Student Information

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Answer 1

a)

Since the time taken to process and send the response, T_A and T_B, are independent and uniformly distributed between [0, 100] milliseconds, we have

$$f_a(t_A) = \begin{cases} \dfrac{1}{100}, & 0 \leq t_A \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

$$f_b(t_B) = \begin{cases} \dfrac{1}{100}, & 0 \leq t_B \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

$$f(t_A, t_B) = \begin{cases} \dfrac{1}{10000}, & 0 \leq t_A \leq 100, \quad 0 \leq t_B \leq 100 \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} F_a(t_A) &= \int_{-\infty}^{t_A} f_a(x)dx \\ &= \int_{-\infty}^{0} f_a(x)dx + \int_{0}^{t_A} f_a(x)dx \\ &= 0 + \frac{t_A}{100} \\ F_b(t_B) &= \int_{-\infty}^{t_B} f_b(x)dx \\ &= \int_{-\infty}^{0} f_b(x)dx + \int_{0}^{t_B} f_b(x)dx \\ &= 0 + \frac{t_B}{100} \\ F(t_A, t_B) &= \frac{t_A t_B}{10000} \end{aligned}$$

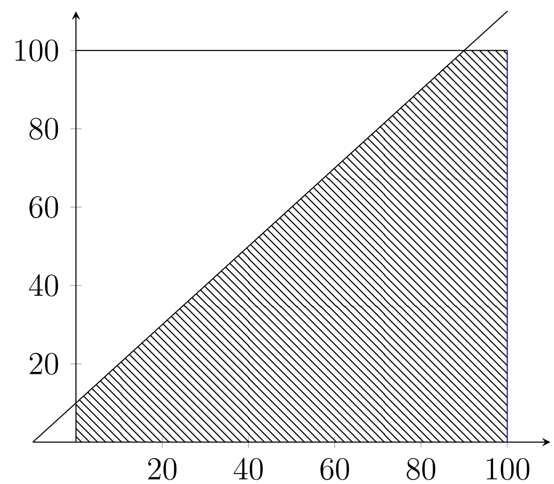
b)

We calculate the probabilities as follows:

$$
\begin{aligned}
P(t_A < 30)P(40 < t_B < 60) = F_a(30)\int_{40}^{60} f_b(x)\,dx \qquad &= F_a(30) \times (F_b(60) - F_b(40)) \\
&= \frac{30}{100} \times \frac{60-40}{100} \\
&= 0.3 \times 0.2 = 0.06
\end{aligned}
$$

c)

Considering the hint given in the question, we plot a coordinate system in which x axis corresponds to t_B and y axis corresponds to t_B.

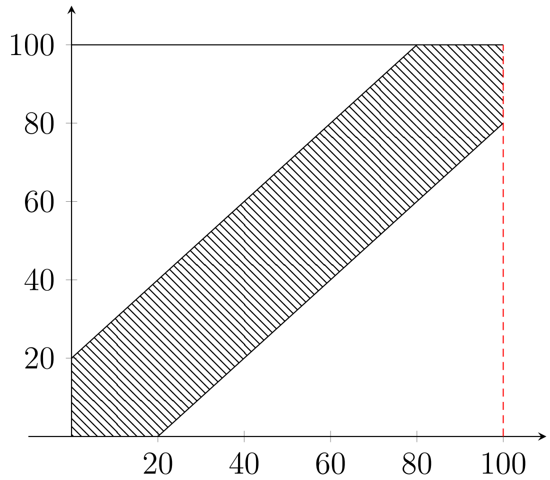


The desired probability is the part of the area under the line $t_A = t_B + 10$ inside the [0, 100] $\times$ [0, 100] square, divided by the area of the [0, 100] $\times$ [0, 100] square.

$$
\frac{\frac{(10+100)\times 90}{2} + 10 * 100}{100 \times 100} = 0.595
$$

d)

In order to find the probability of passing the task, we take the area between the lines $t_A = t_B + 20$ and $t_A = t_B - 20$ intersected with the [0, 100] $\times$ [0, 100] square divided by the area of the [0, 100] $\times$ [0, 100] square. The desired area is shown below.



