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Problem 1: Each part is 3 points.

1.1. For which of two signals with given Fourier Series (FS) coefficients, the FS approximation of the signal converges faster to the original signal as we add more FS coefficients? (Explain your reasoning briefly):

i) $a_k = \begin{cases} \frac{1}{j\pi k}, k = \mp1, \mp3, \ldots \\ 0, k = \mp2, \mp4, \ldots \\ \frac{1}{2}, k = 0 \end{cases}$ ii) $a_k = \begin{cases} \frac{1}{j\pi^2 k^2}, k = \mp1, \mp3, \ldots \\ 0, k = \mp2, \mp4, \ldots \\ \frac{1}{2}, k = 0 \end{cases}$

The signal ii) needs less coefficients to converge to 0 because of $\frac{1}{j\pi^2 k^2}$. Therefore it converges faster to the original signal

1.2. Rate of oscillation of discrete time signal increases as ω increases up to ……..

1.3. The reason of Gibbs phenomenon is .trying to come up with a continuous signal using discrete coefficients.

1.4. What are the frequency values f_1 and f_2 of the following beat signal: $\sin(2\pi 210t) + \sin(2\pi 200t) = 2\cos(2\pi f_1 t)\sin(2\pi f_2 t)$?

$\cos(2\pi 210t + \frac{\pi}{2}) + \cos(2\pi 200t + \frac{\pi}{2})$

1.5. Calculate Fourier Transform of $\delta(t)$ using Fourier Transform formula: $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$.

$X(jw) = \int_{-\infty}^{\infty} \delta(t)\, e^{-jwt}$ $\delta(t)\begin{cases}1, & t=0\\ 0, & t\neq 0\end{cases} \Rightarrow X(jw) = \delta(0)\, e^{-j.0.t} = \underline{1}$

1.6. $y(t) = \frac{dx}{dt} + x(t)$, determine system properties: Linear or nonlinear; Time invariant or time varying; Causal or non-causal) The system is causal because it only depends on the current time

$a.x_1(t) \xrightarrow{system} a x(t) + a\frac{dx}{dt} = a.y_1(t)$
$b: x_2(t) \xrightarrow{system} b x_2(t) + b\frac{dx_2}{dt} = b.y_2(t)$ $\rightarrow a y_1(t) + b y_2(t)$

$x_1(t) \xrightarrow{system} y_1(t) \rightarrow \overset{a}{x} \rightarrow a y_1(t)$
$x_2(t) \xrightarrow{system} y_2(t) \rightarrow \overset{b}{x} \rightarrow b y_2(t)$ $\rightarrow a y_1(t) + b y_2(t)$ = Linear

$x(t) \xrightarrow{system} y(t) = \frac{dx}{dt} + x(t) \xrightarrow{delay} \frac{dx}{d(t-t_0)} + x(t-t_0)$

$x(t) \xrightarrow{delay} q(t) = x(t-t_0)$

↓ system

$y(t) = \frac{dx}{dt} + q(t) = \frac{dx}{dt} + x(t-t_0)$ $\neq$ time varying

1.7. Express $u[n+2] + u[n+1] - u[n] - u[n-3]$ in terms of impulse sequences:

$\delta(n+2) + 2\delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$

1.8. Calculate Discrete Fourier Transform coefficients of $x[0] = 8, x[1] = 4, x[2] = 8, x[3] = 4$ (Recall $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$). $N=4$

$X[k] = 8 + 4 e^{-j\frac{\pi}{2}k} + 8e^{-j\pi k} + 4e^{-\frac{3\pi}{2}k}$

The coefficients can be found by setting k's value to the order of the desired coefficient

1.9. Add two sinusoids: $x_1(t) = 2\sin(5\pi t - \frac{\pi}{2})$ and $x_2(t) = 4\cos(5\pi t + \frac{\pi}{6})$ to obtain another sinusoid. Use phasor addition. $x_1(t) + x_2(t) = x_3(t)$

$2\sin(5\pi t - \frac{\pi}{2}) = 2\cos(5\pi t) = \frac{e^{j5\pi t} + e^{-j5\pi t}}{2}\cdot 2$

