BLG 454E Learning From Data (Spring 2018) Homework I

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1 Question 1

The question translates to:

$P(Sat = 1) = 1/4$, $P(Sat = 0) = 3/4$

$P(Sun = 1|Sat = 1) = 1/2$, $P(Sun = 1|Sat = 0) = 1/4$

$P(Sat = 1|Sun = 1) =?$

Solution:

$P(Sat = 1|Sun = 1) = \frac{P(Sun=1|Sat=1).P(Sat=1)}{P(Sun=1)}$

$P(Sat = 1|Sun = 1) = \frac{P(Sun=1|Sat=1).P(Sat=1)}{P(Sun=1|Sat=0).P(Sat=0)+P(Sun=1|Sat=1).P(Sat=1)}$

$P(Sat = 1|Sun = 1) = \frac{(1/2)(1/4)}{(1/4)(3/4)+(1/2)(1/4)} = 2/5(Result)$

2 Question 2

I grouped the moves with same probabilities

- Starting from A:

  $\frac{1}{7}$ (Starting from A counts as finishing in 0 moves)

- B to A and F to A:

  $2 * \frac{1}{7} * \frac{1}{3} = \frac{2}{21}$

- B to G to A and F to G to A:

  $2 * \frac{1}{7} * \frac{1}{3} * \frac{1}{6} = \frac{2}{126}$

- G to A:

  $\frac{1}{7} * \frac{1}{6} = \frac{1}{42}$

- G to B to A and G to F to A:

  $2 * \frac{1}{7} * \frac{1}{6} * \frac{1}{3} = \frac{2}{126}$

- C to B to A and E to F to A:

  $2 * \frac{1}{7} * \frac{1}{3} * \frac{1}{3} = \frac{2}{63}$

- C to G to A, E to G to A and D to G to A:

  $3 * \frac{1}{7} * \frac{1}{3} * \frac{1}{6} = \frac{3}{126}$

Probability of reaching point A in 2 moves or less:

$\frac{1}{7} + \frac{2}{21} + \frac{2}{126} + \frac{1}{42} + \frac{2}{126} + \frac{2}{63} + \frac{3}{126} = \frac{44}{126}(Result)$

3 Question 3

Likelihood function for the normal distribution:

$L(x_1..x_n|\mu, \sigma^2) = (2\pi\sigma^2)^{-n/2}e^{(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2)}$

When we take the logarithm of this expression, we get:

$l(x_1..x_n|\mu, \sigma^2) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$

We need to choose the μ and σ^2 parameters that maximize this function. To do this, we set the partial derivatives of this function in respect to μ and σ^2 to zero.

$\frac{\partial}{\partial\mu}l(x_1..x_n|\mu, \sigma^2) = 0$

$\frac{\partial}{\partial\mu}(-\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2) = 0$

$\frac{1}{\sigma^2}\sum_{j=1}^{n}(x_i - \mu) = 0$ (The sum part of the equation has to be zero)

$\sum_{j=1}^{n} x_i - n\mu = 0$ Therefore,

$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$ (Estimator for μ)

$\frac{\partial}{\partial\sigma^2}l(x_1..x_n|\mu, \sigma^2) = 0$

$\frac{\partial}{\partial \sigma^2}(-\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2) = 0$

$-\frac{n}{2\sigma^2} + \frac{1}{\sigma^4}\frac{1}{2}\sum_{i=1}^{n}(x_i - \mu)^2 = 0$

$\frac{1}{2\sigma^2}(-n + \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2) = 0$

