$ and e^{-3t} is bounded between 0 and 1. Therefore, this system is bounded and stable.

2 Question 2

$h(t) * x(t) = H(jw).X(jw)$

$H(jw) = \int_{-\inf}^{\inf} h(t)e^{-jwt}dt$

$= \int_{-\inf}^{\inf} 5e^{-0.5(t-3)}[u(t-3) - u(t-11)]e^{-jwt}dt$

$= \int_{3}^{11} 5e^{-(0.5+jw)t+1.5}dt$

$= \int_{3}^{11} 5e^{-(0.5+jw)t+1.5}dt$

$= \frac{5e^{1.5}(e^{-11(0.5+jw)} - e^{-3(0.5+jw)})}{-0.5-jw}$

$X(jw) = \int_{-\inf}^{\inf} u(t-2)e^{-jwt}dt$

$= \int_{2}^{\inf} e^{-jwt}dt$

$= \frac{e^{-2jw}}{jw}$

$H(jw).X(jw) = \frac{5e^{1.5-2jw}(e^{-11(0.5+jw)} - e^{-3(0.5+jw)})}{(-0.5-jw)jw}$

3 Question 3

The implementation FFT is in q3.py. I analyzed and took the implementation from https://jakevdp.github.io/blog/2013/08/28/understanding-the-fft/. This implementation is a recursive approach to implementing the FFT. Normally, matrix multiplication is used to implement the regular discrete fourier transform formula. However, fast fourier transform algorithm divides this problem into subproblems using the equations below:

$$X_k = \sum_{n=0}^{N-1} x_n.e^{-i2\pi kn/N}$$

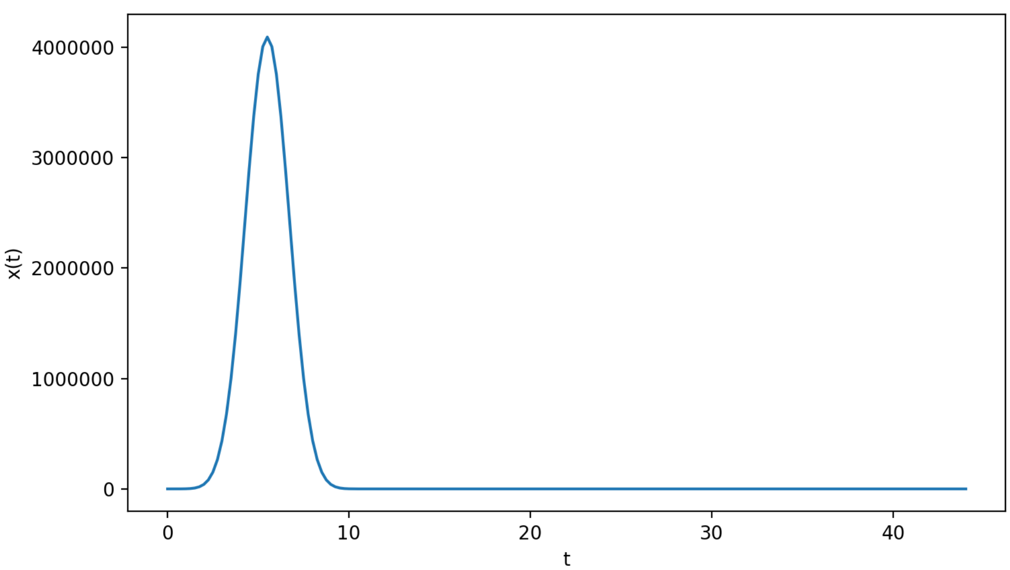
$$= \sum_{m=0}^{N/2-1} x_{2m}.e^{-i2\pi k(2m)/N} + \sum_{m=0}^{N/2-1} x_{2m+1}.e^{-i2\pi k(2m+1)/N}$$

$$= \sum_{m=0}^{N/2-1} x_{2m}.e^{-i2\pi km/(N/2)} + e^{i2\pi k/N} \sum_{m=0}^{N/2-1} x_{2m+1}.e^{-i2\pi km)/(N/2)}$$

With this divide and conquer approach, the complexity is reduced to O(NlogN) and FFT algorithm is achieved. The implementation im q3.py is still slow compared to the built-in function. The reason for this is that the built-in FFT function is highly optimized and uses vectorization instead of recursion.

