Student Information

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Answer 1

Expected value for the blue die is equal to the sum of each possible probability multiplied by its corresponding value.

a)

For the blue die:

$$\begin{aligned}E(B) &= \sum bp(b) \\ &= \frac{1}{6}\cdot 1+\frac{1}{6}\cdot 2+\frac{1}{6}\cdot 3+\frac{1}{6}\cdot 4+\frac{1}{6}\cdot 5+\frac{1}{6}\cdot 6 \\ &= \frac{1}{6}\cdot 21 \\ &= 3.5\end{aligned}$$

For the yellow die:

$$\begin{aligned}E(Y) &= \sum yp(y) \\ &= \frac{1}{8}\cdot 1+\frac{1}{8}\cdot 1+\frac{1}{8}\cdot 1+\frac{1}{8}\cdot 3+\frac{1}{8}\cdot 3+\frac{1}{8}\cdot 3+\frac{1}{8}\cdot 4++\frac{1}{8}\cdot 8 \\ &= \frac{1}{6}\cdot 24 \\ &= 3\end{aligned}$$

For the red die:

$$\begin{aligned}E(R) &= \sum rp(r) \\ &= \frac{1}{10}\cdot 2+\frac{1}{10}\cdot 2+\frac{1}{10}\cdot 2+\frac{1}{10}\cdot 2+\frac{1}{10}\cdot 2+\frac{1}{10}\cdot 3+\frac{1}{10}\cdot 3+\frac{1}{10}\cdot 4+\frac{1}{10}\cdot 4+\frac{1}{10}\cdot 6 \\ &= \frac{1}{10}\cdot 30 \\ &= 3\end{aligned}$$

b)

The expected value for rolling three blue dice is equal to $E(B)+E(B)+E(B)$ ($E(B)$ is calculated above.) $= 3.5+3.5+3.5 = 10.5$ // On the other hand the total expected value for rolling three

different dice is
$E(B) + E(Y) + E(R) = 3 + 3 + 3.5 = 9.5$.
Since I want to maximize the total value, I would choose the one with the highest expectance.

c)

Then the expected value for rolling three blue dice would still be same (10.5, as calculated in part b). However, given the fact that the yellow die's value will be 8, the expected value for rolling three different dice would be
$8 + E(B) + E(R) = 8 + 3.5 + 3 = 14.5$. Since the expected total value for rolling three different dice is higher than rolling three blue dice, I would choose rolling three different dice.

d)

Let R be event of rolling a red die, B be event of rolling a blue die, Y be event of rolling a yellow die, T be the event of rolling a die with value 3. We want to find $P(R|T)$

$$P(R) = P(B) = P(Y) = \frac{1}{3} \quad \text{(Since each color has equal probability)}$$

Using the Law of Total Probability, we have:

$$\begin{aligned} P(T) &= P(T|R)P(R) + P(T|B)P(B) + P(T|Y)P(Y) \\ &= \frac{1}{3} \cdot \frac{2}{10} + \frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{3}{8} \\ &= \frac{89}{360} \end{aligned}$$

Using conditional probability,

$$\begin{aligned} P(R|T) &= \frac{P(T|R)P(R)}{P(T)} \\ &= \frac{\frac{1}{3} \cdot \frac{2}{10}}{\frac{89}{360}} \approx 0.2697 \end{aligned}$$

e)

Let $P((a, b))$ represent $P(a|B) \cdot P(b|Y)$ ($i.e.$ Probability of blue die showing value a and yellow die showing value b)
Since rolling a blue die and a yellow die is independent events, the probability of their intersection can be computed as stated above.
Given the dice, we have three possibilities:
$P((1, 4)) = \frac{1}{6} \cdot \frac{1}{8}$

$P((2,3)) = \frac{1}{6} \cdot \frac{3}{8}$

$P((4,1)) = \frac{1}{6} \cdot \frac{3}{8}$

($P((3,2))$ is not included since there are no side in yellow die with value 2)

To find the probability of union, we add the probabilities together. The final answer is $P((1,4)) + P((2,3)) + P((4,1)) = \frac{7}{48} \approx 0.1458$

Answer 2

a)

We have 80 companies (80 trials), at least 4 successes and 0.025 probability for each trials ($p = 0.025$ and $q = 1 - p = 0.975$). We can calculate this probability using binomial distribution. Putting the values in, we have

$$\begin{aligned}
P(X \geq 4) &= 1 - P(X < 3) \\
&= 1 - (C(80,3)p^3q^77 + C(80,2)p^2q^{78} + C(80,1)p^1q^{79} + C(80,0)p^0q^{80}) \\
&= 1 - 0.8594 = 0.1406
\end{aligned}$$

b)

We can find this probability by calculating the probability of no discount from Company A and no discount from Company B in two days. ($P(X = 0) \cdot P(Y = 0)$) and taking the complement of it. We can calculate both probabilities using binomial distribution.
For Company A, we have 160 trials in 2 days with $p = 0.025$ and $q = 0.975$
For Company B, we have 2 trials in 2 days with $p = 0.1$ and $q = 0.9$

$$\begin{aligned}
P(X = 0) &= C(160,0)p^0q^{160} \\
&= 1 \cdot 1 \cdot (0.975)^{160} \\
&= (0.975)^{160} \\
P(Y = 0) &= C(2,0)p^0q^2 \\
&= C(2,0)(0.1)^0(0.9)^2 \\
&= (0.9)^2
\end{aligned}$$

Finally, we take the complement of the product of these probabilities:

$$1 - (0.975^{160} \cdot 0.9^2) \approx 0.9859$$

Answer 3

```
n = 1000;
blue = [1 2 3 4 5 6];
yellow = [1 1 1 3 3 3 4 8];
red = [2 2 2 2 2 3 3 4 4 6];
count = 0;
first_options = zeros(1, n);
second_options = zeros(1, n);
for i = 1:n
   first_options(i) = blue(randi(length(blue))) + yellow(randi(length(yellow))) +
      red(randi(length(red)));
   second_options(i) = blue(randi(length(blue))) + blue(randi(length(blue))) +
      blue(randi(length(blue)));
end
for i= 1 : n
  if first_options(i) < second_options(i)
    count += 1;
  endif
end
avg_first = mean(first_options);
avg_second = mean(second_options);
percentage = (count/n)*100;
disp("Average total value for the first option (rolling a single die of each
    color):"), disp(avg_first), disp("\n");
disp("Average total value for the second option (rolling three blue dice):") ,
    disp(avg_second), disp("\n");
disp("Percentage of the cases where the total value of the second option is greater
    than the first option:") , disp(percentage);
```

```
>> stat_octave

Average total value for the first option (rolling a single die of each color):
9.6580


Average total value for the second option (rolling three blue dice):
10.486


Percentage of the cases where the total value of the second option is greater than the first option:
54.600
```

Figure 1: output of the above code

The average value of for the first option (9.658) is very close to the expected value for the first option, which is 9.5. This demonstrates the accuracy of the code.
The average value of for the second option (10.486) is very close to the expected value for the first

option, which is 10.5. This, again demonstrates the accuracy of the code.
Percentage of the cases where the total value of the second option is greater than the first option (54.6) shows the approximate probability of second option being greater than the first option.