In order to find the area of the painted field, we add the areas of the triangles above and below the field and subtract the result from the area of the 100x100 square.

$$\frac{100 \times 100 - (\frac{80 \times 80}{2} + \frac{80 \times 80}{2})}{100 \times 100} = 0.36$$

Answer 2

a)

We can consider it as 150 Bernouli trials with $n = 150, p = 0.6, q = 0.4$. Therefore we have mean $\mu = E(x) = p = 0.6$ and $\sigma^2 = pq = 0.24, \sigma = \sqrt{0.6 \cdot 0.4}$.
We need

$$\begin{aligned} P(S_{150} \geq 150 \cdot 0.65) &= 1 - P(S_{150} < 150 \cdot 0.65) \\ &= 1 - P(\frac{S_{150} - 150 \cdot \mu}{\sigma\sqrt{n}} < \frac{150 \cdot 0.65 - 150 \cdot 0.6}{\sqrt{150 \cdot 0.6 \cdot 0.4}}) \\ &= 1 - \Phi(1.25) \\ &= 1 - 0.8944 \\ &= 0.1056 \end{aligned}$$

b)

We can consider it as 150 Bernouli trials with $n = 150, p = 0.1, q = 0.9$. Therefore we have mean $\mu = E(x) = p = 0.1$ and $\sigma^2 = pq = 0.09, \sigma = \sqrt{0.09} = 0.3$.
We need

$$
\begin{aligned}
P(S_{150} \leq 150 \cdot 0.15) &= P(\frac{S_{150} - 150 \cdot \mu}{\sigma\sqrt{n}} \leq \frac{150 \cdot 0.15 - 150 \cdot 0.1}{\sqrt{150} \cdot 0.3}) \\
&= \Phi(2.04124) \\
&= 0.9793
\end{aligned}
$$

Answer 3

We need $P(170 < X < 180)$ with $\mu = 175$ and $\sigma = 7$. After we standardize, we have

$$
\begin{aligned}
P(170 < X < 180) &= P(\frac{170 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{180 - \mu}{\sigma}) \\
&= P(\frac{170 - 175}{7} < Z < \frac{180 - 175}{7}) \\
&= P(\frac{-5}{7} < Z < \frac{5}{7}) \\
&= \Phi(5/7) - \Phi(-5/7) \\
&= 0.7625 - 0.2375 \\
&= 0.525
\end{aligned}
$$

Answer 4

a)

```
pkg load statistics
mu = 175;
sigma = 7;
n = 1000;
ans_a = normrnd(mu, sigma, n,1)
hist(ans_a)
```