$4\cos(5\pi t + \frac{\pi}{6}) = 4\left(\frac{e^{j(5\pi t + \frac{\pi}{6})} + e^{-j(5\pi t + \frac{\pi}{6})}}{2}\right)$

$x_3(t) = 2e^{j(5\pi t + \frac{\pi}{6})} + 2e^{-j(5\pi t + \frac{\pi}{6})} + e^{j5\pi t} + e^{-j5\pi t}$
$= e^{j5\pi t}(2e^{\frac{\pi}{6}} + 1) + e^{-j5\pi t}(2e^{\frac{\pi}{6}} + 1) = (2e^{\frac{\pi}{6}} + 1)(e^{j5\pi t} + e^{-j5\pi t}) = (4e^{\frac{\pi}{6}} + 2)\cos(5\pi t)$

1.10. Fourier Transform of $x(t) = \sin(\omega_0 t)$ is equal to $\frac{j}{2}\delta(f - \frac{w_0}{2\pi}) + \frac{j}{2}\delta(f + \frac{w_0}{2\pi})$

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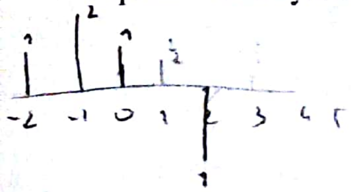
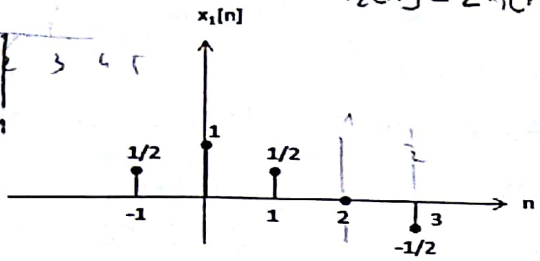
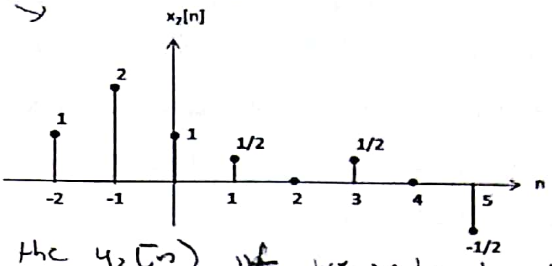
Problem 2: Problem parts (a) and (b) are not related.

2. (a) Consider a linear time-invariant system whose response to $x_1[n]$ is the signal $y_1[n]$. Sketch carefully the response of the system $y_2[n]$ to the input $x_2[n]$ in the given axis below. Justify your reasoning clearly.

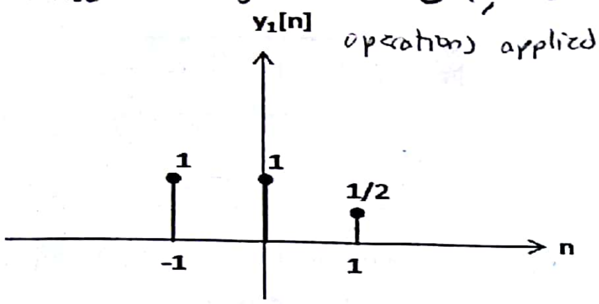


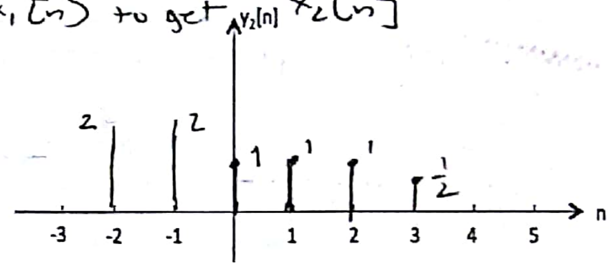
$x_2[n] = 2x_1[n+1] + n[n-2]$





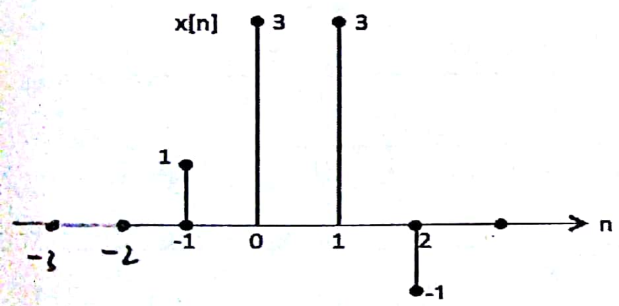
Since the system is LTI, we can find the $y_2[n]$ if we apply y_1 the same operations applied to $x_1[n]$ to get $x_2[n]$