$n = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$

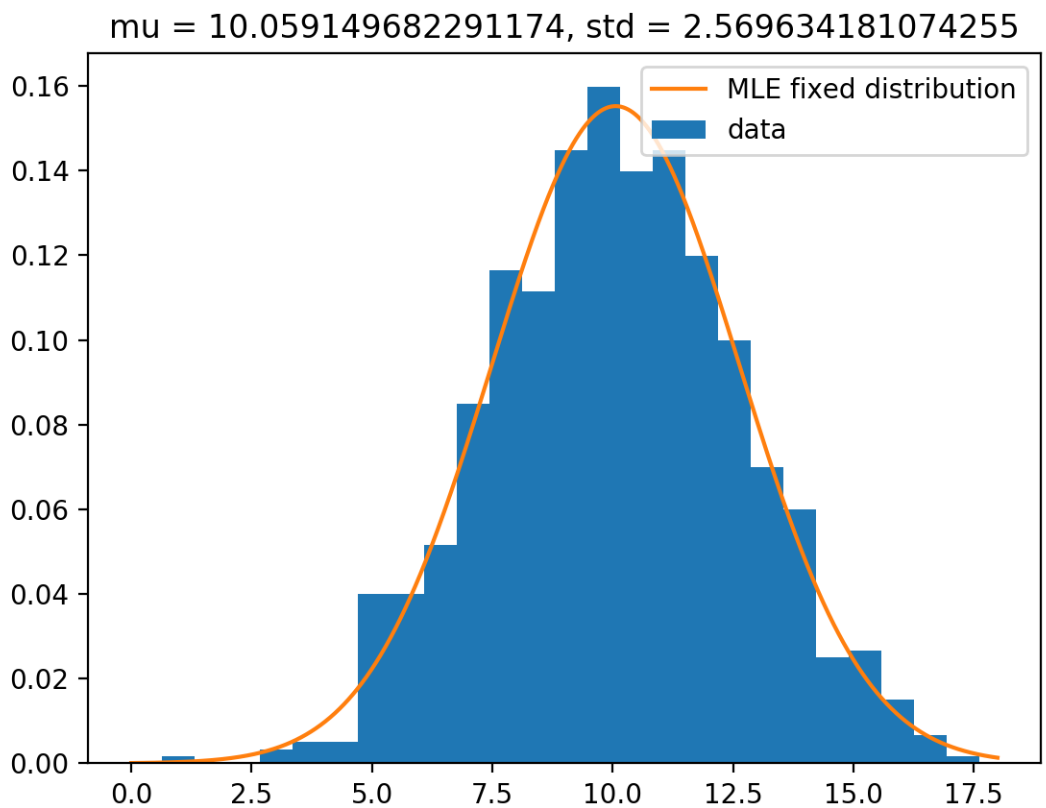
$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$

Estimates for the given data:

$\mu(\text{mean}) = 10.059149682291174$

$\sigma(\text{std}) = 2.569634181074255$

For the estimation, I used python. I used the matplotlib module to create the plot and numpy module to use only the arange() function. This is the result:



4 Question 4

a)

$P(y=-)=P(y=+)=\frac{1}{2}$

$P(x_1=0|y=-)=\frac{3}{5}$, $P(x_1=0|y=+)=\frac{2}{5}$

$P(x_1=1|y=-)=\frac{2}{5}$, $P(x_1=1|y=+)=\frac{3}{5}$

$P(x_2=0|y=-)=\frac{3}{5}$, $P(x_2=0|y=+)=\frac{3}{5}$

$P(x_2=1|y=-)=\frac{2}{5}$, $P(x_2=1|y=+)=\frac{2}{5}$

$P(x_3=0|y=-)=\frac{4}{5}$, $P(x_3=0|y=+)=\frac{1}{5}$

$P(x_3=1|y=-)=\frac{1}{5}$, $P(x_3=1|y=+)=\frac{4}{5}$

$P(y|x)=\frac{P(x|y)P(y)}{P(x)}$

b)

$P(y=-|x)=\frac{P(x_1=1|y=-)P(x_2=1|y=-)P(x_3=1|y=-)P(y=-)}{P(x_1=1|y=-)P(x_2=1|y=-)P(x_3=1|y=-)P(y=-)+P(x_1=1|y=+)P(x_2=1|y=+)P(x_3=1|y=+)P(y=+)}$

$P(y=-|x)=\frac{\frac{2}{5}\frac{2}{5}\frac{1}{5}\frac{5}{10}}{\frac{2}{5}\frac{2}{5}\frac{1}{5}\frac{5}{10}+\frac{3}{5}\frac{2}{5}\frac{4}{5}\frac{5}{10}}=\frac{1}{7}$

$P(y=+|x)=\frac{P(x_1=1|y=+)P(x_2=1|y=+)P(x_3=1|y=+)P(y=+)}{P(x_1=1|y=-)P(x_2=1|y=-)P(x_3=1|y=-)P(y=-)+P(x_1=1|y=+)P(x_2=1|y=+)P(x_3=1|y=+)P(y=+)}$

$P(y=+|x)=\frac{\frac{3}{5}\frac{2}{5}\frac{4}{5}\frac{5}{10}}{\frac{2}{5}\frac{2}{5}\frac{1}{5}\frac{5}{10}+\frac{3}{5}\frac{2}{5}\frac{4}{5}\frac{5}{10}}=\frac{6}{7}$

Since $P(y=+|x)>P(y=-|x)$, we predict $y=+$ for $(x_1=1,x_2=1,x_3=1)$

c)

$P(x_1=1)=\frac{1}{2}, P(x_2=1)=\frac{4}{10}$

$P(x_1=1,x_2=1)=P(x_1=1|x_2=1)P(x_2=1)$

$P(x_1=1,x_2=1)=\frac{2}{4}*\frac{4}{10}=\frac{1}{5}=P(x_1=1)P(x_2=1)$

Therefore, x_1 and x_2 are independent.