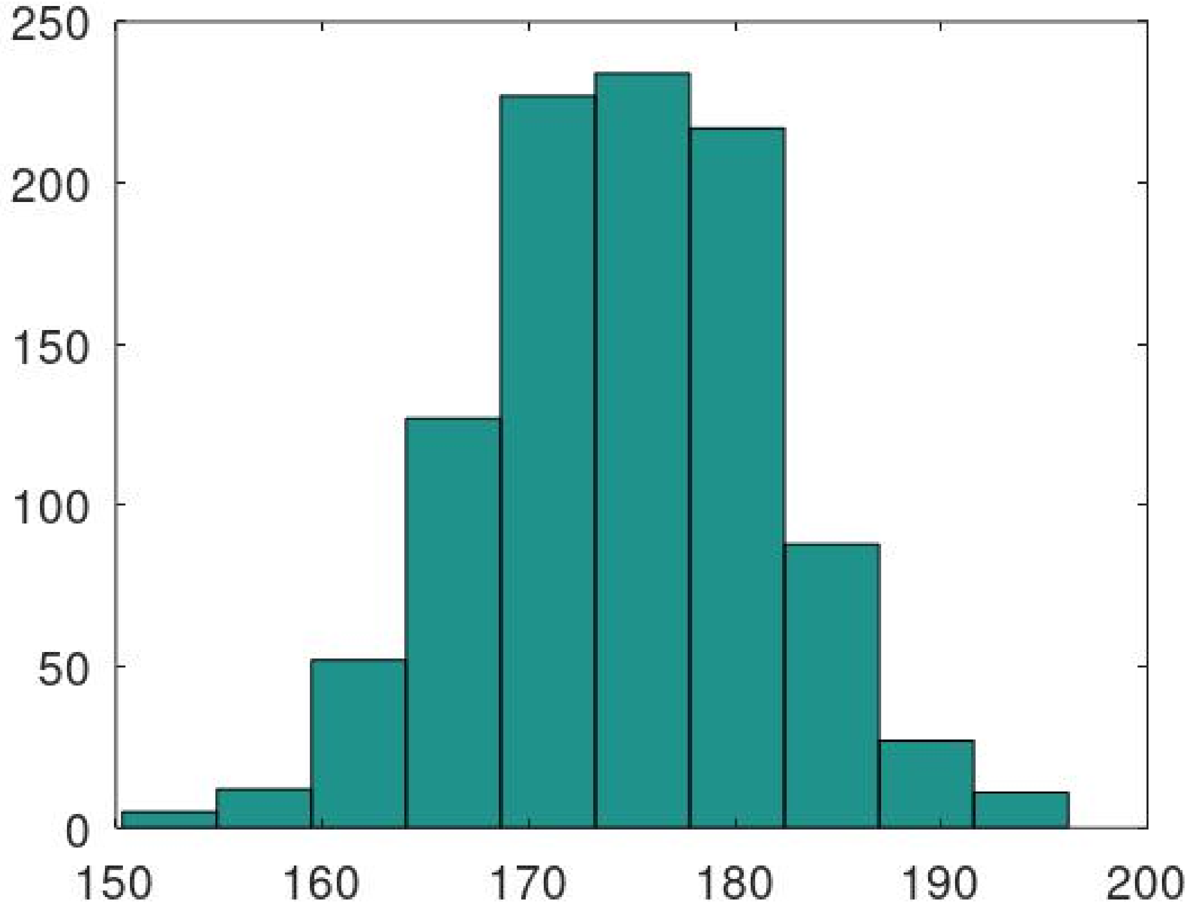


Figure 1: Output of the above code

The distribution of the data forms a shape of normal distribution, as expected.

b)

```
n = 1000;
mu = 175;
sigma1 = 6;
sigma2 = 7;
sigma3 = 8;
x1 = mu-4*sigma1:0.1:mu + 4*sigma1;
x2 = mu -4*sigma2:0.1: mu+4*sigma2;
x3 = mu -4*sigma3:0.1:mu +4*sigma3;
y1 = normpdf(x1, mu, sigma1);
y2 = normpdf(x2, mu, sigma2);
y3 = normpdf(x3, mu, sigma3);
```

```
plot(x1, y1, 'r', x2, y2, 'g', x3, y3, 'b')
legend('\sigma=6', '\sigma=7', '\sigma=8')
```