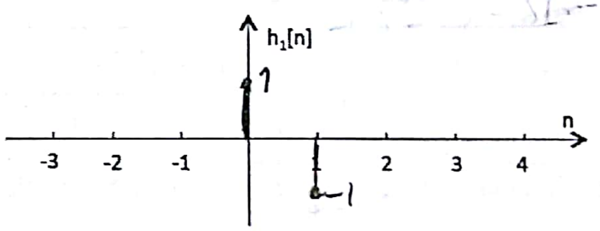


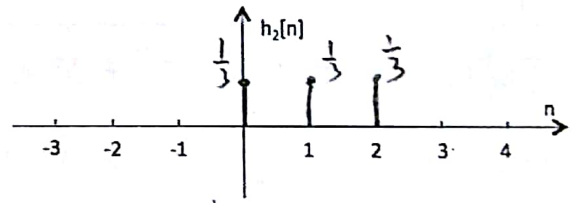


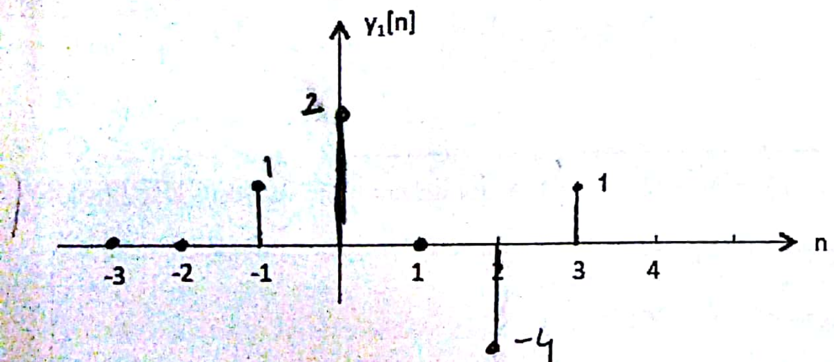
(b) A discrete time signal $x[n]$, whose plot is given below, is provided as an input to two systems that you will define:

S1: a 2-pt filter that detects abrupt changes in the signal; S2: a 3-pt filter that smooths the signal. Plot the impulse responses $h_1[n]$ and $h_2[n]$ corresponding to S1 and S2, respectively, in the given axes below. Also, plot the output of the system S1, i.e. $y_1[n]$, and output of S2, $y_2[n]$, in response to input x[n] in the given axes below. Justify your solution.