4 Question 4

For the signal, I used the list [1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0]. This list goes from t=0 to t=4 (both are included) with steps of 0.25. When this signal is convolved with itself 10 times, we get the output signal below. The code used to obtain this signal is in q4.py



As the signal is convolved with each other, the values in the signal get higher and the shape is smoothened.

5 Question 5

a) Converting to frequency domain:

$\delta(t-2) \rightarrow e^{-jw2}$

$-0.2e^{-0.2(t-2)}[u(t-2)] \rightarrow \frac{-0.2.e^{-jw2}}{0.2+jw}$

$H(jw) = e^{-jw2} + \frac{-0.2.e^{-jw2}}{0.2+jw} = \frac{jw.e^{-jw2}}{0.2+jw}$

b) Magnitude squared response:

$|H(jw)|^2 = H(jw).H^*(jw)$

$= \frac{jw.e^{-jw2}}{0.2+jw}.\frac{-jw.e^{-jw2}}{0.2-jw} = \frac{w^2e^{-4jw}}{0.04+w^2}$

c) $x_1(t) = 5$

$Y_1(jw) = H(j0)5 = 0.5 = 0$

$x_2(t) = 10\cos(0.2t)$

$Y_2(jw) = -10(\pi\delta(w-0.2) + \pi\delta(w=0.2))H(0.2j)$

$Y_2(jw) = -10(\pi\delta(w-0.2) + \pi\delta(w=0.2))\frac{0.2je^{-0.4j}}{0.2+0.2j}$

$x_3(t) = u(t)$

$Y_3(jw) = (\pi\delta(w) + \frac{1}{jw})H(0) = 0$

$Y(jw) = Y_1(jw) + Y_2(jw) + Y_3(jw) = 0 + Y_2(jw) + 0$

$Y(jw) = Y_2(jw) \rightarrow y(t) = F^{-1}Y_2(jw)$

6 Question 6

a) $\frac{1}{2\pi}\int_{-\inf}^{\inf}\frac{\sin(10w)^2}{2w^2}e^{jwt}dw$

b) $\frac{1}{25+w^2} = \frac{A}{5-jw} + \frac{B}{5+jw}$

$\frac{1}{25+w^2} = \frac{A(5+jw)+B(5-jw)}{25+w^2}$

$5A + Ajw + 5B - Bjw = 1 \rightarrow A = B = \frac{1}{10}$

$\frac{1}{25+w^2} = \frac{1}{10}(\frac{1}{5-jw} + \frac{1}{5+jw})$

$\frac{1}{25+w^2} = \frac{1}{10}(e^{5t}u(-t) + e^{-5t}u(t))$

c) $m(t) = e^{-a(t-2)}u(t-2)$

$p(t) = cos(w_0 t)$

$m(t)p(t) = \frac{1}{2\pi}M(jw) * P(jw)$

$m(t)p(t) = \frac{1}{2\pi}M(jw) * (\pi\delta(w-w_0) + \pi\delta(w+w_0))$

$m(t)p(t) = \frac{1}{2}M(jw) * \delta(w-w_0) + \frac{1}{2}M(jw) * \delta(w+w_0)$

$m(t)p(t) = \frac{1}{2}M(jw) * \delta(w-w_0) + \frac{1}{2}M(jw) * \delta(w+w_0)$

$m(t)p(t) = \frac{1}{2}M(j(w-w_0)) + \frac{1}{2}M(j(w+w_0))$

$m(t) = e^{-a(t-2)}u(t-2) \rightarrow M(jw) = \frac{1}{a+jw}e^{-j2w}$

$m(t)p(t) = \frac{1}{2}\frac{e^{-j2(w-w_0)}}{a+j(w-w_0)} + \frac{1}{2}\frac{e^{-j2(w+w_0)}}{a+j(w+w_0)}$

7 Question 7

The midterm can be found in the zip archive.