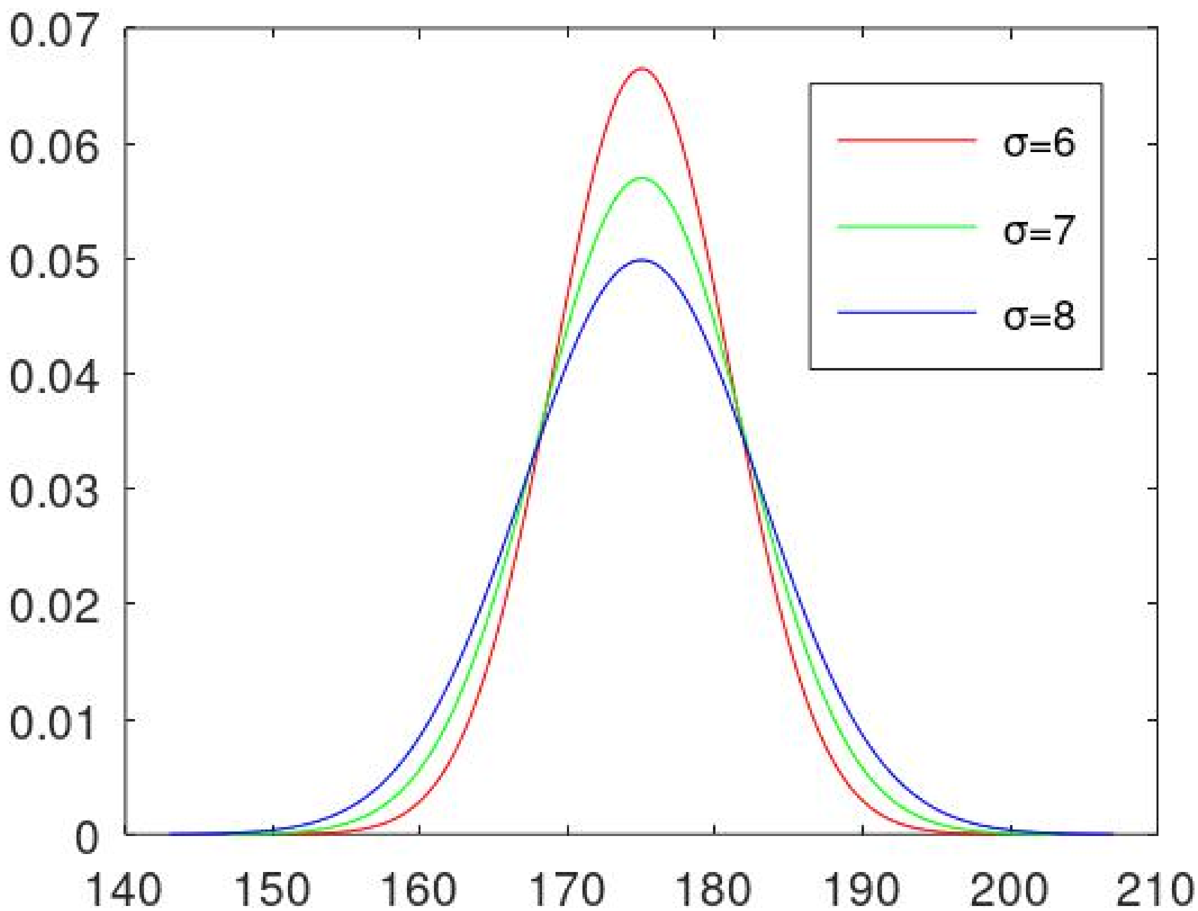


Figure 2: Output of the above code

As sigma gets higher, the curve gets a little flattened. This is expected because as the standard deviation increases, the data will be more widespread.

c)

```
pkg load statistics
n = 1000;
mu = 175;
sigma = 7;
count_45 = 0;
count_50 = 0;
count_55 = 0;
```

```
for i = 1:n
  dataset = normrnd(mu, sigma, 150, 1);
  count = sum(dataset > 170 & dataset < 180);

  if count >= 150 * 0.45
    count_45 += 1;
  endif
  if count >= 150 * 0.5
    count_50 += 1;
  endif
  if count >= 150 * 0.55
    count_55 += 1;
  endif
endfor

prob_45 = count_45 / n;
prob_50 = count_50 / n;
prob_55 = count_55 / n;

disp("Probability of having at least 45% of adults with heights between 170 cm and 180
    cm: "), disp(prob_45), disp("\n");
disp("Probability of having at least 50% of adults with heights between 170 cm and 180
    cm: "), disp(prob_50), disp("\n");
disp("Probability of having at least 55% of adults with heights between 170 cm and 180
    cm: "), disp(prob_55), disp("\n");
```

```
Probability of having at least 45% of adults with heights between 170 cm and 180 cm:
0.9640


Probability of having at least 50% of adults with heights between 170 cm and 180 cm:
0.7770


Probability of having at least 55% of adults with heights between 170 cm and 180 cm:
0.2890
```

Figure 3: Output of the above code

The probability that a randomly selected adult will have a height between 170 cm and 180 cm was calculated as 0.525 in Answer 3. Therefore, we expect around 52.5% of them to be between 170 cm and 180 cm. The simulation result are accurate when we consider the expectance.
The probability of having at least 45% of adults with heights between 170 cm and 180 cm is very high considering the threshold of percentage is way lower than expected percentage.
The probability of having at least 50% of adults with heights between 170 cm and 180 cm is 0.777.

This is accurate considering the threshold of percentage is still lower than expected percentage. The probability of having at least 55% of adults with heights between 170 cm and 180 cm is 0.289. This is also accurate considering the threshold of percentage is higher than expected percentage, we are less likely to have at least 55%.