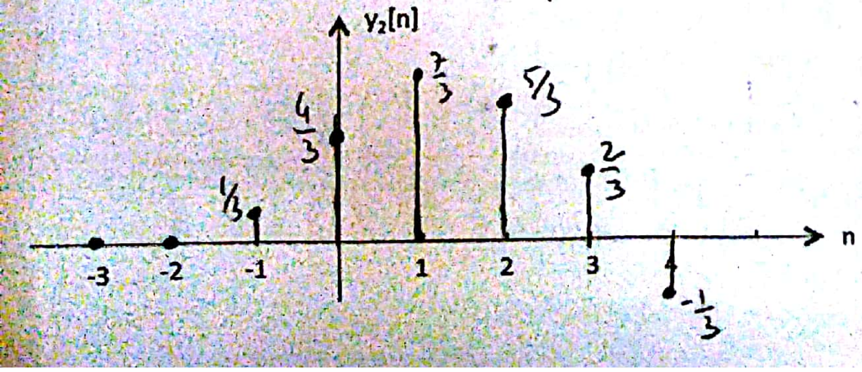




$y_1[n] = \sum_k h[k]\, x[n-k]$

$y_1[n] = h[0].x[n] + h[1].x[n-1]$

$= x[n] - x[n-1]$



$y_2[n] = \sum_k h[k]\, x[n-k]$

$= h[0].x[n] + h[1].x[n-1] + h[2].x[n-2]$

$= \frac{1}{3}(x[n] + x[n-1] + x[n-2])$

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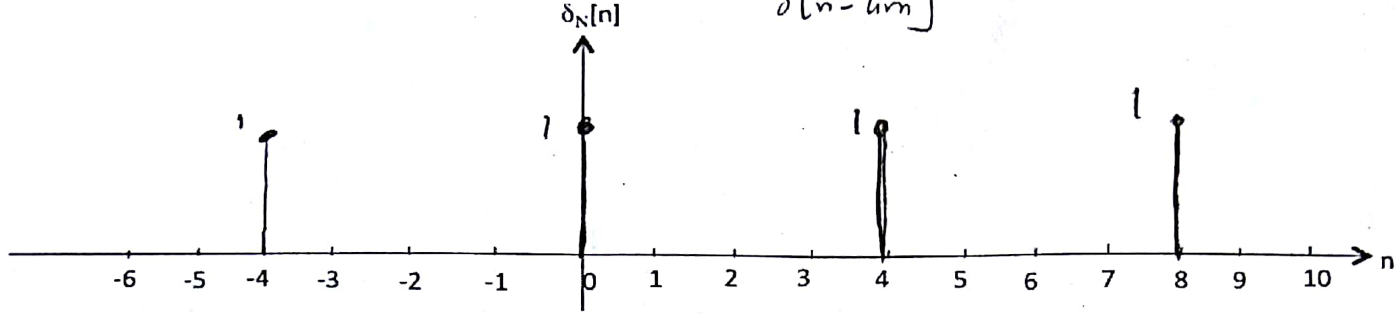
Problem 3. Consider the periodic impulse train: $\delta_N[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN]$.

3.1. Plot $\delta_N[n]$ for N=4.

$\delta[n-4m]$



3.2. Calculate the Discrete Time Fourier Series coefficients c_k of $\delta_N[n]$, for all k integers: $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$

$$c_0 = \frac{1}{4}\sum_{n=0}^{3} x[n] = \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 = \frac{1}{4}$$

$$c_1 = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-j\frac{\pi}{2}n} = \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 = \frac{1}{4}$$

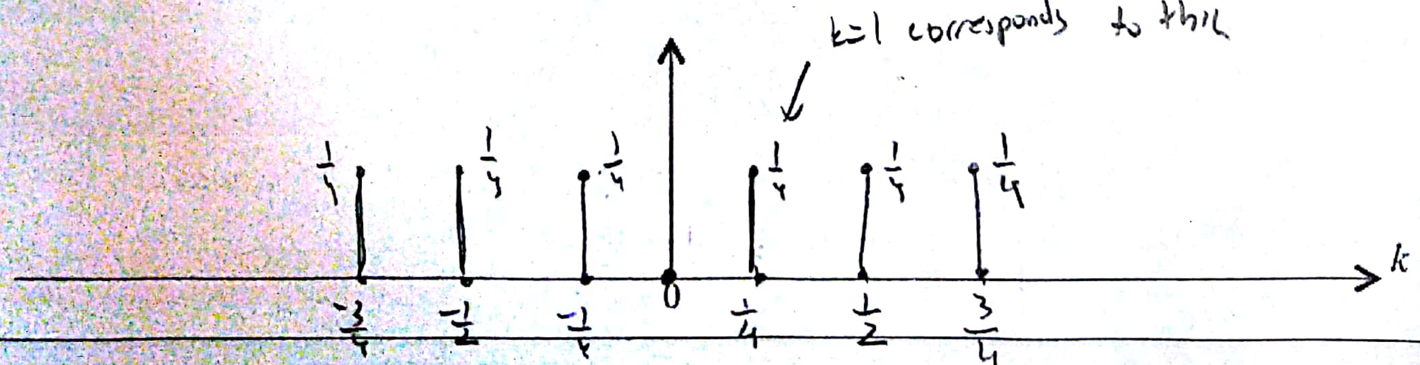
$$c_2 = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-j\pi n} = \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 = \frac{1}{4}$$

$$c_3 = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-j\frac{3\pi}{2}n} = \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 = \frac{1}{4}$$

3.3. Using the coefficients calculated in 3.2, write down the Fourier series representation of $\delta_N[n]$.

$$\delta_N[n] = \sum_{k=0}^{3} c_k\, e^{j\frac{\pi}{2}kn} = \frac{1}{4} + \frac{1}{4}\cdot e^{j\frac{\pi}{2}n} + \frac{1}{4} e^{j\pi n} + \frac{1}{4} e^{j\frac{3\pi}{2}n}$$

3.4. Plot the frequency spectrum of $\delta_N[n]$. State which frequency k=1 corresponds to?



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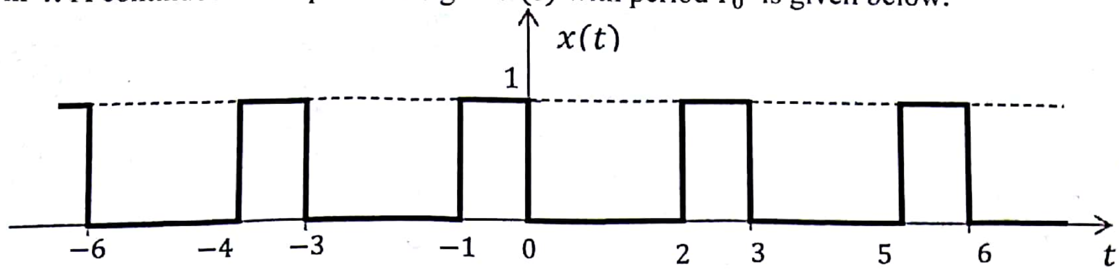
Problem 4: A continuous time periodic signal $x(t)$ with period T_0 is given below:



$T_0 = 3$

$f_0 = \frac{1}{3}$

a) Derive a general formula for the Fourier series coefficients: a_0 and a_k (k: integers, $k \neq 0$) for $x(t)$. $a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-j2\pi f_0 kt}dt$

$a_0 = \frac{1}{3}\int_0^3 x(t).e^0 dt$

$a_0 = \frac{1}{3}\int_2^3 dt$

$a_0 = \frac{1}{3} t\Big|_2^3 = \frac{1}{3}$

$a_k = \frac{1}{3}\int_2^3 e^{-j\frac{2\pi}{3}kt}dt$

$a_k = \frac{1}{3}\left(\frac{e^{-j\frac{2\pi}{3}kt}}{-j\frac{2\pi}{3}k}\right)\Bigg|_2^3$

$= \frac{1}{3}\left(\frac{e^{-j2\pi k} - e^{-j\frac{4\pi}{3}k}}{-j\frac{2\pi}{3}k}\right) = \frac{1}{-j2\pi k}\cdot\left(e^{-j2\pi k} - e^{-j\frac{4\pi}{3}k}\right)$

b) Compute the Fourier series coefficients a_k for $-3 \leq k \leq 3$ in <u>polar form</u> and plot the spectrum for those harmonics. (Use $\theta \triangleq \operatorname{atan}\left(\frac{\sqrt{3}}{3}\right)$ if needed.)

$a_k = -\frac{1}{2\pi k j}\left(e^{-j2\pi k} - e^{-j\frac{4\pi}{3}k}\right)$

$a_{-3} = \frac{1}{6\pi}\left[e^{j6\pi} - e^{j6\pi}\right] = 0$

$a_{-2} = \frac{1}{4\pi}\left[e^{j4\pi} - e^{j\frac{8\pi}{3}}\right] = \frac{1}{4\pi}e^{j\theta}$

$a_{-1} = \frac{1}{2\pi}\left[e^{j2\pi} - e^{j4\pi/3}\right] = \frac{1}{2\pi}e^{j\theta}$

$a_0 = \frac{1}{3}\int^3 x(t).e^0 dt = \frac{1}{3}\int_0^3 x(t)dt = \frac{1}{3}$

$a_1 = -\frac{1}{2\pi}\left[e^{-j2\pi} - e^{-j\frac{4\pi}{3}}\right] = -\frac{1}{2\pi}e^{-j\theta}$

$a_2 = -\frac{1}{4\pi}\left[e^{-j4\pi} - e^{-j\frac{8\pi}{3}}\right] = -\frac{1}{4\pi}e^{-j\theta}$

$a_3 = \frac{1}{6\pi}\left[e^{-j6\pi} - e^{-j6\pi}\right] = 0$

Spectrum of x(t)